DEVELOPMENT AND APPLICATION OF THE P-VERSION OF THE
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**Title:** Development and Application of the P-Version of the Finite Element Method

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**Abstract:**

The p-version of the finite element method is a new, important, computationally efficient approach to finite element analysis. It is more robust than the conventional h-version and its rate of convergence, for domains with corners and for other singularity problems, is twice that of the h-version.

Hierarchic elements which implement the p-version efficiently have been formulated so as to enforce $C^0$ or $C^1$ continuity in the planar case, and so as to enforce $C^1$ continuity in three dimensions.
DEVELOPMENT AND APPLICATION OF THE P-VERSION OF THE FINITE ELEMENT METHOD, AFOSR-82-0315

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## CONTENTS

<table>
<thead>
<tr>
<th>Section</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. DISCUSSION</td>
<td>1</td>
</tr>
<tr>
<td>2. PROFESSIONAL PERSONNEL</td>
<td>3</td>
</tr>
<tr>
<td>3. PAPERS PUBLISHED AND PRESENTED SINCE THE START OF THE PROJECT</td>
<td>4</td>
</tr>
</tbody>
</table>
Two approaches to finite element analysis are now widely recognized in the engineering and mathematical communities. In both approaches the domain $W$ is divided into simple convex subdomains (usually triangles or rectangles in two dimensions, and tetrahedra or bricks in three dimensions) and over each subdomain the unknown (displacement field) is approximated by a (local) basis function (usually a polynomial of degree $p$). Basis functions are required to join continuously at boundaries of the subdomains in the case of planar or 3 dimensional elasticity, or smoothly in the case of plate bending. The difference between the two approaches lies in the manner in which convergence is achieved. These two approaches are:

1. The **h-version** of the finite element method. In this approach the degree $p$ of the approximating polynomial is kept fixed, usually at some low number such as 2 or 3. Convergence is achieved by allowing $h$, the maximum diameter of the convex subdomains, to go to zero. Estimates for the error in energy have long been known. In all of these estimates $p$ is assumed to be fixed and the error estimate is asymptotic in $h$, as $h$ goes to zero.

2. The **p-version** of the finite element method. In this approach the subdivision of the domain $\Omega$ is kept fixed but $p$ is allowed to increase until a desired accuracy is attained. The p-version is reminiscent of the Ritz method for solving partial differential equations but with a crucial distinction between the two methods. In the Ritz method a single polynomial approximation is used over the entire domain $\Omega$ (in general, is not convex). In the p-version of the finite element method polynomials are used as approximations over convex subdomains. This critical difference gives the p-version a much more rapid rate of convergence than either the Ritz method or the h-version.
The p-version of the finite element method requires families of polynomials of arbitrary degree \( p \) defined over different geometric shapes. Polynomials defined over neighboring elements join either continuously (are in \( C^0 \)) for planar or three dimensional elasticity, and smoothly (are in \( C^1 \)) for plate bending. In order to implement the p-version efficiently on the computer, these families should have the property that computations performed for an approximation of degree \( p \) are re-usable for computations performed for the next approximation of degree \( p + 1 \). We call families possessing this property hierarchic families of finite elements.

The h-version of the finite element method has been the subject of intensive study since the early 1950's and perhaps even earlier. Study of the p-version of the finite element method, on the other hand, began at Washington University in St. Louis in the early 1970's. Research in the p-version (formerly called The Constraint Method) has been supported in part of the Air Force Office of Scientific Research.

Recent Research Accomplishments are:

1. The proof a theorem which establishes that for plate bending problems (in which \( C^1 \)-continuity is required) the rate of convergence of the p-version is twice that of the h-version. This supplements an earlier theorem which was proved for the \( C^0 \) case.

2. Hierarchic families of \( C^1 \)-triangular elements have been implemented on the computer using pre-computed arrays.

3. The difficult problem of the finite element analysis of a rhombic plate has been solved using both the p-version and the h-version.

4. Work has begun on an efficient post-processing technique to compute accurate displacements and stresses, and to calculate stress intensity factors.
3. PROFESSIONAL PERSONNEL

1. I. Norman Katz, Professor of Applied Mathematics and Systems Science, Washington University

2. Barna A. Szabo, A. P. Greensfelder Professor Civil Engineering, Washington University

3. Xing-Ren Ying, Research Assistant, doctoral candidate in the Department of Systems Science and Mathematics, Washington University

4. Hong-Cai Wang, Research Assistant, doctoral candidate in the Department of Systems Science and Mathematics, Washington University
3. PAPERS PUBLISHED AND PRESENTED SINCE THE START OF THE PROJECT (1977)

3.1 Published Papers:


3.2 Presented Papers:


