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Concern for predicting aural detectability of complex natural sounds has generally focused on steady state tonal and broadband signals. Procedures for calculating detectability of such signals are reasonably well established and validated. Large errors of prediction may occur, however, when these procedures are applied to other classes of signals, such as high crest factor, short duration, impulsive wavetrains. The experimentation described here was intended to support analyses of detection performance that could lead to generate quantitative procedures for predicting the audibility of repetitive, periodic impulses.
ABSTRACT (cont.)

Signals. Three separate studies were conducted using similar procedures. The first study quantified the detectability of individual impulses. The second study was directed toward the detectability of multiple impulses of varying periodic repetition rates. The third study explored hypotheses about detection strategies suitable for modelling the audibility of repetitive impulsive wave trains.
The Detectability of Repetitive, Periodic Impulses

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MAY, 1983

SUBMITTED TO:
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U.S. ARMY RESEARCH OFFICE
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I. INTRODUCTION

Concern for predicting aural detectability of complex natural sounds has generally focused on steady state tonal and broadband signals. Procedures for calculating detectability of such signals (see, for example, Fidell, Pearsons, and Bennett, 1974, or Fidell, Teffeteller, Horonjeff and Green, 1979) are reasonably well established and validated. Large errors of prediction may occur, however, when these procedures are applied to other classes of signals, such as high crest factor, short duration, impulsive wavetrains (Abrahamson, 1975). One of the most common sources of such noise is helicopters, although other transportation and industrial equipment may also produce such noise emissions (cf. Fidell and Horonjeff, 1981a).

Procedures for estimating audibility of impulsive waveforms may be useful for several purposes. First, there is some reason to believe that annoyance of such signals may be related to their audibility (Fidell and Horonjeff, 1981b). Hence, ability to predict the audibility of this class of sound might lead toward an improved ability to predict their annoyance (cf. Fidell et al., 1979). Second, more accurate and precise predictions of audibility of impulsive sounds could be useful in a number of real world detection range estimation problems. The experimentation described below was intended to support analyses of detection performance that could lead to general quantitative procedures for predicting the audibility of repetitive, periodic impulsive signals.

Three separate studies were conducted using similar procedures. The first study quantified the detectability of individual impulses. The second study was directed toward the detectability of multiple impulses of varying periodic repetition rates. The third study explored hypotheses about detection strategies suitable for modelling the audibility of repetitive impulsive wavetrains.
II. LITERATURE REVIEW AND INTERPRETATION

Fidell and Horonjef (1981) and Plomp (1961) have examined the effects of impulse fundamental waveform frequency and repetition rate on the detectability of an impulse wavetrain imbedded in broadband Gaussian noise. These experiments covered a range of 100 to 1000 Hz in waveform frequency, and from 5 Hz to the waveform frequency in repetition rate. Plomp (1961) presented sinusoidal waveforms of 250 and 1000 Hz fundamental frequency to observers monaurally, via headphones. Fidell and Horonjef (1981) presented sinusoidal waveforms of 100, 200, 400 and 1000 Hz fundamental frequency, as well as triangular and rectangular waveforms of 200 and 1000 Hz fundamental frequency, via loudspeaker in an anechoic listening environment. Plomp's observation interval was 1000 ms while Fidell and Horonjef's was 500 ms. Both investigators tested to a specified level of detection performance. Plomp (1961) used 50 percent correct in a modified free response paradigm, while Fidell and Horonjef (1981) tested to 75 percent correct in a trialwise adaptive, two-alternative forced choice procedure (Levitt (1970)).

Findings of these experiments are summarized in Figure 1. Signal power to noise power ratio for constant detection performance is plotted against impulse repetition rate for a family of waveform frequencies. Plotting signal amplitude in terms of power differs somewhat from the customary use of peak level. This manner of plotting rotates the peak level detection curve by 3 dB per doubling of repetition rate clockwise. It also accentuates the discontinuity in the curve, by producing an abrupt change in slope. The absolute signal-to-noise ratios specified on the ordinate apply only to the open data points and solid least
squares fit lines of Fidell and Horonjeff. Since Plomp reported only relative threshold levels, his data were adjusted in amplitude to fit the Fidell and Horonjeff data of nearest fundamental waveform frequency. Agreement between the two sets of data is excellent. Worst case discrepancies appear to be on the order of probable experimental error.

The most striking observation to be made from these data is the non-monotonicity in the relationship between signal-to-noise ratio and repetition rate for constant detection performance. At low repetition rates, increased signal power is required for increasing repetition rates. However, at higher repetition rates the signal power needed for constant detection performance decreases. The nominal low repetition rate slope is +1.5 dB in signal-to-noise ratio per doubling of repetition rate. For high repetition rates, the nominal slope is -1.5 dB per doubling. The dashed line shown for the 100 Hz data reflects the uncertainty in slope due to a lack of downward trend data points between the 20 and 100 Hz repetition rates.

A second observation that may be made about the data seen in Figure 1 is the orderly trend in the repetition rate at which the discontinuity in the relationship occurs as a function of waveform frequency.* The repetition rates at which the discontinuities occur are 32, 30, 40, and 55 Hz for the 100, 200, 400, and 1000 Hz waveform frequencies, respectively.

An interesting aspect of this trend is the close agreement between the discontinuity repetition rate and the effective auditory masking bandwidth at the impulse waveform center frequency.

*The discontinuity in the 100 Hz curve is probably best estimated by extrapolating backward from the 100 Hz repetition rate data point at the nominal slope of 1.5 dB per doubling.
Figure 1. Signal power to noise power ratios for constant detection performance of impulse wavetrains of 100, 200, 250, 400 and 1000 Hz sinusoidal waveform frequencies.
(Fletcher, 1940; Hawkins and Stevens, 1950; Green, et al., 1959; Fidell and Horonjeff, 1982). Since the wavetrain frequency spectrum peaks very near the individual waveform frequency, it could be argued that in marginal signal-to-noise ratio conditions the signal will be detectable only in a relatively narrow frequency range about the waveform frequency. The effective auditory masking bandwidth does not vary a great deal within this range. These observations provide reason to believe that people may adopt different listening strategies for repetition rates above or below the auditory masking bandwidth in the frequency region of highest detectability.

A fundamental question, of course, is why masking bandwidths should influence listening strategy. The theory of signal detectability (Swets, 1964; Green and Swets, 1966) provides a framework for examining this issue, and has been shown to work well in describing how people listen for broadband, non-impulsive signals in the presence of noise. The theory, combined with basic empirical data, sheds considerable light on this auditory masking bandwidth dependency and in fact predicts the observed -1.5 dB per doubling slope for high repetition rates.

A modern model based on this theory suggests in effect that people listen for the signal through a bank of closely spaced, narrow band filters; make independent likelihood assessments of the presence or absence of the signal at the output of each filter; and orthogonally vector sum the likelihoods to obtain a composite likelihood estimate across the frequency spectrum. Mathematically this process is conveniently described by equations 1 and 2. With noise alone applied to the input, the output of any given filter exhibits a time varying mean square
(given a finite averaging time, say a few hundred milliseconds) with a long term mean value \( \mu \) and a variance \( \sigma^2 \). If a small amount of signal is added to the input the long term mean square value at the output will also increase, with negligible impact on \( \sigma \). Detection, however, depends not on the absolute change in mean square level, \( \Delta \mu \), but its relation to the variance, \( \Delta \mu / \sigma \) (Green and Swets, 1966). An analogy in which band limited Gaussian noise is applied to a meter illustrates the point. If the meter reading is fluctuating only 0.5 dB it is far easier to detect a 1 decibel change to its input than if the meter is fluctuating 5 dB.

At the filter output \( \Delta \mu \) depends on the signal-to-noise ratio, and \( \sigma \) on the filter bandwidth (the wider the bandwidth, the smaller the \( \sigma \)). The ratio of \( \Delta \mu / \sigma \) is thus an index of detectability, \( d' \), that may be conveniently calculated for a frequency band of finite width by

\[
d' = \eta W^{1/2} S/N
\]  

where \( S/N \) is the average mean square signal-to-noise ratio at the filter output, \( W \) is the effective (or equivalent rectangular) bandwidth of the filter, and \( \eta \) is an efficiency factor (taken to be approximately 0.4 for present purposes but shown to be somewhat frequency dependent by Fidell, Horonjeff, Teffeteller, and Green (1983)).

Note that equation 1 behaves in a manner consistent with the foregoing discussion. Holding background noise and bandwidth constant, \( d' \) is directly proportional to signal level (at least for small signal levels where the variance of the signal plus
noise condition is not significantly different from the case with noise alone). If, on the other hand, signal-to-noise ratio is held constant $d'$ also increases with increasing bandwidth (reflecting the decrease in variance associated with the increased number of degrees of freedom).

Although equation 1 provides a model for predicting detection performance within a single auditory masking band, it does not address the issue of how detection information in multiple bands is combined. Garner (1974) postulated (and Green et al. (1959) later confirmed) that people are much better energy detectors if the signal to be detected is concentrated in a narrow spectral region. Within an auditory filter band power summation appears to be perfect, but across bands a less efficient statistical summation process is probably employed. Equation 2 reflects the empirical findings of Green et al. by treating the $d'$ values of each filter as independent observations and combining them as the sum of the squares. The composite detectability, $d'_{c}$, is computed as the square root of the sum of the squares

$$d'_{c} = \left[ \sum_{i=1}^{N} (d'_{i})^2 \right]^{1/2} \quad (2)$$

where $d'$ is the detectability index in the $i$th frequency band and $N$ is the number of frequency bands. This model accounts for the earlier findings of Schafer and Gales (1949) and Green (1958).
To a first approximation, the auditory filter bank may be considered as a set of non-overlapping (i.e., uncorrelated) filters separated in center frequency by the average bandwidth of two adjacent filters. Recent evidence suggests that the filter set may be conveniently modeled as constant percentage bandwidth, on the order of 1/6 to 1/10th octave in width. Over a limited frequency range, however, this narrow width implies that adjacent filters are of nearly constant bandwidth.

The Fourier transform of an impulse wavetrain produces discrete spectral lines of frequency spacing equal to the repetition rate. Thus, at low repetition rates the line spacing will be rather dense, and at high repetition rates there will be considerable separation between lines. The envelope of these lines, which remains invariant with repetition rate, defines (for a sinusoidal impulse waveform) a \( \sin(x)/x \) function which peaks at or slightly below the waveform frequency.

A specific example of processing a wavetrain of repeated 1 kHz single cycle sinusoids by the auditory filter bank illustrates how \( d' \) varies with increasing repetition rate of a constant power signal. In the frequency domain the spectrum will peak (and also be most detectable in the presence of Gaussian noise) at about 1000 Hz, where the auditory filter band is approximately 60 Hz wide. At a low repetition rate, for example 10 Hz, the auditory filters in the frequency region of greatest detectability will each contain approximately 6 spectral lines. Figure 2 illustrates this relationship over a very limited frequency range.
FIGURE 2. CONCEPTUAL RELATIONSHIP BETWEEN FOURIER TRANSFORMS OF EQUAL POWER IMPULSE WAVETRAINS AT (a) 10 Hz, (b) 20 Hz, (c) 60 Hz, (d) 120 Hz AND (e) AN ASSUMED AUDITORY FILTER BANK OF NOMINAL 60 Hz BANDWIDTHS, OVER A LIMITED SPECTRAL RANGE.
If the repetition rate is doubled to 20 Hz and constant power is maintained, the signal spectrum will contain half the number of lines, but each will have twice the power as the 10 Hz spectrum lines. This leaves only three lines in each auditory filter, but since each has twice the power the output of each filter remains unchanged. Following equation 1, the $d'$ values for each band remain constant since the S/N's are unchanged, and $d'_c$ in equation 2 remains unchanged as well. This process continues up to the point at which the repetition rate is equal to the auditory bandwidth (60 Hz in this example). At this point there is exactly one spectral line per auditory filter band, and both $d'$ and $d'_c$ remain unchanged as long as constant power is maintained.

At higher repetition rates the picture changes. Increasing the repetition rate from 60 to 120 Hz now increases the value of $d'_c$. As before, doubling the repetition rate halves the number of spectral lines, but those that remain contain twice the power. In contrast to the lower repetition rate examples, only every other auditory filter band contains any signal energy, but those that do contain twice as much. It follows from equation 1 that $d'$ doubles for those bands that do contain signal energy, and becomes zero for those that do not. Performing the summation by equation 2, $d'_c$ increases by the square root of 2 (or 1.5 dB).

Theoretically, this process continues until the repetition rate reaches the waveform frequency, at which point the signal becomes a steady sine wave, the Fourier transform has one spectral line, and only one auditory filter contains any energy. Since $d'_c$ increases at the rate of 1.5 dB per doubling of repetition rate for a constant power signal, and since $d'$ is directly propor-
tional to S/N, it follows that the signal-to-noise ratio required to support constant detection performance must diminish at the rate of 1.5 dB per doubling of repetition rate.

Figure 3 summarizes the preceding discussion by showing the predicted relative signal-to-noise ratio required for constant detection performance over a broad range of repetition rates. The relationship is flat for repetition rates less than the auditory bandwidth, and decreases at higher rates by 1.5 dB per doubling. The breakpoint in the relationship plotted in Figure 3 therefore depends on the auditory bandwidth in the frequency range of greatest detectability, which in turn depends on absolute frequency. Therefore, for approximately sinusoidal waveforms the breakpoint will be monotonically related to the fundamental waveform frequency.

Comparison of Figure 3 with Figure 1 suggests that equations 1 and 2 account for two of the major empirical observations (the breakpoint dependency on waveform frequency and the rolloff rate at high repetition rates), but do not provide insight into the third observation (the positive going slope at low repetition rates). The equations predict that detection should remain constant at lower repetition rates, but the data do not support the prediction. The data indicate that if power is held constant, lower repetition rates become easier to detect. This suggests that observers take advantage of the periodic nature of the signal at low repetition rates.

The three experiments described below explored hypotheses about the low repetition rate effect. Compelling arguments exist for such exploration in both the frequency and time domains.
FIGURE 3. SIGNAL TO NOISE POWER RATIO FOR CONSTANT DETECTION PERFORMANCE OF AN IMPULSE WAVETRAIN PREDICTED BY STATISTICAL SUMMATION MODEL.
The fact that the tent-shaped curves of Figure 1 are peaked suggests that auditory bandwidth may be an important determinant not only for the onset of the high repetition rate effect, but also for the offset of the low repetition rate effect. On the other hand, the +1.5 dB per doubling slope suggests a time domain explanation: observers may listen for the individual impulses in the wavetrain, assign a likelihood to the presence or absence of each, and perform a vector summation over time in accordance with equation 2 (in which the d'_i represent detectability values assigned to each impulse). In such a model, holding peak amplitude constant, a doubling of repetition rate affords the observer twice the number of opportunities to detect the signal in a finite observation interval. In other words, N in equation 2 is doubled. The average value of d'_i remains constant, however, and d'_c increases by a factor of \( \sqrt{2} \). Since the peak value has been held constant, the signal power has doubled for this \( \sqrt{2} \) increase in detectability. Thus, to hold detection performance constant the signal power must increase by \( \sqrt{2} \) (or 1.5 dB). This reasoning is in agreement with the empirical results of Plomp (1961) and Fidell and Horonjeff (1981).

Implicit in this temporal summation model is the assumption that the observer knows exactly when to listen for each impulse in the wavetrain. This may not be an easy task in marginal signal-to-noise ratio conditions. As the interpulse interval increases, human observers may find increasing difficulty in positioning a narrow temporal listening window to coincide with signal plus noise rather than noise alone. This is a potential confounding effect, since it suggests that detection performance degrades rather than improves with decreasing repetition rate. The first experiment described in this paper addresses this issue.
Viewed from a slightly different perspective, the temporal summation model implies that equal numbers of impulses are equally detectable and that repetition rate could perhaps be traded for observation interval duration (preserving constant $N$) with no change in detection performance. If people are truly impulse counters, then this relationship should hold, at least over some limited range of observation interval duration. This hypothesis forms the basis of the second experiment.

A third experiment was also designed to study yet another hypothesis: that people might be sensitive to very short term signal statistics. Both the short term mean and the variance of the impulsive wavetrains vary systematically with repetition rate, and could therefore support detection decisions. For example, because the standard deviation of an impulsive wavetrain is so great, adding an impulsive wavetrain to Gaussian noise might increase the variance of the signal plus noise distribution more than it increases its mean. By studying the audibility of band-limited impulsive wavetrains in the third experiment, an opportunity was provided for exploring any such potential effects.
III. STUDY 1: SINGLE IMPULSE, VARIABLE STARTING TIME EXPERIMENT

A. Method

Four observers (one male, three female, aged 19-27) were paid an hourly wage and a performance bonus to participate in this study. All were audiometrically screened to within 10 dB of ISO audiometric zero. All but the male observers had previous experience in detecting sinusoids in noise. A trialwise adaptive procedure (a combination of the "2 down, 1 up" and "3 down, 1 up" block up and down methods described by Levitt and Rabiner (1967) and Levitt (1970)) was used to estimate the signal-to-noise ratio associated with 75% (d' = 1) correct detection performance in two alternative forced choice trials. Observation, intratrial, and intertrial intervals were all 500 ms. The response interval was 750 ms. The a priori probability of signal occurrence was equal in the two observation intervals. Illuminated push buttons were used to identify the observation intervals, record observers' responses, and provide feedback during the response interval.

At the start of a determination, the signal presentation level was approximately 10 decibels greater than the level expected to be necessary to support 75% correct detection performance. The initial step size was 2.8 decibels. The step size was halved on succeeding reversal cycles until the minimum step size (1 decibel) was achieved. All data from trials other than those with minimum step sizes were ignored for estimating the signal-to-noise ratio corresponding to 75% correct detection performance. Each determination required approximately 20 reversal cycles to achieve a stopping criterion of 0.5 dB
for the 95% confidence interval on the cycle midpoint sound level.

Data were collected in a total of twelve signal conditions during two-hour long daily sessions with frequent rest breaks. All data were collected under free field listening conditions by observers seated individually in an anechoic chamber with a cutoff frequency of approximately 125 Hz. Continuous white Gaussian noise with a nominal spectrum level of 30 dB was present at all times. The overall level of this masking noise from 50 to 4000 Hz was approximately 65 dB re 20µPa.

Single cycles of waveforms with fundamental frequencies of 100, 200, and 1000 Hz were detected at four fixed onset delay times (60, 100, 200, and 300 ms) and one random onset delay time. In the random onset delay condition, the delay was selected from a rectangular distribution of onset times ranging from 60 to 300 ms. For the 200 Hz waveform, data were collected only in the 100 ms and random delay conditions. All onset delays were measured from the start of the observation interval, marked by the illuminated response switches. The response switch lamps required less than 40 ms from application of voltage for their filaments to become highly visible in the ambient lighting conditions.

Figure 4 shows the temporal waveforms of the single cycle signals. These acoustic measurements were made at the observer's head position (in the absence of the observer and seat), using a Brüel and Kjaer 1" condenser microphone (type 4145), a Genrad microphone preamplifier (type 1560-P42), a Brüel and Kjaer precision impulse sound level meter (type
FIGURE 4. SINGLE IMPULSE WAVEFORMS, EXPERIMENT #1
2209), and a Spectral Dynamics Model SD360 Digital Signal Processor (FFT analyzer). Figures 5 and 6 are frequency domain representations (on different scales) of the single impulse waveform spectra of Figure 4, and of the masking noise.

The order of presentation of experimental conditions was randomized over observers by randomly selecting a waveform frequency, and then testing each delay (minimum of 3 determinations) in random order before proceeding to the next waveform. The entire test was then repeated with a different randomization schedule. Training consisted of at least one, two-hour long session for each new waveform. Observers were required to repeat three consecutive determinations of the detectability of a single cycle waveform to within a 1.5 dB range. Upon completion of the test/retest, a minimum of six determinations of the signal-to-noise ratio corresponding to \( d' = 1 \) was made in each experimental condition.

B. Results

Table 1 contains absolute signal energy levels \( (E) \) and signal energy to noise power density ratios \( (E/N_0) \) observed for 75% correct detection performance in all waveform frequency and onset delay conditions. These \( E/N_0 \) values were calculated by performing FFT analyses (5 Hz resolution) on both the signal and noise. Broadband signal energy \( (E) \) was determined by capturing the transient signal with the FFT analyzer in a 1024 point record, obtaining the Fourier transform, energy summing all spectral components of the transform, and multiplying by the record length (to account for duration). Noise power
FIGURE 5. SINGLE IMPULSE AND MASKER SPECTRAL CONTENT FOR 100 Hz AND 200 Hz WAVEFORMS, EXPERIMENT # 1
FIGURE 6. SINGLE IMPULSE AND MASKER SPECTRAL CONTENT FOR 1000 Hz WAVEFORM, EXPERIMENT # 1
### TABLE I. SIGNAL ENERGY AND SIGNAL ENERGY TO NOISE POWER DENSITY RATIOS FOR 75 PERCENT CORRECT DETECTION PERFORMANCE -- EXPERIMENT #1

<table>
<thead>
<tr>
<th>Waveform Frequency (Hz)</th>
<th>Delay (msec)</th>
<th>Subject 1 Signal Energy (E)</th>
<th>Subject 2 Signal Energy (E)</th>
<th>Subject 3 Signal Energy (E)</th>
<th>Subject 4 Signal Energy (E)</th>
<th>Subject Mean Signal Energy (E)</th>
<th>Masker N₀ (dB/Hz)</th>
<th>Subject Mean E/N₀</th>
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<tr>
<td>100</td>
<td>60</td>
<td>36.1</td>
<td>35.9</td>
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<td>36.7</td>
<td>36.1</td>
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<td>7.8</td>
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<tr>
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<td>-</td>
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<td>38.9</td>
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<td>39.0</td>
<td>38.6</td>
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<td>39.8</td>
<td>39.3</td>
<td>-</td>
<td>11.5</td>
</tr>
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1) average over 50-170 Hz range  
2) average over 60-310 Hz range  
3) average over 360-1500 Hz range
density \( (N_o) \) was calculated by obtaining the long term average power spectrum and averaging the spectral component amplitudes over the frequency range delimited by the upper 10 decibels of the signal spectrum. Detection performance averaged over observers is displayed in Table 1, and also in Figure 7, normalized for convenience of plotting. For each waveform, the signal-to-noise ratio sufficient for 75% correct detection performance is plotted relative to average signal-to-noise ratios in the 100 ms and random onset delay conditions (common to all waveforms presented). This normalization makes it clear that there are no appreciable differences in the detectability of any of these waveforms as a function of onset delay, within the temporal range investigated. The pooled estimate of the 95% confidence interval for each plotted points is approximately \( \pm 0.4 \) dB.

The dashed line in Figure 7 connects the mean normalized signal-to-noise ratios necessary for 75% correct detection performance at all frequencies, for each onset delay. A trend toward deterioration in detection performance (i.e., a need for greater signal-to-noise ratios to sustain constant 75% correct detection performance) is apparent at the longest onset delay. This trend is consistent in direction with that observed in other investigations of the effects of longer starting time uncertainties (e.g., Egan, Schulman, and Greenberg, 1961). The data for the random onset delay condition are consistent with this trend as well, since they are intermediate in signal-to-noise ratio between the short (<200 ms) and long (300 ms) onset delay conditions.
FIGURE 7. SIGNAL THRESHOLD AS A FUNCTION OF SIGNAL PRESENTATION DELAY FROM START OF OBSERVATION INTERVAL
IV. STUDY 2: MULTIPLE IMPULSE, VARIABLE REPETITION RATE EXPERIMENT

A. Method

The same four observers who participated in the first study also participated in this study, under identical trial procedures except as noted below. The signal consisted of periodically repeated impulsive waveforms of 1000 Hz fundamental frequency (see Figure 4). The background noise was identical in both level and spectral content to the first study. The 5 Hz lower bound on repetition rate was chosen to achieve a minimum of two impulses in observation intervals of moderate duration (500 milliseconds). The upper bound of 40 Hz was chosen to avoid entering a different listening regime in which the growth of detectability with increasing repetition rate reverses sign for a constant energy signal (Fidell & Horonjeff [1981], Plomp [1961]). The bounds on observation interval were dictated by the auditory integration time at the lower end and practical considerations regarding experimentation time and listener fatigue at the high end. The repetitive impulsive waveform was generated at rates of 5 to 40 Hz, in observation intervals of durations from 250 to 2000 ms. Data was collected in 15 signal conditions representing various combinations of repetition rate and signal duration. The wavetrain was continuously generated asynchronously from the observation intervals. Portions of the wavetrain beginning with positive-going zero crossings were gated into the observation intervals, so that (for example), a 1000 ms observation interval would contain 20 impulses on average at the 20 Hz repetition rate. Each of the individual impulses was of identical waveshape to that seen in Figure 1 at all repetition rates tested.
B. Results

Table II contains mean values of absolute signal energy (E) and signal-to-noise ratio (E/N₀) necessary to maintain 75% correct detection performance under all conditions in which data were collected. Each tabled value represents an arithmetic mean of at least 24 determinations of the 75% correct detection point. A minimum of six determinations was made by each of four observers. A pooled estimate of the width of the 95% confidence interval for each value is ±0.6 dB. The value of N₀, 27.8 dB, is an average over 360 to 1500 Hz (the frequency range of the upper 10 dB of the Fourier transform of the impulse train). Figure 8 plots these data as a function of repetition rate.

An orderly family of curves (parametric in observation interval duration) for the different repetition rates may be seen in Figure 8. The lines connecting the points are least squares regression lines, from which no data point deviates by more than 0.2 dB. The slopes of these relationships between E/N₀ for constant detection performance and repetition rate do not differ appreciably from one another over the 8:1 range in observation interval durations. The nominal slope in E/N₀ per decade of repetition rate is 5 dB (1.5 dB per doubling).

Figure 9 replots the same data as a function of observation interval duration in a family of repetition rate curves. The ordinate and data points (from Table II) remain the same. Once again, an orderly relationship is observed, with all slopes nominally 8 dB in E/N₀ per decade of observation interval duration (2.4 dB per doubling). The displacements between curves
<table>
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<th>Impulse Repetition Rate (Hz)</th>
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<th>250</th>
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FIGURE 8. OBSERVED SIGNAL TO NOISE RATIOS (E/N₀) AS A FUNCTION OF IMPULSE REPETITION RATE FOR FOUR DIFFERENT OBSERVATION INTERVAL DURATIONS -- EXPERIMENT #2
FIGURE 9. OBSERVED SIGNAL TO NOISE RATIOS (E/N₀) AS A FUNCTION OF OBSERVATION INTERVAL DURATION FOR FIVE DIFFERENT IMPULSE REPETITION RATES -- EXPERIMENT # 2
for doublings of repetition rates are almost exactly 1.5 dB.

Since the displacements between curves in Figures 8 and 9 are virtually constant for a doubling in the curve parameter, it is reasonable to replot these data once more, as shown in Figures 10 and 11, by adjusting each individual datum by
\[ -5 \log_{10}(\text{repetition rate}) \] and
\[ -8 \log_{10}(\text{duration}), \]
respectively. The lines connecting the points in Figures 10 and 11 are least squares regressions that account for 99 and 98 percent, respectively, of the variance in the data sets. The slope of this relationship in Figure 10 is 4.73 dB in $E/N_0$ per decade of repetition rate. In Figure 11, the regression line slope is 7.96 dB per decade of observation interval duration.
FIGURE 10. OBSERVED SIGNAL TO NOISE RATIOS ($E/N_0$) AS A FUNCTION OF IMPULSE REPETITION RATE COLLAPSED OVER OBSERVATION INTERVAL DURATION BY $8 \log_{10} (D)$
FIGURE 11. OBSERVED SIGNAL TO NOISE RATIOS (E/N₀) AS A FUNCTION OF OBSERVATION INTERVAL DURATION COLLAPSED OVER REPETITION RATE BY 5 log₁₀ (RR)
V. STUDY 3: MULTIPLE IMPULSE, NARROW BANDWIDTH EXPERIMENT

A. Method

Four observers who had not participated in earlier studies were paid to listen for repetitive impulsive wavetrains bandpassed through one-third octave band filters. The individual filtered impulses were single cycles of sinusoids of 200 Hz and 1 kHz fundamental frequency. The acoustic waveforms at the observer's listening location are shown in Figure 12. These impulses were repeated at periodic rates ranging from 5 Hz to 200 Hz, as in Studies 1 and 2. Detection performance was also tested for continuous sine waves at 200 Hz and 1 kHz, as well as for one-third octave bands of white, Gaussian noise centered at these frequencies. The broadband (50 Hz to 4 kHz) background noise in which the various signals were embedded was very similar to that seen in the lower panel of Figure 5. As in Studies 1 and 2, the background noise was continuously present whenever the observer was in the anechoic chamber.

The primary goal of the third study was to explore hypotheses about how observers could use information other than the energy of the repetitive wavetrains to make detection decisions. The signals were filtered to avoid confounding of detection performance by frequency domain effects. The narrow bandpass of the external filter, comparable to that of hypothetical internal auditory filter (cf. Fidell et al., 1983 and Patterson and Nimmo-Smith, 1980), precluded listening in frequency regions other than that of the peak of the waveform spectrum. Trial procedures were identical to those of Studies 1 and 2.
FIGURE 12. IMPULSE WAVEFORMS, EXPERIMENT #3
B. Results

Figure 13 shows long term rms signal-to-noise ratios for all of the signals at the 75% correct detection point (averaged over observers), in a format similar to that of Figure 1. They are, in fact, more similar to the slope of the ideal relationship seen in Figure 3. Since the only difference between the signals heard in Studies 1 and 3 (i.e., those displayed in Figures 1 and 13) was the one third octave band filtering applied to signals in Study 3, it appears that restriction of the frequency range over which observers could listen to the repetitive impulsive signals was responsible for the flattening of the slopes in Figure 13.

The relative positions of the data points for the continuous sinusoids in the two figures serve as convenient reference points for comparisons of detection performance in Studies 1 and 3. In particular, the difference in signal to noise ratio between the 1 kHz continuous sine wave and the 5 Hz repeated impulses is 3 dB in both cases. The corresponding 200 Hz data in the two studies, however, fails to agree by about two decibels.
B. Results

Figure 13 shows long term rms signal-to-noise ratios for all of the signals at the 75% correct detection point (averaged over observers), in a format similar to that of Figure 1. They are, in fact, more similar to the slope of the ideal relationship seen in Figure 3. Since the only difference between the signals heard in Studies 1 and 3 (i.e., those displayed in Figures 1 and 12) was the one third octave band filtering applied to signals in Study 3, it appears that restriction of the frequency range over which observers could listen to the repetitive impulsive signals was responsible for the flattening of the slopes in Figure 13.

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FIGURE 13. SIGNAL POWER TO NOISE POWER RATIOS FOR 75 PERCENT CORRECT DETECTION PERFORMANCE AS A FUNCTION OF IMPULSE REPETITION RATE FOR TWO IMPULSE WAVEFORMS -- EXPERIMENT # 3
VI. DISCUSSION

A. Study 1

The first study was intended to determine whether or not inter-pulse interval has a major influence on the detectability of repeated impulses. The hypothesis under test was that knowledge of when to listen allows human observers to eliminate unwanted noise from time domain detection decisions. Such knowledge is of the sort yielded by cross-correlating a signal specified exactly and the signal plus noise. It is reasonable to speculate that as the interpulse interval increases, positioning a narrow time window in just the right place becomes increasingly difficult for observers. This difficulty could arise from either of two phenomena, both of which degrade detection performance: (1) the narrow window misses the pulse on a sizeable number of occasions, so that a correlation is made against noise alone instead of signal plus noise; or (2) the window must be increased in width to accommodate the positioning uncertainty at the cost of more noise entering the detector. In this case, most of the multiple correlations made in rapid succession are against noise alone.

An ideal experimental design for quantifying such an effect would be a dynamic one, in which the detectability of the last pulse in periodic impulsive wavetrains would be measured in the context of the previous impulses.* Fidel and Horonjeff (1981) demonstrated an effect of 4 dB in signal-to-noise ratio associated with a range of 5:1 in impulse repetition rate. This effect might arguably have been attributed to the effect of interpulse interval. Since this is a relatively large effect, it was

*Use of aperiodic repetitive wavetrains complicates analyses by confounding temporal and frequency domain effects.
believed that it would also be discernable in a study of the present sort, in which observers had to detect a single "last" impulse of variable temporal spacing from the start of an observation interval identified by a visual cue.

The facts that significant differences in detectability observed as a function of implied interpulse interval in Study 1 occurred at intervals greater than 250 ms, and that these differences were on the order of only 0.5 dB, suggest that interpulse interval per se has little to do with the detectability of the repetitive impulsive wavetrains employed in either the earlier (Fidell and Horonjeff, 1981) or current studies. Since a measurable effect (on the order of 0.5 dB) could be found only at intervals greater than 250 ms, one could argue that any effects on the detectability of repetitive wavetrains would be limited to repetition rates lower than 4 Hz (a repetition rate of 5 Hz was the lowest used in the current or earlier studies). The magnitude of this effect, as well as the implied repetition rate below which it occurs, is consistent with the observations of Garner (1947) in the frequency domain. Using short tone bursts (1 to 50 millisecond duration) of a 1000 Hz sinusoid presented at a range of repetition rates (0.25 to 50 Hz), Garner found a discontinuity in the threshold vs. repetition rate curves between 2 and 5 Hz. This discontinuity is observed as a vertical shift in the threshold curve on the order of 0.5 dB.

B. Study 2

The hypothesis under test in Study 2 was that equal numbers of impulses are of equal detectability: that is, duration can be traded directly for repetition rate to maintain constant de-
tection performance. If this hypothesis were correct then the $E/N_0$ values in Table II corresponding to constant products of repetition rate and duration would be the same (or at least be devoid of any residual correlation with repetition rate or duration). The difference between the slopes in Figures 10 and 11, however, suggests this is not the case.

Figure 14, prepared from the data of Table II, illustrates the failure of compensation between wavetrain duration and repetition rate. The least-squares regression lines connect data points of equal numbers of impulses (the product of repetition rate and observation interval duration). These data show a strong residual correlation with repetition rate. A similarly strong correlation is observed if these data are plotted against observation interval duration. It is clear that the lines in Figure 12 do not have a slope of zero; in fact, the average slope is $-3.2$ dB in $E/N_0$ per decade of repetition rate, which is the difference in the slopes of Figures 10 and 11. The current data thus infirm the hypothesis that equal numbers of impulses are equally detectable, at least for observation durations greater than the integration time of the human auditory system (nominally 250 ms).

Taken separately, however, the effects of the two variables, repetition rate and observation interval duration, agree well with prior findings. The $5 \log_{10}(\text{repetition rate})$ relationship is in excellent agreement with the low repetition rate observation of Plomp (1961) and Fidell and Horonjeff (1981). The current data, however, expand the earlier findings by demonstrating that the $5$ dB/decade ($1.5$ dB/doubling) slope is independent of duration over an 8:1 range of 250 to 2000 milliseconds.
FIGURE 14. OBSERVED SIGNAL TO NOISE RATIOS ($E/N_0$) AS A FUNCTION OF IMPULSE REPETITION RATE FOR OBSERVATION INTERVALS CONTAINING EQUAL NUMBERS OF IMPULSES -- EXPERIMENT # 2
Likewise, the $8 \log_{10}(\text{duration})$ relationship also appears to be invariant with repetition rate over an 8:1 range of 5 to 40 Hz. This slope of 8 dB/decade (2.4 dB/doubling) suggests that people make very little use of the additional signal energy as duration increases (at least above 250 milliseconds). If people were perfect energy detectors above 250 milliseconds, the expected value of the slope would be zero. Alternatively, if people were only performing as power detectors (making no use of increased signal duration) the expected value of the slope would be $10 \text{ dB/decade}$ (3 dB/doubling). The observed slope, however, is in good agreement with the findings of Green (1957) and Fidell et al. (1981). Both of these studies found that detection performance degraded at the rate of 2 dB per doubling of duration for signal durations in excess of 250 to 300 milliseconds for constant energy sine wave signals. At lower durations, listeners were observed to be near perfect energy detectors.

Green (1957) suggested a compromise detection model to explain these observations for long duration signals. According to Green, people combine both energy summation and statistical summation in the detection process. That is, they exploit energy summation to the fullest extent possible by dividing the total signal duration into 250 millisecond intervals, and perform perfect energy integration within each. After assessing signal detectability (i.e., computing a $d'$ value) in each interval, the detectabilities are combined as independent observations by equation 2, where the summation is a temporal (instead of spectral) one, and $N$ is the number of 250 millisecond intervals in the total signal duration.
This model predicts a slope of 5 dB/decade (1.5 dB/doubling), less than the observed 8 dB/decade. One interpretation of the available evidence, is that human observers may try to use this strategy, but are unable to perform the statistical summation process in an efficient manner. Perhaps the individual d' determinations decay rapidly with time, so that only the most recent 250 millisecond intervals contribute substantially to the signal's detectability at any particular point in time. Further exploration of this signal duration effect, however, is beyond the scope of this paper.

It is a matter of conjecture whether the original hypothesis would have been rejected in the temporal region below 250 milliseconds, since human observers appear to be perfect energy detectors for non-impulsive signals in this region. Although possible, an experiment at these short durations would of necessity require careful control. Since relatively high repetition rates would be necessary to achieve an expectation of two or greater impulses per observation interval, the danger exists of entering the high repetition rate listening regime and confounding results.

The high correlations apparent in Figures 10 and 11 suggest that a general expression could be developed for predicting the signal-to-noise ratios (E/N₀) observed in this experiment as a function of repetition rate and observation duration. Using constant coefficients of 5 and 8 for log(replication rate) and log(duration), respectively, the following expression predicts
E/N_o for 75% correct detection performance in a 2AFC task with a 95% confidence interval of ±0.4 dB:

$$10 \log_{10}(E/N_o) = 12.88 + 5 \log_{10}(\text{rep rate}) + 8 \log_{10}(\text{duration})$$ (3)

Or, alternatively,

$$E/N_o = (19.41)(\text{repetition rate})^{0.5}(\text{duration})^{0.8}$$ (4)

Use of the two variables, repetition rate and observation interval duration, accounts for over 98 percent of the variance in the currently observed E/N_o values for constant detection performance. Variance unaccounted for by this relationship is within the uncertainty of individual data points, whose 95% confidence interval of ±0.6 dB was noted earlier. The potential applicability of this relationship beyond the ranges explored in the current experiments is discussed below. Also discussed is the significance of the constant terms in equations 3 and 4, and their apparent relation to the detectability of a single impulse in the wavetrain.

C. Comparison of Data from Studies One and Two

Use of the same 1 kHz impulse waveform and the same masking noise in both experiments provides the basis for a direct analysis of the ability to predict wavetrain detection performance (experiment 2) from single impulse detection performance (experiment 1).
Figure 15 directly compares the results of both experiments on a single graph, by displaying signal-to-noise ratio ($E/N_o$) for 75 percent correct detection performance as a function of the number of impulses in the observation interval. The results from experiment 1 are plotted as the solid triangular data point in the lower lefthand corner. The remainder of the data are from experiment 2, shown as families of open data points of constant observation interval duration.

Figure 15 differs from Figure 8 only in that each observation interval family has been moved horizontally to vertically align data points with equal numbers of impulses in the observation interval. The long dashed lines are least squares fits through the 500 and 1000 ms data. The short dashed line is the mean slope of all experiment 2 data (4.7 dB/decade from Figure 10) fit to the 250 millisecond data and extrapolated to one impulse per observation interval.

The strength of the argument for extrapolation of the 250 millisecond data lies with the strong linear relationship of the 500 millisecond data down to 2.5 impulses per observation interval and the observed 250 to 300 millisecond auditory integration time for non-impulsive signals. The latter point suggests that additional data points at durations of less than 250 milliseconds would fall on the short dashed line extrapolation through the 250 millisecond data. Since the single impulse is simply a limiting case of a short duration signal, it is not unreasonable to assume that this data point would fall on the 250 millisecond extrapolation line as well.
FIGURE 15. OBSERVED SIGNAL TO NOISE RATIOS (E/N₀) AS A FUNCTION OF NUMBER OF IMPULSES FOR THREE DIFFERENT OBSERVATION INTERVALS IN EXPERIMENT # 2 COMPARED WITH THE SINGLE IMPULSE OF EXPERIMENT # 1
Figure 15 confirms the expected: the single impulse data point falls within 0.2 dB of this line. This close agreement suggests that equation 3 could be rewritten in a form which predicts repeated impulse results from the single impulse observations (or vice versa). This can be accomplished by replacing repetition rate in equation 3 with the number of impulses in the observation interval, $N$, divided by the observation interval duration, and normalizing the duration to 250 milliseconds in conformance with the auditory integration time for non-impulsive signals (Green, 1957; Fidell et al., 1981). Equation 3 thus becomes

$$10 \log_{10}(E/N_o) = 11.07 + 5 \log_{10}(N) + 3 \log_{10}(\text{duration}/0.250)$$

(5)

where the original constant of 12.88 in equation 3 has been reduced by $3 \log_{10}(1/0.250)$ to preserve equality on the right hand side of the equation. The resulting constant of 11.07 is well within the experimental error of the average 11.2 dB signal-to-noise ratio observed in the single impulse experiment at 1000 Hz.

Equation 6 restates equation 5 as a general expression to predict $E/N_o$ for a repeated impulse wavetrain based on $E/N_o$ for a single impulse

$$10 \log_{10}(E/N_o)_{ri} = 10 \log_{10}(E/N_o)_{si} + 5 \log_{10}(N) + 3 \log_{10}(\text{duration}/0.250)$$

(6)

where $(E/N_o)_{ri}$ and $(E/N_o)_{si}$ are the signal-to-noise ratios for equal detection performance of the repeated and single impulses, respectively.
If, in fact, the 11.2 dB signal-to-noise ratio observed in the single impulse experiment is substituted for the first term in equation 6 the $E/N_0$ values for every condition in experiment 2 are predicted within ±0.6 dB.

D. Study 3

The detection performance summarized in Figure 13 is characteristic of that which could be expected from a near-ideal energy detector with a time constant of 200 ms or longer. It is not clear why simply restricting the bandpass of the repetitive impulsive wavetrains should have produced such an effect. The filter bandwidths employed were very close to those of the hypothetical internal auditory filter (Fidell et al., 1983) at 200 Hz, and within a factor of two at 1 kHz.

For the filtered 200 Hz sinusoidal impulses, there is essentially no dependence of detection performance on repetition rate below 40 Hz. Further, with the exception of an apparently anomalous point at 40 Hz, there is only minor improvement in detection performance at higher repetition rates. This improvement in detection performance at higher repetition rates is readily explained by the inherent concentration of energy in a few spectral components within a band narrower than that of the external filter, as discussed in Section II of this report.

For the 1 kHz data, the relationship is intermediate between the idealized slopes of Figure 3, and the data of Figure 1. This is apparently a consequence of the less restrictive bandwidth of the external filter with respect to that of the internal filter at this frequency recall that the external filter at
1 kHz was twice as wide as the internal filter). As a result, the 1 kHz data show a slightly greater dependence on repetition rate than do the 200 Hz data.

An alternative explanation for these data was sought (without success) in the time domain. The hypothesis explored was that observers may have employed a detection strategy involving subdivision of the 500 ms observation interval into a set of shorter listening epochs. If these listening epochs were independent, their duration should have been no shorter than the reciprocal of the bandwidth of the internal filter. In each of these listening epochs, observers could logically have been modeled as computing the mean square energy, as well as the means and variances of this statistic across epochs. Either the mean or the variance could have served as the basis for a decision about which observation interval contained the signal.

To explore such hypotheses, the same filtered impulsive wavetrains were generated at the average level at which observers achieved 75% correct detection performance, and mixed with the masking noise used in Study 3. These acoustic waveforms were filtered through a one-third octave band filter centered at the impulse waveform frequency, sampled at a 10 kHz rate (12 bit analog-to-digital conversion accuracy), and stored in digital form. The digitized information was analyzed into epoch lengths equal to the reciprocal of the one-third octave bandwidths (21.7 ms for the 200 Hz wavetrain and 4.3 ms for the 1000 Hz wavetrain). Mean square values, in units of pascal²-seconds (products of signal power and epoch length) were then calculated in each epoch.

The results of these analyses may be seen in the distribution functions of Figures 16 and 17. These functions show the relative
Figure 16. Probability density functions of mean square energy (21.7 msec epochs) for masker alone and signal plus masker at 75 percent correct detection level for 200 Hz impulse waveform -- Experiment # ?
FIGURE 17. PROBABILITY DENSITY FUNCTIONS OF MEAN SQUARE ENERGY (4.3 MSEC EPOCHS) FOR MASKER ALONE AND SIGNAL PLUS MASKER AT 75 PERCENT CORRECT DETECTION LEVEL FOR 1000 Hz IMPULSE WAVEFORM -- EXPERIMENT # 3
positions of the noise alone and signal plus noise distributions at the point at which the observers achieved 75% correct detection performance in the two alternative, forced choice task. Figures 16 and 17 are therefore best estimates of the probability density functions that observers heard, and could have based their detection decisions on.

Various degrees of skewness can be seen in these density functions, corresponding to their degrees of freedom. For epoch lengths equal to the reciprocal of the analysis bandwidth the distribution will be nominally chi-square with two degrees of freedom. Figures 16 and 17 also indicate the means and standard deviations of these sampling distributions. As is expected for Chi-square distributions with small numbers of degrees of freedom, the means and standard deviations are comparable in magnitude.

The dependence of the mean and variance on repetition rate is seen more clearly in Figure 18. The upper panel of Figure 18 plots means and standard deviations of the observed distributions of signal plus noise for 75% correct detection performance for the 200 Hz waveforms. The lower panel shows the same information for 1000 Hz waveforms. In both panels, solid points represent values of means, while open points represent values of standard deviations. The three horizontal sections of each panel summarize all information collected in the third experiment. From left to right, the sections show masking noise alone (in a 1/3 octave analysis bandwidth); the filtered repetitive impulsive wave-trains; and the two steady state signals (the continuous tone and noise alone). The regression lines in the center sections of the two panels connect data points at repetition rates lower than bandwidth of the internal filter: 46 Hz for the 200 Hz case, 60-100 Hz in the 1 kHz case.
FIGURE 18. MEANS AND STANDARD DEVIATIONS OF MEAN SQUARE ENERGY DISTRIBUTIONS FOR SIGNAL PLUS MASKER AT 75 PERCENT CORRECT DETECTION LEVEL
Several trends are evident in the two panels. Note, for example, that the differences between means and standard deviations increase linearly with repetition rate in both cases. However, the standard deviation values for the 1 kHz wavetrains are essentially independent of repetition rate, whereas they decrease with repetition rate for the 200 Hz wavetrains. No simple, consistent interpretation of these data suggests itself. Nonetheless, the orderliness of the trends could in principle support some forms of decision making based on variance as well as means of the short term distributions.
VII. CONCLUSIONS

The combined findings of the two experiments lend support to two major inferences, applicable to human detection of low repetition rate impulse wavetrains. A different listening strategy is apparently employed at higher repetition rates (Fidell and Horonjeff, 1981) for which spectral components are spaced at frequency intervals greater than masking bandwidths.

First, for an observation interval of any duration, the detection process can be conveniently modeled as an independent (orthogonal) vector summation of the detectabilities of the individual impulses in the observation interval. Thus, $d'_c$ increases by a factor of $\sqrt{2}$ per doubling of numbers of impulses. Expressed another way, two equally detectable signals differing in repetition rate by a factor of 2 would be expected to differ in energy by $\sqrt{2}$, or 1.5 dB (5 dB per decade). The observed 4.7 dB per decade relationship observed is quite close to this rate.

The second major inference that the current data suggest is that the vector summation process of individual impulse detectabilities is a "leaky" one over time, just as is the energy summation process for steady state signals. That is, the summation that human observers perform is a non-ideal one, that fails to preserve all available information. Both the leakage rate and the critical signal duration beyond which leakage starts seem very similar for the two cases. The curve shown in Figure 11 indicates that after repetition rate has been taken into account, the duration of the observation interval has an
important effect on signal detectability: the longer the observation, the greater the signal energy needed to maintain constant detection performance.
REFERENCES


