EFFECTS OF MEAN FLOW ON THE
DYNAMIC CHARACTERISTICS OF
FLUID TRANSMISSION LINES

THESIS

Mark S. Briski
First Lieutenant, USAF

AFIT/GAE/AA/83D-2

DEPARTMENT OF THE AIR FORCE
AIR UNIVERSITY
AIR FORCE INSTITUTE OF TECHNOLOGY

Wright-Patterson Air Force Base, Ohio

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CHARACTERISTICS OF FLUID TRANSMISSION LINES

THESIS

Presented to the Faculty of the School of Engineering
of the Air Force Institute of Technology
Air University
In Partial Fulfillment of the
Requirements for the Degree of
Master of Science in Aeronautical Engineering

Mark S. Briski, B.S.
First Lieutenant, USAF

December 1983

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The purpose of this thesis was to study the effects of turbulent mean flow on the frequency response of round air filled transmission lines. The report was based heavily on past theses and dissertations performed at the Air Force Institute of Technology, specifically those written by Malanowski, Vining, and Moore.

The computer program used in this study was originally written by Malanowski and the model used to predict turbulent characteristics was developed by Moore with modifications suggested by Dr. Franke.

I wish to thank the people who were helpful in the completion of this study. Captain Mark Ross devoted much on a thesis with a similar topic and was very helpful in almost all phases of this study. Dr. Milton Franke, my thesis advisor, was always helpful whenever I ran into problems. Harley Linville was always ready to help when equipment problems cropped up. I also wish to thank my wife, Theresa, for putting up with me during the final months of study and typing.

Mark S. Briski
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</tr>
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<td>A</td>
<td>line cross-sectional area</td>
</tr>
<tr>
<td>Beta</td>
<td>phase angle</td>
</tr>
<tr>
<td>C</td>
<td>capacitance/unit length</td>
</tr>
<tr>
<td>$C_a$</td>
<td>adiabatic capacitance/unit length; $A/\gamma P$ for ideal gas</td>
</tr>
<tr>
<td>D</td>
<td>line diameter</td>
</tr>
<tr>
<td>f</td>
<td>friction factor</td>
</tr>
<tr>
<td>$g$</td>
<td>dynamic pressure ratio, $P_r/P_s$</td>
</tr>
<tr>
<td>$g_{ct}$</td>
<td>line gain</td>
</tr>
<tr>
<td>G</td>
<td>conductance/unit length</td>
</tr>
<tr>
<td>$h_v$</td>
<td>velocity distribution parameter</td>
</tr>
<tr>
<td>$h_t$</td>
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</tr>
<tr>
<td>$j$</td>
<td>$\sqrt{-1}$</td>
</tr>
<tr>
<td>L</td>
<td>inertance/unit length</td>
</tr>
<tr>
<td>$L_a$</td>
<td>adiabatic inertance/unit length $\rho/A$ for ideal gas</td>
</tr>
<tr>
<td>l</td>
<td>line length</td>
</tr>
<tr>
<td>$\bar{P}$</td>
<td>DC (mean) Pressure at beginning of the line</td>
</tr>
<tr>
<td>P</td>
<td>AC pressure</td>
</tr>
<tr>
<td>Q</td>
<td>volumetric flow rate (DC)</td>
</tr>
<tr>
<td>Symbol</td>
<td>Description</td>
</tr>
<tr>
<td>--------</td>
<td>---------------------------------------------------</td>
</tr>
<tr>
<td>R</td>
<td>resistance/unit length</td>
</tr>
<tr>
<td>r</td>
<td>line radius</td>
</tr>
<tr>
<td>Re</td>
<td>Reynolds number based on diameter</td>
</tr>
<tr>
<td>Rg</td>
<td>real gas constant</td>
</tr>
<tr>
<td>Rr</td>
<td>turbulent viscous resistance</td>
</tr>
<tr>
<td>Rv1</td>
<td>laminar viscous resistance</td>
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<tr>
<td>T</td>
<td>temperature</td>
</tr>
<tr>
<td>u</td>
<td>mean velocity</td>
</tr>
<tr>
<td>x</td>
<td>distance downstream in line</td>
</tr>
<tr>
<td>Y</td>
<td>shunt admittance/unit length</td>
</tr>
<tr>
<td>Z</td>
<td>impedance/unit length</td>
</tr>
<tr>
<td>Zc</td>
<td>characteristic impedance/unit length</td>
</tr>
<tr>
<td>Zt</td>
<td>terminal or end impedance</td>
</tr>
<tr>
<td>α</td>
<td>attenuation/unit length</td>
</tr>
<tr>
<td>β</td>
<td>phase shift/unit length</td>
</tr>
<tr>
<td>Γ</td>
<td>propagation constant/unit length</td>
</tr>
<tr>
<td>γ</td>
<td>ratio of specific heats</td>
</tr>
<tr>
<td>μ</td>
<td>dynamic viscosity</td>
</tr>
<tr>
<td>Symbol</td>
<td>Description</td>
</tr>
<tr>
<td>--------</td>
<td>-------------</td>
</tr>
<tr>
<td>( \nu )</td>
<td>kinematic viscosity</td>
</tr>
<tr>
<td>( \rho )</td>
<td>density</td>
</tr>
<tr>
<td>( \sigma )</td>
<td>Prandtl number</td>
</tr>
<tr>
<td>( \Omega )</td>
<td>non-dimensional frequency ( r^2\omega/\nu )</td>
</tr>
<tr>
<td>( \Omega_b )</td>
<td>non-dimensional break frequency</td>
</tr>
<tr>
<td>( \omega )</td>
<td>angular frequency</td>
</tr>
<tr>
<td>( \omega_{b, DC} )</td>
<td>break frequency of model with DC resistance</td>
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<tr>
<td>( \omega_{b, AC} )</td>
<td>break frequency of model with AC resistance</td>
</tr>
<tr>
<td>( \omega_v )</td>
<td>viscous characteristic frequency; ( \frac{8\nu}{\pi V/A} )</td>
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<tr>
<td>( \omega_t )</td>
<td>thermal characteristic frequency; ( \omega^* \sqrt{c^*} )</td>
</tr>
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</table>

**Subscripts**

- \( \text{a} \): adiabatic
- \( \text{A} \): absolute
- \( \text{r} \): recieving end properties
- \( \text{s} \): sending end properties
- \( \text{t} \): turbulent
Abstract

Theoretical predictions of the small signal frequency response of round pneumatic transmission lines with turbulent mean flow were compared with experimental results. The frequency response curves were found for lines varying in length from 24 to 36 in. with inside diameters of 0.195, 0.119, and 0.041 in. The lines were tested at Reynolds numbers of 2000, 5000, and 10000.

Theoretical solutions were obtained using Nichelich equations as modified by Krishnaiyer and Lechner. Solutions were also found using several different modifications of a constant LRC model developed by Moore. The results were mixed: for the 0.195 and 0.119 in. lines the prediction of gain was good but for the 0.041 in. lines the results were poor. The accuracy and applicability of the constant LRC model was explored along with its various modifications. The constant LRC model with the AC resistance showed potential for predicting the gain in fluid transmission lines with turbulent flow. The limitations and applicability of the constant LRC models was studied.
THE EFFECTS OF MEAN FLOW ON THE DYNAMIC CHARACTERISTICS OF FLUID TRANSMISSION LINES

I. Introduction

Background

The propagation of small signals in fluid lines has been of interest to scientists and engineers for many years and, particularly since the advent of fluidic devices in the 1950's. Many investigations have been conducted, both analytical and experimental, in an effort to develop models that can accurately predict the frequency response of fluid-filled transmission lines. Equations, developed by Nichols (Ref 1) and modified by Krishnaiyer and Lechner (Ref 2), have proven useful in predicting the frequency response under laminar flow conditions.

Brown, Margolis, and Shah (Ref 3) using two and three region viscosity profiles predicted increased attenuation with Reynolds number in lines with fully developed turbulent flow. These models however resulted in relatively complicated solutions. Moore (Ref 4) used a simple constant LRC model as suggested by Brown et. al. (Ref 3) and Funk and Wood (Ref 5) to predict this increased attenuation.

Objectives

The following objectives were established to
investigate the effects of turbulent mean flow and oscillatory signals in pneumatic transmission lines.

1. To experimentally determine the small signal response of lines with developed turbulent mean flow.
2. To modify existing computer programs to incorporate new resistance expressions.
3. To develop models for phase shift and attenuation.
II. Theory

Many previous investigations of fluid-transmission lines have successfully used a pneumatic-electrical analogy in their analysis with pressure and volumetric flow rate analogous to voltage and current respectively. Using this analogy the following equations describe the relationship between pressure and volumetric flow:

\[
\frac{dP}{dx} = ZQ = (R + j\omega L)Q \tag{1}
\]

and

\[
\frac{dQ}{dx} = iP = (G + j\omega C)P \tag{2}
\]

where the complex terms Z and Y are defined as the series impedance and shunt admittance of the line.

Krishnaiyer and Lechner (Ref 2) present Z and Y in round lines with blocked or laminar flow as

\[
Z = \frac{5NWh}{A^2} \left[ \frac{ER}{V} \right] + j \left[ \frac{UwC}{A} + \frac{5Nw}{A^2} \left[ E - \frac{w}{j} \right] \right] \tag{3}
\]

and

\[
Y = -\frac{w(\gamma-1)A}{4\pi P} \left[ \frac{DG}{DC} \right] \left[ \frac{DG}{DC} \right]^2 + j\omega \left[ \frac{A}{\gamma P} + \frac{(\gamma-1)A}{4\pi P} \left[ \frac{DC}{DC} \right]^2 + \left[ \frac{DG}{DC} \right]^2 \right] \tag{4}
\]

where the terms DR, DL, DG, and DC are given as
\[ DR = 3/8 + h_v/4 + 3/(h_v) \]  
\[ DL = h_v/4 - 15/(64 h_v) \]  
\[ DG = h_t/2 - 1/(4 h_t) \]  
\[ DC = 1/4 + h_t/2 + 1/(4 h_t) \]  

with \( h_v \) and \( h_t \) defined as velocity and temperature distribution parameters given by equations (9) and (10)

\[ h_v = \frac{1}{\sqrt{\omega / \omega_v}} \]  
\[ h_t = \sigma h_v \]  

\( \omega_v \) is a characteristic frequency defined by Nichols (Ref 1) as

\[ (\nu)_v = \frac{\omega}{\omega_v} \]  

These equations are accurate over a frequency range of \( 0.1 \omega_v < \omega < \infty \) for blocked lines and for lines with fully developed laminar flow.

Transmission lines can be described by a propagation operator per unit length \( \Gamma \) and a characteristic impedance \( Z_0 \) given by equations (12) and (13)

\[ \Gamma = (Z \gamma)^{1/2} = \alpha + j\beta \]  
\[ Z_0 = (Z / \gamma)^{1/2} \]

where \( \alpha \) is defined as the attenuation and \( \beta \) is the phase.
angle per unit length. The input impedance of a line is given as

\[ Z_S = Z_0 \frac{(Z_T + Z_0)e^{\Gamma l}}{(Z_T + Z_0)e^{\Gamma l} - (Z_T - Z_0)e^{\Gamma l}} \]  

(14)

The ratio of the receiving pressure to the sending pressure is

\[ \frac{P_r}{P_S} = \frac{2Z_T}{(Z_T + Z_0)e^{\Gamma l} + (Z_T - Z_0)e^{\Gamma l}} \]  

(15)

The gain is the magnitude of the complex pressure ratio given by equation (15)

\[ g = \frac{|P_r|}{|P_S|} \]  

(16)

and in decibels

\[ g_{db} = 20 \log_{10} g \]  

(17)

The phase shift between the receiving and sending points on the line is the angle formed by the ratio of the imaginary to the real parts of the pressure ratio

\[ \beta \theta = \tan^{-1} \frac{\text{Im}(P_r/P_S)}{\text{Re}(P_r/P_S)} \]  

(18)

**Turbulent Flow Effects**

In order to predict the attenuation for lines with fully developed turbulent mean flow, a constant LRC model is
used following the method of Moore (Ref 4). This model uses the turbulent resistance of the line, the adiabatic inertance of the line, and the isothermal capacitance (adiabatic capacitance with $\gamma = 1.0$) of the line to determine the propagation operator and characteristic impedance. The turbulent resistance is defined as

$$R_{vt} = \frac{f \rho \mu}{2 \Delta C}$$

(23)

The adiabatic inertance is

$$L_{\bar{\omega}} = \frac{\rho}{A}$$

(24)

and the adiabatic capacitance is

$$C_{\bar{\omega}} = \frac{1}{\gamma \rho}$$

(25)

The turbulent impedance is a complex expression consisting of a real term, the turbulent resistance, and an imaginary term, the adiabatic inertance.

$$Z_T = R_{vt} + j \omega L_{\bar{\omega}}$$

(26)

Using the adiabatic inertance introduces a small error into the model but greatly simplifies it and reduces computation time. The actual turbulent inertance is slightly greater than the adiabatic inertance, Moore (Ref 4) shows that the turbulent inertance is approximately 3.5% larger than the
adiabatic inertance at a Reynolds number of 5,000. The turbulent inertance approaches the adiabatic inertance as the Reynolds number increases.

The turbulent admittance is a complex expression consisting of a conductance (real part) and a capacitance (imaginary part). The capacitance is thought to be isothermal at low frequencies so the expression for the adiabatic capacitance is used (Eq. (25)) with \( \gamma = 1 \). The conductance is assumed to be zero for the low frequencies at which this model is used

\[
Y_t = j\omega C_0
\]  

(27)

Moore (Ref 4) uses the Blasius relation, equation (28), as a simple method of relating \( fRe \) to the Reynolds number for turbulent flow.

\[
fRe = 0.3164 Re^{3/4}
\]  

(28)

For laminar flow the expression relating \( fRe \) to Reynolds number is Eqn. (29).

\[
fRe = 64
\]  

(29)

The turbulent resistance then becomes

\[
R_{vt} = \frac{0.3164 Re^{3/4} \mu}{2AD^2}
\]  

(30)
and the laminar resistance for this model becomes

$$R_{vl} = \frac{32\mu}{AD^2}$$  \hspace{1cm} (31)

The resistance used in this model is a DC resistance, Franke (Ref 6) suggested that using the DC resistance might cause the constant LRC model to predict lower attenuations than would actually be encountered. The constant LRC was modified to account for a possible higher AC resistance. The AC resistance, as suggested by Franke (Ref 6), is

$$R_{vl,AC} = \frac{P\mu}{AD} \left\{ f + \frac{u}{2} \frac{\partial f}{\partial u} \right\}$$  \hspace{1cm} (32)

After carrying out the differentiation and simplifying the expression, the AC resistance becomes

$$P_{\ell_{vl,AC}} = 1.75 R_{vl}$$  \hspace{1cm} (33)

while the expression for the laminar resistance will not change.

The attenuation predicted by the constant LRC model, using both the DC and AC resistance, is nondimensionalized using the method of Brown (Ref 3) and plotted against a nondimensional frequency $\Omega$ in Fig 1. The attenuation predicted by Nichols equations, as approximated by Krishnaiyer and Lechner (Ref 2), is also shown as a reference. Fig. 1 shows that the constant LRC models predict attenuations higher than those predicted by Nichols below
certain frequencies, but reach constant attenuations that result in under prediction at higher frequencies. The frequency at which the turbulent and laminar curves intersect is denoted as the break frequency \( \Omega_l \). The dimensional break frequency \( \omega_b \) is obtained from \( \Omega_l \). The actual behaviour of lines with turbulent flow has been experimentally proven by Brown, et. al. (Ref 3) and Funk and Wood (Ref 5) to be laminar at frequencies higher than the break frequency. The behaviour in the vicinity of the break frequency is uncertain, experiments by Margolis and Brown
(Ref 7) produced large deviations in attenuation in the vicinity of the break frequency.

The computer program used in this study employed the turbulent constant LRC model at frequencies below the break frequency, \( \omega_b \), and the laminar model (Eqns. (3) and (4)) at frequencies greater than the break frequency. The program uses these models to determine the propagation operator and characteristic impedance (Eqns. (12) and (13)) then uses Eqns. (14) and (15) to determine the theoretical pressure ratio. The program then uses Eqns. (16), (17), and (18) to compute the theoretical gain and phase shift.

**Mean Pressure Drop**

Schlichting expresses the relation between pressure drop and distance in a constant area duct as

\[
\Delta \bar{P} = -f \frac{l}{D} \frac{\rho \mu^2}{\bar{z}}
\]  

(34)

If equation (28) is used to determine \( f \) and substituted into equation (34) then the pressure drop can be expressed as a function of velocity and Reynolds number. Further manipulation with the velocity and density can result in equations (35) and (36) which express the pressure drop as a function of \( l, D, \) Re, \( \rho, \) and \( \mu \) for turbulent and laminar flows respectively.

\[
\Delta \bar{P} = -0.15 \varepsilon \bar{z} \frac{l}{D} \frac{\mu^2}{\rho \bar{z}} e^{1.75}
\]  

(35)
\[ \Delta \bar{P} = -32 \frac{1}{D} \frac{\mu^2}{\rho D^2} Re \]  

Equations (35) and (36) are only applicable for incompressible flow. To find the pressure drop in small diameter lines at high Reynolds numbers, Eqns. (35) and (36) are rewritten as infinitesimal pressure drops for a small distance \( dx \). The ideal gas law is used to express density in terms of pressure and temperature, then assuming that Reynolds number and temperature are constant in the line the equations can be integrated to get Eqns. (37) and (38).

\[ \Delta \bar{P} = \bar{P}_1 - \left[ 0.154 \frac{1}{\rho^{3/2} T_{avg}} + \frac{\mu^2}{\rho D^2} Re + \frac{2}{\rho D^2} \right] \frac{1}{2} \]  

(37)

\[ \Delta F = \bar{F}_1 - \left[ 0.3 \frac{1}{\rho^{3/2} T_{avg}} + \frac{2}{\rho D^2} \right] \]  

(38)
III. Experimental Apparatus

The experimental apparatus consisted of pneumatic signal generating equipment and signal analysis instrumentation shown schematically in Fig. 2. A rotometer was also used to verify that the mean flow was consistent with the pressure drop.

The wave analyzer connected to the sending dynamic pressure transducer was used to generate a sinusoidal signal. This signal was then amplified and sent to the pneumatic signal generating equipment which amplified it further and sent it to the pneumatic signal generator. A frequency counter was used to accurately determine the actual frequency of the signal generated by the wave analyzer.

The signals in the line were measured by two dynamic pressure transducers. These signals were sent to charge amplifiers. The amplified signal was then passed to the dual beam oscilloscope and to the two wave analyzers. The oscilloscope was used to monitor the signal and to measure the phase delay. The wave analyzers were used to measure the RMS output voltage of the signals.

The mean flow rate was established using the rotometer. Once the mean flow rate was established the rotometer was removed and the flow rate was monitored using the static pressure transducer. The losses associated with the rotometer were small enough that its removal caused
negligible changes in flow rate. The rotometer was calibrated before any tests were conducted.

**Test Configurations**

Six different line configurations were used for the experiment. A schematic of the basic line configurations is shown in Fig 3. Each test line consisted of plexiglas fixtures at each end which contained the transducer cavities and a smooth steel line of constant diameter between the two plexiglas blocks. The lines were smooth enough to accurately predict the pressure drop in the line using equation (37) for turbulent flow and equation (38) for laminar flow.

**Classification of Cases**

The cases were designated using a two digit system. The first digit indicates the line being tested and corresponds to the line numbers used in Table I. The second digit refers to the Reynolds number at which the line is being tested. A one indicates a Reynolds number of 2000, a two indicates a Reynolds number of 5000, and a 3 indicates a Reynolds number of 10000. For example case 52 is line 5 tested at a Reynolds number of 5000.

Line 1 was tested in several different configurations before taking data for all of the lines at the various Reynolds numbers. One test was to verify that the length of the line from the pneumatic signal generator to the test lines did not have an adverse effect on the signals and the
All lines have a circular cross section

All dimensions are in inches

For values of $L_p$, $l$, $\ldots$, and $L_1$ see Table I

Figure 3 Test Line Configuration
Table I

Line dimensions for different configurations

<table>
<thead>
<tr>
<th>Line number</th>
<th>D</th>
<th>D</th>
<th>H</th>
<th>H</th>
<th>L</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.251</td>
<td>0.195</td>
<td>0.181</td>
<td>0.029</td>
<td>24.00</td>
</tr>
<tr>
<td>2</td>
<td>0.190</td>
<td>0.119</td>
<td>0.190</td>
<td>0.029</td>
<td>24.00</td>
</tr>
<tr>
<td>3</td>
<td>0.065</td>
<td>0.041</td>
<td>0.202</td>
<td>0.029</td>
<td>24.00</td>
</tr>
<tr>
<td>4</td>
<td>0.251</td>
<td>0.195</td>
<td>0.186</td>
<td>0.028</td>
<td>36.00</td>
</tr>
<tr>
<td>5</td>
<td>0.190</td>
<td>0.119</td>
<td>0.182</td>
<td>0.026</td>
<td>36.00</td>
</tr>
<tr>
<td>6</td>
<td>0.065</td>
<td>0.041</td>
<td>0.187</td>
<td>0.027</td>
<td>36.00</td>
</tr>
</tbody>
</table>

All dimensions are in inches
other to determine the effect of the volume immediately beneath the dynamic transducers. These two cases were tested at Reynolds number 2000 on line 1 and were designated cases 11A and 11B respectively.
IV. Experimental Procedure

The same procedure was used consistently to collect data for each test. Equation (37) or (38) was used to calculate a pressure drop for the line at the Reynolds number desired. The output of the static pressure transducer was then used to set up the equivalent pressure drop. The rotometer was then used to verify that the flow rate was correct for the case. The rotometer was then removed and the static transducer was used to monitor the mean pressure for any variations in flow rate. Test runs were made at Reynolds numbers of 2000, 5000, and 10000 for the four smaller diameter lines. The two 0.195 diameter lines were run at Reynolds numbers of 2000 and 5000 due to the mass flow constraints of the pneumatic signal generator.

The barometric pressure and temperature in the room were recorded before each test run. The desired flow rate was set using the procedure outlined above then the wave analyzer was used to set the frequency of the signal. The RMS output of the sending and receiving transducers was recorded from the wave analyzers and the phase lag was measured on the oscilloscope.
V. Results and Discussion

The computer program used to predict the theoretical gain and phase shift was based on that used by Malanowski (Ref 9). It was extensively modified to incorporate the constant LRC turbulent models. In order to verify that the original laminar section still functioned, the program was used to predict the gain and phase shift in a blocked line. Then, the program was used to predict the gain and phase shift for each of the six lines with laminar mean flow. Table II gives a summary of test conditions and some experimental and theoretical data for the cases with Reynolds numbers of 2000. Figures 4-18 show the experimental and theoretical gains and phase shifts for these cases.

There are no phase diagrams for line 6 at any of the Reynolds numbers tested; this is due to the fact that the noise in line 6 made accurate collection of phase lag impossible. This is also true of several of the other lines at higher Reynolds numbers.

Figures 4-18 verify that Nichols equations are very good at predicting both the gain and phase shift for lines with laminar mean flow. Only cases 31 and 61 show major discrepancies, occurring mainly at lower frequencies. This may be partially due to noise problems, described above, which were associated with the two smallest lines. All of the laminar curves however, showed a 1-2 db error at
### Table II

**Test conditions**

<table>
<thead>
<tr>
<th>Case Number</th>
<th>P psig</th>
<th>Q cis</th>
<th>Re</th>
<th>( \bar{u} ) fps</th>
<th>( \omega ) rad/sec</th>
<th>( P_\infty ) psia</th>
<th>T( _\infty ) deg F</th>
</tr>
</thead>
<tbody>
<tr>
<td>11</td>
<td>0.016</td>
<td>7.61</td>
<td>2012</td>
<td>21.2</td>
<td>20.8</td>
<td>14.37</td>
<td>75.2</td>
</tr>
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<td>11A</td>
<td>0.016</td>
<td>7.61</td>
<td>2012</td>
<td>21.2</td>
<td>20.8</td>
<td>14.37</td>
<td>75.2</td>
</tr>
<tr>
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<td>7.61</td>
<td>2012</td>
<td>21.2</td>
<td>20.8</td>
<td>14.37</td>
<td>75.2</td>
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<tr>
<td>12</td>
<td>0.116</td>
<td>18.93</td>
<td>5003</td>
<td>52.8</td>
<td>20.8</td>
<td>14.30</td>
<td>75.2</td>
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<tr>
<td>21</td>
<td>0.069</td>
<td>4.58</td>
<td>1992</td>
<td>34.3</td>
<td>55.7</td>
<td>14.39</td>
<td>75.2</td>
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<tr>
<td>22</td>
<td>0.493</td>
<td>11.18</td>
<td>4944</td>
<td>83.8</td>
<td>54.8</td>
<td>14.39</td>
<td>75.2</td>
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<tr>
<td>23</td>
<td>1.554</td>
<td>21.29</td>
<td>9695</td>
<td>159.5</td>
<td>53.2</td>
<td>14.39</td>
<td>77.0</td>
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<td>1.50</td>
<td>2012</td>
<td>94.7</td>
<td>448.5</td>
<td>14.53</td>
<td>77.0</td>
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<tr>
<td>32</td>
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<td>3.03</td>
<td>5037</td>
<td>191.3</td>
<td>361.5</td>
<td>14.45</td>
<td>77.9</td>
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<tr>
<td>33</td>
<td>23.385</td>
<td>4.42</td>
<td>9814</td>
<td>279.0</td>
<td>270.7</td>
<td>14.40</td>
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<tr>
<td>42</td>
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<td>18.66</td>
<td>4974</td>
<td>52.1</td>
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<td>14.39</td>
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<tr>
<td>51</td>
<td>0.099</td>
<td>4.58</td>
<td>1986</td>
<td>34.3</td>
<td>55.9</td>
<td>14.34</td>
<td>77.0</td>
</tr>
<tr>
<td>52</td>
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<td>446.6</td>
<td>14.37</td>
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<tr>
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<td>2.46</td>
<td>4059</td>
<td>155.3</td>
<td>363.5</td>
<td>14.39</td>
<td>78.8</td>
</tr>
<tr>
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<td>3.48</td>
<td>7457</td>
<td>219.7</td>
<td>280.3</td>
<td>14.39</td>
<td>78.8</td>
</tr>
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</table>
Figure 4  Experimental and Theoretical Gain vs Frequency for Case 11
Figure 6: Experimental and Theoretical Gain vs Frequency for Case II A
Figure 9: Experimental and Theoretical Phase vs Frequency for Case 11B
Figure 12 Experimental and Theoretical Gain vs Frequency for Case 31
Figure 14 Experimental and Theoretical Gain vs Frequency for Case 41
Figure 15 Experimental and Theoretical Phase vs Frequency for Case 41
Figure 16 Experimental and Theoretical Gain vs Frequency for Case 51

Reynolds number = 1986
Figure 18 Experimental and Theoretical Gain vs Frequency for Case 61

Reynolds number = 1996

Experiment
Theory
frequencies below approximately which approaches the low frequency limit for which Krishnaiyer and Lechners equations are applicable.

Turbulent Flow

Figures 19-34 show the gain and phase shift for Reynolds numbers from 5,000 to 10,000. Table III gives the value of the nondimensional frequency (used in Fig. 1) which corresponds to the highest tested frequency of 1000 hz for each case. The value of , which corresponds to the intersection of the laminar and turbulent attenuation curves is also found in Table III for both AC and DC resistance.

The method used to predict the gain and phase shift employed the turbulent model up to the break frequency . From this frequency up Krishnaiyer and Lechners equations were used to predict the gain and phase shift. Because of this the curves representing the constant LRC models are only plotted up to their break frequency.

Figures 19-26 show the theoretical gain and phase shift and the experimental gain and phase shift for four of the lines at a Reynolds number of 5,000. The figures show that Nichols theory is accurate above the break frequency. Figures 27-28 show large discrepancies in all of the theories although the constant LRC with AC resistance is good at high frequencies. The constant LRC models appear to be inadequate to predict the phase shift for any Reynolds number tested while Krishnaiyer and Lechners equations

36
Table III

Values of $\Omega_{\text{max}}$, $\omega_{b,\text{DC}}$, and $\omega_{b,\text{AC}}$ for the Turbulent cases

<table>
<thead>
<tr>
<th>Case Number</th>
<th>$\Omega_{\text{max}}$ (rad/sec)</th>
<th>$\omega_{b,\text{DC}}$ (rad/sec)</th>
<th>$\omega_{b,\text{AC}}$ (rad/sec)</th>
</tr>
</thead>
<tbody>
<tr>
<td>12</td>
<td>2416</td>
<td>260</td>
<td>878</td>
</tr>
<tr>
<td>22</td>
<td>919</td>
<td>684</td>
<td>2311</td>
</tr>
<tr>
<td>23</td>
<td>946</td>
<td>2005</td>
<td>6639</td>
</tr>
<tr>
<td>32</td>
<td>141</td>
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<td>42</td>
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<td>52</td>
<td>931</td>
<td>675</td>
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<td>53</td>
<td>976</td>
<td>1945</td>
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<td>62</td>
<td>140</td>
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<td>63</td>
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<td>10420</td>
<td>34503</td>
</tr>
</tbody>
</table>
Figure 19 Experimental and Theoretical Gain vs Frequency for Case 12
Figure 21 Experimental and Theoretical Gain vs Frequency for Case 22

Experiments by Krishnanayar & Lechners Eqs.

Constant LR model with DC Resistance

Constant LR model with AC Resistance

Reynolds number = 4944
Figure 22 Experimental and Theoretical Phase vs Frequency for Case 22
Figure 23 Experimental and Theoretical Gain vs Frequency for Case 42
Figure 24 Experimental and Theoretical Phase vs Frequency for Case 42
Figure 25 Experimental and Theoretical Gain vs Frequency for Case 52
Figure 26 Experimental and Theoretical Phase vs Frequency for Case 52
Figure 27 Experimental and Theoretical Gain vs Frequency for Case 32
Figure 28: Experimental and Theoretical Gain vs Frequency for Case 62

- Constant LRC model with DC resistance
- Constant LRC model with AC resistance

Reynolds number = 4059

Experiment

Krishnaiyer & Lechners Eqs.
Figure 29 Experimental and Theoretical Gain vs Frequency for Case 23
Figure 30 Experimental and Theoretical Phase vs Frequency for Case 23
Figure 31 Experimental and Theoretical Gain vs Frequency for Case 53
Figure 33 Experimental and Theoretical Gain vs Frequency for Case 33

Reynolds number = 9814
Figure 34 Experimental and Theoretical Gain vs Frequency for Case 63
are accurate for phase information for most of the lines tested.

Figures 29 and 31 best demonstrate how Krishnaiyer and Lechners equations predict attenuations which are too low. The constant LRC model with DC resistance also predicts attenuations which are too low while using the AC resistance results in much better predictions in attenuation and gain. Both of the constant LRC models are shifted on the frequency axis in the figures. The phase shift predicted is also very far off. This may be due to the assumption made in the models to use the isothermal capacitance. Moore (Ref 4) showed that the capacitance in blocked lines is isothermal at low frequencies and adiabatic at high frequencies. The author knows of no method at this time to determine the frequency dependence of the capacitance for turbulent mean flows.

Cases 33 and 63 are similar to cases 32 and 62 in that the theory does not predict the gain very well. There are very large pressure drops associated with these cases. The models were all developed for incompressible flow; this may be one of the reasons the theoretical curves are off so much. In addition, Mach number effects were not accounted for. There is no phase data available for these two cases because of the noise problem discussed earlier.
VI. Conclusions

1. The constant LRC model with AC resistance can accurately predict the attenuation but cannot predict the phase shift as it is used here. Because of this the model cannot be used to predict the gain because it shifts the frequency of the harmonic peaks. At this time no method exists to predict the difference in capacitance due to turbulent boundary layers. The changes in capacitance may be important in matching theoretical solutions with data.

2. The constant LRC model is most accurate when the AC resistance is used. Using the DC resistance causes the attenuation to be underestimated and prevents the model from being used to predict gain.

3. None of the models or equations used performs well in cases with large pressure drops (compressible flow).

4. The propagation characteristics of fluid transmission lines with turbulent flow exhibits laminar characteristics at high frequencies and increased attenuations at low frequencies.
VII. Recommendations

1. Studies of the effects of turbulent boundary layers on capacitance should be conducted. If possible simple models should be developed to predict the capacitance.

2. Studies on the effects of compressible flow on the propagation characteristics of fluid transmission lines should be conducted.

3. Experimental and analytical research should be conducted on the low frequency characteristics of transmission lines with turbulent mean flow.
Bibliography


Appendix

Computer Program for the Solution of the Frequency Response of Pneumatic Transmission Lines With Turbulent Mean Flow
Computer Program Input Sequence

Basic Program

1. Read ICAS: Reads the experiment case number.

2. Read TF, PG, AMU, RE, GAM, SIG:
   - TF is the line temperature in deg F
   - PG is the atmospheric pressure
   - AMU is the dynamic viscosity
   - RE is the gas constant
   - GAM is the ratio of specific heats
   - SIG is the square root of the Prandtl number.

3. Read N: N is the number of lines.

4. Read DI(I), AD(I), P(I):
   - DI is the line diameter
   - AD is the line length
   - P is the line pressure.

Calcomp Plotter Subroutine

5. Read CASE(I): CASE same as ICAS.

6. Read NPTS, LSMB:
   - NPTS is the number of experimental input points
   - LSMB is the plotter symbol used to represent the experimental data

7. Read FREQ, PS, PR, PHT
   - FREQ is the experimental frequency at which the data was recorded.
   - PS is the RMS voltage of the sending dynamic transducer
   - PR is the RMS voltage of the receiving dynamic transducer
   - PHT is the phase lag between the sending and receiving ports in milliseconds.
Program MSB(Input, Output, Tape5=Input, Tape6=Output, Tape7, Tape9)

C TRANSMISSION LINE STUDY/Z-IN METHOD WITH BRANCH SHORT OUTPUT

* DIMENSION DBT(300), DI(100), AD(100), CV(100), RVT(100),
  1OMT(100), CMA(100), CALM(100), FNM(100), RN(100), GN(100),
  2ALN(100), CN(100), AN(100), BTN(100), AMC(100), ARZ(100), BZRN(100),
  3AZIN(100), BZIN(100), DC(100), DG(100), DD(100), P(100), RHO(100),
  4ANU(100), CA(100), TCA(100), OMG(300), RTP(100), GP(100), ETA(100),
  5AZOT(100), BZOT(100), REY(100), Q(100), BET(100), TURB(100),

* DIMENSION DB(300), OMGX(300), OMP(300), PHASE(300), PH(300), CASE(100)

DATA PI/3.1415926/
DATA TPI/6.2831853/

C*************************************************************************
C*************************************************************************
C*************************************************************************

C CALL PLITS(0., 0., 9)

1 READ(5,*) ICAS
110 READ(5,*) TF, PG, AMU, RE, GAM, SIG
   READ(5,*) N

C*************************************************************************
C*************************************************************************
C*************************************************************************

C AD IS THE LINE LENGTH
C DI IS THE LINE DIAMETER
C P IS THE INLET PRESSURE
C A ONE FOR NLINE OR MLINE INDICATES THE LINE IS OPEN AND ANY OTHER
C NUMBER INDICATES A BLOCKED END.

C*************************************************************************

DO 898 I=1, N
   READ(5,*) DI(I), AD(I), P(I)
898 CONTINUE

NPG=0
ICT=1
IND=1
NNN=1
M=0

C*************************************************************************
C*************************************************************************
C*************************************************************************

C DETERMINE REYNOLDS NUMBER FOR LINE
TOL=0.0
DO 2 I=3, N
   TOTL=TOTL+AD(I)
2 CONTINUE

RHO(N)=(PG+P(N))/(RE*(TF+460))
REYT=(PG*P(N)+.5*(P(N)**2.))*(DI(N)**3.)/32./AMU/AMU
REYT=REYT/TOTL/RE/(TF+460.)
IF (REYT.LT.2300.) GOTO 3
REYT=(REYT*32./.1582)**(4./7.)
3 QM=REYT*DI(N)*AMU/4.
DO 23 I=1, N
   PBR=P(I)+PG
   TBR=TF+460.
   RHO(I)=PBR/(RE*TRB)
   AMU(I)=AMU/RHO(I)
\[ CA(I) = \sqrt{PBR \cdot GAM / RHO(I)} \]
\[ AR(I) = PI \cdot DI(I) / 4 \cdot DI(I) \]
\[ CV(I) = (8 \cdot PI \cdot ANU(I)) / AR(I) \]
\[ OMT(I) = CV(I) / (SIG * SIG) \]
\[ CNA(I) = (8 \cdot PI \cdot AMU) / (AR(I) * AR(I)) \]
\[ Q(I) = QM / RHO(I) \]

CONTINUE

Q(1) = 0.0
Q(2) = 0.0
GAM1 = 0.5 \cdot (GAM - 1)

DO 26 I = 1, N

REY(I) = (4 \cdot RHO(I) \cdot Q(I)) / (PI \cdot DI(I) \cdot AMU)

PBR = P(I) + PG

TEMP = GAM1 / (GAM \cdot PBR)

GMA(I) = TEMP / AR(I)

AGM(I) = AR(I) / (GAM \cdot PBR)

QTCA(I) = 0.25 \cdot CA(I)

RVT(I) = 0.3164 \cdot AMU / (AR(I) \cdot DI(I) / DI(I))

RVT(I) = RVT(I) \cdot (REY(I) ^{0.75})

26 FN(I) = QTCA(I) / AD(I)

NST = 1

MW = 0

Y = 1

DO 80 J = 1, NST

M = M + 1

Y = Y + MW

W = TPI * Y

DO 27 I = 1, N

ARG = 5 \cdot SQRT(W / CV(I))

27 RN(I) = CNA(I) \cdot (0.375 + ARG + (0.375 / (4 \cdot ARG)))

DO 28 I = 1, N

DC(I) = 0.25 \cdot SQRT(W / OMT(I)) + 0.125 \cdot SQRT(OMT(I) / W)

DG(I) = SQRT(W / OMT(I)) - 0.125 \cdot SQRT(OMT(I) / W)

DD(I) = DC(I) + DC(I) + DG(I) + DG(I)

DO 29 I = 1, N

ARG = 5 \cdot SQRT(CV(I) / W)

29 GN(I) = W \cdot (GAM - 1) \cdot AGM(I) / DD(I)

DO 29 I = 1, N

ARG = 5 \cdot SQRT(CV(I) / W)

29 ALN(I) = RHO(I) \cdot (1 + ARG - (ARG \cdot (15 \cdot CV(I) / (W \cdot 64)))) / AR(I)

TEMP = GAM1 / W

DO 30 I = 1, N

TURB(I) = 0

30 CN(I) = AGM(I) \cdot (1 + (GAM - 1) \cdot DC(I) / DD(I))

TEMP = W

DO 31 I = 1, N

TEM1 = RN(I) \cdot GN(I) + TEMP \cdot ALN(I) \cdot CN(I)

TEM2 = RN(I) \cdot CN(I) + GN(I) \cdot ALN(I)

CALL RTCMP(ARG1, ARG2, TEM1, TEM2)

AN(I) = ARG1

BTN(I) = ARG2

TEMP = W \cdot RHO(I) / (P(I) + PG)

TEMP2 = W \cdot AR(I) \cdot RVT(I) / (P(I) + PG)

CALL RTCMP(ARG1, ARG2, TEM1, TEM2)

TAN = ARG1
TBTN=ARG2
IF(AN(I).GE.TAN) GO TO 31
AN(I)=TAN
BTN(I)=TBTN
TURB(I)=1
31 ANC(I)=TPI/BTN(I)
C CALCULATE ZO
DO 32 I=1,N
TEM1=W*ALN(I)
TEM2=W*CN(I)
TEM3=RN(I)
TEM4=CN(I)
CALL CMPDV(ARG1,ARG2,TEM3,TEM1,TEM4,TEM2)
CALL RTCMP(AZRN(I),BZRN(I),ARG1,ARG2)
AZRN(I)=AZRN(I)
BZRN(I)=BZRN(I)
IF(TURB(I).EQ.0) GO TO 32
TEM1=RHO(I)*(P(I)+PG)/AR(I)/AR(I)
TEM2=RT(V(I))*(P(I)+PG)/V/AR(I)
CALL RTCMP(ARG1,ARG2,TEM1,TEM2)
AZRN(I)=ARG1
BZRN(I)=ARG2
32 CONTINUE
C CALCULATE Z IN 1
I=1
TEM=AN(I)*AD(I)
IF(TEM.GT.88.) GO TO 80
ARG1=COSH(TEM)
ARG2=SINH(TEM)
TEM5=BTN(I)*AD(I)
TEM=COS(TEM5)
TEM1=ARG1*TEM
TEM3=ARG2*TEM
TEM4=ARM1*TEM
TEM5=BTN(I)*AD(I)
CALL CMPDV(ARG1,ARG2,TEM3,TEM1,TEM4,TEM5)
TEM1=AZRN(I)
TEM2=BZRN(I)
CALL CMPMP(TEM3,TEM4,TEM1,TEM2,ARG1,ARG2)
AZRN(I)=TEM3
BZRN(I)=TEM4
C CALCULATE Z IN 3
I=3
TEM1=AZRN(I)
TEM2=BZRN(I)
TEM3=0.
TEM4=0.
TEM5=AN(I)
TEM6=AD(I)
TEM7=BTN(I)
CALL CALZIN(AARG,BARG,TEM1,TEM2,TEM3,TEM4,TEM5,TEM6,TEM7)
AZRN(I)=AARG
BZIN(I) = BARG
C CALCULATE Z IN 4, 5
DO 34 I = 4, 5
   TEM1 = AZRN(I)
   TEM2 = BZRN(I)
   TEM3 = AZIN(I - 1)
   TEM4 = BZIN(I - 1)
   TEM5 = AN(I)
   TEM6 = AD(I)
   TEM7 = BTN(I)
   CALL CALZIN(AARG, BARG, TEM1, TEM2, TEM3, TEM4, TEM5, TEM6, TEM7)
   AZIN(I) = AARG
34 BZIN(I) = BARG
C CALCULATE Z IN 2
   I = 2
   TEM1 = AZRN(I)
   TEM2 = BZRN(I)
   TEM3 = AZIN(I - 1)
   TEM4 = BZIN(I - 1)
   TEM5 = AN(I)
   TEM6 = AD(I)
   TEM7 = BTN(I)
   CALL CALZIN(AARG, BARG, TEM1, TEM2, TEM3, TEM4, TEM5, TEM6, TEM7)
   AZIN(I) = AARG
   BZIN(I) = BARG
C CALCULATE RECEIVING Z FOR LINE 6
   I = 6
   TEM1 = AZIN(I - 1)
   TEM2 = BZIN(I - 1)
   TEM3 = AZIN(I - 4)
   TEM4 = BZIN(I - 4)
   CALL ZEBRA(AZOTI, BZOTI, TEM1, TEM2, TEM3, TEM4)
   AZOT(I) = AZOTI
   BZOT(I) = BZOTI
C CALCULATE Z IN 6 INCLUDING R-TRANSUDER
   TEM1 = AZRN(I)
   TEM2 = BZRN(I)
   TEM3 = AN(I)
   TEM4 = AD(I)
   TEM5 = BTN(I)
   CALL CALZIN(AZINI, BZINI, TEM1, TEM2, AZOTI, BZOTI, TEM3, TEM4, TEM5)
   AZIN(I) = AZINI
   BZIN(I) = BZINI
C CALCULATE Z IN 7, 8, 9, AND 10 INCLUDING R-TRANSUDERS
DO 39 I = 7, 10
   TEM1 = AZRN(I)
   TEM2 = BZRN(I)
   TEM3 = AZIN(I - 1)
   TEM4 = BZIN(I - 1)
   TEM5 = AN(I)
   TEM6 = AD(I)
   TEM7 = BTN(I)
   CALL CALZIN(AZINI, BZINI, TEM1, TEM2, TEM3, TEM4, TEM5, TEM6, TEM7)
AZIN(I)=AZINI
39 BZIN(I)=BZINI
C CALCULATE P5/P6 INCLUDING R-TRANSDUCER
  I=6
  TEMP=BTN(I)*AD(I)
  CSBI1=COS(TMP)
  SNBI1=SIN(TMP)
  TEMP=AN(I)*AD(I)
  ARG1=COSH(TMP)
  ARG2=SINH(TMP)
  TEM1=ARG1*CSBI1
  TEM2=ARG2*SNBI1
  AZOTI=AZOT(I)
  BZOTI=BZOT(I)
  AZINI=AZIN(I)
  BZINI=BZIN(I)
  CALL CMPDV(TEM7,TEM8,AZOTI,BZOTI,AZINI,BZINI)
  CALL CMPMP(TEM3,TEM4,TEM7,TEM8,TEM1,TEM2)
  TEM5=ARG2*CSBI1
  TEM6=ARG1*SNBI1
  TEM9=AZRN(I)
  TEM10=BZRN(I)
  CALL CMPDV(TEM7,TEM8,AZOTI,BZOTI,TEM9,TEM10)
  CALL CMPMP(TEM1,TEM2,TEM7,TEM8,TEM5,TEM6)
  TEM1=TEM3-TEM1
  TEM2=TEM4-TEM2
  TEMP=TEM1*TEM1+TEM2*TEM2
  BETA(I)=ATAN2(TEM2,TEM1)
  BET(I)=(180./PI)*BETA(I)
  RTP(I-1)=SQRT(TEMP)
  GP(I-1)=20.*ALOG10(RTP(I-1))
C CALCULATE P6/P7, P7/P8, P8/P9, AND P9/P10 INCLUDING R-TRANSDUCER
DO 43 I=7,10
  TEMP=BTN(I)*AD(I)
  CSBI1=COS(TEMP)
  SNBI1=SIN(TEMP)
  TEMP=AN(I)*AD(I)
  ARG1=COSH(TEMP)
  ARG2=SINH(TEMP)
  TEM1=ARG1*CSBI1
  TEM2=ARG2*SNBI1
  AZINII=AZINI(I-1)
  BZINII=BZIN(I-1)
  AZINI=AZIN(I)
  BZINI=BZIN(I)
  CALL CMPDV(TEM7,TEM8,AZINII,BZINII,AZINI,BZINI)
  CALL CMPMP(TEM3,TEM4,TEM7,TEM8,TEM1,TEM2)
  TEM5=ARG2*CSBI1
  TEM6=ARG1*SNBI1
  TEM9=AZRN(I)
  TEM10=BZRN(I)
  CALL CMPDV(TEM7,TEM8,AZINII,BZINII,TEM9,TEM10)
  CALL CMPMP(TEM1,TEM2,TEM7,TEM8,TEM5,TEM6)
TEM1 = TEM3 - TEM1
TEM2 = TEM4 - TEM2
TEM = TEM1 * TEM1 + TEM2 * TEM2
BETA(I) = ATAN2(TEM2, TEM1)
BET(I) = (180. / PI) - BETA(I)
RTP(I-1) = SQRT(TEM)

43 GP(I-1) = 20. * ALOG10(RTP(I-1))
RPT = RTP(5) * RTP(6) * RTP(7) * RTP(8) * RTP(9)
GPT = 20. * ALOG10(RPT)

35 IF (BETAR .LE. 0.) GOTO 36
BETAR = BETAR - TPI
GOTO 35

36 IF (BETAR .GT. -TPI) GOTO 37
BETAR = BETAR + TPI
GOTO 36

37 BETAD = (180. / PI) * BETAR
WRITE(6,700) Y, BETAD, RTP, GPT

700 FORMAT (2X, 6HFREQ, 3X, 11HBETA( Deg), 3X, 11PE12.4, 1X, PHASE(I), 1PE12.4, 2HDB)

80 FORMAT(1HO, 13X, 6HLENGTH, 9X, 8HDIAmETER, 7X, 8HPRESSURE, 9X, 5HC ADB, 10X
1, 3HFN, 7X, 7HDENSITY, 5X, 11HREYNOLDS NO)

802 FORMAT (1HO, 1X, 5HLINE, 12, 3X, 8(1PE12.4, 2X))

803 WRITE(6,802) I, AD(I), DI(I), P(I), CA(I), FN(I), RHO(I), REY(I)

NNN = 2
80 CONTINUE
GO TO (85, 611), IND

85 Y = 0.
IW = 5.
NST = 200
IND = 2
GO TO 40

C*************************************************************************************
C*************************************************************************************
C*************************************************************************************
C*************************************************************************************
C*************************************************************************************
C*************************************************************************************
C*************************************************************************************
C THIS SECTION IS FOR THE CALCOMP PLOTTER. ONE SET OF EXPERIMENTAL DATA MUST BE INCLUDED WITH EACH RUN.
C
C NPTS IS THE NUMBER OF EXPERIMENTAL POINTS TO BE INPUT.
C LSMB IS THE PLOTTER SYMBOL TO BE USED FOR GPX AND PHD.
C SEE A CALCOMP PLOTTER USERS MANUAL FOR DESCRIPTION.
C FREQ, PS, PR, AND PHD ARE THE INPUT EXPERIMENTAL VALUES.
C
C**********
C
611 IF (ICAS) 502, 502, 610
610 CALL FACTOR(0.625)
       OMG(M+1)=0.
       OMG(M+2)=100.
       DBT(M+1)=-30.
       DBT(M+2)=5.0
       CALL AXIS(0.,0.,17HFREQUENCY (HERTZ),-17,10.0,0.,OMG(M+1),
                  +OMG(M+2))
       CALL AXIS(0.,0.,15HGAIN (DECIBELS),15,8.,90.,DBT(M+1),
                  +DBT(M+2))
       CALL LINE(OMG,DBT,M,1,0,4)

204 FORMAT(6X,1A2)
205 READ(7,204)CASE(1)
206 READ(7,*)NPTS,LSMB
207 WRITE(6,207)
       FORMAT(1HO,10X,HN4X,4HFREQ,10X,2HPS,1X,2HPR,1X,4HGAIN,10X,
              +5HPHASE)
       J=0
       DO 69 I=1,NPTS
612 READ(7,*)FREQ,PS,PR,PHT
       GPX=20.*ALOG10(PR/PS)
       PHD=PHT*FREQ*.36
       WRITE(6,209)FREQ,PS,PR,GPX,PHD
209 FORMAT(7X,15,5F12.5)
       DB(I)=GPX
       OMGX(I)=FREQ
       IF(PHD.GT.0) GOTO 69
       J=J+1
       PH(J)=PHD
       OMPG(J)=FREQ
       JMAX=J
69 CONTINUE
       OMPX(NPTS+1)=OMG(M+1)
       DB(NPTS+1)=DBT(M+1)
       OMPX(NPTS+2)=OMG(M+2)
       DB(NPTS+2)=DBT(M+2)
       CALL LINE(OMGX,DB,NPTS,1,-1,LSMB)
       CALL PLOT(15.0,0,0,-3)
       CALL FACTOR(0.625)
       OMG(M+1)=0.
       OMG(M+2)=100.
       PHASE(M+1)=-360.
       PHASE(M+2)=45.
       CALL AXIS(0.,0.,17HFREQUENCY (HERTZ),-17,10.0,0.,OMG(M+1),
                  +OMG(M+2))
       CALL AXIS(0.,0.,21PHASE ANGLE (DEGREES),21,8.,90.,PHASE(M+1),
                  +PHASE(M+2))
       CALL LINE(OMG,PHASE,M,1,0,4)
       OMPG(JMAX+1)=OMG(M+1)
       OMPG(JMAX+2)=OMG(M+2)
```
PH(JMAX+1)=PHASE(M+1)
PH(JMAX+2)=PHASE(M+2)
CALL LINE(OMGP,PH,JMAX,1,-1,LSMB)
CALL SYMBOL(15.,5.,165,5HCASE,0.,5)
CALL SYMBOL(16.,5.,165,CASE(1),0.,2)
1003 CALL PLOT(10.,0.,-3)
C
C ..........................................................
C
C
C
501 CALL PLOTE
502 STOP
END
C CALCULATES ROOT OF A COMPLEX NUMBER
SUBROUTINE RTCMP(X,Y,A,B)
CALL ANGL(TEMP,A,B)
TEMP=.5*TEMP
Y=A*A+B*B
X=SQR(T(Y))
X=SQR(T(X))
Y=X*SIN(TEMP)
X=X*COS(TEMP)
RETURN
END

C MULTIPLIES TWO COMPLEX NUMBERS
SUBROUTINE CMPMP(X,Y,A1,A2,B1,B2)
X=A1*B1-A2*B2
Y=A1*B2+A2*B1
RETURN
END

C FINDS THE QUOTIENT OF TWO COMPLEX NUMBERS
SUBROUTINE CMPDV(C1,C2,A1,A2,B1,B2)
C1=A1*B1+A2*B2
C1=C1/TEMP
C2=C2/TEMP
RETURN
END

SUBROUTINE HSINX(ARG,X)
A=EXP(X)
B=EXP(-X)
A=A-B
ARG=.5*A
RETURN
END

SUBROUTINE HCOSX(ARG,X)
A=EXP(X)
B=EXP(-X)
A=A+B
ARG=.5*A
RETURN
END

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```
SUBROUTINE ANGL(C, A, B)
DATA PI/3.1415926/
C=ABS(B/A)
C=ATAN(C)
IF (A.GT.0.) GO TO 5
IA=1
GO TO 7
IA=0
IF (B.GT.0.) GO TO 10
IB=2
GO TO 15
IB=0
IA=IA+IB+1
GO TO (35, 30, 25, 20), IA
C=C-PI
GO TO 35
C=C
GO TO 35
C=PI-C
RETURN
END

SUBROUTINE CALZIN(AZIN1, BZIN1, AZRN1, BZRN1, AZIN2, BZIN2, AN1, DI1, BTN1)
TEKP=AN1*DI1
CALL HCOSX(ARG1, TEMP)
CALL HSINX(ARG2, TEMP)
TEMP=BTN1*DI1
CSBI1=COS(TEMP)
SNBI1=SIN(TEMP)
ZR=0.
CALL CMPMP(TEM1, TEM2, AZIN2, BZIN2, ARG1, ZR)
CALL CMPMP(TEM3, TEM4, AZRN1, BZRN1, ARG2, ZR)
CALL CMPMP(TEM5, TEM6, AZIN2, BZIN2, ARG2, ZR)
CALL CMPMP(TEM7, TEM8, AZRN1, BZRN1, ARG1, ZR)
A1=TEM1+TEM3
B1=TEM2+TEM4
A2=TEM5+TEM7
B2=TEM6+TEM8
CALL CMPMP(TEM1, TEM2, A1, B1, CSBI1, ZR)
CALL CMPMP(TEM5, TEM6, A2, B2, CSBI1, ZR)
CALL CMPMP(TEM7, TEM8, A1, B1, SNBI1)
CALL CMPMP(TEM3, TEM4, A2, B2, SNBI1)
TEM3=TEM3+TEM1
TEM4=TEM4+TEM2
TEM7=TEM7+TEM5
TEM8=TEM8+TEM6
CALL CMPDV(TEM1, TEM2, TEM3, TEM4, TEM7, TEM8)
CALL CMPPH(AZIN1, BZIN1, TEM1, TEM2, AZRN1, BZRN1)
RETURN
END

SUBROUTINE ZEBRA(C1, C2, A1, A2, B1, B2)
D1=1.
D2=0.

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CALL CMPDV(ARG1, ARG2, D1, D2, A1, A2)
CALL CMPDV(ARG3, ARG4, D1, D2, B1, B2)
ARG1 = ARG1 + ARG3
ARG2 = ARG2 + ARG4
CALL CMPDV(C1, C2, D1, D2, ARG1, ARG2)
RETURN
END
Mark S. Briski the son of Mathew and Dorothy Briski. He graduated from Elgin High School in Elgin, Illinois, in 1965. He enlisted in the Air Force in April of 1974 and was awarded an AFROTC scholarship in 1975 to attend the Illinois Institute of Technology. Upon graduation he was awarded a Bachelor of Science degree in Mechanical-Aerospace Engineering and was commissioned into the Air Force Reserve. In February of 1980 he entered active duty and was assigned to the Foreign Technology Division at Wright-Patterson AFB, Ohio. He was assigned to the Air Force Institute of Technology in June 1982.

Permanent Address: [Redacted]
Title: THE EFFECTS OF MEAN FLOW ON THE DYNAMIC CHARACTERISTICS OF FLUID TRANSMISSION LINES

Thesis Chairman: Dr. Milton E. Franke
Theoretical predictions of the small signal frequency response of round pneumatic transmission lines with turbulent mean flow were compared with experimental results. The frequency response curves were found for lines varying in length from 24 to 36 in. with inside diameters of 0.195, 0.119, and 0.041 in. The lines were tested at Reynolds numbers of 2000, 5000, and 10000.

Theoretical solutions were obtained using Nichols' equations as modified by Krishnaiyer and Lechner. Solutions were also found using several different modifications of a constant LRC model developed by Moore. The results were mixed: for the 0.195 and 0.119 in. lines the prediction of gain was good but for the 0.041 in. lines the results were poor. The accuracy and applicability of the constant LRC model was explored along with its various modifications. The constant LRC model with the AC resistance showed potential for predicting the gain in fluid transmission lines with turbulent flow. The limitations and applicability of the constant LRC models was studied.