ULTRASONIC NDE OF BONDS IN MULTILAYERED SYSTEMS

BY PALMER L. EDWARDS

RESEARCH AND TECHNOLOGY DEPARTMENT

1 AUGUST 1983

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NAVAL SURFACE WEAPONS CENTER

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ULTRASONIC NDE OF BONDS IN MULTILAYERED SYSTEMS

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Presented here is a report on a program to develop a procedure to use ultrasonics to nondestructively detect flaws in the bonding of layered materials. The program was begun by the Materials Evaluation Branch in 1981, and up to the present time the work has been done primarily through contracts with Tetra Tech, Inc., of Arlington, Virginia. The first contract period involved the development of a computer model utilizing normal-incidence beams, the design of the experiments to test the model, the performance...
of the experiments in collaboration with personnel at NSWC, and the analysis of the experimental results. At the end of the first contract period Tetra Tech recommended that angled beams be used rather than the normal-incidence beams that had been used, and this was done in the second year. The computer model developed under the first contract was modified to be used with angled beams. Experiments performed at 5 MHz with a bronze-epoxy-aluminum structure gave results consistent with the computer model relative to transit times, but questions exist relative to the model's determination of reflection coefficients at the structure's interfaces. Experiments performed at 1 MHz with a simulated bronze-epoxy-rubber-epoxy-bronze structure showed observable signals due to disbonds from as deep into the structure as the bottom side of the rubber plate. The computer model, however, did not correctly predict the transit time, and there are questions concerning the calculated reflection coefficients. These differences may have been due to the fact that the bronze is not isotropic, or that the ultrasonic pulses might undergo transposition upon reflection at an interface. In the next step in the program the computer model should be modified, if possible, to include anisotropic properties of materials and the possibility of transposition.
FOREWORD

This report discusses the present status of a program to develop a procedure for the use of ultrasonic techniques to nondestructively detect flaws in the bonding of layered materials of interest to the U. S. Navy. The program was begun in 1981 by the Materials Evaluation Branch of the Materials Division at NSWC with the main portion of the program being contracted out to Tetra Tech, Inc., of Arlington, Virginia. The theoretical development was done by Tetra Tech in Arlington and the experimental work was done at NSWC, White Oak in collaboration with NSWC personnel. Dr. Ramesh Shankar was the Tetra Tech program manager and principal investigator, and was assisted by Dr. Stephen S. Lane and Mr. Thomas J. Paradiso. Mr. Clifford Anderson of NSWC was the program monitor, and Mr. Jeffrey M. Warren, Ms. Susan N. Vernon, and Dr. P. L. Edwards collaborated in the laboratory experiments and data collection.

Approved by:

JACK R. DIXON, Head
Materials Division
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INTRODUCTION

The U. S. Navy makes use of multilayered structures in a number of areas; and has an interest in the nondestruction evaluation of such structures. The purpose of this publication is to report on the status of a program undertaken to use ultrasonics to nondestructively detect flaws in the bonding of structures consisting of alternate layers of bronze and rubber with epoxy bonding. The purpose of the program was to develop a generic model to provide information as to the best signal processing techniques and instrumentation to use in the analysis of multilayered structures. This program was undertaken by the Materials Evaluation Branch of the Materials Division of the Research and Technology Department in 1981, and the work up to the present time has been done primarily through contracts with Tetra Tech, Inc., of Arlington, Va.

PROGRAM PLANS AND ACCOMPLISHMENTS

Tetra Tech described their proposed contribution to the program in "PROPOSAL TO DEVELOP ACOUSTIC NDE OF MULTILAYERED COMPOSITES" dated 22 June 1981. The report states "The specific objectives of the study are to:

- to develop a computer simulation model to determine the acoustic response at any intermediate layer of a multilayered structure, for arbitrary layer dimensions, acoustic parameters, and driving source function,
- design an experiment, collect and record ultrasonic data from rubber-bronze composite specimens containing flaws in their intermediate layers,
- process the digital signals using spectral and cepstral methods to enhance flaw response and minimize reverberations, and
- utilize 2-D image processing based software developed at Tetra Tech to analyze ultrasonic C-scan data to enhance the spatial flaw response".

The proposal also states that "The model should also provide information on the effects of medium anisotropies and inhomogeneities on signal response."

The initial contract included the following Tasks, as described in the 22 June 1981 proposal:

**Task 1: DEVELOPMENT OF AN ACOUSTIC MODEL FOR MULTILAYERED STRUCTURES**

- **Subtask 1.1:** Literature Search
- **Subtask 1.2:** Computer Implementation of Acoustic Models
- **Subtask 1.3:** Model Response for Flaws in Intermediate Layers

**Task 2: DESIGN OF EXPERIMENT FOR ULTRASONIC DATA COLLECTION FROM RUBBER-BRONZE COMPOSITE SPECIMENS.**

- **Subtask 2.1:** Experimental Verification of Model
- **Subtask 2.2:** Design of Experiment for Data Collection
- **Subtask 2.3:** Data Verification Preliminary Analysis

The proposal also states that "based on model results and preliminary data analysis results, experiments will be designed to collect and record ultrasonic data from a number of "flawed" and "unflawed" specimens. The experimental protocol will include the type of instrumentation, driving force function, transducers to be used, sampling rate and any prefiltering that may be required... The data will be collected at NSWC’s facility in White Oak, Md."

In the work following the initiation of the program, Tetra Tech personnel have developed the computer models, have designed the experiments to test the models, have performed the experiments at NSWC, White Oak, with the collaboration and assistance of NSWC personnel and have analyzed the experimental results. A bronze-epoxy-rubber multiple layer structure was used to check the model and to determine suitable signal processing methods. The signal processing methods included compensating filters, deconvolution for bandwidth enhancement and resolution improvement, and the cepstrum to separate the overlapping response from different interfaces. The studies dealt only with pulses normal to the interfaces, and the bronze and rubber layer thickness were such that the pulse travel time through the bronze and rubber layers were approximately the same. In the final report for the first year’s contract it was recommended that angled beams be used for such inspection rather than the usual normal incidence, pulse-echo methods.

The second contract had as the main heading on the Work Statement the following: "Development and Analysis of Angled Beam Methods for Multilayered Struc-

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The theoretical model developed under the previous contract was to be modified for angled beam use. It was then to be used to predict optimum transducer placement for use with rubber-epoxy-bronze structures. The model predictions were to be compared with experimentally determined values. The factors to be compared were mode conversion effects, transmission and reflection coefficients as a function of angle, attenuation, multiple path interference, and phase changes and interference due to thin layers. Experimental data from bronze-rubber-and other specimens would be used as needed to check the model.

The final report for the second contract period—the PHASE 2 report—is dated March 1983. It reports on the modification of the theoretical model developed in PHASE 1 to handle angled beams. The model-simulated reflection series is compared to experimental data collected using a bronze-epoxy-aluminum structure for the purpose of checking the model. The angle-beam model was then used to design a procedure to inspect bronze-epoxy-rubber multilayered structures for "flaw" and "no-flaw" conditions, and the procedure was used to collect data in the laboratory under flaw and no-flaw conditions. Various parts of this report will be considered below. The report has a number of errors in it due to typographical mistakes, errors in the scales of some figures, etc. A number of them are listed with their corrections in Appendix A.

In the first section of the PHASE 2 report the computer based model for normal acoustic beams in multilayered systems was modified to be used with off-normal beams. The off-normal beams are complicated by mode conversion, which leads to their being four reflection and four transmission coefficients that must be known for each interface. Tetra Tech has taken this into account, and has developed a procedure to handle this problem. The mathematical procedures used are quite complicated and have not, as of this time, been completely verified. The model allows these inputs for each layer: the layer thickness, density, the longitudinal and shear velocities, the attenuation, and the mode (shear or longitudinal) of propagation in the layer. Different layers may have different modes of propagation. The model generates a complex reflection series as a function of time.

In the second section of the PHASE 2 report, experimental data is obtained and interpreted as validating the following four aspects of the model:

1.) transit time in each layer,
2.) mode of propagation,
3.) amplitude of interface response, and
4.) phase angle of interface response.

The multilayered specimen used consisted of a bronze-epoxy-aluminum specimen with the bronze 6.35 mm thick, the aluminum 25.40 mm thick, and the epoxy bonding them being nominally 1.00 mm thick. Two 5 MHz transducers were used in the pitch-catch mode. The crystal faces were about 6.4 mm by 6.4 mm, and they were mounted on

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plastic wedges to produce a 33° shear wave in the bronze and a 45° shear wave in the aluminum.

The transit time through the bronze determined experimentally was 6.64 µsec, which agreed well with their corrected* theoretical value of 6.42 µsec. The experimental transit time through the aluminum was determined as 23.21 µsec, in good agreement with their theoretical value of 23.23 µsec.

Although the theoretical and experimental values of the transit time quoted above are in good agreement, the theoretical values were calculated assuming the bronze to be isotropic, and were based on a shear velocity of 2.36 mm/µsec provided to Tetra Tech by NSWC. Further laboratory studies at NSWC have shown that the shear velocity in the bronze varies as much as 10% with direction, and it may vary more than this. Also, in laboratory studies at NSWC it was found that at 1 MHz the agreement between the model's prediction of the transit times and those found experimentally for bronze plates were not in agreement. These measurements are described in Appendix B. The lack of agreement at 1 MHz may be the result of the reflected beams being transposed relative to the incident beam due to the phase shift during reflection or to the anisotropy of the bronze, or to a combination of these and/or some other effects. Good agreement at 1 MHz was found, however, between the theoretical and experimental values for a 0.261 inch thick aluminum plate, and also for a 1.0 inch thick brass plate.

The model developed by Tetra Tech predicted a shear-wave propagation mode for the bronze-aluminum sample, and the agreement between the model-predicted transit time and the experimental values for the bronze-aluminum at 5 MHz validated this propagation mode at this frequency, but questions still remain in the 1 MHz frequency range.

The validation of the signal magnitude predicted by the model was done by computing the power spectrum of the transmitted signals and then measuring the power around the center frequency. Peak amplitude measurements could not be used because of phase shifts that occurred upon reflection and transmission. The average decay per round trip in bronze was measured to be -2.9 dB, whereas their theoretical value was -3.5 dB. The equations used by Tetra Tech to calculate the reflection and transmission coefficients are given in Appendix C along with another set from a different source. Tetra Tech gave, via telephone, the calculated reflection coefficient for a shear wave in the bronze at the bronze-epoxy interface at an incident angle of 33° as 0.648 e^{-168.2°}. We were not able to confirm this value using either set of equations in Appendix C.

Tetra Tech’s method used to validate the signal phase given by their model

*See Appendix A

5Krautkramer, J., and Krautkramer, H. Ultrasonic Testing of Materials

   104.
involved experimentally determining the phase spectra of different responses and comparing with the model predictions. The first set of equations in Appendix C were used in the model predictions. The responses considered were for pulses that had made n trips back and forth across the bronze, and identified as the \((n,0,0)\) response, where \(n\) equaled 3, 4, or 5. Pulses were also considered that had made one, two or three trips back and forth through the bronze and one trip back and forth through the epoxy and aluminum, and they are identified as the \((1,1,1)\), \((2,1,1)\), and \((3,1,1)\) response, respectively.

A comparison of their theoretical and experimental phase determinations are given here. The angles indicated are measured relative to the \((3,0,0)\) response.

**TABLE 1. SIGNAL PHASE RELATIVE TO THE \((3,0,0)\) RESPONSE**

<table>
<thead>
<tr>
<th>Response</th>
<th>Phase (degrees)</th>
</tr>
</thead>
<tbody>
<tr>
<td>((4,0,0))</td>
<td>Experiment: -161</td>
</tr>
<tr>
<td>((5,0,0))</td>
<td>Experiment: -322</td>
</tr>
<tr>
<td>((1,1,1))</td>
<td>Experiment: -35</td>
</tr>
<tr>
<td>((2,1,1))</td>
<td>Experiment: +169</td>
</tr>
<tr>
<td>((3,1,1))</td>
<td>Experiment: +8</td>
</tr>
</tbody>
</table>

Tetra Tech’s conclusion was that these results verify their model; however, the fact that the phase angles reported for the bronze-epoxy reflection were not consistent with the calculations made as reported in Appendix C raises some questions. Also, the experimental measurements are discussed in Appendix D, and some questions concerning them are raised there.

Tetra Tech next considered the configuration that would be optimum for locating flaws in the lower epoxy layer of a bronze-epoxy-rubber-epoxy-bronze structure. On the basis of their analysis the conclusion was reached that the optimum configuration for this structure would involve a \(32^\circ\) shear wave in the upper bronze plate and that the responses to be observed were the \((3,1,1,1)\) response for the no-flaw case and the \((3,1,1)\) response for the flaw case. Contributing to the conclusion were the following ideas and observations:

- **a.** The \((3,1,1,1)\) and \((3,1,1)\) responses occur roughly midway between the \((3,0,0)\) and \((4,0,0)\) responses, and therefore interference with other reflections is minimized.

- **b.** Both responses would be large enough to be detected, so that the flaw or no-flaw determination would be made on characteristics of a measured signal rather than the absence of a signal, which absence might be due to some other reason.

- **c.** The phase difference between the two signals was calculated to be \(130^\circ\) apart which should allow for good discrimination between them.
d. Under these conditions only a shear wave would be propagated through the bronze, and although both shear and longitudinal waves would propagate through the rubber and epoxy, the shear wave would be so highly attenuated in the rubber that it could be neglected, and the epoxy layer being so thin the shear waves in it were considered negligible. Under these conditions, there would be only one important wave in each layer, and therefore, the developed model would be applicable.

In considering an operating frequency it was necessary to take into account that resolution is increased with higher frequencies, but so is attenuation, which can be quite high in rubber. Taking these facts into account an operating frequency of 1 MHz was chosen for the operation.

Experimental results were obtained using two 1 MHz transducers on a sample consisting of a 6.35 mm thick bronze plate bonded to a 3.175 mm thick rubber plate. A bond to the bottom of the rubber was simulated by pressing a couplant-covered bronze plate against the rubber plate. The transducers were clamped in place so that their position and coupling were unchanged as the lower bronze plate was put into contact with and removed from the rubber plate. The change in the waveforms with and without the bronze plate were easily detected by the eye, which means that the signals vary depending on whether or not there is bonding at the bottom of the rubber plate, and that an appropriate signal processing system may be developed to detect bonding and disbonding at this position in the sample. It should be noted however, that changing the direction of the transducers resulted in the pulses traveling in different directions in the bronze and rubber, and because of the anisotropy of the bronze this lead to a variation of the received signal, which can complicate the determination of bonding and disbonding.

CONCLUSIONS AND RECOMMENDATIONS

In the development of an ultrasonics procedure to detect and locate faults in multi-layered structures, the PHASE 1 undertaking considered normal-incident longitudinal waves in the pulse-echo mode, and a computer-based model was developed to predict the phases and responses of reflections from such a structure. It was recognized that in some cases the normal-incidence configuration would be complicated by overlapping pulse responses, especially for the case where the transit times through different layers were approximately the same.

The PHASE 2 of the development involved the modification of the computer model developed in PHASE 1 so that it could be used with off-normal, or angled, ultrasonic pulses. For the angled beam the ultrasonic system would operate in the pitch-catch mode. Analysis showed that to determine if a fault existed at a given interface, then the beam angle should be selected so that the transmitting and receiving transducer could be placed so that the following conditions would be met:

1. The reflected signal from the interface would not be received at the same time as other signals.

2. The signals from the interface of interest must be large enough to be detected both if there is a fault present or if there is not a fault present.
3. The signal phase shift upon reflection where there is a flaw should differ from the reflection where there is no flaw by an angle as close to 180° as possible.

The Tetra Tech computer model was designed to be able to determine the incident angles and transducer spacings at which the above needs would best be met. However, because of the disagreements between the model and experimental results as discussed in Appendix B, it cannot be expected that the model would give the proper angles to accomplish the above three objectives. Tetra Tech did show, however, that the difference between a flaw and no-flaw signal—other factors being unchanged—could be detected from the second epoxy layer in a bronze-epoxy-rubber-epoxy-bronze layered structure.

The next step in the program should be a modification of the computer model in order to take into account the anisotropy of materials and the possibility of signal transposition. The model should then be checked experimentally as reported in Appendix B, to see if it determines transit times correctly. This could turn out to be extremely complicated since the property directions in materials to be tested would not usually be known. Although it appears rather unlikely, procedures might be developed in which the existence of anisotropy is relatively unimportant to the end result. Also, if a testing procedure is needed for a given layered structure made of specified materials, a testing procedure might be developed for that particular case taking into account the specific properties of the materials involved.
APPENDIX A

FINAL REPORT TYPING AND CORRECTIONS

Below are listed some typographical errors, errors in diagrams, etc., in the final report A-1 for the second contract, and their corrections.

Page 2
R_k and R'_k are reflection coefficients, not acoustic impedances as indicated down about one-third from the top of the page.

Page 5
The equation \( u = v_0 e^{i \omega x} \)
should be \( u = u_0 e^{(i \omega x)/v} \).

Page 10
In Section 1.3.3, second paragraph, the time locations of the (1,0,0), (2,0,0) and (3,0,0) responses are incorrectly given as 1.25, 2.50, and 3.75 \( \mu \)sec, whereas they should be* 6.42, 12.84, and 19.26 \( \mu \)sec using the corrected transit time of 6.42 \( \mu \)sec on page 16. The first return from the bottom of the aluminum occurs at 29.65 \( \mu \)sec.

Page 11
The time scale of Figure 1.4 should be increased by a factor of five in order for the corrected time locations (Page 10) to be properly indicated in the plot.

Page 12
The time scale on Figure 1.5 should be increased by a factor of five.

Page 16
The equation \( t_{\text{Bronze}} = \frac{2 \times 6.35}{2.2 \cos(33)} = 6.58 \mu \)sec
should read* \( t_{\text{Bronze}} = \frac{2 \times 6.35}{2.36 \times \cos(33)} = 6.42 \mu \)sec.

* Corrected values furnished by Tetra Tech, Inc.

A-1
where the shear velocity has been taken as 2.36 mm/μsec.

The first sentence in the last complete paragraph on this page should read:

The average decay per round trip is 2.9 db which is obtained from \((2.7 + 6.0/2)/2 = 2.9\).

The table of values:

<table>
<thead>
<tr>
<th></th>
<th>(V_p) (mm/μs)</th>
<th>(V_s)</th>
<th>Density (gm/cc)</th>
<th>(Q_p)</th>
<th>(Q_s)</th>
<th>Thickness (mm)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Bronze</td>
<td>4.85</td>
<td>2.20</td>
<td>8.9</td>
<td></td>
<td></td>
<td>3.175</td>
</tr>
<tr>
<td>Epoxy</td>
<td>2.45</td>
<td>--</td>
<td>--</td>
<td>--</td>
<td>--</td>
<td>1.000</td>
</tr>
<tr>
<td>Rubber</td>
<td>2.20</td>
<td>1.05</td>
<td>1.37</td>
<td>10</td>
<td>--</td>
<td>6.35</td>
</tr>
</tbody>
</table>

should read*

<table>
<thead>
<tr>
<th></th>
<th>(V_p) (mm/μs)</th>
<th>(V_s)</th>
<th>Density (gm/cc)</th>
<th>(Q_p)</th>
<th>(Q_s)</th>
<th>Thickness (mm)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Bronze</td>
<td>4.85</td>
<td>2.36</td>
<td>8.9</td>
<td></td>
<td></td>
<td>6.35</td>
</tr>
<tr>
<td>Epoxy</td>
<td>2.42**</td>
<td>--</td>
<td>--</td>
<td>--</td>
<td>--</td>
<td>1.00</td>
</tr>
<tr>
<td>Rubber</td>
<td>2.20</td>
<td>1.05</td>
<td>1.37</td>
<td>10</td>
<td>--</td>
<td>3.175</td>
</tr>
</tbody>
</table>

The time scales in Figures 3.1, 3.2 and 3.3 should be increased by a factor of 2 so that they cover the range from 0 to 50 μsec.

The beam angles* for the epoxy and rubber are also incorrect in the figure, as are some of the indicated velocities. The beam angles may be calculated using Snell's Law.

* Corrected values furnished by Tetra Tech.

** An alternate longitudinal velocity for the epoxy of 2.55 mm/μsec has also been given.
The time scales* in Figures 4.1, 4.2, 4.3, and 4.4 should be increased by a factor of 2 so that they cover the range from 0 to 20 µsec.

The beam angle* in the rubber should be 33° rather than the indicated 28°.

Figures 4.1 and 4.2 are reproduced on pages vii and viii, and the above corrections should be made there also.

* Corrected values furnished by Tetra Tech, Inc.
APPENDIX B

ANGLE BEAM MEASUREMENTS IN BRONZE AT 1 MHz

The transducer-plate configuration is shown in Figure B-1. The transmitter beam passes through the wedge to the interface with the plate at the angle $\theta$, and enters the plate at the angle $\phi$. It passes through the plate reflecting from the top and bottom surfaces. According to the model theory the angles $\theta$ and $\phi$ satisfy Snell's law, and at the reflections within the plate the angle of reflection is equal to the angle of incidence and the reflected wave begins where the incident wave strikes the surface.

In the experimental arrangement the transmitter is in a fixed position and the receiver is moved away from it and the ultrasonic signals it receives are observed on an oscilloscope, and may also be recorded for analysis. The distance between the points at which successive peaks are expected to be received is indicated by $L$ in Figure B-1, with

$$L = 2D \tan \phi \quad (B-1)$$

where $D$ is the plate thickness.

The transit time $t$ through the plate is defined as the time required for the pulse to follow its path from one side of the plate, across to the other side of the plate, and then return to the first side, so that

$$t = \frac{L}{V \sin \phi} \quad (B-2)$$

where $V$ is the pulse velocity.

Tetra Tech, in its validation of transit time, section 2.3.1 of The PHASE 2 report measured, using 5 MHz transducers, the time for successive peaking of the pulses as the receiver was moved away from the transducer and obtained an average transit time of 6.64 $\mu$sec. Substituting Equation (B-1) into Equation (B-2) gives

$$t = \frac{2D}{V \cos \phi} \quad (B-3)$$

For the 6.35 mm thick bronze plate, an angle $\phi$ of 33$^\circ$, and a velocity of 2.36 mm/$\mu$sec, the model equation gives a transit time of 6.42 $\mu$sec.

---

FIGURE B-1. TRANSDUCER-PLATE CONFIGURATION
Experimental values of transit times $t$ and distances $L$ between successive peaks, using 1 MHz transducers, were made by NSWC personnel using a 10 inch by 30 inch bronze plate with a thickness of 6.73 mm. Some measurements were made with wave path along the 10-inch direction (say the A direction) and others with the wave path along the 30 inch direction (say the B direction). The average values obtained for the various experimental runs are given in Table B-1.

Table B-1. EXPERIMENTAL PULSE MEASUREMENTS ON BRONZE PLATES

<table>
<thead>
<tr>
<th>Run</th>
<th>Direction</th>
<th>Average Transit Time $t$ (usec)</th>
<th>Average Distance $L$ (mm)</th>
<th>Velocity ($\text{mm/\mu sec}$)</th>
<th>Angle $\phi$ ($^\circ$)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>A</td>
<td>9.8</td>
<td>24.5</td>
<td>2.83</td>
<td>61.2</td>
</tr>
<tr>
<td>2</td>
<td>A</td>
<td>9.6</td>
<td>24.0</td>
<td>2.85</td>
<td>60.7</td>
</tr>
<tr>
<td>3</td>
<td>B</td>
<td>9.1</td>
<td>19.4</td>
<td>2.59</td>
<td>55.3</td>
</tr>
<tr>
<td>4</td>
<td>B</td>
<td>8.8</td>
<td>19.2</td>
<td>2.68</td>
<td>55.0</td>
</tr>
<tr>
<td>5</td>
<td>B</td>
<td>8.8</td>
<td>18.5</td>
<td>2.59</td>
<td>54.0</td>
</tr>
</tbody>
</table>

The angle $\phi$ given in Table B-1 was obtained using the experimental value of $L$ and Equation (B-1). Using Snell’s law, with the incident angle in the wedge being $39^\circ$ and the wave velocity of 2.73 mm/\mu sec in the wedge, the value of $\phi$ for a velocity of 2.83 mm/\mu sec calculates to be $40.7^\circ$ and for 2.59 mm/\mu sec is $36.7^\circ$. The theoretical transit time using Equation (B-3) with a $\phi$ of $33^\circ$, a velocity of 2.36 mm/\mu sec, and the plate thickness of 6.73 mm is 6.80\mu sec.

From the above it is obvious that the computer model does not give the same transit time as was determined experimentally. There are other discrepancies also. The experimentally determined velocities using the angled beam model do not agree with the values previously determined, and the experimental angles $\phi$ do not agree with the values given by Snell’s law.

It is not known at this time just what causes this disagreement. Further velocity measurements were made, and it was found that shear waves normal to the surface polarized in one direction had a velocity of 2.35 mm/\mu sec and polarized perpendicular to that direction had a velocity of 2.61 mm/\mu sec. There were no pieces of the bronze available for velocity measurements in other directions, and so there may be greater velocity variations than the 2.35 and 2.61 mm/\mu sec noted above. Thus, variations of velocity with direction and polarization may be a contributing factor to the above differences, as may the transposition of the reflected waves as discussed by Krautkrämer and Krautkrämer\textsuperscript{B-2}, and by Brekhovskikh\textsuperscript{B-3}.

Because of the disagreements in the velocity and angle measurements in the bronze, similar measurements were made with a 0.261-inch-thick aluminum plate, and the agreement there was quite good. The velocities measured with the path as in


Figure B-1 gave an average velocity of 3.11 mm/µsec and an average angle $\phi$ of 48.6°. Snell's law gives a value of $\phi$ equal to 47.3°. The shear velocities measured for a path normal to the plate surface and with polarizations at 0°, 45° and 90° relative to the edge of the plate were 3.18, 3.19, and 3.19 mm/µsec. Thus there appears to be negligible dependence of the shear velocity on polarization direction, in contrast to the observations relative to bronze.
APPENDIX C

REFLECTION COEFFICIENTS AT BRONZE-EPoxy INTERFACE

In a monthly report C-1 for March, 1982, Tetra Tech gave the following information relative to the calculation of the reflection and transmission coefficients at a bronze-epoxy interface.

"In general, when a plane wave is incident on the boundary between two solids, reflected and transmitted waves of both types—shear and longitudinal—will be created. In order to calculate their amplitudes, four equations are required. These are provided by the statements of continuity of normal and transverse stress, and of continuity of normal and transverse displacement. Ewing, Jardetsky & Press C-2 (1957) have presented these equations, and their notation will be used here.

"The linearity of the system makes it possible to consider incident longitudinal waves separately from incident shear waves. We wish to find the following coefficients:

\[ R_{pp} = \text{reflection coefficient for P waves (longitudinal waves) without phase change} \]

\[ R_{ps} = \text{reflection coefficients for P waves with phase change to S waves (shear waves)} \]

\[ T_{pp} = \text{transmission coefficient for P waves without phase change} \]

\[ T_{ps} = \text{transmission coefficient for P waves with phase change to S waves} \]

The transmission and reflection coefficients for incident S waves are similarly defined.

Considering incident P waves first, the development of Ewing, Jardetsky, and Press (EJP) can be used to show that the matrix


equation relating the reflection and transmission efficiencies (sic) to the material properties and to the angle of incidence is:

\[
\begin{bmatrix}
-1 & 0 & m_{13} & m_{14} \\
0 & 1 & m_{23} & m_{24} \\
1 & 0 & m_{33} & m_{34} \\
0 & -1 & m_{43} & m_{44}
\end{bmatrix}
\begin{bmatrix}
R_{pp} \\
R_{ps} \\
T_{pp} \\
T_{ps}
\end{bmatrix}
= \begin{bmatrix}
1 \\
0 \\
1 \\
0
\end{bmatrix}
\] (1)

To evaluate the unknown matrix elements \( m_{ij} \), let \( \alpha, \beta, \) and \( \rho \) be respectively, the longitudinal wave velocity, transverse wave velocity, and density. The angle of all wave vectors is measured with respect to the normal to the interface and is denoted by \( \theta \). Then the phase velocity \( c \) is the same in all media for all wave types, and is given by

\[
c = \text{(wave velocity)}/\sin (\theta)
\] (2)

We also define the quantity \( g \) such that

\[
g_{br} = \frac{2}{c^2} \beta_b \left( \beta_b^2 \rho_b - \beta_r^2 \rho_r \right)
\] (3)

with a similar definition for \( g_{rb} \) obtained by interchange of subscripts. The coefficients \( A_i \) and \( B_i \) are defined to be

\[
A_i = \sqrt{\frac{c^2}{\beta_i^2} - 1} \quad \text{and} \quad B_i = \sqrt{\frac{c^2}{\beta_i^2} - 1}
\] (4)

where \( i \) can refer to either \( b \) (for bronze) or \( r \) (for rubber). With this notation the matrix elements can be written in the following form when the wave is incident onto the rubber from the bronze:

\[
\begin{align*}
m_{13} &= g_{br} + \frac{\rho_r}{\rho_b} \\
m_{23} &= \frac{1}{B_b} \left(1 - m_{13}\right) \\
m_{33} &= \frac{A_r}{A_b} \left(1 - g_{br}\right) \\
m_{43} &= -A_r g_{br} \\
m_{14} &= B_r g_{br} \\
m_{24} &= \frac{B_r}{B_b} \left(1 - g_{br}\right) \\
m_{34} &= \frac{1}{-A_b} \left(1 - m_{13}\right) \\
m_{44} &= m_{13}
\end{align*}
\] (5)
Standard matrix methods can be used to solve for the reflection and transmission coefficients. The matrix elements $m_{ij}$ for the case where the incident wave is in rubber can be found from those above by interchanging the subscripts $r$ and $b$ wherever they appear.

"Only a slight modification is required to allow for the case where a shear wave is incident. Denoting the square $4\times4$ matrix on the left side of Equation (1) by $M$, we have in this case

$$M = \begin{bmatrix} R_{sp} & 0 \\ R_{ss} & 1 \\ T_{sp} & 0 \\ T_{ss} & 1 \end{bmatrix} \tag{6}$$

The elements of $M$ are the same as before. They are evaluated in the same way as before, depending on whether the incident wave is in bronze or rubber."

In the PHASE 2 report C-3 the following information and equations are given relative to calculating the reflection coefficients:

"... the potentials $\phi$ and $\psi$ are introduced whose derivatives give rise to compressional and shear waves as follows:

$$\begin{align*}
u &= \partial \phi / \partial x - \partial \psi / \partial z \\
w &= \partial \phi / \partial z + \partial \psi / \partial x
\end{align*} \tag{1.2}$$

where $u$ and $w$ are displacements in $x$ and $z$, respectively. If the form

$$u = u_0 e^{i\omega x / v}$$

is assumed, the derivatives of equation 1.2 are equivalent to multiplication by $i\omega / v$.

The equations for continuity of displacement and stress at the boundary take the form:

---

Here $A$ refers to the amplitude of the compressional potential and $B$ to that of the shear potential. The subscript 1 refers to the upper layer, and 2 to the lower layer. Reflected waves (present in the upper layer only) are denoted by a prime sign ('). The coefficients $M_{ij}$ are functions of the material properties only, and are detailed in [2].

"Only one incident wave at a time need be considered in the upper layer. In the case above, only a shear wave is incident there. Then the incident p wave amplitude $A_1$ is zero, and $B_1$ is the amplitude of the incident wave potential. We divide each of the Equations 1.3 by $B_1$, and make the following identifications:

$$
\frac{A_1}{B_1} = r_{sp} \\
\frac{A_2}{B_1} = t_{sp} \\
\frac{B_1}{B_1} = r_{ss} \\
\frac{B_2}{B_1} = t_{ss}
$$

Then the equations 1.3 can be written in matrix notation as

$$
\begin{bmatrix}
0 \\
1 \\
0 \\
1
\end{bmatrix} =
\begin{bmatrix}
-1 & 0 & M_{13} & M_{14} \\
0 & 1 & M_{23} & M_{24} \\
1 & 0 & M_{33} & M_{34} \\
0 & -1 & M_{43} & M_{44}
\end{bmatrix}
\begin{bmatrix}
r_{sp} \\
r_{ss} \\
t_{sp} \\
t_{ss}
\end{bmatrix}
\text{1.4}
$$

Denoting the square matrix of coefficients on the right by $H$, the solution for the desired reflection and transmission coefficients is
A complex matrix inversion routine is required to evaluate this expression on a digital computer.

"Finally, to get reflection coefficients, the coefficients given above must be multiplied by an appropriate ratio of velocities. Thus, if \( V_s \) is the shear velocity in layer 1 and \( V_p \) the compressional velocity in layer 2,

\[
r_{sp} \quad r_{ss} \quad t_{sp} \quad t_{ss} \quad = H^{-1}\begin{bmatrix} 0 \\ 1 \\ 0 \\ 1 \end{bmatrix}
\]

and so on in accordance with equation 1.2."

According to Tetra Tech, the capitalized transmission coefficients in Equations (1) and (6) should be lower case as they are in Equations 1.4 and 1.5, and the above Equation (7) is then applicable.

In calculating the transmission and reflection coefficients for the bronze-epoxy interface, the following parameters were used:

<table>
<thead>
<tr>
<th>Material</th>
<th>Longitudinal Velocity</th>
<th>Shear Velocity</th>
<th>Density</th>
</tr>
</thead>
<tbody>
<tr>
<td>Bronze</td>
<td>4.85 mm/\mu sec</td>
<td>2.36 mm/\mu sec</td>
<td>8.89 gm/cm^3</td>
</tr>
<tr>
<td>Epoxy</td>
<td>2.42</td>
<td>1.05</td>
<td>2.42</td>
</tr>
</tbody>
</table>

Using these values and a 33° incident angle in the bronze, the reflection coefficient for an incident shear wave in the bronze at the bronze-epoxy interface was calculated to be \( 0.462e^{160.40^\circ} \), which does not agree with the Tetra Tech value of \( 0.648e^{168.5^\circ} \).

It should be noted that the quantities \( r_{sp}, r_{ss}, \) etc. are reflection coefficients for the displacement potentials, and the corresponding quantities \( R_{sp}, R_{sp}, \) etc., are reflection coefficients for the pressure.
Because of the poor agreement between the above calculated values, the following equations were used to calculate the transmission and reflection coefficients for displacement waves for the case of a shear wave incident in Medium 1 to an interface with Medium 2:

\[
\begin{bmatrix}
Z \sin 2\theta_t & -\sin 2\theta & Zf_2 \\
Z \cos 2\theta_t & \cos 2\theta & 2Z \sin \theta_t \cos \theta_{\ell t} \\
-\cos \theta_t & \cos \theta & -\sin \theta_{\ell t} \\
\sin \theta_t & \sin \theta & -\cos \theta_{\ell t}
\end{bmatrix}
\begin{bmatrix}
f_1 \\
r \\
r \\
T
\end{bmatrix}
= 
\begin{bmatrix}
\sin 2\theta \\
\cos 2\theta \\
\cos \theta \\
\sin \theta
\end{bmatrix}
\begin{bmatrix}
t \\
r \\
r \\
T
\end{bmatrix}
\]

where

\[
t = \frac{S_t}{S_1} \\
r = \frac{S_r}{S_1} \\
T = \frac{L_t}{S_1} \\
R = \frac{L_r}{S_1}
\]

\(S_t\) is the amplitude of the incident shear wave,
\(S_r\) is the amplitude of the reflected shear wave,
\(S_t\) is the amplitude of the transmitted shear wave,
\(L_t\) is the amplitude of the transmitted longitudinal wave, and
\(L_r\) is the amplitude of the reflected longitudinal wave.

Also:

\[
f_1 = \frac{v_{p1}}{v_1} \left[ 1 + 2 \left( \sin^2 \theta - \frac{1}{v_{p1}^2} \right) \right]
\]

\[
f_2 = \frac{v_{p2}}{v_2} \left[ 1 + 2 \left( \frac{\sin^2 \theta}{v_1^2} - \frac{v_{p2}^2}{v_2^2} \right) \right]
\]

\[
Z = \frac{\rho_{2}v_2}{\rho_{1}v_1}
\]

---


C-6
where in medium 1

\[ V_1 \] is the shear wave velocity,
\[ V_{p1} \] is the longitudinal wave velocity,
\[ \rho_1 \] is the medium 1 density,
\[ \theta \] is the shear wave incident and reflected angle,
\[ \theta_{lr} \] is the angle of the reflected longitudinal wave,

and in medium 2

\[ V_2 \] is the shear wave velocity,
\[ V_{p2} \] is the longitudinal wave velocity
\[ \rho_2 \] is the medium 2 density,
\[ \theta_t \] is the transmitted shear wave angle, and
\[ \theta_{lt} \] is the transmitted longitudinal wave angle.

For the case of the incident shear wave at 33° with the normal, as before, the reflection coefficient \( r \) was calculated to be \( 0.916e^{-j54.6°} \), which does not agree with either of the previously obtained values.
APPENDIX D

EXPERIMENTAL SIGNAL PHASE MEASUREMENTS

The signal phase measurements referred to in the PHASE 2 report\textsuperscript{D-1} involves the phase shift that the pulse signal undergoes due to the two reflections it makes in a trip back and forth across the plate. In Figure D-1, for example, the signal phase shift of the (4,0,0) response relative to the (3,0,0) response would be the phase shift due to the reflection at the top of the plate added to that due to the reflection at the bottom of the plate; the phase shift due to the path distance is not included.

Let the distance that the pulse travels in getting to where the (3,0,0) response is received be \( x_3 \), and to the (4,0,0) response is received be \( x_4 \). Consider (3,0,0) and (4,0,0) responses as shown in Figure D-2 on a common time scale \( t \). In order to determine the (3,0,0) signal phase, the signal is "windowed", with \( t_3 \) as the time from the beginning of the window. Similar windowing is done for the (4,0,0) response, as well as for the other responses. The windowed signals were about 1 \( \mu \)sec duration, and zeros were added to bring the period up to 10 \( \mu \)sec. The phase spectra were then computed using an FFT algorithm.

In order to get the phase difference between two responses, the Tetra Tech procedure was to window the responses by observing on an oscilloscope screen so that the signals began at the beginning of the window. The Fourier transforms were then calculated, and the phase differences obtained from the transforms. To understand under what conditions this will give the phase difference consider the following analysis.

The Fourier series for the propagating pulse may be written as:

\[
F(t) = \frac{A_0}{2} + \sum_{n=1}^{\infty} C_n \cos[n\omega(t - \frac{x}{v_n}) + \phi_n] \quad (D-1)
\]

where

\( \omega \) is the fundamental angular frequency, 6.28x10\(^5\) rad/sec,

\( n \) is the harmonic number, and

\( \phi_n \) is the phase angle for the \( n \)th harmonic at \( x \) equal zero.

\textsuperscript{D-1} Tetra Tech, Inc., ACOUSTIC NDE OF MULTILAYERED COMPOSITES - PHASE 2: ANGLE BEAM VALIDATION OF BRONZE-RUBBER STRUCTURES, Tetra Tech TC-6139, Mar 1983.
FIGURE D-1. TRANSDUCER-RECEIVER POSITIONS FOR SIGNAL PHASE MEASUREMENTS
Figure D-2. Windowing of (3,0,0) and (4,0,0) responses.
Let the window for the \((3,0,0)\) response begin at \(T_3\). The signal at \(x_3\) is then, as a function of \(t\)

\[
F(t) = \frac{A_0}{2} + \sum_{n=1}^{\infty} C_n \cos[n\omega(t - \frac{x_3}{v_n}) + \phi_{3n}]
\]  
(D-2)

and as a function of \(t_3\), where \(t_3 = t - T_3\),

\[
F_3(t_3) = F(t_3 + \tau_3) = \frac{A_0}{2} + \sum_{n=1}^{\infty} C_n \cos[n\omega(t_4 + \tau_3 - \frac{x_3}{v_n}) - \tau_{3n}]
\]  
(D-3)

Where \(\phi_{3n}\) equals \(\phi_n\) plus the phase shifts that occur upon the surface reflections. The phase angle as a function of \(n\) at \(x = x_3\) is

\[
\psi_{3n} = n\omega(\tau_3 - \frac{x_3}{v_n}) + \phi_{3n}
\]  
(D-4)

and this is the phase angle at \(x_3\) that is given by the FFT algorithm. By a similar procedure, the phase angle given by the algorithm at \(x_4\) is

\[
\psi_{4n} = n\omega(\tau_4 - \frac{x_4}{v_n}) + \phi_{4n}
\]  
(D-5)

The angle \(\phi\) is equal to \(\phi_{2n}\) plus the signal phase shift \(\Delta\phi\) due to the reflections at the surfaces in going from \(x_3\) to \(x_4\), thus

\[
\psi_{4n} = n\omega(\tau_4 - \frac{x_4}{v_n}) + \phi_{3n} + \Delta\phi
\]  
(D-6)

and

\[
\Delta\psi_n = \psi_{4n} - \psi_{3n} = n\omega(\tau_4 - \tau_3 - \frac{x_4 - x_3}{v_n}) + \Delta\phi
\]  
(D-7)

Theoretically, \(\Delta\phi\) is independent of frequency, and for that reason it has not been given a subscript \(n\). A basic assumption made by Tetra Tech was that by visual observation they could window the response so that \(\Delta\psi_n = \Delta\phi\). For this to be the case it is necessary that

\[
\tau_4 - \tau_3 - \frac{x_4 - x_3}{v_n} = 0
\]  
(D-8)

Since \(\tau_3, \tau_4, x_3,\) and \(x_4\) are constants, this means that \(v_n\) must not vary with
frequency. Letting
\[ \Delta \tau = \tau_4 - \tau_3 \]  
and  
\[ \Delta x = x_4 - x_3, \]  
Equation (D-8) becomes
\[ \Delta \tau = \frac{\Delta x}{v}, \]  
where \( v \) is the wave velocity in the medium, and \( \Delta \tau \) is the time between the beginning of the two windows. If \( \Delta \tau \) does not satisfy Equation (D-11), and it may be difficult to begin the two windows so that it does, then Equation (D-8) is not satisfied, and setting
\[ \tau_4 - \tau_3 - \frac{x_4 - x_3}{v} = C \]  
then equation (D-7) becomes
\[ \Delta \psi_n = Cn + \Delta \phi, \]  
in which case the measured signal phase shift is a linear function of frequency.

Tetra Tech has assumed that they could select \( \tau_4 \) and \( \tau_3 \) such that \( C = 0 \), but their data from the PHASE 2 report \( D-1 \) which is shown here appears to have a linear variation with frequency. In all the three cases of TABLES D-1 and D-2, the phase difference is due to one transit path across and back through the bronze.

Using linear regression to determine the curves, the linear equations for the three cases become:

for (3,0,0) and (4,0,0) responses,
\[ \Delta \psi_n = 8.5 f - 190.8^\circ \]  
(D-14)

for (4,0,0) and (5,0,0) responses,
\[ \Delta \psi_n = 12 f - 230^\circ \]  
(D-15)
for \((1,1,1)\) and \((2,1,1)\)

\[
\Delta \psi_n = 18 f - 241.2^\circ, \quad (D-16)
\]

where \(\Delta \psi\) is measured in degrees and \(f\) is in megahertz. The coefficient of correlation is 0.986 or better for each of the equations. The units of the coefficients of \(f\) are degrees per megahertz, and to convert the units to seconds as is needed for use in Equation (D-13) they must be multiplied by \(2.78 \times 10^{-9}\) sec MHz per degree.

**TABLE D-1. PHASE DIFFERENCES FOR \((3,0,0)\) - \((4,0,0)\) AND \((4,0,0)\) - \((5,0,0)\) RESPONSES**

<table>
<thead>
<tr>
<th>Phase Difference (Degrees)</th>
<th>Frequency (MHz)</th>
<th>((3,0,0))</th>
<th>((4,0,0))</th>
<th>((4,0,0))</th>
<th>((5,0,0))</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>4.8</td>
<td>-150</td>
<td>-172</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>5.0</td>
<td>-148</td>
<td>-170</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>5.2</td>
<td>-147</td>
<td>-168</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>5.4</td>
<td>-145</td>
<td>-166</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>5.6</td>
<td>-143</td>
<td>-162</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Average Difference</td>
<td></td>
<td>-147\pm3</td>
<td>-168\pm4</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

**TABLE D-2. PHASE DIFFERENCES FOR \((1,1,1)\) - \((2,1,1)\) RESPONSES**

<table>
<thead>
<tr>
<th>Phase Difference</th>
<th>Frequency</th>
<th>((1,1,1)) and ((2,1,1))</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>3.6</td>
<td>-177</td>
</tr>
<tr>
<td></td>
<td>3.8</td>
<td>-172</td>
</tr>
<tr>
<td></td>
<td>4.0</td>
<td>-169</td>
</tr>
<tr>
<td></td>
<td>4.2</td>
<td>-166</td>
</tr>
<tr>
<td></td>
<td>4.4</td>
<td>-162</td>
</tr>
<tr>
<td>Average Difference</td>
<td></td>
<td>-169 \pm 6</td>
</tr>
</tbody>
</table>
The above equations then become

\[
\Delta \psi_n = 2.36 \times 10^{-8} \tau - 190.8^\circ \quad \text{(D-17)}
\]

\[
\Delta \psi_n = 3.33 \times 10^{-8} \tau - 230^\circ \quad \text{(D-18)}
\]

\[
\Delta \psi_n = 5.00 \times 10^{-8} \tau - 241.2^\circ \quad \text{(D-19)}
\]

A comparison of these equations with Equation (D-13) shows that the constant term of the equations is just \( \Delta \phi \) of Equation (D-13), which is just the signal phase shift in going from the first pulse under consideration to the second one. They may be compared with Tetra Tech's experimental and theoretical values shown in Table 1. There is disagreement in both cases. The disagreement with the experimental value results from the assumption of an error in windowing, and the disagreement with the theoretical value may be due to the reflection calculations considered in Appendix C.

Comparison of the equations with equation (D-13) also shows that \( C \) is equal to the multiplier of \( \tau \) in the equations. From Equations (D-8) and (D-12) it is seen that a non-zero \( C \) is due to, and equal to, errors in the values of \( \Delta \tau \), which results from errors in windowing; that is, in errors in the beginning times of the windows. From the Equations (D-17), (D-18), and (D-19) it is seen that windowing errors of the responses considered vary from 0.0236 to 0.0500 microseconds. This implies an error in the timing of the beginning of the window that is 2% to 5% of the window duration. In looking at the windowed pulses on pages 17 and 18 of the PHASE 2 report, it appears that such errors could easily be made.
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* (1)