ELEMENTS OF AN ICEBERG DETERIORATION MODEL

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Avery Point Groton, Connecticut 06340

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Final Report
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[Signature]

[Name]
Technical Director
U.S. Coast Guard Research and Development Center
Avery Point, Groton, Connecticut 06340
A model of iceberg deterioration using loss of mass has been developed which is based upon gross iceberg characteristics and environmental factors. The significant iceberg characteristics have been determined to be waterline length, height above water, and class (drydock, pinnacle, tabular, or domed). The environmental factors used are water temperature, wind speed, and wave height and period. The reduction of mass has been assumed to be the result of forced convective melting of the underwater portion, wave erosion at the water line, and calving of ice undercut by the waves. This model is designed to be in a form simple enough so that the deterioration can be easily computed.
**METRIC CONVERSION FACTORS**

**Approximate Conversions to Metric Measures**

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°F = °C * 9/5 (after subtracting temperature) + 32
°C = (°F - 32) * 5/9

#1 in = 2.54 (exactly). For other exact conversions and more detailed tables, see USAS Model. Publ. 256, Units of Weights and Measures. Price $2.50. SD Catalog No. C13.10.288.
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1.0 INTRODUCTION

A model of iceberg deterioration must attempt to use the best available formulations of ice melt in sea water while allowing for a high degree of uncertainty in nearly all other parameters. This uncertainty starts with the size and shape of the iceberg itself. The iceberg's size and shape are reported by an observer in an aircraft flying at various altitudes, under varying conditions of visibility, and at widely differing ranges to the iceberg. Only one side of the iceberg is typically seen. Length and height estimates are approximate. Nothing of the underwater shape or form is known to the casual observer, although a number of sonar studies have been conducted of the underwater portion (Robe, 1975, and Russell et al., 1977). These reports detail the underwater characteristics of a small sample of icebergs and from this sample idealized icebergs can be constructed.

To this uncertainty of the shape and size must be added the uncertainty of the determination of environmental forces which account for the deterioration. In this model we plan to use only three environmental factors; sea surface temperature, wave height and period, and wind speed. The almost total lack of reporting weather stations in the Labrador Sea and on the Grand Banks results in the data on these environmental parameters being extracted from large-scale computer simulations which are accurate in the large-scale, but lack the detail often needed for deterioration computations.

The model developed here does not attempt to be more rigorous than the data justifies or that requirements demand. It draws freely on rules of thumb, generalities, approximations, and experience; without apology. From the uncertain data input for the iceberg characteristics and environmental parameters a judgement must be made as to what the initial mass of the iceberg should be at the initial point of the deterioration model.

The approach in this report is more pragmatic than that used by Kollmeyer (1966) although the use of turbulent heat transfer is similar. Kollmeyer constructed a heat budget for the iceberg which balanced the heat need for melting with heat sources.
2.0 ICEBERG CLASSES

Icebergs have roughly been separated into types or classes based on form. These classes as presented by Murray (1968) are given in Table 1.

Table I
Iceberg Shape Classes

(a) BLOCKY - Steep precipitous sides with horizontal or flat top. Very solid berg. Length-height ratio less than 5:1
(b) DRYDOCK - Eroded such that a large U-shaped slot is formed, with twin columns or pinnacles. Slot extends into the waterline or close to it.
(c) DOME - Large smooth rounded top. Solid type berg.
(d) PINNACLED - Large central spire or pyramid of one or more spires dominating shape. Less massive than dome-shaped berg of similar dimensions.
(e) TABULAR - Horizontal or flat-topped berg with length-height ratio greater than 5:1.
(f) BERGY BIT - A mass of glacial ice smaller than a berg but larger than a growler, about the size of a small cottage. Small berg or large growler is preferred usage.
(g) GROWLER - A mass of glacial ice that has calved from a berg or is the remains of a berg. A growler has a height of less than three meters and a length of less than six meters.

These categories are arbitrary since in reality the shape classes strongly overlap each other (Farmer and Robe, 1977). They do however describe certain gross characteristics of icebergs which relate mainly to stability. In this report we limit our consideration to four types which combine the characteristics of the above listed classes. The four types are defined using the classes in Table 1: (1) the tabular, which combines the tabular and blocky classes; (2) the domed, which combines the domed and the growler; (3) the pinnacle; and (4) the drydock which are the same as in Table I.
3.0 DETERIORATION OF UNDERWATER AREA FOR ICEBERG CLASSES

In order to melt the underwater surface of an iceberg by turbulent heat transfer one needs to know not only the rate at which heat passes across the ice/water interface, but also the surface area across which the heat is transferred. Since an observer sees only the shape, height, and width of the above water portion, the task of deriving the underwater surface area can neither be straightforward nor exact. Since we are trying to derive surface area I have chosen to relate the underwater surface area to the square of the mean waterline length so that the result is a dimensionless ratio.

The iceberg types will be treated as stereotypical constructs from which deterioration factors can be extracted using above water dimensions alone. The deterioration factors which are of importance are waterline length exposed to wave erosion and the underwater surface area which is exposed to forced convective melting. The waterline length is obtained directly, but the underwater surface area is derived from the length, width, and height of the above water portion of the iceberg and depends on the assumptions which are defined in the following discussion of the stereotypical iceberg types.

a. Tabular Iceberg Type

In Figure 1 a perfectly regular rectangular iceberg is portrayed with a, b, and c being the width, length, and thickness, respectively. The above water height is represented by h which is equal to 1/8 c. For a blocky iceberg we let a = b.
The average waterline length $L_w$ squared is given by

$$L_w^2 = .5a^2 + .25b^2 + .5\sqrt{a^2 + b^2}$$ (2)

Dividing equation (1) by equation (2) we arrive at a ratio of underwater surface area to the square of the waterline length of

$$\text{Ratio} = \frac{ab + 1.75ac + 1.75bc}{.5a^2 + .25b^2 + .5\sqrt{a^2 + b^2}}$$ (3)

Let $b = Na$, then (3) becomes

$$\text{Ratio} = \frac{4N}{2 + N^2 - 2\sqrt{1 + N^2}} + \frac{7(N+1)}{2 + N^2 + 2\sqrt{1 + N^2}} \left(\frac{c}{a}\right)$$ (4)
The ratio of thickness to width \( C/a \) normally falls in the range

\[ 0.4 < \frac{C}{a} < 1 \] (Robe, 1975)

Choosing a middle value of

\[ \frac{C}{a} = 0.7 \]

(4) becomes

\[ \text{Ratio: } \frac{4N}{2 + N^2 + 2 \sqrt{1 + N^2}} + \frac{4.9(N+1)}{2 + N^2 + 2 \sqrt{1 + N^2}} \] (5)

Therefore the underwater surface area can be recovered by knowing the ratio \( N \) of the longest and shortest side of the rectangular iceberg and by multiplying equation (5) by the square of the observed waterline length. This result will be used later in the development of forced convective melting.

b. Pinnacle Iceberg Type

We now follow the treatment of the tabular iceberg with the more complicated double pyramid shape in Figure 2. In Figure 2 refers to the horizontal dimension at the largest iceberg horizontal section, \( L' \) is the waterline length, \( h \) is the vertical half thickness of the iceberg, while \( h' \) is the above water height. We first need to know where the waterline lies on this shape. What is the value of \( h' \)? Starting with the fact that 1/8th of the volume is above water we have the total volume \( V \)

\[ V = \frac{2}{3} L'^2 h \] (6)

The subaerial volume \( V_a \) is

\[ V_a = \frac{1}{3} n L'^2 nh \] (7)

where \( L' = nL \) and \( h' = nh \)
FIGURE 2. An idealized representation of an iceberg of the pinnacle type. The base of the double pyramid is square with side length $l'$. The altitude of each of the double pyramids is $h$. The above water dimensions are height $h'$ and waterline length $l'$.

The above water volume based on $\frac{1}{8}$th of the total volume from (6) is

$$V_a = \frac{1}{12} l'^2 h$$

Equating (7) and (8)

$$\frac{1}{3} n^3 = \frac{1}{12}$$

or

$$n = 0.63$$
The total surface area of the double pyramid is given by

\[ A = 4l \sqrt{\frac{l^2}{4} + h^2} \]  

and the submerged area is given by

\[ A_s = 3.2l \sqrt{\frac{l^2}{4} + h^2} \]  

The average waterline length is

\[ L_w = \frac{\sqrt{2}l' + l'}{2} = 1.21l' \quad \text{or} \quad .76l \]

The squared average waterline length is then

\[ L_w^2 = .58l^2 \]  

The ratio of the submerged surface area to the square of the average waterline length is given by

\[ \text{Ratio} = \frac{5.52 \sqrt{\frac{l^2}{4} + h^2}}{l} \]  

Converting to \( l' \) and \( h' \), (12) becomes

\[ \text{Ratio} = 5.52 \sqrt{.25 + \left(\frac{h'}{l'}\right)^2} \]

A reasonable value of \( \frac{h'}{l'} \) is 0.3, Budd (1977). If a value of \( \frac{h'}{l'} = 0.3 \) is chosen the ratio of the submerged area to the square of the waterline length becomes

\[ \text{Ratio} = 3.2 \]  

The submerged surface area is recovered by multiplying the ratio by the square of the observed waterline length.
c. **Drydock Iceberg Class**

The drydock iceberg is the most stable of the deteriorated iceberg forms. Time is required to cut the notch through the center portion of the iceberg and during this time the iceberg must not rotate. The drydock berg is idealized in Figure 3 where the width is $a$, the thickness $c$, and the above water height $b$.

**FIGURE 3.** An idealized representation of an iceberg of the drydock type. The iceberg is assumed square with side $a$ and total thickness $c$. The slabs which form the drydock sides and the drydock cut are each taken as one third $a$. The height of the above water portion is represented by $b$. 
A berg which is drydocked will be assumed to have a length nearly equal to its width. Following the arguments in the preceding sections we have from Figure 3 that, if \( b \) is the depth of the drydock cut, the above water volume is

\[ V_a = \frac{2}{3} a^2 b \quad (14) \]

and the below water volume is

\[ V_s = a^2 (c - b) \quad (15) \]

If, as is required, the above water ice volume is one eighth of the total volume then using equations (14) and (15)

\[ \frac{2}{3} a^2 b^2 = \frac{1}{7} a^2 (c - b) \quad (16) \]

or

\[ 5.66 b = c \quad (17) \]

The below water surface area then becomes

\[ A_s = a^2 + 4a(c - b) \quad (18) \]

and substituting from (17)

\[ A_s = a^2 + 4a(4.66b) \quad (19) \]

The average waterline length for this berg is

\[ L_w = \frac{a + \sqrt{2a}}{2} \quad \text{or} \quad 1.21a \quad (20) \]

The average waterline length squared becomes

\[ L_w^2 = 1.46a^2 \]
The ratio of the underwater surface area to the square of the average waterline length is given by

\[ \text{Ratio} = .68 + 12.79 \frac{b}{a} \]  \hspace{1cm} (21)

The value of \( \frac{b}{a} \) can easily be obtained from observation.

If, for example \( \frac{b}{a} = .5 \) then

\[ \text{Ratio} = 7.08 \]  \hspace{1cm} (22)

The underwater surface area subject to forced convection can then be found by multiplying this ratio in (22) by the square of the observed waterline length.

d. Domed Iceberg Class

This is the class of icebergs which is more or less spherical. They are so because they are in an advanced state of deterioration and roll over at the slightest disturbance. The domed class includes the smaller bergs and growlers as well as an occasional large iceberg which happens to have gone spherical early. To represent the domed iceberg a sphere was chosen, Figure 4, and the computations were made using spherical coordinates. The radius is represented by \( \rho \), the vertical rotation by \( \phi \) and the horizontal rotation by \( \theta \). The sphere's center lies a distance \( d \) below the water surface. From Figure 4 we can compute the above water surface area from

\[ A_a = 4 \int_0^{\frac{\pi}{2}} \int_0^{\frac{\pi}{2}} r^2 \sin \phi \, d\phi \, d\theta \]

\[ A_a = 2\pi \rho^2 (1 - d/\rho) \]  \hspace{1cm} (23)
FIGURE 4. An idealized representation of an iceberg of the domed type. The iceberg is assumed spherical with the sphere having a radius. The waterline is a distance $d$ above the sphere center. Spherical coordinates are used for calculations.

Subtracting (23) from the total surface area of a sphere one obtains the below water surface area.

$$A_s = 4\pi \rho^2 - 2\pi \rho^2(1-d/\rho)$$  \hspace{1cm} (24)

Since one eighth of the total volume is above water it is possible to compute $d$ or the distance from the sphere center to the waterline (Figure 5).

The shaded portion of the circle is proportional to the volume of that portion of a sphere represented by the shaded area. The area is given by

$$A_d = 2\int x \, dy$$  \hspace{1cm} (25)

where  \hspace{1cm} $x = \sqrt{1+(y+d)^2}$
so that (25) becomes

$$A_\alpha = 2 \int_0^{1-d} \sqrt{1-(y+d)^2} \, dy$$

(26)

let $u = y+d$ and $du = dy$

substitute into (26)

$$A_\alpha = 2 \int_0^{1-d} \sqrt{1-u^2} \, du$$

(27)

integrate and substitute back to get

$$A_\alpha = 2 \left[ \frac{\pi}{4} - \frac{d}{2} \sqrt{1-d^2} - \frac{1}{2} \sin(d) \right]$$

(28)

Equation of a unit circle

$$(y+1)^2 + x^2 = 1$$

FIGURE 5. The above water fraction of a domed iceberg is calculated by integrating the volume elements $x \, dy$. 

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The area of a circle of unit radius is equal to \( \pi \). The 1/8th portion would then be \( \pi/8 \), and from (28)

\[
\pi/8 = \frac{4\pi}{8} - d\sqrt{1 - d^2} - \text{Asin}(d)
\]

or \( d = 0.6347 \) (30)

for a unit sphere.

Waterline length as a function of the radius is then

\[
\rho^2 - (0.6347\rho)^2 = (L_w/2)^2
\]

or

\[
L_w = 1.54\rho
\]

(32)

The waterline length squared becomes

\[
L_w^2 = 2.39\rho^2
\]

(33)

To compute the ratio of the submerged surface area to the square of the waterline length we use (24) and (33).

\[
\text{Ratio} = \left[4\pi\rho^2 - 2\pi\rho^2(1 - d/\rho)\right]/2.39
\]

or using (30)

\[
\text{Ratio} = 4.29
\]

(35)

To recover the underwater surface area multiply (35) by the square of the observed waterline length.
4.0 ICEBERG MASS ESTIMATE

The total mass of an iceberg is difficult to obtain except by using stereo photography and a great deal of computational work. An estimate of the mass is necessary, however, as an initial point for a deterioration scheme. Farmer and Robe (1977) were able to develop an extremely simple method for estimating the mass of icebergs of random shape and size. Their method, which is based on a number of icebergs measured carefully using stereo aerial photography, depends only on the length, width, and height of an iceberg. Their relationship gives the iceberg mass \( M \) in metric tons \( (10^3 \text{kg}) \) as:

\[
M = 3.01 \times \text{Length} \times \text{Width} \times \text{Height}
\]  

with the units of length, width and height all in meters.

5.0 ENVIRONMENTAL FACTORS

Only three environmental factors are considered in this model: (1) wind, which accounts for a portion of the berg motion relative to the water; (2) sea surface temperature, which will provide the thermal driving force for melting; and (3) wave height and period, which creates a very turbulent near surface zone eroding the ice rapidly near the water line.

a. Wind

Wind forces applied to the above water portion of the iceberg will cause the iceberg to move relative to the water thereby inducing forced convective melting of the iceberg. Icebergs are generally in motion relative to the water at any particular depth, even in a zero wind situation, because of the change of current velocity with depth and the fact that the iceberg moves in response to the integrated force applied to it by water at all depths. A good value for this background relative motion \( \Delta V \) between iceberg and water is 0.05 m/s. The relative motion caused by the wind was given by White et al. (1980) as:
where $B$ is shape factor for different icebergs and $W$ is the steady wind speed. The shape factor was estimated as follows:

- Pinnacle berg: $B = 44 \pm 5$
- Dome or drydock: $B = 66 \pm 8$
- Tabular berg: $B = 75 \pm 9$

If the mid value of $B = 65$ is chosen for equation (37) and the 0.05 m/s background relative motion is added in (37) becomes:

$$\Delta V = \frac{W + 3.30}{66}$$

where $\Delta V$ and $W$ are in m/s.

b. Sea Surface Temperature

Sea water temperatures in the region around the Grand Banks of Newfoundland are highly variable. The water temperature can vary many degrees in short distances and over a few meters depth. The sea surface temperature can either be higher or lower than the surface layer which surrounds the iceberg. However, lacking the ability to sample water profiles in this area, sea surface temperature remains the best available indicator of the ability of the water to melt an iceberg. The difference between the water temperature and the melting temperature of ice in 30°/oo seawater (approximately -1.63 °C) provides the thermal driving force for melting the iceberg ice.

c. Wave Height and Period

The erosion of the waterline by wave action is one of the most forceful mechanisms of iceberg deterioration. The deterioration takes place directly by wasting and indirectly by calving of ice from the undercut sides of the iceberg. Both the wave height and wave period, together with the water temperature, play a large role in developing the wave cut notch. Short period waves tend to deepen the wave cut notch faster than waves of longer periods.
6.0 DETERIORATION RATES

The turbulent heat transfer between the seawater and the iceberg will now be calculated. The transfer of heat is dependent on the surface area exposed, the seawater temperature and the relative ice/water speed. The loss of mass through wave undercutting and calving will also be considered.

a. Side wall melting

Modifying the development in White, et al. (1980), the rate of melting \( V_m \) in \( \text{cm} \, s^{-1} \, \circ C^{-1} \) is:

\[
V_m = \frac{g_w \rho_f \Gamma}{K} \tag{39}
\]

where \( \rho_f \) is the density of ice (0.9 gm cm\(^{-3}\)), \( \Gamma \) is the latent heat of fusion of water (344 J gm\(^{-1}\)), and \( g_w \) is the heat transfer rate per unit area per degree C in J cm\(^{-2}\) s\(^{-1}\) \( \circ C^{-1} \).

The local Nusselt number which relates the local heat transfer rate to the thermal conductivity is given by:

\[
\text{Nu}_{x} = \frac{g_w x}{K \Delta T} \tag{40}
\]

where \( x \) is the downstream dimension in (cm), \( g_w \) is given above, \( K \) is the kinematic thermal conductivity Js\(^{-1}\) cm\(^{-1}\) \( \circ C^{-1} \) (we will use \( K = 0.006 \text{ Js}^{-1} \text{cm}^{-1} \circ C^{-1} \) which is the value for fresh water at 17.5 \( ^\circ C \) and varied only slightly with the temperature and salinity), and \( \Delta T \) is the thermal driving force or water temperature, \( T_w \), minus the ice melting temperature, \( T_f \).

Presenting the Nusselt number in a different form where it is based on the waterline length.

\[
\text{Nu}_{x} = 0.056 \text{Re}_{x}^{0.8} \text{Pr}^{0.4} \tag{41}
\]
Where $L_0$ is the waterline length in (cm), $\text{Re}_{L_0}$ is the waterline Reynolds number ($\text{Re} = \frac{\text{AV}L_0}{\nu_w}$), $\Delta V$ is the relative iceberg/water velocity (cm s$^{-1}$), $\nu_w$ is the kinematic viscosity of sea water (cm$^2$ s$^{-1}$) which for water of 30 $^\circ$/oo and density ($\rho_w = 1.0275$ gm cm$^{-3}$) is represented by

$$\nu_w = 0.183 - 6.18 \times 10^{-4} T + 1.51 \times 10^{-5} T^2 - 2.0 \times 10^{-7} T^3$$  \hspace{1cm} (42)

with only a slight salinity dependence, and $Pr$ is the Prandtl number ($Pr = \nu/\kappa$) where $\kappa$ is the thermal diffusivity. The Prandtl number can be represented by

$$Pr = 1.13 - 0.431T - 0.000893T^2 + 0.00521T^3$$  \hspace{1cm} (43)

Combining (40) and (41) and solving for $\nu_w$ we obtain:

$$\nu_w = 0.056 \frac{K AT}{L_0}$$  \hspace{1cm} (44)

substituting in (39)

$$\nu_m = \frac{0.056}{\rho_e T} \left( \frac{K AT}{L_0} \right) \left( \frac{\Delta V L_0}{\nu_w} \right)^{0.8} Pr^{0.4}$$

or

$$\nu_m = \left( \frac{0.006 \times 0.056}{0.9 \times 344} \right) \left( \frac{K AT}{L_0} \right) \left( \frac{\Delta V L_0}{\nu_w} \right)^{0.8} Pr^{0.4}$$  \hspace{1cm} (45)

if the freezing point for ice in 30$^\circ$/oo seawater is equal to $-1.63^\circ$C then (converting centimeters to meters)

$$\nu_m = 1.09 \times 10^{-8} \left( \frac{I_w + 1.63}{L_0^{0.2}} \right) \left( \frac{\Delta V}{\nu_w} \right) Pr^{0.4}$$  \hspace{1cm} (46)

in ms$^{-1}$ $^\circ$C$^{-1}$.

When computing $\Delta V$ use the wind-driven relative velocity plus 0.05 m/s which accounts for the relative velocity of the iceberg and water in a zero wind situation.
b. **Calving**

The direct melting by wave turbulent heat transfer at the waterline will not be treated separately, but will be lumped in with the problem of calving as a result of wave undercutting. Calving occurs when an undercut slab of ice on the sidewall of an iceberg breaks off under its own weight. The frequency with which this happens depends upon wave height $H$, wave period $t$, and the thermal driving force. The amount of ice that is lost depends upon the shape of the iceberg, the length of the face exposed to waves and to the height of the calving face. An assumption is made that the roughness of the iceberg is nearly 1% of the wave height, i.e. 2 cm surface roughness for waves in the 1 to 3 meter range. From White et al. (1980) the speed with which the water line is cut is then

$$V_w = 0.000058 H/t$$

with $V_w$ being the erosion velocity in ms$^{-1}$, $H$ the wave height in meters, and $t$ the wave period in seconds.

Calving which results from this undercutting is assumed to take place along a front which is one half of the observed waterline length. This factor is chosen because the wave activity tends to concentrate at a few places along the waterline and in any case only one side of the iceberg at a time is exposed to the wave erosion. Rarely does the entire side of an iceberg calve at one time. The amount of ice calved depends not only on the length of the calving wall but also on the wall height or the calving slab thickness. For a given slab thickness above a wave cut notch a criterion has to be developed for when the notch has been cut deeply enough into the side of the iceberg to cause calving. However, first a guideline for the calving slab thickness must be stated as a function of the total above water height. For tabular and drydocked icebergs with their fairly vertical sides a value of 80% of the observed height is chosen from observation. For pinnacle icebergs with their more sloping side wall a value of 30% of the iceberg above water height is chosen. Domed icebergs, because their instability leads to frequent changes in waterlines, are not permitted to calve in this model.
The critical wave cut waterline notch depth $L_r$ which gives rise to calving comes from White et al. (1980) and is given by:

$$L_r = 0.33(3.75H + S_t^2)^{1/2}$$  \hspace{1cm} (48)

where $H$ is the wave height in meters and $(S_t)$ is the slab thickness in meters. When the depth of the wave cut notch developing at a velocity $V_w$ given in (47) equals or exceeds $L_r$ in equation (48) calving occurs. In addition the dimensions of the iceberg are reduced by the value $L_r$.

7.0 THE MODEL

The elements of a practical iceberg deterioration model have been constructed in the preceding sections of this report. The application then becomes a matter of using the formula developed to estimate the speed with which the iceberg is destroyed. The steps are as follows, and in Figure 6:

a. Collect the data on size and shape of the iceberg together with the wind speed, sea surface temperature, and wave height and period for the iceberg's location. (Source: International Ice Patrol, Governors Island, New York, and FNOC, Monterey, California.)

b. Determine the initial mass of the iceberg in metric tons using equation (36).

c. For the appropriate iceberg class compute the underwater surface area from the square of the waterline length using equations (5), (13), (22), and (35).

d. From equation (38) compute the speed of the iceberg relative to the water using the wind speed.

e. Enter the speed of the iceberg relative to the water, the waterline length, and the sea surface temperature into equation (46) to give the melting speed in m s\(^{-1}\) °C\(^{-1}\). Multiply this result by the thermal driving force, the sea surface temperature plus 1.63. Multiply this in turn by the underwater surface area. This provides the forced convective melting in
m³ s⁻¹. Then multiply this result by the deterioration time available in seconds and by the density of ice to obtain the mass lost in metric tons by forced convective melting.

f. Use the time available for deterioration together with the sea surface temperature and wave height and period in equation (47) to determine the extent of wave undercutting. If this value exceeds the critical value from equation (48) allow the iceberg to calve along 1/2 of the iceberg length. If the undercutting exceeds twice the critical value in equation (48) allow the iceberg to calve twice, etc.

g. Subtract the mass lost by forced convective melting and calving from the original mass and shorten the iceberg by the length of the wave cut notch.

h. Compute a new underwater surface area and repeat the calculations for a new time period.
FIGURE 6. Flow diagram for preliminary iceberg deterioration model.
8.0 VERIFICATION AND TESTING

A model of a physical real-world system must be verified and tested in order to be practical. Testing of the model must account for the totality of the deterioration effects and not each detail. The important outcome is whether the iceberg as a whole melts on time and not whether each particular input to the deterioration is exact. The verification can only be statistical for what in essence is a statistical process.

Since verification and testing of the model must proceed from the full-scale and from the open ocean environment, the testing is likely to be difficult. A statistical sample is required. The sample size has to be large enough to be significant, but small enough to be manageable. A sample of the order of 10 for each iceberg type is considered satisfactory.

A verification experiment should track ten to fifteen icebergs of each of the four types from the time they reach open water outside the ice pack until they entirely melt. The measurements collected for the individual icebergs must correspond with the measurements which would actually be used in the model. The measurements should be repeated at two or three-day intervals for the life of the iceberg.

Measurements are critical during the rapid deterioration that takes place in water warmer than 5°C and during stormy intervals.

As an initial phase of testing, those icebergs in the southern portions of the Ice Patrol area which are undergoing the most rapid melting should be followed.
REFERENCES


