REFRACTION DUE TO SHOCK WAVES

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**REFRACTION DUE TO SHOCK WAVES**

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A simple procedure for estimating the refractive error produced by the density profile behind a nuclear blast wave is developed for use in radar system analysis codes such as ROSCOE/NORSE.
## TABLE OF CONTENTS

<table>
<thead>
<tr>
<th>Section</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>1 INTRODUCTION</td>
<td>3</td>
</tr>
<tr>
<td>2 INDEX OF REFRACTION</td>
<td>5</td>
</tr>
<tr>
<td>3 SHOCK-INDUCED REFRACTIVE ERRORS</td>
<td>9</td>
</tr>
<tr>
<td>4 A MODULE TO PREDICT REFRACTION AND PRELIMINARY RESULTS</td>
<td>24</td>
</tr>
<tr>
<td>REFERENCES</td>
<td>29</td>
</tr>
</tbody>
</table>

### Appendix

<table>
<thead>
<tr>
<th>I</th>
<th>ABSORPTION</th>
<th>31</th>
</tr>
</thead>
<tbody>
<tr>
<td>REFERENCES</td>
<td></td>
<td>42</td>
</tr>
<tr>
<td>II</td>
<td>SUBROUTINE LISTINGS</td>
<td>43</td>
</tr>
</tbody>
</table>
SECTION I
INTRODUCTION

The modification of the index of refraction by a shock wave may produce refraction of electromagnetic signals, thus shock waves have the potential of introducing errors into radar system measurements of target locations. At high altitudes, i.e., in the ionosphere, refraction is determined by the amount of ionization present, but at low altitudes the increased electron-neutral collisional frequency causes the accompanying absorption to dominate any refraction so produced. Therefore any significant refraction produced at low altitude will be due to the changes in the atmospheric density and temperature. The atmospheric index of refraction as a function of these parameters is reviewed in Section 2.

In this approach to shock wave induced refraction it will be assumed that the given parameters include the actual target, burst and emitting source (radar) locations and that the resulting angular refractive error is the desired output. This is consistent with the current ROSCOE/NORSE code structure. It is also a more difficult problem to solve than predicting the arrival angle error of a specified ray, since the arrival angle of the ray that originates at the target is not a priori known. The errors expected are fortunately relatively small (< 1°) thus a method of iteration beginning with the unrefracted ray, will be used. Section 3 presents these procedures for four possible cases, defined by the locations of the source and of the target relative to the shock front. The computer program and some preliminary results are described in Section 4. Program listings are included as an appendix.
Limits on when refraction due to spherically expanding shock waves need be considered have been generated. The early time limit to the importance of refractive effects is set by the cessation of absorptive effects. Exact values are of course sensitive to geometry, however the relatively strong dependence of ionization on temperature limits the temperature range of interest to between about 800° K for a long path to 1200° K to 1400° K for relatively short paths. This is further discussed in Appendix I where the absorption limit is discussed in detail. An absolute upper limit to the temperature of 1500° is suggested. The location of this temperature contour varies with time. At the shock front, the corresponding shock strength is about 25 psi overpressure, which occurs at a scaled range of $56(W_{KT})^{1/3}$ meters. The fireball however is significantly hotter than the shock and will move outward to almost twice this range.

The lower limit to when refraction need be predicted is a function of both the accuracy required and the location of the burst point relative to the sight path. For example, if an instantaneous refractive error of 1 milliradian is significant and if the shock wave becomes tangent to the sight path at about its mid point, then a density increase of 0.4 percent is sufficient and this corresponds to a shock overpressure of 0.03 psi, i.e., a relatively weak shock.
SECTION 2
INDEX OF REFRACTION

The index of refraction at any point is a function of both the amount of ionization present and of the density and temperature of the various neutral constituents. In analyzing refractive errors for propagation paths at low altitude, it is only necessary to consider the changes to the neutral constituents. This is because the absorption of electromagnetic energy is also proportional to the amount of ionization present and to the electron-neutral collisional frequency, which is proportional to pressure. Therefore at the lower altitudes high levels of absorption will occur on typical propagation paths that have even moderate amounts of refraction. This can be seen by comparing the angular deviation of the ray path, \( \psi \) in degrees, to the one-way absorption, \( A \) in dB. Reference 1 gives the approximate relation* for the case of spherically stratified ionization

\[
\frac{\psi}{A} = \frac{10^7}{\nu r}, \quad */\text{db}
\]  

(2-1)

where \( \nu \) is the electron-neutral collision frequency (sec\(^{-1}\)) in the region and \( r \) is a characteristic dimension in (km). Using typical values, \( \nu = 10^{11} \text{ sec}^{-1} \) and \( r = \frac{1}{10} \text{ km} \), we obtain \( \psi = 10^{-3} \) degree per db or 16 micro-radians per db, i.e., the absorption along a path that yields 1 milliradian deflection due to an ionization gradient will also yield 60 db of

* See for example Page 6-6 of Reference 1.
absorption. In the Appendix it is shown that when the temperature exceeds about 1000° K - 1500° K, then the quasi-equilibrium ionization resulting from delayed fission-product radiations will produce such high levels of absorption. Therefore when predicting refraction caused by a shock wave it is only necessary to consider the impact of density and temperature changes on the index of refraction, and only for temperatures below 1500°K.

A best fit to the data on the atmospheric refractive index at radio frequencies was determined at NBS and reported in Reference 2 to be (their Equation 7)

\[ N = (n-1)10^6 = 77.6 \frac{P}{T} - 6 \frac{e}{T} + 3.75 \times 10^5 \frac{e}{T^2} \]  

(2-2)

where \( P \) is the total pressure, in millibars, \( T \) is the absolute temperature, in °K, and \( e \) is the partial pressure of water vapor, in millibars. A form that contains the wavelength dependence can be used to show that this dependence is negligible at radio frequencies. This form is as given by Allen (Reference 3) at STP as

\[ (n-1) \times 10^6 = 64.328 \frac{29498.1}{146 - \left(\frac{1}{\lambda}\right)^2} + \frac{255.4}{41 - \left(\frac{1}{\lambda}\right)^2} \], at 15°C (2-3)

where \( \lambda \) is the vacuum wavelength in microns. Since we are not interested in wavelengths less than one mm or 10^3 microns, the corrections are always negligible and this reduces to \((n-1) \times 10^6 = 272.6\), consistent with Equation 2-2 above.

It is convenient to simplify Equation 2-2. We introduce the relative partial pressure of water vapor, \( e' = e/p \). The gas law allows
\( p/T \) to be replaced by the density, \( \rho \), in \( \text{gm/cm}^3 \), which then can be factored out. Following Reference 3, we can also combine the second term with the third term with only small error for the temperature range of interest – especially since the ambient value of \( e' \) is highly variable.

\[
N = (n-1)10^6 = 2.2 \times 10^5 \rho \left( 1 + 4.8 \times 10^3 \frac{e'}{T} \right)
\]  
(2-4a)

or

\[
\eta = 1 + 0.22 \rho \left( 1 + 4.8 \times 10^3 \frac{e'}{T} \right).
\]  
(2-4b)

Later it will be convenient to consider the ratio of the indices of refraction across a shock front. Using the subscripts \( o \) and \( s \) to signify the ambient and shocked conditions, this ratio is

\[
\frac{n_s}{n_o} = \frac{1 + 0.22 \rho_s \left( 1 + 4.8 \times 10^3 \frac{e'}{T_s} \right)}{1 + 0.22 \rho_o \left( 1 + 4.8 \times 10^3 \frac{e'}{T_o} \right)}
\]  
(2-5)

which can be closely approximated by

\[
\frac{n_s}{n_o} = \frac{1 + 0.22 \rho_s \left( 1 + 4.8 \times 10^3 \frac{e'}{T_s} \right)}{1 - 0.22 \rho_o \left( 1 + 4.8 \times 10^3 \frac{e'}{T_o} \right)}
\]  
(2-6)

In this form we see that an accurate knowledge of the water vapor content is usually not important. In much of the range of interest the compression ratio is about 40% greater than the shock temperature ratio.
Using an ambient temperature of 288° K the second term in the bracket becomes 6.7 \text{ e'}. Typical values of e' are less than 0.03, yielding a value of 0.2 or less, to be compared with the shock overdensity ratio, \((\rho_s/\rho_0 - 1)\) \equiv (\mu - 1), which is generally much greater.

For discussion purposes, it is convenient to use an ambient density of 10^{-3} g/cm^3 (and a value corresponding to an altitude of about 6000 ft above sea level) and a relatively high value of water vapor partial pressure of 3% (corresponding to saturated air at about 25° C). Equation 2.6 then becomes

\[
\frac{\rho_s}{\rho_0} = 1 + 2.2 \times 10^{-4} [(\mu - 1) + \frac{1}{2} (\mu \frac{T_0}{T_s} - 1)] . \tag{2-7}
\]

It is shown in the Appendix that if the shock temperature exceeds about 4 times ambient temperature then absorption is the dominant process. The corresponding compression ratio, \(\mu = \frac{\rho_s}{\rho_0}\), is 4.7. Inserting these values into Equation 2-7 yields an upper limiting value of

\[
\frac{\rho_s}{\rho_0} = 1 + 2.2 \times 10^{-4} [3.7 + \frac{1}{2} \left(\frac{4.7}{4} - 1\right)] = 1 + 8.3 \times 10^{-4} \tag{2-8}
\]
SECTION 3
SHOCK-INDUCED REFRACTIVE ERRORS

The geometry of the shock wave related refraction problem is shown in Figures 1 and 2. The ambient (external) atmosphere is assumed to be uniform and the shock wave is assumed to be spherical. Therefore the source, target and burst points define the plane of these figures. There is no refraction out of this plane. Figure 1 shows the larger view of this plane. A source (e.g., radar) is located at the bottom of the figure, tracking a target at the top which is moving to the left. A burst occurs at a range $R_B$ from the source. The angle at the source between the sight lines to the burst and to the real location of the target is $\theta_1$. A shock wave is expanding from the burst, at a radius $S_R$. The density increase within the shock wave produces an increase in the local index of refraction. Refractive effects will occur only after the target passes behind the shock front, i.e., not until after it passes point $T_0$. When viewed through the shocked region the apparent target location $T'$ will be to the right (in this figure) of the real location, $T$. As the shock wave intercepts the line of sight there will be a portion of the trajectory that is not visible. Of course, in the general case the current shock location could be beyond either the target or source location or beyond both.

Figure 2 illustrates a possible propagation path within the shocked region. The insert at the top of the figure shows a possible radial profile of density. This profile, including the current shock radius is assumed to be given. For example it may be obtained from the Nuclear Blast Standard (1 KT) (Reference 4), the LAMB code (which includes
Figure 1. Geometric definitions outside the shock.
Figure 2. Geometric definitions for refraction within a spherically stratified region.
In Figure 2 the reference axis has been chosen along the radial between the burst point and the entry point of the ray path. Note that the location of this entry point, and thus also the angle $\psi_0$ are to be determined, and when known represents the solution of the problem.

The location of the target, labeled $T$, may be anywhere along the path, including inside the shock. The apparent location, $T'$, will be along the dashed extension of the incident ray, at different range. The propagation path starts from $S$ at the angle ($\theta_1 + \delta \theta_1$) and enters the shock at point $A$ where it makes the angle $\psi_0$ with a radial from the burst point. The actual ray to the target is refracted towards this radial through an angle $\delta \psi$. When both the radar and target are outside the shocked region the ray path exits the shock at point $C$ where it is refracted away from the radial through $C$ by the same angle $\delta \psi$, and arrives at the target location at an angle $\delta \theta_2$ off of the line $S-T$. In this case of spherical symmetry it is possible to replace the actual curved path by a straight line connecting the entry and exit points. This is equivalent to approximating the radially varying value of the index of refraction by some 'effective' value, $\bar{n}$, yet to be defined.

**REFRACTION IN A SPHERICALLY STRATIFIED REGION**

Referring to Figure 2, the polar coordinates of an arbitrary point along the ray path interior to the shock as measured from the burst point are $(r, \phi)$, and as measured in polar coordinates from the entry point are $(\rho, \psi)$. The angle between the vector $r$ and the local direction of the ray path is $\zeta$. Note from the geometry of these definitions that

\[ \rho \sin \psi = -r \sin \phi \]

and

\[ \rho \cos \psi = \frac{S_R}{r} \cos \phi \]
thus

\[ \psi = \tan^{-1} \left( \frac{r \sin \phi}{S_R - r \cos \phi} \right). \]  

(3-1)

The change in ray direction upon shock entry (and when appropriate upon exit) is obtained from Snell's law. Note that at entry point \( \psi = \zeta_S \). Thus

\[ n_0 \sin \psi_0 = n_S \sin \psi_1 = n_S \sin \zeta_S. \]  

(3-2)

The solution for the ray path within a spherically stratified region has been given by Archer (References 5 and 6). The form of Snell's law for a spherically stratified medium is

\[ r \ n(r) \sin \zeta = K \]  

(3-3)

The radial variation of \( n \) is obtained by substituting the given radial variations of density, \( \rho(r) \), and of temperature, \( T(r) \), into equation (2-4b) of the previous section. The constant \( K \) may be defined by values either upon entry or at the point of closest approach where, \( \zeta = \pi/2 \) and \( r = R_m \). Combining equations 3-2 and 3-3 provides a relation between \( R_m \) and the (as yet unknown) angle of incidence outside the shock, yields

\[ K = r \ n(r) \sin \zeta \]  

(3-4a)

\[ = R_m \ n(R_m) \]  

(3-4b)

\[ = S_R \ n_S \sin \zeta_S = S_R \ n_S \sin \psi_1 \]  

(3-4c)

\[ = S_R \ n_0 \sin \psi_0 \]  

(3-4d)

13
The differential equation of the ray path within the shocked region is

\[ d\phi = \pm \frac{K \, dr}{r \sqrt{r^2 - n^2(r) - K^2}}. \] (3-5)

The integral of this equation between two points \((r_1, \phi_1)\) and \((r_2, \phi_2)\) provides the angle \(\phi = \phi_2 - \phi_1\) between these points, i.e.,

\[ \phi = \int_{r_1}^{r_2} \frac{K \, dr}{r \sqrt{r^2 - n^2(r) - K^2}} \] (3-6)

where we have used the symbol \(\phi\) to indicate the integral is to be taken along a specific path and not just between the radial points \(r_1\) and \(r_2\). Specifically, for paths which pass through the point of closest approach this integral runs from \(r_1\) to \(R_m\) to \(r_2\). In the following, the point \(r_1\) is either the known location of the source when the source is within the shock or the entry point of the ray path when the source is outside. Note that while the radius to that entry point is known (i.e., \(S_R\)) its angular position is not known. Similarly \(r_2\) refers to either the target location or the exit point of the ray path.

Unfortunately the constant \(K\) in equation 6 contains \(R_m\) which depends on the ray path, or equivalently the angle \(\psi_0\) which depends on the entry point. Therefore it is necessary to solve the set of equations iteratively. Fortunately, we are interested only when the refractive error is small, thus the undeviated path provides a first estimate of \(R_m\) or \(\psi_0\). From Figure 1 this can be seen to be

\[ R_m = R_B \sin \theta_1 \] (3-7)
Using this value of \( r \), a density and temperature are obtained from the given shock wave profile. These are then used in equation 2-4b to obtain the local index of refraction. This together with the radius then provides the initial estimate of \( K \).

**BEARING ERROR PREDICTIONS**

There are four possible geometric situations that may occur depending on the relative location of the shock to the source and to the target. Each requires a slightly different method of solution, although the principle remains the same. Using the assumption that the refractive error is small, \( K \) as obtained above is used to determine a first estimate of the angular extent of the ray path interior to the shock, \( \phi \). In three of these cases, geometric considerations then provide an estimate of the shock front entry or exit angle, which provides a second estimate of \( K \) and leads to a situation that may be iterated. In the fourth case, which we shall discuss first, the iteration is on the location of \( R_m \) since the shock front is never reached.

**Source and Target Both Within the Shocked Zone**

In this case the angular extent, \( \phi \), must be equal to the angle at the burst point between the radii to the source and the target, i.e.,

\[
\phi = \pi - (\theta_1 + \theta_2)
\]  

(3-8)

The end points of the integral are also determined by geometry, thus the only available parameter is \( K \), or equivalently the location of the minimum radius to the propagation path. Given that parameter the initial direction of the ray is obtained from equation 3-4a. The refractive error is then the difference between this angle and \( \theta_1 \), i.e.,
\[ \delta \theta_1 = \zeta(R_B) - \theta_1 \] \hspace{1cm} (3-9a)

\[ = \sin^{-1} \left[ \frac{K}{R_B \eta(R_B)} \right] - \theta_1 \] \hspace{1cm} (3-9b)

An estimate of the correction to \( K \) necessary to make the calculated value of \( \phi \) agree with the geometric value may be obtained by differentiating equation 3-6 w.r.t \( K \).

\[
\frac{d\phi}{dk} = \frac{d}{dk} \left[ \frac{r^2}{r_1} \frac{K dr}{\sqrt{r^2n^2(r) - K^2}} + F(r_2,K) \frac{dr_2}{dk} - F(r_1,K) \frac{dr_1}{dk} \right] 
\]

\[
= \frac{r^2}{r_1} \left[ 1 + \frac{K^2}{r^2n^2(r) - K^2} \right] \frac{dr}{r_1 \sqrt{r^2n^2(r) - K^2}} + 0 - 0 
\]

\[
= \frac{r^2}{r_1} \frac{r n^2(r) dr}{(r^2n^2(r) - K^2)^{3/2}} \] \hspace{1cm} (3-10)

**Source Outside and Target Inside**

This situation is illustrated in Figure 3. The initial estimate of \( \phi \) is based on \( K \) as obtained from \( R_m \), equation 3-4b. This then defines a revised entry point and thus a new value for \( \psi_0 \) from geometry, i.e., two sides \( (R_B, S_R) \) and the included angle \( (\beta_3 - \phi) \) of the triangle S-B-O are known. The revised value of \( K \), obtained from equation 3-4d, is then used to recalculate \( \phi \). The process is iterated for a consistent set \((\phi, \psi_0)\). The refractive error, by geometry is then

\[ \delta \theta_1 = \psi_0 - \theta_1 - (\beta_3 - \phi) = \psi_0 + \theta_2 - \pi + \phi \] \hspace{1cm} (3-1)
Figure 3. Source outside and target inside the shocked region.

Note that the curved nature of the path interior to the shock does not enter directly.

Source Inside, Target Outside

This is essentially the inverse of the previous case with one major difference, since the refractive error occurs on the curved portion of the path which extends to the source. The method of solution is essentially the same, except that once the consistent set \((\phi, \psi_0)\) is obtained the corresponding value of \(K\) is used in equation 3-4a to obtain the instantaneous ray path angle \(\zeta\) at the radius \(R_B\). The refractive error is then

\[
\delta \theta_1 = \zeta(R_B) - \theta_1
\]

which is the same as equation 3-9a.
Target and Source Both Outside

When the propagation path completely traverses the shocked region as depicted in Figures 1 and 2, then the radial from B to D will be perpendicular to this path and the angle \( \psi_0 \) (or B-A-C) is the complimentary of the angle \( \phi_m \) (or A-B-D), where \( \phi_m \) is defined as one-half the total angular extent of the ray path inside the shock, i.e.,

\[
\phi_m = \frac{\int_{S}^{R_m} \frac{K \ dr}{r \sqrt{r^2 n^2(r) - K^2}}}{3-12}
\]

The effective value of the index of refraction which corresponds to a straight line path inside the shock, is

\[
\eta_e = \frac{n_0 \ sin \ \psi_0}{\ sin \ \psi_e} = n_0 \ sin \ \psi_0 / \ cos \ \phi_m
\]

(3-13)

Consider the triangles formed by the unrefracted extension of the external ray paths. From the small triangle the angle between lines S-A-D and D-C-T is \( 2\delta\psi \). From the large triangle (S,T,D) that angle is \( \delta\theta_1 + \delta\theta_2 \). Thus

\[
\delta\theta_1 + \delta\theta_2 = 2(\psi_0 - \psi_e) = 2\delta\psi
\]

(3-14)

The angle between the radial BD and the radial that is perpendicular to ST can be seen to be \( (\delta\psi - \delta\theta_1) \) which is also \( \frac{1}{2} (\delta\theta_2 - \delta\theta_1) \).

Applying the law of sines to the triangles S-E-D and D-E-T yields

\[
(R_2-d) \ sin \ \delta\theta_2 = \ (R_2-d) \ sin \ (90-\delta\psi) = (R_2-d) \ sin \ \delta\theta_2
\]

(3-15)
where

\[ R_1 = R_B \cos \theta_1 \]
\[ R_2 = R_T \cos \theta_2 \]
\[ d = R_B \sin \theta_1 \cdot \tan\left(\frac{\delta \theta_2 - \delta \theta_1}{2}\right) \]

\( \delta \theta_1 \) and \( \delta \theta_2 \) are small angles, thus the sines in equation 3-15 may be approximated by the angles, yielding

\[ (R_1 + d) \delta \theta_1 = (R_2 - d) \delta \theta_2 . \]

Using this in Equation 3-14 to eliminate \( \delta \theta_2 \) yields

\[ (1 + \frac{R_1 + d}{R_2 - d}) \delta \theta_1 = 2 \delta \psi . \]

\( d \) can also be neglected, being much smaller than either \( R_1 \) or \( R_2 \), yielding

\[ (1 + \frac{R_1}{R_2}) \delta \theta_1 = 2 \delta \psi . \tag{3-16} \]

A relation between the viewing angle, \( \theta_1 + \delta \theta_1 \) and the angle of incidence, \( \psi \), of the ray SA at the shock front is obtained by applying the law of sines to triangle S-B-A.

\[ \frac{\sin (\theta_1 + \delta \theta_1)}{R} = \frac{\sin (180 - \psi)}{R_B} = \frac{\sin \psi_0}{R_B} \tag{3-17} \]
We now apply Snell's law in the form

\[ n_0 \sin \psi_0 = n_e \sin (\psi_0 - \delta \psi) \]  

(3-18)

where \( n_e \) is the "effective" value behind the shock. Expanding the left side of Equations 3-18 and rearranging yields

\[ \tan \psi_0 = \frac{\sin \delta \psi}{\cos \delta \psi - n_0/n_e} \]  

(3-19)

To eliminate \( \psi_0 \) we rewrite \( \sin \psi_0 \) in Equation 3-17 in terms of the tangent \( \psi_0 \) and substitute from Equation 3-19, thus

\[ \frac{R_B}{S_R} \sin (\theta_1 + \delta \theta_1) = \sin \psi_0 = \tan \psi_0 \left( \frac{\tan \psi_0}{\tan^2 \psi_0 + 1} \right) = \frac{\sin \delta \psi}{\sqrt{\sin^2 \delta \psi + \cos^2 \delta \psi - 2 (n_0/n_e) \cos \delta \psi + (n_0/n_e)^2}} \]

\[ = \frac{\sin \delta \psi}{\sqrt{(1 - \frac{n_0}{n_e})^2 + 2 \frac{n_0}{n_e} (1 - \cos \delta \psi)}} \]  

(3-20)

Equation 3-16 is used to eliminate \( \delta \psi \) yielding an equation containing \( \delta \theta_1 \) and known quantities,

\[ \frac{R_B}{S_R} \sin (\theta_1 + \delta \theta_1) = \frac{\sin (\frac{R_1 + R_2}{2R_2} \delta \theta_1)}{\sqrt{(1 - \frac{n_0}{n_e})^2 + 2 \frac{n_0}{n_e} [1 - \cos (\frac{R_1 + R_2}{2R_2} \delta \theta_1)]}} \]  

(3-21)
We expect $\delta \theta_1$ to be a small angle and do not expect the factor $\frac{R_1+R_2}{2R_2}$ to prevent a small angle approximation, then

$$\frac{R_B}{S_R} \left( \sin \theta_1 + \delta \theta_1 \cos \theta_1 \right) = \frac{\left( \frac{R_1+R_2}{2R_2} \right) \delta \theta_1}{\sqrt{\left( 1 - \frac{n_0}{n_e} \right)^2 + \frac{n_0}{n_e} \left[ \frac{(R_1+R_2)}{2R_2} \delta \theta_1 \right]^2}} \quad (3-22)$$

From Equation 2-8 we expect the first term in the square root to be of the order of, but less than $(8.3 \times 10^{-4})^2$. This generally will be smaller than the second term whenever refraction exceeds a few milliradians, therefore as a first estimate we neglect the first term, yielding

$$\frac{R_B}{S_R} \left( \sin \theta_1 + \delta \theta_1 \cos \theta_1 \right) = \frac{n_e}{n_0} \quad (3-23)$$

or

$$\delta \theta_1 = \frac{S_R \sqrt{\frac{n_e}{n_0} - \sin \theta_1}}{\cos \theta_1} \quad (3-24)$$

**REAPPEARANCE**

As the line of sight to the target passes behind the shock front the target will disappear (ignoring diffraction) and will not reappear until the external rays S-A and C-T are tangent to the shock, i.e., the angle of incidence, $\psi_0$ is $90^\circ$. From equation 3-18 and 3-16,

$$\frac{n_0}{n_e} = \sin \left( \frac{\pi}{2} - \delta \psi \right) = \cos \delta \psi$$

$$= \cos \left( \frac{R_1+R_2}{2R_2} \delta \theta_1 \right) \quad (3-25)$$
Again in the small angle approximation
\[
\frac{n_0}{n_e} = 1 - \frac{1}{2} \left[ \frac{R_1+R_2}{2R_2} \delta \theta_1 \right]^2
\]  
(3-26)

Inverting,
\[
\delta \theta_1 = \frac{2R_2}{R_1+R_2} \sqrt{2\left(1 - \frac{n_0}{n_s}\right)}
\]  
(3-27)

Substituting the index of refraction ratio from equation (2-6) gives
\[
\delta \theta_1 = \frac{2R_2}{R_1+R_2} \sqrt{2\left(\mu - 1\right) + 4.8 \times 10^3 \frac{\mu}{T_0} \left(\frac{T_0}{T_s} - 1\right)}
\]  
(3-28)

for \(\rho_0 = 10^{-3}\)
\[
\delta \theta_1 = \left(\frac{2R_2}{R_1+R_2}\right) (0.02) \left[\left(\mu - 1\right) + 4.8 \times 10^3 \frac{\mu}{T_0} \left(\frac{T_0}{T_s} - 1\right)\right]^{1/2}
\]  
(3-29)

when the shock first becomes transparent at \(T_s = 4T_0\) and \(\mu = 4.7\), the square root term yields about a factor of 2 thus the refraction error will be about 40 milliradians times the factor involving the relative locations (which is usually of the order of unity).

If we define the minimum value of interest for \(\delta \theta_1\) we can then determine the minimum shock strength and therefore the maximum shock radius of interest. We can convert equation 3-29 into terms of the relative shock overpressure, \(\pi = \frac{P}{P_0} - 1\), using the Hugoniot relation
\[
\mu = \frac{7 + 6\pi}{7 + \pi}
\]
and the ideal gas law

\[ \frac{T_0}{T_S} = \frac{P_0}{p} \cdot \frac{\rho}{\rho_0} = \mu_{1+\pi} . \]

Substitution of these into equation (3-29) yields

\[
\delta \theta_1 = \left( \frac{2R_2}{R_1 + R_2} \right) (0.02) \left[ \frac{5\pi}{7 + \pi} + \frac{16}{(7 + \pi)^2} \left( 21 \pi - \frac{\pi^3}{1 + \pi} \right) \right]^{1/2} \tag{3-30}
\]

which for weak shocks reduces to

\[
\delta \theta_1 = \left( \frac{2R_2}{R_1 + R_2} \right) (0.02) \left[ \frac{5\pi}{7} (1 + 10\pi') \right]^{1/2} , \pi \ll 1 \tag{3-31}
\]

For example, to produce the above estimate to, say, 1 milliradian, would require that the factor within the square root be only \( \frac{1}{400} \) which occurs at about an overpressure of about 0.03 psi, i.e., in the far field of the shock wave.
SECTION 4
A MODULE TO PREDICT REFRACTION
AND PRELIMINARY RESULTS

A set of subroutines was prepared with the intent that an appropriate subset could easily be adopted into larger programs. These subroutines and their functions are described in the following paragraphs.

Program BLAST and subroutine GETINPUT provide our stand-alone driver that would be replaced by a calling procedure within the larger program. Inputs that are expected, units used, and where appropriate default values, are:

1) \( \rho_A \) = Ambient air density (\( \text{gm/cm}^3 \), default = 1.225\( \times 10^{-3} \))
2) \( T_{E} \) = Ambient air temperature (°K, default = 288)
3) \( W \) = Relative partial water vapor pressure (default = .01)
4) \( W \) = Effective blast yield (kilotons)
5) \( T_{M} \) = Time of interest after burst (sec)
6) \( R_{BS} \) = Actual range between source and burst (cm)
7) \( R_{TS} \) = Actual range between source and target (cm)
8) \( \theta_{I} \) = Actual angle between \( R_{BS} \) and \( R_{TS} \) (radians)

In the stand-alone version these are obtained via a common block from subroutine GETINPUT in which the default values are stored.

SUBROUTINE REFRACT

Subroutine REFRACT is the heart of the calculational procedure. The first step is to scale ranges and time to equivalent one kiloton.
values so that the shock location (i.e., radius) and density profile data can be obtained from the AFWL Nuclear Blast Standard (1 KT) (Reference 4) via a call to DENSITY. Then a set of tests is performed to determine the relative locations of the source and the target relative to the shock front and whether or not the LOS extends to the point of closest approach (RM1P) of the LOS to the burst. These tests define the approximation procedures and the integration limits.

The primary output provided is the refractive error, in radians, in the plane formed by the burst point, target and source: that is, the increase in the angle THETA1 caused by the refraction of the ray path in passing through the density profile of the blast wave. Additional outputs that are available include PSIO, the angle of incident at the shock front (when the source is outside); relative locations of target or source with respect to the shock, and the interior angle \( \phi \); and \( n \), the effective index of refraction (when both target and source are outside).

**SUBROUTINE ETA**

Subroutine ETA returns the index of refraction within the shocked region according to Equation 2-4b. The input is the scaled radial dimension of the point of interest. This subroutine then calls the 1 KT blast model to obtain the overdensity via the common block /WFRT/. A temperature is needed in Equation 2-4b as part of the water vapor correction. The 1 KT blast model does not provide a temperature or internal energy profile behind the blast wave. Although a temperature could be obtained from the overpressure profile and the equation of state through an iteration procedure, a simple approximation has been used instead. The temperature is estimated by assuming a gamma law expansion (at \( \gamma = 1.4 \)) from the current shock front density, but limited to be at least ambient temperature. This is rationalized as being sufficient since the shock
temperature does not exceed 1200° K, thus any error in the temperature will probably be less than the uncertainty in the partial pressure of the water vapor.

SUBROUTINE INTEGRT

This subroutine performs the integration of Equation 3-6 to provide the angle $\phi$ between radial limits supplied to it by subroutine REFRACT. The procedure used is to subdivide this interval into steps within which the value of eta is essentially constant. Equation 3-6 can then be integrated analytically to give the increment, $\Delta \phi$, over each step, $\Delta r$, i.e.,

$$\Delta \phi_i = \frac{K}{n_i} \int_{r_i}^{r_i+\Delta r} \frac{dr}{r/r^2 - (K/n_i)^2} = \cos^{-1} \left( \frac{K}{n_i r_i} \right) - \cos^{-1} \left( \frac{K}{n_i (r_i + \Delta r)} \right)$$

where $n_i$ is the value of the index of refraction at the center of the $i$th interval. The approximate number of steps to be taken is specified by a data statement as NUM, however the step size can be decreased or increased internally based on a test of the relative change of the index of refraction within each step.

The calling sequence for this subroutine expects

- RS = First integration limit of scaled radius
- RM = Second integration limit
- CK = Constant $K$ of equation 3-6
- ETANEW = Value of eta at starting point of integration
- DPDK = A trigger value which causes a calculation of $\frac{d\phi}{dk}$ when positive or zero, but skips this calculation when set negative

26
Upon return, in addition to providing the values of $\phi$ and when requested $\frac{d\phi}{dK}$, the subroutine stores the most recent values of $n_1$ in ETANEW for use in the second integration when needed.

SUBROUTINES DENSITY AND AIRPT

Subroutine DENSITY contains only those portions of the 1 KT blast standard that are required for the refraction prediction. These were extracted from Reference 4 and are carried as a separate routine so that when the more complete set of blast subroutines is used elsewhere within a larger code, this subroutine can be deleted and that set used. An initial call to DENSITY (TIME) at each time of interest sets the following parameters.

- **PRAD** = Shock front radius (cm)
- **OPPK** = Peak overpressure, at PRAD (dyne/cm²)
- **ODPK** = Peak overdensity, at PRAD (gm/cm³)
- **RDZ** = Radius at which overdensity passes through zero (cm)
- **TEMPK** = Shock front temperature (°K)

Subsequent calls at the same time use entry DENS(RAD) and obtain ODR, the overdensity at the specified radius, RAD. Outputs from DENSITY are transferred via the common block /WFRT/. Subroutine DENSITY requires the air equation of state to calculate ODPK, from the prediction value of OPPK.

The DOAN-NICKLE equation of state of air as given in subroutine AIRPT($E,G,P,T$) of the MDAC version of LAMB has been used, since it includes the temperature and pressure thus providing TEMPK. This subroutine appears to include the subroutine AIR($E,R,G$) of the 1 KT standard, which may be accessed through an entry call.
TYPICAL RESULTS

These subroutines have been exercised for the several conceivable types of sight paths, depending on the relative location of the radar target, shock wave and point of closest approach to the burst point of the sight path. For those cases in which absorption dominates or where the shock is not intersected a message to that effect is produced without the prediction of refraction. Table 1 describes eight possible sight paths through the shock wave from a one kiloton burst at one second and lists the calculated refractive error.

Table 1. Sight path parameters and results.

<table>
<thead>
<tr>
<th>Case</th>
<th>RBS cm</th>
<th>RTS cm</th>
<th>Radians</th>
<th>milliradians</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1.3E5</td>
<td>1E5</td>
<td>.1</td>
<td>8.4E-4</td>
</tr>
<tr>
<td>2</td>
<td>1.3E5</td>
<td>1.6E5</td>
<td>.15</td>
<td>2.9E-2</td>
</tr>
<tr>
<td>3</td>
<td>1.3E4</td>
<td>2E4</td>
<td>.7</td>
<td>2.5E-4</td>
</tr>
<tr>
<td>4</td>
<td>3E4</td>
<td>5E4</td>
<td>.72</td>
<td>-6.0E-5</td>
</tr>
<tr>
<td>5</td>
<td>2.5E4</td>
<td>3E4</td>
<td>1.9</td>
<td>658.(?)</td>
</tr>
<tr>
<td>6</td>
<td>4E4</td>
<td>2E5</td>
<td>.7</td>
<td>0.136</td>
</tr>
<tr>
<td>7</td>
<td>3E4</td>
<td>1.5E5</td>
<td>2.1</td>
<td>-1058.(?)</td>
</tr>
<tr>
<td>8</td>
<td>1.5E5</td>
<td>2.5E5</td>
<td>.22</td>
<td>16 to 104(*)</td>
</tr>
</tbody>
</table>

(?) These values are abnormally large; indicating that the "small derivation" approximation is invalid but do show that refraction will be an extremely severe problem.

(*) This case did not converge but oscillated between these values, again indicating a serious refractive error.
REFERENCES


APPENDIX I
ABSORPTION

When the electron concentration within a shock wave is sufficiently high, the absorption of electromagnetic waves becomes so great that refraction effects can be ignored. Here we will generate estimates of the conditions at which such circumstances occur and identify those cases wherein refraction might be important. Except where noted, the basic equation used below are from The Aids for the Study of Electromagnetic Blackout (Reference 1).

The differential absorption of an electromagnetic wave of angular frequency, \( \omega \) (radians/sec), can be expressed as

\[
Ab = \frac{46 \, g \nu}{(g \nu)^2 + (h \omega)^2} \, N_e \, \text{db/m} \tag{I-1}
\]

where \( N_e \) is the local electron concentration \((\text{e/cm}^3)\) and \( \nu \) is the electron collision frequency \((\text{sec}^{-1})\). At altitudes below about 100 km the collision frequency of importance is that with neutral particles, which is

\[
\nu = 1.7 \times 10^5 \, p \, \text{sec}^{-1} \tag{I-2}
\]

where \( p \) is the local air pressure \((\text{dynes/cm}^2)\). At sea level the ambient pressure is \( p_0 = 10^6 \, \text{dyne/cm}^2 \) hence \( \nu_0 = 1.7 \times 10^{11} \, \text{sec}^{-1} \) which corresponds to an operating frequency, \( f = \frac{\omega}{2\pi} = \frac{\nu_0}{2\pi} = 2.7 \times 10^{10} = 27 \, \text{Ghz} \). The operating frequencies of interest may be on either side of the collision frequency.
The factors \( g \) and \( h \) in Equation I-1 above are correction factors to account for the velocity dependence of electron-neutral collisions. For representative calculations values of these factors, as obtained from graphs in Reference 1, are given in Table I-1 below.

### Table I-1. Typical values of \( g \) and \( h \).

<table>
<thead>
<tr>
<th>Condition</th>
<th>( g )</th>
<th>( h )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \omega &lt; \nu )</td>
<td>0.6 to 0.65</td>
<td>&gt; 1.7</td>
</tr>
<tr>
<td>( \omega = 0.4 \nu )</td>
<td>0.75</td>
<td>1.3</td>
</tr>
<tr>
<td>( \omega &gt; \nu )</td>
<td>1</td>
<td>1</td>
</tr>
</tbody>
</table>

For \( \omega < \nu \) absorption is independent of the operating frequency and Equation (I-1) can be written

\[
Ab = 75 \frac{N_e}{\nu}, \quad \omega < \nu, \quad \text{db/m} \quad (I-3)
\]

Using the sea level, ambient value of \( \nu \) this becomes

\[
Ab = 4.5 \times 10^{-10} N_e, \quad f \ll 27 \text{ Ghz}, \quad \text{db/m} \quad (I-3a)
\]

At 10 Ghz Equation (I-1) becomes

\[
Ab = \frac{61 N_e}{\nu \left[ 1 + \left( \frac{0.64}{p} \right)^2 \right]} , \quad f = 10 \text{ Ghz}, \quad \text{db/m} \quad (I-4)
\]
Using sea-level conditions in Equation I-4 yields

\[ Ab = 2.5 \times 10^{-10} N_e , \quad \text{db/m} . \]  

(I-4a)

Note that the second factor in the denominator of Equation I-4 reduced the absorption by about 40\%, but will contribute much less in a strongly shocked region.

For frequencies well above the collision frequency, i.e., millimeter waves, Equation (I-1) becomes

\[ Ab = 46 \frac{\nu}{\omega^2} N_e , \quad \omega \gg \nu \quad \text{db/m} \]  

(I-5)

Probably the minimum path length through the shocked region that is ever of interest will be a few tens of meters. Normally the path will be much longer, i.e., one-hundred or more meters. Equation I-3a indicates that an electron concentration of \(10^9\) e/cm\(^3\) would yield about 10 db (one-way) or 20 db (two-way) absorption over a 20 meter path. Equation I-4a would require a 40 meter path or twice the electron concentration. Similarly, Equation I-5 indicates that \(10^{10}\) e/cm\(^3\) will produce high levels of absorption at a frequency of 95 Ghz. One thus concludes that if the electron concentration exceeds \(10^9\), or perhaps \(10^{10}\) depending on geometry and operating frequency, absorption will be the dominant effect. Thus refraction is only of interest when the electron concentration is below this range.

**ELECTRON CONCENTRATION**

To maintain an electron concentration of \(10^9\) e/cm\(^3\) by thermal collisions alone requires a local temperature, \(T\), of about 2500° K for
sea level conditions. However, the ionization generated by the neutrons and delayed gamma rays from the fission debris can maintain this concentration to a much lower temperature. The gamma source is in general the more significant, although at early times and close-in the neutrons can be of equal significance. Reference 1 gives the gamma ray ionization source as

$$q_y = \frac{2 \times 10^{19} \, W_F \, \rho \, e^{-\mu/r} \, \rho \, d\rho}{4\pi R^2 (1 + t)^{1.2}}$$

where

- $W_F$ = fission yield (MT)
- $\rho$ = air density at the field point (gm cm$^{-3}$)
- $R$ = range from the source to the field point (km)
- $t$ = time after detonation (sec).
- $\mu$ = mass absorption coefficient (cm$^2$ gm$^{-1}$)

In the following, we are interested in the shocked region as the shock becomes transparent. This occurs at a shock temperature which is only weakly dependent on yield, thus the shock radius to be used in calculating $q$ scales approximately as the cube root of the yield. The fraction of the total yield which is fission is of course an unknown, but $1/2$ is a reasonable nominal value for megaton class yields. This fraction tends to be larger for small yields. The intervening absorption ($e^{-\mu/r}$) increases with yield causing a decrease in $q$ as the yield increases. As a result of these various factors, $q/\rho$ in the region of interest will vary less rapidly than the cube root of total yield. Furthermore, the prediction of $\bar{N}_e$ and absorption will be shown to vary as the square root of $q$. Thus the final conclusion is only weakly dependent on the inputs chosen for Equation 6. To represent a nominal one-megaton surface burst we will choose $W_F = \frac{1}{2}$ MT, $R = 1$ km, $t = .3$ sec, and $e^{-\mu/r} = \frac{1}{2}$, and obtain

$$q_y = 3 \times 10^{17} \rho.$$
To approximately account for neutrons we will double this and use

\[ q = 6 \times 10^{17} \rho, \quad \text{ion pairs cm}^{-3}\text{sec}^{-1}. \] (I-7)

The quasi-equilibrium solution of the rate equations for the electron concentration, as given in Reference 1, is *

\[ N_e = \frac{\sqrt{q}}{\sqrt{\alpha}} \frac{\sqrt{q\alpha + D}}{\sqrt{q\alpha + D + A}}, \quad \text{cm}^{-3} \] (I-8)

where

\[ \alpha = \frac{A\alpha_i + D\alpha_d}{A + D}, \quad \text{cm}^3\text{sec}^{-1} \] (I-9)

\[ \alpha_i = \text{ion-ion recombination coefficient} \]
\[ = 3 \times 10^{-8} + 6 \times 10^{-6} \frac{P}{T^{2.5}}, \quad \text{cm}^3\text{sec}^{-1} \] (I-10)

\[ \alpha_d = \text{electron-ion recombination coefficient} \]
\[ = \frac{9 \times 10^{-5}}{T}, \quad \text{cm}^3\text{sec}^{-1} \] (I-11)

\[ A = \text{electron attachment rate} \]
\[ = 9.7 \times 10^3 \frac{p^2}{T^3} \exp(-\frac{600}{T}) + 0.9 \frac{p^2}{T^2}, \quad \text{sec}^{-1} \] (I-12)

* In these equations, \( p \) is the pressure in dynes per cm\(^2\) and \( T \) is the absolute temperature in degrees Kelvin.
\[ D = \text{electron detachment rate which is the sum of collisional detachment, } D_c, \text{ resulting from high temperatures and photo detachment } D_p \text{ caused by energetic photons emitted by the fireball. The collisional detachment coefficient is given in Reference 1 as} \]

\[ D_c = 2.4 \times 10^4 \frac{P}{\sqrt{T}} \exp(-\frac{5590}{T}) + 2.1 \sqrt{T} \exp(-\frac{4990}{T}) \text{, sec}^{-1} \quad (I-13) \]

The photo detachment coefficient is given in Reference 2 as

\[ D_p = \left(\frac{R_F}{R}\right)^2 1.36 \times 10^{-16} T_F^{5.4} \text{, sec}^{-1} \quad (I-14) \]

where \(T_F\) is the effective radiating temperature (°K) and \(\frac{R_F}{R}\) is the ratio of the fireball radius to the range to the point of interest (which we shall take to be unity.)

To compute values for the above reaction rates and estimate the relative importance of the various terms we will use ambient conditions corresponding to surface values for the mid United States, i.e., an altitude of about 4000 ft, where the ambient density is about \(1.1 \times 10^{-3}\) gm/cm\(^3\) and the ambient pressure is \(9 \times 10^5\) dynes/cm\(^2\). Earlier studies have shown that temperatures greater than about 800° K lead to high levels of absorption. We will use a shock temperature of 1000° K and the corresponding values of shock overpressure and density. These are an overpressure ratio, \(\Delta p/p\), of 15 and a density compression ratio of 4.4. Then the pressure at the shock front is \(1.5 \times 10^7\) dyne/cm\(^2\). This yields

\[ A = 9.7 \times 10^3 \frac{(1.5 \times 10^7)^2}{(10^3)^2} \exp(-\frac{600}{1000}) + 0.9 \frac{(1.5 \times 10^7)^2}{(10^3)^2} \]

\[ = 1.2 \times 10^9 + 2 \times 10^8 = 1.4 \times 10^9 \text{, sec}^{-1} \]
(Note that the first term dominates for these conditions)

\[ \alpha_i = 3 \times 10^{-8} + 6 \times 10^{-6} \frac{1.5 \times 10^7}{(10^3)^{2.5}} \]

\[ = 3 \times 10^{-8} + 2.85 \times 10^{-6} = 2.9 \times 10^{-6}, \text{ cm}^3/\text{sec} \]

(Note that the second term dominates)

\[ \alpha_d = \frac{9 \times 10^{-5}}{10^3} = 9 \times 10^{-8}, \text{ cm}^3/\text{sec} \]

\[ D_c = 2.4 \times 10^4 \left( \frac{1.5 \times 10^7}{\sqrt{10^3}} \exp\left(-\frac{5590}{1000}\right) + \right. \]

\[ 2.1(1.5 \times 10^7) \sqrt{10^3} \exp\left(-\frac{4990}{1000}\right) \]

\[ = 4.25 \times 10^7 + 6.8 \times 10^6 = 4.9 \times 10^7, \text{ sec}^{-1} \]

\[ D_p = 1.36 \times 10^{-16} (10,500)^{5.4} \approx 10^6, \text{ sec}^{-1} \]

The last two equations show photo detachment can be ignored (within the shocked zone).

Substituting the above values into Equation 1-8 yields an electron concentration at the above specified shock condition of \( N_e = 1.1 \times 10^9 \) e/cm\(^3\), showing we are in the range of conditions that are of interest.

**TEMPERATURE DEPENDENCE OF ABSORPTION**

We may obtain the dependence of \( N_e \) and thus absorption on local temperature and pressure by noting the dominant terms in these equations.
In equation 1-9 for \( \alpha \), \( A\alpha_i \) dominates the numerator and \( A \) dominates the denominator, i.e.,

\[
A\alpha_i = (1.4 \times 10^9)(2.9 \times 10^{-6}) = 4 \times 10^3
\]

\( \gg \) \( D\alpha_d = (5 \times 10^7)(9 \times 10^{-8}) = 5 \)

and \( A = 1.4 \times 10^9 \gg D = 5 \times 10^7 \).

Thus Equation 1-9 can be closely approximated by

\[
\alpha = \alpha_i = 3 \times 10^{-6}
\]  \hspace{1cm} (I-10a)

We may also simplify Equation (1-8) by noting that

\[
\sqrt{q\alpha} = \left[6 \times 10^{17} (4.4) 1.1 \times 10^{-3} (3 \times 10^{-6}) \right]^{1/2} = 9 \times 10^4 \ll D
\]

Thus Equation (1-8) becomes

\[
N_e = \frac{D}{A} \frac{q}{\alpha_i}, \text{ cm}^{-3}
\]  \hspace{1cm} (I-15)

Now by using only the dominant terms in \( D, A, \) and \( \alpha_i \)

\[
N_e = \frac{2.4 \times 10^4 \frac{p}{\sqrt{T}}}{9.7 \times 10^3 \frac{p^2}{T^3}} \exp\left(-\frac{5590}{T}\right) \left[\frac{6 \times 10^{17} \frac{p}{T}}{6 \times 10^{-6} \frac{p}{T^{2.5}}}\right]^{1/2}
\]

\[
= 2.5 \frac{T^{2.5}}{p} \exp\left(-\frac{4990}{T}\right) \left[10^{23} \frac{p}{T^{2.5}}\right]^{1/2}
\]
we may use the gas law to simplify the square-root term i.e.,

\[
\frac{p}{p_0} T = \frac{2.9 \times 10^6}{M} T
\]

and that term becomes

\[
\left[ \right]^{1/2} = 2 \times 10^8 T^{3/4}.
\]

The resulting approximate formula yields predictions that are a few percent higher than those obtained using all terms. To obtain agreement at 1000° K we will use

\[
N_e = 4.3 \times 10^8 \frac{T^{3.25}}{p} \exp\left(- \frac{4990}{T}\right), \text{ cm}^{-3}
\]

\[
= 4.3 \times 10^8 \frac{F(T)}{p}
\]

where

\[
F(T) = T^{3.25} \exp\left(- \frac{4990}{T}\right).
\]

For convenience this function is plotted in Figure I-1.

Equation I-16 may be substituted into the absorption equations to relate absorption directly with temperature, pressure and frequency. From Equation (I-1), (I-2) and (I-16)

\[
Ab = \frac{8.5 \times 10^{13} g F(T)}{7.3 \times 10^8 (g p)^2 + (h f)^2}, \text{ db/m}
\]

(I-18)

when \( \omega \ll \nu \), then this reduces to

\[
Ab = 2 \times 10^5 \frac{F(T)}{p^2}, \quad \omega \ll \nu, \quad \text{db/m}
\]

(I-19)
and in the other limit of \( \omega > \nu \), then

\[
Ab = 8.5 \times 10^{13} \frac{F(T)}{f^2}, \quad \omega \ll \nu, \quad \text{db/m} \quad (I-20)
\]

**MAXIMUM TEMPERATURE FOR REFRACTION**

In order to specify the temperatures above which absorption dominates we must make worst case assumptions about the path and the system. The path length will be shortest for a small yield - but even the shock of a 5 kt burst is about 250 meters in radius when it becomes transparent. Perhaps one-tenth of this is a minimum path length. If we also choose 20 db two-way loss as a system limiting factor, then for typical radar frequencies we will use \( Ab = \frac{1}{2} \) db/m. By inverting Equation 19 and using a shock pressure of about 15 atmospheres we obtain \( F(T) = 5 \times 10^8 \). Figure 1-1 shows this corresponds to a temperature of 1400° K. Note that this high shock pressure also implies an increase in the applicable frequency range of Equation 1-19, since \( \nu \) is proportional to \( p \).

The above temperature is higher than previously suggested as an upper limit primarily because the path length chosen is quite small. When the path length through the shock region is chosen as 100 meters, then this temperature drops to 1130° K. We have also assumed the absorption is uniform along the path in the shock. Since it is not - i.e., it depends on \( p^{-2} \), then the above temperature is an overestimate. For convenience in setting up the refraction calculation we will somewhat arbitrarily use 1200° K as our upper cut off.

There are several assumptions in the above discussion that should not strongly limit the more general applicability of the result. For example, the ranges considered and the fission yield and the doubling to account for the neutrons are all consistent with a nominal 1 MT near-surface burst. Shifting to a nominal small yield would cause several nearly compensating changes in the numbers, but the resulting shock temperature to give specified db levels would change only slightly.
REFERENCES


APPENDIX II
SUBROUTINE LISTINGS

PROGRAM KLAST
C
C DRIVER FOR TEST OF REFRACTIVE ERROR
C
INCLUDE 'GIVEN.C'
LOGICAL CONTINUE
OPEN(UNIT=2,NAME='USER1:REFRACT.UNIT',STATUS='NEW')
CONTINUE = .FALSE.
TCASE = 1

19 TCASE = TCASE + 1
WRITE(2,105A)ICASE
105H FORMAT(' INPUT FOR CASE NUMBER ',I3)

CALL GETINPUT(CONTINUE)
CALL HFRACT(DTHETA)

WRITE(2,105A)DTHETA
105H FORMAT(' REFRACTIVE ERROR IS ',1PF1.6)

C ANOTHER SET OF INPUT DATA
IF (CONTINUE) 10 TO 11

C END OF INPUT DATA
CLOSE(UNIT=2)
END
SUBROUTINE GETINPUT(MOREDATA)

READS INPUT FROM REFRACT.DAT
TRANSFERS THE DATA VIA THE COMMON BLOCK GIVEN

LOGICAL END, MOREDATA
INCLUDE 'GIVEN.CMN'

DIMENSION IHOL(19), IN(2), IGHOA(5), ITEMPA(6), IWATER(6),
1    IRHS(4), INTS(4), ITHETA1(7), ICOMMENT(3),
2    ITEND(4), ITIME(5)

DATA IN/1, IHW/0,
1    IGHOA/A, IHW/0, IHM/1, IHA/0,
2    ITEMPA/A, IHT/1, IHE/1, IHP/1, IHA/0,
3    IWATER/A, IHM/1, IHT/1, IHE/1, IHR/1,
4    IRHS/3, IHM/1, IHE/1, IH/1,
5    INTS/3, IHM/1, IHT/1, IHE/1,
6    ITHETA1/A, IHT/1, IHM/1, IHE/1, IHA/1,
7    ICOMMENT/2, IHT/1,
8    ITEND/3, IHE/1, IH/1,
9    ITIME/4, IHT/1, IHM/1, IHE/1

DATA RHOA/1, 02E-1, ITEMPA/2HR/WATER/1.5E+1

IF (MOREDATA) GO TO 10

OPEN(UNIT=1, NAME='USER1:REFRACT.DAT', STATUS='OLD', READONLY)

GET ONE LINE OF INPUT

10 READ(1, I50, END=14) IHUL

I50 FORMAT(1A1)

SKIP THE LINE IF IT IS A COMMENT

CALL HOLFWEL(IHUL, ICOMMENT, EQUAL, FALSE, NSKIP)
IF (EQUAL) GO TO 10

CALL HOLFWEL(IHUL, IGHOA, EQUAL, TRUE, NSKIP)
IF (EQUAL) GO TO 11

CALL HOLFWEL(IHUL, ITEMPA, EQUAL, TRUE, NSKIP)
IF (EQUAL) GO TO 12

CALL HOLFWEL(IHUL, IWATER, EQUAL, TRUE, NSKIP)
IF (EQUAL) GO TO 13

44
CALL HOLEUL(INUL,INHS,EQUAL,'TRUE',NSKIP)
IF (EQUAL) GO TO 17K

CALL HOLEUL(INUL,IRTS,EQUAL,'TRUE',NSKIP)
IF (EQUAL) GO TO 1M

CALL HOLEUL(INUL,ITHETA1,EQUAL,'TRUE',NSKIP)
IF (EQUAL) GO TO 1N

CALL HOLEUL(INUL,ITIME,EQUAL,'TRUE',NSKIP)
IF (EQUAL) GO TO 20

CALL HOLEUL(INUL,INEND,EQUAL,'FALSE',NSKIP)
IF (EQUAL) GO TO 30

END OF DATA SET

CALL HOLEUL(INUL,INEND,EQUAL,'FALSE',NSKIP)
IF (EQUAL) GO TO 30

ERROR = UNRECOGNIZABLE INPUT
WRITE(2,1069)HUL
1069 FORMAT(' UNRECOGNIZABLE INPUT',A,A,5X,A)
STOP

READ M (YIELD)

10N RACKSPACE 1
READ(1,1070) M
1070 FORMAT('NSKIP',X,2H4.0)
IF (M.EQ.1) WRITE(2,1075)
1075 FORMAT('WARNING! INPUT YIELD IS LESS THAN 1 TON')
IF (M.EQ.0) THEN
WRITE(2,1076)
1076 FORMAT('ERROR: YIELD >= 1, IF 0.4 IS NEGATIVE OR ZERO, PROGRAM 1 EXITS')
STOP
ENDIF
GO TO 1M

READ KMDA (DENSITY)

11R RACKSPACE 1
READ(1,1071)RMDA
IF (RMDA.LT.1.E-6) WRITE(2,1076)
1074 FORMAT('WARNING! INPUT DENSITY IS LESS THAN 1E-6,
11LAST MODEL IS SUSPECT')
GO TO 16
READ TEMPA (AMBIENT TEMP.)

12H RACKSPACE 1
READ(1,1476) TEMPA
IF (TEMPA.LT.1.00) THEN
WRITE(2,1495)
1095 FORMAT('1 WARNING: INPUT AMBIENT TEMP IS VERY SMALL OR
1 NEGATIVE. NOMINAL VALUE OF 298K USED.')
TEMPA=298.0
ENDIF
GO TO 10

READ WATER

13H RACKSPACE 1
READ(1,1474) WATER
IF (WATER.LT.0.0) THEN
WRITE(2,1494)
1094 FORMAT('1 WARNING: INPUT VALUE OF WATER VAPOR IS NEGATIVE,
1 MY AIR IS USED.')
WATER=0.0
ENDIF
GO TO 10

READ RNS (RANGE FROM SOURCE TO HURST)

17H RACKSPACE 1
READ(1,1470) RNS
IF (RNS.LT.0.0) THEN
WRITE(2,1493)
1093 FORMAT('1 ERROR: RANGE FROM SOURCE TO HURST =*1,PF10.4,* IS
1 NEGATIVE. PROGRAM ENDS.')
STOP
ENDIF
GO TO 10

READ HTS (RANGE FROM SOURCE TO TARGET)

18H RACKSPACE 1
READ(1,1472) HTS
IF (HTS.LT.0.0) THEN
WRITE(2,1492)
1092 FORMAT('1 ERROR: RANGE FROM SOURCE TO TARGET =*1,PF10.4,* IS
1 NEGATIVE. PROGRAM ENDS.')
STOP
ENDIF
GO TO 10
READ THETA1 (ANGLE BETWEEN VECTORS RHS AND RTS)

19A BACKSPACE 1
READ(I,1127) THETA1
IF (THETA1.LT.-.0.9) THEN
WRITE(2,1203) THETA1
205A FORMAT(' ERROR: THETA1 = ',PE14.4,' IS NEGATIVE, PROGRAM EXITS.')
STOP
ENDIF
GO TO 16

READ TIME

25A FORMAT(' ERROR: TIME = ',PE14.4,' IS NEGATIVE, PROGRAM EXITS.')
STOP
ENDIF
GO TO 16

END STATEMENT

50A CONTINUE
MOREDATA = ' TRUE.
GO TO 45

END OF FILE

60A CONTINUE
MOREDATA = ' FALSE.
CLOSE(UNIT=11)
GO TO 45

WRITE(2,9116)
911A FORMAT(' 1 )
WRITE(2,9117)
911B FORMAT(' )
WRITE(1,9111)  'INPUT DATA 1')
WRITE(2,9112)  'INIT'(2,0,171)
91171 FORMAT(' w (YF10) .............. ',IPE13.6)
WRITE(2,9113) 'HUM
91172 FORMAT(' RHDA (DENSITY) .............. ',IPE13.6)
WRITE(2,9114) 'TEMP
91173 FORMAT(' TEMPA (AMBIENT TEMP.) ...........,IPE13.6)
WRITE(2,9115) 'MATH
SUBROUTINE HOLE.N(IONE,ITNO,ANS,LENGTH,SKIP)

MODIFICATION OF HNCSIM SUBROUTINE ; TESTS FOR EQUIVALENCE.

BETWEEN ARRAYS IONE AND ITNO; IF LENGTH IS 'TRUE', IT
RETURNS THE NUMBER OF CHARACTERS FROM THE BEGINNING OF THE
RECORD UNTIL AN '=' CHARACTER IS REACHED.

LOGICAL LENGTH, ANS
DIMENSION IONE(1),ITNO(1)
DATA IMLANK/I1M/,IEMU/I1M/
ANS=FALSE.
SKIP=1
J=1
N=ITNO(1)+1
10 IF(IONE(J),EMU,IMLANK) GO TO 2
11 IF(IONE(J),RE,ITNO(1)) GO TO 13
12 CONTINUE
ANS=TRUE.
13 IF(LENGTH) GO TO 15
RETURN
15 CONTINUE
20 IF(IONE(J),EU,IFWH) GO TO 21
21 CONTINUE
22 CONTINUE
30 CONTINUE
100 RETURN
END
SUBROUTINE REFRACT(UTHETA)

INCLUDE 'GIVEN,CMNT
INCLUDE 'WFRIT,CMNT
DATA PI/3.1415926536/TEMPZ/4000./HALFPI=ASIN(1./O)
PI=2.*HALFPI

DTHETA=0.
W3B(*RH2/RHNA*TEMPZ/TMPA)**(1./3.)
STIME=TIME/3.
CALL DENSITY(STIME)

CALL FOR INITIZATION
IF (TEMPK.LE.12.) GO TO 11
WRITE(2,8)
8NOM FORMAT(1) SHOCK STRONG ENOUGH TO CAUSE ABSORPTION, REFRACTION
GO TO 8N

R1 IS THE RANGE FROM SOURCE TO POINT OF CLOSEST APPROACH
10 R1=PHS*COS(THETA1)
SHAD=PHAD
RHS=SQRT(RHS**2+RTS**2-2*RHS*RTS*COS(THETA1))
RMIN=RHS*SIN(THETA1)
LASTIME=*

IF SHOCK HAS NOT REACHED LINE OF SIGHT, NO NOTHING.
1 IF (RMIN.GE.RHAD) GO TO 4K
IF SHOCK HAS NOT CROSSED LINE OF SIGHT, NO NOTHING.
2 IF ((WTH.GE.RHAD).AND.(RTS.LT.R1)) GO TO 4K
IF ((RHS.GE.RHAD).AND.(R1.LT.0)) GO TO 4K
GO TO 5K
4K CONTINUE
WRITE(2,9)
8NOM FORMAT(1) SHOCK HAS NOT REACHED LINE OF SIGHT (')
GO TO 4K

5K CONTINUE
SIPSIN(PHIS/RHAD)
PSIP=ASIN(SIPSIN)
ETAX=1.*RHS*R2*RHOA*(1.+4.*BE+3.*WATER/TMPA)
GETDEF RTAX,MAX,AND CONST
STRT=AMINI(SPA),AMAXI(RHS,RTA))
STRT=STRT/3.
CALL ETA(STRTS,RHUI,TEMP1,ETA5)
SHL=AMINI(SRAI),AMINI(RHS,KTH))
SHL=SHL/3.

49
CALL ETA(SMLS,RH01,TEMPI,ETAM)
CONMAX=SMLS*ETAM
IF (SHT. EQ. SHA0) THEN
  CON=PRAD*ETA*SIPS1
ENDIF

FLSF
IF (SPL. EQ. WHS) THEN
  CON=CONMAX*SIN(THETA1)
ELSEF
  CALL ETA(SHHS,RH01,TEMPI,ETASRHS)
  CON=SHHS*ETASRHS*SIN(THETA1)
ENDIF

END

00 LOOPC01

01 IF (SHA0 .GT. AMIN(RHS,RTH)) GO TO 04

02 COMPLETE TRANSAL = BOTH TARGET AND SOURCE ARE OUTSIDE SHOCK

03 INTERSECT FROM SHA0 TO RMN

04 RECALC RMN USING ETA AT RMN

05 SRM1=RMN/RTH

06 CALL ETA(SRM1,RH01,TEMPH,ETAMIN)

07 RMN=RMN*ETAMIN/ETAM

75 DUMP=PRAD

76 DUM2=RMN/RTH

77 ETANEM=ETA

78 CALL INTEGR( DUM1, DUM2, CON, ETANEM, PHI, =1)

79 ETAMIN=ETANEM

80 ETAH=CON/PRAD/COS(PHI)

81 THETAH=(SHA0*SUM(ETAH)-RMN)/(RHS*COS(THETA1))

82 IF (LASTTH. EQ. 1) GO TO 90

83 SIPS1=RHS/SHA0*SIN(THETA1+THETA)

84 PRASIN(SIPS1)

85 LOOPC=LOOPC+1

86 CON=CON

87 CON=ETAM*PRAD*SIPS1

88 RMN=CON/ETAMIN

89 IF (CON .GT. CONMAX) THEN
  CON=CONMAX
  RMN=CON/ETAMIN
  LASTTIME=1
ENDIF

GO TO 75

50
INTEGRATE ALL CASES WHERE
TARGET IS SHOCKED UP WITH AFF INSIDE SHOCK
WH \theta TA = (\text{RTS}/RTF \times \sin(\text{THETA}))
\text{ASIN} \text{RETURNS VALUES BETWEEN } \pm \text{HALFPI AND } \text{HALFPI}
CHECK IF \theta TA SHOULD BE GREATER THAN \text{HALFPI}
IF ((K1 \geq L1), \text{IN}, (\text{RTS}, \text{LE}, K1)) GO TO 150
\text{THE} \theta \text{TA} = \text{ASIN}((\text{KHS}/\text{RTF} \times \sin(\text{THETA})))
IF ((\text{THETA} < 2 \times \text{PI} \times \text{THETA}) \text{HALFPI}) \theta TA = \pi \text{THETA}3

95 CONTINUE
150 \text{NUM1} = \text{MIN}1((\text{KHS}, \text{SHAD}) \times 3
\text{NUM}2 = \text{MIN}2((\text{SHAD}) \times 3
FTANWETAS
\text{DPHINT} = 1.
CALL INTEGRATE\text{MIN}(1, \text{NUM}1, \text{NUM}2, \text{CONST}, \text{ETANWF}, \phi, 16)
\text{FTANWETAS}
\text{DPHINT} = 1.

LINE OF SIGHT CRUSSES POINT OF CLOSEST APPROACH
ADDITIONAL INTEGRATION
\text{NUM1} = \text{MIN}1((\text{KHS}, \text{KTH}) \times 3
\text{NUM}2 = \text{MIN}2((\text{SHAD}) \times 3
\text{DPHINT} = 1.
CALL INTEGRATE\text{MIN}(1, \text{NUM}1, \text{NUM}2, \text{CONST}, \text{ETANWF}, \phi, 16)
\text{DPHINT} = 1.
\text{FTANWETAS}
\text{DPHINT} = 1.

15. IF (\text{KHS} \leq \text{L1}, \text{SHAD}) GO TO 175
SHOCK IS OUTSIDE AND TARGET IS INSIDE SHOCK
\text{L9} = \text{RTF}/(\text{KHS} + 2 \times \text{SHAD} + 2 \times \text{KHS} \times \text{SHAD} \times \cos(\text{THETA} + \phi))
\text{THE} \theta \text{TA} \text{ASIN}((\text{SHAD} \times 1 \times \sin(\text{THETA} + \phi)) \times \text{THETA})
IF (\text{LAST} \text{THETA} = 1) GO TO 175
\text{PS1} = \text{THETA}3 \times \phi \times \text{THETA}1 \times \text{DTHETA}

150 IF (\text{UP} = \text{UP1}) GO TO 150
\text{CONST} = \text{CONST}
\text{CONST} \times \text{PHASE} \times \text{ETA} \times \sin(\text{FST4})
\text{RH\text{NE}} = \text{CONST} \times \text{TARM1N}
IF (\text{CONST} \times \text{GFC} \times \text{GHMAX}) THEN
\text{CONST} = \text{GHMAX}
\text{RH\text{NE}} = \text{CONST} \times 3 \times \text{ETANM} \times \text{LSTM} = 1
\text{ENM1F}
GO TO 150
C
175 IF (RTH+LI, SPA) GO TO 210
C
SOURCE IS INSIDE AND TARGET IS OUTSIDE SHOCK
C = SQRT(RTH + 2 * SPA + 2 * RTH * SPA + COS(THETA + PHI))
PSI = ASIN(RTH/COS(THETA + PHI)) - HALFPi
IF (LASTTIME, EQ. 1) GO TO 180
C
ITERATION LOOP
LOOPC = LOOPC + 1
CONST = CONST + 1
CONST = PHA * ETA + PSI
RM = CONST / ETA
IF (CONST GE COMAX) THEN
CONST = COMAX
RM = CONST / ETA
LASTTIME = 1
ENDIF
GO TO 180
C
180 RHS = RHS / 3
CALL ETA(SHS, PHII, TEI, ETAH), DHS = ASIN((SHS * ETA + PSI) / (RHS * ETAH)) = THETA
GO TO 180
C
SOURCE AND TARGET ARE INSIDE SHOCK
200 IF (LASTTIME. EQ. 1) GO TO 250
C
ITERATION LOOP
LOOPC = LOOPC + 1
CONST = CONST + 1
CONST = THETA + PHI / 2 / M + CONST + 1
RM = CONST / ETA
IF (CONST GE COMAX) THEN
CONST = COMAX
RM = CONST / ETA
LASTTIME = 1
ENDIF
GO TO 180
C
250 RHS = RHS / 3
CALL ETA(SHS, PHII, TEI, ETAH), DHS = ASIN(CONST / (RHS * ETAH)) = THETA
C
800 RETURN
END
SUBROUTINE INTEGR(HS,RM,CK,ETANEW,PHI,DPDK)

INPUT
HS = LIMIT OF INTEGRATION
RM = LIMIT OF INTEGRATION
CK = CONSTANT
ETANEW = INDEX OF REFRACTION AT THE UPPER LIMIT OF INTEGRATION

OUTPUT
PHI = ANGLE BETWEEN THE VECTORS RS AND RM
DPDK = THE DERIVATIVE OF PHI WITH RESPECT TO THE CONSTANT

INCLUDE 'FRT.CHM'
INCLUDE 'GIVEN.CHM'
DATA NUM/5'/,TESTL/0',1'/,TESTS/0',RM2/

ETA=ETANEW
R1=AAX1(HS,RM)
R2=AAX1(HS,RM)
I*P
SUM1=*
SUM2=P

IF (ETA<NE,ETAC)
STEP=(R1-R2)/F(LAT(NUM)
RAD1=R1
10 RAD2=AAX1(R2,RAD1-STEP)
IF (RAD2.EQ.R2) STEP=RAD1-WAN2
WADC=(RAD2-RAD1)/2
I=1+1
IF (1.GT.NUM+2) G0 T0 24
CALL ETA(RADC,X1,X2,ETAC)

TEST=ABS(ETA-ETAC)/(ETA+1.)
IF (TEST.LF.TESTL) G0 T0 20

STEP=STEP/2.
G0 T0 10

20 C1=CK/ETAC/RAD1
C2=AAX1(C1,CK/ETAC/AAX1(R2,RAD2))
D1=ACUS(C1)-ACUS(C2)
SUM1=SUM1+D1
IF (AAX1(C1,C2).GE.1.) G0 T0 30
IF (DPDK.LT.0.) G0 T0 25
D2=1./SQR(A1./C1**2-1.)=1./SQR(1./C2**2-1.)
SUM2=SUM2+D2

25 IF (RAD2.LF.RP2) G0 T0 31
ETA=ETAC
RAD1=RAD2

53
C
IF (TEST,LF,TESTS) STEP=STEP+2.
IF (C2,LT,1,WH) GO TO 1H
3H PH=SUM1
DRN=SIMDGCK
RTAN=ETAC
RETURN

C
20H CONTINUE
WHITE(2,141V)
141V FORMAT(' ERROR: INTEGRATION ROUTINE HAS EXCEEDED THE MAXIMUM
! NUMBER OF LOOPS; PROGRAM EXITS. !
STOP
END

54
SUBROUTINE ETA(SH, RHOR, TEMPR, ETA)

INPUT

SH = SCALF) RANGE (CM)

OUTPUT

RHor = DENSITY (UNN/ECH2)

TEMPP = TEMPERATURE (DEGREES K)

ETA = INDEX OF REFRACTION

CALCULATES RHOR AND TEMPP FROM 1 KT STANDARD, AND THEN ETA AT SCALED RANGE

INCLUDE 'GIVEN.CMN'

INCLUDE 'DENSITY.CMN'

DATA RHOR/1.225E-3/

CALL DENS(SH)

RHor=RHor*(1.+00R/RHOR)

TEMPP=MAX1(TEMPP, TEMPP*(1.+00R)/(1.+00R))**n.4)

ETA=1.+n.22+RHor*(1.+4.006/0ER/TEMPR)

RETURN

END
SUBROUTINE AIRPT (EEF, RHR, GMONE, PRES, TEMP)
C
DOAIN-NICKLE EQUATION OF STATE OF AIR (SEMI-PHYSICAL FIT)
AS EXTRACTED FROM MDAC EER H2 VERSION OF LAMH
C
INPUT
EEF = ENERG Y (EHGS/GM)
RHR = DENSITY (GM/CM3)
OUTPUT
GMONE = GAMMA = 1
PRES = PRESSURE (DYNE/CM2)
TEMP = TEMPERATURE (DEGREES K)
C
IGO=1
GO TO 1
C
THIS ENTRY RETURNS ONLY GMONE
ENTRY AIR(EEF, RHR, GMONE)
IGO=0
C
1  E=EEE*1.0F-1.0
RHO=RHR/1.204E-3
E1=(EE1*F)/.975
IF (ABS(F1) .LT. 5.0) GO TO 3
C
IF (E1 .GT. 6.0) GO TO 2
C
FON1=0
FON2=EXP(-E/6.03)
WS=0.0
GO TO 4
C
2  FON2=EXP(-E/4.4n)
FON1=0
WS=1.0
GO TO 4
C
3  NE1=.975*(RHI)E+5
FE1=RHR*1.55*42*ALUG(RHI)
F1=(EE1*F)/1.0F1
WS1/(EXP(F1)+1.0)
FON1=EXP(-F/4.4n)*WS
FON2=EXP(-E/6.03)*(1.0+WS)
C
4  NE1=0
IF (E.LE.1.4) GO TO 5
56
C
RE1AN(6.9447E-03*6.5*1.3*974E-02)*LUG(E)
F2=(E=4.8)*3
IF (AH(E2),LT,5.1) GO TO 7
C
TF (E2,GT,1.1) GO TO 0
C
5
FNY(*1)
WS=1.1
GO TO R
C
6
FNY=EXP(-6/25.5)
WS=1.1
GO TO R
C
7
F2=(E=4.5)*RH0**E*1.8
F2=(E=4.5)*RH0**E*1.5
F2=(E=4.5)*DE2
WS=1.1*EXP(E2)+1.1
FNY=EXP(-F/25.5)*WS
C
8
F2=(E=16.1)*1.1
HETB=BF(TA*(1.0=5.0)+1.5)*WS
FNY(*1)
IF (E3,GT,5.1) FE=1.5*(EXP(E3)+1.1)
GMONE=(1.0=161.0=255.0=28.0=FUNO.0=37.0=45.0=55.0)*RH0**HET
C
IF (161.0=11.0) GO TO 1
C
IF (161.0=11.0) GO TO 1
C
PRES=GMONE*EFF*PR
C
TEMPERATURE
C
RHULN=ALG(EH)
F(6.52549E-03*6.5*1.5*974E-02)*RHULN=2.71434E-03*RHULN=9.72E-1
G=923123E-7*RHULN=0.1*RHULN=9.72E-1
H=923123E-7*RHULN=9.71549E-01*RHULN=9.711E-1
CON=3.4R0.0*CMONE*E
CON2=(H=1.0)*FF
HET=(E=3.0)/M.06
C
IF (HET,GT,1.0) GO TO 9
C
IF (HET,GT,1.0) GO TO 9
C
RETURN
FND
10
RETURN
FND
57
SUBROUTINE DENSITY (T)

PROVIDES BLAST PARAMETERS AT INPUT TIME "T" BASED ON
NUCLEAR BLAST STANDARD = 1 KT, AF: L=TR=73-55, REV APRIL 1975

INITIAL CONDITIONS ASSUMED TO BE STP, I.E.
AMBIENT PRESSURE = 1,125E6 DYN/CM2
AMBIENT DENSITY = 1,225E-3 GM/CM3

OUTPUT VIA COMMON BLOCK ON INITIAL CALL AT TIME T
PRAD = SHOCK FRONT RADIUS (CM)
OPPA = SHOCK FRONT OVER-PRESSURE (DYNE/CM2)
UPRK = SHOCK FRONT OVER-DENSITY (GM/CM3)
TEMPK = SHOCK FRONT TEMPERATURE (K) = FROM EQUATION OF STATE
OUTPUT ON SUBSEQUENT CALLS AT SAME TIME VIA ENTRY DENS
OUR = OVER-DENSITY AT RADIUS R (GM/CM3)

INCLUDE 'WFRT.CHN'

DATA MAV=26.25, TPNH=1/2, JTN=977/10, TPRR=9/79,
1 AC/2,1H1/2, AN/2, QE1/2, ASTO/2, NSTO/2, 4,45E4,
2 RPM/1,225E-3,
3 MAV/329/1, C/1, INH/2, CZ/3397/10, HZ/849/10,
4 BR/1, N3497/10, CC/1, N36/10,
5 TPNH/2.3/1

IF (T.EQ.TTOLU) GO TO 9

DETERMINE WFPR2 (RADIUS)
1 IF (T.LE.TVH) GO TO 14

DETERMINE RZP
RZP=(1.9*H+611)*H2

IF (T.LT.TVH) WFPR2=2421W*ET*TPWH0*(1.9*(1.23+TVH,123)*
((-1.9=EXP(TA3+ET*TPWH2)))
IF (T.LT.TVH,1) GO TO 7

LAT=ALNG(T)
ALFT=ALT3.391/2
RFNTH=TPR3,45E4+2,15L3*ALFT*(9,E3/ALFT=0,8E3)/ALFT

IF (T.LT.TVH) RNF=RFNTH*TVH,1*WFPR2*(TVH,2+T))/TVH,18

3 IF (T.EQ.TTV) GO TO 7

PRADEK
Determining DOPK

1. \texttt{RH=1.0/VH}
\texttt{RT1=0.2, 24517F-bnH}
\texttt{CF=SUNT(4LONC+T1+0.4*EXP(-4LONC/T1+0.7))/4LONC}}
\texttt{OPPK=(IF(COMPA+RH)*SHAT/CF)*HR}

Determining DOPK

2. \texttt{OP=OPPK}
\texttt{RT1=ORP/1.0, 125Fb}
\texttt{P=OPR/1.0, 125Fb}
\texttt{G=LONF, 64}
\texttt{GAMMAN=1.4}
\texttt{GAMMA=GAMMA}

3. \texttt{DIC(1.0, 24249, 3.8, (2.0*GAMMA*(GAMMA+1.0)*R*T1))}
\texttt{EE=EE*(GAMMA+1.0)}
\texttt{CALL ATPT(KE, RH, INF, DOP, TEMPK)}
\texttt{GAMMA=GAMMA}
\texttt{GAMMA=GAMMA+(2.0*GAMMA+1.0)*R)
\texttt{IF \texttt{(ABS(GAMMA-GAMMA0)<1.0e-5), GO TO 6}}
\texttt{CONTINUE}

4. \texttt{DOPK=RH/1000}

Determining Hz

5. \texttt{RZ=0.0}
\texttt{IF \texttt{(T+L+0.265), RZ=2.0*RH, T+0.265, 395}}
\texttt{IF \texttt{(T+G+0.265), RZ=(1.0*RH)+T+0.265, CE+0.56}}
\texttt{RETURN}

6. \texttt{ENTRY DENS(HZ, R=PAD)}

Determining DOPK

7. \texttt{RPK=RPK1}
\texttt{IF \texttt{(T+L+0.2), GO TO 11}}

8. \texttt{RPK=RPK1, F=5}
\texttt{RF=1.0, F=5}
\texttt{IF \texttt{(T+P+1.0), GO TO 11}}
C
RN3ORPMk
RN1A
NaJ5U1PI
9LN
MaULPKw.
AL No q
fMNLta
~A LhoP
AFAL
NFL13aA
(0011 ILP'K
MNF
mL
*C 0
J1 0
N
4-M
1tMN)
A LPHA.a
(WNP
I
(IO'4YIIi)
252
StlP
/TNLE
ALPHA(hRNAPsiPN*RPLS/ALPHA+1.e/UPK)
FNGZ*EXP/RPLS
RNORAPhARPhNP+HFTA
RCMNLH=RMNLH*UPK/UPK)
CRMLH=ALPHAPhARPhNP*(RMNLH*UPK)
(CGZ=(META*RMNLH*RHZ)/RMNLH*/(FNGZ*CRMLH*H2*H*H*FNGZ))
C+EXP(CGZL)
CGZL=CGZL*(H*P*FNGZ)
EXP(CGZL)
FPL
E=0
E=0
IF (T > T1,1.0) GO TO 13
IF (T > T2,1.0) GO TO 12
A=1.0*E=3
C=ALG(R/0+UPK*A)/(R7=0+UPK)
A=(0+UPK*A)*EXP(.0*UPK)
C
TF (T1*E1,1.0) NDX=1
TF (T > T1,1.0) NDX=1
TF (T > T2,1.0) NT=DML*UPK*(1-e)+UPK*(T=1.25)
TTOL=1
RETURN
C
WHITE(2,1,UPK)
STOP
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