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ADVANCED EXPERIMENTAL TECHNIQUES IN CRACK TIP MECHANICS

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ABSTRACT

Advanced experimental techniques in crack tip mechanics are discussed under three categories of 2- and 3-D linear elastic, 2-D elasto-plastic and 2-D dynamic fracture mechanics. Specific techniques which were discussed are acousto-elasticity, frozen stress-moire technique, isodyne photoelasticity, moire technique, laser speckle method, hybrid experimental-numerical analysis and caustic method.

INTRODUCTION

The experimental techniques for crack tip mechanics of the 1970's were governed by the practical requirements for determining accurately 2- and 3-D stress intensity factors in linear elastic fracture mechanics (LEFM). The extensive applications of three-dimensional frozen-stress phototelasticity [1], interferometry [2] and moire method [3] yielded static stress intensity factors for complex boundary value problems, such as a corner flaw at a nozzle-cylinder junction and at a through hole [4] and a compact specimen [5]. Dynamic stress intensity factors determined by the extensive use of dynamic photoelasticity [6, 7] and dynamic caustics [8, 9] provided considerable insight to the controversial criteria for dynamic fracture and crack arrest. Dynamic photoelasticity and dynamic caustics were also used to establish dynamic crack curving and branching criteria [10, 11], which are also applicable to static and quasi-static crack problems, and to repudiate the proposed fracture mechanics interpretation of the V-notched Charpy data [12]. These experimental techniques, which are constantly being improved to determine well-defined physical quantities, i.e., the stress intensity factors, have contributed to the credibility which linear elastic fracture mechanics commands, in postmortem failure analysis and life-time prediction of structural components.
The inevitable extensions of linear elastic fracture mechanics to fracture of composites, fatigue crack extension and stable crack growth as well as ductile fracture have imposed a new role onto the above experimental techniques. The experimental results are now also used to identify the physical laws and associated physical parameters governing these fracture phenomena. The search for these unknown physical parameters requires increased experimental accuracy as well as advanced data processing technique in the presence of geometric and material nonlinearities encountered in nonlinear fracture mechanics.

The purpose of this paper is to review the advances made in the established experimental techniques as well as to report on new experimental techniques for analyzing the traditional as well as new problems in crack tip mechanics. The techniques are discussed under three categories of 2- and 3-D linear elastic, 2-D elasto-plastic and 2-D dynamic fracture mechanics.

2-D Linear Elastic Fracture Mechanics

Acousto-elasticity

Acousto-elasticity, which was hailed as an analog to photoelasticity for opaque materials in 1959 [13], failed to achieve wide acceptance due to the unresolved transducer coupling effect and high sonic attenuation [14]. The resurgence of acousto-elasticity in the 1980's is due in parts to improvements in the instrumentation techniques but is mainly attributed to the ability for processing large amounts of ultrasonic data by a computer-controlled scanning system [15]. Since longitudinal ultrasonic waves provide information only on plane-stress isopachics (sum of principal stresses), the use of shear waves measurements, which are referred to as acoustic birefringence, is more widely used today. Influence of the inherent acoustic anisotropy (texture) in the material can be modeled by orthotropic elasticity theory which involves three acousto-elastic constants [16, 17]. The acoustic birefringence equation becomes

\[ B = \left( [B_0 + M_1 (\sigma_1 + \sigma_2) + M_2 (\sigma_1 - \sigma_2) \cos 2\theta]^2 + [M_3 (\sigma_1 - \sigma_2) \sin 2\theta]^2 \right)^{1/2} \]  

(1)
where $B_0$ is the initial birefringence of the unstressed state. $M_1$, $M_2$ and $M_3$ are the three acousto-elastic constants.

The angle between the initial and stressed acousto-elastic axes, which in general do not coincide with the principal stress axes, is

$$\tan 2\phi = \frac{M_3(\sigma_1 - \sigma_2)\sin 2\theta}{B_0 + N_1(\sigma_1 + \sigma_2) + M(\sigma_1 - \sigma_2)\cos 2\theta} \quad (2)$$

The shear stress in the $xy$ plane is then given by

$$\sigma_{xy} = \frac{B \sin 2\phi}{2M_3} \quad (3)$$

Clark, Mignogna and Sanford [18] used the above relations to measure the stress intensity factor in a 2024-T351 aluminum compact specimen shown in Figure 1. A pulse-echo-overlap system, as shown in Figure 2, was used to determine point-by-point, the orientation of the acoustic axes and the acoustic birefringence in the $51 \times 51$ mm square region shown in Figure 1. A 10 MHz ac-cut quartz shear-wave transducer of 1.8-mm diameter was used in a manual scanning process. The estimated accuracy were approximately $5\%$ and $\pm 2$ degrees in birefringence and $\phi$ measurements, respectively.

The acousto-elastic birefringence generated from 66 data points in the square region was reconstructed and Sanford's procedure [19] was used to compute five coefficients in the LEFM crack tip stress field by averaging the results of 100 computations using 20 randomly selected data points each time. Good agreement between the corresponding coefficients, which were obtained from a similar photoelasticity experiment, were noted. The stress intensity factor was computed from the coefficient of the first term, or the $1/\sqrt{r}$ term, in the above polynomial crack tip stress field.

The acousto-elastic technique is one of the few static, stress analysis
techniques available for opaque materials. As in 2-D photoelasticity, the thickness-averaged acoustic birefringence is not subject to the plane stress constraint of the caustic method. Obvious improvement in the technique can be made by incorporating an automated scanning procedure with real-time data processing which has been used by others [15]. Yet to be explored is the physical significance of acoustic birefringence associated with the crack tip plastic region associated with ductile fracture.

3-D Linear Elastic Fracture Mechanics

Frozen Stress-Moire Technique

The hybrid technique, which utilizes both frozen stress, 3-D photoelasticity and moire interferometry, provides the complete information for characterizing the crack tip state [20]. The procedure is redundant in that the in-plane displacement field, which is determined by the high resolution moire technique [21], also defines the strain and stress fields. The isochromatics, however, can be used to verify the accuracy of the stresses which are obtained by numerically differentiating the displacements. Such optimum use of the redundant experimental data is yet to be explored.

The procedure consists of applying an aluminum reflective grating to the slices cut from the frozen-stress 3-D photoelastic model and returning the slice to its unloaded stage by annealing through its critical temperature. The in-plane displacements are obtained by moire interferometry of the deformed grating superimposed onto an undeformed virtual grating with a grating density of 2400 lines per mm. Figure 3 shows experimental setup for viewing the Moire fringes. The in-plane displacements of \( u_n \) and \( u_z \) are related to the stress intensity factor by:

For plane strain,

\[
\begin{align*}
\frac{u_n}{G} &= \frac{K_{AP}}{\sqrt{2\pi r}} \cos \frac{\theta}{2} \left[ 1 - 2\nu \sin^2 \frac{\theta}{2} \right] \\
\frac{u_z}{G} &= \frac{K_{AP}}{\sqrt{2\pi r}} \sin \frac{\theta}{2} \left[ 2 - 2\nu \cos^2 \frac{\theta}{2} \right]
\end{align*}
\]

(4)
For plane stress,

\[
\begin{align*}
  u_n &= \frac{K_{AP}}{G} \sqrt{r} \sin \frac{\theta}{2} \left[ \frac{1 - v}{1 + v} + \sin^2 \frac{\theta}{2} \right] \\
  u_z &= \frac{K_{AP}}{G} \sqrt{r} \sin \frac{\theta}{2} \left[ \frac{2}{1 + v} - \cos^2 \frac{\theta}{2} \right]
\end{align*}
\]

(5)

where \( G \) is shear modulus of elasticity, \( v \) is Poisson's ratio.

The photoelastic-moire technique was used to determine the variation in stress intensity factor along a straight crack front in a four-point bend specimen of 279.4 \( \times \) 25.7 \( \times \) 13.3 mm size after ASTM E399. Figure 4 shows the stress intensity factors at the center slice of this cracked beam determined by both photoelasticity and moire interferometry for a crack depth to beam ratio of 0.5. The reference \( K_{th} \) in Figure 4 was determined by 2-D plane strain analysis [22]. Figure 5 shows the variation of stress intensity factor through the thickness of the beam. A state of plane stress and the presence of a \( 1/\sqrt{r} \) singularity were assumed in the data reduction process.

While the uncertainties in the relaxation mechanism as well as the resultant state of stress associated with the annealing process require further studies, the frozen stress-moire techniques provides a mean for complete and detailed stress analysis of the crack tip state in 3-D linear elastic fracture mechanics.

Isodyne Photoelasticity

Isodyne represents curves of constant intensity of the normal forces acting on the characteristic curves in a plane stress field and are thus related to the first derivatives of the Airy stress function. Two isodyne fields related to two orthogonal characteristic curves completely define the elastic state of plane stress [23]. When modeled optically with the integrated polariscope, shown in Figure 6 [24], the photoelastic isodynes resemble...
the isochromatics generated by scattered light photoelasticity. Similar to scattered light photoelasticity, optical inhomogeneity generated by the high stress gradient in the vicinity of the crack tip may distort the photoelastic isodyne. The requirement for a plane stress state, which is not a prerequisite in scattered light photoelasticity, can be modulated by the "semi-plane stress state" used by Pindera et al. [25] who then determined the stress intensity factor at the midsection, i.e. plane of symmetry, of a four-point bend specimen shown in Figure 7. Also shown in Figure 7 is the variation in the stress intensity factor computed for various crack tip distance where a pronounced effect of the near-tip nonlinearity and crack tip bluntness are noted.

Assuming that the influence of optical inhomogeneity in the scattered light path can be quantified, the photoelastic isodyne technique share the same advantage of 3-D scattered photoelasticity which can be used to analyze the crack tip state of stress under live load. The stress intensity factor can be computed more accurately if K is expressed directly in terms of the isodyne value thus eliminating the extra numerical differentiation process in obtaining the stresses.

2-D Elasto-Plastic Fracture Mechanics

The experimental techniques listed in this section obviously can be used for elastic analysis but unlike the above, are not limited to elastic analysis.

Moire Technique

The use of moire technique in elasto-plastic fracture mechanics is not new [3, 26]. Despite its obvious application to high temperature, nonlinear problems in fracture mechanics, literature is relatively sparse in the fracture mechanics interpretation of the crack tip displacement field determined by the moire method. Exception to the above is the analysis of externally notched rings sliced from a Type 304 stainless tube, 7.1-mm O.D. and 0.38 mm thick, with electro-etched cross-line gratings of 40 lines per mm and subjected to a simulated internal pressure at 1100 F [27]. Figure 8 shows the experimental setup for recording the distorted grating which was analyzed by master gratings of 4 and 8 lines/mm. From the resultant u and v moire fringe patterns, COD for slow-crack growth initiation was found to be
COD = 0.976 \cdot a \cdot \sigma^{5.78} \tag{6}

where the crack length \(a\) and the applied hoop stress \(\sigma\) are represented in terms of mm and KN, respectively.

Figure 9 shows that the initiation COD in this experiment remained relatively constant despite the changes in the crack tip bluntness. Sciammarella then estimated the J-integral for the initiation of slow crack growth by the following approximate formula after Rice et al. [28].

\[
J_{in} = \frac{1}{b \cdot t} \int_{0}^{bcr} P d\delta_{cr} - P r\delta_{cr} \tag{7}
\]

where \(b\) is the ligament length, \(t\) is the specimen thickness, \(\delta_{cr}\) is the displacement due to the presence of the crack between two reference sections for the load at the moment of crack initiation and

\[
p = \sigma A \tag{8}
\]

where \(\sigma\) is the hoop stress and \(A\) the specimen cross-sectional area. The values of \(\delta_{cr}\) were obtained as

\[
\delta_{cr} = \delta_{total} - \delta_{nocr} \tag{9}
\]

where \(\delta_{total}\) is the displacement between two reference cross sections and \(\delta_{nocr}\) is the displacement given by

\[
\delta_{nocr} = \Delta \epsilon_{h} \tag{10}
\]

Moire method, which was limited in its applications to fracture problems involving large scale yielding due its low sensitivity, can be used in the high sensitivity region of linear elastic fracture mechanics by the recent developments in high density line gratings upto 4000 lines per mm with grating sizes upto 100 x 63 mm [29]. The use of virtual grating, which was described previously, eliminates the need for physical contact of the reference grating. Its use at elevated temperature testing, such as that described above, or under an explosive loading condition may be in doubt since the long optical paths, which is required in the experimental setup, may be distorted by the
Table 1 shows the excess variations in the J estimated by this procedure thus leading this author to conclude that COD is a better criterion for predicting the initiation of slow crack growth.
moving air current or shock waves.

The moire fringes can be generated by holographic interferometry. Referred to as "intrinsic holographic moire", these fringes can be recorded by using the basic setup shown in Figure 10 [30]. The reference state is obtained by a single exposure of the unloaded specimen. Rigid body motions of the loaded specimen are compensated by displacing the reference state and observing the fringe contrast in the TV monitor. The $u$ and $v$ fringe patterns are recorded on tape or alternatively photographed directly.

Laser Speckle Method

Despite its many implied applications in fracture mechanics [31, 32], literature is void of useful data which has been generated by the speckle method. With its high sensitivity, i.e. $u$ and $v$ displacement measurements of the order of 0.005 mm, the laser speckle method should find wide ranging applications in experimental fracture mechanics. By using the digital imaging technique [33, 34] to cross correlate the two speckle images generated by the unloaded and loaded specimens, the method provides an efficient procedure for processing the immense amount of data and for easy access to graphic peripherals.

Hybrid Experimental-Numerical Analysis

One of the major obstacles, which hinders the progress of experimental ductile fracture research, is the undefined crack-tip states of stress and strain in the presence of large scale yielding. Since the $1/\sqrt{r}$ singular state in linear elastic fracture mechanics is a physical impossibility which successfully models brittle fracture, similar phenomenological model could be developed for a crack under large scale yielding. A popular and possibly over-exploited such model is the Dugdale strip yield zone which conveniently reduces the elastic-plastic crack-tip state to an elastic one. The Dugdale strip yield model used in a recent analysis [37] is a modification of the classical Dugdale model where higher order terms were added to increase the number of disposable parameters. Experimental data is then used to fit the disposal parameter associated with the Dugdale model, which is modified to fit the complex state associated with large scale yielding, just as the stress intensity factor is determined from photoelasticity and moire fringe data. The adequacy of such model can be verified by the matching other crack-
tip data which is not used in the fitting process but which is generated numerically by the Dugdale model and independently by the experiment. The extensive numerical experimentation necessary for this verification study in essence replaces the finite element or boundary element method used in the traditional hybrid experimental-numerical stress analysis technique [36]. The verified modified Dugdale model through the generation mode of hybrid experimental-numerical analysis can then be used to generate numerically various fracture parameters for evaluation.

The utility of the hybrid experimental-numerical analysis is demonstrated by a recent investigation on stable-crack growth under mixed-mode loading [37]. Isochromatics in a 1.6-mm thick polycarbonate tensile specimens with central slanted crack were recorded during a continuing stable crack growth period. The resultant Z-shaped crack was modeled by a straight Dugdale crack, which was modified to account for the residual stresses left behind in the wake of the rapidly extending crack, as shown in Figure 11. The modification consisted of two unknown tangential forces acting at the physical crack tip. Lengths of the Dugdale strip yield zones ahead of the crack tip were measured from the photoelastic records [37]. These lengths coincided with the length of the theoretical values of the horizontal crack thus justifying the use of the model of Figure 11 to represent the Z-shaped cracks. The crack-tip stress field which is represented by a polynomial stress function of the crack-tip coordinates together with the two unknown tangential forces were fitted to the recorded elastic isochromatics surrounding the plastic region using an overdeterministic fitting routine [38]. Figure 12 shows the near- and far-field isochromatics which were regenerated by using the modified Dugdale model and those obtained by photoelasticity. Figure 13 shows the crack tip opening angle (CTOA), which was computed by using the modified Dugdale model, for the two initial crack geometries to be almost constant during the stable crack growth process.

While the hybrid experimental-numerical technique may not provide the micromechanics insight to crack-tip mechanics, it can be used to effectively extract fracture parameters which otherwise cannot be measured directly.

Caustic Method

The method of caustics is becoming a popular technique for measuring the static and dynamic stress intensity factors for plane-stress problems in
linear elastic fracture mechanics. Caustic can also be generated by any
deformed specimen surface including the obvious dimpling surrounding a ductile
-crack. Rosakis and Freund [39] used an asymptotic elastic-plastic analysis
to relate this dimpling to a plastic intensity factor. By postulating an HRR
singularity, J-deformation theory of plasticity and the separation of theta
and r, the plastic strain in the thickness direction is obtained as

$$\varepsilon_{33}^p = - (\varepsilon_{rr}^p + \varepsilon_{\theta\theta}^p)$$  \hspace{1cm} (11)

where the in-plane plastic strain components are given in terms of the stress
components as

$$\varepsilon_{rr}^p = \frac{\alpha_0}{E} \left[ \frac{JE}{\alpha_0^2 n r} \right] \frac{n}{n+1} \left( \frac{\sigma_e}{\sigma_0} \right)^n \left( \Sigma_{rr} - \frac{1}{2} \Sigma_{\theta\theta} \right)$$  \hspace{1cm} (12)

The resultant caustic generated by the thickness direction strain of equation
(12) is shown in Figure 14. J-integral value can then be determined by

$$\varepsilon_{\theta\theta}^p = \frac{\alpha_0}{E} \left[ \frac{JE}{\alpha_0^2 n r} \right] \frac{n}{n+1} \left( \frac{\sigma_e}{\sigma_0} \right)^n \left( \Sigma_{\theta\theta} - \frac{1}{2} \Sigma_{rr} \right)$$  \hspace{1cm} (13)

where \(z_0\), \(d\) and \(\sigma_0\) are the screen distance, specimen thickness and tensile
yield stress, respectively.

While further verification study is necessary, the caustic method
promises to provide an experimental procedure with which, the J-value can be
determined directly using crack tip measurements in contrast to the ASTM
designated far-field procedure which is based on many simplifying assumptions.

2-D Dynamic Fracture Mechanics

As mentioned in the Introduction, literature is abundant with experimental results on linear elastic dynamic fracture using dynamic photoelasticity and dynamic caustics. Experimental as well as data processing procedures for these two techniques are continually being improved and their domain of application is being extended. One such extension is the use of the hybrid experimental-model analysis for modeling the Dugdale strip yield zone ahead of a rapidly tearing crack [37]. Likewise, the caustic method with its asymptotic elastic-plastic solution could be extended with relative ease to analyze problems involving rapid tearing.

CLOSING COMMENTS

While no claim is made for completeness, most of the significant new experimental techniques for crack tip mechanics hopefully have been mentioned in this paper. The potential of applying some of the 2-D techniques, which were listed under specific fields in crack tip mechanics, to other fields obviously must be explored.

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REFERENCES


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Fig. 1 Schematic diagram of the modified compact tensile specimen. The region where acoustic measurements were made is labelled 'area of interest'. Also shown are the initial acoustic axes, $X_0$ and $Y_0$.

Fig. 2 Block diagram of pulse-echo-overlap system.
Fig. 3 Setup for Moire interferometry.

Fig. 4 Comparison of Photoelastic and Moire Results.
Fig. 5 SIF distribution across the thickness (a/W = 0.50)

Fig. 6 Integrated Polariscope for isodyne photoelasticity
MATERIAL: HOMALITE 100

\[ l_0 = 160 \quad h = 30 \]
\[ l_1 = 140 \quad b = 9.5 \]
\[ l_2 = 110 \quad a = 3 \]

MEASUREMENT PLANE: \((x, y, \varnothing)\)

\[ K_t \left[ \text{Nmm}^{-3/2} \right] \]

\[ K_t = K_t(r) \]

\[ (K_t)_{th} = 55.2 \text{Nmm} \]

\[ \frac{1}{3} K_t = \sigma_{xx}(2\pi r)^{1/2} \]

\[ r \left[ \text{mm} \right] \]

Fig. 7 Four point bend beam with sharp notch. \( K_t(r) \) for central plane was determined by isodyne technique.

Fig. 8 Optical setup for high-temperature studies.
Figure 9  C.O.D. for the initiation of slow crack growth as a function of original notch-root diameter.

Fig. 10 Schematic representation of the recording system in image plane holography.
Fig. 11 Modified Dugdale strip yield model.
Fig. 12 Computer generated and actual (solid curves) Isochromatics 30° SCN specimen
Fig. 13 CTOA during stable crack growth of 0° CN and 30° SCN.

Fig. 14 Geometric construction of the predicted initial curve (dashed) and caustic curve (solid).
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