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A GOAL FOCUSING APPROACH TO ANALYSIS OF INTERGENERATIONAL TRANSFERS OF INCOME: THEORETICAL DEVELOPMENT AND PRELIMINARY RESULTS

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ABSTRACT

Existing analytical models of intergenerational transfers of income developed by economists deal with the criterion of equity of retirement systems but not with their stability. These models frequently assume a steady growth in population. However, present and projected population dynamics are characterized by very large changes in the age structure of the population. Governmental and private projection models used for analysis of government policies for social security and other age-determined transfer programs do rely on the current population projections, but they assume independence of future economic growth from the retirement system.

Moreover, none of these models are formulated in a way that makes them operational for studying the numerous dynamic trade-offs and relative possibilities inherent in income transfer programs, for example, the trade-off between taxes and subsidies, between participation rates and income for different groups over time, and the possible effects of cross-subsidization among groups of different compositions and sizes. Such information is central in striving to attain simultaneously the socially desirable goals of maintaining or improving the standards of living of both the working and retired populations and of maintaining an equitable and stable intergenerational transfer system.

This paper proposes such an operationally implementable method of analysis employing the new approach of goal focusing, derived from ideas of goal programming and efficient point analysis. A basic theoretical model is developed and illustrated by numerical example, and extensions in several directions are described. Such models will provide considerable assistance in obtaining substantive knowledge of the prospective magnitudes of the trade-offs involved and their behavior within a set of plausible trends over the next ten to thirty years.
Introduction

The policy decision which will have fundamental implications for the economic and social stability of the United States well into the next century concerns the balancing of three desirable social goals:

1. Maintaining or improving the standards of living of the working population;
2. Maintaining or improving the standards of living of the retired population; and
3. Maintaining a stable inter-generational transfer system which permits members of successive generations to be supported in retirement by their successors in the workforce.

Standards of living are measured over time by the per capita disposable income for the respective groups. The principal indicator for the functioning of the inter-generational transfer system is the maintenance of a non-negative rate of return between the real payments made to the transfer system in the aggregate by the members of each generation and the payments received by them later.

Historically, transfer payments, and in particular transfer payments to the aged, have been the most rapidly growing component of the government budget expenditures and personal income receipts. In the federal budget, these transfer payments include pensions of government employees, military pensions and the social security programs including disability and medicare benefits. The largest and the most fundamental component of government expenditure, and the one with the most universal ramifications, is the social security program.

The importance of social security as a basic program for income support in the United States is likely to increase rather than to decline in the
future, because the real value of private pension funds has been eroded by slowdown in the economy and by inflation. Also, many companies find themselves unable to expand or even to continue their pension programs and some even may be unable to cover the unfunded pension liabilities from their future profits.

The problem of transfer payments and especially of social security benefits is that the current and anticipated growth rates in the economy do not permit funding of increases in transfer payments at the rate at which they have been growing in the past. It may not even be feasible to provide the future retirees the combination of early retirement age and increases in entitlements which have been built into the existing retirement program without reducing the real disposable income of active workers at the same time. The expected economic growth rates would certainly not support the introduction of new benefit programs, comparable to the past additions of the disability and medicare provisions or automatic cost-of-living adjustments, without imposing large strain on the disposable income of the working population. The need to balance the future incomes of workers and of retirees is the prime policy need. However, the conditions for achieving this balance are not well understood.

Trade-offs between the standards of living of the two populations at a point in time could be established from data on the levels of transfers of real purchasing power from workers to retirees, the size of the two populations and their average disposable incomes. However, such data are not available at present and can only be approximated with some effort. Moreover, the dynamic trade-offs among these goals are much more important than trade-offs at a point in time. They are much more difficult to understand because they are embedded in the processes of economic growth, and they depend on
individual choices between work and leisure which presumably depend on the opportunities for both available to persons at different ages. They involve the issue of preservation of the inter-generational transfer system and require a three-way analysis among these goals. These dynamic trade-offs have not been previously analyzed.

The social significance of this topic is heightened by recent demographic changes. The decline in the birth rate over the past ten years in the United States, which also occurred in other industrialized nations, has changed the population age structure in a way which may raise problems of social and economic stability. Specifically, as the proportion of families with children has declined, participation by women in the labor force has increased rapidly at the same time as the large post-World War II age cohort has been entering the labor force. The resulting growth of the labor force permitted continuation of economic growth in spite of diminished growth in productivity. A relatively high growth in the labor force and employment may continue into the next decade. But, this growth will continue to slow down and, barring massive immigration of workers to the United States, may virtually stop at the end of the century as the bulge of the population will approach retirement age (see Table 1).

The more immediate and pressing problem, however, is the imbalance in the growth rates in disposable real income of the two groups. The average growth rate in real transfer incomes has tended to exceed the growth in earnings since 1960, and in the 1970's the growth in real earnings of the working population even before taxes has been at a virtual standstill while the growth in the average transfer payment has accelerated (see Table 2).

As the number of the retirees and other beneficiaries of the public transfer system grows, so does the burden of taxes placed on the working
### TABLE 1
SELECTION INDICATORS OF CHANGES IN THE AGE STRUCTURE OF POPULATION UNITED STATES, ACTUAL AND PROJECTED, 1960–1995

<table>
<thead>
<tr>
<th>Period</th>
<th>Persons Age Indicated Annual Average, Thousands</th>
<th>Average Annual Net Change in the Labor Force</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>17</td>
<td>30</td>
</tr>
<tr>
<td>1960–64</td>
<td>2,992</td>
<td>2,256</td>
</tr>
<tr>
<td>1965–69</td>
<td>3,577</td>
<td>2,337</td>
</tr>
<tr>
<td>1970–74</td>
<td>4,014</td>
<td>2,812</td>
</tr>
<tr>
<td>1975–79</td>
<td>4,213</td>
<td>3,413</td>
</tr>
<tr>
<td>1980–84</td>
<td>3,865</td>
<td>3,910</td>
</tr>
<tr>
<td>1985–89</td>
<td>3,555</td>
<td>4,322</td>
</tr>
<tr>
<td>1990–94</td>
<td>3,199</td>
<td>4,337</td>
</tr>
</tbody>
</table>

p. = preliminary NPA projections
n.a. = not available

TABLE 2

SELECTED INDICATORS OF REAL INCOME AND OF CHANGES IN REAL INCOME OF RETIRED AND WORKING POPULATIONS, UNITED STATES, 1940-1976

<table>
<thead>
<tr>
<th>Period</th>
<th>Federal Pension Beneficiaries</th>
<th>State and Local Pension Beneficiaries</th>
<th>Social Security OASDI Beneficiaries</th>
<th>Average Gross Weekly Earnings of Production Workers</th>
<th>Pre-tax Earnings Per Person Employed</th>
</tr>
</thead>
<tbody>
<tr>
<td>1940</td>
<td>$4,101</td>
<td>$2,669</td>
<td>$—</td>
<td>$—</td>
<td>$4,721</td>
</tr>
<tr>
<td>1950</td>
<td>2,857</td>
<td>1,915</td>
<td>817</td>
<td>92.33</td>
<td>5,952</td>
</tr>
<tr>
<td>1960</td>
<td>2,605</td>
<td>2,158</td>
<td>1,102</td>
<td>113.96</td>
<td>7,673</td>
</tr>
<tr>
<td>1970</td>
<td>3,715</td>
<td>2,797</td>
<td>1,316</td>
<td>129.10</td>
<td>9,262</td>
</tr>
<tr>
<td>1976</td>
<td>4,755</td>
<td>3,142</td>
<td>1,742</td>
<td>128.93</td>
<td>9,596</td>
</tr>
</tbody>
</table>

A. Real Income per Recipient in 1972 Dollars

B. Annual Rates of Change in Real Income (percent)

<p>| | | | | | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>1940-50</td>
<td>-3.7%</td>
<td>3.4%</td>
<td>-</td>
<td>-</td>
<td>2.3%</td>
</tr>
<tr>
<td>1950-60</td>
<td>-0.9</td>
<td>1.2</td>
<td>3.0%</td>
<td>2.1%</td>
<td>2.6</td>
</tr>
<tr>
<td>1960-70</td>
<td>3.6</td>
<td>2.4</td>
<td>1.8</td>
<td>1.3</td>
<td>1.9</td>
</tr>
<tr>
<td>1970-76</td>
<td>4.2</td>
<td>2.3</td>
<td>4.7</td>
<td>0.0</td>
<td>0.6</td>
</tr>
</tbody>
</table>

1Includes Federal government civilian and military pensions.
2Total private nonagricultural.
3Total earnings in the economy and income of proprietors as given in the national income accounts deflated by the consumer price index and divided by total employment including armed forces.

population (and factors of production generally) quite apart from the growth in the average transfer benefit received. As a result of these economic and demographic trends, barring an unprecedented and improbable upsurge in productivity, the issue of trade-offs between the incomes of the working population and incomes of the retired population, which has already been felt politically, is likely to become progressively more acute. However, no substantive knowledge exists regarding the prospective magnitudes of these trade-offs or their behavior within a given set of plausible trends over the next 10 to 30 years.

This paper seeks to address this deficiency by developing a systematic and operationally implementable method for the analysis of inter-generational transfers of income. To accomplish this we will employ an approach referred to as "goal focusing," a new approach to assessing complex relationships and problems.

The ideas of goal focusing derive in part from those of goal programming from management science and operations research [2]. However, the usual goal programming approach is here joined to the concepts of efficient point analyses and optimality as formulated in economics [3]. That is, in goal focusing one seeks the closest "efficient point" instead of only the "closest point" to the specified goals.

In doing so, it also supplies a variety of other conveniences. Its linear programming equivalents afford computational efficiency and an easy access to sensitivity analyses, etc. The approach also extends to constraint adjunc-tions, other kinds of model alterations, and other such devices for exploration that may appear to be appropriate.
If desired, these goal focus ideas can be extended to goal intervals when ranges of possibilities are to be considered rather than specific levels or targets [2].

In the following sections we present our basic theoretical development and the results of an illustrative example. Extensions and modifications of our model, already underway, are discussed in a concluding section.

The Basic Model

Consider an economy in which the population is grouped demographically by, for example, age, income, etc., and in which tax payments are made and tax subsidies received by these groups. Let

- \( p(t) \) = vector of demographic (population) groups in period \( t \),
- \( a(t) \) = vector of tax-paying groups in period \( t \),
- \( b(t) \) = vector of tax subsidy-receiving groups in period \( t \), and
- \( R(t) \) = matrix of participation rates in period \( t \).

We then have the relations

1. \( a(t) = R(t) p(t) \) = vector of economically active population groups in period \( t \), and
2. \( b(t) = (I - R(t)) p(t) \) = vector of economically inactive population groups in period \( t \), where \( I \) represents the identity matrix.

Further, let \( x_i(t) \) be the "tax" per person paid by the economically active members of the \( i \)th tax-paying group in period \( t \) and let \( y_j(t) \) be the "subsidy" given to the economically inactive members of the \( j \)th subsidy-receiving group in period \( t \). Then over a specified horizon \( t = 1, \ldots, T_0 \), we might require that "collections" be at least equal to "payments" for each time period as in, for example, the social security system. That is, we require
\[
\sum_{i} x_i(t)s_i(t) > \sum_{j} y_j(t)b_j(t), \quad t = 1, \ldots, T_0
\]

or in matrix notation

(3) \[ x^T(t)a(t) > y^T(t)b(t), \quad t = 1, \ldots, T_0 \]

Substituting relationship (1) and (2) in (3), we obtain

\[ x^T(t)R(t)p(t) > y^T(t)(I-R(t))p(t), \quad t = 1, \ldots, T_0 \]

or equivalently

(4) \[ (x^T(t) + y^T(t))R(t)p(t) > y^T(t)p(t), \quad t = 1, \ldots, T_0 \]

which defines a convex set in the non-negative variables \( x_i(t), y_j(t) \) and \( r_{ij}(t) \) (the elements of \( R \)).

If \( p(t) \) were unknown, we might assume that the vector of demographic groups in any period is obtained from the corresponding vector in the previous period by means of a known transition matrix \( M \) invariant over time. Thus

\[ p(t) = Mp(t-1) = \cdots = M^Tp(0). \]

An example of such a matrix is given by the cohort survival matrix (see [5], [7], and [9]).

\[
M = \begin{bmatrix}
  b_1 & b_2 & b_3 & \cdots & b_{n-1} & b_n \\
  s_1 & s_2 & s_3 & & & \\
  & & & & \ddots & \\
  & & & & & & s_{n-1}
\end{bmatrix}
\]

where \( b_i \) and \( s_i \) are age specific birth and survival rates, respectively. In any given period \( t \), \( b_i \) denotes the rate at which group \( i \) produces children who
will then become members of the first group in period t+1; and \( s_i \) denotes the percentage of group \( i \) that survives to become members of group \( i+1 \) in period t+1. Note that here we do not allow the members of any group to remain in that group from one period to the next.

In this case, projecting from time \( t=0 \) with the transition matrix \( M^t = \cdots M \) (t times), we can rewrite (4) as

\[
(4') \quad (x(t) + y(t))R(t)M^p(0) > y(t)M^p(0), \quad t = 1, \ldots, T_0
\]

As an indication of the further conditions which should be imposed on the vectors \( x(t) \) and \( y(t) \), we might require that the subsidies be at least equal to (and the taxes no greater than) some specified amounts,

\[
(5) \quad y(t) > y(t) \text{ and } x(t) < x(t), \quad t = 1, \ldots, T_0.
\]

In addition, we might require that the participation rates be kept within prescribed bounds,

\[
(6) \quad R(t) < R(t) \text{ and } R(t) > R(t), \quad t=1, \ldots, T_0.
\]

Equity might require that a cohort over its life cycle should receive benefits that are at least equal to the payments made by this cohort into the system. In order to illustrate how this kind of principle might be incorporated we choose a horizon that contains one whole cohort life cycle. We shall then impose this equity condition as a goal for a cohort that completes such a life cycle within the horizon.

Since upper bounds on payments to society as a whole are already provided by (3), our goal for the life cycle cohort will be to maximize its total lifetime benefits.
The expression (7) represents the life cycle benefits for the cohort starting at τ=1. (Equity considerations can be developed for other cohorts by similar expressions but we shall not undertake this here.)

To determine our R(t) values endogenously we can begin by postulating an aggregate, economy-wide scenario of, say, GNP growth over time, which is exogenous to our system. This is done only for simplicity since explicit treatment of the dynamic interactions between participation rates and GNP and between tax rates, investment and GNP would unduly complicate the development we are undertaking here. With current estimates of productivity and population demographics we can project "reasonable" target levels for the R(t) values. We can then provide a clear picture of a simplified use of the mechanism of goal focusing as part of an aggregative analytical method for obtaining participation rates corresponding to the time dependent scenarios we wish to investigate.

A reasonable objective for our choice of tax rates might be to minimize the maximum tax rate over the horizon. In addition we have the goal of maximization of the life cycle benefits to the cohort starting at τ=1 plus the

1The pay-as-you-go conditions (4'), will ensure an upper bound to this maximum.
goals associated with goal focusing determination of the participation rates. For simplicity we shall denote the latter by $G(t, x(t), y(t))$. Thus, through the goal focusing mechanism, the $R(t)$ will be determined in an endogenous manner that is dependent on the $x(t)$ and $y(t)$ as determined from, say, regression estimates.

We now proceed to formulate our goal focusing model in the following time dependent fashion. Let

$$G(t, x(t), y(t)) = \text{matrix of "goal" participation rates in period } t$$

$$G^+(t), G^-(t) = \text{matrices of positive and negative deviations, respectively, from } G(t, x(t), y(t)).$$

$$W^+(t), W^-(t) = \text{matrices of weights associated with } G^+(t) \text{ and } G^-(t),$$

respectively.

Then our goal focusing model can be written as

$$\text{(8) } \min \lambda - W_0 \sum_{t=1}^{T} y^T(t)(I-R(t))^{t-1} e_1 . e_1^T \rho(0) + \sum_{t=1}^{T_0} e^T W^+(t) G^+(t) e +$$

$$+ \sum_{t=1}^{T_0} e^T W^-(t) G^-(t) e$$

subject to

$$\lambda e^T - x^T(t) > 0, \quad t=1, \ldots, T_0$$

$$y(t) > y(t), \quad t=1, \ldots, T_0$$

$$-x(t) > -x(t), \quad t=1, \ldots, T_0$$

$$-R(t) > -R(t), \quad t=1, \ldots, T_0$$

$$R(t) > R(t), \quad t=1, \ldots, T_0$$

$$x^T(t) + y^T(t) R(t) p(t) - y^T(t) p(t) > 0, \quad t=1, \ldots, T_0$$

$$R(t) - G^+(t) + G^-(t) = G(t, x(t), y(t)), \quad t=1, \ldots, T_0$$

$$x(t), y(t), R(t), G^+(t), G^-(t) > 0, \quad t=1, \ldots, T_0$$
Before proceeding to an illustrative numerical example we show that we can linearize the above nonlinear programming problem by making the following approximations.

We first introduce the new variable $z(t)$ defined by

\[(9) \quad z(t) \equiv x(t) + y(t)\]

and rewrite (4) as

\[(10) \quad z^T(t)R(t)p(t) - y^T(t)p(t) > 0, \quad t=1, \ldots, T_0.\]

We can then approximate the nonlinear inequality (10) by

\[(11) \quad z^*(t)R^*(t)p(t) + [(z^T(t) - z^*(t))R^*(t) + z^*(t)(R(t) - R^*(t))]p(t) - y^T(t)p(t) > 0, \quad t=1, \ldots, T_0\]

where $z^*(t)$ and $R^*(t)$ are suitably chosen reference points for $z(t)$ and $R(t)$. Inequality (11) then simplifies to

\[(11') \quad z^T(t)R^*(t)p(t) + [z^*(t)R(t)p(t) - y^T(t)p(t) > z^*(t)R^*(t)p(t)], \quad t=1, \ldots, T_0.\]

By a similar approximation our objective function in (8) may be linearized to

\[
\text{Min} \lambda - W_0 \sum_{t=1}^{T} [y^T(t)(I - R^*(t)) - y^*(t)]R(t) + y^*(t)R^*(t)] \eta e_1. e_1 \text{Mp}(0)
\]

\[+ \sum_{t=1}^{T_0} e^{T+t}(t)G^+(t)e + \sum_{t=1}^{T_0} e^{T-t}(t)G^-(t)e.\]
Finally, we take the "goal" participation rates \( G(t, x(t), y(t)) \) to be linear in \( x(t) \) and \( y(t) \) with variations around \( R(0) \), the participation rates at time \( t = 0 \). Thus we have

\[
(12) \quad e^T G(t, x(t), y(t)) = e^T R(0) - x^T(t) A - y^T(t) B.
\]

Substituting (9), (11') and (12) in (8), a linear approximation to our goal focusing model can then be written as

\[
(13) \quad \min \lambda - W_0 \sum_{t=1}^{T} \left[ y^T(t)(I - R^*(t)) - y^*(t) R(t) + y^*(t) R^*(t) \right] + e^T \sum_{t=1}^{T} e^T(t) e
\]

subject to

\[
\begin{align*}
\lambda e^T & - x^T(t) > 0, & t = 1, \ldots, T_0 \\
y(t) & > y^*(t), & t = 1, \ldots, T_0 \\
-x(t) & > -x^*(t), & t = 1, \ldots, T_0 \\
-R(t) & > -R^*(t), & t = 1, \ldots, T_0 \\
R(t) & > R^*(t), & t = 1, \ldots, T_0 \\
x(t) + y(t) & = 0, & t = 1, \ldots, T_0 \\
x^T(t) + y^T(t) & = 0, & t = 1, \ldots, T_0 \\
-x^T(t) A - y^T(t) B + e^T R(t) & = e^T R(0), & t = 1, \ldots, T_0 \\
x(t), y(t), z(t), R(t), G^+(t), G^-(t) & > 0, & t = 1, \ldots, T_0
\end{align*}
\]

**Illustrative Example**

To illustrate our procedure consider an economy in which the population is grouped by age into four categories as shown below.
<table>
<thead>
<tr>
<th>Group</th>
<th>Age Yrs.</th>
<th>Population p(0)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0-19</td>
<td>70.5 m.</td>
</tr>
<tr>
<td>2</td>
<td>20-39</td>
<td>70.0</td>
</tr>
<tr>
<td>3</td>
<td>40-59</td>
<td>46.1</td>
</tr>
<tr>
<td>4</td>
<td>60+</td>
<td>35.5</td>
</tr>
</tbody>
</table>

We assume that births occur only within the second group, i.e., among those between 20 and 39 years of age. The yearly birth rate for that group is taken to be 3 million per 70 million or 0.043, so that the birth rate is 0.86 over the twenty year period.

The matrix of cohort survival rates is given below and is to be interpreted in the following way. In any period t, the entry in row 1 column 2 is the birth rate for the second group, all other birth rates are zero; while the entry in row 3 column 2 indicates that 0.97 of the population of group 2 will survive to become the members of group 3 in the subsequent period. For completeness we assign a survival rate of e (the entry in row 4 column 4) to the oldest age group but assume it is insignificant for our analysis here.

**Cohort Survival Matrix**

<table>
<thead>
<tr>
<th>From Group</th>
<th>To Group</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0</td>
<td>0.86</td>
<td>0</td>
<td>0</td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>0.97</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td></td>
</tr>
<tr>
<td>3</td>
<td>0</td>
<td>0.97</td>
<td>0</td>
<td>0</td>
<td></td>
</tr>
<tr>
<td>4</td>
<td>0</td>
<td>0</td>
<td>0.87</td>
<td>e</td>
<td></td>
</tr>
</tbody>
</table>

We shall take our time horizon to be four periods each of twenty years duration so that we may follow the youngest cohort over its entire life cycle.
Participation rates at time $t = 0$ are given below.

\[
\begin{array}{cccc}
1 & 2 & 3 & 4 \\
1 & 0.14 & 0 & 0 & 0 \\
2 & 0 & 0.75 & 0 & 0 \\
3 & 0 & 0 & 0.70 & 0 \\
4 & 0 & 0 & 0 & 0.25 \\
\end{array}
\]

We assume that the participation rate for any group is influenced by the levels of tax and subsidy for that group but we here exclude any possible cross-group effects of taxes and subsidies, so that the matrices $A$ and $B$ of (12) and (13) have the particular (diagonal) forms

\[
A = \begin{bmatrix}
    a_{11} & 0 & 0 & 0 \\
    0 & a_{22} & 0 & 0 \\
    0 & 0 & a_{33} & 0 \\
    0 & 0 & 0 & a_{44}
\end{bmatrix}, \quad
B = \begin{bmatrix}
    b_{11} & 0 & 0 & 0 \\
    0 & b_{22} & 0 & 0 \\
    0 & 0 & b_{33} & 0 \\
    0 & 0 & 0 & b_{44}
\end{bmatrix}
\]

We estimate that

\[
\begin{align*}
    a_{11} &= 0.02 \times 10^{-6}; & b_{11} &= 0.465 \times 10^{-6} \\
    a_{22} &= 0.115 \times 10^{-6}; & b_{22} &= 1.12 \times 10^{-6} \\
    a_{33} &= 0.67 \times 10^{-6}; & b_{33} &= 2.7 \times 10^{-6} \\
    a_{44} &= 3.9 \times 10^{-6}; & b_{44} &= 6.5 \times 10^{-6}
\end{align*}
\]

The model was run with different sets of values for the bounds on taxes, subsidies and participation rates, and for the objective function weights.
where, for simplicity, the values were held constant over time. For illustrative purposes, we report the results of the (typical) case shown below.

Participation Rates

<table>
<thead>
<tr>
<th></th>
<th>Lower Bound</th>
<th>Upper Bound</th>
</tr>
</thead>
<tbody>
<tr>
<td>$R_L(t)$</td>
<td>0 0 0 0</td>
<td>0.2 0 0 0</td>
</tr>
<tr>
<td>$R_U(t)$</td>
<td>0.6 0 0</td>
<td>0 0.8 0 0</td>
</tr>
</tbody>
</table>

Minimum levels for subsidies were taken to be the same for all groups. We assumed a value of $5,200 per year which is equivalent to $0.104m. over a twenty year period.

Lower Bounds on Subsidies $m$

$$y(t) = \begin{bmatrix} 0.104 \\ 0.104 \\ 0.104 \\ 0.104 \end{bmatrix} , \text{Vt.}$$

Maximum tax levels per year and their equivalents over a twenty year period were taken to be as follows:

- group 1: $4,000/yr.$
- group 2: 6,666
- group 3: 10,000
- group 4: 5,000
Upper Bounds on Taxes $m.$

\[
\begin{bmatrix}
0.08 \\
0.13332 \\
0.2 \\
0.1 \\
\end{bmatrix}, V_t.
\]

The weights \( W^+(t) \) and \( W^-(t) \) were taken to be unity for all groups in all time periods, while \( W_0 \), the weight on maximizing life time benefits for the single cohort, was taken to be 0.1.

**Results**

The results for our illustrative example are given below, but we note here the following observations. As required, total tax collections exceed subsidy payments; however, the differences range between $1.2b and $48.4b annually. We expect appropriate selection of time dependent weights and relative weights would bring collections and payments closer in line.

As expected, lifetime benefits to our single cohort greatly exceed tax receipts from the cohort, by a ratio of almost 2:1 in our example. This may occur because our model allows each economically inactive member of society to receive a subsidy, which is not the reality for the youngest group. Based on our results, if we were to remove the subsidy to our illustrative cohort in period one, the taxes paid by that cohort over its life cycle would exceed the benefits it receives. However, if we employ this procedure for the youngest group in each period, the gap between collections and payments for society as a whole widens considerably. We expect such discrepancies would be diminished significantly by including in the objective function life-time-benefits for each cohort rather than for only our illustrative one. This, of course, is the appropriate situation for any real application, and also provides valuable
## Illustrative Example Results

**Group 1**

<table>
<thead>
<tr>
<th></th>
<th>Period 1</th>
<th>Period 2</th>
<th>Period 3</th>
<th>Period 4</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Population M</strong></td>
<td>60.200000</td>
<td>58.811100</td>
<td>50.218840</td>
<td>49.060220</td>
</tr>
<tr>
<td><strong>Participation Rate</strong></td>
<td>0.049871</td>
<td>0.089520</td>
<td>0.089520</td>
<td>0.089520</td>
</tr>
<tr>
<td><strong>Lower Bound</strong></td>
<td>0.000000</td>
<td>0.000000</td>
<td>0.000000</td>
<td>0.000000</td>
</tr>
<tr>
<td><strong>Upper Bound</strong></td>
<td>0.200000</td>
<td>0.200000</td>
<td>0.200000</td>
<td>0.200000</td>
</tr>
<tr>
<td><strong>Participants M</strong></td>
<td>3.002234</td>
<td>5.264770</td>
<td>4.495591</td>
<td>4.391871</td>
</tr>
<tr>
<td><strong>Tax/Year $</strong></td>
<td>4000</td>
<td>4000</td>
<td>4000</td>
<td>4000</td>
</tr>
<tr>
<td><strong>Collections/Year $B</strong></td>
<td>12.008937</td>
<td>21.059079</td>
<td>17.982362</td>
<td>17.567483</td>
</tr>
<tr>
<td><strong>Collections/Period $T</strong></td>
<td>0.240179</td>
<td>0.421182</td>
<td>0.359647</td>
<td>0.351350</td>
</tr>
<tr>
<td><strong>Non Participants M</strong></td>
<td>57.197766</td>
<td>53.546330</td>
<td>45.723249</td>
<td>44.668349</td>
</tr>
<tr>
<td><strong>Subsidy/Year $</strong></td>
<td>9418</td>
<td>9418</td>
<td>9418</td>
<td>9418</td>
</tr>
<tr>
<td><strong>Lower Bound</strong></td>
<td>5200</td>
<td>5200</td>
<td>5200</td>
<td>5200</td>
</tr>
<tr>
<td><strong>Payments/Year $B</strong></td>
<td>538.688558</td>
<td>278.440918</td>
<td>237.760897</td>
<td>232.275414</td>
</tr>
<tr>
<td><strong>Payments/Period $T</strong></td>
<td>10.773771</td>
<td>5.568818</td>
<td>4.755218</td>
<td>4.645508</td>
</tr>
</tbody>
</table>

**Group 2**

<table>
<thead>
<tr>
<th></th>
<th>Period 1</th>
<th>Period 2</th>
<th>Period 3</th>
<th>Period 4</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Population M</strong></td>
<td>68.385000</td>
<td>58.394000</td>
<td>57.046767</td>
<td>48.712275</td>
</tr>
<tr>
<td><strong>Participation Rate</strong></td>
<td>0.800000</td>
<td>0.617522</td>
<td>0.800000</td>
<td>0.670586</td>
</tr>
<tr>
<td><strong>Lower Bound</strong></td>
<td>0.600000</td>
<td>0.600000</td>
<td>0.600000</td>
<td>0.600000</td>
</tr>
<tr>
<td><strong>Upper Bound</strong></td>
<td>0.800000</td>
<td>0.800000</td>
<td>0.800000</td>
<td>0.800000</td>
</tr>
<tr>
<td><strong>Participants M</strong></td>
<td>54.708000</td>
<td>36.059580</td>
<td>45.637414</td>
<td>32.665770</td>
</tr>
<tr>
<td><strong>Tax/Year $</strong></td>
<td>6666</td>
<td>6666</td>
<td>6666</td>
<td>6666</td>
</tr>
<tr>
<td><strong>Upper Bound</strong></td>
<td>6666</td>
<td>6666</td>
<td>6666</td>
<td>6666</td>
</tr>
<tr>
<td><strong>Collections/Year $B</strong></td>
<td>364.683528</td>
<td>240.373158</td>
<td>304.218999</td>
<td>217.570020</td>
</tr>
<tr>
<td><strong>Collections/Period $T</strong></td>
<td>7.293671</td>
<td>4.807463</td>
<td>6.084380</td>
<td>4.355000</td>
</tr>
<tr>
<td><strong>Non Participants M</strong></td>
<td>13.677000</td>
<td>22.334420</td>
<td>11.409353</td>
<td>16.046505</td>
</tr>
<tr>
<td><strong>Subsidy/Year $</strong></td>
<td>5200</td>
<td>5200</td>
<td>5200</td>
<td>5200</td>
</tr>
<tr>
<td><strong>Lower Bound</strong></td>
<td>5200</td>
<td>5200</td>
<td>5200</td>
<td>5200</td>
</tr>
<tr>
<td><strong>Payments/Year $B</strong></td>
<td>71.120400</td>
<td>116.138986</td>
<td>59.328638</td>
<td>83.441828</td>
</tr>
<tr>
<td><strong>Payments/Period $T</strong></td>
<td>1.422408</td>
<td>2.322780</td>
<td>1.186573</td>
<td>1.668837</td>
</tr>
</tbody>
</table>
### Illustrative Example Results

<table>
<thead>
<tr>
<th>Group 3</th>
<th>Period 1</th>
<th>Period 2</th>
<th>Period 3</th>
<th>Period 4</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Population</strong> M</td>
<td>67.900000</td>
<td>66.333450</td>
<td>56.642180</td>
<td>55.335364</td>
</tr>
<tr>
<td><strong>Participation Rate</strong></td>
<td>.800000</td>
<td>.600000</td>
<td>.600000</td>
<td>.600000</td>
</tr>
<tr>
<td><strong>Lower Bound</strong></td>
<td>.600000</td>
<td>.600000</td>
<td>.600000</td>
<td>.600000</td>
</tr>
<tr>
<td><strong>Upper Bound</strong></td>
<td>.800000</td>
<td>.800000</td>
<td>.800000</td>
<td>.800000</td>
</tr>
<tr>
<td><strong>Participants</strong> M</td>
<td>54.320000</td>
<td>53.066760</td>
<td>33.985308</td>
<td>44.268291</td>
</tr>
<tr>
<td><strong>Tax/Year</strong> $</td>
<td>.10000</td>
<td>10000</td>
<td>10000</td>
<td>10000</td>
</tr>
<tr>
<td><strong>Upper Bound</strong></td>
<td>10000</td>
<td>10000</td>
<td>10000</td>
<td>10000</td>
</tr>
<tr>
<td><strong>Collections/Year</strong> $B</td>
<td>543.200000</td>
<td>530.667600</td>
<td>339.853080</td>
<td>442.682912</td>
</tr>
<tr>
<td><strong>Collections/Period</strong> $T</td>
<td>10.864000</td>
<td>10.613352</td>
<td>6.797062</td>
<td>8.853658</td>
</tr>
<tr>
<td><strong>Non Participants</strong> M</td>
<td>13.580000</td>
<td>13.266690</td>
<td>22.656872</td>
<td>11.067073</td>
</tr>
<tr>
<td><strong>Subsidy/Year</strong> $</td>
<td>5200</td>
<td>5200</td>
<td>5200</td>
<td>5200</td>
</tr>
<tr>
<td><strong>Lower Bound</strong></td>
<td>5200</td>
<td>5200</td>
<td>5200</td>
<td>5200</td>
</tr>
<tr>
<td><strong>Payments/Year</strong> $B</td>
<td>70.616000</td>
<td>68.986788</td>
<td>117.815734</td>
<td>57.548779</td>
</tr>
<tr>
<td><strong>Payments/Period</strong> $T</td>
<td>1.412320</td>
<td>1.379736</td>
<td>2.356315</td>
<td>1.159076</td>
</tr>
</tbody>
</table>

| Group 4 | |
|---------|---------|---------|---------|---------|
| **Population** M | 40.107000 | 59.073000 | 57.710101 | 49.278697 |
| **Participation Rate** | 0.000000 | 0.059948 | 0.180504 | 0.000000 |
| **Lower Bound** | 0.000000 | 0.000000 | 0.000000 | 0.000000 |
| **Upper Bound** | .400000 | .400000 | .400000 | .400000 |
| **Participants** M | 0.000000 | 3.541308 | 10.416904 | 0.000000 |
| **Tax/Year** $ | 0 | 0 | 0 | 0 |
| **Upper Bound** $ | 5000 | 5000 | 5000 | 5000 |
| **Collections/Year** $B | 0.000000 | 0.000000 | 0.000000 | 0.000000 |
| **Collections/Period** $T | 0.000000 | 0.000000 | 0.000000 | 0.000000 |
| **Non Participants** M | 40.107000 | 55.531692 | 47.293197 | 49.278697 |
| **Subsidy/Year** $ | 5200 | 5200 | 5200 | 5200 |
| **Lower Bound** $ | 5200 | 5200 | 5200 | 5200 |
| **Payments/Year** $B | 208.556400 | 288.764797 | 245.924626 | 256.249222 |
| **Payments/Period** $T | 4.171128 | 5.775296 | 4.918493 | 5.124984 |
| **Total Collections/Year** $T | .919892 | .792100 | .662054 | .678000 |
| **Total Payments/Year** $T | .888981 | .752331 | .660830 | .629515 |
| **Total Collections/Period** $T | 18.397849 | 15.841997 | 13.241089 | 13.560008 |
| **Total Payments/Period** $T | 17.779627 | 15.046630 | 13.216598 | 12.590305 |
| **Total Cohort Collections** $T | 61.040943 | 58.633160 | 58.633160 | 58.633160 |
| **Total Cohort Payments** $T | 11.844703 | 20.577850 | 20.577850 | 20.577850 |
insight into the possible effects of cross-subsidization, between groups and across time.

The preceding comments also go far in explaining the following results.

With one exception, that of group one in the first period, subsidies were consistently at their lower bounds; on the other hand, with the exception of the oldest group in each period where taxes were zero, taxes were at their upper bounds. Participation rates for the youngest and oldest groups were consistently close to their lower bounds of zero, while for the middle two groups the rates either were at their upper bounds or close to their lower bounds depending on the period considered.

The results from our illustrative example provide some preliminary indications of the scope of the model and the kinds of insights to be obtained. Clearly, the experience gained here suggests further developments, along the lines outlined in the next section. Subsequently, more realistic applications can be made.

Extensions and Modifications

In current work, the basic model is being extended and modified in several directions and more comprehensive applications are being analyzed.

The procedure for determining the vector of demographic groups in any period will be refined. This will involve elaboration of the transition matrix M. In particular, we can allow for members of any group to remain in that group from one period to the next, although this is contingent on how narrowly we define a group and a time period for any given application.

Explicit treatment of the dynamic interactions between participation rates and GNP and between tax rates, investment and GNP need be made. That is, the model should incorporate the effects of economic growth on the retirement system.
There exist two negative connections between the levels of transfer payments and economic growth. One is that the taxes raised to pay for the transfer payments may reduce the investment in fixed capital by more than the expenditures made by the benefit recipients contribute to stimulate investment through growth in the demand for output. The second channel by which transfer payments adversely affect economic growth is through their impact on labor force participation of older population groups. Thus, since participation rates are determined endogenously in our model through the goal focusing mechanism, we need improved estimates of the relationship between participation rates and the levels of taxes and subsidies. In this context, we might require that taxes and subsidies for certain groups stand in particular relationship to those for other groups; and that tax and subsidy levels be allowed to increase only at certain minimum and maximum rates over time.

Alternative criteria of stability can be explored rather than simply requiring "collections" to at least equal "payments" in each time period. We would require this to be so over a given time horizon, but there is scope to relax this condition for particular time periods and analyze the trade-off involved.

Similar analyses are possible with alternative criteria of equity. The present model requires that a cohort over its life cycle should receive benefits that are at least equal to the payments made by this cohort into the system. A more comprehensive example may extend this requirement to all groups, but the general approach also allows for modification of this condition for particular groups over specified time horizons; that is, it affords analysis of the trade-off involved in cross-subsidization. These equity considerations are built in as part of the goal functional along with...
reasonable objectives such as minimizing the maximum tax rate or maximizing the minimum subsidy rate over the time horizon.

Summary

This paper has proposed an operationally implementable method for the analysis of intergenerational transfers of income employing the new approach of goal focusing. This development improves on existing models through its capability for analysis of the numerous trade-offs and relative possibilities inherent in an income transfer program. Its linear programming equivalents afford computational efficiency and easy access to sensitivity analyses.

A basic theoretical model has been presented and an illustrative example has demonstrated the general procedure involved and the kinds of insights to be obtained. Extensions in several directions, some currently underway, have been described.

Such models will provide considerable assistance in obtaining substantive knowledge of the prospective magnitudes of the trade-off involved and their behavior within a set of plausible trends over the next ten to thirty years. Such information is central in striving to attain simultaneously the socially desirable goals of maintaining or improving the standards of living of both the working and retired populations and of maintaining an equitable and stable intergenerational transfer system.
REFERENCES


A Goal Focusing Approach to Analysis of Intergenerational Transfers of Income: Theoretical Development and Preliminary Results

This report has been approved for public release and sale; its distribution is unlimited.

Existing analytical models of intergenerational transfers of income developed by economists deal with the criterion of equity of retirement systems but not with their stability. These models frequently assume a steady growth in population. However, present and projected population dynamics are characterized by very large changes in the age structure of the population. Governmental and private projection models used for analysis of government policies for social security and other age-determined transfer programs do rely on the cur-
rent population projections, but they assume independence of future economic growth from the retirement system.

Moreover, none of these models are formulated in a way that makes them operational for studying the numerous dynamic trade-offs and relative possibilities inherent in income transfer programs; for example, the trade-off between taxes and subsidies, between participation rates and income for different groups over time, and the possible effects of cross-subsidization among groups of different compositions and sizes. Such information is central in striving to attain simultaneously the socially desirable goals of maintaining or improving the standards of living of both the working and retired populations and of maintaining an equitable and stable intergenerational transfer system.

This paper proposes such an operationally implementable method of analysis employing the new approach of goal focusing, derived from ideas of goal programming and efficient point analysis. A basic theoretical model is developed and illustrated by numerical example, and extensions in several directions are described. Such models will provide considerable assistance in obtaining substantive knowledge of the prospective magnitudes of the trade-offs involved and their behavior within a set of plausible trends over the next ten to thirty years.