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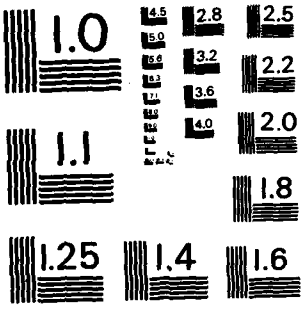

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STRUCTURES NOTE 485

INITIAL ESTIMATES OF DAMPING RATES AND FREQUENCIES

by

T. G. RYALL

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STRUCTURES NOTE 485

INITIAL ESTIMATES OF DAMPING RATES AND FREQUENCIES

by

T. G. RYALL

ABSTRACT

A simple, quick method for obtaining frequencies and damping ratios from Fourier transforms has been devised.

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1. INTRODUCTION

In any estimation process there are four quantities which may have to be taken account of:

(i) bias of estimates;

(ii) variance of estimates;

(iii) time taken to obtain estimates on machine that is available;

(iv) memory and storage requirements of method.

The particular problem, for which the following method was generated, is to estimate the poles of a linear dynamical system in close to real time. The linear system is the representation of an aircraft in flight and the poles are determined for stability reasons.

It is required to know how these poles vary as a function of forward speed and altitude.

In this instance the aircraft will fly at a constant speed for a period of time. Five minutes will be available for calculations then the speed will be incremented or altitude changed.

All calculations have to be done on a minicomputer which has a storage capacity of 16K and can calculate a 2048-point Fast Fourier Transform (FFT) in approximately 1·5 minutes.

Any non-linear method is effectively ruled out due to this time constraint.

In a more general setting, where "time" is not so crucial a factor, even a non-linear method like maximum likelihood or least squares needs some initial estimates. This method will give good initial estimates for any non-linear method.

Although the title of this note may imply that the estimates are both quick and inaccurate, this is not the case.

Resolution is much better than that of power spectral methods as are estimates of dampings and frequencies.

Providing that the maximum number of frequencies of decay which are "close" together is two, then the greatest level of mathematical complexity is the solving of two simultaneous equations and the solving of a quadratic equation with complex coefficients.

More complicated situations require the solution of larger sets of simultaneous equations and higher order polynomial equations; the essential simplicity of the method is then lost and it is then preferable to go to a full maximum likelihood method.[1]

2. THE PROBLEM

Given a signal $\Sigma A_i e^{-\alpha_i t} \cos(\omega_i t + \phi_i)$ it is desired to estimate ω_i and α_i where ω_i is the decay frequency and α_i is the decay rate.

It is common to estimate f_i and α_i/ω_i where f_i is the frequency in hertz ($2\pi f_i = \omega_i$) and α_i/ω_i is normally expressed as a percentage. The reason for this choice of parameters is simply that for stability reasons we would like the poles of the system to lie within a certain sector in the complex domain. Care must be taken with this concept especially for low frequencies.

2.1 Single Pole–Single Zero Approximation

In a small frequency band the Fourier transform is assumed to behave as

$$\frac{N_0 + N_1 s}{D_0 + s} \tag{2.1.1}$$

where $s = i\omega$ and N_0, N_1, D_0 may be complex. Conjugate poles are ignored as neighbouring frequencies will have, in general, a greater influence.

Only three frequency points are needed to obtain N_0, N_1 and D_0. Guidelines will be presented later as to how these points may be selected.

Let the three frequency points be s_1, s_2, s_3 and let the corresponding values of the transform be Z_1, Z_2, Z_3. Equating observed values with expected values it follows that

$$(N_0+N_1s_1) = Z_1(s_1+D_0), \tag{2.1.2}$$

$$(N_0+N_1s_2) = Z_2(s_2+D_0), \tag{2.1.3}$$

$$(N_0+N_1s_3) = Z_3(s_3+D_0). \tag{2.1.4}$$

Subtracting (2.1.3) from (2.1.2) and dividing by (s_1-s_2) it follows that

$$N_1 = \frac{s_1Z_1-s_2Z_2}{s_1-s_2} + D_0\frac{Z_1-Z_2}{s_1-s_2}. \tag{2.1.5}$$

Performing a similar operation on (2.1.3) and 2.1.4) it follows that

$$N_1 = \frac{s_2Z_2-s_3Z_3}{s_2-s_3} + D_0\frac{Z_2-Z_3}{s_2-s_3}. \tag{2.1.6}$$

Let

$$Y_1 = \frac{s_1Z_1-s_2Z_2}{s_1-s_2}, \qquad Y_2 = \frac{s_2Z_2-s_3Z_3}{s_2-s_3}.$$

$$X_1 = \frac{Z_1-Z_2}{s_1-s_2}, \qquad X_2 = \frac{Z_2-Z_3}{s_2-s_3}.$$

Y_1 and Y_2 are the so-called divided differences of the function sZ; X_1 and X_2 are the divided differences of the function Z.

Subtracting (2.1.5) from (2.1.6) and solving for D_0 it follows that

$$D_0 = -(Y_1-Y_2)/(X_1-X_2). \tag{2.1.7}$$

D_0 contains the parameters of interest and is of the form $\alpha-i\omega$. If required N_1 and N_0 may be found by back substitution in equations (2.1.6) and (2.1.4).

2.2 Equation of Nyquist Locus

The complex response $Z(i\omega)$ may be rewritten in the form

$$Z(i\omega) = N_1 + \frac{N}{D_0+i\omega},$$

where $N = N_0-N_1D_0$.

Now let $N = A+iB$, $D_0 = \alpha-i\omega$, $N_1 = X_0+iY_0$, $X(i\omega)+iY(i\omega) = Z(i\omega)$. Then

$$X(i\omega) = X_0 + \frac{A\alpha+B(\omega-\omega_1)}{\alpha^2+(\omega-\omega_1)^2}. \tag{2.2.1}$$

$$Y(i\omega) = Y_0 + \frac{B\alpha-A(\omega-\omega_1)}{\alpha^2+(\omega-\omega_1)^2}. \tag{2.2.2}$$

After some algebra it can be shown that

$$\left(X-X_0-\frac{A}{2\alpha}\right)^2 + \left(Y-Y_0-\frac{B}{2\alpha}\right)^2 = \frac{A^2+B^2}{4\alpha^2}. \tag{2.2.3}$$

Equation (2.2.3) is recognised as the equation of the circle: centre $\left(X_0+\frac{A}{2\alpha}, Y_0+\frac{B}{2\alpha}\right)$, radius $\frac{\sqrt{A^2+B^2}}{2\alpha}$. It should be noticed that it is possible to have a poor estimate of the circle, which has parameters involving the zero as well as the pole, whilst still obtaining good estimates of the pole. That is, it is more important to determine how 'fast' the Fourier transform moves along the circle than to define the circle precisely.

2.3 Two Zeros–Two Poles Approximation

If two damped frequencies are "close" together then a two zero–two pole approximation is called for:

$$Z(s) = \frac{N_0+N_1s+N_2s^2}{1+D_1s+D_2s^2}. \tag{2.3.1}$$

where $s = i\omega$. Equation (2.3.1) has five complex parameters hence five frequency points are needed.

Equating expected values with observed values gives five equations

$$Z_i\{1+D_1s_i+D_2s_i^2\} = N_0+N_1s_i+N_2s_i^2 \qquad i = 1, 2, \ldots, 5. \tag{2.3.2}$$

Define

$$X_2(i) = \frac{1}{S_i-S_{i+2}}\left\{\frac{Z_i-Z_{i+1}}{S_i-S_{i+1}} - \frac{Z_{i+1}-Z_{i+2}}{S_{i+1}-S_{i+2}}\right\},$$

$$Y_2(i) = \frac{1}{S_i-S_{i+2}}\left\{\frac{S_iZ_i-S_{i+1}Z_{i+1}}{S_i-S_{i+1}} - \frac{S_{i+1}Z_{i+1}-S_{i+2}Z_{i+2}}{S_{i+1}-S_{i+2}}\right\},$$

$$Z_2(i) = \frac{1}{S_i-S_{i+2}}\left\{\frac{S_i^2Z_i-S_{i+1}^2Z_{i+1}}{S_i-S_i} - \frac{S_{i+1}^2Z_{i+1}-S_{i+2}Z_{i+2}}{S_{i+1}-S_{i+2}}\right\}.$$

$X_2(i)$, $Y_2(i)$, $Z_2(i)$ are the second divided differences of Z, sZ and s^2Z respectively.

The equations (2.3.2) can now be reduced to

$$X_2(i)+D_1Y_2(i)+D_2Z_2(i) = N_1 \qquad i = 1, 2, 3. \tag{2.3.3}$$

If we now let

$$X_i' = X_2(i)-X_2(i+1);\; Y_i' = Y_2(i)-Y_2(i+1);\; Z_i' = Z_2(i)-Z_2(i+1) \qquad i = 1, 2$$

Then (2.1.3) reduces to

$$X_i + D_1Y_i'+D_2Z_i' = 0 \qquad i = 1, 2. \tag{2.3.4}$$

Hence

$$\left.\begin{aligned} D_2 &= \frac{X_2'Y_1'-X_1'Y_2'}{Z_1'Y_2'-Z_2'Y_1'} \\ D_1 &= \frac{X_1'Z_2'-X_2'Z_1'}{-Y_1Z_2'+Y_2'Z_1'} \end{aligned}\right\} \tag{2.3.5}$$

The required poles are now the solution of $1+D_1s+D_2s^2 = 0$, i.e.

$$s = -\frac{D_1\pm\sqrt{D_1^2-4D_2}}{2D_2}. \tag{2.3.6}$$

$$D_2 \neq 0.$$

It should be noticed that (2.3.6) involves a complex square root.

No attempt will be made to determine the locus in the Nyquist Plot as the parametric representation (2.3.1) is probably the simplest. Furthermore it is not the curve itself but how "fast" the Fourier Transform moves along the curve which is important. Back substitution will determine N_0, N_1 and N_2 if required.

3. SELECTION OF FREQUENCY POINTS—SOME GUIDELINES

The representations of the response are only local representations and hence do not cover a wide frequency band. Ideally the points selected should cover the damped frequency. Furthermore as successive differences are calculated only points with large signal-to-noise ratios should be used. From equation (2.1.7) (single-pole case)

$$D_0 = -(Y_1-Y_2)/(X_1-X_2).$$

To reduce sensitivity it is necessary that X_1-X_2 is large, i.e.

$$\left|\frac{Z_1-Z_2}{s_1-s_2}-\frac{Z_2-Z_3}{s_2-s_3}\right|$$

is large. For equally-spaced points this means searching for local maxima of $|Z_1-2Z_2+Z_3|$.

Interpolated points in a Fourier transform should not be used as these will not be statistically independent.

Interpolated points in a Fourier transform come from using the shifting theorem, i.e. by taking the FFT of $[X(t)e^{-i\omega t}]$ (ω small) rather than the FFT of $X(t)$.

4. DISCUSSION

The method presented was devised because traditional methods (e.g. half-power points and rapid phase changes) have a number of disadvantages.

The disadvantages of the half-power method are that:

(i) it uses only information from the power spectrum and ignores the phase spectrum;

(ii) the peak power may in fact be missing as may the half-power points since the FFT is discrete;

(iii) the method does not allow for the presence of a nearby zero in which case the peak power does not occur at the frequency of decay.

A nearby zero can occur quite frequently especially if the transfer function is minimum phase (the response is measured quite close to the excitation point).

Similar criticisms may be levelled against methods which use only phase information.

The method devised in this note, which uses both amplitude and phase information, allows up to two zeros and two poles to be close together. The stability of the estimates may be checked by seeing if points other than those used in the estimation lie near the Nyquist plot.

The stability of the estimates may also be checked by using a different selection of points.

The situation may arise where it is not clear if there are any poles in a certain frequency band or not. The stability of the estimate is then quite important for deciding an answer to this question.

It has been assumed throughout that the frequencies of interest are well away from the Nyquist frequency. By using the Fourier sum rather than the Fourier integral it is possible to change the method so as to work near the Nyquist frequency.

5. EXAMPLES

Figures 1*a*, 2*a*, 3*a* are the time history, amplitude spectrum and Nyquist plot respectively of a signal without noise. The time history is generated mathematically and contains decay frequencies of 20 Hz, 55 Hz and 60 Hz with decay rates of 2, 1 and 5% of the corresponding decay frequency respectively.

A single-pole curve is fitted to points at 17·5 Hz, 20 Hz and 22·5 Hz (the position of the first point is indicated by a "1" in Figure 3*a*). The decay frequency and damping as determined by equation (2.1.7) are 20 Hz and 2·02%.

A two-pole curve is fitted to points at 52·5, 55, 57·5, 60 and 62·5 Hz (the position of the first point is indicated by a "2" in Figure 3*a*). The two decay frequencies as determined from equation (2.3.6) are 55 and 60·01 Hz, and the dampings are 1 and 5·04% respectively.

Figures 1*b*, 2*b*, 3*b* are the corresponding plots with noise added to the signal. The single- and double-pole curves are fitted to the same frequency points as for the original signal.

The r.m.s. value of the noise is $A/\sqrt{3}$ where $A = \frac{1}{5}$ largest|Amplitude|. This implies that the signal-to-noise ratio is always $\leqslant 5\sqrt{3}$. The estimated decay frequencies are 20, 55·09 and 61·4 Hz while the corresponding dampings are 1·8, 1·11 and 5·07%.

6. CONCLUSION

A simple procedure for determining initial estimates of decay frequencies from Nyquist plots has been determined. The process gives acceptable results even in the presence of a moderate amount of noise. These initial estimates may then be refined by using some non-linear method if required.

7. ACKNOWLEDGMENT

Thanks are due to Ms P. Cox who wrote the necessary computer programs and prepared the figures.

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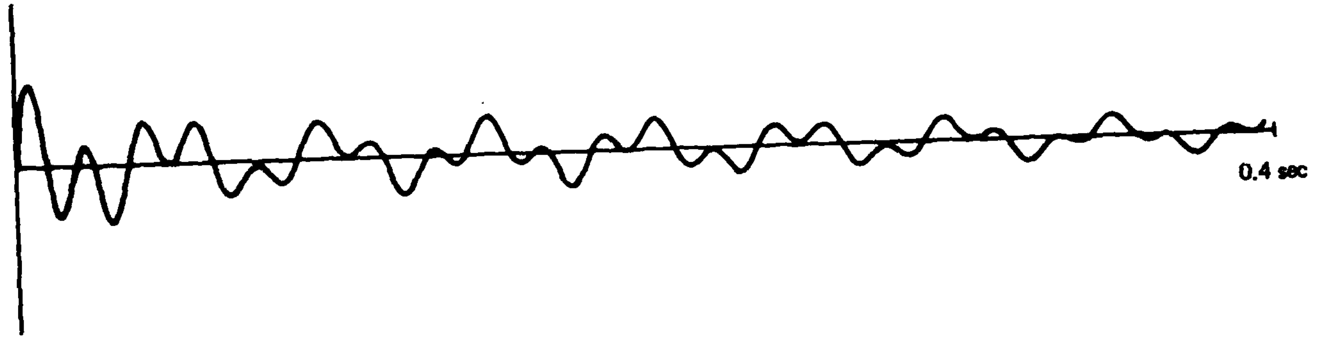

FIG. 1(a) TIME HISTORY WITHOUT NOISE

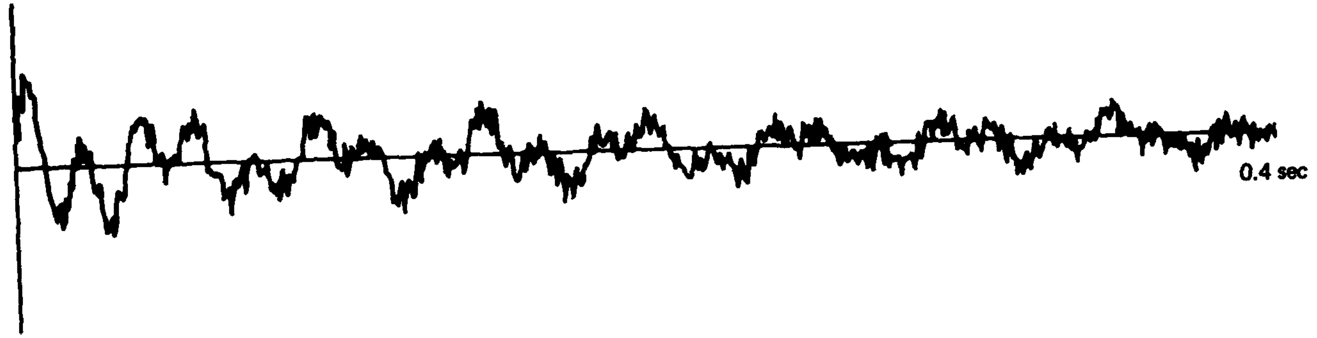

FIG. 1(b) TIME HISTORY WITH NOISE

FIG. 2(a) AMPLITUDE SPECTRUM OF TIME HISTORY WITHOUT NOISE

FIG. 2(b) AMPLITUDE SPECTRUM OF TIME HISTORY WITH NOISE

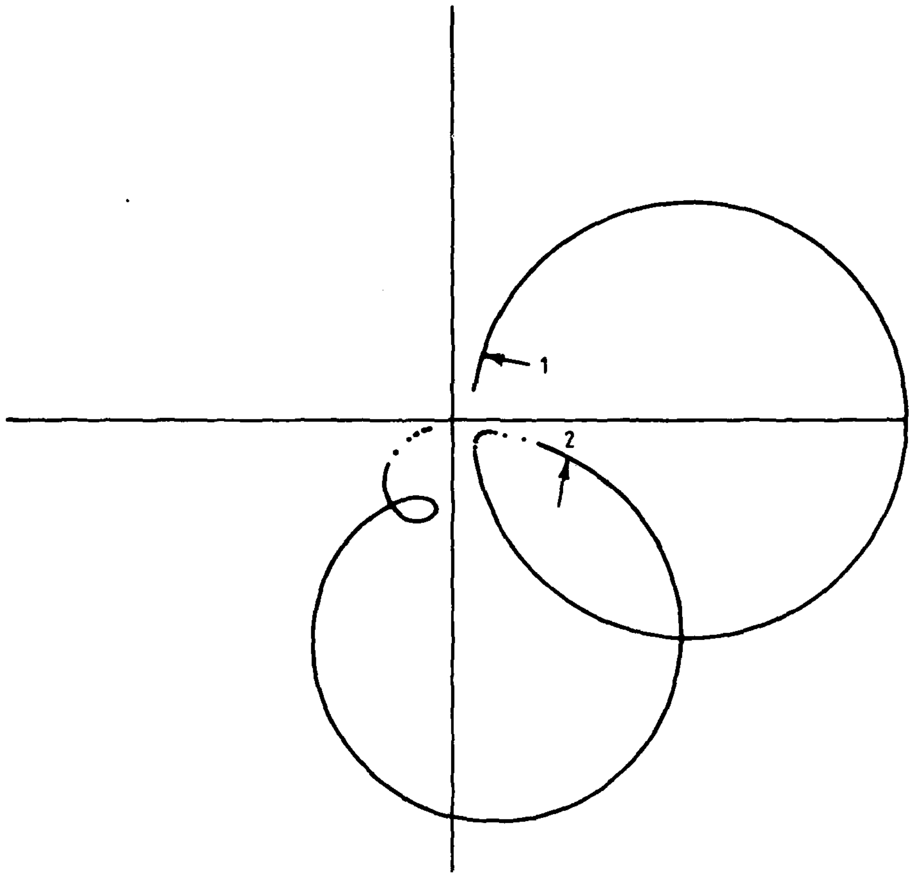

FIG. 3(a) NYQUIST PLOT FROM TIME HISTORY WITHOUT NOISE

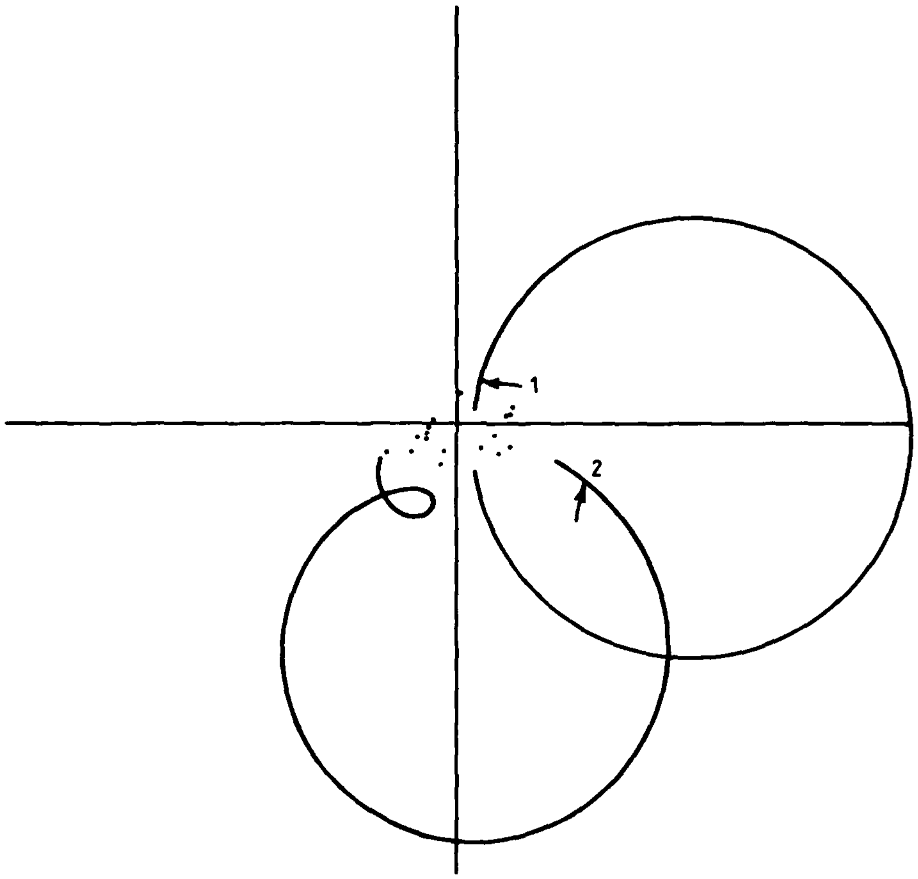

FIG. 3(b) NYQUIST PLOT FROM TIME HISTORY WITH NOISE

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16. Abstract

A simple, quick method for obtaining frequencies and damping ratios from Fourier Transforms has been devised.

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