REAL TIME OPTICAL INTERFEROMETRIC IMAGE ADDITION AND SUBTRACTION BY WAVE POLARIZATION(U) LUMIN INC
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Real Time Optical Interferometric Image
Addition and Subtraction by Wave Polarization

by

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Prepared for

U.S. Army Missile Command
Redstone Arsenal, Al. 35898

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REAL-TIME OPTICAL INTERFEROMETRIC IMAGE ADDITION
AND SUBTRACTION BY WAVE POLARIZATION

ABSTRACT

A real-time coherent optical interferometric image addition and subtraction system based on a technique similar to an earlier work by Kunzmann and Spitschan is extensively investigated. Theoretical analysis of the system has been compared with experimental data obtained. Distortion and resolution power of the system with respect to the size and positions of the input images have been analyzed in detail.
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REAL-TIME OPTICAL INTERFEROMETRIC IMAGE ADDITION
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1. Introduction

Optical image addition and subtraction by holography and interferometry have been investigated by various researchers. In fact, in the interferometric methods, complex amplitudes of two images from different light paths of an interferometer are recombined by a grating or an ordinary holographic beam splitter. By introducing a phase difference of -π into one of the paths, the subtraction between the two images can be achieved. However, in the method used by Kunstmann and Spitschak, an analyzer was used to produce opposite phases for the two spatially separated images, and a Wollaston prism was used for the recombination of the two images. The method is comparatively simpler and offers the following advantages. It is easier to change the phase difference between the imaging paths of the two images since this can be achieved by simply rotating the analyzer. The imaging paths of the two images are relatively closer to each other than those in the interferometric Michelson interferometer. Hence the method is less vulnerable to environmental vibrations. Furthermore, no holographic beam splitters or gratings are required. The birefringent Wollaston prism is commercially available. However, although Kunstmann and Spitschak reported that subtraction has been performed in an area of 3 mm by 3 mm in their experiment, no theoretical analysis of their experimental results with regard to the various optical components such as the lens and Wollaston prism have been given. For example, they have not taken the aberrations of the Wollaston prism and the precision of the coincidence of the two images into
consideration. The spherical aberration of the Wollaston prism in their
system can interfere only in the subtraction and addition and the limit
offinterference in the system is low. For this reason, we have investigated
an improved system. The details of the theory, experimental results, and
experimental data analysis of the improved system are described in the
following sections.

II. Theoretical Discussion

The basic principle of the technique of performing real-time image
subtraction and addition using polarization modulation can be illustrated
with the assistance of Fig. 1. Besides ordinary polarizers, a special
polarization element used is called a Wollaston prism.

In Fig. 1, S is a coherent point light source, C is a collimator, and
$S_1$ and $S_2$ are two transparencies with their amplitude transmittances $S_1(x_1, y_1)$
and $S_2(x_1, y_1)$ representing respectively the two input signals being
processed. The center of $S_1$ is located at $(x_0, 0)$ and that of $S_2$ is at $(-x_0, 0)$
in the input plane $L_1$, and both of them are inclined at an acute angle to
the z-axis as shown in Fig. 1. The input plane $L_1$ is located at the first
focal plane of the lens $L_1$, and the output plane of the system is at the
back focal plane of lens $L_2$. $P$ is a polarizer and $AN$ is an analyzer.
$P_1$ and $P_2$ are two modulation polarizers located in front of the input trans-
parencies $S_1$ and $S_2$ respectively. The unit-vectors along the directions of
the principal transmittance of $P$, $P_1$, $P_2$, and $AN$, represented by $P_1$, $P_2$, and $A$ respectively, are shown in Fig. 2. $W$ is a Wollaston prism which is
located at the vicinity of the back focal plane of $L_1$ and the front focal
plane of $L_2$ and has its two optical axes parallel to $P_1$ and $P_2$ respectively.

For simplicity, we shall assume that the two lenses $L_1$ and $L_2$ have
the same focal length $f$ and have perfect image formation properties with
the input plane \( I_1 \) and output plane \( I_2 \) as a pair of conjugate planes.

Since the spherical waves radiated by the points at the plane \( I_1 \) are transformed into plane waves by lens \( L_1 \), the Wollaston prism though can introduce distortions but will not induce any other monochromatic aberrations for objects at plane \( I_1 \).

The complex, vectorial amplitude distribution of the illumination light at \( I_1 \) may be represented by \( P \). As a result of the polarization modulation of \( S_1 \) and \( S_2 \), the amplitude distribution \( I_1 \) immediately behind the input plane is

\[
I_1 = S_1(x_1, y_1) \cdot P_1 + S_2(x_1, y_1) \cdot P_2
\]

If there is no Wollaston prism in the system, the different polarization states of \( S_1 \) and \( S_2 \) would not affect their image locations, and the two images would still be separated at the output plane \( I_2 \). However, when the prism is in place, the situation is different. The Wollaston prism will deflect the light beams passing through it according to the polarization state. As can be seen in Appendix A, the values of the angles of deflection for beams polarized along \( P_1 \) and \( P_2 \) are both approximately equal to

\[
\tan A \approx n_0 - n_e
\]

where \( A \) is the apex angle of the Wollaston prism defined in the Appendix, \( n_0 \) and \( n_e \) are the refractive indexes of the prism corresponding to the ordinary and extraordinary beams respectively. However, the directions of the deflections of these two beams are opposite to each other. Thus, if we adjust the distance \( x_0 \) to make the angle of inclination \( \theta \) be equal to the angle \( \theta \), the centers of the images of \( S_1 \) and \( S_2 \) can both be shifted to be on the z-axis in the output plane and hence the two images will
coincide, limited by distortions, with each other (see Appendix A). Start
from an approximately uniform amplitude attenuation and constant vis-
perturbation (see (3A)). 1.1,..., start from a complex constant. The output
emitted at one end is then obtained without an analyzer A being placed in
front of F3, and then the relations are:

\[ E = \frac{1}{2} \left[ S_1(x_2, y_2) + S_2(x_2, y_2) \right] A. \]

When an is placed in front of F3 as shown in Fig. 1; due to the
relationship among P, P1, P2 and A (Fig. 2), the output becomes

\[ E = \frac{1}{2} \left[ S_1(x_2, y_2) - S_2(x_2, y_2) \right] A. \]  

(3)

The image subtraction between S1 and S2 can thus be achieved.

Moreover, the flexibility of the proposed system can be seen by the
fact that broader focal plane preprocessing capabilities can be attained by
simply manipulating the relative angular positions of P and P1. First,
we notice that the sagittal slit in Eq. (2) is caused by a "crossed" slit
procedure, i.e., \( E = \frac{1}{2} \) as shown in Fig. 1. If we rotate \( \alpha \) by 90°, a non
collinear slit will be required and the output then becomes

\[ E = \frac{1}{2} \left[ S_1(x_2, y_2) + S_2(x_2, y_2) \right] \tilde{A}. \]  

(4)

and hence image addition can be achieved. Secondly, if a linear combina-
tion of the two images is desired, we can simply rotate \( \tilde{A} \) or \( P_\perp \)
to obtain the following output.

\[ 0 = \frac{1}{2} \left[ S_1(x_2, y_2) \cos \alpha + S_2(x_2, y_2) \sin \alpha \right] \tilde{A}. \]  

(5)

where \( \alpha \) is the angle between \( \tilde{A} \) and \( P_\perp \) and the "-" and "+" signs correspond
to the geometries shown in Fig. 2(a) and Fig. 3(b) respectively.

For simple and convenient technique for the linear combination of these images has its practical importance. For example, in image subtraction, the two transparencies usually do not have the same transmittance over the supposed-to-be identical portion of the two images due to the differences in emulsions or the exposure and development conditions. Thus, in order to carry out a true subtraction, a "weighted" subtraction or a linear combination is required for the removal of the pseudo-differences.

III. Experimental Results

A. The experimental set-up

The experimental setup is basically constructed according to the diagram shown in Figures 1 with slight modifications. A 50 mA He-Ne laser was used as the light source, the polarizer P is placed in the raw laser beam before the collimating lens C. The polarized beam, after being expanded through a pin-hole spatial filter, was collimated by a 6-inch aperture 18-inch focal-length telescopic lens C. It was found that the polarizers P₁ and P₂ were not required. In that case, each input image produces two spatially separated images at the output plane, and the useless one can be easily blocked. The rest of the system remained the same except that the imaging lens L₂ was inserted between the analyzer AN and the final output image Iₙ. Five different lenses have been used at the position of L₁ for the purpose of finding out the influence of the Fourier transform on the effect of subtraction. The important characteristics of the lenses are listed in Table 1. The Wollaston prism is made of calcite, with its front surface area of 10 mm by 10 mm, total thickness of 6 mm, and the split angle of 9° (the apex angle A = 0.245 rad).
Interference fringes resulted from the subtraction of two open spaces.

During the experimental process, the lenses listed in Table 1 were one by one placed at $L_1$ of Fig. 1 and the output beam at $L_2$ of the interferometer was observed directly. When there was no specified images being placed at $L_1$ plane, the subtraction was performed between two open spaces $L_1$ and $L_2$ of Fig. 1. It has been found that the two possible axis orientations of the Wolfson transform lens has a significant influence on the result of subtraction. The subtraction between two open spaces yields interference fringes which will be analyzed in the next section. When the Wolfson transform lens is placed with one of its two surfaces facing toward the light source (say Orientation 1) broader interference fringes can be obtained than when the surface is placed facing away from the light source (say Orientation 2). The phenomenon, common to all the lenses listed in Table 1, is further illustrated in Figs. 4-9. Figures 4 and 5 present the photographic recordings of the interference fringes where the Wolfson transform lens $L_1$ was placed along Orientation 1 and 2 respectively. The various parts in these figures indicate the different positions of the Wolfson prism along the optical axis. When the Wolfson prism was placed at position a, the dark fringe has a minimum area in the interference fringe pattern. It can be seen that this is true for Figures 4-9. In Figs. 6-9 are photographic recordings of
the interference fringes where \( I_{14}, I_{15}, I_{16}, \) and \( L \) were placed with orientation 1 and the Wollaston prism was so positioned that the dark fringes were produced. The other parts of the same figure correspond to the areas where those fringes were placed along Orientation 1. As in the system but with the Wollaston prism position reversed, it has been found that along Orientation 2, no matter how one adjust the position of the Wollaston prism, there was no way to obtain a greater dark fringe area than those obtained with the lens along Orientation 1.

It is important to note that the dark fringes caused by the destructive interference of the areas in the two open spaces at the input image plane are the usable areas for the performance of the subtraction. Therefore, we have attempted to maximize the dark fringes in the interference fringe patterns created by \( I_{14} \) through \( I_{16} \) each along the two possible axial orientations. The results of computations of the maximum \( x \)- and \( y \)-directions of the dark areas from one of the photographs of Figures 4 through 9 to its best accuracy are tabulated in Table II. Although the maximum area does not equal \( xy \) for each of the cases listed in Table II, it seems that \( I_{14} \) placed along Orientation 1 would yield the maximum area for subtraction in comparison with the other lenses.

C. Results of subtraction between two specific input images — a penny and a metal washer.

Since \( I_{14} \) placed along Orientation 1 has yielded the maximum area of the dark fringe, it was chosen for the Fourier transform lens.
In order to understand the subtraction system thoroughly, the interference fringe patterns as shown in Figs. 4-10 require a theoretical interpretation and the degree of accuracy of the subtraction results needs to be analyzed. We shall discuss these two effects of the system as follows.
A. Interpretation of the interference fringes

The theoretical model that we propose to interpret the interference fringe pattern assumes that all optical components used in our system are optically perfect, i.e., the lenses are free of aberrations. The reason that the fringes appear at the output plane is mainly due to the fact that two different optical rays, one with the other, two different optical points of view, create a certain interference at the output plane. The whole system is used to locate these two distinct paths of the light rays, to find the location of the image point on the output plane based on the light ray concept. The light ray concept is based on the approximation of geometrical optics, since we assume that the amount of light diffraction may be negligible in the present case. More details of the model are presented below with the help of some illustrations.

A part of the image processing system of Fig. 1 is shown in more fine details in Fig. 2. Point $p_3$ is a certain point with coordinates $(x_3, y_3)$ on the output plane. Based on the principle of geometrical optics, it is not difficult to find the locations of the two input plane points $p_1$ and $p_2$ that correspond to the output point $p_3$. Since in the idealized case, we make the assumption that the system is free of aberrations and diffractions, $p_1$ and $p_2$ are a pair of conjugated object/image points. The optical paths between $p_1$ and $p_2$ can be evaluated along any ray passing through both of them. To be specific, we choose the rays originating from $p_1$ and $p_2$ that are parallel to the optical axis before they reach the Fourier transform lens. Hence the two rays would intercept at the back focal point of the lens.
...the optical interference fringe patterns, of the Wollaston prism, is shown for the optical paths of the rays that re-
trough the Wollaston prism. From eq. 11, it can be seen that the
path length difference between \( p_1 \) and \( p_2 \) is equal to the path
length difference between \( (3\phi_0-1) \) and \( (3\phi_0+1) \). Based on this path
length difference, we have calculated on a digital computer the interfer-
ence fringe patterns. Other pertinent data used in the calculations in-clud-
the wavelength \( \lambda = 632.8 \text{ nm} \); the thickness of the Wollaston prism \( t =
6.0 \text{ mm} \); the apex angle of the Wollaston prism \( \phi = 0.245 \text{ radians} \); the
ordinary refractive index \( n_0 = 1.5654 \); the extraordinary refractive
index \( n_e = 1.6164 \); the focal lengths of the Fourier lens and the Wol-
aston lens are both equal to \( f = 750 \text{ mm} \); the ranges of the position of
points \( p_1 (x, y) \) are \(-700 \text{ mm} \leq x \leq 700 \text{ mm} \); \( y_3 = -150 \text{ mm} \); and
the focal length \( f_3 \) varies with \( x_3 = 2 \text{ mm} \); and \( y_3 = 2 \text{ mm} \). In
finally, the distance \( d \) between the front surface of the Wollaston prism and
the back focal point \( Y \) of the Fourier transform lens is taken as a parameter
which varies from \( 1.7 \text{ mm} \) to \( 2.0 \text{ mm} \) with a step increment of \( 0.2 \text{ mm} \) to account
for the micro-adjustment of the Wollaston prism in the
cyber.

The results of the calculations are plotted by the computer and
shown in Fig. 14 where the interference fringes are represented by dots
and B's. The dots denote that the absolute value of a fraction of the
path length difference introduced by the Wollaston prism at \( p_2 \) is less
than \( 1/4 \) or greater than \( 3\pi/4 \) and the letter B denotes that the absolute

* These indices are for wavelength \( \lambda = 589.3 \text{ nm} \); for \( \lambda = 632.9 \text{ nm} \) the
calculated patterns would be slightly different.
value of the path length difference is between \( \frac{3}{4} \) and \( \frac{1}{4} \). Thus, if the
analyser is in the air path, it does not introduce any additional phase difference to the data plane for the 'bright' fringe and the 'dark' fringe. In contrast, if the analyser is in air, it introduces a distortion, so the first of the fringe patterns will be misaligned. The second analyser
aligned as specified.

Finally, we can see from Fig. 1 that around \( \lambda = 1.1 \) m of the zero
time is at minimum and the relatively uniform area of the zero
fringe is at its maximum. The maximum dark area is best for performing
image subtraction. The theoretically predicted phenomenon has been
qualitatively verified by the experimental results. Any quantitative
difference between the computed and experimental data probably is caused by
the imperfections of the Collaston prism and the aberrations of lenses.
The phenomenon that the fringe patterns are different for a certain level
at the two different orientations can also be explained by the fact that
the orientation is dependent on the orientation.

The distortion at the output plane.

It is the fact that the amount of deflection of the light rays is
dependent on the relative phase of the incident angle at the front surface
of the prism. If two images are superimposed at the output plane cannot totally
coincide with each other. Both images suffer from different amount of
distractions. The resolution power of the subtraction technique is limited by
these distractions. In the following, we estimate how serious the dis-
tortion is and try to find out the optimum positions of placing the two
input images where the distortions can be minimized. For simplicity, the
calculation is done only for one-dimensional case. We assume that the
Collaston prism has the same parameters as those in the A part of this...
Section one that both the center beam and the image beam have the same focal length \( f \) (arbitrary unit), thus the magnification of the whole system is 1.

The calculation is based on the configuration as shown in Fig. 1. First we choose a point \( C_3 \) on the output image and then trace back along the geometrical optical paths to two corresponding points \( C_1 \) and \( C_2 \) on the two input transparencies. We shall call the points \( C_1 \), \( C_2 \), and \( C_3 \) the centers of the respective images. Likewise, for any point \( C_3 \) on the output, the corresponding input image points \( P_1 \) and \( P_2 \) on the input can also be found. We further let \( P_1 = \frac{f}{H_1} C_1 \), \( P_2 = \frac{f}{H_2} C_2 \), and \( H_1 - H_2 \) where the positive sign is chosen if the vector is parallel to the positive axis. If there is no distortion, \( P_1 = P_2 = P_3 \). The difference \( H_1 - H_2 \) and \( H_1 + H_2 \) can be used to measure the amount of distortion. The difference \( H_1 - H_2 \) can be used to determine the resolution of the distortion. For example, according to our calculated results, if the point at the intersection of the output plane and the optical axis is 0 cm as \( C_3 \), then \( P_1 \) and \( P_2 \) will have their \( x \)-coordinate 0.1060 and 0.1075, respectively, where all units are considered to be the same as that of the focal length. An output image point \( P_3 \) with \( H_2 = 0.100 \) has its corresponding object points \( P_1 \) with \( H_1 = 0.1050 \) and \( P_2 \) with \( H_2 = 0.1010 \). In this case, the difference \( H_1 - H_2 = 0.004 \) is the minimum resolvable distance when the image height is around 0.1. The overall resolution data are illustrated further by two additional figures. Figure 16 is a plot of \( H_2 - H_1 \) versus \( H_2 \) when \( C_3 \) is located on the axis. It can be seen that the difference value of \( \|H_3\| \) and \( \|H_3\| = 0.01 \), the difference \( H_2 - H_1 \) is much greater for positive \( H_3 \) than for negative \( H_3 \). Thus, in order to minimize distortion and simultaneously maximize the size of the image to be subtracted one has to choose an optimum set of centers of images (\( C_1 \), \( C_2 \), \( C_3 \))
For example, if the maximum tolerable difference ($H_2 - H_1$) is 0.061 and we place the upper end point of each of the input images 0.34 above its center at $y = -0.10$, the total height of the image can be subtracted would be $0.04 - (-0.16) = 0.19$. If instead, when the middle point of the image is placed at the ‘center’, the total height of the image that can be subtracted within the tolerance is then reduced to $0.04 - (-0.04) = 0.08$.

Figure 17 shows a different case in which the point $C_3$ is not located at the origin axis but at $x_3 = -0.06$. Then it can be seen that the values of $H_2, H_1$ are almost distributed symmetrically around the center at $H_3 = 0.07$. While the maximum tolerable $H_2 - H_1$ is 0.064 and the middle points of the two input transmittances are placed at the centers $C_3$ and $C_4$, the total height of the image 0.18 is will reduced to $0.04 - (-0.16) = 0.16$. Here, here in this case, the distortion around the center emerges very visibly and have a considerable value. It is also interesting to find out the relative distortion as defined by $\frac{H_2 - H_1}{H_3}$ excluding $H_3 = 0$. Two additional pictures corresponding to the above-described two cases are plotted and shown in figures 18 and 19 respectively. We can see that for the first case ($C_3$ located on axis), the relative distortion is greater at the vicinity of the center than at the lower part of the image. For the second case ($C_3$ at $x_3 = -0.06$), the minimum relative distortion appears at the vicinity of the center. The total height for the first case within the tolerance of $[H_2 - H_1, H_3]$ is about $(0.04) - (-0.16) = 0.12$ while the total height for the second case is about $0.06 - 0.04 = 0.14$. The computer programs for the calculations are listed in Appendix C and D respectively.
V. Conclusions

We have presented the principle, experimental data, and data analysis of a real-time image subtraction technique by using the properties of wave polarization. The theoretical analysis predicts that interference fringes should be produced in the system and the experimental data demonstrated the phenomenon. The dark (or destructive interference) portion of the fringes can be used for performing image subtraction. The maximum area that can be used for subtraction in the image constructed is about 26\texttimes{}18 mm, which is about two times the half coherence of magnitude greater than that reported earlier in similar applications. In addition, in the present system, the destructive area and the light intensity is higher in comparison with other systems. A detailed calculation of the distortion of the image due to aberrations also has revealed the optimum positions where the input lenses should be placed at the input plane when the tolerance of the difference of distortion error is set. From the experimental data, it was found that the Gaussian lenses with the smallest D/S yield the greatest usable subtraction area. This is expected since in general lenses with greater focal lengths have less aberrations.

With regard to future work, it seems that two feasible ideas should be investigated. One is to incorporate the subtraction technique in a electro-optical system that includes the use of a Hughes liquid crystal light valve (CL) for real-time image subtraction in a continuous mode. The other is to apply the Wollaston prism for performing real-time and color image subtraction or additions. Both of these ideas could lead to important real-time applications.
APPENDIX A

THE FUNCTION OF A WOLLASTON PRISM

Based on the nature of a Wollaston prism, light beam traveling through the prism will gain an amount of path-length according to the beam's orientation of polarization. To illustrate the effect, a x-z plane cross-section of a Wollaston prism which consists of two triangle prisms is shown in Fig. A1. Each triangle has the apex $A = \tan^{-1}(h/t)$, where $h$ is the thickness and $t$ is the height of the prism. The two triangle prisms are made of the same kind of birefringent crystal but have different optic axes. For our purpose, if the crystal is a "negative" one, such as the calcite, the optic axis of the left half, as shown by $P_1$ in Fig. A1, is parallel to y-axis (or $P_1$) and that of the right half ($P_2$) is parallel to x-axis (or $P_2$). Thus, for the light beam polarized along $P_2$, the path length increment $\delta_1(x)$ is approximately,

$$\delta_1(x) = \frac{1}{2} x \tan A (n_0 - 1) + \frac{1}{2} x \tan A (n_e - 1) \quad \text{and} \quad \delta_2(x) = (n_0 - n_e)(x \tan A) \quad \text{(A1)}$$

Similarly, for the light beam polarized along $P_1$, the increment $\delta_2(x)$ can be written as

$$\delta_2(x) = (n_0 - n_e)(x \tan A) \quad \text{and} \quad \delta_2(x) = (n_0 - n_e)(x \tan A) \quad \text{(A2)}$$

Based on the assumptions represented by equations (A1) and (A2), two important conclusions can be deduced. First, any plane wave after
travelling through a Wollaston prism will be deflected by an angle \( \alpha \), which is approximately equal to \((n_o - n_e) \tan \alpha\). Because we have \(n_o > n_e\) for a negative crystal, \(\alpha\) has a positive value, i.e., all plane waves coming from the points of \(S_1\) will be deflected downward by an angle \(\alpha\), since they are polarized along \(P_1\). The image of \(S_1\) at the output plane will then also be moved downward. It will be centered on the \(z\)-axis, if we make \(\alpha = 0\). Since all light waves from \(S_2\) are polarized along \(P_2\), the image of \(S_2\) will move upward and coincide with the image of \(S_1\).

Furthermore, the amplitude distribution at the Wollaston prism will not affect the calculated path length difference; yet, due to the focusing effect of \(L_1\), a Wollaston prism of relatively small aperture can be used.
EXPERIMENTS


Table 1. Characteristics of the five types of transmission lenses used in Experiment 1.

<table>
<thead>
<tr>
<th>Lens No.</th>
<th>Total Length (cm)</th>
<th>Diameter of Aperture D (cm)</th>
<th>μ</th>
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<tbody>
<tr>
<td>1</td>
<td>48.75</td>
<td>9.525</td>
<td>0.107</td>
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<tr>
<td>2</td>
<td>76.25</td>
<td>10.16</td>
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<td>6.125</td>
<td>0.2</td>
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<tr>
<td>4</td>
<td>38.60</td>
<td>4.635</td>
<td>0.184</td>
</tr>
<tr>
<td>5</td>
<td>22.50</td>
<td>5.08</td>
<td>0.148</td>
</tr>
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</table>
Table 1: The shiftable x and y directions (denoted by X and Y respectively and in units of mm) of the dark fringes with the fibers aligned along a certain axial orientation.

<table>
<thead>
<tr>
<th>Class</th>
<th>Def</th>
<th>orientation</th>
<th>X</th>
<th>Y</th>
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<tr>
<td>1</td>
<td>1</td>
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<td>8.1</td>
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<td></td>
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<td>2</td>
<td>11.9</td>
<td>2.0</td>
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1. Photograph of the polarization modulation white-light image system.

2. Principal transmittance unit-vectors of the polarizers \( P, P_1, P_2 \) and the analyzer \( A \).

3. Principal transmittance unit-vectors of \( P, P_1, P_2 \) and \( A \) for the linear combination of two images.

4. Photograph of interference fringes with \( L_1 \) along Orientation 1 and the Wollaston prism at positions a and b.

5. Photograph of interference fringes with \( L_1 \) along Orientation 2 and the Wollaston prism at positions a, b, c, and d.

6. Photograph of interference fringes with \( L_1 \) along Orientation 1 and \( L_2 \) along Orientation 3.

7. Photograph of interference fringes with \( L_1 \) along Orientation 1 and \( L_3 \) along Orientation 4.

8. Photograph of interference fringes with \( L_1 \) along Orientation 2, \( L_2 \) along Orientation 3, and \( L_3 \) along Orientation 4.

9. Photograph of interference fringes with \( L_1 \) along Orientation 4 (a) and Orientation 2 (b).

10. Photographs of output interference fringes with \( L_1 \) placed alone Orientation 2 and the Wollaston prism placed at four different positions a, b, c, and d along the axis. No specific images were placed at the input plane of the system.

11. Photographs of the subtraction between a penny and a metal washer with the distance of separation between the two images varied. \( L_{11} \) was placed in the system along Orientation 1 and the Wollaston prism was placed at position a as shown in Fig. 10.

12. Light rays originating from points \( p_1 \) and \( p_2 \) at the input plane reaching the point \( p_3 \) at the output plane through the Wollaston prism.

13. The detailed paths of the light rays passing through the Wollaston prism.
FIGURE CAPTIONS (Cont'd)

14. Theoretically computed interference fringe patterns versus the on-axis position of the Wollaston prism.

15. Relative positions of \( p_1 \) and \( p_2 \) at the input plane and \( p_3 \) at the output plane for the computation of image distortion.

16. A plot of \( H_2 - H_1 \) versus \( H_3 \) with \( C_3 \) located on-axis.

17. A plot of \( H_2 - H_1 \) versus \( H_3 \) with \( C_3 \) located off-axis.

18. A plot of \( \log \left( \frac{|H_2 - H_1|}{H_3} \right) \) with \( C_3 \) located on-axis.

19. A plot of \( \log \left( \frac{|H_2 - H_1|}{H_3} \right) \) with \( C_3 \) located off-axis.

Ai. A x-y plane cross-sectional view of a Wollaston prism.