RANDOM CHOICE SOLUTIONS
FOR WEAK SPHERICAL SHOCK-WAVE TRANSITIONS OF N-WAVES IN AIR
WITH VIBRATIONAL EXCITATION

BY

H. HONMA AND I. I. GLASS

JULY, 1983
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**Abstract:**

In order to clarify the effects of vibrational excitation on shock-wave transitions of weak, spherical N-waves, which were generated by using sparks and exploding wires as sources, the compressible Navier-Stokes equations were solved numerically, including a one-mode vibrational-re-laxation equation. A small pressurized air-sphere explosion was used to simulate the N-waves generated from the actual sources. By employing the random-choice method (RCM) with an operator-splitting technique, the effects...
of artificial viscosity appearing in finite-difference schemes were eliminated and accurate profiles of the shock transitions were obtained. However, a slight randomness in the variation of the shock thickness remains. It is shown that a computer simulation is possible by using a proper choice of initial parameters to obtain the variations of the N-wave overpressure and half-duration with distance from the source. The calculated rise times are also shown to simulate both spark and exploding-wire data. It was found that, in addition to the vibrational-relaxation time of oxygen, both the duration and the attenuation rate of a spherical N-wave are important factors controlling its rise time.

The effects of the duration and the attenuation rate of a spherical N-wave on its rise time, which are designated as the \textit{N-wave effect} and the \textit{nonstationary effect}, respectively, are discussed in more detail pertaining to Lighthill's analytical solutions and the RCM solutions for nonstationary plane waves and spherical N-waves. It is also shown that the duration and the attenuation rate of a spherical N-wave are affected by viscosity and vibrational nonequilibrium, so that they can deviate from the results of classical, linear acoustic theory for very weak spherical waves.
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List of Symbols

\(a\) speed of sound

\(a_1\) undisturbed speed of sound

\(a_e\) equilibrium speed of sound

\(a_f\) frozen speed of sound

\(c_j\) normalized vibrational specific heat for \(j\)-molecule \((= c'_j/R)\)

\(C\) viscous term in Eq. (4.1)

\(e\) internal energy

\(E\) total energy

\(h\) absolute humidity

\(H_c\) spherical correction of convection term in Eq. (4.1)

\(H_R\) vibrational relaxation term in Eq. (4.1)

\(H_v\) spherical correction of viscous term in Eq. (4.1)

\(j\) \(j = 0,\) plane wave; \(j = 2,\) spherical wave [Eq. (4.1)]

\(k\) coefficient in Eq. (3.24)

\(m\) \((a_f^2 - a_e^2)/a_c^2\)

\(M_e\) equilibrium Mach number

\(M_f\) frozen Mach number

\(M_s\) shock Mach number

\(n\) decay index of \((\Delta p)_{\text{max}}\) for spherical wave

\(P\) pressure

\(P_n\) normal pressure, 101.3 KPa

\(p_1\) undisturbed pressure

\(P_{\text{sat}}\) partial pressure of water vapour at saturation

\(P_{41}\) initial diaphragm pressure ratio

\(Pr\) Prandtl number

\(\tilde{P}\) similarity variable for plane \(N\)-waves, defined by Eq. (3.12)

\(\hat{\delta}_{\text{max}}\) maximum value of \(\tilde{P}\)

\((\Delta p)\) overpressure \((= p - p_1)\)

\((\Delta p)_2\) equilibrium overpressure behind steady plane shock wave

\((\Delta p)_{\text{max}}\) maximum overpressure of \(N\)-wave

\((\Delta p)_f\) overpressure immediately behind frozen shock wave

\((\Delta p)_{\text{cr},j}\) critical overpressure for \(j\)-molecule

\(r\) radial distance

\(r_0\) radius of pressurized sphere

\(r^*\) normalized radial distance \((= r/r_0)\)
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<td>Re</td>
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<td>RH</td>
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<td>( \Delta r )</td>
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<td>( t_r )</td>
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<td>( t_{r,n} )</td>
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<td>( t_s )</td>
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<tr>
<td>( t^* )</td>
<td>normalized time ( = a_1 t / r_0 ) or ( a_1 t / x_0 )</td>
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<td>( t_{d}^* )</td>
<td>normalized half duration of N-wave ( = a_1 t_d / r_0 )</td>
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<td>( \tilde{t} )</td>
<td>normalized time ( = t / t_d )</td>
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<td>( T )</td>
<td>temperature</td>
</tr>
<tr>
<td>( T_0 )</td>
<td>normal temperature</td>
</tr>
<tr>
<td>( T_1 )</td>
<td>undisturbed temperature</td>
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<td>( T_{d1} )</td>
<td>initial diaphragm temperature ratio</td>
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<td>( T_{e,j} )</td>
<td>vibrational temperature for ( j )-molecule</td>
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<td>( \Delta T )</td>
<td>over-temperature ( = T - T_1 )</td>
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<td>( u )</td>
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<td>( u_2 )</td>
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<tr>
<td>( \mathbf{u} )</td>
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<td>( u_1 )</td>
<td>flow velocity ahead of steady shock wave</td>
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<td>( u_2 )</td>
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<td>( \Pi )</td>
<td>nonstationary term in Eq. (4.1)</td>
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<td>( \Pi_s )</td>
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<td>( v )</td>
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<td>( \mathbf{v} )</td>
<td>flow velocity in a moving coordinate system, Eq. (3.24)</td>
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<tr>
<td>( v_0 )</td>
<td>absolute value of ( v ) at upstream and downstream infinity</td>
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List of Symbols - Continued

\( x \) distance
\( x_0 \) length of high-pressure chamber of shock tube
\( x_d \) half distance of N-wave corresponding to \( t_d \)
\( x_s \) characteristic shock-thickening distance
\( x^* \) normalized distance \( = x/x_0 \)
\( X \) coordinate defined as \( X = x - a_1 t \)
\( x_d \) half distance of N-wave, defined for \( X \)
\( X_n \) node of N-wave \( (u = 0) \), defined for \( X \)
\( \Delta x \) increment of \( x \)
\( \Delta x^* \) increment of \( x^* \)
\( \left( \Delta x \right)_0 \) Taylor thickness for 10-90\% equilibrium overpressure
\( \Delta X \) shock thickness of N-wave, defined for \( X \) [Eq. (3.14)]
\( y \) time in Eq. (3.24)
\( \Xi \) distance parameter defined by Eqs. (3.5), (3.16), (3.21), (3.35), (3.36), (3.39)
\( \Xi_d \) duration parameter defined by Eq. (3.15)
\( \Xi^* \) thickness parameter defined for \( \Xi \)
\( \left( \Xi^* \right)_0 \) Taylor-thickness parameter defined by maximum slope of velocity
\( \left( \Xi^* \right)^0 \) Taylor-thickness parameter defined by 10-90\% equilibrium overpressure
\( \left( \Xi^* \right)^n \) Taylor-thickness parameter defined by 5-95\% equilibrium overpressure
\( \gamma \) ratio of specific heats
\( \gamma \) diffusivity defined by Eq. (3.2)
\( \gamma_j^v \) diffusivity based on vibrational bulk viscosity for \( j \)-molecule
\( \gamma_j \) molar concentration for \( j \)-molecule
\( \gamma_j^* \) characteristic vibrational temperature for \( j \)-molecule
\( \gamma_j \) thermal conductivity
\( \nu \) viscosity
\( \nu_r \) bulk viscosity for rotational relaxation
\( \nu_j^v \) bulk viscosity for vibrational relaxation of \( j \)-molecule
\( \nu_j \) kinematic viscosity
\( \rho \) density
\( \rho_j^v \) vibrational energy for \( j \)-molecule
\( \rho_j^* \) equilibrium vibrational energy for \( j \)-molecule

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<td>$\tau$</td>
<td>time parameter defined by Eq. (3.21)</td>
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<td>$\tau_s$</td>
<td>characteristic-time parameter for shock thickening</td>
</tr>
<tr>
<td>$\tau_j$</td>
<td>relaxation time for j-molecule</td>
</tr>
<tr>
<td>$\xi$</td>
<td>similarity parameter for plane N-wave, defined by Eq. (3.12)</td>
</tr>
<tr>
<td>$\xi_d$</td>
<td>$\xi$ corresponding to half duration of N-wave</td>
</tr>
<tr>
<td>$\xi_m$</td>
<td>$\xi$ for $\tilde{P}_{\max}$</td>
</tr>
<tr>
<td>$\Delta_\xi$</td>
<td>shock thickness defined for $\xi$</td>
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### Subscripts

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<th>Subscript</th>
<th>Description</th>
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<tr>
<td>N</td>
<td>nitrogen</td>
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<tr>
<td>O</td>
<td>oxygen</td>
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1. INTRODUCTION

The pressure waves generated by supersonic transport aircraft (SSL) and from explosions in air are often observed as weak X-waves far from the source. Such pressure waves are heard as sonic booms. The loudness of these waves depends on their maximum overpressures and rise times. The X-waves with short (microseconds) rise times are perceived as louder and more startling than the ones with long (milliseconds) rise times. As a consequence, X-wave rise times were investigated extensively for SSL sonic booms and for explosions in air (Refs. 2, 4). However, the observed SSL rise times were often found to be larger than those which were estimated from classical theory for viscous shock structures of steady, plane waves, derived by Taylor (Ref. 3). A recent review of this matter may be found in Ref. 6.

This discrepancy was attributed mainly to the effects of atmospheric turbulence (Refs. 7-10), and real-gas effects arising from the vibrational excitation of the analyzed oxygen and nitrogen air molecules (Refs. 11, 12). However, the decisive factor for this increased rise time was still in question. There were difficulties in providing correlations between the observed and analytically estimated rise times, owing to a lack of information regarding the ambient temperature, humidity and air turbulence. Such quantities are not always readily available. It was therefore necessary to carry out some simulation experiments under controlled conditions where known atmospheric conditions could be obtained.

Holslander (Ref. 6) was able to generate well-formed weak spherical X-waves by using sparks or exploding wires as a source in a still-air dome, usually used for air-cushion experiments (Ref. 13). In this manner he wanted to clarify the vibrational effects on the rise time of SSL X-waves. He found that the observed rise times were much shorter than the rise times estimated from the analysis of plane, fully dispersed waves (Ref. 12). The results could not be explained by any existing analysis. The object of this report is to provide a theoretical basis for explaining Holstjensen's data, which will be outlined in Section 2.

The processes involved in the generation of X-waves by exploding sparks and wires are very complex and are not readily predicted. Consequently, it is necessary to assume a reasonable source model in order to simulate the explosions. In this paper it is assumed that the expanding plasma can be simulated by a pressurized sphere of small radius at room temperature. The computer simulation requires adjusting the radius of the pressurized sphere and the imaginary diaphragm pressure ratio to fit the experiments for maximum overpressure and half-duration of the X-wave with distance from the source. It is then possible to determine the initial energy of the source. The latter is of academic interest as it is not possible to determine the actual energy release from the voltage and capacitance of the discharge without a great deal of additional time-dependent measurements.

The nonstationary, spherical-symmetric Navier-Stokes equations were solved numerically, including the equation of one-mode vibrational relaxation for explosions of pressurized spheres in atmospheric air. An operator splitting technique was used in which, at the first stage of calculation, the solutions for inviscid, frozen flow were obtained by applying the Random Choice Method (RCM) and then the effects of viscosity and vibrational nonequilibrium were evaluated by using an explicit finite-difference method.

The RCM is a numerical method which was developed by Glimm (Ref. 14), Chorin (Ref. 15) and God (Ref. 16) for flow problems including shock waves. In this method, a Riemann problem is solved for each spatial mesh at each time step and then one of its solutions is chosen at random as a solution for the next time step by using a random sampling technique. It is the great merit of this method that shock waves and contact surfaces can be expressed as discontinuous surfaces without smearing arising from artificial viscosities inherent in all finite-difference methods. This is the main reason for adopting the RCM for the present analysis. The algorithm is based on a program developed by Saito and Glass (Ref. 17). The application of the operator-splitting technique to the first-order N-terms Navier-Stokes equations was first introduced by MacCormack (Ref. 18). In his analysis, the mixed solutions were obtained using a characteristic method. Recently, Sato(19) and Shimizu (Ref. 19) have tried to solve the Navier-Stokes equations for a shock-time problem by applying the RCM with an operator-splitting technique. In the present analysis, the RCM with an operator-splitting technique was extended to include vibrational relaxation effects for spherically-symmetric waves.

It will be shown subsequently that the rise times of weak, spherical X-waves generated by sparks and exploding wires are some two factors which never appear in steady plane waves. These are designated as an \( \omega \)-effect and a \( \gamma \)-effect, respectively. The \( \omega \)-effect means that the rise times of weak X-waves are affected by the expansion of the flow immediately behind the shock front. The nonstationary effect means that the rise times of weak shock waves respond to changes in shock strength so slowly that their transient behaviors must be considered. The fundamental analytical ideas about these effects were provided by Lighthill (Ref. 20) for both viscous X-waves and impulsively-generated viscous plane waves. In Section 3, his results are re-examined for use in the present study.

In order to consider the effects of vibrational excitation of oxygen and nitrogen air molecules, the papers of Polyaeva et al. (Ref. 21) and Johannsen and Hodgson (Ref. 12) for plane, dispersed waves are also re-examined in Section 5, and an approximate relation is derived for the rise time of a fully or partly-dispersed wave. Furthermore, the modified Lighthill and Lightill solutions for fully-dispersed waves are discussed.

In Sections 4.1 and 4.2, the basic equations and the numerical method for solution are described. In Section 4.3, the results of solution for nonstationary shock transitions, RCM solutions for nonstationary viscous and dispersed plane waves are compared with analytical solutions described in Section 3. As for solutions for spherical waves (Section 4.4), some numerical results for weak spherical X-waves in air are presented for the following three cases: (i) formation of X-waves in
the near-field of a pressurized sphere, (ii) comparison between perfect-inviscid, perfect-viscous, real-inviscid and real-viscous solutions, (iii) effects of vibrational relaxation time or ambient temperature and humidity, (iv) effects of N-wave duration or radius of pressurized sphere, and (v) effects of nitrogen vibrational relaxation. The observed rise times of spark and exploding-wire generated N-waves are also compared with those obtained from the analytical simulations.

In this report, the usual definition of rise time is followed, and is taken as the time-interval for the overpressure to vary from 10% to 90% of its peak value. This definition is especially useful for actual SST signatures, as discussed in Ref. 6. Figure 1.1 illustrates the definition of an N-wave rise time $\tau_r$ and its half-duration $\tau_d$. Figure 1.2 also illustrates the definition of a plane-wave rise time $\tau_p$. The corresponding shock thickness $\lambda$ and half-duration length $x_d$ may approximately be given by

$$\lambda = a_1 \tau_r, \quad x_d = a_1 \tau_d$$

where $a_1$ is the undisturbed speed of sound, since we consider only very weak waves.

2. SPARK AND EXPLoding-WIRE DATA

In this section, the spark and exploding-wire experiments which were carried out by Holst-Jensen (Ref. 6) and the resulting data are summarized. The purpose of these experiments was to generate weak, fully-developed N-waves with overpressure below 100 Pa in air, which would have interference-free shock fronts. This was accomplished by using sparks and exploding-wire. The wire used in these experiments and the UTIAS air cushion vehicle (ACV) circular track facility (Ref. 15) was used as a still-air reservoir for part of the experiments. Its major internal diameter is about 42.5 cm. This provided waves free from interference with walls and other objects.

For detecting weak shocks in the overpressure range 5-100 Pa, a condenser microphone was used [Buel & Kjaer 4155 free field 6.5 mm (1/4 in.) dia]. Amplification of the microphone signal was provided by a preamplifier B&K 2619. The response of the microphone system was tested in the UTIAS Travelling-Wave Sonic-Bow Simulato (Ref. 21). When measuring without its protective grid at zero angle of incidence, the microphone has an approximate minimum rise time $\tau_r = 2.9 \mu s$. The oscilloscopes used were Tektronix types 555 and 555 with a type U plug-in that has a bandwidth better than 300 KHz. The microphone was calibrated with a B&K pistophone type 4220, which gives a sound pressure level at 250 Hz of 124 dB.

In the first series of experiments, sparks were used as a source of N-waves. The sparks were generated by the energy released from a charged 1.5 kF capacitor. The maximum charging voltage was 8 kV and the discharge device was a thyratron. A microphone was placed ahead of the measuring microphone in parallel to get the trigger signal for the oscilloscope. The source and microphone were set up at 1.8 m above the floor to avoid interference from reflected signals.

Fairly extensive measurements were done by using sparks at temperatures of 273-277 K and relative humidities of 50-75%. Five source-receiver distances (4.1 m, 9.0 m, 9.8 m, 15.0 m and 21.0 m) were employed with four different charging voltages of 4.4 KV, 5.0 KV, 5.4 KV and 6.0 KV. This series of measurements is termed Series-I. Another series of measurements (Series-II) was also done at a temperature of 289 K and relative humidity of 50% for the distance range of 11.8-19.0 m and a charging voltage of 4.4 KV.

Exploding wires were used to produce N-waves by replacing the resistor in the spark circuit by a thin nickel wire 0.125 mm dia. and optimum length of 5 cm. The sudden discharge of energy superheated the wire. The expansion of the metal vapour generated an N-wave in the far field. The measurements were done at two conditions for Series-II (I) $\tau_1 = 27^\circ$, $\tau_2 = 55^\circ$, $r = 6.7 m$, $22.8 m$, $24.5 m$, $S = 4.6 KV$, $6.0 KV$, and Series-IV (I) $\tau_1 = 200^\circ$, $\tau_2 = 87.5^\circ$, $r = 24.3 m$, $29.3 m$, $S = 4.6 KV$, $6.0 KV$. Here $I$ is the room temperature, $\tau_1$ the relative humidity, $r$ the distance from the source and $S$ the charging voltage.

The vibrational relaxation times for oxygen and nitrogen were evaluated by using the empirical relation obtained from the absorption of sound waves by Bass and Shields (Ref. 23), as tabulated in Table 2.1. The vibrational relaxation time at room temperature strongly depends on the absolute humidity of the atmosphere, as water molecules significantly reduce its value.

Representative oscillograms from sparks and exploding wires are shown in Fig. 2.1. It can be seen that both a spark and an exploding-wire source make it possible to produce well-established N-waves far from the source, as the spark source. The N-waves were much cleaner than those generated by a spark, especially with regard to the rear shock. It was found that the wire length $l$ plays a significant role in shaping the rear shock pressure profile. After testing several wire lengths, a wire length $l = 5.0$ cm proved to generate the most symmetrical N-waves, and was used in all subsequent runs. The microphones were set up normal to the wire to minimize any line-source effect.

In Figs. 2.2 - 2.4, the maximum (peak) overpressure $(\rho_p)^{\text{max}}$, the half-duration $\tau_d$ and the rise time $\tau_r$ are plotted against the distance from the source $r$. Figure 2.5 shows plots of $\tau_r$ vs $(\rho_p)^{\text{max}}$. For the Series-I experiment, the data are plotted only for $S = 4.4 KV$ and $6.0 KV$ to avoid confusion.

In Figs. 2.2, the lines indicate the curves of $(\rho_p)^{\text{max}} = C r^n$, which are drawn from the arbitrary points to fit the experimental data, where $n$ is termed the decay index of maximum overpressure. The solid and broken lines correspond to the curves for $n = 1$ and 1.4, respectively. For 100 - $(\rho_p)^{\text{max}}$ 20 Pa both spark and exploding-wire data show that maximum overpressures decay nearly inversely proportional with distance from the source, as estimated from linear-acoustic theory. On the other hand, the spark data show that the decay index increases below 20 Pa. This deviation from linear-acoustic theory can be attributed to real-gas effects arising from
vibrational excitation of oxygen (see Section 3.4). It is noted that the same input energy does not result in the same decay of $|\Delta p|_{\text{max}}$ for different energy sources. The exploding-wire source makes for a stronger explosion in air than the spark source for the same discharge voltage. It should also be noted that the overpressure decays are different for the different series of spark experiments despite the same discharge voltage.

In Figs. 2.3 - 2.5, the broken lines indicate the tendency of the experimental data. The half-duration $t_g$ increases with $r$. The durations for the exploding-wire experiment (85-155 $\mu$sec) are longer than those for the spark experiments (60-90 $\mu$sec), while the maximum overpressure decreases with $r$. It should be noted from Fig. 2.4 that the rise times $t_r$ are different for the different series of experiments and supply voltages at the same maximum overpressure.

3. SOME ANALYSES FOR WEAK SHOCK TRANSITIONS

In this section, some analytical solutions for weak shock transitions are reviewed and discussed in connection with the spark and exploding-wire data, which were shown in Section 2. In Sections 3.1 - 3.3, some analytical solutions for viscous shock transitions are shown in cases of steady planar waves, quasi-stationary $N$-waves and nonstationary planar waves, respectively. The analytical solution for steady planar waves was derived by Taylor (Ref. 21), and will be designated as the Taylor solution or the Taylor shock transition. The analytical solutions for quasi-stationary $N$-waves and nonstationary planar waves were defined by Lighthill (Ref. 20), and will be designated as the Lighthill solutions or the Lighthill shock transition, respectively. In Section 3.4, solutions for dispersed waves with vibrational excitation are shown for a steady plane wave, and an approximate expression is derived for the rise time of a fully or partly dispersed wave. The Taylor and Lighthill solutions are extended to dispersed waves with vibrational relaxation by using a bulk-viscosity concept, and the extended solutions will be designated as the modified Taylor solution and the modified Lighthill solution, respectively. Some insight is also given into the structures and rise times of weak spherical $N$-waves.

3.1 Classical Taylor Plane Shock-Wave Transitions

In the following three sections, Sections 3.1 - 3.3, the classical or $p$-waves shock transitions are considered, where the vibrational mode of molecular internal energy is assumed to be $\gamma \neq \infty$. Viscous, steady shock waves are formed as a result of a balance between the wave form-strengthening tendency due to the finite-amplitude compression connections effects and the wave form-easing tendency due to the viscous diffusion effects. This balances determines the thickness of a steady shock wave and depends on the shock strength.

The classical Taylor solution (Ref. 5) for weak plane shock wave transitions is expressed by Lighthill (Ref. 20) as

$$
\frac{\Delta p}{\Delta p_0} = \left(1 + \exp \left[\frac{\gamma - 1}{\gamma \cdot \gamma - 1} \frac{x}{\gamma - 1} \right] \right)^{-1}
$$

(3.1)

for a shock wave travelling with steady profile at a constant speed $u_x$, where $x$ = flow velocity relative to the ground; $v_2$ = flow velocity at $x = \infty$; $\gamma$ = ratio of specific heats; $x$ = distance, $t$ = time, $\xi$ = diffusivity of sound, defined by

$$
\frac{\gamma - 1}{\gamma \cdot \gamma - 1} \frac{x}{\gamma - 1} \xi = \frac{1}{2} \left[ \frac{\gamma - 1}{\gamma + 1} \right] \frac{\gamma}{\gamma - 1} \frac{\xi}{\gamma - 1}
$$

(3.2)

where $\nu$ = kinematic viscosity, $\mu$ = viscosity, $\lambda = \mu / \rho$ = bulk viscosity due to rotational relaxation, $\operatorname{Pr} = \frac{\nu}{\lambda}$ = Prandtl number. All the thermodynamic and transport coefficients, $\gamma$, $\mu$, $\nu$, $\lambda$, and $\operatorname{Pr}$, may be assumed to be constant throughout the flow, since the shock waves are weak. The original Taylor solution did not include the bulk viscosity due to rotational relaxation as it appears in Eq. (3.2). However, in the present paper, the term $\gamma \cdot \gamma - 1 = \frac{\gamma - 1}{\gamma + 1}$ is used when it includes only the effects of rotational relaxation in order to distinguish from the $\frac{\gamma - 1}{\gamma \cdot \gamma - 1}$ which includes both the effects of rotational and vibrational relaxation.

From the weak-wave assumption, we have

$$
\frac{\Delta p}{\Delta p_1} = \frac{1}{2} \left[ \frac{\gamma - 1}{\gamma + 1} \right] \frac{\xi}{\gamma - 1}
$$

(3.3)

where $\Delta p$ is the overpressure $|\Delta p| = p - p_0$; $a_1$, the undisturbed speed of sound; $p_1$, the undisturbed pressure. Then Eq. (3.3) can be rewritten as

$$
\frac{|\Delta p|_2}{|\Delta p|_1} = \frac{1}{2} \left[ \frac{\gamma - 1}{\gamma + 1} \right] \frac{\xi}{\gamma - 1}
$$

(3.4)

where $|\Delta p|_2$ is the overpressure at $x = \infty$. Define a dimensionless variable

$$
\frac{\Delta p}{p_1} = \frac{a_1(x+1)}{p_1} (\frac{|\Delta p|_2}{|\Delta p|_1})
$$

(3.5)

Then

$$
\frac{|\Delta p|_2}{p_1} = \frac{1}{2} \left[ \frac{\gamma - 1}{\gamma + 1} \right] \frac{\xi}{\gamma - 1}
$$

(3.6)

or

$$
\frac{2}{\gamma - 1} \frac{|\Delta p|_2}{p_1} = \frac{1}{2} \left[ \frac{\gamma - 1}{\gamma + 1} \right] \frac{\xi}{\gamma - 1}
$$

(3.7)

Figure 3.1 exhibits the Taylor velocity or pressure profile in a plot of $x/\xi$ or $|\Delta p|/p_1$ against $x$. The variable $x$ is a similarity variable, since the velocity or pressure profile can be obtained as a unique curve against $x$ for shock waves with different strength $|\Delta p|_2/p_1$, and it will be termed the distance parameter.

Three different definitions of shock thickness for $x$ are also shown in Fig. 3.1. The thickness $2|x|_1$ is defined by
This thickness corresponds to the velocity or density-based thickness, and it has been used in some literature for shocks of moderate strength. The thicknesses \( \langle x \rangle_0 \) and \( \langle x \rangle_1 \) are defined by the distances for the overpressure to vary from 10\% to 90\%, and from 5\% to 95\%, respectively, of its equilibrium value behind the shock. The last definition was used by Lighthill (Ref. 20) for the shock thickness derived from the velocity profile. From Eq. (5.6) or (5.7), we can evaluate the values of \( \langle x \rangle_0 \), \( \langle x \rangle_1 \), and \( \langle x \rangle_2 \) as

\[
\langle x \rangle_0 = 4.00 \, \text{cm}, \quad \langle x \rangle_1 = 5.12 \, \text{cm}, \quad \langle x \rangle_2 = 6.87 \, \text{cm}
\]

These will be termed the thickness parameters. The second definition of the shock thickness (10-90\% overpressure) is used throughout this report because it can give a reasonable criterion for evaluating the thickness of a shock wave with an antisymmetric structure, which is found in N-waves and in partly or fully dispersed plane waves.

The actual Taylor thickness \( \langle x \rangle_0 \) and the Taylor rise time \( t_{12} \) (10-90\% overpressure) can be related to the Taylor thickness parameter \( \langle x \rangle_2 \) as

\[
\frac{t_{12}}{t_{12}^0} = \frac{\langle x \rangle_0}{\langle x \rangle_2} = \frac{\langle x \rangle_1}{\langle x \rangle_2} = \frac{\langle x \rangle_2}{\langle x \rangle_2} \quad (3.8)
\]

from Eq. (3.5), where \( t_{12} \) is the Taylor rise time corresponding to the Taylor thickness \( \langle x \rangle_2 \). We assume \( t_{12}^0 = \langle x \rangle_0 / a_1 \), since the wave speed is nearly equal to \( a_1 \) for very weak waves.

In Fig. 5.2, the Taylor thickness \( \langle x \rangle_0 \) or the Taylor rise time \( t_{12}^0 \) are plotted in a nondimensional form against \( \langle \rho \rangle_0 / \rho_0 \) for a range of \( \langle \rho \rangle_0 / \rho_0 = 10^{-3} - 10^{-5} \) or \( \langle \rho \rangle_1 / \rho_1 = 1 \) Pa - 100 Pa in the atmosphere. At NTP for \( a_0 \), \( \rho_0 = 1.293 \times 10^{-3} \) m\( \cdot \)s\(^{-1} \), \( \rho_0 = 2.73 \times 1 \), \( \rho_1 = 1.4 \), \( \rho_0 = 0.7 \) and, from Eq. (3.2), \( \langle \rho \rangle_0 = 3.13 \times 10^{-5} \) m\( \cdot \)s\(^{-1} \). Using \( a_0 = 1.1 \times 10^{-3} \) m\( \cdot \)s\(^{-1} \), the characteristic length and time are

\[
\langle x \rangle_0 = 1.05 \times 10^{-3} \, \text{cm}, \quad \langle x \rangle_1 = 3.1 \times 10^{-3} \, \text{sec}
\]

Therefore, for \( \langle \rho \rangle_0 / \rho_0 = 1 \) or \( \langle \rho \rangle_1 / \rho_1 = 10 \) Pa at NTP, then \( \langle x \rangle_0 = 5.3 \, \text{mm} \) and \( t_{12}^0 = 16.8 \, \text{sec} \), from Fig. 5.2. The Taylor thickness or rise time is inversely proportional to the shock strength \( \langle \rho \rangle_2 / \rho_2 \). As the shock speed is weakened, the Taylor thickness increases and tends to infinity as \( \langle \rho \rangle_2 \rightarrow 0 \).

As mentioned at the beginning of this section, the balance between the finite-amplitude (nonlinear) compression effects and the viscous-diffusion effects determines the thickness of a steady shock wave. As the wave is weakened, the nonlinear effects are gradually diminished, while the viscous-diffusion effects remain unchanged regardless of the shock strength. Therefore, for very weak shocks, the diffusion effects exceed overwhelmingly the nonlinear compression effects and broaden the shock thickness to very large values in the limit of \( \langle \rho \rangle_2 \rightarrow 0 \), the nonlinear effects disappear and only the diffusion effects remain, so that the thickness tends to infinity. However, in an actual case, the steady structure of such a very weak wave would not be realized because it requires an infinitely long time for the wave to reach a steady state through viscous diffusive action. In the case when the shock strength increases, the nonlinear effects are strengthened, while the diffusive effects remain unchanged. However, the shock thickness cannot be less than the molecular mean-free-paths, since the shock compression process is after all a result of molecular collisions. In other words, for strong shocks, the shock thickness has a lower limit which is controlled by molecular-collision processes.

Figure 3.3 shows a comparison between the experimental and theoretical (Taylor) rise time \( t_{\text{exp}} \) vs the maximum overpressure \( \langle \rho \rangle_{\text{max}} \). The Taylor curves shown in Fig. 3.2 are reproduced for \( \rho_1 = 273 \, \text{K} \) and 290 K. As seen from Fig. 3.3, the rise times for the spark data (Series I and II) are shorter than the Taylor rise times for the same maximum overpressure, while the rise times for the exploding-wire data (Series III and IV) are longer. Both data do not coincide with the Taylor curves. It is clearly seen that the Taylor rise times for steady viscous shocks can give no reasonable explanation for the observed rise times for weak spherical N-waves. Therefore, another analysis is required for this purpose.

3.2 Viscous Plane N-Waves

In this section, consideration is given to the case of a balanced N-wave, which is produced by moving a piston forward and then retracting it to its original position in a tube. The generated plane N-wave gradually decays due to viscous effects as it proceeds. Lighthill (Ref. 20) solved this problem and obtained a similar solution for weak plane N-waves, where the velocity profile is given as

\[
u = \frac{X}{1 + \exp[(X/2)^2]}/\exp(Re - 1) \quad (3.9)
\]

where \( X \) is a coordinate measured in a frame of reference which moves in the same direction as the waves, with an undisturbed speed of sound \( a_1 \), and is defined as \( X = x - a_1 \); \( u \) is the local speed of sound, \( v \), the particle velocity; \( Re \) is a Reynolds number of each half of the N-waves, which is defined in terms of the mass flow in that half. For example, for the front half

\[
Re = \frac{1}{t_1} \int_0^{t_1} \frac{dX}{X} \quad (3.10)
\]

for the half.

From the nonlinear wave relation,

\[
u = \frac{-X}{v} \quad (3.11)
\]
Using eqs. (3.5) and (3.11) and defining the similarity variables
\[ \tilde{\bar{p}} = \frac{\bar{p}}{P_1} \quad \tilde{t} = \frac{t}{\tau} \quad \tilde{Z} = \frac{Z}{Z_d} \]
then from Eq. (3.9),
\[ \tilde{\bar{p}} = \left[ 1 + \frac{\exp\left[ -\frac{\tilde{t}^2}{2}\right]}{\exp(\tilde{\tau}) - 1} \right]^{-1} \quad (3.15) \]
Figure 3.4 shows the pressure profiles for several different Reynolds number Re in a plot of \( \tilde{\bar{p}} \) against \( \tilde{t} \).

For a given Reynolds number Re, we can obtain \( \tilde{\bar{p}}_{\text{max}} \) (the maximum value of \( \tilde{\bar{p}} \)), \( \tilde{Z} \) (the shock thickness defined by 10-90% overpressure) and \( \tilde{t}_d \) (the half length of the N-wave measured from the origin to the point of 10% overpressure in the wave front). Then the following parameters can be obtained:
\[ \tilde{Z} = \frac{2}{1 + \frac{1}{\tau}} \quad \tilde{t}_d = \frac{a_1 X_d (\tilde{p})_{\text{max}}}{P_1} \quad (3.14) \]
\[ \tilde{Z}_d = \frac{2}{1 + \frac{1}{\tau}} \quad \tilde{t}_d = \frac{a_1 X_d (\tilde{p})_{\text{max}}}{P_1} \quad (3.15) \]
where \( X \) is the shock thickness corresponding to \( \tilde{Z} \), \( X = \sqrt{\tilde{Z}^2 + Z_d^2} \); \( X_d \), the half length of the N-wave corresponding to \( \tilde{t}_d \), \( X_d = \frac{t_d}{\sqrt{\tau}} \); \( (\tilde{p})_{\text{max}} \), the maximum value of \( \tilde{p} \). The parameters \( \tilde{Z} \) and \( \tilde{Z}_d \) correspond to the shock thickness and the flow duration of the N-wave with reference to the dimensionless variable \( Z \), which is defined similarly to Eq. (3.5) as
\[ Z = \frac{Z}{Z_d} \quad (3.16) \]
\( \tilde{Z} \) is the thickness parameter defined in the previous section and \( \tilde{Z}_d \) will be termed the duration parameter. Details of the derivation of \( \tilde{p}_{\text{max}}, \tilde{Z} \) and \( \tilde{Z}_d \) are given in Appendix A.

Figure 3.5 exhibits the pressure profiles for the same cases as shown in Fig. 3.4 in a plot of \( (\tilde{p})/\tilde{p}_{\text{max}} \) against \( \tilde{Z} \), where \( \tilde{p}_{\text{max}} \) is the \( \tilde{p} \) at \( (\tilde{p})/\tilde{p}_{\text{max}} = 0.5 \). The solid line indicates the Taylor solution for steady plane waves, which is given by Eq. (3.3) or (3.7). The Lighthill N-wave solution approaches the Taylor solution as \( \tilde{Z} \to \infty \) or Re \( \to \infty \). This can also be seen from Eq. (3.13) as follows. Assume that \( \tilde{p} \) reaches its maximum \( \tilde{p}_{\text{max}} \) at \( \tilde{t} = \tilde{t}_m \) for large Re. Then, approximately,
\[ \tilde{p}_{\text{max}} = \frac{\bar{p}}{P_1} \quad \tilde{t} = \tilde{t}_m / 2 \]
Putting \( \tilde{t} = \tilde{t}_m + \tilde{t} \) \( (\tilde{t} \ll \tilde{t}_m) \), then
\[ \tilde{p} = \tilde{p}_m \left[ 1 + \exp\left( \frac{\tilde{t}}{\tilde{t}_m} \right) \right]^{-1} = \tilde{p}_{\text{max}} \left[ 1 + \exp\left( \tilde{p}_{\text{max}} \tilde{t}/P_1 \right) \right]^{-1} \quad (3.17) \]
in the limit of Re \( \to \infty \). Equation (3.17) has the same form as Eq. (3.6), the Taylor solution, since \( \tilde{p}_{\text{max}} \) can be replaced by \( Z = Z_d \), where \( Z_d \) is the \( Z \) at \( \tilde{t} = \tilde{t}_m \). It should be noted that the shock thickness decreases as the Reynolds number Re or the duration parameter \( \tilde{t}_d \) decreases for the same maximum overpressure.

In Fig. 3.6, the ratio of the thickness parameter \( (\tilde{Z})/(\tilde{Z})_0 \) is plotted against the duration parameter \( \tilde{t}_d \), where \( (\tilde{Z})_0 \) is the \( (\tilde{Z}) \) for \( \tilde{Z}_d \) = (Taylor solution) and is given by \( (\tilde{Z})_0 = 5.127 \). This figure clearly shows the dependence of the shock thickness on the duration of the N-wave. As the duration or the maximum overpressure increases, the shock thickness approaches the Taylor value. As the duration or the maximum overpressure decreases, the deviation from the Taylor value increases.

In Fig. 3.7, the normalized shock thickness \( X/\sqrt{\tau/\tilde{a}} \) or the normalized rise time \( t_r/\sqrt{\tau/\tilde{a}} \) is plotted against the normalized maximum overpressure \( \tilde{p}_{\text{max}}/P_1 \) for the normalized duration \( X_d/\sqrt{\tau/\tilde{a}} \) or \( t_d/\sqrt{\tau/\tilde{a}} \) constant. It can also be seen from Fig. 3.7 that the shock thickness or rise time decreases for a fixed maximum overpressure \( \tilde{p}_{\text{max}} \) as the duration of N-wave decreases. This is the \( \tilde{Z}_d \) vs \( \tilde{Z}_d \) described in the Introduction.

In Fig. 3.8, the experimental data of Ref. 6 are compared with the Lighthill solutions for N-waves. The rise time \( \tilde{t}_r \) is plotted against the maximum overpressure \( \tilde{p}_{\text{max}}/P_1 \). The solid lines exhibit the N-wave solutions for \( t_d = 50 \) sec and a 0-sec which correspond to the half-durations in the spark experiments. The Taylor rise time for \( \tilde{t}_r = 27.5 \) sec is also plotted against \( \tilde{p}_{\text{max}}/P_1 \). The figure shows that the rise times obtained in the spark experiments are adequately explained by the Lighthill model of viscous (frozen) N-wave shocks though the measured rise times slightly deviate from the theoretical curves in the range of the lower overpressure.

In Fig. 3.9, the experimental data are plotted on a figure showing the ratio of the thickness parameters \( (\tilde{Z})/(\tilde{Z})_0 \) vs the duration parameter \( \tilde{t}_d \), shown in Fig. 3.6. The data cover the range of \( \tilde{Z}_d = 10-100 \), in which the shock data lie between \( \tilde{Z}_d = 10 \) and 60 and the exploding-wire data lie between \( \tilde{Z}_d = 50 \) and 100. Using the duration parameter \( \tilde{t}_d \), the data may be categorized into three domains. Above \( \tilde{Z}_d = 50 \), the measured \( (\tilde{Z}) \)-values deviate from the Lighthill curve and steeply increase with increasing \( \tilde{Z}_d \). In the range \( \tilde{Z}_d = 15-50 \), the measured \( (\tilde{Z}) \)-values nearly coincide with the Lighthill curve, a scatter of the data exists. Below \( \tilde{Z}_d = 15 \), the measured \( (\tilde{Z}) \)-values also deviate from the curve and steeply decrease with decreasing \( \tilde{Z}_d \). The broken lines are drawn to stress the tendency of the data.

Figure 3.10 shows a comparison between the observed and Lighthill N-wave pressure profiles. Typical profiles in the Series I-IV are plotted by the broken lines in comparison with the corresponding analytical ones, which are evaluated from Eq. (3.13) to have the same maximum overpressure \( (\tilde{p})_{\text{max}} \) and the same half-duration \( t_d \) as the experimental ones, and plotted by the solid lines to fit each other at the nodes of the N-waves. As seen from the figure, the pressure profiles observed in the spark experiments [Series I and II; Figs. 3.10(a) and (b)] nearly coincide with the analytical
ones, while the pressure profiles observed in the exploding-wire experiments [Series III and IV; Fig. 3.10(c) and (d)] deviate from those predicted analytically. The main difference between both experiments is that of the half-duration of the N-wave. Figure 3.11, as well as Figs. 3.8 and 3.9, suggests that the Lighthill viscous N-wave model does not always explain the rise times of N-waves over the entire range of t or Z.

3.3 Nonstationary Viscous Plane Waves

In this section, consideration is given to a nonstationary plane wave, which is generated by the impulsive motion of a piston in a tube. The initially discontinuous wave-front is smoothed out due to viscous diffusion and it tends to form a final steady profile. It will be shown in the succeeding sections that this process of shock thickening \(|z| \times \text{exp} \left( -\frac{x^2}{2t^2} \right) \) plays an important role in determining the rise times of weak spherical N-waves.

Lighthill (Ref. 20) has given a solution for the nonstationary plane wave by solving Burger's Equation. He obtained the following result:

\[ u(X,t) = \frac{u_2}{1 + \exp \left( -\frac{x - u_2 t}{\sqrt{2\alpha t}} \right)} \cdot \text{erf} \left( \frac{x - u_2 t}{\sqrt{2\alpha t}} \right) \]  

(3.18)

in which the initial wave form is given by

\[ u(X,0) = u_2 \text{ for } X > 0, \text{ and zero for } X = 0 \]  

(3.19)

where \( u_2 \) is the excess wavelet velocity for \( X > \).

Using Eqs. (3.3) and (3.11),

\[ \frac{(p)}{p_1} = \left[ 1 + \exp \left( \frac{1}{2\sigma_0^2} \right) \right] \cdot \text{erf} \left( \frac{x - u_2 t}{\sqrt{2\sigma_0^2 \alpha_0^2 t}} \right) \]  

(3.20)

where \( \sigma \) and \( t \) are the distance and time parameter, respectively, defined by

\[ \sigma = \frac{X - \frac{1}{2} u_2 t}{p_1} \quad \sigma_0 = \frac{X - \frac{1}{2} u_2 t}{p_1} \]  

(3.21)

The complementary error function is defined by

\[ \text{erf}_c(X) = \frac{1}{2} \int_X^{\infty} e^{-y^2} dy \]  

Note that the shock strength \( \frac{(p)}{p_1} \) depends on the piston velocity \( u_2 \) \( \times \) \( \text{exp} \left( -\frac{x^2}{2t^2} \right) \) and \( \text{erf} \left( \frac{x - u_2 t}{\sqrt{2\alpha t}} \right) \) throughout the process. When \( t \to \infty \), Eq. (3.20) becomes

\[ \frac{(p)}{p_1} = 1 + \exp \left( \frac{x - u_2 t}{\sqrt{2\sigma_0^2 \alpha_0^2 t}} \right) \]  

(3.22)

which is the Taylor solution for steady plane waves, Eq. (3.0).

Figure 3.11 shows the pressure profiles for several different time parameters in a plot of \( \frac{(p)}{p_1} \) against the distance parameter \( Z \). The pressure profile approaches the Taylor profile as \( t \to \infty \). It can be seen that the shock thickness \( t \) increases as \( \frac{(p)}{p_1} \) increases [whether based on maximum slope or 10-90% of \( \frac{(p)}{p_1} \)].

In Fig. 3.12, the ratio of the thickness parameters \( \frac{(\sigma)}{(\sigma_0^2 \alpha_0^2 t)} \) is plotted against \( x \). If we define a characteristic-time parameter of shock thickening \( s \), as at \( (\sigma) = 0.99 \), then \( s = 5.5 \) or \( \sigma = 30.25 \) from which the corresponding time \( t \) and distance \( x \) are obtained from Eq. (3.21) as

\[ \frac{t}{x} = \frac{x}{s} = \frac{(p)}{p_1} \]  

(3.23)

which are designated as the shock-thickening time and distance, respectively. These are inversely proportional to the square of the shock strength \( \frac{(p)}{p_1} \). This means that it takes a progressively longer time and distance to reach a final steady state for weaker shock waves or for lower \( \frac{(p)}{p_1} \). Physically, this tendency of longer shock-thickening time or distance for weaker shocks is attributed to the decline of shock steepening due to nonlinear (convective) effects.

In Fig. 3.13, the normalized shock-thickening time \( \frac{t}{x} \) against distance \( x \) is plotted against the shock strength \( \frac{(p)}{p_1} \). The time scale on the right hand side indicates the shock thickening time at NTP in air. For \( \frac{(p)}{p_1} = 10^{-4} \) or \( \frac{(p)}{p_1} = 10 \text{ Pa}, t_s = 1 \text{ sec} \) or \( x_s = 330 \text{ m} \). These values suggest that the nonstationary effect on the rise time or the shock thickness becomes very important for weak shock waves, for it takes a long time or a large distance to reach a steady state. This result is of value in interpreting Fig. 3.4 or 3.5, which provides solutions for quasi-stationary N-waves at the final values after a very long time without specifying how long it may actually take. The above solution quantifies the time or distance in specific cases. The spark and exploding-wire generated N-waves, described in Section 2, are also expected to be affected by this nonstationary effect, since the maximum overpressures are below 20 Pa only over a distance of 10m.

3.4 Shock Transitions with Vibrational Excitation

The structure and thickness of shock waves with vibrational excitation in air will be considered now. The analytical results of Polyakova, Solyan and Khokhlov (Ref. 21) and Johannsen and Hodgson (Ref. 12) for plane dispersed waves are re-examined and compared with Holst-Jensen's data (Ref. 6). Furthermore, extensions of Lighthill solutions for N-waves and nonstationary waves to shock transitions with vibrational excitation are
made possible by using a bulk-viscosity concept.

For weak shock waves with vibrational excitation, steady shock waves are formed as a result of a balance between the wave-form-steepening tendency due to finite-amplitude-compression effects and the wave-easing tendency due to both effects of viscous diffusion and vibrational relaxation. For very weak waves, the compression effects diminish and the wave-form-easing effects become predominant. As discussed in Section 3.1 for inviscid or inviscid shock transitions, in the limit of $\left(\frac{\gamma}{\gamma+1}\right) = 0$, the nonlinear compression effects disappear and the wave-form-easing effects remain, so that the wave thickness tends to infinity. For weak shocks whose strengths are slightly above the limit of zero overpressure, the vibrational relaxation is more effective than the viscous diffusion for the wave-easing tendency. In this case, the compression process is so slow that the energy dissipation due to vibrational nonequilibrium becomes predominant compared with that due to translational and rotational nonequilibrium which requires a more rapid change of the flow properties. As the wave strength increases, the shock thickness decreases owing to the increase in nonlinear-compression effects. When the nonlinear-compression effects overcome the wave-easing effects due to vibrational relaxation, the frozen shock transition appears in the compression process of the wave.

Figure 3.14 illustrates these two types of shock transition with vibrational excitation through pressure and temperature profiles. The vibrational temperature $T_v$ is also plotted to show the vibrational energy excitation. The former wave dominated by the vibrational excitation is a fully dispersed wave, and the latter wave including the frozen sound (relatively sharp, viscous) shock transition is called a partly dispersed wave. For strong shocks, the nonlinear compression mainly balances with the viscous diffusion, though it is accompanied by the slower process of vibrational excitation. As shown in Fig. 3.14, for stronger shocks, the temperature goes up to the maximum (Rankine-Hugoniot) value through the frozen sound shock compression and then falls to the final equilibrium state through the equilibrium $T_v = \text{const}$ as vibration attains its share of energy.

Polyakova et al. (Ref. 11) have obtained an analytical solution for the structure of steady, plane dispersed waves for nonviscous and nonconductive gases as

$$\frac{y + y_0}{v^2} = \left(\frac{v_0 + v}{v_0 - v}\right)^{k-1} \left(\frac{v_0}{v_0 - v}\right)^{k+1}$$

where $\gamma = t - \frac{\gamma}{\gamma+1}$; $t$ = Lagrangian coordinate, $v_0 = \text{equilibrium speed of sound}; y_0 = \text{constant of integration}; k = \text{vibrational relaxation time for j-molecule}; v = \text{velocity in a moving coordinate system}; v_0 = \text{absolute value of the velocity at the spatial coordinate $t = \text{const}$; } k = \text{frozen speed of sound}; c = \left(\frac{1}{\gamma+1}\right)$. 

In order to rewrite Eq. (3.24) using the normalized overpressure $\left(\frac{\gamma}{\gamma+1}\right)$ and the distance parameter $2$, which were introduced in the previous sections, introduce two quantities: the bulk viscosity and a critical overpressure.

The bulk viscosity $\nu_j$ for the $j$-molecule can be expressed as

$$\nu_j = \frac{\gamma}{\gamma+1} \left(\frac{\gamma}{\gamma+1}\right) = \gamma_{\text{ma}}^2$$

for processes sufficiently slow, where $\gamma_{\text{ma}}$ is the equilibrium density of the medium. Then the diffusivity $(\nu_j)$ for $j$-molecule with a bulk viscosity $(\nu_j)$ can be expressed as

$$\nu_j = \frac{\gamma_{\text{ma}}^2}{\gamma_{\text{ma}}^2}$$

This diffusivity will be used as a reference physical property. It should be noted that the use of this property does not mean that the vibrational relaxation processes can always be replaced by the bulk viscosity, which is valid only for processes sufficiently slow.

The critical overpressure is defined as the equilibrium overpressure behind a plane dispersed wave whose wave velocity is equal to the frozen speed of sound. When the equilibrium overpressure exceeds the critical overpressure, the steady plane wave is a partly dispersed wave with a frozen (viscous) shock front, which is followed by the vibrational relaxation region. When the vibrational overpressure is below the critical overpressure, the steady plane wave is a fully dispersed wave with a smooth transition, which is controlled by the vibrational excitation of the molecules.

The equilibrium overpressure across a normal shock wave with vibrational excitation can be given as

$$\frac{\gamma}{\gamma+1} = \frac{2\left(\gamma_{\text{ma}}^2 - 1\right)}{\left(\gamma_{\text{ma}}^2 + 1\right)}$$

where $\gamma_{\text{ma}}$ is the frozen Mach number, $c_j$ the vibrational specific heat for $j$-molecule normalized by the gas constant $c_j$. It is assumed to be constant across the shock wave. The equilibrium specific heats for $O_2$ and $N_2$ in air may be written as

$$C_1 = 0.209, \quad C_2 = 0.761 \left(\frac{T}{T_0}\right)^{-\gamma - 2}$$

where $T_0$ is the initial gas temperature (room temperature), $\gamma_j$ the vibrational characteristic temperature $\gamma_j = 2259.1 K, \gamma_j = 3552 K$. For $\gamma_j = 1$, we have the critical overpressure for the $j$-molecule as

$$\frac{\gamma}{\gamma+1} = \frac{2(\gamma_{\text{ma}}^2 - 1)}{\left(\gamma_{\text{ma}}^2 + 1\right)}$$

for $\gamma_{\text{ma}} = 1$, which is usually valid for atmospheric air, as very little vibrational excitation can exist at nearly room temperature. The critical overpress-
In Fig. 3.15, the critical overpressures \( \langle \rho p \rangle_{c_r,j} \) and \( \langle \rho p \rangle_{c_{r,cr},j} \) are plotted against \( T_1 \). The lines denoted by \( O_2 \) and \( U_2+N_2 \) are calculated this way. Eqs. (3.31) and (3.32) are given in Appendix B. Further details can be found in Appendix B. Equations (3.31) and (3.32) are also given in Appendix B.

Johannsen and Hodgson (Ref. 12) have also obtained an exact solution for steady plane dispersed waves for nonviscous and non-conductive gases, as follows:

\[
M^2_e [1 + (\gamma+1)\rho \lambda] = \frac{2(\gamma+1)c_0^2}{1 - \rho/\rho_1^2}
\]

\[
\frac{1}{\rho_1} = \frac{1}{\rho} - \frac{\gamma+1}{\gamma-1} \rho
\]

\[
2 \left( \frac{\rho_1^2}{\rho^2} \right) = \frac{1}{\gamma-1} \rho_1 - \frac{1}{\gamma-1} \rho
\]

\[
\frac{1}{\rho_1} = \frac{1}{\rho} - \frac{\gamma+1}{\gamma-1} \rho
\]

where \( \gamma \) is the flow velocity, \( \dot{u}_1 \), \( \dot{u}_2 \) are the flow velocities at \( x = 0 \). Using the relations

\[
1 = \frac{1}{\dot{u}_1} = \frac{1}{\dot{u}_2} + \frac{\gamma+1}{\gamma-1} \rho
\]

\[
\frac{1}{\dot{u}_1} - \frac{1}{\dot{u}_2} = \frac{\gamma+1}{\gamma-1} \rho
\]

and neglecting the higher order terms of \( \rho c_j \), the same equation as Eq. (3.34) is obtained, which was derived from the Polyakov et al (Ref. 21) formula, by using the distance parameter defined by

\[
Z = \frac{a_k x}{\dot{u}_j \gamma} + \rho \frac{\gamma+1}{\gamma-1} \rho
\]

Further details can be found in Appendix B. Equation (3.34) will be used as a solution for steady plane dispersed waves.

In the limit of a weak wave \( \langle \rho p \rangle_2 = 0 \), Eq. (3.34) tends to

\[
\gamma+1 \left( \frac{\rho c_j}{\rho} \right) = \left( 1 - \frac{\gamma+1}{\gamma-1} \rho \right)
\]

This has the same form as the Taylor solution, Eq. (3.7), in which the diffusivity \( \gamma \) is replaced by \( \gamma_j \). In the limit of weak shocks, the shock compression process is infinitely slow, so that the bulk viscosity concept may be applied to the vibrational relaxation process. The solution, in which the diffusivity \( \gamma \) is replaced by \( \gamma_j \), will be called the modified Taylor solution.

Figure 3.16 shows the pressure profiles for several different values of \( \rho c_j \) in a plot of \( \langle \rho p \rangle_{c_{r,cr},j} \) against \( Z_0 \). The curve for \( \langle \rho p \rangle_{c_{r,cr},j} = 0 \) corresponds to the modified Taylor solution. For partly-dispersed waves \( \langle \rho p \rangle_2 \neq \langle \rho p \rangle_{c_{r,cr},j} \), there appears a discontinuous shock front. The overpressure \( \langle \rho p \rangle_f \) immediately behind the frozen shock is given by

\[
\langle \rho p \rangle_f = \frac{1}{\rho} \frac{\gamma+1}{\gamma-1} \rho
\]
In Fig. 3.16, the chain curve indicates the pressure profile for \((\rho\text{p})_2/(\rho\text{p})_{cr,j} = 2\), in which the discontinuous shock strength at \(\tau = 2\) is \((\rho\text{p})_1 = 0.5(\rho\text{p})_2\).

The thickness parameter \((\rho\text{p})_{cr,j}\) is defined by the 10-90\% equilibrium overpressure, and can be related to the rise time \(t_I\) as

\[
\tau = \left(\frac{1}{\rho\text{p}}\right)_{cr,j} \frac{1}{j} \left(\frac{\rho\text{p}}{\rho\text{p}}\right)_{cr,j} (3.42)
\]

For fully-dispersed waves where \((\rho\text{p})_2/(\rho\text{p})_{cr,j}\), then from Eq. (3.34),

\[
\tau = \left(\frac{1}{\rho\text{p}}\right)_{cr,j} \frac{1}{\rho\text{p}} = 5.12 = (12)^{1/2} (3.45)
\]

regardless of the value of \((\rho\text{p})_2/(\rho\text{p})_{cr,j}\). That is, the thickness or rise time of a fully-dispersed wave, which is based on the 10-90\% equilibrium overpressure, has the same value of the thickness parameter as the Taylor thickness or rise time, if the diffusivity \(\eta^2\) is used instead of \(\eta\).

In Fig. 3.17, the ratio of the thickness parameter \((\rho\text{p})/(\rho\text{p})_{cr,j}\) is plotted against the equilibrium overpressure normalized by the critical overpressure for fully and partly-dispersed waves. It can be seen in the figure that the effect of dispersion on \((\rho\text{p})_2/(\rho\text{p})_{cr,j}\), remains up to \((\rho\text{p})_2 = 10^{11}(\rho\text{p})_{cr,j}\). This means that the rise times for steady plane waves are affected by the vibrational relaxation up to \((\rho\text{p})_2 = 500-1,000\) Pa in air, since \((\rho\text{p})_{cr,j} = 50-100\) Pa in the usual range of ground temperatures (see Fig. 3.15).

The Lighthill solutions for N-waves (Section 3.2) and nonstationary waves (Section 3.3) may be applied to fully-dispersed waves for small \((\rho\text{p})_2/(\rho\text{p})_{cr,j}\) by replacing the diffusivity \(\eta^2\) with the vibrational diffusivity \(\eta^2\) in order to provide a rough estimate of the shock thickness or rise time for plane waves with vibrational excitation.

Assume that,

\[
\rho\text{f} = 33.1\text{ m/s}, \quad j = 10^{-5} \text{ sec}, \quad (\rho\text{p})_{cr,j} = 50 \text{ Pa}, \quad p_1 = 101.3 \text{ kPa}
\]

then, from Eq. (3.31),

\[
(\rho\text{p})_2/(\rho\text{p})_{cr,j} = 4\times 10^{-5} \text{ m}^2/\text{sec}
\]

(Compare with \(\eta/\rho\text{f} = 4\times 10^{-5} \text{ m}^2/\text{sec}\), that is, the dispersed shock structure is entirely controlled by the vibrational relaxation.)

\[
(\rho\text{p})_2/(\rho\text{p})_{cr,j} = 14 x 10^{-5} \text{ sec}, \quad (\rho\text{p})_2/(\rho\text{p})_{cr,j} = 45 x 10^{-10} \text{ sec}
\]

(Compare with \(\eta/\rho\text{f} = 1.03 x 10^{-5} \text{ m}^2/\text{sec}\) and \(\eta/\rho\text{f} = 3.1 x 10^{-10} \text{ sec} \text{ noted above.}\)) These values are about ten times as long as the times evaluated for viscous shocks in Section 3.1. This means that the thickness or rise time of a plane dispersed wave is about ten times as long as that of a viscous shock wave for the same shock strength \((\rho\text{p})_2/p_1\). The shock-thickening time or distance of an impulsive step wave is also tenfold greater for a dispersed than for a viscous wave, as seen from Eq. (3.25). For \((\rho\text{p})_2/p_1 = 10^{-4}\) or \((\rho\text{p})_2 = 10\) Pa, \(t_I = 15\) sec or \(x_I = 5\) km. This shows that it is very difficult to obtain a plane dispersed wave in a steady state on a laboratory scale.

As for the N-wave effect, the dispersed wave is affected by the expansion behind the shock front more seriously than the viscous wave, since the former has a larger thickness than the latter for the same duration and maximum overpressure. Therefore, both the N-wave and nonstationary effects will seriously modify N-waves with vibrational nonequilibrium.

In Fig. 3.18, the exploding-wire data are compared with several theoretical curves in a plot of the rise time \(t_I\) against the maximum overpressure \((\rho\text{p})_{max}\). The chain lines indicate the Taylor and the modified Taylor rise times. The broken lines indicate the modified Lighthill rise times for N-waves of \(t_I = 100\) and 120 \(\text{ sec}\). The vibrational diffusivity \(\eta^2\) for oxygen is used for the modified Lighthill and the modified Lighthill solutions. All costs are evaluated for the gas temperature \(T = 280\) K and the relative humidity \(R = 8.7\%\) (Series IV). The corresponding vibrational-relaxation time and the critical overpressure of oxygen are about 3.75 \(\text{ sec}\) and 61 Pa, respectively. The measured rise times are much shorter than the modified Lighthill rise times for fully-dispersed N-waves. This discrepancy can be attributed to the N-wave effect.

Figure 3.19 shows a comparison between the observed and modified Lighthill N-wave pressure profiles in a similar way to Fig. 3.10 for viscous N-waves. Typical profiles from Series I-IV are plotted using broken lines in comparison with the corresponding analytical ones shown as solid lines, which are evaluated from Eq. (3.13), with \(\rho\text{p}_2/(\rho\text{p})_{cr,j}\) and \(\rho\text{p}_2/(\rho\text{p})_{cr,j}\) as the experimental ones, and fit at the nodes of the N-waves. By contrast to Fig. 3.10, the discrepancy between the observed and analytical profiles is clear.

In conclusion, this section considers the characteristic feature of weak N-waves with vibrational nonequilibrium. Figure 3.20 illustrates a classification of weak N-waves by their degree of vibrational nonequilibrium. The profiles of gas and vibrational temperatures are plotted under the following assumptions: (i) the maximum (peak) overpressure is below the critical overpressure for steady plane waves; (ii) the maximum overpressure is the same for all cases in Fig. 3.20; (iii) only one mode of vibrational excitation is considered. As seen, the N-waves can be classified into five categories: 

(a) quasi-equilibrium wave, 
(b) moderately-non-equilibrium wave, 
(c) highly-non-equilibrium wave, 
(d) nearly-frozen wave, 
(e) quasi-frozen wave.

The degree of excitation of vibrational energy is denoted by the vibrational temperature \(T_v\), which is plotted by broken lines in Fig. 3.20. The time lag between the gas and vibrational temperatures corresponds to the vibrational relaxation time \(\tau_v\). In a quasi-equilibrium wave, the vibrational temperature nearly follows the gas temperature. This is the case where the concept of bulk viscosity is valid and the modified Lighthill solution for N-waves may be applied. The structure of the shock front is
controlled by the vibrational relaxation, that is, the wave is a fully-dispersed wave. In a moderately-nonequilibrium wave, an appreciable deviation of the vibrational temperature from the gas temperature can be seen. In this case, the concept of bulk viscosity cannot be applied to the vibrational relaxation, though the structure is still controlled by the vibrational relaxation. This wave can also be considered as a fully-dispersed wave. In a highly-nonequilibrium wave, the front structure is controlled by both processes of vibrational excitation and viscous dissipation. The wave becomes a partly-dispersed wave in the sense that the front structure is partly controlled by viscous effect. The structure of a nearly-frozen-flow frozen wave is mainly controlled by viscous effect, though vibrational excitation still remains in the rest of the flow field. In a quasi-frozen wave, the vibrational excitation is marginal so that the whole flow field can be considered as frozen.

The discrepancy between the observed and analytical rise times and pressure profiles described in the preceding sections may be explained by considering the above classification for N-waves. The N-waves generated by sparks could be highly nonequilibrium or nearly-frozen waves, since the front structures seem to be mainly controlled by viscous effect. The N-waves generated by exploding wires could be moderately-nonequilibrium waves. The coupling of the N-wave and nonstationary effects would make the situations even more complex.

4. RANDOM-OCCURRING ANALYSIS FOR WEAK SHOCK TRANSITION

4.1 Basic Equations

The analysis is based on the following assumptions:

(a) The flow is a nonstationary one-dimensional (planar or spherically symmetric) viscous, compressible air flow.

(b) The viscosity and thermal conductivity are assumed to be constant, as the shock waves are weak.

(c) The gas is assumed to be thermally perfect; the equation of state for a thermally-perfect gas is used.

(d) Both cases of calorically-perfect and imperfect gases are analysed. For calorically-imperfect gases (referred to as real gases), the vibrational relaxation of air molecules are taken into account. However, for most of the analyses, only the vibrational relaxation of oxygen is taken into account, since the vibrational-relaxation time of nitrogen is much longer than the duration of most N-waves analysed in this study. The effects of nitrogen vibrational relaxation are discussed only in the last part of this section. The harmonic-oscillator approximation is applied to the vibrational energy level.

(e) The rotational relaxation is taken into account through the bulk-viscosity concept. The bulk viscosity due to the rotational relaxation is assumed to be \( \dot{\rho} = 12/3 \).

Then the basic flow equations can be written as:

\[
\begin{align*}
\frac{\partial \rho}{\partial t} + \frac{\partial (\rho u)}{\partial x} &= 0 \\
\frac{\partial (\rho u)}{\partial t} + \frac{\partial (\rho u^2 + p)}{\partial x} &= \mu \frac{\partial^2 u}{\partial x^2} + \frac{1}{\rho} \left( \frac{\partial \rho u}{\partial x} \right) \\
\frac{\partial (\rho e)}{\partial t} + \frac{\partial (\rho u e)}{\partial x} &= \frac{1}{\rho} \left( \frac{\partial p}{\partial x} \right) + \frac{1}{\rho} \frac{\partial \mu}{\partial x} \frac{\partial^2 u}{\partial x^2} + \frac{1}{\rho^2} \left( \frac{\partial \rho u}{\partial x} \right)^2 \\
\end{align*}
\]

where \( \dot{\rho} = \frac{\partial \rho}{\partial t} \) for plane flows and \( \dot{\rho} = \frac{\partial \rho}{\partial t} \) for spherical flows, \( \rho \) - density, \( u \) - velocity, \( p \) - pressure, \( T \) - temperature, \( e \) - total energy, \( \gamma \) - specific heat ratio, \( \gamma_N \) - vibrational energy for the \( N \)-molecule, \( \gamma_O \) - vibrational energy for the \( O \)-molecule, \( \tau_N \) - vibrational relaxation time for the \( N \)-molecule.

Based on the harmonic-oscillator approximation, the equilibrium vibrational energy for the \( O \)-molecule \( \epsilon_{y,i} \) can be expressed as

\[
\epsilon_{y,i} = \frac{R}{\mu} t \epsilon = \frac{\hbar \omega}{\gamma_{y,i}} \exp \left( \frac{\hbar \omega}{kT} \right)
\]

where \( \omega \) is the characteristic vibrational temperature for the \( y \)-molecule: \( \omega = 2150 \) K, \( \gamma_{y,i} \) - molar concentration for the \( y \)-molecule: \( \gamma_{y,i} = 0.209 \), \( \gamma_{y,i} = 0.78 \). The vibrational temperature \( T_{v,i} \) for the \( y \)-molecule can also be defined as

\[
T_{v,i} = \exp \left( \frac{(y_{i}-1)kT}{(y_{i}-1)E_{v,i}} \right)
\]

The vibrational relaxation times for oxygen \( \tau_{O} \) and nitrogen \( \tau_{N} \) are evaluated using the empirical relations obtained from the absorption of sound waves by Bass and Shields (Ref. 23), as follows:

10
\[ \dot{p} = \frac{1}{\gamma} \left( \frac{p_0}{p} \right) \left( 24 + 4.4 \times 10^4 h \right) \frac{0.05 + h}{0.391 + h} \]  (4.5)

\[ \lambda = \frac{1}{2} \frac{\rho_0}{p} \left( 1 - \frac{9 \times 350h}{\gamma} \exp \left( -0.142 \left( \frac{T_0}{T} - 1 \right) \right) \right) \]  (4.6)

where \( p_0 = 101.3 \text{ KPa}, T_0 = 293.15 \text{ K}, h = \text{ absolute humidity of air} \). As seen in Eqs. (4.5) and (4.6), the vibrational relaxation times for oxygen and nitrogen strongly depend on the absolute humidity of air. In Fig. 4.1, \( T_0 \) and \( \lambda \) are plotted as functions of the absolute humidity for \( p = 101.3 \text{ KPa} \). The relaxation time for nitrogen is two or three orders longer than the relaxation time for oxygen. The relative humidity is defined as

\[ RH = \frac{h(p/p_{sat})}{MR} \]  (4.7)

where \( p_{sat} \) is the partial pressure of water vapour at saturation, and given by the Goff-Gratch equation (Ref. 24) as

\[ \log_{10}(p_{sat}/p_0) = 10.7986\left[ 1 - (T_0/T) \right] + 5.02808 \log_{10}(T/T_0) \]

\[ + 0.10476\times10^{-4}\times10^{-8}\times26921(1/T_0)-1 \]

\[ + 0.42873\times10^{-5}\times10^{-8}\times76955(1-T_0/T) - 1 \]

\[ = 2.195983 \]  (4.8)

4.2 Numerical Method

An operator-splitting technique was applied to Eq. (4.11). The calculation is done for each spatial mesh in each time step using the following procedure:

1. The hyperbolic equations are solved for an inviscid frozen flow,

\[ \frac{\partial \mathbf{u}}{\partial t} = -\mathbf{J} \frac{\partial \mathbf{H}}{\partial \mathbf{u}} \]  (4.9)

where the subscript 1 indicates the solution of step (1).

2. The spherical corrections are made by using the values of the physical properties evaluated in step (1).

\[ \frac{\partial \mathbf{u}_s}{\partial t} = -j(H_1)^{-1} \]  (4.10)

3. The viscous diffusion equations are solved by using the values of the physical properties evaluated in step (2).

\[ \frac{\partial \mathbf{u}_v}{\partial t} = -\frac{2}{\gamma - 1} \left( \frac{\partial}{\partial r} \left( \frac{\mathbf{u}_v}{\mathbf{H}_v} \right) \right) \]  (4.11)

4. The vibrational relaxation equations are solved by using the values of the physical properties evaluated in step (3).

\[ \frac{\partial \mathbf{u}_v}{\partial t} = (H_1)^{-1} \]  (4.12)

The final solutions are obtained in step (4).

The RCM is applied to step (1) and the explicit method of finite difference is applied to steps (2) and (3). In step (4), the integrated relation was used. If one step is passed over among steps (2)-(4), then the following solutions result: plane flow, an inviscid nonequilibrium flow or a viscous frozen flow, respectively. These are termed as a plane solution, a real-inviscid solution and a perfect-viscous solution, respectively. The full solution including both effects of vibrational excitation and viscosity is called a real-viscous solution.

An outline of RCM is described below. Figure 4.2 shows an illustrative diagram for grid construction and sequence of the sampling procedure. The notations \( r \) and \( \cdot t \) are increments of space and time, respectively. For arbitrary integers \( n \) and \( i \), the properties \( U_{n+1/2} \) at time \( n+1/2 \) are calculated from the properties \( U_n \) at time \( n \). The intermediate values \( U_{n+1/2} \) are evaluated at time \( n+1/2 \). In the region of \( i \), the equation of \( \iota \) at point \( (n+1/2) \iota \) can be divided into five subregions, if the initial condition is solved for the discontinuous initial values in this region. Then, for example, the solution consists of a shock wave \( S \), an expansion wave \( R \) and a contact surface \( C \), as shown in Fig. 4.2. At time \( (n+1/2) t \), the region \( (i-1) \iota \) can be divided into four subregions (1)-(4) for five subregions, where the interior of the expansion fan is taken into account, and the physical properties in each subregion can be determined from the solution of the shock-tube problem for the initial condition \( (i-1) \iota \). The values \( U_{n+1/2} \) are evaluated from the values \( U_{n+1/2} \) at point \( P \) \( = r_\iota \iota \). The choice of \( P \) is made by a random-sampling technique in such a way that the sampled points are uniformly distributed within a finite-sampling frequency. In a similar way, the values \( U_{n+1/2} \) are obtained from the initial values \( U_{n} \) and \( U_{n+1} \). At the second half-time step, the values \( U_{n+1/2} \) are evaluated from the values \( U_{n+1/2} \) and \( U_{n+1/2} \) as initial ones. Godunov's iterative technique is applied to solve the Riemann problem. As the vibrational energies are assumed to be frozen, they are invariant across the waves, and keep their initial values, whose boundary is the contact surface.

In the second and third steps of the operator-splitting technique, explicit finite-difference schemes are employed. The finite-difference forms of Eqs. (4.10) and (4.11) reduce to

\[ \frac{\partial \mathbf{u}_s}{\partial t} = [H_1^{-1} + \tau_1] (H_1^{-1} \mathbf{u}_s) \]  (4.14)
on the free boundary. That is, at the free boundary.

\[ U^{n+1} = \frac{1}{2} \left( C_{3}^{n+1} + C_{2}^{n+1} \right) \]

the multiple time step is used to evaluate \( U_{3i}^{n+1} \) to improve accuracy. At the intermediate substep

\[ U_{i}^{n+1} = \frac{1}{2} \left( C_{3}^{n+1} + C_{2}^{n+1} \right) \]

are solved in each spatial mesh under the assumption of constant temperature and pressure, thereby yielding the analytical solution

\[ \left( C_{3}^{n+1} \right)_{i} = \left( C_{3}^{n} \right)_{i} \exp \left( -\frac{x}{\lambda} \right) \]

The finite-difference schemes with multiple time steps, similar to Eq. (4.17), were also applied to Eq. (4.18). In order to reduce the computation time, Eq. (4.18) was used for most of the calculations. As described in Section 4.1, in the present study, only the vibrational relaxation equation for oxygen was solved (except Section 4.4.b). Furthermore, the bulk viscosity concept was applied, instead of Eq. (4.11), to the vibrational relaxation for oxygen in Sections 4.4.5 and 4.4.6, in which the \( 3 \)-waves with long durations were analysed.

The condition of symmetry is imposed on the wall boundary and at the centre of the sphere. That is, at the boundary

\[ \left( C_{3}^{n+1} \right)_{0} = \left( C_{3}^{n} \right)_{0} \]

The condition of continuity of gradient is imposed on the free boundary. That is, at the free boundary

\[ \left( C_{3}^{n+1} \right)_{i} = \left( C_{3}^{n} \right)_{i+1} \left( i = 0, 1, \ldots k \right) \]

The entire programs are given in Appendix C.

4.3 Solutions for Plane Waves

As a check on the method, the one-dimensional shock-tube problem was solved for a perfect-inviscid flow, perfect-viscous flows and real-inviscid flows. The thickness and structure of the shock waves are compared with those obtained analytically in Chapter 3.

4.3.1 Perfect-Inviscid Solution

Figure 4.5 shows a computer plot of a perfect-inviscid solution of overpressure (\( \rho \)) against distance \( x \) for several time intervals for a diaphragm pressure ratio \( P_{41} = 2 \) and initial temperature ratio \( T_{41} = 2 \). The overpressure (\( \rho \)) is normalized by the initial pressure \( P_{0} \) and the distance \( x \) is normalized by the length of the high-pressure chamber \( x_{0} \) \( x = x_{0} \). The diaphragm is placed at \( x = 1 \). The time \( t \) is normalized by \( x_{0}/a_{1} (t = x_{0}/a_{1}) \). After starting the calculation or the removal of the diaphragm, a shock wave as a discontinuous front propagates towards the right hand side, and a rarefaction wave propagates towards the left hand side. When \( t \) = 1 the head of the rarefaction wave arrives at the end wall of the high-pressure chamber. The shock Mach number \( M_{0} \) is about 1.16, and the normalized equilibrium overpressure or the shock strength \( (\rho J)_{2}/(\rho J)_{1} \) is about 0.405. It should be noted that, unlike finite-difference schemes, the shock wave as a discontinuous front occupies one mesh jump without smearing, where the normalized one-mesh size \( x_{0} = 1/40 \).

4.3.2 Perfect-Viscous Solutions

Figure 4.4 shows a computer plot of a perfect-viscous solution for the same case as Fig. 4.3. The rarefaction wave reflects at the end wall (\( x = 0 \)) and proceeds towards the right hand side. As expected, smooth shock transitions due to actual viscosity are obtained. In order to show these smooth transitions clearly, a hypothetical chamber length \( x_{0} = 0.001 \) cm was assumed at an initial pressure and temperature of \( P_{0} = 0.13 \) kPa and \( T_{1} = 293.15 \) K. Consequently, \( t = 0.106 \) sec for \( t = 3.506 \).

Here, it was not necessary to obtain the whole flow field. The fine structure of the shock front was important. Therefore, in order to save computational time, the calculation was done only in a confined region near the front for the wave far from the diaphragm, neglecting the behaviour of the rarefaction wave. Figure 4.5 illustrates the region of calculation and a plot of the shock-front path in the \( x-t \) plane. In the calculation, 30-80 mesh points around the front were used, and the physical properties at each mesh point were transferred back to two points in the computational space as the wave proceeds over two points in physical space. The condition of continuity, Eq. (4.20), is imposed on the free boundary of the region of calculation. In Fig. 4.5, the white circles indicate the perfect-viscous RCM solutions, in which the position of the shock front is defined as the position of 50% of \( (\rho J)_{2} \). The solid and broken lines indicate analytical shock and sound-wave paths, respectively. The numerical solution for the shock path is in excellent agreement with
Figures 4.6(a)-(c) show perfect-viscous numerical solutions for the shock-tube problem described above by comparison with Taylor's and Lighthill's analytical solutions for the shock thickness, which is defined by 10-90% of $(\Delta p)^2$. The ratio of the thickness parameters $(\Delta p)/\Delta t$ at the shock front, which corresponds to the thickness or rise time normalized by the Taylor thickness or rise time, is plotted against the time parameter $\Delta t$ as defined by Eq. (3.21). The figures indicate that the step wave with zero thickness is reduced to a plane wave with a smooth transition owing to viscous action, as the wave proceeds. The broken and solid lines indicate Taylor's and Lighthill's solutions, respectively. The various numerical solutions are indicated by symbols. All calculations were carried out for the same case as Fig. 4.4 with $\Delta t = 1.0, p_1 = 101.3$ kPa, $T_1 = 1.16^\circ$, $(\Delta p) = 50.0$ KPa.

Figure 4.6(a) shows the effect of multiple time steps for viscous correction. The mesh size is $x^* = 1.25\times 10^{-5}$ cm. The black and white circles indicate the cases for $k = 1$ and 10 in Eq. (4.10), respectively. The $k = 10$ result for the transient behaviour of the shock thickness is closer to Lighthill's solution. It is seen that the multiple time step for viscous correction improves the result for the transient behaviour of the shock thickness. The random walk due to the random sampling in RCM and the overshoot of the thickness value near Taylor's value can be seen. The multiple time step of $k = 10$ was used for all calculations described below.

Figure 4.6(b) shows the effect of the choice of random numbers. The mesh size is $x^* = 1.25\times 10^{-5}$ cm. The black and white circles indicate the cases using the random numbers by maximum-length linearly recurring sequence and linear congruential sequence, respectively. It can be seen that the latter method is in better agreement. Therefore, linear congruential sequence was used for all other calculations in the present study, as well as by Saito and Glass (Ref. 17). It is also seen in Fig. 4.6(b) that the result is improved by reducing the mesh size, in comparison with the result in Fig. 4.6(a).

Figure 4.6(c) shows the comparison between the RCM and MacCormack's finite-difference method (MFM), which is shown in Appendix D. The MFM solution is in poor agreement with Lighthill's solution. Its thickness or rise time values are much larger than the analytical ones owing to the effect of artificial viscosity. The RCM solution with operator-splitting techniques is superior to the MFM solution for the same mesh size, although random scattering of the thickness or rise time values do occur. Better agreement with Lighthill's solution was attained by using a finer RCM mesh as shown. Computer costs would limit the ultimate mesh size to be used.

In Fig. 4.7, the normalized overpressure $(\Delta p)/p_1$ is plotted against the distance parameter $x$ at times $t = 0.99, 4.50$ and $58.3$ for cases of $x^* = 1.25\times 10^{-5}$ cm (white circle in Fig. 4.6(c)). The origin of $x$ is taken at the place of $(\Delta p)/p_1 = 0.5$. The solid lines indicate Lighthill's solution for the final steady state at $x^* = 0.99$ and Taylor's solution for the final steady state at $x^*$. The RM pressure profiles show very good agreement with the analyses. This result suggests that the RCM with the operator-splitting technique may be applied to analyze the transient behaviour of a viscous shock structure, though some random walks and overshoot above the Taylor value were observed for the thickness or rise time data.

4.3.3 Real-Inviscid Solution

The initial conditions $(\rho_1 = 1.0018, T_1 = 1.0, p_1 = 101.3$ kPa, $T_1 = 303.15^\circ$ K and RH = 90%) were chosen so as to give a fully-dispersed wave in the final steady state for a real-inviscid flow, and to give a fast approach to the steady state in order to reduce the computational cost. Only the vibrational excitation for oxygen molecules was taken into account for atmospheric air. The corresponding relaxation time for oxygen is $\nu = 1.04$ sec and the characteristic time using the bulk viscosity $(\nu^2)/p_1$ for oxygen is $(\nu^2)/p_1 = 8.4\times 10^{-4}$ sec. The equilibrium shock Mach number $M_{se} = 1.0004$ and the equilibrium overpressure is $(\rho_2)/p_1 = 0.1$. The initial step wave is shown in Fig. 4.5. The initial step wave is shown in Fig. 4.5. The calculation was also carried out only for a confined region near the front for the wave far from the diaphragm, similar to the perfect-viscous flow as shown in Fig. 4.3. The initial step wave is smoothed out owing to the dissipative effect of the vibrational relaxation. This result suggests that the process which smears the wave is largely different from that of the viscous wave. This tendency of smoothing has been shown analytically for linear waves (Ref. 25) and for nonlinear waves (Ref. 26). In a transient state, the wave is a partly-dispersed wave with a frozen shock front, even if the equilibrium shock pressure is below the critical overpressure. This suggests that the nonstationary effect is more important for dispersed waves than for viscous shocks.

Figure 4.9 shows plots of $(\Delta p)/p_1$ vs $x$ for real-inviscid shocks. The solid and broken lines indicate the modified Lighthill solution and the modified Taylor solution, respectively. The symbols indicate the RM solutions for $x = 0.025$ cm and $0.0125$ cm, respectively. The latter case corresponds to the one in Fig. 4.8. The RM solutions of a shock thickness show random walks and overshoot above the Taylor value, similar to the viscous solutions shown in Fig. 4.6. The thickness tends to approach the modified Taylor value using the bulk viscosity for oxygen vibrational relaxation. It should be noted that the shock-thickening time of the RCM solution is nearly the same as that of the modified Lighthill
solution, although the (12,12) vs. - plot of the RM solution deviates from Lighthill's solution owing to the difference in the transient-wave profiles between the two solutions shown in Figs. 3.11 and 4.8. That is, the shock-thickening time based on the modified Lighthill solution provides a reasonable estimate.

In Fig. 4.10, the normalized overpressure \( p_{\text{over}} / p_{\text{in}} \) is plotted against the distance parameter \( x \) at 25.0 and 27.6 for the case of \( x = 0.0125 \) on (white circle in Fig. 4.9), where \( x \) is also defined using the bulk viscosity for oxygen vibrational relaxation. The solid line indicates the analytical solution for \( p_{\text{over}} / p_{\text{in}} \) evaluated from Eq. (5.54) for steady dispersed waves. The RM solution profile for \( x = 25.6 \) shows very good agreement with analysis, but the \( x = 27.6 \) solution shows a slight deviation from the analytical one at the upstream side of the front. This deviation would be attributed to the randomness associated with the RM solution. However, in general, the RM solution for real-inviscid flow provides very reasonable results.

1.3 Solutions for Spherical Waves

As described in Chapter 3, the shock structures of spherical waves may be affected by N-wave and nonstationary effects and would be different from those of plane waves in some situations. The purpose of this section is to show some characteristic features of transient behaviour of shock structures of spherical waves through the RM analysis associated with the spark and exploding-wire experiments.

Twenty-three cases of numerical results are presented in this section for spherical waves, and termed as cases A1, A2, ... , B1, B2, ... , C1, C2, ... , D1, D2, ... , respectively. The A-series (A1, A2, ...) corresponds to perfect-inviscid solutions; B-series, perfect-viscous solutions; C-series, real-inviscid solutions; and D-series, real-viscous solutions. The parameters, which should be given as initial conditions, are the radius of the pressurized sphere \( r_0 \), the pressure and temperature ratios \( p_{11} \) and \( T_{11} \), the initial inner pressurized air and the ambient atmosphere, the atmospheric pressure \( p_{01} \) and temperature \( T_{01} \), and the relative humidity \( R_H \). These are tabulated for each case in Table 4.4. We assumed \( p_{01} = 101.5 \text{ kPa} \) for all cases. The relaxation time \( \tau \) and the spatial meshes \( r^* \) and \( r^{{\star}} \) are also tabulated in Table 4.1. The atmospheric conditions \( T_{01} \) and \( R_H \) are chosen from data in the spark and exploding-wire experiments described in Chapter 2 (Series 1).

In the C- and D-series analyses (real gases), the vibrational excitation is taken into account only for oxygen except case B1, in which both vibrational excitation for oxygen and nitrogen are included. In cases B5 through B8, the vibrational relaxation for oxygen is evaluated by using the bulk viscosity concept instead of solving the relaxation equation for oxygen.

1.4.1 Near-field Solutions for Perfect Inviscid Flows

In this section, perfect-inviscid solutions for spherical waves are shown in the near field of the pressurized sphere. The process of N-wave formation from an explosion of a pressurized air sphere and the effects of the pressure and temperature ratios are discussed.

Case A1 is a perfect-inviscid solution for \( P_{11} = 2 \) in the near-field of a pressurized sphere. Figures 4.11(a)-(c) exhibit computer plots of overpressure distribution at various times after the explosion.

Figure 4.11(a) shows the initial process of explosion of a pressurized air sphere. The front shock, which is formed immediately after bursting the sphere, decays as it propagates outwards, leaving an expanding flow behind it. The rarefaction wave, which propagates inwards into the sphere, reflected at the centre of the sphere and produces a highly rarefied region behind it. A second imploding shock wave of ever increasing strength is formed at the boundary between the inner and outer expansion regions. Some "noise" in the pressure profiles in the expansion region can be attributed to the random walk inherent in the RM. The comparison between near-field solutions of the explosion of a pressurized air sphere using Lax, MacCormack and Rannou's methods for a perfect-inviscid flow is given in Appendix D.

The succeeding process of N-wave formation is shown in Fig. 4.11(b). The imploding second shock reflects at the centre of the sphere and produces a highly compressed region around it. The reflected second shock is initially very strong, but rapidly decays at it proceeds outwards. It follows the front shock and forms the rear shock of an N-wave. Figure 4.11(c) exhibits the established N-wave, which maintains a similar profile as it propagates outwards. Its maximum peak overpressure decays gradually and the duration increases slowly.

Figures 4.12(a-d) show a comparison of established N-waves for cases A1-A4. Figure 4.12(a) exhibits a pressure profile for the same case as Fig. 4.11(c), though the mesh size is increased to \( r^* = 1.10 \) to be compared with cases A2-A4. In case A2, the temperature ratio \( T_{11} \) is twice that for case A1. In case A3, the pressure ratio \( P_{11} \) is increased from 2 to 9. In case A4, both pressure and temperature ratios are increased. Figure 4.12(b) shows that case A2 results in a more symmetric N-wave than case A1 owing to the hotter sphere, which enables the second shock to form sooner. This suggests that the half-duration of the negative overpressure of an N-wave can be controlled through a choice of \( T_{11} \). Figures 4.12(c) and 4.12(d) show that for higher \( P_{11} \) and \( T_{11} \) the N-waves generated by a spark or an exploding wire cannot be simulated using a pressurized-sphere explosion model.

As seen in Figs. 4.12(a-d), the overpressure profiles of the positive phase show only a slight change in shape regardless of \( P_{11} \) and \( T_{11} \) (although \( p \) and the durations are different). However, the negative phases strongly depend on these ratios. The length of the positive side is of the order of \( r^* = 1 \) or \( r^* = 1 \) in each case. That is, the half-duration of an N-wave is determined mainly by a choice of the sphere radius within the range of \( P_{11} \) and \( T_{11} \) considered here. In the following, use is made of \( r^* = 1 \), in order to simplify the analysis, since attention is focused on the front-shock structures of the N-waves in this work.
1.1.2 **Comparison Between Perfect-Inviscid, Perfect-Viscous, Real-Inviscid and Real-Viscous far-Field Solutions**

The calculation for cases A5, B1, C1 and D1 were carried out for the same parameters in order to make the comparison clear between perfect-inviscid, perfect-viscous, real-inviscid and real-viscous solutions in the far field. The vibrational excitation for oxygen was taken into account for real cases C1 and D1. The ambient conditions correspond to the series-1 experiment, and the relaxation time \( \gamma = 1.50 \pm 0.05 \) sec.

The results are shown in Figs. 1.1-4.17. Figure 1.13 shows the path of the shock front by plotting the centre of the front [0.5] [4pal]. The normalized radius \( r^* \) and the normalized time \( t^* \) are defined by \( r^* = r/r_0 \) and \( t^* = a_1 t/r_0 \), respectively. The solid line indicates the path of a sonic line. It is seen that away from the explosion the shock path nearly coincides with the sonic-line path. This result indicates the validity of the method of solution with regard to the propagation of the wave. The calculations were also carried out only in a confined computational region near the front in the far field as well as the calculations for plane wave shown in Fig. 4.3.

The maximum overpressure \( P_{\text{max}} \) for spherical waves is the integral of the overpressures due to perfect-inviscid, perfect-viscous, real-inviscid and real-viscous solutions at the centre. According to classical acoustic theory, \( P_{\text{max}} = \frac{r}{r} \) for weak spherical waves. However, as shown in the following, the loss of the maximum overpressure can deviate from classical theory if the effects of viscosity and vibrational nonequilibrium are taken into account. In order to readily evaluate the decay rate of the maximum overpressure, the decay index \( n \) is introduced, where \( n \) is defined locally as \( P_{\text{max}} \). In general, the value of \( n \) varies with \( r \), while \( n = 1 \) applies to spherical acoustic waves.

Figure 1.14 shows the decay of the maximum overpressure for four cases as a function of the distance \( r \). In the perfect-inviscid solution case A5, the maximum overpressure decays at a rate inversely proportional to \( r^* \) for \( P_{\text{max}} \approx 100 \) Pa, though \( n = 1 \) for \( P_{\text{max}} \approx 100 \) Pa. In other cases, B1, C1 and D1, the decay indices are higher and the overpressures are obtained for \( P_{\text{max}} \approx 100 \) Pa for all cases including case A5, at \( r = 20m \), \( n = 1.25 \) for case B1 and \( n = 1.465 \) for cases C1 and D1. The deviation from the classical acoustic theory for \( P_{\text{max}} \approx 100 \) Pa is attributed to the nonlinear effects in a wave of time \( t = 0 \) sec.

Figure 1.15 exhibits the half-duration \( t_g \) as a function of distance \( r \). The rapid increase of \( t_g \) near the centre is attributed to nonlinear effects. In case A5, \( t_g \) is constant for \( P_{\text{max}} \approx 100 \) Pa, while in cases B1, C1 and D1, \( t_g \) increases with \( r \) due to dissipative effects of viscosity and vibrational nonequilibrium.

Figure 1.16 shows the rise times \( t_p \) as a function of distance \( r \). Figures 1.17 shows the pressure profiles at several locations for cases A5, B1, C1 and D1. The perfect-inviscid solution results in a discontinuous front so that \( t_p = 0 \) in this case, unlike the smoothing caused by artificial viscosity in finite-difference methods. As seen in case C1, the effect of vibrational nonequilibrium contributes to \( t_p \) only for weak waves. The rise time of the perfect-viscous case D1 are almost the same as the rise times of the perfect-viscous case B1, until the effect of vibrational nonequilibrium becomes noticeable. The viscous effect plays a dominant role in determining the rise time in these cases. However, the vibrational nonequilibrium plays an important role in reducing the maximum overpressure.

The profile of the perfect-viscous transition at \( r = 20m \) [Fig. 4.17] is not similar to either the profile for a steady plane wave (Section 3.1), the quasi-stationary N-wave for moderate Reynolds number (Section 2.1), or the nonstationary, plane wave (Section 3.2). This shows the characteristic feature of the nonstationary effect for spherical N-waves. Figure 4.17(c) indicates that the wave is a partly dispersed wave with a discontinuous front, even though the steady plane wave becomes a fully dispersed wave with a smooth transition for the corresponding overpressure at \( r = 20m \) (Section 3.1). Again, this is a nonstationary effect for dispersed waves, which is discussed in Section 2.1.3. The nonstationary dissipative effects due to viscosity and vibrational nonequilibrium are coupled in the real-viscous solution [Fig. 4.17(d)].

The results for cases A5, B1, C1 and D1 show that the decay behaviour of the maximum overpressure, the half-duration, the rise time (for the shock thickness) and the pressure profile of a weak spherical N-wave can be affected by both viscosity and vibrational nonequilibrium. This shows that both effects must be taken into account when analysing the shock structure of a weak spherical N-wave.

1.4.3 **Simulations for Spark and Exploding-Wire-generated N-Waves**

In this section, the numerical simulations are shown for the spherical N-waves, which were generated from spark and exploding-wire sources, described in Chapter 2. A requirement was set for the \( r = 0 \) of weak spherical N-waves that the calculated maximum overpressure \( P_{\text{max}} \) and the half-duration \( t_g \) should coincide with the experimental values at a specified location \( r \). This requirement can be fulfilled by giving appropriate values to the initial pressure ratio \( P_0 \) and the radius \( r_0 \) of the pressurized sphere. However, in practice, the adjustment of the values of \( P_0 \) and \( r_0 \) is a laborious task in order to match required values of \( P_{\text{max}} \) and \( t_g \) at a specified location. Several trial calculations were needed to get the final results. Cases B2, C2, D1, B2 and B3 are the results of simulations for the spark and exploding-wire wire.

In Figs. 4.18-4.21, the results of the numerical calculations are compared with the experimental data by plotting \( P_{\text{max}} = \gamma t_g \) vs. \( r \) and \( t_p \) vs. \( r \). In these figures, the experimental results are plotted by black symbols and the numerical data by white symbols. The solid and broken lines denote the interpolating lines for the numerical and experimental data, respectively. In Fig. 4.21, the
broken lines denote the lightfall rise times for X waves with $\theta_{1c}$ and $\theta_{2c}$, and the chain lines denote the Taylor rise time for steady, plane waves. The abrupt changes in rise time are attributed to the randomness inherent in the R003.

Figures 1.18 and 1.19 show that one can simulate the change of $U \max$ and $t_d$ against $r$ by a proper choice of $r_1$ and $F_{21}$. Curves B2 and C2 in cases 1.20 and 1.21 indicate that the perfect-viscous (12) and real-inviscid (12) solutions cannot simulate the nonstationary behavior of the rise time, even if $U \max$ and $t_d$ can be simulated against $r$. Curves B1, B2, and B3 indicate that the real-viscous solutions simulate the experimental results reasonably well, when one considers the flow complexities at the spark discharge and exploding wire. Curves B1, B2, and B3 almost simulate the data for cases 1.11 and 1.12, respectively.

The general features of the results can be summarized as follows:

- The decay index of $\pi \max$ evaluated from the series 1 and 11 spark data is about 1.45, while for the simulated spark case 81, $n = 1.30$ at $r = 2.0 m$ and $n = 1.42$ at $r = 10 m$. The deviation of the former is the linear acoustic theory ($n = 1$) is simulated reasonably well and can be attributed to the dissipative effects of viscosity and vibrational nonequilibrium for oxygen.

- The half duration $t_d$ increases with $r$ (Fig. 1.20). Its rate of increase is about 1.2% sec/m and simulates the experimental data. The increase of $t_d$ may also be attributed to dissipative effects of viscosity and vibrational non-equilibrium for oxygen.

- The spark data (series 1 and 11) and their simulation for rise time (cases 10 and 112) show that the shorter relaxation time $t_d$, see, case 82, gives the longer rise time and the longer relaxation time $t_d$, see, case 84, gives the shorter rise time. This tendency is due to the nonstationary effect. The long relaxation time gives the slow rate of change of the shock thickness, as discussed in sections 3.8.1 and 3.8.3, so that the rise time remains short even for weak waves. Further discussion about the effect of relaxation time on $t_d$ will be given in the succeeding section.

- The exploding wire data (series 11 and their simulation case 112) show, by comparison with the spark data, that the stronger explosion and longer duration give the longer rise time for the same overpressure (Fig. 1.21). This is again due to the nonstationary effects. The strong explosion gives a slower rate of change of the maximum overpressure for the same overpressure (see Fig. 1.18) so that the rise time has enough time to increase. Furthermore, a longer duration provides a margin for increasing the rise time. This effect will be discussed in more detail in Section 4.1.3.

Figures 1.22(a-c) show the pressure, temperature, and vibrational-temperature profiles at several locations for real viscous cases, 81, 82, and 83, respectively. The solid lines indicate the pressure and temperature profiles, which are the same for weak waves in normalized forms of $U \max$ and $t_d / t_d \max$. The broken lines indicate the vibrational temperature in a normalized form of $U \max$ where $U \max$.

Figure 1.22(a) shows the simulation for the spark (series 11) experiment. The wave profiles are shown at $r = 0.2, 2.0, 3.5, 6.3, 15.0, 18.0, 21.0, and 25.0 m$, and the maximum overpressures are 0.007 Pa, 135 Pa, 56.0 Pa, 25.0 Pa, 17.2 Pa, 15.2 Pa, 10.4 Pa and 8.5 Pa, respectively. As seen, the peak pressure and temperature become gradually blunted due to the energy transfer from the translational and rotational modes to the vibrational mode, while the back expansion pressure and temperature profile becomes gradually rounded due to the reverse energy transfer from the vibrational mode to the translational and rotational modes. This shows that the relaxing behaviour of the vibrational energy and leads to an elongation of the half duration. The shock thickness or the rise time is mainly controlled by the dissipative effect of viscosity, though it is only partly affected by the energy transfer from the translational and rotational modes to the vibrational mode for waves at $r = 18.0$ and 21.0 m. In a sense that the shock front is mainly controlled by viscous dissipation, these waves are called fully dispersed waves.

Figure 1.22(b) exhibits the simulation for the spark (series 11) experiment. The wave profiles are shown at $r = 0.0, 0.1, 0.2, 0.3, 0.4, 0.5, 0.6, 0.7, 0.8, 0.9, 1.0, 1.5, 1.7, 1.9, 2.0, 3.5, 5.0, 17.7 Pa, 15.2 Pa, 10.4 Pa, 8.5 Pa and 6.35 Pa, respectively, approximately in maximum with the maximum overpressures in Fig. 1.22(a). The difference in profiles at between cases 81 and 82 can readily be seen. In case 82, the process of peak blunting occurs between $r = 1.5$ and $1.58$ or $t_d \max = 5.5$ Pa and 135 Pa, while in case 81, it occurs between $r = 2.5$ and 2.0 m or $t_d \max = 5.5$ and 5.1 Pa. This can be attributed to the differences in vibrational relaxation time for oxygen and initial temperature: $\tilde{t} = 1.6 \text{ sec}$, $\tilde{t} = 25 \text{ sec}$, and $\tilde{t} = 25 \text{ sec}$ in case 81 and in $\tilde{t} = 1.6 \text{ sec}$, $\tilde{t} = 25 \text{ sec}$ and $\tilde{t} = 25 \text{ sec}$ in case 82. In case of the shorter relaxation time, the peak blunting occurs in the earlier stage when the shock thickness is still relatively thin. Furthermore, in case of the higher initial temperature, more energy is required to excite the vibrational mode so that the effect of vibrational excitation appears for waves at higher maximum overpressure. In the range $r = 8.5 \text{ m}$ to $8.5 \text{ m}$ or $t_d \max = 17.2 \text{ Pa}$, the front structures are mainly controlled by vibrational excitation and the wave profiles nearly follow the vibrational temperature profiles owing to the energy transfer to the vibrational mode. In this sense, the waves may be called fully dispersed waves in this range. However, it should be noted that the viscous dissipation also plays an important role in increasing the shock thickness or rise time, by contrast with weakly dispersed, plane waves, as seen in Figs. 1.20 and 1.21 in which the real viscous solutions (82) are compared with the perfect viscous and real Inviscid solutions (81, 82).

Figure 1.22(c) shows the simulation for the exploding wire (series 11) experiment. The wave profiles are shown at $r = 0.0, 0.1, 0.2, 0.3, 0.4, 0.5, 0.6, 0.7, 0.8, 0.9, 1.0, 1.5, 1.7, 1.9, 2.0, 2.3, 2.5$ and 2.8 m, respectively. The solid lines indicate the pressure and temperature profiles, which are the same for weak waves in normalized forms of $U \max$ and $t_d / t_d \max$. The broken lines indicate the vibrational temperature in a normalized form of $U \max$ where $U \max$.
from the ones in Fig. 4.22(h). The peak-blotting occurs between \( r = 4.13 \) and \( r = 6.26 \). Figures 4.22a-c show that wide variations of wave profiles are possible depending on combinations of relaxation time, initial temperature, half-duration, and strength of explosion.

In Figs. 4.23 and 4.24, the calculated pressure profiles are compared with those observed at several locations for series I and II, respectively. Figure 4.23 shows a comparison between cases B-1 and series II profiles at \( r = 4.13m \), \( 10m \) and \( 19m \), while Fig. 4.24 shows a comparison between case B-2 and series II profiles at \( r = 1.17m \) and \( 19m \). The solid and broken lines indicate the numerical and experimental pressure profiles, respectively. The shock transition and reattachment profiles are simulated reasonably well, if we consider the difficulty of adjusting \( \rho_1 \) and \( \rho_2 \) to get the required values for \( \rho \) and \( \tau \) at the desired positions. Compare with Figs. 5.10 and 5.11 to the light illness and pressure profiles.

In Figs. 4.25-4.27, the full X-wave profiles of pressure, temperature and vibrational temperature are plotted at the longest distances of observation in series I, II and B for cases B-1A, B-2A and B-3A. In order to save computation time, the full X-wave solutions were obtained with larger mesh sizes, which were two or three times as large as those for the half X-wave solutions for B-1, B-2 and B-3 shown above. These figures show that the transition profiles and rise times of the rear shock are different from the front shock due to the difference in vibrational nonequilibrium.

In Fig. 4.28, the calculated full X-wave profile of pressure is compared with the observed one at \( 20.93m \) for series II, in which the full X-wave profiles were obtained (Ref. 61). Although the calculated half-duration \( \tau_2 \) is 20% longer than the observed one, both profiles are similar. The precise simulation for full X-waves would require an adjustment of the initial temperature ratio \( \rho_1 \) in addition to the timer adjustment of \( \rho_1 \) and \( \rho_2 \). This may be done in a future study.

### 4.1.4 Effects of Vibrational Relaxation Time

The purpose of this section is to show the effect of vibrational relaxation time more clearly by comparing cases B-2 and B-3. The initial pressure ratio \( \rho_1 \) and the sphere radius \( r_0 \) are the same for both cases, but the initial temperature and humidity are different and give rise to a wave of 5.54 and 15.4 sec., respectively. (The initial temperature and humidity of case B-3 correspond to case B-1.)

Figures 4.29-4.31 show \( \rho \) at \( \rho_{\text{max}} \), \( \tau_2 \) and \( \tau_4 \) as functions of \( r \). The discontinuous change of \( \tau_4 \) in Fig. 4.31 is also attributed to the randomness which appears in the RCM solutions. Figure 4.32 shows a comparison of both pressure profiles at the same distance of \( 19m \).

The abrupt change of \( \rho_1 \) at \( \rho_{\text{max}} \) (Fig. 4.29) is slightly affected by the vibrational relaxation time \( \tau_2 \). Figure 4.31 shows a comparison of both pressure profiles at the same distance of \( r = 20m \).

The abrupt change of \( \rho_1 \) at \( \rho_{\text{max}} \) (Fig. 4.29) is slightly affected by the vibrational relaxation time \( \tau_2 \). Figure 4.31 shows a comparison of both pressure profiles at the same distance of \( r = 20m \).

The abrupt change of \( \rho_1 \) at \( \rho_{\text{max}} \) (Fig. 4.29) is slightly affected by the vibrational relaxation time \( \tau_2 \). Figure 4.31 shows a comparison of both pressure profiles at the same distance of \( r = 20m \). As clearly seen in Figs. 4.29-4.31, \( \rho \) at \( \rho_{\text{max}} \) and \( \tau_4 \) are not affected appreciably by the difference in \( \rho_1 \), but \( \tau_2 \) is very much affected by it. The rise time for case B-3 with a longer relaxation time is shorter at a fixed distance than the rise time for case B-2 with a shorter relaxation time. This conclusion can be explained by the nonstationary effects.

For steady, dispersed plane waves, as shown in Section 3.4, the longer relaxation time gives a thicker transition or a longer rise time, since \( \tau_2 \) is proportional to \( \rho \) [see Eq. (4.3)]. However, the shock thickening time, which was defined by the time of approach of an impulsive step wave to the stead state, is also proportional to \( \rho \) in the modified lightfield solution for a nonstationary fully dispersed wave. Furthermore, it was shown in Section 4.3.3 that this shock thickening time is in close agreement with the HWM solution. That is, the longer the relaxation time, the slower is the rate of change of shock thickness. For case B-3 with the longer relaxation time, the wave still remains a partly dispersive wave whose shock transition is mainly controlled by viscous action, while for case B-2 with a shorter relaxation time the wave becomes a fully dispersive wave whose transition is mainly controlled by vibrational nonequilibrium. This is the reason why the longer relaxation time gives a shorter rise time for the weak spherical waves in contrast with steady plane waves.

### 4.1.5 Effects of X-wave Duration

In this section, the effects of the duration of the X-wave on the decay rate of \( \rho_{\text{max}} \), the rate of increase of \( \rho_{\text{max}} \) and the rise time \( \tau_4 \) are investigated by changing the radius \( r_0 \) of the pressurized sphere. In cases B-5 and B-6, real-viscous solutions are obtained for the same conditions as case B-3 except for the sphere radius. The radii for cases B-5 and B-6 are ten and fifty times, respectively, as large as the radius for B-3. Consequently, the half-durations of the generated X-waves for B-5 and B-6 are about ten and fifty times as long as the half-duration for B-3. Furthermore, the distances travelled by the wave fronts in cases B-5 and B-6 are about ten and fifty times as long as the distance in case B-3 to reach nearly the same maximum overpressure.

In cases B-5 and B-6, the vibrational relaxation time for oxygen is much shorter than the half-durations of the X-waves, and \( \tau_4 \) is about 10.4 sec. for B-5 and 20.9 sec. for B-6, where \( \rho \) at \( r = 19m \). According to the classification described in the last part of Section 3, the former cases correspond to quasi-equilibrium waves, while the latter correspond to a moderate nonequilibrium wave. In this section, the bulk-viscosity concept is introduced to evaluate the vibrational relaxation for oxygen instead of solving the relaxation equation for oxygen. This assumption is necessary for these cases due to the fact that the relaxation time or length for oxygen is much shorter than the characteristic flow time or length, such as the half-duration of X-wave length. In practice, a typical time step \( t \) for B-5 and B-6 becomes longer than \( \frac{1}{t} \) for B-5 and B-6. In the basic equations (Eq. (4.1)), the coefficient of viscosity \( \rho \) was replaced by \( \rho \rho \) for B-5 and B-6, and the method of solving the equations for perfect viscos flows was used. This is, only spherical and viscous corrections were carried out in the operator-splitting technique. Details of the bulk viscosity analysis are shown in Appendix 1.

Figure 4.35 shows the attenuation of \( \rho_{\text{max}} \) vs.
the normalized distance $r^*$ for cases $0$, $0.25$, $0.5$, and $1$. Case $0$ is a perfect-inversion solution for the same $1$ as cases $0.25$, $0.5$, and $1$. In the initial stage of $1_{	ext{max}}$, the decay indices are almost the same for all cases at higher $1_{\text{max}}$ but vary as the waves weaken. At $r^* = 2000$, $1_{\text{max}} = 1$ for case $0$, $1.13$ for case $0.25$, $1.15$ for case $0.5$, and $1.06$ for case $1$. The decay index $n$ decreases at a fixed distance as the half-duration $t_1$ increases. That is, the effect of vibrational non-equilibrium on the decay rate of $1_{\text{max}}$ as the wave approaches equilibrium. Since $n = 1$ for a weak frozen wave (case $0$), the maximum value of $n$ would exist for a moderate non-equilibrium wave. Among the cases shown in this section, the maximum value of $n$ was obtained for case $0.25$.

Figure 4.34 shows the normalized half duration $t_1$ as a function of $r^*$, where $t_1^*$ is defined by $t_1^* = \frac{t_1}{r^*}$. The half durations rapidly increase in the initial stage for all cases due to the non-linear effect, but are quite different at the later stage of weakened waves depending on the degree of vibrational non-equilibrium. For wave waves below $100 \text{ Pa}$, the rate of increase is zero for a frozen wave (case $0$), but positive for non-equilibrium waves (cases $0.25$, $0.5$, $1$). The maximum rate of increase of $t_1^*$ as $r^*$ is obtained for the moderate non-equilibrium wave case $0.25$ as well as for the decay rate of $1_{\text{max}}$ (Fig. 1.35).

Figure 4.35 shows the rise time $t_r$ as a function of $r^*$ for $1_{\text{max}}$. The broken line indicates the modified solution for steady, real viscous, plane waves, in which the coefficient of viscosity $\kappa = \text{coefficient of viscosity}$ is used instead of $\nu$, in the Taylor solution. The chain lines indicate the modified Taylor solutions for real viscous waves with $t_g = 0 \text{ sec}$, $1 \text{ sec}$, and $2 \text{ sec}$, respectively, which correspond to the half durations at $r^* = 2000$ in cases $0.25$, $0.5$, and $1$. It can be seen in Fig. 4.35 that the rise time increases with increasing $t_1$ at a fixed $1_{\text{max}}$ and approaches the modified Taylor or Lighthill value. In case $0$, the rise time $t_r$ overestimates the modified Taylor value for the higher $1_{\text{max}}$. This overshoot would correspond to the overshoot of $t_1$ above the Taylor value for an impulsively started shear motion as described in Section 1.3.2, and can be improved by using a larger mesh size. As described for plane waves in Section 4.3, the present method of calculation gives good results for the transient behaviour of $t_1$, but has the defect that there appear some overshoots of $t_1$ above the Taylor value for quasi-steady waves. Some improvement will be required in the calculations in a future study. The nonstationary effect clearly appears in Fig. 4.34 for $r^* = 10 \text{ Pa}$ in case $0.25$ and for $1_{\text{max}} = 10 \text{ Pa}$ for case $1$. So that the $t_1$ values tend to freeze and have lower values than the modified Taylor and Lighthill values.

Figure 4.36 shows the rise time $t_r$ as a function of $1_{\text{max}}$ for comparison between cases $0$ and $1$. In case $0$, the relaxation equation for oxygen was solved without using the bulk viscosity concept for the same parameters of case $0$. The broken line indicates the modified solution. The rise time of $1_{\text{max}}$ in the initial stage of weakened waves is higher over that of the modified Taylor values than those of $0$. This is the reason why the bulk viscosity concept was used in the analysis of $N$ waves of long durations.

Figure 4.37 shows the normalized overpressure profiles ($1_{\text{max}}$) plotted against the normalized time ($t_1$) for the same maximum overpressure for cases $0.25$, $0.5$, and $1$. As the wave approaches equilibrium, the peak rise time becomes sharp and the back becomes straight.

1.4. Effects of Vibrational Relaxation

The effects of vibrational relaxation for oxygen at the interaction of $1_{\text{max}}$, the half duration $t_1$, and the rise time $t_r$ are investigated by introducing the vibrational relaxation equations for nitrogen into the governing equations used in the previous section. The vibrational relaxation for oxygen is taken into account through the bulk viscosity concept. The real viscous calculation was carried out by introducing the real gas correction for nitrogen in the operator splitting technique. The details can be seen in Appendix 1.

The lower initial pressure ratio of $1_{\text{max}} = 0.8$ was chosen for cases $0$, $0.25$, and $0.5$, since it was expected that the effect of vibrational relaxation would be less for lower overpressure cases $0$. $0_{\text{max}}$ and $1_{\text{max}}$ are real viscous solutions for the same conditions as cases $0$, $0.25$, and $1_{\text{max}}$, but only the vibrational relaxation for oxygen is taken into account by using the bulk viscosity concept. $1_{\text{max}}$ is a real viscous solution for the same conditions as case $1$, including the vibrational relaxation for nitrogen.

Initially, it was decided to carry out the calculation for the interaction of $1_{\text{max}}$, including the vibrational relaxation for nitrogen, to verify a few facts. However, it was found that it took three exponential time for the wave to reach the maximum overpressure. Enough to show the effect of nitrogen. Instead, the initial pressure ratio was reduced to attune this in a reasonable computational time.

Figures 4.38 and 4.39 show plots of $1_{\text{max}}$, $t_1$, and $t_r$, respectively, for cases $0$, $0.25$, and $0.5$. The broken and chain lines in Fig. 4.38 indicate the modified Taylor rise time and the modified Lighthill rise time, respectively, for real viscous waves, respectively. The Taylor rise time is used for cases $0$ and $0.25$.

Figure 4.40 shows that the initial decay of $1_{\text{max}}$ is almost the same for both $0$ and $0.25$, but the decay rate increases for $1_{\text{max}}$ below $20 \text{ Pa}$ in $0.5$ due to vibrational non-equilibrium in nitrogen. At $r^* = 1000$, $1_{\text{max}} = 1$ for $0$ and $1.5$ for $0.25$. In Fig. 4.39, the increasing rate of $t_1$ is slightly larger in $0.25$ than in $0$ due to the effect of vibrational non-equilibrium for nitrogen. However, as easily seen from Fig. 4.39, the rise time $t_r$ is not affected by it.

Figure 4.41 shows a comparison between the pressure profiles at $t_1$ for case $0$ and $0.25$, and Fig. 4.42 shows profiles of temperature and vibrational temperature for nitrogen at the same location for $0$. These figures indicate that the wave of $0_{\text{max}} = 1$ corresponds to a highly non-equilibrium wave for vibrational relaxation of nitrogen, and thus the shock transition is mainly controlled by the vibrational relaxation of oxygen. The nitrogen non-equilibrium acts only to lower the maximum overpressure. It can be said that the dissipative effect of vibrational non-equilibrium for nitrogen in case $0_{\text{max}}$ plays a role like oxygen in
conclusions.

The foregoing results can be summarized as follows:

1. It was shown that the transient shock structures of weak plane and spherical waves in air can be
analyzed by solving the unsteady, compressible
Navier-Stokes equations with a vibrational-relaxation
equation for oxygen or nitrogen, using the
random-choice method (RCH) with an operator-splitting
technique.

2. The perfect-viscous and real-inviscid solutions for
impulse step-waves that show the spreading
processes due to dissipative effects of viscosity
and vibrational nonequilibrium for shock fronts are
in reasonable agreement with analysis. However,
there is some randomness in the shock thickness or
the rise time value and there are overshoots above
the steady-state value.

3. The initial X-wave formation process was established
for a perfect-inviscid wave for explaining
pressurized air spheres. It was found that the
attenuation of the maximum peak overpressure and
the half-duration of an X-wave in the far field
is controlled by a proper choice of sphere
radius and initial diaphragm-pressure ratio.

4. The perfect-inviscid, perfect-viscous, real-
inviscid and real-viscous far-field solutions for
weak spherical waves in air were compared. It was
found that the dissipative effects of viscosity and
vibrational nonequilibrium of oxygen on the decay
of the maximum overpressure, half-duration and
X-wave rise time become distinguishable for values of
$\Gamma_{\text{max}} > 100$ Pa.

5. The numerical simulations were carried out for
weak spherical X-waves generated in atmospheric air
from sparks and exploding wires. The numerical
results show good agreement with the experimental
data with regard to the decay rate of $\Gamma_{\text{max}}$, the
increasing rate of $t_d$, the rise time $t_r$ and the
wave profiles. The results indicate that the
observed shock structures of weak spherical X-waves are
controlled by the coupled dissipative effects of
viscosity and vibrational nonequilibrium of
oxygen.

6. The calculated and observed rise times for
shock thicknesses for weak spherical X-waves are
mostly much smaller than those predicted analyti-
cally for steady plane waves. It is found that
this phenomenon is attributed to the $\gamma$-value and the
$\alpha$-value, combined with vibrational
relaxation of oxygen. The shorter half-duration
ave shorter rise times for the same maximum over-
pressures due to flow expansion behind the
front shock of an X-wave (X-wave effect). A more rapid
decline of the maximum overpressure also results
in a shorter rise time for the same maximum
overpressure, since the shock-thickening time becomes
increasingly long as a wave is weakened, so that
the increase of shock thickness cannot follow the
change in maximum overpressure (nonstationary
effect). Furthermore, a longer relaxation time
results in a shorter rise time in contrast to a
steady wave, since the shock-thickening time is
nearly proportional to the relaxation time for
dispersed waves (nonstationary effect).

7. As the duration increases, the rise time
approaches the modified Taylor value for steady
plane waves or the modified Lighthill value for
quasi-stationary X-waves, which is obtained by
introducing the bulk viscosity concept into the
vibrational-flow analysis, for waves with longer
durations, the rise time appears only for lower
maximum overpressures.

8. The decay index $\eta$, which denotes the local
decay rate of $\Gamma_{\text{max}}$, defined by $\Gamma_{\text{max}} r_0^\eta$, is equal
to unity for a classical, linear acoustic wave, but
increases due to the dissipative effects of viscosity
and vibrational nonequilibrium for moderate non-
equilibrium, weak spherical X-waves. It approaches
unity for quasi-equilibrium waves of long duration.

9. The effects of $\gamma$ vibrational nonequilibrium on
$\Gamma_{\text{max}}$, $t_d$ and $t_r$ are found to be similar to those of $\eta$,
such as an increase in decay index and half-
duration, and a narrowing of the pressure peak. These
effects appear only at lower maximum overpressure
below 50 Pa for waves of long duration.

Finally, as noted in section 1, the original
motivation for the present study was to answer the
question whether X-waves generated by sparks or
exploding wires can simulate SS1-generated X-waves
including the dissipation effects on X-wave rise times.
This question was answered in section 3.

Nonetheless, the present study is important since
it has succeeded in providing appropriate
explanations for the rise time of a spark and exploding-wire
genereated X-waves by using the concepts of the
$\gamma$-value and the $\alpha$-value with the aid of
very good RCH simulations of the actual experiments.

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Table 2.1

Vibrational Relaxation Times for Oxygen (O) and Nitrogen (N)

<table>
<thead>
<tr>
<th>Series</th>
<th>Method</th>
<th>T (s)</th>
<th>RH (%)</th>
<th>A (cm)</th>
<th>O (usec)</th>
<th>N (usec)</th>
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<tr>
<td>1</td>
<td>Spark</td>
<td>2.73</td>
<td>0.58</td>
<td>0.45</td>
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<td>1.05-1.22</td>
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<td>111</td>
<td>DW</td>
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<td>9.1</td>
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<tr>
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<td>DW</td>
<td>2.86</td>
<td>0.88</td>
<td>5.7</td>
<td>0.52</td>
<td></td>
</tr>
</tbody>
</table>

*Room temperature, RH-relative humidity,
All absolute humidity, DW-exploding wire

Table 1.1

Parameters for Computation of Spherical Waves

(a) Perfect-Inviscid Flows

<table>
<thead>
<tr>
<th>Case</th>
<th>Pr</th>
<th>T1</th>
<th>r*</th>
<th>r</th>
<th>r</th>
<th>r</th>
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<tr>
<td>A1</td>
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<td>1/80</td>
<td>1/10</td>
<td>0.0385</td>
<td></td>
</tr>
<tr>
<td>A2</td>
<td>2.0</td>
<td>1.0</td>
<td>1/10</td>
<td>1/10</td>
<td>0.0385</td>
<td></td>
</tr>
<tr>
<td>A3</td>
<td>9.0</td>
<td>1.0</td>
<td>1/10</td>
<td>1/10</td>
<td>0.0385</td>
<td></td>
</tr>
<tr>
<td>A4</td>
<td>9.0</td>
<td>1.0</td>
<td>1/10</td>
<td>1/10</td>
<td>0.0385</td>
<td></td>
</tr>
<tr>
<td>A5</td>
<td>2.34</td>
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<td>1/10</td>
<td>1/50</td>
<td>0.0385</td>
<td></td>
</tr>
<tr>
<td>A6</td>
<td>1.8</td>
<td>1.0</td>
<td>1/10</td>
<td>1/50</td>
<td>0.0385</td>
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(b) Perfect-Viscous Flows

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<th>r</th>
<th>r</th>
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<td>2/3</td>
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<td>1/40</td>
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<td>2/3</td>
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<td>1/40</td>
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(c) Real-Inviscid Flows

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<th>r</th>
<th>r</th>
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<th>r</th>
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<td>13.6</td>
<td>1.15</td>
<td>1/50</td>
<td>0.0385</td>
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<tr>
<td>C2</td>
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<td>1.0</td>
<td>273</td>
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<td>13.6</td>
<td>1.15</td>
<td>1/50</td>
<td>0.0385</td>
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(d) Real-Viscous Flows

<table>
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<th>r</th>
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<th>r</th>
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<td>1/50</td>
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<td>1.0</td>
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<td>5.54</td>
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<td>1/30</td>
<td>1.91</td>
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FIG. 1.1 DEFINITION OF RISE TIME $t_r$ AND HALF-DURATION OF AN N-WAVE $t_d$.

FIG. 1.2 DEFINITION OF RISE TIME $t_r$ OF A PLANE WAVE.
FIG. 2.1 SPARK AND EXPLODING-WIRE-GENERATED N-WAVES.

(a) SERIES I - SPARK
$S = 6.0 \text{kV}, \ r = 21.6 \text{m}; (\Delta p)_{\text{max}} = 8.2 \text{ Pa},$
$t_d = 72 \mu s, \ t_r = 11.9 \mu s.$

(b) SERIES IV - EXPLODING WIRE
$S = 6.0 \text{kV}, \ r = 29.3 \text{m}; (\Delta p)_{\text{max}} = 20.2 \text{ Pa},$
$t_d = 122 \mu s, \ t_r = 15.2 \mu s.$
\[(\Delta p)_{\text{max}} (\text{Pa})\]

\[r (\text{m})\]

**FIG. 2.2** MAXIMUM OVERPRESSURE DATA \((\Delta p)_{\text{max}}\) vs \(r\).

\[n = 1\]

\[n = 1.4\]

SPARK: SERIES I - ○ AND II - △;

EXPLODING WIRES: SERIES III - ◊ AND IV - □.
FIG. 2.3 HALF-DURATION DATA $t_d$ VS $r$.

SPARK: SERIES I - ○ AND II - △;
EXPLODING WIRES: SERIES III - ◊ AND IV - □.
**FIG. 2.4** RISE TIME DATA $t_r$ vs $r$.

SPARK: SERIES I - O AND II - △;
EXPLODING WIRE: SERIES III - ◇ AND IV - □.

**FIG. 2.5** RISE TIME DATA $t_r$ vs $(\Delta p)_{\text{max}}$.

SPARK: SERIES I - O AND II - △;
EXPLODING WIRE: SERIES III - ◇ AND IV - □.
FIG. 3.1 TAYLOR VELOCITY OR PRESSURE PROFILE AND DEFINITIONS OF SHOCK THICKNESS.
FIG. 3.2 NORMALIZED TAYLOR THICKNESS $(\Delta x)_0/8/a_1$ OR NORMALIZED TAYLOR RISE TIME $t_{\tau_0}/(\sqrt{\rho_1}a_1)$ PLOTS AGAINST SHOCK STRENGTH $(\Delta p)_2/p_1$. 
FIG. 3.5 COMPARISON BETWEEN EXPERIMENTAL AND THEORETICAL TAYLOR RISE TIMES PLOTTED AGAINST MAXIMUM OVERPRESSURE.

SPARK: SERIES I - ○ AND II - △;
EXPLODING WIRE: SERIES III - ◊ AND IV - □.
FIG. 3.4 LIGHTHILL N-WAVES (ASYMPTOTIC FORMS OF PULSES WITH ZERO MASS FLOW) FOR REYNOLDS NUMBERS Re = 50, 10, 5, 1, 0.5 (REF. 20).
FIG. 5.5 LIGHTHILL N-WAVES IN A PLOT OF THE NORMALIZED OVERPRESSURE
\( \frac{p_1}{C p_{\text{max}}} \) VS THE DISTANCE PARAMETER \( Z \) FOR DURATION
PARAMETERS \( Z_d = 5.8, 66.4, 20.9, 5.87, 1.93 \).
Fig. 5.6  Thickness parameter (ζ), normalized by Taylor thickness parameter (ζT₀ = 5.127), as a function of duration parameter Z_d for Lighthill N-waves.
FIG. 3.7 NORMALIZED SHOCK THICKNESS ΔX/(δ/a₁) OR NORMALIZED RISE TIME tᵣ/(δ/a₁²) VS NORMALIZED MAXIMUM OVERPRESSURE (Δp)ₘₐₓ/p₁ FOR NORMALIZED DURATION tₙ/(δ/a₁²) = 5x10⁵, 2x10⁵, 10⁵, 5x10⁴ FOR LIGHTHILL N-WAVES.
Fig. 3.8: Comparison between experimental and theoretical Taylor growth rate versus maximum pressure in pure nitrogen.

Fig. 3.9: Comparison between experimental and theoretical Lighthill N-wave front parameters, normalized by Taylor thickness parameter $\Delta z_0$, against duration parameter $\Delta$.
(a) SERIES I - SPARK DATA
Lighthill: \((\Delta p)_{\text{max}} = 8.52 \text{ Pa}, t_r = 12 \mu s, t_d = 72 \mu s.\)
Series I: \(r = 21.6 \text{ m}, (\Delta p)_{\text{max}} = 8.52 \text{ Pa}, t_r = 11.4 \mu s, t_d = 72 \mu s.\)

(b) SERIES II - SPARK DATA
Lighthill: \((\Delta p)_{\text{max}} = 5.83 \text{ Pa}, t_r = 16.7 \mu s, t_d = 76.8 \mu s.\)
Series II: \(r = 19.0 \text{ m}, (\Delta p)_{\text{max}} = 5.83 \text{ Pa}, t_r = 15.5 \mu s, t_d = 76.8 \mu s.\)

FIG. 3.10 COMPARISON BETWEEN EXPERIMENTAL ---- AND THEORETICAL ——
(LIGHTHILL) PRESSURE PROFILES OF N-WAVES.
(c) SERIES III - EXPLODING-WIRE DATA
LIGHTHILL: \((\Delta p)_{\text{max}} = 17.0 \text{ Pa}, t_r = 7.97 \mu\text{s}, t_d = 113.6 \mu\text{s}\).
SERIES III: \(r = 27.6\text{ m}, (\Delta p)_{\text{max}} = 17.0 \text{ Pa}, t_r = 10.5 \mu\text{s}, t_d = 113.6 \mu\text{s}\)

(d) SERIES IV - EXPLODING-WIRE DATA
LIGHTHILL: \((\Delta p)_{\text{max}} = 15.3 \text{ Pa}, t_r = 8.68 \mu\text{s}, t_d = 105.3 \mu\text{s}\).
SERIES IV: \(r = 29.3\text{ m}, (\Delta p)_{\text{max}} = 15.3 \text{ Pa}, t_r = 18.7 \mu\text{s}, t_d = 105.3 \mu\text{s}\).

FIG. 3.10 - CONTINUED
COMPARISON BETWEEN EXPERIMENTAL --- AND THEORETICAL ———-
(LIGHTHILL) PRESSURE PROFILES OF N-WAVES.
Fig. 3.11 Pressure profiles \( \Delta p / (\Delta p)_0 \) vs distance parameter \( Z \) for various time parameters \( \tau = 0, 0.1, 0.5, 2.0, 10, \) for Lighthill nonstationary viscous plane waves.

Fig. 3.12 Ratio of thickness parameter \( (\Delta Z)/(\Delta Z)_0 \) plotted against square root of time parameter \( \sqrt{\tau} \) for Lighthill nonstationary viscous plane waves.
Fig. 5.13 Normalized shock-thickening time $t_s/(\Delta a_1^2)$ or distance $x_s/(\Delta a_1)$ plotted against shock strength $(\Delta p)_2/p_1$ for nonstationary viscous plane waves.
FIG. 3.14 PRESSURE $p$, AND TEMPERATURE (TRANSLATION + ROTATION $T$, AND VIBRATION $T_v$) PROFILES OF SHOCK TRANSITIONS WITH VIBRATIONAL EXCITATION FOR FULLY AND PARTLY DISPERSED WAVES.
FIG. 5.15 CRITICAL OVERPRESSURE $(\Delta p)_{cr, O}$ AND $(\Delta p)_{cr, O+N}$ AS FUNCTIONS OF INITIAL TEMPERATURE $T_1$ FOR AIR.
FIG. 3.16 PRESSURE PROFILES OF DISPERSED WAVES USING NORMALIZED OVERPRESSURE 
$(\Delta p)/(\Delta p)_2$ VS DISTANCE PARAMETER $Z-Z_0$ FOR OVERPRESSURE RATIOS 
$(\Delta p)/(\Delta p)_{cr,j} = 0, 0.5, 1.0, 2.0, 4.0$. 
Fig. 3.1: Thickness parameter \( (\Delta Z) \), normalized by Taylor-thickness parameter \( (\Delta Z)_0 = 5.127 \), as a function of overpressure ratio \( (\Delta p)_2 / (\Delta p)_{cr} \) for fully and partly-dispersed waves.
FIG. 5.18 COMPARISON BETWEEN EXPERIMENTAL (SERIES IV), DW■ AND THEORETICAL (TAYLOR, MODIFIED TAYLOR, MODIFIED LIGHTHILL X-WAVES) RISE TIMES $t_r$ PLOTTED AGAINST MAXIMUM OVERPRESSURE $(\Delta p)_{max}$. $T_1 = 280$ K, RH = 87.5%, $\omega = 5.75$ μSEC.
FIG. 3.19 COMPARISON BETWEEN EXPERIMENTAL AND THEORETICAL (MODIFIED LIGHTHILL) PRESSURE PROFILES OF N-WAVES.
(c) Series III: Exploding-wire data

(c) SERIES III: EXPLODING-WIRE DATA
MODIFIED LIPTIIILL: ($\Delta p$)$_{\text{max}}$ = 17 Pa, $t_r$ = 42.8 $\mu$s, $t_d$ = 115.6 $\mu$s.
SERIES III: $r$ = 27.6 m, ($\Delta p$)$_{\text{max}}$ = 17 Pa, $t_r$ = 105.5 $\mu$s, $t_d$ = 115.6 $\mu$s.

(d) Series IV: Exploding-wire data

(d) SERIES IV: EXPLODING-WIRE DATA
MODIFIED LIGHTHILL: ($\Delta p$)$_{\text{max}}$ = 15.3 Pa, $t_r$ = 57.4 $\mu$s, $t_d$ = 105.5 $\mu$s.
SERIES IV: $r$ = 29.3 m, ($\Delta p$)$_{\text{max}}$ = 17 Pa, $t_r$ = 18.7 $\mu$s, $t_d$ = 105.5 $\mu$s.

FIG. 5.19 - CONTINUED - COMPARISON BETWEEN EXPERIMENTAL AND THEORETICAL (MODIFIED LIGHTHILL) PRESSURE PROFILES OF N-WAVES.
FIG. 3.20  CLASSIFICATION OF S-WAVES ACCORDING TO THE DEGREE OF NONEQUILIBRIUM.
**Fig. 3.1** Relaxation times of oxygen and nitrogen in atmospheric air in the relative humidity range 10—100%. (Ref. 100.) (Ref. 12.)

**Fig. 4.2** Illustration of random choice method. S = Shock wave; C = Contact surface, R = Rarefaction wave.
FIG. 4.3 SHOCK-TUBE PROBLEM USING RANDOM-CHOICE METHOD FOR A PERFECT-INVISCID FLOW ($p_{11} = 2.0$, $T_{11} = 1.0$, $x^* = 1/40$).
FIG. 4.4 SHOCK-TUBE PROBLEM USING RANDOM-CHOICE METHOD FOR A PERFECT-VISCOUS FLOW ($P_{41} = 2.0$, $T_{41} = 1.0$, $\Delta x^* = 1/40$, $x_0 = 0.001$ cm).
FIG. 4.5 REGION OF CALCULATION AND SHOCK-FRONT PATH.
O - RANDOM-CHOICE METHOD (ΔX = 1.25\times10^{-5} \text{ cm}),
---, ------ ANALYTICAL.
Fig. 4.6 Transient behaviour of shock thickness for shock-tube problem using RCM for a perfect viscous flow \( \rho_1 = 2.0, \mu_1 = 1.0, \rho_1 = 273 \text{ K}, p_1 = 101.3 \text{ kPa}. \)
FIG. 4.6 - CONTINUED - TRANSIENT BEHAVIOUR OF SHOCK THICKNESS FOR SHOCK-TUBE PROBLEM USING RCM FOR A PERFECT-VISCOUS FLOW

\( P_{11} = 2.0, T_{11} = 1.0, T_1 = 273 \text{ K}, p_1 = 101.3 \text{ kPa} \) (Taylor).

\[ \Delta P / \Delta P_2 \]

\( \tau \to \omega \) (Taylor)

\( \tau = 0.99 \)

--- LIGHTHILL, \( \cdot \tau = 0.99 \), \( \Delta \tau = 45.0 \), \( \circ \tau = 58.3 \).

(c) COMPARISON BETWEEN RCM AND MFM (MacCORMACK'S FINITE-DIFFERENTIAL METHOD)

\( + X = 1.25 \times 10^{-5} \text{ cm} \) (MacCORMACK)

\( \circ X = 1.25 \times 10^{-5} \text{ cm}, \Delta \tau = 0.75 \times 10^{-5} \text{ cm} \) (RANDOM-CHOICE)
FIG. 4.8 TRANSIENT SHOCK PRESSURE AND TEMPERATURE PROFILES FOR SHOCK-TUBE PROBLEM USING RCM FOR A REAL-INVISID FLUID \[ P_4 = 1.0018, T_4 = 1.0, p_1 = 101.3 \text{ kPa}, T_1 = 303.15 \text{ K}, \text{RH} = 90\%, S_0 = 1.0004, \]
\[ (\Delta p)_2 = 91.1 \text{ Pa}, (\Delta T)_2 = 0.0777 \text{ K}, \theta = 1.04 \text{ sec}, x_0 = 0.5 \text{ cm}, \]
\[ x = 0.0125 \text{ cm}. \]

\[ \frac{\Delta p}{(\Delta T)_2}, \frac{(\Delta T)_2}{(\Delta T)_2}, \frac{(\Delta T)_2}{(\Delta T)_2}, \]

- \( \frac{(\Delta p)}{(\Delta T)_2} \) or \( \frac{(\Delta T)}{(\Delta T)_2} \),
- \( \frac{(\Delta T)_2}{(\Delta T)_2} \),
- FROZEN SHOCK FRONT.
FIG. 4.9 TRANSIENT BEHAVIOUR OF SHOCK THICKNESS FOR SHOCK-TUBE PROBLEM USING RCM FOR A REAL-INVISID FLOW \([p_1] = 1.0018, T_1 = 1.0, p_1 = 101.3 \text{ kPa}, T_1 = 505.15 \text{ K}, \text{RH} = 90\%, M_c = 1.0004, \langle p \rangle_2 = 91.1 \text{ Pa}\).

\(\triangle x = 0.025 \text{ cm}, \circ x = 0.0125 \text{ cm}\).

FIG. 4.10 STEADY SHOCK-PRESSURE PROFILE FOR SHOCK-TUBE PROBLEM USING RCM FOR A REAL-INVISID FLOW \([p_1] = 1.0018, T_1 = 1.0, p_1 = 101.3 \text{ kPa}, T_1 = 505.15 \text{ K}, \text{RH} = 90\%, M_c = 1.0004, \langle p \rangle_2 = 91.1 \text{ Pa}, \langle \nabla T \rangle_2 = 0.0777 \text{ K},
\n\lambda_0 = 1.04 \text{ sec}, x_0 = 0.5 \text{ cm}, x = 0.0125 \text{ cm}\).

——— ANALYTICAL, \(\bullet x = 25.0, \circ x = 27.6\)
FIG. 4.10  NUMERICAL SOLUTION OF EXPLOSION OF A PRESSURIZED AIR SPHERE USING RANDOM CHOICE METHOD FOR A PERFECT INCOMPRESSIBLE CASE $\gamma = 1.4$, $p_0 = 1$ atm, $r_0 = 10$ cm.

the initial process of explosion

FIG. 4.11 CONTINUED
NUMERICAL SOLUTION OF EXPLOSION OF A PRESSURIZED AIR SPHERE USING RANDOM CHOICE METHOD FOR A PERFECT INCOMPRESSIBLE CASE $\gamma = 1.4$, $p_0 = 1$ atm, $r_0 = 10$ cm, $t = 10$ sec.

the formation of X-ray
FIG. 4.11 - CONTINUED - NEAR-FIELD SOLUTION OF EXPLOSION OF A PRESSURIZED AIR SPHERE USING RANDOM-HOLE METHOD FOR A PERFECT-INVIScid FLOW BASED AT, $P_{11} = 2.0$, $T_{11} = 1.0$, $r' = 1.101$.

(c) PROPAGATION OF SPHERICAL N-WAVE.
(a) $P_{41} = 2.0$, $T_{41} = 1.0$ (Case A.1A)

FIG. 4.12 EFFECTS OF INITIAL PRESSURE AND TEMPERATURE RATIOS $P_{41}$ AND $T_{41}$ ON N-WAVE PROFILE ( $r^* = 1/10$).

(a) $P_{41} = 2.0$, $T_{41} = 1.0$ (CASE A.1A)
Fig. 4.12 - Continued - Effects of initial pressure and temperature ratios $P_{41}$ and $T_{41}$ on N-wave profile ($r^* = r/r_0$).

(b) $P_{41} = 2.0$, $T_{41} = 2.0$ (Case A.2)
FIG. 4.12 - CONT. EFFECTS OF INITIAL PRESSURE AND TEMPERATURE RATIOS $P_{i1}$ AND $T_{i1}$ ON WAVE PROFILES ($CT^* = 1.101$)

(c) $P_{i1} = 9.0$, $T_{i1} = 1.0$ (Case A3)
Fig. 4.12 - Continued - Effects of Initial Pressure and Temperature Ratios $p_{41}$ and $T_{41}$ on N-Wave Profile ($r^* = 1/10$).

(d) $p_{41} = 9.0$, $T_{41} = 9.0$ (Case A.4)
FIG. 4.13 PATH OF SHOCK FRONT FOR A PERFECT-INVISCID FLOW (CASE A5).

FIG. 4.14 COMPARISON FOR PERFECT-INVISCID (A5 - △), PERFECT-VISCOSUS (B1 - ○), REAL-INVISCID (CI - +), AND REAL-VISCOSUS (D1 - ●), FAR-FIELD RCM SOLUTIONS OF ATTENUATION OF MAXIMUM OVERPRESSURE.
FIG. 4.15 COMPARISON BETWEEN PERFECT-INVISCID (A5 - △), PERFECT-VISCOUS (B1 - ○), REAL-INVISCID (C1 - +), AND REAL-VISCIOUS (D1 - ●), FAR-FIELD RCM SOLUTIONS: HALF-DURATION $t_d$ VS DISTANCE $r$.

FIG. 4.16 COMPARISON BETWEEN RISE TIMES $t_r$ VS DISTANCE $r$ FOR PERFECT-INVISCID ($t_r = 0$), PERFECT-VISCIOUS (B1 - ○), REAL-INVISCID (C1 - +), AND REAL-VISCIOUS (D1 - ●), FAR-FIELD RCM SOLUTIONS: RISE TIME $t_r$ VS DISTANCE $r$. 
FIG. 4.17 COMPARISON OF PERFECT-INVISCID, PERFECT-VISCOURS, REAL-INVISCID AND REAL-VISCOURS PRESSURE PROFILES AT SEVERAL LOCATIONS FOR REAL-VISCOURS, FAR-FIELD RCM SOLUTIONS.
Fig. 4.18: Computer simulation of maximum overpressure attenuation for spark and exploding wire data.

Numerical: ▼ D1, ◆ C2, ● B1, △ B2, ■ D2
Experimental: ○ SERIES 1, △ SERIES 11, □ SERIES 12

Fig. 4.19: Computer simulation of time-duration as a function of distance for spark and exploding wire data.

Numerical: ▼ B2, ◆ C2, ● D1, △ B2, ■ D3
Experimental: ○ SERIES 1, △ SERIES 11, □ SERIES 12
FIG. 2.10 COMPUTER SIMULATION OF RISE TIME AS A FUNCTION OF DISTANCE FOR SPARK AND EXPLODING-WIRE DATA.

**Numerical:** ▾ E, ◆ E2, ○ D1, △ D2, ■ D3
**Experimental:** ○ SERIES I, △ SERIES II, □ SERIES IV

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**FIG. 2.11** COMPUTER SIMULATION OF RISE TIME AS A FUNCTION OF MAXIMUM OVERPRESSURE FOR SPARK AND EXPLODING-WIRE DATA.

**Numerical:** ▾ E, ◆ E2, ○ D1, △ D2, ■ D3
**Experimental:** ○ SERIES I, △ SERIES II, □ SERIES IV
Fig. 1.22(a) Transient Shock Pressure and Temperature Profiles for Spherical N-Waves in Air Solved by RCM for a Real-Viscous Flow: Simulation for Series 1 Experiment (Spark).

\[ \frac{\Delta p}{\Delta p_{\text{max}}}, \frac{\Delta T}{\Delta T_{\text{max}}}, \frac{\Delta T_v}{\Delta T_{\text{max}}} \]

- \( p_1 = 2.44 \text{ kPa}, T_1 = 1.0 \text{ K}, \rho_1 = 101.3 \text{ kPa}, T_1 = 273 \text{ K}, \text{RH} = 67\%, r_0 = 1.15 \text{ cm}, r = 0.0585 \text{ cm}, \mu_0 = 15.6 \text{ m.sec}, (\rho p)_{\text{CR},0} = 52.2 \text{ Pa} \)

--- (\( \rho p \))_{\text{MAX}} or \( \frac{T}{T_{\text{MAX}}} \), --- (\( T/A \))_{\text{MAX}}
Fig. 4.22(b) Transient shock pressure and temperature profiles for spherical N waves in air solved by ROM for a real-viscous flow: Simulation for series of experiments -4 N M.5.

\[ \frac{(\Delta p)}{(\Delta p)_{\text{max}}}, \frac{(\Delta T)}{(\Delta T)_{\text{max}}}, \frac{(\Delta v)}{(\Delta T)_{\text{max}}} \]

- 0.20 m, 1641 Pa
- 1.50 m, 134 Pa
- 4.58 m, 360 Pa
- 6.27 m, 250 Pa
- 8.37 m, 177 Pa

1 cm
10 \( \mu \)s

1067 m, 132 Pa
129 m, 104 Pa
151 m, 85 Pa
190 m, 633 Pa
FIG. 4.23: COMPARISON OF SPARK PRESSURE PROFILES OF COMPUTED X-RAY WITH EXPERIMENTAL DATA: SERIES 1 VS. CASE 1a.
\[ \Delta P (\text{Pa}) \]

At \( r = 11.7 \text{m} \)
--- NUMERICAL: \( \Delta P_{\text{max}} = 11.8 \text{ Pa}, \ t_f = 10.56 \mu\text{s}, \ t_d = 65.0 \mu\text{s} \)
--- EXPERIMENTAL: \( \Delta P_{\text{max}} = 11.77 \text{ Pa}, \ t_f = 10.8 \mu\text{s}, \ t_d = 64.8 \mu\text{s} \)

\[ \Delta P (\text{Pa}) \]

At \( r = 19 \text{m} \)
--- NUMERICAL: \( \Delta P_{\text{max}} = 6.55 \text{ Pa}, \ t_f = 16.78 \mu\text{s}, \ t_d = 69.3 \mu\text{s} \)
--- EXPERIMENTAL: \( \Delta P_{\text{max}} = 5.88 \text{ Pa}, \ t_f = 15.5 \mu\text{s}, \ t_d = 70.8 \mu\text{s} \)

**Fig. 1.34** COMPARISON OF MAXIMUM PRESSURE PROFILES OF COMPUTED X-RAY WITH EXPERIMENTAL DATA: SERIES 11 AS CASE D2.
FIG. 4.5  FULL X-WAVE PROFILES OF PRESSURE, TEMPERATURE AND VIBRATIONAL TEMPERATURE AT r = 21.6 cm FOR CASE 1A.

[\text{\text{P}_{\text{end}} = 6.29 \text{ Pa}}, \text{ \tau_r = 12.59 \text{ ms}}, \text{ \tau_d = 88.8 \text{ ms}}]
Fig. 4.26 Full X-wave profiles of pressure, temperature, and vibrational temperature at r = 19m for Case D2A.
\[ p_{\text{max}} = 5.87 \text{ Pa}, \ t_T = 20 \mu s, \ t_d = 75.5 \mu s \]
FIG. 4.27 FULL N-WAVE PROFILES OF PRESSURE, TEMPERATURE AND VIBRATIONAL TEMPERATURE AT $r = 29.5\text{m}$ FOR CASE D4A.

$[C_p]_{\text{max}} = 14.5 \text{ Pa}, \quad t_r = 27.5 \mu\text{s}, \quad t_d = 156.5 \mu\text{s}$
Fig. 4.28: Comparison of pressure profiles at \( r = 29.5 \text{m} \) of computed X-wave with experimental data: Series IX vs. Case D5.

- Numerical: \( \Delta P_{\text{max}} = 11.5 \text{ Pa}, t_f = 27.5 \text{ s}, t_d = 105.5 \text{ s} \)
- Experimental: \( \Delta P_{\text{max}} = 15.5 \text{ Pa}, t_f = 18.7 \text{ s}, t_d = 105.5 \text{ s} \)

Fig. 4.29: Comparison of \( \Delta P_{\text{max}} \) vs. \( r \) for cases D2 and D4 for different vibrational relaxation times (D2: 3.54 \( \mu \text{sec} \); D4: 15.6 \( \mu \text{sec} \) for \( \tau_0 \)).
FIG. 4.50 COMPARISON OF $t_d$ VS $r$ FOR CASES D2 AND D4 FOR DIFFERENT VIBRATIONAL RELAXATION TIMES (D2: 5.51 sec, D4: 15.6 sec for $\rho_0$).

FIG. 4.51 COMPARISON OF $t_r$ VS $r$ FOR CASES D2 AND D4 FOR DIFFERENT VIBRATIONAL RELAXATION TIMES (D2: 5.51 sec, D4: 15.6 sec for $\rho_0$).
FIG. 4.52 COMPARISON OF $p$ VS $t$ FOR CASES D2 AND D4 FOR DIFFERENT VIBRATIONAL RELAXATION TIMES (D2: 5.54 usec, D4: 15.6 usec for $o$).

D2: $r = 19m$, $|p|_{\text{max}} = 6.55 \text{ Pa}$, $t_f = 16.78 \text{ usec}$, $t_d = 69.5$ usec
D4: $r = 19m$, $|p|_{\text{max}} = 6.0 \text{ Pa}$, $t_f = 7.5 \text{ usec}$, $t_d = 66.3 \text{ usec}$

FIG. 4.53 COMPARISON OF $(\Delta p)_{\text{max}}/p_1$ VS $r^*$ FOR PERFECT-INVISCID CASE A6 $\blacklozenge$ AND REAL VISCOUS CASES D2 $\blacktriangledown$, D5 $\blacklozenge$ AND D6 $\blacksquare$ $(p_{41} = 1.8$, $r_0 = 1.15 \text{ cm for D2})$.
**FIG. 4.34** COMPARISON OF \( t_d^* \) VS \( r^* \) FOR CASES A₀ (PERFECT-INVISID), D₂, D₅ AND D₆ (REAL-VISCOUS) FOR DIFFERENT HALF DURATIONS 
(\( P_{41} = 1.8, \ r_0 = 1.15 \text{ cm for D₂, D₅: 11.5 cm, D₆: 57.5 cm} \)).

**FIG. 4.35** COMPARISON OF \( t_r \) VS \( (\Delta P)_{max} \) FOR DIFFERENT HALF DURATIONS 
FOR REAL-VISCOUS CASES D₂, D₅ AND D₆ (\( P_{41} = 1.8, \ r_0 = 1.15 \text{ cm for D₂. D₅: 11.5 cm, D₆: 57.5 cm} \)).
FIG. 4.36 COMPARISON OF RELAXATION EQUATION VS. BULK VISCOSITY FOR REAL VISCOUS CASES. 100 AND 200 USING RELAXATION EQUATION AND BULK VISCOSITY CONCEPT SOLUTIONS FOR A WAVE OF LONG DURATION (t = 1.8, r = 57.5 cm).

FIG. 4.37 COMPARISON OF PRESSURE PROFILES (P/P₀) VS. T FOR REAL VISCOUS CASES D2, D5 AND D6 FOR DIFFERENT IMPULSE DURATIONS (t = 1.8, r = 1.5 cm for D2, D5; 11.5 cm for D6; 16.5 cm).

D2: r = 2.5, 1₀ max = 7.98 Pa, t = 11.98, 1 = 15.91, 2 = 0.97
D5: r = 4.5, 1₀ max = 7.98 Pa, t = 29.3, 1 = 57.5, 2 = 0.97
D6: r = 2.2, 1₀ max = 7.98 Pa, t = 159.1, 1 = 2006, 2 = 0.97
FIG. 4.38 PLOT OF (ΔP)\text{max}/P₁ VS r* FOR COMPARISON OF REAL-VISCOSUS CASES D6, D7 AND D8 FOR EFFECT OF VIBRATIONAL RELAXATION OF NITROGEN (P₁₁ = 1.8 FOR D6; D7, D8: 1.08, r₀ = 57.5 cm; D6, D7: O₂ only, D8: O₂₂N₂).

FIG. 4.39 COMPARISON OF t_d* VS r* FOR CASES D6, D7 AND D8 FOR EFFECT OF VIBRATIONAL RELAXATION OF NITROGEN (P₁₁ = 1.8 FOR D6; D7, D8: 1.08; r₀ = 57.5 cm; D6, D7: O₂ only, D8: O₂₂N₂).
FIG. 4.40 COMPARISON OF \( t_r \) VS \( C'p'_{\text{max}} \) FOR CASES D6, D7 AND D8 FOR EFFECT OF VIBRATIONAL RELAXATION OF NITROGEN \( \tau_{\text{r1}} = 1.8 \) FOR D6; \( \tau^* \), D8: 1.08; \( r_0 = 57.5 \text{ cm} \); D6, D8: \( O_2 \) ONLY, D8: \( O_2+N_2 \).

--- MODIFIED TAYLOR, --- MODIFIED LIGHTHILL

FIG. 4.41 COMPARISON OF PRESSURE PROFILES \( \Delta p/p_1 \) VS \( \bar{t} \) FOR CASES D7 AND D8 FOR EFFECT OF VIBRATIONAL RELAXATION OF NITROGEN \( \tau_{\text{r1}} = 1.08 \):
\( \tau_0 = 57.5 \text{ cm} \); \( \tau^* \), D8: \( O_2 \) ONLY, D8: \( O_2+N_2 \).

D6: \( \tau^* = 1950, C'p'_{\text{max}} = 1.55 \text{ Pa}, t_F = 1^{2}4.7 \text{ s}, t_d = 18^{2}6.5 \text{ s} \)
D7: \( \tau^* = 1950, C'p'_{\text{max}} = 1.25 \text{ Pa}, t_F = 191.6 \text{ s}, t_d = 1949.4 \text{ s} \)
FIG. 4.12 PROFILES OF TEMPERATURE \( \Delta T \) AND VIBRATIONAL TEMPERATURE \( (\Delta T_v)_N \) VS \( t = t/t_d \) FOR NITROGEN AT \( \tau^* = 1950 \) FOR REAL-VISCOUS CASE 18.

GAS TEMPERATURE, \( \cdots \cdots \cdot \) VIBRATIONAL TEMPERATURE
the centre of the shock front of an N-wave can be given by

\[ \rho = \frac{1}{2} \rho_0 \exp(-k) \]  \hspace{1cm} \text{(A.1)}

which is derived from Eq. (5.15), where \( \rho_0 \) is the value at \( \rho = 0.5 \) in the shock front.

The peak point in an N-wave can be obtained from \( \rho^2 \), where \( \rho_0 \) is the initial value of \( \rho \). This gives the relation

\[ \exp(k) = \frac{1}{2} \left( \rho_0 \exp(-k) \right)^{1/2} \]  \hspace{1cm} \text{(A.2)}

where \( \rho_0 \) is the value at \( \rho = 0.5 \). Equation (A.2) is rewritten as

\[ \rho^2 = \frac{1}{2} \rho_0 \exp(-k) \]  \hspace{1cm} \text{(A.3)}

to evaluate \( \rho \). The method of successive iteration is used to solve Eq. (A.3) for \( \rho^2 \), the centre value \( \rho_0 \) was used for the initial value of iteration. The value of \( \rho_{\text{max}} \) is obtained by substituting \( \rho_{\text{max}} \) into Eq. (5.13).

The shock thickness \( d \) and the half-duration \( t_d \) are given from

\[ d = \rho_0 \exp(-k) t_d \]  \hspace{1cm} \text{(A.4)}

where \( t_d \) and \( t_d \) are the values at \( \rho = 0.1 \) and \( 0.9 \rho_{\text{max}} \), respectively. The values of \( t_d \) and \( t_d \) are calculated from the iterative equations, which are derived from Eq. (5.15). Thus

\[ t_d = \rho_0 \exp(-k) \]  \hspace{1cm} \text{(A.5)}

\[ t_d = \rho_0 \exp(-k) \]  \hspace{1cm} \text{(A.6)}

where \( k \) is the number of iteration. The initial values of iteration need were \( t_d = \frac{1}{2} \rho_{\text{max}} \) and \( \rho_{\text{max}} \).

The values of \( \rho \) and \( t_d \) are obtained by substituting \( \rho_{\text{max}} \) and \( t_d \) into equations (5.11) and (5.15). The computer program for obtaining \( \rho_{\text{max}} \) and \( t_d \) is given in listing A.1, and a plot of \( t_d \) vs \( R_t \) is shown in Fig. A.1.

In order to compare the experimental results with the inline N-wave solutions, it is necessary to establish the procedure for obtaining the corresponding inline N-wave rise time that the observed values of the maximum overpressure \( \rho_{\text{max}} \) and the observed half-duration \( t_d \). At first, \( t_d \) is determined from Eq. (5.16). Then the approximate value of \( \rho_{\text{max}} \) is read out from the \( \rho_{\text{max}} \) chart in Fig. A.1. By repeating the process of Eqs. (A.1) and (A.2), an approximate value of \( \rho_{\text{max}} \) is obtained. The iterative process is repeated until \( \rho_{\text{max}} \) so obtained does not coincide with the exact \( \rho_{\text{max}} \). Finally, the corresponding value of \( \rho_{\text{max}} \) is determined and \( \rho_{\text{max}} \) is obtained. The actual shock thickness is obtained from Eq. (5.14). The computer program for this procedure is given in listing A.1.
APPENDIX B

DERIVATION OF ANALYTICAL RELATIONS IN SECTION 3.1

1. Derivation of Eq. 3.3.201: Critical Overpressure

Consider a steady normal shock wave in a gas with vibrational excitation and assume the equilibrium states for both sides of the shock front. Then, the equations of continuity, momentum and energy are given by

\[ \rho_1 U_1 = \rho_2 U_2 \quad \text{(18.1)} \]

\[ P_1 = P_1^0 \quad \text{(18.2)} \]

\[ C_1 = C_1^0 \quad \text{(18.3)} \]

where the subscripts 1 and 2 denote the states ahead of and behind the shock front, respectively; \( \rho \), density; \( u \), velocity; \( P \), pressure; \( C \), temperature; \( \epsilon \), vibrational energy; \( C_v \), specific heat at constant pressure for translational and rotational energy, assumed constant. The equation of state and the vibrational energy are assumed to be expressed by

\[ P = \rho R T \quad \text{(18.4)} \]

\[ \epsilon_1 = \epsilon_1^0 \quad \text{(18.5)} \]

where \( R \) is the gas constant; \( \epsilon_1^0 \), vibrational specific heat for the j molecule, assumed constant across the shock.

From Eqs. (18.1), (18.2), and (18.3)

\[ \frac{P_2}{P_1} = \left(\frac{\rho_1}{\rho_2}\right)^{\frac{1}{2}} \left(\frac{u_1}{u_2}\right) \quad \text{(18.6)} \]

Taking the ratio of Eqs. (18.1) and (18.2) and using the relation (18.5), it can be written as

\[ \frac{P_2}{P_1} = \left(\frac{\rho_1}{\rho_2}\right)^{\frac{1}{2}} \left(\frac{u_1}{u_2}\right) \quad \text{(18.7)} \]

Substituting Eq. (18.8) into Eq. (18.7) obtains

\[ \frac{P_2}{P_1} = \left(\frac{\rho_1}{\rho_2}\right)^{\frac{1}{2}} \left(\frac{u_1}{u_2}\right) \quad \text{(18.8)} \]

From Eq. (18.9), if \( \frac{P_2}{P_1} = 1 \), the expression for the critical overpressure \( P_{cr, 1} \) is

\[ \frac{P_2}{P_1} = \left(\frac{\rho_1}{\rho_2}\right)^{\frac{1}{2}} \left(\frac{u_1}{u_2}\right) \quad \text{(18.9)} \]

8.2 Derivation of Eq. 3.3.121: Buffington

Equation 3.200 for \( \epsilon_1^0 \) can be rewritten as

\[ \frac{\rho_1}{\rho_2} = \frac{u_1}{u_2} \quad \text{(18.10)} \]

where \( \gamma_1 \) is the ratio of specific heat at constant pressure to specific heat at constant volume for translational and rotational energy.

Using the relation (18.10), obtain from Eq. 3.200

\[ \frac{\rho_1}{\rho_2} = \frac{u_1}{u_2} \quad \text{(18.11)} \]

8.3 Derivation of Eq. 3.3.123: Parameter k

The parameter \( k \) appearing in Eq. 3.201 can be rewritten as

\[ k = \frac{\epsilon_1^0}{\epsilon_1} = \frac{\epsilon_1^0}{\epsilon_1} \quad \text{(18.12)} \]

After the notations for a normal shock wave, the

\[ \epsilon_1 \quad \epsilon_1^0 \]
AD-A135 903  RANDOM CHOICE SOLUTIONS FOR WEAK SPHERICAL SHOCK-WAVE TRANSITIONS OF N-WA. (U) TORONTO UNIV DOWNSVIEW (ONTARIO) INST FOR AEROSPACE STUDIES H HONMA ET AL.
UNCLASSIFIED JUL 83 UTIAS-253 AFOSR-TR-83-1041 F/G 20/1 NL
\[
\frac{\ddot{u}}{a_e} = \frac{\ddot{u}}{a_1} = \frac{1}{2} \frac{\ddot{u}_1 - \ddot{u}_{12}}{a_1} = \frac{1}{2N_f} \frac{\langle \dot{u}\rangle_p}{P_1} + \frac{1}{2\gamma} \frac{\langle \dot{u}\rangle_p}{P_1} \tag{B.13}
\]

Substituting Eq. (B.13) into Eq. (B.12), obtain
\[
\frac{1}{k} = \frac{1}{2\gamma} \frac{y}{P_1} \frac{\gamma}{(y-1)^2 c_j} = \frac{\langle \dot{u}\rangle_p}{P_1} \frac{1}{\langle \dot{u}\rangle_p} = \frac{\langle \dot{u}\rangle_p}{P_1} \tag{B.32}
\]

**B.4 Derivation of Eq. (3.34) from Eq. (3.24):**

Poljakov et al. (Ref. 21)

Equation (3.24) can be rewritten as
\[
\frac{y_0}{kt_{ij}} = \left[ 1 - \frac{1}{k} \right] \ln \left[ 1 + \frac{1}{y_0} \right] - \left[ 1 + \frac{1}{k} \right] \ln \left[ 1 - \frac{1}{y_0} \right] \tag{B.14}
\]

Using the notations for a normal shock in B.1
\[
\frac{y_0}{k} = \frac{1}{\langle \dot{u}\rangle_p} = 2 \frac{\langle \dot{u}\rangle_p}{\langle \dot{u}\rangle_p} = 2 \left( \frac{P_1}{\langle \dot{u}\rangle_p} \right) \tag{B.33}
\]

Using the expression (B.32) for \( k \)
\[
\frac{y_0}{k_{ij}} = \frac{\gamma - 1}{2\gamma} \frac{2\gamma}{P_1} - \frac{1}{(y-1)^2 c_j} - \frac{1}{(y+1)^2 c_j} \tag{B.16}
\]

where \( z \) is defined as
\[
z = - \frac{a_1^2 \gamma \langle \dot{u}\rangle_p^2}{P_1} \tag{B.35}
\]

Substituting Eqs. (B.33), (B.16) and (B.35) into Eq. (B.14) obtains
\[
\frac{y_0}{k} = \frac{1}{\langle \dot{u}\rangle_p} = 2 \left( \frac{P_1}{\langle \dot{u}\rangle_p} \right) \ln \left[ 1 + \left( \frac{\langle \dot{u}\rangle_p}{P_1} \right) \right] - \left[ 1 - \left( \frac{\langle \dot{u}\rangle_p}{P_1} \right) \right] \ln \left[ 1 - \left( \frac{\langle \dot{u}\rangle_p}{P_1} \right) \right] \tag{B.34}
\]

where
\[
z_0 = \frac{2\gamma}{y+1} \left[ \frac{y_0}{k_{ij}} \right] + \frac{2}{k} \ln 2 = \text{const} \tag{B.17}
\]

**B.5 Derivation of Eq. (3.34) from Eq. (3.37):**

Johannsen and Hodgson (Ref. 12)

From Eq. (B.10)
\[
M_f^2 = \frac{[y_0+2(y-1)c_j] \langle \dot{u}\rangle_p}{P_1} + 2[y_0(y+1)c_j]}{2(y+1)(y-1)c_j} \tag{B.18}
\]

introducing the relation (3.29), then
\[
1 - M_f^2 = \frac{y_0}{y+1} \left[ 1 - \left( \frac{\langle \dot{u}\rangle_p}{P_1} \right) \right] \ln \left[ 1 - \left( \frac{\langle \dot{u}\rangle_p}{P_1} \right) \right] \tag{B.34}
\]

The terms in Eq. (3.37) can now be rewritten as
\[
\left[ 1 - M_f^2 \right] \left[ 1 - \left( \frac{\langle \dot{u}\rangle_p}{P_1} \right) \right] \ln \left[ 1 - \left( \frac{\langle \dot{u}\rangle_p}{P_1} \right) \right] \tag{B.20}
\]

and
\[
\left[ 1 - M_f^2 \right] \left[ 1 - \left( \frac{\langle \dot{u}\rangle_p}{P_1} \right) \right] \ln \left[ 1 - \left( \frac{\langle \dot{u}\rangle_p}{P_1} \right) \right] \tag{B.21}
\]

Thus, Eq. (3.37) can be rewritten as
\[
\frac{y_0}{k} = \frac{1}{\langle \dot{u}\rangle_p} = 2 \left( \frac{P_1}{\langle \dot{u}\rangle_p} \right) \ln \left[ 1 + \left( \frac{\langle \dot{u}\rangle_p}{P_1} \right) \right] - \left[ 1 - \left( \frac{\langle \dot{u}\rangle_p}{P_1} \right) \right] \ln \left[ 1 - \left( \frac{\langle \dot{u}\rangle_p}{P_1} \right) \right] \tag{B.22}
\]

Define
\[
z = - \frac{2y}{y+1} \frac{x}{a_1 J_{ij}} \frac{\langle \dot{u}\rangle_p}{P_1} \tag{B.39}
\]

\[
z_0 = \frac{2y}{y+1} \frac{2\langle \dot{u}\rangle_p}{P_1} \ln \left[ 1 + \left( \frac{\langle \dot{u}\rangle_p}{P_1} \right) \right] \tag{B.24}
\]

then obtain
\[
\frac{y_0}{k} = \frac{1}{\langle \dot{u}\rangle_p} = 2 \left( \frac{P_1}{\langle \dot{u}\rangle_p} \right) \ln \left[ 1 + \left( \frac{\langle \dot{u}\rangle_p}{P_1} \right) \right] - \left[ 1 - \left( \frac{\langle \dot{u}\rangle_p}{P_1} \right) \right] \ln \left[ 1 - \left( \frac{\langle \dot{u}\rangle_p}{P_1} \right) \right] \tag{B.34}
\]
B.6 Derivation of Eq. (3.41): Frozen-Shock Overpressure

Generally, the frozen overpressure can be expressed as

\[
\frac{(\Delta p)_f}{P_1} = \frac{2\gamma}{\gamma + 1} \left( M_{F}^{*} + 1 \right)
\]

(Eq. 8.25)

Substituting Eq. (8.19) into Eq. (8.25), obtain

\[
\frac{(\Delta p)_{cr,j}}{P_1} = -\frac{(\Delta p)_{cr,j}}{P_1} - \frac{(\Delta p)_2}{P_1} - \frac{(\Delta p)_3}{P_1}
\]

(Eq. 3.41)
APPENDIX C

PROGRAM LISTING FOR RANDOM-CHOICE METHOD

The program for solving Eq. (4.1) using the RCM with an operator-splitting technique is given in this section. The normalized variables used for computation are:

\[ E' = \frac{E}{(\rho_1 RT_1)}, \quad \nu' = \frac{\nu}{\sqrt{RT_1}}, \quad p' = \frac{p}{\rho_1}, \]

\[ \rho' = \frac{\rho}{\rho_1}, \quad T' = \frac{T}{T_1}, \quad \sigma'_j = \frac{\sigma_j}{(RT_1)}, \]

\[ r' = \frac{r}{L_0}, \quad t' = a_1^{-1}(\sqrt{L_0}) \]

where \( L_0 \) is a reference length, taken as \( L_0 = 5r_0 \), in most calculations.

The time step is determined from the maximum value for the local stability criterion (CFL condition) at each time step:

\[ \Delta t' = \max\left[\Delta r' / \left(\nu' + \sqrt{\frac{\nu' p'}{\rho'_j}}\right)\right] \]

The program listing given below was used for the computations of real-viscous spherical waves.
RANDOM CHOICE METHOD VARIATION H2.

* SPHERICAL WAVE *

* REAL * VISCIOUS *

IMPLICIT REAL*8 (A-H,P-Z)
DIMENSION KT1(10),PT1(10),TT1(10),TV1(10)
REAL XARRAY(416),YARRAY(416)
COMMON/IK/K1,KL,ISTP,KR,NP1,ITT,N
COMMON/DT,UL,PL,R,U,E,RR,U1,P,R,X1,Y,GAM,SOL,SOS,SOR
COMMON/OUT/TIME,DX,RHO(416),PRE(416),UX(416),ENG(416),XR(416)
COMMON/RAD/ETCH,PRA,TA(416),U2(416)
COMMON/RELAX/SO(416),U2(416),E1,TH1,TAU
COMMON/TSU/ISK,ISS,ILM
INTEGER TSTP

DATA READING
READ(5*81) NPRINS
READ(5*81) ISTART
READ(5*81) NQQT
READ(5*81) IQ
READ(5*81) NSTOP
READ(5*81) JCT
READ(5*81) JD
READ(5*81) N
READ(5*81) NHALF
READ(5*81) NQQ
READ(5*81) IXYP
READ(5*81) INCR
READ(5*81) ISK
READ(5*81) ISS
READ(5*82) TMAX
READ(5*82) TMIN
READ(5*82) PMAX
READ(5*82) PMIN
READ(5*82) XP1
READ(5*82) XP2
READ(5*82) XFAC
READ(5*82) RMAX
READ(5*82) PRFAC
READ(5*82) ESS
READ(5*82) ETA
READ(5*82) WL
READ(5*82) PL
READ(5*82) RL
READ(5*82) TO
READ(5*82) RH
READ(5*82) COEP
READ(5*82) COET
---
81 FORMAT(10)
82 FORMAT(F15.7)

COEFFICIENT OF XYPLOT
YP1=PMIN
YP2=(PMAX+PMIN)/12.0
YP3=TMIN
YP4=(TMAX+TMIN)/12.0
JCTM=JCT-JD
LMT=1
NP1=N+1
NP2=N+15
NPM=N-1
NPX=N-5

C-2
APPENDIX D

PROGRAM OF MacCORMACK’S FINITE-DIFFERENCE METHOD

In Section 4.3.2, the RCM solutions are compared with MacCormack’s solution for a perfect-viscous plane wave. In this section, the scheme and the program of the MacCormack method are given for the perfect-viscous plane wave.

The basic equation (4.1) can be written for perfect-viscous plane waves as

\[
\frac{\partial U}{\partial t} + \frac{\partial F}{\partial x} - \frac{\partial C}{\partial x} = 0
\]

where

\[
U = \begin{bmatrix} \rho \\ \rho v \\ E \end{bmatrix}, \quad F = \begin{bmatrix} \rho v \\ \rho v^2 + p \\ (E+p)v \end{bmatrix}, \quad C = \begin{bmatrix} 0 \\ 2\mu v \\ \lambda T + \mu \nu^2 \end{bmatrix}
\]

The corresponding finite difference scheme of the MacCormack method are the predictor step:

\[
\tilde{U}^{n+1} = U^n + \frac{\Delta t}{\Delta x} (F^n - F^n_1) + \frac{\Delta t}{(\Delta x)^2} (C^n_{1+1} - 2C^n_1 + C^n_{1-1})
\]

and the corrector step

\[
U^{n+1}_1 = \frac{1}{2} [U^n + U^n_1] - \frac{\Delta t}{2\Delta x} (F^n_{1+1} - F^n_{1-1})
\]

\[
+ \frac{\Delta t}{(\Delta x)^2} (C^n_{1+1} - 2C^n_1 + C^n_{1-1})
\]

The normalized variables used for computation are

\[
E' = E/(c_1 RT_1), \quad v' = v/\sqrt{\gamma RT_1}, \quad p' = p/p_1,
\]

\[
\rho = \rho / \rho_1, \quad T' = T/T_1, \quad x' = x/L_0, \quad t' = a_1 t/L_0
\]

where \(L_0\) is the reference length, and put as \(L_0 = 5x_0\).

The time step is determined from 90% of the maximum value for the local stability criterion (CFL condition) at each time step:

\[
t' = \max [0.9 \Delta x'/(\gamma T' + |v'|)]
\]
* USERID = TN30000
* PROCEDURE = LOGON2
* TSLOG STARTED TIME=10:57:0R DATE=82-12-08
* READY

E MAC F7(F1)

E LIST

00010 C
00020 C
00030 C
00040 C
00050 C
00060 C
00070 C
00080 C
00090 C
00100 C
00110 C
00120 C
00130 C
00140 C
00150 C
00160 C
00170 C
00180 C
00190 C
00200 C
00210 C
00220 C
00230 C
00240 C
00250 C
00260 C
00270 C
00280 C
00290 C
00300 C
00310 C
00320 C
00330 C
00340 C
00350 C
00360 C
00370 C
00380 C
00390 C
00400 C
00410 C
00420 C
00430 C
00440 C
00450 C
00460 C
00470 C
00480 C
00490 C
00500 C
00510 C
00520 C
00530 C
00540 C
00550 C

D-2
C.MESH

C 0 J=1 K=1

C INITIAL CONDITIONS

DO 207 J=1,KJ2

U1(J+1)=PX(J-1)

V2(J)=0.0

V2(J)=0.0

TC(J)=T4

U1(J+3)=G1BP4

207 PA(J)=1.0

DO 208 J=1,KJ2

U1(J)=0.5*(U1(J-1)+U1(J-2))

PA(J)=0.5*(PA(J-1)+PA(J-2))

TC(J)=0.5*(TC(J-1)+TC(J-2))

Y=0.0

CALL PL(F4.0,9.0,-3)

CALL SCALE(XARRAY,12@59KJ,1)

CALL SCALE(YARRAY,15@5,KJ,1)

CALL AXISC(0.0,0.0,6HX-AX1S,-6,12.5,00Q

CALL SYMPCL(1.2,17.,0.,33)

CALL SYMPBCL(1.2,16.0,0.3,33)

C 1,ACCOFMACK

DO 209 I=1,3

U2(J+1)=U1(J+1)

KL1=KL0

DO 360 h=1,MAX

CFL1=1.0

DO 390 J=KL1,KS1

CFL2=1.0/DSQRT(V2(J))+DSQRT(DABS(PA(J)*U1(J)))

IF(CFL1<LT.CFL2) GO TO 300

CFL1=CFL2

J=J

300 CALL T1,UE

D-3
01220   UTX=CFL1*MCFA
01230   DT=DTX*DX
01240   D=2*DX*DT
01250   D=0.5*wx
01260   C=DTX/DTX*RE0
01270   D=0.4*FPFR
01280   C=0.5x
01290   D=0.5x
01300   Y=Y+8
01310   G0 302  J=KL1,KS1
01320   DCONEXT
01330   U2(J,1)=U1(J,1)-DTX*U1(J,1)*2+DNLH*U1(J,1)*2
01340   U2(J,2)=U1(J,2)-DTX*U1(J,1)*1+DNLH*U1(J,1)*1
01350   1 (PA(J,1)-PA(J,1))/GF
01360   1 (VC(J,1)=2.0*VR*VC(J,1)+VC(J,1))
01370   U2(J,3)=U1(J,3)-DTX*U1(J,1)*2+U1(J,3)+PA(J,3)/GF
01380   D*U2(J,1)0.5
01390   U2(J,3)=DNLH*U1(J,1)-2.0*NLH*U1(J,1)-2.0*NLH*U1(J,1)
01400   302 CONTINUE
01410   DO 303  J=KL1,KS1
01420   310 V=U2(J,1)/U2(J,1)
01430   IF(DABS(VB).LT.(0.1P-9)) GO TO 347
01440   3010 V2(J)=V*2
01450   3010 V2(J)=0.0
01460   3010 VC(J)=0.0
01470   3010 TC(J)=0.0
01480   3010 PA(J)=0.0
01490   3010 U1(J,1)=U1(J,1)
01500   3010 U1(J,1)=U1(J,1)
01510   3010 U1(J,1)=U1(J,1)
01520   304 U2(J,1)=U2(J,1)
01530   304 U2(J,1)=U2(J,1)
01540   304 U2(J,1)=U2(J,1)
01550   V2(J)=V2(J)
01560   PA(J)=PA(J)
01570   TC(J)=TC(J)
01580   DO 306  J=KL1,KS1
01590   DCON3
01600   U1(J,1)=0.5*5U1(J,1)-U2(J,1)-DNLH*U2(J,1)-DNLH*U2(J,1)
01610   U1(J,1)=0.5*5U1(J,1)-U2(J,1)-DNLH*U2(J,1)-DNLH*U2(J,1)
01620   1 HVZ(J,1)=3*PA(J)-PA(J,1))/GF
01630   1 (DA*VC(J,1)+2.0*VC(J,1)+VC(J,1))
01640   1 (DA*VC(J,1)+2.0*VC(J,1)+VC(J,1))
01650   1 (DA*VC(J,1)+2.0*VC(J,1)+VC(J,1))
01660   1 (DA*VC(J,1)+2.0*VC(J,1)+VC(J,1))
01670   306 CONTINUE
01680   DO 307  J=KL1,KS1
01690   310 V=U1(J,1)/U1(J,1)
01700   IF(DABS(VB).LT.(0.1P-9)) GO TO 347
01710   V2(J)=V*2
01720   309 G0 TO 348
01730   347 V2(J)=0.0
01740   348 PA(J)=G2*U1(J,1)-U2(J,1)+PA(J)/GF
01750   VC(J)=2.0*VR*VC(J)
01760   WC(J)=PA(J)/U2(J,1)
01770   307 CONTINUE
01780   DO 314  I=KL1
01790   314 U1(I,1)=U1(I,1)
01800   U1(I,1)=U1(I,1)
01810   V2(I,1)=V2(I,1)
01820   PA(I)=PA(I,1)
01830   VC(I)=VC(I)
01840   TC(I)=TC(I)
01850   DO 3145  I=KL1,KS1
01860   J=KS1+I
01870   PA=PA(J)-1.0000

D-4
01470 IF(PA<GT*FS1) GO TO 3150
01490 U1(J1)=1E0000
01900 U1(J2)=0.000000
01910 U1(J3)=G1
01920 PA(J)=1E0000
01930 VC(J)=0.000000
01940 TC(J)=1E0000
01950 3145 V2(J)=0.000000
01960 C IF STEP CONTROL X
01970 3150 IF(KL1=EQ*2) GO TO 315
01980 KL1=KL1-1
01990 3150 IF(KS1=GT*KJ1) GO TO 316
02000 KS1=KS1-1
02010 3100 IF(AND(KL1*EQ*1) GO TO 310
02020 IF(KL1=LT*1000) GO TO 310
02030 WRITE(6*2001) *I;Y;NT;JCFL
02040 2001 F(J)=F1*1+17;5*2+V=+F1*5;5*4+NT=j;F1*5;5*5,j;5H;JCFL=j;IC)
02050 KJ1=S1=KLI
02060 K=1=K*1
02070 KM2=KM1*6
02080 I=0=0=00=0
02090 J=2400 J=1=10
02110 KT1(J)=IK*18(J-1)
02120 2400 GT1(J)=FAC*PA(KT1(J))=I*100
02130 IF(AND(KL1*EQ*TP1(J))=J*100)
02140 2501 CONTINUE
02150 CONTINUE FORMAT(14*F0.4+14*F0.4+14*F0.4+14*F0.4)
02160 C IF PLOT 2 X
02170 IF(PA*PA*1.0000)
02180 IF(PA*GT*ESS) GO TO 999
02190 U=240 I=1+J
02200 YARRAY(I)=PA(I*1)+I
02210 CONTINUE
02220 CALL LINE(XARRAY,YARRAY,KJ,1,0,3)
02230 IF(MIN(K+CA*1.E+10) GO TO 360
02240 CA=CA+1
02250 CA=CA+1
02260 I1C=1C+1
02270 360 CONTINUE
02280 999 CALL VSTP(0*6)
02290 CALL GSPLT
02300 C IF EN
02310 WRITE(6*611)
02320 611 FORMAT(///40X,15H8888 END 888888888
02330 STOP
02340 END
END OF DATA

INPUT

END S
SAVLE TO DATA SET (/T:30000,MAC*FORT*)
READY
TSLUG END

USERIN = T:3000
PROCEDURE = LOGP2

TSLOG ENDED TIME=10:58:30 DATE=82-12-08
APPENDIX E

COMPARISON BETWEEN NEAR-FIELD SOLUTIONS
OF THE EXPLOSION OF A PRESSURIZED AIR SPHERE
USING LAX, MACCORMACK AND RANDOM-CIRCLE METHODS (RCM)
FOR A PERFECT-INVIScid FLOW

In the initial stage of the present study, several numerical methods were tried to solve the problem of the explosion of a pressurized air sphere. Some of the results are presented here to show the superiority of the RCM over other methods for analysing shock-transitions of spherical N-waves.

The near-field solutions using Lax, MacCormack and RCM for the same case as $A_1$ ($P_{31} = 2.0$, $T_{31} = 1.0$) are shown in Figs. E.1, E.2 and E.3, respectively. In Figs. E.1 and E.2 (Lax and MacCormack methods), the time steps were selected to be 80% of the CFL condition to avoid undesirable oscillations of numerical values. As seen in Figs. E.1(a) and E.2(a), the Lax and MacCormack solutions give smoothed shock-transitions due to the effect of artificial viscosity in a rough mesh size of $\Delta \tau = 1/80$. By using the finer mesh sizes ($\Delta \tau = 1/320$, Figs. E.1(b) and E.2(b)), this smoothing is improved, and the Lax method gives a better result. However, the smoothing at the front shock still remains. The RCM solutions [Figs. E.2(a) and (b)] show discontinuous shock fronts irrespective of mesh sizes ($\Delta \tau = 1/40, 1/80$), though some randomnesses appear in the expansion part of a pressure profile. In our analysis of shock transition, it is necessary to clarify the effects of viscosity and vibrational nonequilibrium on shock thickness without the effect of artificial viscosity. Consequently, we adopted the RCM.
FIG. E.1(a) NEAR-FIELD SOLUTION OF EXPLOSION OF A PRESSURIZED AIR SPHERE USING LAX METHOD FOR PERFECT-INVISCID FLOW (CASE A1). MESH SIZE $\Delta r^* = 1/80$. 
FIG. 1(b) NEAR-FIELD SOLUTION OF EXPLOSION OF A PRESSURIZED AIR SPHERE USING LAX METHOD FOR A PERFECT-INVIScid FLOW (CASE A1). MESH SIZE $Ax = 1/320$. 
FIG. E.2(a) NEAR-FIELD SOLUTION OF EXPLOSION OF A PRESSURIZED AIR SPHERE USING MACCORMACK METHOD FOR A PERFECT-INVISCID FLOW (CASE A1). MESH SIZE Δr* = 1/80.
FIG. 1.2(b) NEAR FIELD SOLUTION OF EXPLOSION OF A PRESSURIZED AIR SPHERE USING MACCORMACK METHOD FOR A PERFECT INVIScid FLOW (CASE A1). MESH SIZE \( r^* = 1.520 \).
FIG. E.5(a) NEAR-FIELD SOLUTION OF EXPLOSION OF A PRESSURIZED
AIR SPHERE USING RANDOM-CHOICE METHOD FOR A
PERFECT-INVISCID FLOW (CASE A1). MESH SIZE \( r^* = 1/40 \).
Fig. 1.3(b) Near-field solution of explosion of a pressurized air sphere using random-choice method for a perfect-inviscid flow (case 1). Bush data at $r^* = 1.80$. 
APPENDIX F

BULK VISCOSITY ANALYSIS FOR VIBRATIONAL RELAXATION FOR OXYGEN

In Sections 4.4.5 and 4.4.6, the bulk viscosity concept is introduced to evaluate the vibrational relaxation for oxygen instead of solving the relaxation equation for oxygen. The basic equations are shown in some detail as follows:

\[
\begin{align*}
\frac{\partial U}{\partial t} + \nabla \cdot (\mathbf{F} - \mathbf{PV}) &= \mathbf{C} \\
\mathbf{F} &= \begin{bmatrix} \frac{2}{3} \mathbf{r}^2 + \frac{1}{2} \mathbf{r} \mathbf{V} \\ \mathbf{V} \\ \mathbf{E} + \frac{1}{2} \mathbf{V}^2 + \mathbf{p} \\ \mathbf{N} \end{bmatrix}, \\
\mathbf{C} &= \begin{bmatrix} 0 \\ 2 \mathbf{e} \mathbf{v} \\ \mathbf{e} \mathbf{v}^2 + \mathbf{p} \\ \mathbf{v}^2 \mathbf{N} \end{bmatrix}, \\
\mathbf{H} &= \begin{bmatrix} \frac{1}{2} \mathbf{e} \mathbf{v} \\ 0 \\ \mathbf{e} \mathbf{v}^2 \\ 0 \end{bmatrix}, \\
\mathbf{H}_R &= \begin{bmatrix} 0 \\ 0 \\ 0 \\ e^{-(\gamma N)} e^{-\gamma N/\gamma N} \end{bmatrix}, \\
p &= \gamma \mathbf{R} T, \\
\mathbf{e} &= \gamma \frac{5}{2} \mathbf{R} T + \gamma \mathbf{e} + \gamma \mathbf{J} \\
\gamma &= \frac{1}{2} \mathbf{r}^2 + \frac{1}{2} \mathbf{r} \mathbf{V} \\
\gamma_0 &= \frac{1}{2} \mathbf{r}^2 + \frac{1}{2} \mathbf{r} \mathbf{V} \\
\gamma_0 &= \frac{1}{2} \mathbf{r}^2 + \frac{1}{2} \mathbf{r} \mathbf{V} \\
\gamma &= \frac{1}{2} \mathbf{r}^2 + \frac{1}{2} \mathbf{r} \mathbf{V} \\
\gamma_0 &= \frac{1}{2} \mathbf{r}^2 + \frac{1}{2} \mathbf{r} \mathbf{V} \\
\gamma &= \frac{1}{2} \mathbf{r}^2 + \frac{1}{2} \mathbf{r} \mathbf{V} \\
\gamma_0 &= \frac{1}{2} \mathbf{r}^2 + \frac{1}{2} \mathbf{r} \mathbf{V}
\end{align*}
\]

Instead of Eqs. (4.1) and (4.2), where \( \gamma_0 \) is an effective viscosity including the bulk viscosity \( \gamma_0 \) for oxygen, defined by

\[
\gamma_0 = \gamma + (\gamma_0)^{1/2}.
\]

The bulk viscosity \( \gamma_0 \) is evaluated from Eq. (3.25):

\[
\gamma_0 = \gamma + (\gamma_0)^{1/2}.
\]

where

\[
\gamma = \frac{1}{2} \mathbf{r}^2 + \frac{1}{2} \mathbf{r} \mathbf{V} \\
\gamma_0 = \frac{1}{2} \mathbf{r}^2 + \frac{1}{2} \mathbf{r} \mathbf{V} \\
\gamma_0 = \frac{1}{2} \mathbf{r}^2 + \frac{1}{2} \mathbf{r} \mathbf{V} \\
\gamma_0 = \frac{1}{2} \mathbf{r}^2 + \frac{1}{2} \mathbf{r} \mathbf{V} \\
\gamma_0 = \frac{1}{2} \mathbf{r}^2 + \frac{1}{2} \mathbf{r} \mathbf{V} \\
\gamma_0 = \frac{1}{2} \mathbf{r}^2 + \frac{1}{2} \mathbf{r} \mathbf{V} \\
\gamma_0 = \frac{1}{2} \mathbf{r}^2 + \frac{1}{2} \mathbf{r} \mathbf{V}
\]

The operator-splitting technique was applied to Eq. (F.1) as well as Eq. (4.1). The effect of vibrational relaxation for oxygen was taken into account in the step of viscous correction [Step 3; Eq. (4.1)] of the operator splitting through Eqs. (F.3)-(F.5). More precisely, in the first step, the RCM solution should be obtained by solving the Riemann problem for oxygen in vibrational equilibrium, since the whole flow field may be considered for oxygen as in quasi-equilibrium. However, in the present report, the effects of oxygen vibrational excitation is taken into account only through the bulk viscosity, since its contribution to the internal energy specific heats of the air molecules may be considered as very small as long as it is nearly in equilibrium at room temperature. Thus, the RCM solutions were obtained by using the inviscid-frozen program, excluding the term \( \gamma_0 \) in Eq. (F.2).
In order to clarify the effects of vibrational excitation on shock-wave transitions of weak spherical waves, which were generated by using spark and exploding wires as sources, the compressible Navier-Stokes equations were solved numerically, including a vibrational-relaxation equation for oxygen or nitrogen. A small spherical air-blast explosion was used to simulate the waves generated from the actual sources. By employing the random-shock method RSM with a splitting technique, the effects of vibrational viscosity appearing in finite-difference schemes were eliminated and accurate profiles of the shock waves were obtained. It was shown that a computer simulation is possible by using a proper choice of initial parameters to obtain the variations of the shock overpressure and half-duration with distance from the source. The calculated results are also shown to simulate both spark and exploding-wire data. It was found that, in addition to the vibrational-relaxation time of oxygen, both the duration and attenuation rate of a spherical wave are important factors controlling its rate type. The effects of the duration and attenuation rate of a spherical wave on its rate type, which are designated as the "source effect" and the "source effect" respectively, are discussed in more detail pertaining to Lighthill's analytical solutions and the RSM solutions for nonstationary plane waves and spherical waves. It is also shown that the duration and attenuation rate of a spherical wave are affected by viscosity and vibrational non-equilibrium, so that it can deviate from the results of classical, linear acoustic theory for very weak spherical waves.

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