# Effort and Accuracy in Choice

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**Abstract:**

Individuals often use several different strategies such as the expected value rule, conjunctive rule, and elimination-by-aspects, to make decisions. It has been hypothesized that strategy selection is, in part, a function of (4) the ability of a strategy to produce an accurate response and (2) the strategy's demand for mental resources or effort. We examine effort and accuracy and their role in strategy selection. Several strategies that may...
be used to make choices under risk are simulated using a production system framework. This framework allows the estimation of the effort required to use the strategy in a choice environment, while simultaneously measuring its accuracy relative to a normative model. A series of Monte-Carlo studies varied several aspects of the choice environments, including the complexity of the task and the presence or absence of dominated alternatives. These simulations identify strategies which approximate the accuracy of normative procedures while requiring substantially less effort. These results, however, are highly contingent upon characteristics the task environment. Finally, we discuss the potential of production system models in understanding task effects in decisions and the learning of effort/accuracy tradeoffs.
INTRODUCTION

One of the major findings of years of decision research is that individuals use many different cognitive processes (strategies) in making a decision. The strategy used is contingent upon characteristics of the decision task such as the number of alternatives available [37]. Many theorists suggest that the selection of decision strategies is, in part, a function of the strategy's accuracy, that is its ability to produce an accurate response, and the strategy's effort, that is its demand for mental resources [3], [19], [24], [41], [42], [48]. A view of strategy selection as involving costs and benefits has several appealing aspects. First, such a principle can maintain the assumption of calculated rationality on the part of the decision maker [29]. The use of an apparently suboptimal rule becomes optimal once again, once the costs of the decision process itself are included in the assessment of rationality. Even the most grievous errors, such as intransitive preferences, may be seen as the outcome of a rational process. As Tversky [49] has noted:

"It seems impossible to reach any definite conclusions concerning human rationality in the absence of a detailed analysis of the sensitivity of the criterion and the cost involved in evaluating the alternatives (p. 45-46)."

Second, because the costs and benefits of decision procedures will vary between tasks, this perspective may partially explain the wide variety of strategies seen in decision behavior [37].

In examining this perspective a major difficulty has been the development of measures defining the costs and benefits associated with various decision procedures. This paper proposes an approach to measuring the costs and benefits associated with decision processes based on production systems models.
and computer simulation. This approach allows the estimation of effort and accuracy as the task and context of decision situations vary. Furthermore, we will argue that production system models are a reasonable framework for exploring how decision strategies may be learned, and for guiding the design of decision support systems [22].

The paper is organized as follows: First we present a task analysis of the choice domain of concern, decisions under risk. Within this task we discuss previous definitions of accuracy and effort, and then develop possible alternative metrics. Next, we present production system representations of several heuristic decision processes, and conduct a series of Monte-Carlo experiments which estimate the accuracy and effort associated with these heuristics. The paper ends with a discussion of the cost-benefit approach to strategy selection and its relationship to learning.

Task Analysis

We focus on decisions under risk for two reasons: First, the task is representative of a large number of real-world decision situations that involve uncertain outcomes. Second, important previous work examining the accuracy of heuristics by Thorngate [48], concerns risky choice.

A risky choice problem consists of three basic components: (1) The alternatives available to the decision maker, (2) Events or contingencies that relate actions to outcomes, and (3) The values associated with the outcomes. These informational elements, along with a goal statement (such as "choose the preferred alternative."), represent the heart of the risky choice task environment presented to a decision-maker. The decision maker's internal representation of this task environment is the individual's problem space, containing the solution (i.e. the preferred alternative) which must be
identified. Newell and Simon [34] present a more detailed discussion of the concept of a problem space.

Risky choice heuristics can be defined as rules which systematically simplify search through the problem space by disregarding some elements of the problem space. Alternative simplifications represent different heuristics: A decision maker can, for example, choose to ignore certain alternatives, either by deciding that an acceptable alternative has already been found (satisficing) or by eliminating an alternative from future consideration because of an objectionable outcome, such as a ruinous loss. Other simplifications consist of selective examination of the outcomes. The Maximin choice procedure, for instance, suggests that a decision maker evaluates alternatives by examining only the outcome for each alternative with the lowest payoff. The alternative with the highest minimum payoff is selected. Another suggestion is that decision makers might ignore some of the event information present in the problem space. Thorngate [48], for example, describes the equiprobable procedure, which ignores probabilities, selecting the alternative with the highest average payoff. Finally, other simplifications can occur in the process of combining attributes, such as calculating differences in payoffs and probabilities, as suggested by the additive difference rule [49].

Of course, actual choice behavior is probably not a straightforward execution of one choice strategy or another. Bettman [4] suggests that choice may be more constructive. That is, "choice heuristics may not be stored in their entirety in memory, but may exist only as fragments-subparts which are put together constructively at the time of processing, at the time of making a decision or a choice [4, p. 33]". Nonetheless, we feel that identifying the characteristics of prototypical strategies such as Maximin and Elimination-by-aspects [50] is a useful first step in understanding why a decision maker utilizes different strategies as a function of task demands [37].
Finally other characteristics besides the accuracy and effort of choice heuristics, may influence strategy use, for example, justification [45]. Accuracy and effort, however, are likely to play a major role in strategy selection.

**Measuring Accuracy**

At a general level, quality of choice can be defined by consistencies in preference, e.g., transitivity. In the case of risky choice, however, more specific criteria have been suggested. The expected utility rule, for example, builds on principles of consistency to provide a specific mechanism for combining value and belief information into a decision. A special case of the EU rule is maximization of expected value. The main advantage of Expected Value as a choice rule is that the values of an individual decision maker are not required to operationalize the rule.

Previous work on the accuracy of heuristics by Thorngate [48] adopted this EV criterion. Using a Monte-Carlo simulation, Thorngate determined the proportion of decisions for which several heuristics selected the alternative with the highest expected value. For purposes of comparison with Thorngate's results, we adopted the same measure of accuracy, and term it proportion accurate choices (PAC).

A limitation of Thorngate's measure of accuracy, and consequently of our PAC measure, is its insensitivity to near misses, such as the selection of an alternative near the best in expected value. We therefore adopt an additional measure of accuracy that allows us to compare the relative performance of heuristics, in terms of EV, to a strict expected value rule and to a baseline response of random choice, which involves no search of the problem space:
Relative Performance = \frac{EV \text{ Heuristic Choice} - EV \text{ Random Choice}}{EV \text{ Optimal Choice} - EV \text{ Random Choice}} \quad [1]

This measure of relative performance is bounded with a value of 1.00 for the expected value maximization strategy, and 0.0 for random selection. The measure has the property that it controls for the chance of an accurate response as a function of number of alternatives, as well as reflecting the relative sizes of errors made by heuristics for each set of alternatives.

Additional measures of choice accuracy are possible. As noted above, for example, the EV rule is just a special case of the maximization of expected utility strategy for risky choice. Consequently, we examine the accuracy of heuristics using a third set of measures based on the expected utility strategy with utility defined by a power function \( U(x) = x^{2/3} \). This type of utility or value function has been recently discussed by Kahneman and Tversky [21].

Finally, we examine a fourth measure of accuracy that is independent of the form of the value or utility functions. That measure is based on the frequency of selection of dominated alternatives [15]. The number of times a dominated alternative (an alternative inferior to another on all attributes) is selected by a heuristic is a useful metric in that it is clear choice error. However, the prevalence of choice sets containing dominated alternatives is not readily apparent.

In summary, several possible definitions of a decision error exist. We use two expected value based measures: (1) The proportion of accurate choices, i.e., those with maximum EV, and (2) relative performance, which reflects the degree of improvement in EV over a random choice. Our focus on expected value based measures is due to the use of the EV concept by Thorngate [48]. However, we also examine both expected utility and dominance-based measures of accuracy. The appropriateness of these different measures of
accuracy will be a function of the choice task. One advantage of simulation, however, is the ease of examining multiple measures of accuracy.

**Measuring Effort**

Mental effort has a long and venerable history as a theoretical construct in decision-making research, and cognitive psychology in general [20], [32], [41], [47]. For example, Russo and Dosher [41] discuss several interpretations of the concept of cognitive effort. They define effort as the total use of cognitive resources required to complete the task. Since this seems a useful approach for decision research, we adopt that definition of effort.

Attempts to compare decision rules in terms of an effort metric are just beginning. Shugan [42], for example, suggested that effort or "the cost of thinking" could be captured by "a measurable (i.e. well-defined and calculable) unit of thought." He proposes the binary comparison of two alternatives on an attribute as that basic unit. The more comparisons made, the more effortful the choice. Unfortunately, Shugan's use of the binary comparison as a fundamental unit of effort restricts his analysis to certain decision rules. An important contribution of Shugan's work, however, is (1) the notion that decomposing decision strategies into components can provides estimates of their relative costs, and (2) the observation that the effort required by a choice rule can be affected by task characteristics such as the covariance between attributes. Similar ideas were also suggested by Wright [51].

Huber [18] and Johnson [19] expand this notion of decomposing choice strategies into a set of components. Drawing on ideas of Newell and Simon [34] they independently suggest that heuristic strategies can be constructed from a small set of elementary information processes (EIP's). Thus a decision rule or strategy could be thought of as a sequence of events, such as reading
the values of two alternatives on an attribute, comparing them, etc. Chase [9] provides a more general discussion of the use of the EIP concept in the analysis of information processing.

The EIP's described by Huber [18] and Johnson [19] for decision strategies are similar to those postulated for other cognitive tasks such as mental arithmetic [10] and problem solving [34]. A hope of those advancing the concept of EIP's is that there exists a small set of elementary processes common to a variety of tasks [9]. Additionally, Newell and Simon [34] have proposed that effort can be measured by the total number of elementary information processes used in a task. A relationship has been shown between the number of EIP's predicted by models and response times for a variety of cognitive tasks. For example, Carpenter and Just [8] use a production system model using elementary information processes to explain latencies in sentence-picture verification. Card, Moran and Newell [7] apply similar techniques to a more complex task, computer text editing.

Our measure of decision effort builds upon the Newell and Simon [34] suggestion. Effort will be measured in terms of the number of elementary information processes used to select an option.

Production Systems as Models: Combining Accuracy and Effort

The decomposition of common decision heuristics into component processes yields insight into the relative complexity of these rules. At the same time, the assumptions necessary to derive simple closed form expressions for estimating effort greatly limit the decision tasks that can be examined [9]. Thus, although one obtains a detailed picture of each decision rule, the picture applies to a small class of possible decision problems.

Another way of estimating effort is to implement heuristics as formal symbolic systems which can be simulated on a computer. One framework is a
production system [34], which consists of a set of productions, a task
environment, and a working memory. The productions specify a set of actions
(EIP'S) and the conditions under which they occur. These are expressed as a
(condition) \(\rightarrow\) (action) pair, and the actions specified in a production are
performed (fire) only when the condition side is satisfied by matching the
contents of working memory. Working memory is a set of symbols, both those
read from the external environment, and those deposited by the actions
performed by previous productions. The set of productions possessed by an
individual can be thought of as being part of long-term memory. Arguments for
the value of production systems as a representation of human cognitive
processes and further descriptions of production systems are presented by
Newell [33]. Note that productions are similar to the types of if-then rules
that are often used to represent knowledge in expert or artificial
intelligence systems designed to aid human judgments [11].

Table I lists the set of elementary processes, similar to those described
by Johnson and Huber, which were used in building the production system
representations of the choice rules. Figure 1 contains the production system
representation of the expected value rule, which selects the alternative with
the highest expectation from the set. This production system contains three
productions, each of which performs the actions listed on the right-hand side
of the figure only when the condition on the left hand side is true. Thus, at
the beginning of the decision, only the third production would be true, and
the production system would then READ the payoff for the first alternative
into working memory, MOVE its attention to the probability of that outcome,
READ it, and use the PRODUCT operator to weight the payoff by is probability.
This result is then ADDED to a running sum for that alternative, and attention
is then MOVED to the next payoff. This production continues to be applied
until all outcomes have been examined. Now the second production fires, and
COMPAREs this alternative to the best found until now, and marks the winner as the current best alternative found. This process repeats until all alternatives have been examined, and the condition side of the first production in the list becomes true, announcing that the alternative which is the current best alternative has been chosen.

There are several things worth noting about this production system: its components. First, although expectation-based decision rules are generally thought to be among the most effortful, the rule can be implemented without making large demands on working memory. This is accomplished by combining the partial results as soon as possible (note the ADD operation in Figure 1). All the decision rules we discuss operate similarly, and do not store results in long-term memory. Additionally, all are designed to minimize the number of operations. Because human decision makers may not necessarily adopt this technique, our implementations represent minimum estimates of the effort required to use each strategy. For example, variations of the strategies that would use long-term memory operations would lead to greater estimates of effort. Recognize also that the adjustment of values by probabilities implied by the product operator, for example, may not involve a literal multiplication of two numbers, rather they may be combined by some analogical process which adjusts the value of one quantity given another [27].

There also are several conflict resolution mechanisms proposed to select a production to execute if more than one is true. Our implementations simply assume that the first production in the list whose condition side is matched fires. Finally, it is worth noting that these elementary processes are similar to those found in studies of other cognitive tasks, and that estimates of the time required for each operation have been made. In principle it should be possible to construct and test estimates of the time necessary to execute these rules. We will turn to this point later.
Implementing production systems as a computer program is straightforward. Through Monte-Carlo techniques, it is possible to observe the choice that would be made by each rule over many trials, as done by Thorngate [48]; while simultaneously counting the number of mental operations required by each heuristic. In the next section, we describe a series of simulations which perform these tasks.

**Simulation**

**Heuristics**

We examine six heuristics which make quite different simplifications of the problem space for risky choice. These rules clearly differ along several dimensions, such as the method used to integrate probability and payoff information. However, they also differ markedly in the amount of available information that they consider. A priori, we might expect this to be an important determinant of both the accuracy and the effort resulting from their use.

At one extreme is the **Expected value** rule, which does not simplify the problem space at all. The selection of an alternative is based on complete search of the available information. The **Equiprobable** heuristic similarly examines all the alternatives and all outcomes. It, however, ignores one of the two outcome attributes, probability, implicitly treating all events as equally likely. To choose a lottery, the Equiprobable heuristic adds the payoffs for the outcomes of each alternative, and chooses the alternative with the highest total. This heuristic is similar to an equal weight model. The **MostLikely** heuristic, in contrast, examines only one outcome for each

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Insert Table 1 About Here

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alternative, the outcome with the highest probability of occurrence, and selects the alternative with the largest payoff for this outcome. Thus, this rule searches each event to find the most-likely outcome, and examines only the payoff associated with that event. This heuristic is similar to a lexicographic rule. The Maximin heuristic ignores probabilities entirely and selects the alternative with the largest minimum payoff. This heuristic is related to the conjunctive rule. Elimination-by-aspects is a choice rule proposed by Tversky [50]. We implement a version discussed by Thorngate [48]

Insert Figure 1 About Here

which attends only to payoff information. Each payoff of a gamble is compared to a cutoff equal to the mean payoff. If a payoff is less than the cutoff, the gamble is eliminated from further consideration. The rule terminates when either (1) one alternative remains or (2) all attributes have been considered, and one must choose randomly from the remaining alternatives.

Elimination-by-aspects ignores probabilities entirely, and performs only partial search of the payoff information. Finally, the Random choice rule serves as a baseline, simply choosing an alternative at random with no search.

Appendix A provides a listing of the production system representation of all rules but the random.

Task and Context Variables

The terms task variables and context variables have often been used interchangeably in the literature. After Payne [37], we adopt the following distinction: Task variables are those associated with general characteristics of the decision problem, such as the number of alternatives, which are not dependent on the particular values of the objects of the decision sets. Context variables, in contrast, are associated with the particular values of
the objects, such as the correlation between attributes. Other possible distinctions between task and context are discussed by Einhorn and Hogarth [12].

A frequently explored task variable is the complexity of the decision problem, usually manipulated through variation in the number of alternatives and outcomes presented by the choice problem. We vary the number of risky alternatives and outcomes at levels of 2, 4, and 8. These levels match previous behavioral and simulation research [36], [48]. We expect the decision strategies to show differential increases in effort as tasks become more complex [19]. We also expect decreases in the accuracy of heuristics as the complexity increases [48]. This makes variations in task complexity particularly interesting: It may be possible, for example, to identify heuristic rules which remain relatively effortless, and substantially accurate, as tasks become more complex.

Context effects have received considerably less attention than task effects in decision research. In part, this is because there is little systematic theory to guide the exploration of the impact of context on the accuracy and effort of choice rules. Indeed, previous work has made general statements about the viability of some decision rules based upon results obtained from a single context. For example, Thorngate [48] suggests that probability information may be relatively unimportant in making accurate risky choices:

"A wide variety of decision heuristics will usually produce optimal, or close to optimal results and can thus be termed relatively efficient. The ... equiprobable heuristic deserves further comment... its high efficiency suggest that 'good' choices can very often be made with scant
regard for the subtleties of accurate probability estimation procedures (p. 223-224)."

Although Thorngate did manipulate task complexity, this generalization is based upon a single context and should be viewed with some caution.

The probabilities in a risky choice must, by definition, sum to 1. Within this constraint, the variance of the distribution of probabilities can vary from a minimum of 0 when all outcomes are equally likely \( p = \frac{1}{m} \) for all \( m \) outcomes to a maximum of \( \frac{1}{m} - \frac{1}{m^2} \) when one of the \( m \) events is certain \( (p = 1) \), the rest impossible \( (p = 0) \). Thorngate's method for constructing gambles ensured that the variance in the probability distribution would be small relative to the variance in payoffs. Since expected value is the product of these two quantities, it is not surprising that probability information had little impact on the performance of his rules. Further, since the tendency of Thorngate's method to produce low variance in probabilities increases exponentially with the number of outcomes, we should be particularly cautious in interpreting his results for more complex environments. In the simulation we implement another method of probability generation which produces larger variances in the probability distributions. Characteristics of the two methods are discussed in Appendix B.

Another context variable which can vary between choice sets is the presence or absence of dominated alternatives. Although random generation itself can produce dominated alternatives, it has been argued that decision makers ignore them, effectively reducing the size of the choice set [23]. On the other hand, dominated alternatives can impact choice [17]. It has also been suggested that the success of one simplified strategy, the equal weighting of attributes, is dependent upon the presence of dominated
alternatives [30]. In the simulations that follow, we examine decision sets with dominated alternatives present and those with dominated alternatives removed.

Method

Each of the six decision rules was applied to 200 randomly generated decision problems in each of 36 conditions defined by a 3 (Number of Alternatives) by 3 (Number of Outcomes) by 2 (Variance of Probabilities) by 2 (Presence or Absence of Dominated Alternatives) factorial. After each trial the alternative selected was recorded along with a tally of each elementary operation used by the decision rule.

Payoffs were randomly selected from a uniform distribution bounded by 0 and 1000 by the multiplicative congruence method using the IMSL subroutine GGUUBS. Probabilities were generated by one of two methods: The low-variance condition replicates Thorngate's [48] procedure. The required number of deviates, \( m \), was generated from a uniform distribution and divided by the sum, normalizing the sum to 1.0. In contrast, the high variance method first selected a deviate from range 0, 1. Each subsequent deviate was randomly selected from the interval \( (0, 1 - \Sigma p_i) \) where \( p_i \) are the previously generated deviates. When \( m - 1 \) probabilities had been generated the procedure halted and the \( m \)th probability was set to \( 1 - \Sigma p_i \).

The presence or absence of dominated alternatives was manipulated by testing for the presence of first-order stochastic dominance. First order stochastic dominance describes a relation between two risky alternatives, \( A \) and \( B \) that ensures that \( A \) will always produce a higher utility than \( B \) for a decision maker with a finite, monotonically increasing utility function. It is analogous to simple dominance for riskless choice. A detailed description of the alternative generation procedure is available in Appendix B.
Note that despite the widely differing characteristics of the four cells created by the two types of context effects, all cells will have the same mean payoff and probability, and that the correlation between payoffs and probabilities will be close to 0. The differences due to context effects are reflected in the variance of the probabilities, and in the covariation of the payoffs across gambles.

All initialization, execution, and recording routines for the simulation are written in PASCAL, with the exception of random number generation performed by the Fortran language IMSL subroutines.Copies of the PASCAL source code are available from the first author.

Analysis

The significance of the results was established by an analysis of variance of the cell means based upon the 200 trails. This five way ANOVA analysed the task effects, that is, number of alternatives (2, 4 or 8), and number of outcomes (2, 4 or 8) and the context effects, that is the presence or absence of dominated alternatives and variance of probabilities (low or high). The final factor in the design was decision rule. For some dependent measures the cells in the analysis contained constants, and subsequently no within-cell variance. For example, the Expected Value strategy always chose the correct answer, resulting in a proportion of correct choice equal to 1.0. To provide an analysis which did not violate the assumption of homogeneous within-cell variance, we used an error-term based upon the within-cell variances of the non-constant cells. Although the resulting test is conservative, the experimental design provides sufficient power for hypothesis testing.

While the large number of trials ensures stable estimates, it also provides overwhelming statistical significance for many effects. Accordingly,
in reporting results it becomes more important to examine the size of each effect relative to the others. Since all factors use the same error term, the magnitude of the $F$ statistic is an index of the size of effects and is a linear function of other measures such as $\omega^2$.

**Results**

The two dependent measures, accuracy and effort, will be discussed sequentially, and we will then discuss their relationship. For each measure, we will start with the low-variance, dominated alternative condition, the cell which most closely replicates Thorngate's [48] results, and then discuss the results for the remaining experimental conditions.

**The Accuracy of Heuristics**

Table 2 presents the proportion of accurate choices, i.e. the proportion of trials in which each heuristic selects the gamble with the highest expected value, while Table 3 presents their relative performance, i.e. the percent improvement in expected value relative to a random choice. The low-variance, dominance present cell, labeled (1) in the tables, resembles the task environment used by Thorngate, and our results closely replicate his. The equiprobable rule, for example, appears to be quite accurate. A decision maker using such a heuristic in this task environment will select the best option about 75% of the time (see Table 2) and will average almost 90 percent of the expected value provided by the normative model relative to random choice see Table 3. In general, the heuristics demonstrate impressive accuracy in this task environment.
Note, however, that increases in task complexity have different effects upon the various rules. Increasing the number of outcomes, for example, does not affect the level of absolute and relative accuracy of the equiprobable heuristic. Other rules, in contrast, show decreases in accuracy as the number of outcomes increases.

The results for the two measures of accuracy, Table 2 and 3, tend to agree on the ranking of the heuristics with respect to accuracy. Both absolute and relative measures of accuracy based on expected utility maximization with utility defined by a power function [21], showed a similar pattern of results. Other utility functions are, of course, possible. Nonetheless, our results do not appear to be limited to just expected value based measures.

A different view of heuristics emerges, however, when the variance of the probabilities, relative to the payoffs, increases. For the high variance, dominated alternatives present condition, labeled (2) in the tables, the Most Likely heuristic is now the most accurate, while the Equiprobable heuristic displays a marked decrease in accuracy. Similarly, the Most Likely rule now appears to be the only rule which remains accurate as the number of outcomes increases. Thus, these results suggest that Thorngate's results are of limited generality. The unimportance of probability information is not apparent in this context where a rule which considers probability information, the Most Likely, is superior to a rule which does not, such as the Equiprobable. These results are particularly important in light of the suggestion made by Beach [2] that Thorngate's results justify deemphasizing the importance of probabilities in decision aids.

The effect of our other context manipulation, the removal of dominated alternatives, is dramatically demonstrated in Tables 2 and 3, conditions (3)
and (4). The Maximin and Elimination-by-aspects heuristics, which were reasonably accurate in the presence of dominated alternatives, now perform at near chance levels. Note also the effect of increases in the number of alternatives and outcomes. As can be seen in the Tables, the removal of dominated alternatives increases the impact of task effects on several of the heuristics. Why is this the case? We must conclude that the apparent accuracy of source of heuristics in the presence of dominated alternatives is due to their ability to screen truly inferior alternatives. Almost all the choice strategies examined successfully avoid dominated alternatives. The only rules selecting a dominated alternative with any frequency were the random and elimination-by-aspects. When dominated alternatives are removed, the heuristics, (except the Most Likely heuristic) become virtually indistinguishable from random selection.

The analyses of variance conducted upon both dependent measures, proportion of accurate choices (Table 2) and relative expected value (Table 3), confirm the significance of the observed differences. The ANOVA's showed a significant, \( p < .0001 \), effect of rules, number of alternatives and outcomes, and context manipulations. In addition, the interactions of rules with number of alternatives, number of outcomes, and both context variables, were significant for both dependent measures, \( p < .0001 \). These analyses also allow the computation of confidence intervals for the two measures. Both a priori (simple t-test) and a posteriori (Tukey's method for pairwise comparisons) confidence intervals are noted in each table.

In summary, while Thorngate correctly suggested that heuristic rules can approximate the performance of normative procedures, he incorrectly suggested that these findings were generalizable. The "right" heuristic to use in a choice task seems to be strongly influenced by context effects. A
decision-maker trying to maximize accuracy using heuristic strategies would need to know (1) several heuristics and (2) the appropriate conditions for their use. Thus, like Newell & Simon (34, p. 139), we conclude that "the effectiveness of particular heuristics is a function of the problem space."

**Effort and Heuristics**

The simulation yields a count of the numbers of each of the elementary processes listed in Table 4. To discuss the overall effort of any choice procedure, however, we need to develop some meaningful procedure for aggregation. We consider two possible schemes for combining the component counts into an overall index: First, if each heuristic contains approximately equal proportions of each elementary information process, their sum would generate a convenient estimate of overall effort (Newell and Simon, [34], p. 130). The ordering of the strategies on this index will be invariant over various estimates of the effort required by individual operations.

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Insert Table 4 About Here

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Second, we could use empirical estimates of effort for each elementary process, and weight the tally by the estimates. One source of such estimates is previous research attempting to parameterize the time necessary to execute similar EIPs. Work in mental arithmetic suggests that simple ADDs and PRODUCTS are well described as fact-retrieval processes. While the time required to perform each is somewhat dependent upon the size of the operands, university students typically average between .8 and 1.1 seconds to perform single digit multiplications or additions [10]. Dansereau suggests that .3 seconds may be required to encode single digits, an operation analogous to our READ parameter. Comparison between two digits (similar to the COMPARE operator) may take .3 seconds [10]. While there is no direct analogy to the
ELIMINATE operator, similar operations in psycholinguistics (the marking of a relation) take between .1 and .4 seconds [9]. The MOVE operator is similar to an eye fixation which has a typical duration of about .23 seconds [40]. These approximations suggest that equal weighting of each elementary operation may not seriously misrepresent effort costs. To explore the sensitivity of these effort estimates to the weights applied to each operator, we compared an equal weight estimate to one based upon the empirical estimates:

Effort = .3 * READ + .23 * MOVE + .3 * ELIMINATE + .3 * COMPARISON + .9 * ADD + 1.2 * MULT.

The resulting high correlation, $r = .97$, suggests that a parameterless, equal weighting model is sufficient to describe these simplified decision tasks.

Table 5 displays the total number of operations required by each rule as a function of the number of alternatives and the number of outcomes. It is apparent from Table 5 that the rules differ in the impact of increasing task complexity upon effort. For example, Elimination-by-aspects is the least effortful of the non-random choice procedures at all levels of complexity, while the Expected Value rule is always the most effortful. The other heuristics we examine, such as the Maximin, Equiprobable and the Most Likely, require approximately equal, intermediate amounts of effort.

Increases in the amount of information presented to the decision-maker affect these heuristics differently: Some rules increase in effort more rapidly than others. For example, Elimination-by-aspects is practically unaffected by an increase in the number of outcomes (26.0 operators for two outcomes, 29.4 for eight), while the Expected Value rule shows a large increase (70.0 vs. 238.0). In general, the effort required to use the heuristics increases more slowly than the effort required to use Expected
Value. Elimination-by-aspects requires only 42 more operators when the number of alternatives increases from 2 to 8. For the Equiprobable rule the equivalent increase in 130 operations. Expected Value requires 186 additional operators. Thus, all other things being equal, the Expected Value rule may seem less attractive, relative to the other rules, as the number of alternatives or outcomes in a choice set increases. This matches empirical results reported in Payne and Braunstein [38]. Finally, it is worth noting that the two heuristics that are quite accurate relative to expected value, the Equiprobable and the Most Likely, require substantially less effort than Expected Value, suggesting that these may be attractive strategies to a decision-maker willing to trade some accuracy for effort.

A striking feature of the effort estimates not apparent from the Table is their invariance across context effects. The effort levels associated with many of the strategies we examine are unaltered by changes in the variance of the probabilities or by the removal of dominated alternatives. This implies that a decision maker who minimizes effort will be relatively insensitive to context effects in the selecting strategies. On the other hand, the accuracy of this set of choice rules is strongly affected by context. This suggests the hypothesis that effort is greatly affected by task variables and not by context variables, while accuracy is greatly affected by context variables and less so by task variables. This is strongly confirmed by the results of the ANOVA conducted upon these results. Although the analysis shows that the impact of the task effects and their interactions are all quite significant, $F > 10,000$ in many cases, the effects of the context effects and their interactions are much smaller, $F < 22$. Again the analysis provides confidence intervals as noted in Table 5.
Trading Accuracy and Effort

Central to a cost-benefit analysis of strategy selection is the existence of an accuracy-effort tradeoff, a continuum of rules in which increases in effort result in increases in accuracy. The estimates of accuracy and effort provided by the simulation allow the construction of such a display, shown in Figure 2. The figure shows the results from the low variance and high variance, dominance present contexts averaged over task variables. Drawn for each context is a line connecting the strategies which, for a given level of total effort, are the most accurate in terms of relative performance (see Table 3). Strategies not on this line are dominated, and are inferior (in terms of accuracy and effort) to those on the frontier. The differences between the two frontiers illustrate an important point: The rules which describe an accuracy-effort tradeoff vary with context. The equiprobable rule decreases greatly in accuracy when the variance of probabilities increases, without a commensurate decrease in effort. As a consequence it falls far below the efficient frontier. It is interesting to note that these shifts seem to result from context effects rather than changes in task effects. An examination of the data shows that the set of efficient strategies does not vary as the number of alternatives or outcomes change. However, as Figure 2 shows, the set does change with the variance manipulation.

Figure 2 about here

This yields an interesting implication for a cost-benefit perspective. Inherent in this perspective is the idea that the importance of the decision will affect the choice of the decision rule. The more important the decision, the more effort a decision-maker will expend (moving to the upper left in an accuracy-effort curve). However, the current data suggest that the curve is not consistent across task environments. Relatively subtle changes in
context, such as the variability in probabilities, or the presence of dominated alternatives, should change preferences for choice strategies.

Discussion

We need to interpret the results of the simulation with some caution. Although we have examined several task environments, many more task and context variations can be investigated. These should include nonrisky and dynamic choice environments. As we have shown, the accuracy and effort associated with a heuristic are sensitive to task environments. For example, the Equiprobable and Most Likely rules reversed in their rank in accuracy as a function of the variance in probabilities. There are however, several generalizations that are suggested by our results: First, the data show that heuristics, in at least some task environments, can approximate the accuracy of normative rules with substantial savings in effort. Second, no single heuristic will do well across all contexts. Instead, if decision makers strive to maintain a high level of accuracy with a minimum of effort, they would choose among a repertoire of strategies. Finally, our results suggest that task effects tend to have greater influence on effort while context effects tend to have greater influence on accuracy.

Combined Decision Rules

The present paper has treated each decision rule as one that would be uniquely applied to a decision problem. There is evidence, however, that decision makers will employ strategies that combine rules. For example, Payne [36] reports that subjects faced with choice task involving a large number of alternatives will first use an elimination-by-aspects process to eliminate alternatives. When the choice problem is reduced to a smaller set of alternatives, e.g., two, decision makers shift to a more compensatory decision process. A number of researchers also have suggested on both theoretical and
empirical grounds that an early stage in a complex decision process might involve the reduction of alternatives [28], [31], [52]. The general rationale seems to be that such a procedure provides a way for the decision maker to simplify a complex task.

We examined one such combined rule suggested by previous empirical evidence: This rule used elimination by aspects until only three alternatives remained, then calculated expected value of the alternatives on their unexamined attributes. This rule showed some improvement over simple elimination by aspects, choosing the alternative with the highest expected value 15% more often. Most importantly, when compared to the other heuristics this rule shows much slower increases in effort when the number of alternatives increases. While the equiprobable heuristic shows a four-fold increase in effort as the number of alternatives increases (43.3 vs. 173.3), the equivalent increase for the phased rule is less than two-fold (39.3 vs. 59.7). Thus the combined rule has two attractive aspects: (1) it increases the accuracy of the elimination strategy while (2) maintaining that strategy relatively low effort in large choice sets. More research on combined decision strategies seems warranted.

Task Effects and Production System Models

In a recent review of decision research, Einhorn and Hogarth [12] note that "The most important empirical results in the period under review have shown the sensitivity of judgment and choice to seemingly minor changes in tasks" (p. 61). In addition to its descriptive interest, the lack of invariance in decision behavior across seemingly similar tasks is a concern to those attempting to improve decision performance. At the least, the lack of invariance raises question about the validity of the judgmental inputs needed to operationalize the normative procedures. (See H., Kunreuther, &
Schoemaker [16] for examples of biases in the assessment of utility functions.}

Decomposing common decision strategies into component processes (EIP's) and simulating them as production systems may offer an appealing way to identify and understand the potential impact of task variables on decision behavior. The present study, for example, shows that increasing numbers of alternatives affect differently the effort associated with expectation and elimination-by-aspects strategies. If effort is a consideration in strategy selection, one should not be surprised that choice behavior is sensitive to the number of alternatives (see Olshavsky, [35]; Payne, [36]; Payne & Braunstein, [38], for empirical evidence and Klayman, [24], for additional evidence from a computer simulation).

Although not investigated in this paper, manipulation of information formats provides additional examples of the potential value of decomposing strategies into EIP's. Huber [18], for instance, reports that the display of information in a verbal form (e.g., very good or poor) as opposed to a numerical form (e.g., 8 on a nine point scale) reduces the use of strategies containing concatenation or summing types of EIP's. Huber explains the result by suggesting that before concatenation "can be performed on verbal information, it somehow has to be transformed, e.g., by counting the verbal steps between two verbal labels" [p. 192]. The transformation process is assumed to involve additional effort (EIP's) and therefore reduces the attractiveness of strategies involving concatenation or summation under verbal displays. Important display effects also are reported by Bettman and Kakkar [5], Payne and Braunstein [38], Russo [39], Yates, Jagacinski, and Faber [53] among others. Many of these effects may be understood in terms of the impact of display variables upon the effort required by EIP's such as READ and MOVE.
Bettman and Kakkar, for example, report that information acquisition tends to proceed in a fashion consistent with the display format. The suggestion that the amount of interdimensional processing increases when the memory load placed on a decision maker is increased [38] is another example. Finally, a result readily apparent in Table 4 is that decision rules make differential use of the various operators. For example, only the Expected Value and Equiprobable heuristics use the arithmetic operations ADD and PRODUCT. This suggests that strategies may be affected differently when an operator becomes more effortful. If the outcomes of a gamble were described by three digit numbers, for example, the literature would suggest that these arithmetic operators would be much more cumbersome, while other operators such as comparisons would be only minimally affected. This should make rules that depend on arithmetic operators like Expected Value or Equiprobable more effortful relative to rules that utilize comparisons such as Elimination-by-aspects. From a cost-benefit perspective, this makes the former rules less attractive relative to the latter.

The importance of task variables in the design of messages which inform people about risk [44] and in the design of decision support systems [22] is clear. Researchers need to continue to conduct experiments identifying task and context effects. In addition, researchers should begin to explore the impact of various types of processing aids on decisions. We believe such research would be facilitated by the decomposition of decision strategies into sets of productions that can be studied under various task conditions through computer simulation.

**Validation**

One method of validating estimates of accuracy and effort would be indirect, through the correct prediction of the impact of task and context
effects upon the selection of decision rules. As we have discussed, the current framework is compatible with several existing results in the literature. However, much more direct tests of the degree of correspondence between the efficient strategies for a given decision problem identified by our simulations and the actual strategies people use need to be conducted. A variety of process tracing techniques may prove useful in such studies [4], [36].

Another approach to validation would use elementary operations to explain and predict decision related behavior such as the total time required to make a decision or self-reports of cognitive effort. The success of these attempts depends upon:

1. The serial nature of human information processing in higher level cognitive tasks, and
2. The assumption that each mental operation, on average, possesses a characteristic amount of effort.

Although such assumptions are clearly false for some cognitive tasks, such as highly practiced visual search, their validity for decision tasks is an empirical question.

To conduct this research, we must first decompose decision strategies and tally the elementary operations required by each strategy. These estimates can then be used in a regression model to explain both total decision time, and self reports of effort. There are two interrelated manipulations which might allow us to estimate effort:

1. Subjects could be instructed to use a given rule, and the simulations' estimates of effort would predict latency and reports of effort. A pilot study by John Conery using a similar procedure is reported in Russo and Dosher [41].
2. Observe, through process tracing techniques, such as verbal reports or information search, the strategies used by untrained decision makers, and infer which elementary processes are used. Whichever method is used, we would hope that such estimates both fit the data well and are consistent across different tasks.

Note that this analysis does not necessarily predict that decision latency and self reports of effort will necessarily agree. Kahneman [20] suggests that two cognitive processes which require the same amount of time may require quite different levels of mental effort. Thus a comparison may take about as long as an addition, but require less cognitive resources or attention. Subjects may report that comparison intensive rules such as the MostLikely are less effortful than addition-intensive rules, such as the Equiprobable, even though the rules may have identical latencies.

Deciding how to choose and learning accuracy/effort tradeoffs

As we have noted, part of the concern with accuracy and effort is motivated by the role these concepts may play in strategy selection. As Einhorn and Hogarth [12] note: "The wide range of strategies one can use in any given situation poses important questions about how one decides how to choose" (p. 69). Accuracy and effort are just some of the considerations which may help determine a decision-maker's selection of a strategy. Strategies themselves may be viewed as multidimensional objects [12], and include additional considerations such as justifiability, speed of decision, and awareness of conflict. Such a perspective suggests several obvious questions about strategy selection: (1) Which dimensions are most important. (2) Is strategy selection itself compensatory or non-compensatory? (3) When, and how often, does the evaluation of potential strategies occur? Let us examine, in closing, these questions in light of the simulation results and current literature in decision making.
As discussed in the preceding section, there are phenomena that appear consistent with the view that decision makers are influenced by effort. There is less evidence demonstrating the influence of accuracy. (See Klein [25] for a discussion of how utility considerations may guide strategy use.) Because the most accurate decision procedure, maximization of expected utility, is often not used in choice, it is difficult to argue that accuracy dominates rule selection.

A cost-benefit model implies a compensatory tradeoff between accuracy and effort that should be related to the importance of the decision. With sufficient incentive, decisions may involve the use of expected value maximization. However, the use of heuristic strategies seems to persist, even in situations involving substantial incentives [14], [26].

An alternate viewpoint is that effort and information processing limitations represent constraints which limit the strategies that can be adopted. Simon [43], for example, views a decision-maker as using heuristics and satisficing "not because he prefers less to more, but because he has no choice (p. 36)." It is important to note, however, that the concepts underlying a process of expected utility maximization, while quite demanding of the information processor, are not inconsistent with our current understanding of human cognition. Such processes, however, could well require inordinate amounts of time, and in practice, be impossible for the unaided decision-maker. Processing constraints, therefore, may impose severe limitations upon the strategies and thereby provide a boundary for the feasible region in which accuracy-effort tradeoffs could be made. The ultimate status of the cost-benefit perspective awaits further research, but it may be necessary to modify the notion to include an upward bound upon processing capacity. Thus, while effort seems securely ensconced as an explanatory variable in strategy selection, the role of accuracy and its
relation to effort seems less clear.

One difficulty with the idea that people deliberately decide how to choose is a potential infinite regress: One has to decide how to choose to decide how to choose.... A more reasonable perspective is that such decisions are not made often but that the relationship between task and context affects and the efficiency of a decision strategy is learned over time. For example, a decision maker may learn over time that a screening phase will substantially reduce effort in large choice sets. This knowledge can exist as part of the conditions which must be met for a production to fire. More generally, a decision maker may develop over time a task specific strategy that is highly accurate while requiring substantially less information processing than a normative rule. Klein [25] suggests the similar idea that a decision maker's use of heuristics may be related to learning about the nature of task environments. The potential importance of learning makes a production system representation especially useful for the study of strategy development in decision making. As Simon [43] notes: "what makes production systems especially attractive for modeling is that it is relatively easy to endow them with learning capabilities-to build so called adaptive production systems. (p. 121)."

However such an approach to strategy selection must come to grips with the nature of outcome feedback in risky choice. Seldom is such feedback immediately available, and in a risky choice, there is no deterministic link between the outcome obtained and the alternative selected. Even if outcome feedback is available, learning may be hampered because the feedback is related to the alternative selected [13]. In the extreme, it has been argued that learning seldom occurs even under optimal presentation of outcome feedback [6].
If outcome feedback is such a problematic mechanism for learning, how else might decision makers change strategies? In addition to outcome feedback, the decision maker has access to a fairly rich data base about the course of their own decision processes. This process feedback could provide information necessary for strategy change. By noticing possible shortcuts in past and current decisions, the decision maker could induce less effortful choice procedures. For example, a decision-maker might induce the Most Likely heuristic by noticing that certain outcomes seem much more probable than others. To evaluate the impact of this change the decision maker might check that the output of the new heuristic is consistent with several general principles of choice. For example, the decision maker might check that the new procedure does not select dominated alternatives, and that it selects alternatives that have satisfactory levels of the other outcomes. Like a problem solver that has induced a new strategy for mental addition, the decision maker evaluates the strategy change by examining the answer for consistency with previous procedures. The notion that learning occurs on the basis of trace information has been discussed in other cognitive tasks by Anzai and Simon [1].

Summary

This paper uses production system models and computer simulation to explore the accuracy and effort of various decision strategies in different choice environments. The results show that heuristic strategies can be highly accurate while substantially reducing effort relative to normative procedures. The accuracy and effort of strategies, however, is highly contingent on characteristics of the choice task. This result provides a partial explanation for the finding of contingent decision behavior [37]. However,
the extent to which decision makers actually tradeoff effort and accuracy, and do so optimally, are open empirical questions. Much more research is required to understand more completely the selection among decision strategies and how one may learn the relationships between task demands and the accuracy/effort properties of choice strategies.
REFERENCES


43. Simon, H.A. The sciences of the artificial (2nd Ed.), Massachusetts Institute of Technology, 1981.


FOOTNOTES

1We would like to thank Jim Bettman, Hillel Einhorn, Josh Klayman, Howard Kunreuther, and J. Edward Russo for their comments on an earlier draft, Ethan Bradford for programming assistance, and Maureen Lahiff and Larry Maloney for helpful discussion. This research was supported by a contract from the Engineering Psychology Program of the Office of Naval Research.
<table>
<thead>
<tr>
<th>Operation</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>READ</td>
<td>Read an alternative's value on an attribute into STM.</td>
</tr>
<tr>
<td>COMPARE</td>
<td>Compare two alternatives on an attribute.</td>
</tr>
<tr>
<td>DIFFERENCE</td>
<td>Calculate the size of the difference of two alternatives for an attribute.</td>
</tr>
<tr>
<td>ADD</td>
<td>Add the values of an attribute in STM.</td>
</tr>
<tr>
<td>PRODUCT</td>
<td>Weight one value by another (Multiply).</td>
</tr>
<tr>
<td>ELIMINATE</td>
<td>Remove an alternative from consideration.</td>
</tr>
<tr>
<td>MOVE</td>
<td>Go to next element of external environment.</td>
</tr>
<tr>
<td>CHOOSE</td>
<td>Announce preferred alternative and stop process.</td>
</tr>
</tbody>
</table>

Table 1: Primitive operations used in simulation.
TABLE 2
Proportion of Accurate Choices

<table>
<thead>
<tr>
<th>Context Condition</th>
<th>Choice Condition</th>
<th>Task Conditions</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Rule</td>
<td>Number of Alternatives, Number of Outcomes</td>
</tr>
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<td></td>
<td></td>
<td>2</td>
</tr>
<tr>
<td>1. Low Variance</td>
<td>Equiprobable</td>
<td>.84</td>
</tr>
<tr>
<td></td>
<td>Dominated</td>
<td>.78</td>
</tr>
<tr>
<td></td>
<td>Maximin</td>
<td>.73</td>
</tr>
<tr>
<td></td>
<td>EBA3</td>
<td>.63</td>
</tr>
<tr>
<td>2. High Variance</td>
<td>Equiprobable</td>
<td>.77</td>
</tr>
<tr>
<td></td>
<td>Dominated</td>
<td>.86</td>
</tr>
<tr>
<td></td>
<td>Maximin</td>
<td>.68</td>
</tr>
<tr>
<td></td>
<td>EBA</td>
<td>.63</td>
</tr>
<tr>
<td>3. Low Variance</td>
<td>Equiprobable</td>
<td>.68</td>
</tr>
<tr>
<td></td>
<td>No Dominated</td>
<td>.72</td>
</tr>
<tr>
<td></td>
<td>Maximin</td>
<td>.58</td>
</tr>
<tr>
<td></td>
<td>EBA</td>
<td>.56</td>
</tr>
<tr>
<td>4. High Variance</td>
<td>Equiprobable</td>
<td>.58</td>
</tr>
<tr>
<td></td>
<td>No Dominated</td>
<td>.81</td>
</tr>
<tr>
<td></td>
<td>Maximin</td>
<td>.50</td>
</tr>
<tr>
<td></td>
<td>EBA</td>
<td>.50</td>
</tr>
<tr>
<td></td>
<td>Expected Value</td>
<td>1.00</td>
</tr>
<tr>
<td></td>
<td>Random</td>
<td>.50</td>
</tr>
</tbody>
</table>

1Percent of EV Maximization Choices

2Differences exceeding .02 are significant, a priori, .047 a posteriori, \( p < .05 \).

3Elimination-by-Aspects Rule.
<table>
<thead>
<tr>
<th>Context Condition</th>
<th>Choice Rule</th>
<th>Task Conditions</th>
<th>Number of Alternatives, Number of Outcomes</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
<td>2</td>
</tr>
<tr>
<td><strong>1. Low Variance</strong></td>
<td>Equiprobable</td>
<td>.83</td>
<td>.90</td>
</tr>
<tr>
<td></td>
<td>Dominated</td>
<td>.75</td>
<td>.75</td>
</tr>
<tr>
<td></td>
<td>Alternatives Maximin</td>
<td>.54</td>
<td>.67</td>
</tr>
<tr>
<td></td>
<td>EBA^3</td>
<td>.35</td>
<td>.55</td>
</tr>
<tr>
<td><strong>2. High Variance</strong></td>
<td>Equiprobable</td>
<td>.68</td>
<td>.65</td>
</tr>
<tr>
<td></td>
<td>Dominated</td>
<td>.93</td>
<td>.94</td>
</tr>
<tr>
<td></td>
<td>Alternatives Maximin</td>
<td>.52</td>
<td>.56</td>
</tr>
<tr>
<td></td>
<td>EBA</td>
<td>.38</td>
<td>.46</td>
</tr>
<tr>
<td><strong>3. Low Variance</strong></td>
<td>Equiprobable</td>
<td>.46</td>
<td>.35</td>
</tr>
<tr>
<td></td>
<td>No Dominated Maximin</td>
<td>.60</td>
<td>.61</td>
</tr>
<tr>
<td></td>
<td>Alternatives Maximin</td>
<td>.18</td>
<td>.11</td>
</tr>
<tr>
<td></td>
<td>EBA</td>
<td>.16</td>
<td>.10</td>
</tr>
<tr>
<td><strong>4. High Variance</strong></td>
<td>Equiprobable</td>
<td>.26</td>
<td>.25</td>
</tr>
<tr>
<td></td>
<td>No Dominated Maximin</td>
<td>.83</td>
<td>.86</td>
</tr>
<tr>
<td></td>
<td>Alternatives Maximin</td>
<td>.05</td>
<td>.03</td>
</tr>
<tr>
<td></td>
<td>EBA</td>
<td>.12</td>
<td>.14</td>
</tr>
<tr>
<td></td>
<td>Expected Value</td>
<td>1.00</td>
<td>1.00</td>
</tr>
<tr>
<td></td>
<td>Random</td>
<td>.00</td>
<td>.00</td>
</tr>
</tbody>
</table>

^1Percentage of Expected Value Gained over Random Choice

^2Differences exceeding .016 are significant, a priori, .036 a posteriori, p < .05.

^3Elimination-by-Aspects.
## TABLE 4

Average Number of Production Operators Utilized by each Rule

<table>
<thead>
<tr>
<th>Choice</th>
<th>Operators&lt;sup&gt;1&lt;/sup&gt;</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Moves</td>
</tr>
<tr>
<td>Equiprobable</td>
<td>52.9</td>
</tr>
<tr>
<td>Most Likely</td>
<td>66.9</td>
</tr>
<tr>
<td>Maximin</td>
<td>52.9</td>
</tr>
<tr>
<td>Elimination-by-Aspects</td>
<td>10.0</td>
</tr>
<tr>
<td>Expected Value</td>
<td>52.9</td>
</tr>
<tr>
<td>Random</td>
<td>1.8</td>
</tr>
</tbody>
</table>

<sup>1</sup>CHOOSE is not listed since it is constant (1) for all rules.
TABLE 5
Total EIP'S by Task Complexity

<table>
<thead>
<tr>
<th>Choice Rule</th>
<th>Task Condition</th>
<th>Number of Alternatives</th>
<th>Number of Outcomes</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>2</td>
<td>4</td>
</tr>
<tr>
<td>Random</td>
<td></td>
<td>.51</td>
<td>1.5</td>
</tr>
<tr>
<td>Elimination-by-aspects</td>
<td></td>
<td>9.1</td>
<td>23.9</td>
</tr>
<tr>
<td>Maximin</td>
<td></td>
<td>43.3</td>
<td>86.7</td>
</tr>
<tr>
<td>MostLikely</td>
<td></td>
<td>48.3</td>
<td>97.7</td>
</tr>
<tr>
<td>Equiprobable</td>
<td></td>
<td>43.3</td>
<td>86.7</td>
</tr>
<tr>
<td>Expected Value</td>
<td></td>
<td>62.0</td>
<td>124.0</td>
</tr>
</tbody>
</table>

\(^1\) Differences exceeding .074 are significant at p < .05 a priori, .134 a posteriori.
(if at the end of alternatives) then (Choose alternative which is currently the best)

(if at the end of the outcomes) then (COMPARE the Current Alternative to the current best; winner becomes current best)

(if not at the end of the outcomes) then (READ the outcome's payoff; MOVE to the probability; READ the outcome's probability; PRODUCT the probability times the payoff; ADD the Result to the Current Alternative; MOVE to the next outcome's payoff)

Figure 1: The Expected Value Rule
Appendix A: Description of Production System Implementations of Choice Rules

This appendix presents English-like representations of the production systems used to implement the decision processes in the simulation. The equivalent representation of the expected value rule is in Figure 1. All the systems assume a specific form of conflict resolution: that the first true production is executed in each cycle. Instances of the operators in Table 1 are noted in capitals.
Production System for Elimination-by-Aspects

(if only one alternative is left) \(\Rightarrow\) (CHOOSE that alternative; stop)

(if at the end of all attributes) \(\Rightarrow\) (CHOOSE randomly from the remaining alternatives)

(if at the end of an attribute) \(\Rightarrow\) (MOVE to the next payoff)

(if the current alternative's payoff is known and is less than the cutoff) \(\Rightarrow\) (ELIMINATE the alternative)

(if the current alternative's payoff is known and is greater than the cutoff) \(\Rightarrow\) (MOVE to next alternative)

(if no cutoff is present) \(\Rightarrow\) (READ cutoff; READ the current alternatives payoff)

(if the current alternative's payoff is not known) \(\Rightarrow\) (READ current alternatives payoff for this attribute; COMPARE to cutoff)
Production System for Most-Likely Heuristic

(if at end of the alternatives) => (CHOOSE the current best)

(if at the end of this alternatives most-likely outcome to
alternatives outcomes) => payoff of the most-likely outcome of the current best;
MOVE to next alternative)

(if not at the beginning of an alternative) => (MOVE to the next probability;
READ probability; COMPARE current alternatives probability to the best-so-far)

(if at the beginning of an alternative) => (MOVE to payoff; READ probability;
Assign to best-so-far)
Production System for

Equiprobable Heuristic  

(if at the end of the alternatives) (CHOOSE the current best)

(if at the end of this alternative) (COMPARE this alternatives subtotal to current best; MOVE to next alternative; MOVE to next payoff)

(if not at end or beginning of an alternative) (READ Payoff; ADD to this alternatives subtotal)

(if at the beginning of an alternative) (READ Payoff - make it this alternatives subtotal)

Maximin is similar, but a COMPARE replaces the ADD in the third production, and the subtotal is replaced by the minimum payoff.
Appendix B: Description of Context Manipulations

The context effects manipulated in the simulation study can be viewed as changes in the distribution of two random variates: \( p \), the probabilities and \( X \) the payoffs. This appendix describes the two versions of each variate which yield the 2 x 2 factorial utilized in the simulation.

Each gamble consists of \( M \) events, and since the probabilities sum to 1, the mean of any distribution of probability will be \( \frac{1}{M} \). Subsequently, the average correlation between all pairs of probabilities \( p_i \), \( p_j \ i \neq j \) will be \( -\frac{1}{M-1} \), and the variance of the distribution can range from a minimum of 0 (all \( p_i = \frac{1}{M} \)) to a maximum of \( \frac{1}{M} \) \( - \frac{1}{M-1} \) (for example one \( p_i = 1 \), the rest 0). Although no closed form exists for the probability generation method used by Thorngate [48], an expansion of the Taylor series results in the approximation \( \frac{1}{M} \). The alternative method used here possesses a variance of \( \frac{M-1}{M^2} \). Subsequently, the two methods yield radically different distributions, and these differences increase with increases in the number of outcomes. For example, the variance in probabilities for the levels used in the current simulation would be:

<table>
<thead>
<tr>
<th>( M )</th>
<th>Low Variance</th>
<th>High Variance</th>
<th>Maximum Possible Variance</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td>.083</td>
<td>.125</td>
<td>.250</td>
</tr>
<tr>
<td>4</td>
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<td>.046</td>
<td>.187</td>
</tr>
<tr>
<td>8</td>
<td>.005</td>
<td>.014</td>
<td>.109</td>
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</table>

Thus, Thorngate's results, equivalent to our low variance environment, may be of limited generality. His method of generalizing probabilities resulted in distributions of low variance, and the range of probabilities incorporated in the resultant gamble become quite small as \( M \) increased.

All payoffs in the current simulation are drawn from a uniform distribution, range 0 to 1000. To remove stochastically dominated
alternatives we used a rejection method. However, dominance is frequent with increases in \( N \), the number of gambles in a choice set. Consequently, to improve the efficiency of generating gambles, we first ensured that the payoffs of all alternatives avoided simple dominance: i.e. for all pairs of gambles, \( a \) and \( b \), \( a \) had a higher payoff than \( b \) on at least one outcome, while \( b \) had a higher payoff than \( a \) on another outcome. While this maintains the same mean and variance of the distribution of probabilities, it does introduce a correlation between the \( X_i \) of the alternatives. For all pairs of alternatives the correlation between payoffs will be \(-1\). Probabilities are then assigned to the gambles and the choice sets were then examined to ensure that no gamble was first-order stochastic dominant over another. If such a pair existed, the choice set was rejected and a new one created.

Since payoffs are independent of probabilities, the average expected value of all gambles generated (and the expected value of a random choice) is \[
\frac{1}{M} \sum_{i=1}^{M} p_i x_i \] or $500. The average maximum will vary, however, as a function of the variance of \( p \) and \( x \).
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