THE MEASUREMENT OF EARTH RESISTIVITY AFFECTING MHD-EMP RESPONSE OF BURIED CABLES

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Magneto-telluric
Earth Resistivity
Magneto-hydrodynamic Electromagnetic Pulse
Resistivity Methods

The response of buried conducting cable to a MHD EMP wave is determined in part by the resistivity of the earth at frequencies ranging from 10 to 0.01 Hz. Three methods of measurement of earth resistivity are presented with their advantages and limitations.
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1. Skin depth curves.
2. Typical spectrum of amplitudes of electromagnetic noise in the extremely low frequency (elf) range (Ref. 1).
3. Example of a magneto-telluric signal with transient micropulsation activity in the 10- to 40-s period range.
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A nuclear detonation at very high altitude can produce an extremely low frequency variation in the earth's magnetic field. The conducting plasma expanding from the explosion pushes the earth's magnetic field lines away. The resultant change in magnetic field extends over very large distances, up to the earth's surface, and is called the Magnetohydrodynamic Electromagnetic Pulse, MHD EMP.

Since the earth is partially conducting, a changing magnetic field at its surface will induce eddy currents in the various layers of conducting strata down to a skin depth. For a given frequency and uniform conductivity, $\sigma$, the skin depth is proportional to $1/\sqrt{\sigma}$. The electric field produced at the surface by a given charge in a magnetic field will be proportional to the resistance of the skin depth layer, i.e., $\sigma/\sqrt{\sigma}$. The problem is more complicated with a realistic stratified earth but the electric field is still a function of the earth's conductivity profile.

A long conductor grounded to the earth at one point will exhibit a potential difference from earth at other points proportional to the electric field and the distance from the ground point. For practical situations these potentials can reach hundreds of volts. Vulnerability assessment of the possibly affected systems and hardening design guidelines require an evaluation of worst case stresses by:

1) Calculating the worst-case magnetic field changes at the earth by exoatmospheric nuclear explosions.
2) Determining the earth conductivity profile.
3) Calculating the worst case voltages induced in specific long-line systems (with particular grounding, connectivity, and geology).

The objective of this effort relates to the second step: the measurement of the earth conductivity profile over a depth range determined by the skin depth in the .01 to 10 Hz frequency range. Similar methods are also useful at higher frequencies (and shallower skin depths) to evaluate the voltages induced in long cables by nuclear surface bursts. This report will address primarily the ELF problem of MHD EMP.

Since the MHD EMP wave penetrates the earth to one skin depth the resistivity must be known to this depth. To measure the bulk resistivity of the earth one must induce a current in the earth and measure the voltage drop along the current path. There are several ways currents can be generated in the earth. The naturally varying magnetic field around the earth (the magneto-telluric effect) induces current in the earth. Currents can also be generated artificially by creating an electric field in the earth. Currents generated both naturally and artificially have been used to measure the earth's resistivity.

The characteristics of the earth must be understood to develop resistivity measurement methods that will give meaningful results. The characteristics presented here include skin depth considerations, the magneto-telluric field, the inhomogeneous nature of the earth and the effect of high conductors on the earth's surface. The two methods most commonly used to measure the earth's resistivity are the magneto-telluric and the four electrode probe methods. These and a third method developed by Mission Research Corporation (MRC) are presented. The advantages and limitations of each method are discussed considering the nature of the MHD EMP wave.
SECTION 2
EARTH CHARACTERISTICS

2.1 SKIN DEPTH CHARACTERISTICS

The region of the earth where the resistivity must be known is determined by the depth to which an electric field at a particular frequency will penetrate the earth's surface. This depth of penetration is known as the skin depth and is the distance where the amplitude of the electric field drops to \( \frac{1}{e} \) of its value at the surface. The skin depth (in meters) for a sinusoidal wave is given by

\[
\delta = \frac{\sqrt{\rho \mu}}{\pi f}
\]  

(1)

where \( \rho \) is the resistivity in ohm-m, \( f \) is the frequency in Hz and \( \mu \) is the permeability, \( 4\pi \times 10^{-7} \) Henry/m. Skin depth curves are shown in Figure 1.

2.2 THE MAGNETO-TELLURIC FIELD

The magneto-telluric field is the time-varying portion of the earth's magnetic field. These fluctuations induce currents that flow in the earth and occur over periods ranging from milliseconds to centuries. A typical amplitude spectrum of magneto-telluric electromagnetic noise is shown in Figure 2. (Ref. 1). This noise is extremely random in character, i.e., noise at one particular frequency may only occur once over a long period of time. In Figure 3 (Ref. 1) typical plots of magneto-telluric current activity are shown. When designing techniques to measure resistivity with either magneto-telluric or artificial current sources, the nature of magneto-telluric noise must be considered.

Figure 1. Skin depth curves.
Figure 2. Typical spectrum of amplitudes of electromagnetic noise in the extremely low frequency (elf) range (Ref. 1).
Figure 3. Example of a magneto-telluric signal with transient micropulsation activity in the 10- to 40-sec period range. Bergen Park, Colorado, October 11-12, 1962 (Mountain Standard Time). The vertical scale is 10 mV per km per division.

2.3 THE INHOMOGENEOUS NATURE OF THE EARTH

Any measurements of the earth's resistivity is going to be an average over regions of varying resistivities up to one skin depth thickness. At higher frequencies (small skin depths) variations in water content and large rocks are responsible for variations in resistivity. The resistivity ranges of several rock types are shown in Figure 4 (Ref. 2). At lower frequencies (large skin depths) the earth's resistivity is more dependent on major geologic formations and underground streams. Figure 5 (Ref. 2) is a resistivity map of the United States at 10 KHz. Measuring the resistivity at a particular depth is a formidable task, however the MHD EMP wave averages the resistivity over one skin depth. The wave not only averages in depth but also in area. The resolution of the wave can be approximated as an area one skin depth deep by two skin depths wide. A meaningful measurement will be obtained if it simulates the MHD EMP wave in terms of its resolution and averaging properties.

2. "Experimental Study to Make MHD/EMP Effects Predictions from Geomagnetic Storm Data," GTE Sylvania meeting at DNA, 13 July 1979.
### Figure 4. Approximate resistivity ranges of some rock types.
The resistivity scale (horizontal) is logarithmic.

<table>
<thead>
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<th>$\Omega \cdot m$</th>
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<tr>
<td>Igneous rocks</td>
<td></td>
</tr>
<tr>
<td>Metamorphic rocks</td>
<td></td>
</tr>
<tr>
<td>Clay</td>
<td></td>
</tr>
<tr>
<td>Soft shale</td>
<td></td>
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<tr>
<td>Sand</td>
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<tr>
<td>Sandstone</td>
<td></td>
</tr>
<tr>
<td>Porous limestone</td>
<td></td>
</tr>
<tr>
<td>Dense limestone</td>
<td></td>
</tr>
</tbody>
</table>

**2.4 THE EFFECT OF HIGH CONDUCTORS ON THE EARTHS SURFACE**

Any good conductors (power lines, railroad tracks, etc.) within the resolution area of the MHD EMP wave will tend to short out a portion of the field and lower the electric field strength surrounding the buried cable of interest by lowering the effective resistivity of the earth. If the effective resistivity of a particular location is desired, then good conductors in the area are indeed a part of the picture and no correction for them is needed when making measurements; however, if the resistivity of the earth itself is desired, good conductors will cause a substantial error in the measurement. The highest electric fields will appear at the surface when the number of nearby good conductors is a minimum, for this reason an area of at least two skin depths wide with no, or very few, conductors is desired for an accurate measurement.
SECTION 3
THE FOUR ELECTRODE PROBE METHOD

One difficulty encountered when artificially inducing currents in the earth with electrodes is that there is a contact resistance between the earth and an electrode. In the equivalent circuit shown in Figure 6 the contact resistance \(R_c\) and the resistance of the earth \(R_e\) cannot be separated. The four electrode probe method solves this problem by setting up a dipole current field with two probes and measuring the voltage with the other two probes as shown in Figure 7.

Figure 6. Equivalent circuit showing earth contact resistance.

Figure 7. Four electrode probe geometry.
Contact resistance losses only occur where current flows at $C_1$ and $C_2$, not at $P_1$ and $P_2$ where virtually no current flows. The current flows radially in a hemispherical distribution out of $C_1$ and similarly into $C_2$ for earth of uniform resistivity, $\rho$. The current distribution and equipotentials are shown in Figure 8. The current density, $j$, at a distance, $r$, from $C_1$ is

$$j = \frac{I}{2\pi r^2} \quad (2)$$

where $I$ is the total current injected at $C_1$. The potential gradient is

$$\frac{\partial V}{\partial r} = \rho j = \rho I/2\pi r^2. \quad (3)$$

Integrating, the potential at distance $r$ is obtained

$$V = \frac{I\rho}{2\pi r} \quad (4)$$

The potential difference due to the current source $C_1$ between $P_1$ and $P_2$ is

$$V = -\frac{I\rho}{2\pi} \left( \frac{1}{C_1P_1} - \frac{1}{C_1P_2} \right) \quad (5)$$

Similarly the potential difference between $P_1$ and $P_2$ due to the current sink, $C_2$ is

$$V = -\frac{I\rho}{2\pi} \left( \frac{1}{C_2P_1} - \frac{1}{C_2P_2} \right). \quad (6)$$

Summing these two voltages the resultant voltage between $P_1$ and $P_2$ is,

$$\Delta V = \frac{I\rho}{2\pi} \left( \frac{1}{C_1P_1} - \frac{1}{C_1P_2} - \frac{1}{C_2P_1} + \frac{1}{C_2P_2} \right). \quad (7)$$
Figure 8. Current distribution and equipotentials for a uniform earth as viewed from above the surface.
Substituting \( K \) for the expression in the brackets and solving for the resistivity, \( \rho \), one obtains,

\[
\rho = 2\pi \frac{\Delta V}{IK}.
\]  

Several important assumptions have been made in the development of this solution and the method in general. The most important of these is that the earth under consideration is homogenous. Consider the possibility that the resistivity of the earth varied as shown in Figure 9 (Ref. 3). This lowers the current density at the probes, lowering the voltage and the calculated resistivity. This error could be substantial if the resistivity nearby is very low.

A second important assumption is that the injector probes, \( C_1 \) and \( C_2 \), are separated by less than one skin depth, resulting with a dipole field distribution. If the separation is less than one skin depth the current will not penetrate into the earth up to a skin depth. If the separation is larger than one skin depth the field distribution is no longer a simple dipole distribution. The current field distribution in a homogeneous medium for current probes separated by several skin depths is shown in Figure 10. Corrections can be made for a non-dipole current distribution, however for the separation required to simulate the skin-depths of the MHD EMP wave the earth is far from homogeneous and the result would still be in error. For these reasons the four electrode probe method is generally used to examine relative surface soil conditions for agricultural purposes. It would be a poor choice for this particular problem.

Figure 9. Lines of current flow (continuous curves) and equipotentials (dashed curves) due to a pair of point electrodes on the earth's surface. A good conductor will perturb the equipotentials and current flow as shown. For reasons of clarity the potential difference between adjacent equipotentials is not the same. If it were, the equipotentials would be tightly crowded near the electrodes and farther apart in the center.
Figure 10. Current distribution for current injection probes separated by several skin depths in a homogeneous media.
SECTION 4
THE MAGNETO-TELLURIC METHOD

The time varying magneto-telluric field induces current in the earth. If the potential difference developed along a current path and the magnetic field strength, $H$, can be measured, the resistivity to one skin depth can be calculated. Similar to the MHD EMP wave, the magneto-telluric field penetrates the earth one skin depth. This will, by Faraday's law, induce a slab of current in the earth one skin depth deep and many skin depths wide. The current density for this geometry is given by

$$j = \frac{2H}{(1.257 \times 10^3) \delta}$$

where $j$ is the current density in amperes/m$^2$, $\delta$ is the skin depth in meters and $H$ is the magnetic field strength in gammas.

By making the appropriate substitutions for $\delta$ and $j$ the resistivity is given by

$$\rho = \left(\frac{E}{H}\right)^2 \frac{0.2}{f}$$

where $E$ is the measured electric field strength in mV/km and $f$ is the frequency in Hz. Due to the vectorial relationship between $H$ and $j$, the magnetic field must be measured orthogonal to the electric field.

To assure that all signals are observed the arrangement of electrodes and magnetometer as shown in Figure 11 is used. (A magnetometer is
a device that measures the magnetic field.) This results in a resistivity in two different directions, $x$ and $y$ where

$$\rho_x = \left( \frac{E_x}{H_y} \right)^2 \frac{0.2}{f}$$

(11)

and

$$\rho_y = \left( \frac{E_y}{H_x} \right)^2 \frac{0.2}{f}.$$ 

(12)

Figure 11. Configuration for the magneto-telluric measurement method.
The magneto-telluric field is very noisy, as mentioned earlier, so special techniques must be used to interpret the data. The simplest method is to measure the period of various oscillations and use this for the frequency as required in equations (11) and (12). Keller (Ref. 1) does this with marginal success as shown in Figure 12 and Table 1. This data shows the limitations in obtaining data at a desired frequency, as the results are random in frequency. To obtain more data one can use a computer to take Fourier transforms of the wave forms. A typical resistivity spectrum as derived by taking transforms of the wave forms is shown in Figure 13 (Ref. 2). These data are inconclusive due to the noise on the spectrum.

One serious problem with the magneto-telluric method for this particular application is that the frequency range of the MHD EMP wave is in an area where the amplitude of the magneto-telluric noise is small, as shown in Figure 2. However, were it not for the random nature of the magneto-telluric noise, this method has significant advantages over the four electrode probe method. There is no problem with skin depth considerations, as the magneto-telluric field is wide spread, creating a slab of current. Since the resolution of this method is determined by the skin depth, as described earlier, the effects of higher conducting regions and high conductors more than one skin depth away form the point of measurement can be neglected. Considering the above, this method is seriously limited for this particular frequency range.
Figure 12. Record of magneto-telluric field variations used for computing apparent resistivity. The numbers indicate consecutive half-periods of the oscillations picked from the record.

Table 1. Keller's Data.

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<tr>
<th>Number of event</th>
<th>$H_y$ gammas</th>
<th>$E_x$ mV/km</th>
<th>$T$ sec</th>
<th>$\rho a$ ohm-m</th>
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Figure 13. Poker flat magneto-telluric data Z2 impedance estimate.
SECTION 5
AC MEASUREMENT METHOD

5.1 APPROACH AND DESIGN CALCULATIONS

The limitations of the four electrode probe and the magneto-telluric methods are of a different nature. This fact enables one to develop a measurement method which combines the two methods in such a way that the limitations of each method are eliminated. This is accomplished by generating a current field with electrodes separated by several skin depths at the lowest frequency of interest. The two injection points are connected by a straight conductor near the surface of the earth (much closer than a skin depth), but insulated from it. Since the conductor must be very long (~100 km) it is chosen to be an existing conductor, such as a communication wire or power line. Current is injected along the conductor and into the earth at various frequencies spanning the range of interest.

At a given frequency the current distribution in the earth will be as illustrated in Figure 10, i.e., far away from the injection points it will extend downward and sideways approximately one skin depth. Since the fields are falling off sideways from the conductor, the ratio of electric and magnetic field is not equal to the value for uniform planar excitation as given in equation 10. The values of $H_z$ and $E_x$ as a function of distance, $y$, away from the conductor at the surface of a uniformly conducting earth have been calculated by Patra and Mallick (Ref. 4). Their result is

\[
\frac{H_z}{H_0} = \frac{2}{\omega^2} \left[ 2 - 2 \sqrt{\mu} K_1(\sqrt{\mu}) - i\omega^2 K_0(\sqrt{\mu}) \right] \tag{13}
\]

\[
\frac{E_x}{E_{x_0}} = \frac{2}{\omega^2} \left[ 1 - \sqrt{\mu} K_1(\sqrt{\mu}) \right] \tag{14}
\]

where

\[H_0 = I/2\pi y\]
\[E_0 = -i\omega \mu I/2\pi\]
\[\rho = y\sqrt{\mu\omega}\]
\[\sigma = \text{earth conductivity}\]
\[\mu = \text{earth permeability}\]
\[\omega = \text{angular frequency}\]
\[I = \text{wire current}\]
\[K_0, K_1 = \text{modified Bessel functions}\]

\(H_0\) is the magnetic field that would exist around the wire at distance \(y\) in the absence of a conducting earth. \(E_0\) is simply a convenient normalization factor. Near the conductor, \((y < 1/\sqrt{\mu\omega})\) \(H_z\) approaches \(H_0\), independent of soil resistivity. Near the conductor \(E\) varies as \(\log y\) with a coefficient that is dependent on \(\sigma\). Figures 14 and 15 illustrate the variation of electric and magnetic fields with \(\rho\).

From these formulas the equivalent uniform resistivity of the earth can be derived in principal from measurements of the amplitude or phase of the electric or magnetic fields at any point in the region excited by the conductor. Limitations are imposed by nonuniformities in the excited region and the signal-to-noise ratio of the excitation.
Figure 14. $E/E_0$ vs. $p$. 
Figure 15. $H/H_0$ vs. $p$. 

\[ H/H_0 \] 

\[ p \]
The electric field in the earth near the conductor is $E = 10 E_0$, requiring a voltage between the ends of the conductor of $V_e = -i10\omega\mu IL/2\pi$, where $L$ is the conductor length in meters. In addition, the power supply must overcome the resistive loss in the conductor. Assuming a resistance of $0.4 \, \Omega/km$, this loss is $V_c = 4 \times 10^{-4} L$. At low frequencies ($\omega < 10 \, {s}^{-1}$) the resistive loss in the wire dominates the power supply voltage requirement.

A reasonable measurability criterion is that the electric and magnetic fields produced by the current induced into the earth should be at least equal to the natural magneto-telluric fields at the same frequency. If this condition is satisfied the lock-in technique can easily extract the signal due to current injection from the magneto-telluric background.

The magneto-telluric noise is represented by the curve $H(f)$ in Figure 2. This will introduce a magnetic field at the measurement location of $\Delta H = H(f) \sqrt{\Delta f}$, where $\Delta f$ is the measurement bandwidth. It will also produce an electric field of

$$\Delta E = \Delta H \sqrt{\frac{\mu_0}{\omega}}$$  (15)

The current must be sufficiently large to make the impressed field comparable to the magneto-telluric field. In order to deduce the soil resistivity from magnetic field measurements, it's necessary to measure $H$ at large enough distances, such that $H/H_0 < 0.8$, or $\rho > 1$. Thus, since for $\rho = 1$ then $y = 1/\sqrt{\mu_0\sigma}$

$$1 > H(f) \sqrt{\Delta f} \ (2\pi y)$$

$$> 2\pi H(f) \sqrt{\frac{\Delta f}{\sigma\mu_0}}$$  (16)
The electric field can be measured at any distance from the wire to determine the resistivity. Its value is $\approx \mu I / 2\pi$. Thus, for a signal noise ratio of at least

$$I > H(f) \sqrt{\frac{\mu \omega \Delta f}{2\sigma}} \left(\frac{2\pi}{\mu \omega}\right)$$

$$> \frac{\pi H(f)}{\sigma \mu \omega} \sqrt{\frac{2\Delta f}{\sigma \mu \omega}}$$

Since this is a factor of $\sqrt{2}$ less than the criterion on the current derived for magnetic field measurements, we will use the former criterion.

For example, if $\sigma = 10^{-3}$ s/m, $f = 0.1$ Hz, $\Delta f = 10^{-2}$ Hz, and from Figure 2, $H(f) = 0.05 \gamma$ at 0.1 Hz.

$$I > 2\pi \times 0.05 \times 8 \times 10^{-4} \sqrt{\frac{10^{-2}}{10^{-3} \times 4\pi \times 10^{-7} \times 2\pi \times 0.1}}$$

$$> 0.9 \, \text{A}$$

### 5.2 MEASUREMENT SYSTEM

A candidate measurement system is illustrated in Figure 16. Two components of the electric field and three components of magnetic field are measured as a check on uniformity and self consistency. This redundancy can also be used to unfold a measure of the spatial distribution of earth resistivity.

The measuring instruments are phase-locked to the current source to improve the signal-to-noise ratio. Each measurement is performed with and without current injected into the earth to check the signal-to-noise
Figure 16. Low frequency conductivity measurement.
ratio. The reference signal will be brought from the current source to the measurement location by a combination of telephone and local transmitter-receiver (radio or fiber optics) links.

Two types of phase locked detection systems will be used. Lock in amplifiers are used at frequencies greater than 0.1 Hz. At frequencies between 0.1 Hz and 0.01 Hz an analog to digital converter fed into magnetic tape will be used. The data will be unfolded using a computer at a later time. Low bandwidths achieved using these techniques are limited only by the length of the recording time.

The smaller bandwidth at the lowest frequency is achieved by using an analog to digital convertor fed into magnetic tape. The magnetic tape will record three axis of the magnetic field, two axis of the electric field and the reference signal. The A/D converter must handle 20 samples/s at 10 bits per sample. The tape must record 500 bits/s. This results with one 10,000 foot roll of tape per 24 hours. Bandwidths as low as $1.16 \times 10^{-5}$ Hz can be achieved. Figure 17 shows the layout of the measurement using magnetic tape. The magnetic tape will be analyzed using a computer. The analysis will involve averaging the in-phase signals over the time equal to the inverse of the bandwidth. From these values the resistivity can be calculated. The signals will also be analyzed in quadrature, 90 degrees out of phase with the signal. This will establish the noise within the bandwidth. If the magneto-telluric noise is large enough, the resistivity can also be calculated from its signal. It is expected that between 0.1 Hz and 10 Hz the noise will be so low that any resistivity measurement resulting from noise would not be very accurate. At lower frequencies the noise could rise to an acceptable level to act as either a check on the driver signal result or, if the noise is larger, a more accurate result. Either way, large, moderate or small noise, a meaningful measurement will be obtained.
Figure 17. Measurement station using magnetic tape.
This is not the only check on internal consistency in this measurement technique. Since the electric fields and phases are both measured each will generate a result which can be compared. Furthermore, the measurement can be performed at various locations in the excitation field.

5.3 POWER REQUIREMENTS

Table 2 presents the current and power required for electric and magnetic field measurements using, a separation between injection points of 10 x 6 in low conductivity ($\sigma = 10^{-3}$ S/m) earth. The voltage is dominated by the conductor resistance at these frequencies (assumed to be 0.4 Ω/km).

The power requirements are very modest down to $f = 0.1$ Hz. At 0.01 Hz the power limitations become high, although achievable with off-the-shelf power amplifiers. The power increases because the current must increase to overcome $H(f)$ and the conductor resistance is high. Some amelioration is possible by relaxing the requirement that the injection be separated by ten skin depths.

Table 2
Current and Power Requirements
$S:N = 1:1$; space = 10 6; $10^3$ Ω m earth

<table>
<thead>
<tr>
<th>Frequency (Hz)</th>
<th>Skin Depth (k/m)</th>
<th>$H(f) \gamma/Hz$</th>
<th>Injection Separation</th>
<th>$\Delta f (Hz)$</th>
<th>I(A)</th>
<th>V(V)</th>
<th>P(W)</th>
</tr>
</thead>
<tbody>
<tr>
<td>10</td>
<td>5</td>
<td>0.01</td>
<td>50</td>
<td>$10^{-2}$</td>
<td>0.02</td>
<td>0.4</td>
<td>0.008</td>
</tr>
<tr>
<td>1</td>
<td>16</td>
<td>0.002</td>
<td>160</td>
<td>$10^{-2}$</td>
<td>0.01</td>
<td>0.6</td>
<td>0.006</td>
</tr>
<tr>
<td>0.1</td>
<td>50</td>
<td>0.05</td>
<td>500</td>
<td>$10^{-2}$</td>
<td>0.9</td>
<td>180</td>
<td>160</td>
</tr>
<tr>
<td>0.01</td>
<td>160</td>
<td>0.5</td>
<td>1600</td>
<td>$10^{-4}$</td>
<td>2.8</td>
<td>1800</td>
<td>5000</td>
</tr>
</tbody>
</table>
5.4 LOCATION REQUIREMENTS

An area free of good conductors over the entire dimensions of the field probably does not exist. Fortunately, if there are smaller regions of the field free of good conductors, the measurement can be performed with little difficulty. This area must be one voltage probe separation long, along the injection electrode axis and two skin depths wide. Any conductors outside this area will have little to no effect on the measurement. Several such areas must exist in the field so that several measurements can be made.
To determine the response of a buried cable to an MHD EMP wave measurement method with the greatest accuracy must be chosen. The four electrode probe and the magneto-telluric methods have fundamental limitations. The AC method proposed by MRC overcomes these limitations. A survey should be conducted to locate conductors of adequate length and conductivity in a desirable location relatively free of interfering conductors. Once a good cable and location are found the AC measurement method could be performed.
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