AN APPLE COMPUTER PROGRAM FOR THE ANALYSIS OF COMPOSITE LAMINATES

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Universal Energy Systems, Inc.
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This technical report has been reviewed and is approved for publication.

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FOR THE COMMANDER

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AN APPLE COMPUTER PROGRAM FOR THE ANALYSIS OF COMPOSITE LAMINATES

The proposed numerical code which is based on lamination plate theory is capable of determining characteristics of general laminates with cores. The laminate may be subjected to mechanical or hygrothermal loadings. The main features of this program are:

a) calculating stiffness and compliance matrices;

b) calculating effective stresses and moments resulting from temperature or moisture content change;
The program is in Applesoft and can be executed from an Apple computer terminal. It is saved on a disk* obtainable from Dr. S. W. Tsai, AFWAL/MLBM, Wright-Patterson AFB, Ohio 45433, Tel: (513) 255-3068. Material properties for five commonly used composites are stored in the program. Output is displayed on a CRT screen as well as on a "hard copy" using a surface printer.

* Different disks are available for Apple I and Apple II computers.
FOREWORD

This report is an inhouse effort conducted in the Mechanics and Surface Interactions Branch, Nonmetallic Materials Division, Materials Laboratory, Air Force Wright Aeronautical Laboratories, Wright-Patterson Air Force Base, Ohio, (AFWAL/MLBM), under the Visiting Scientist program with Universal Energy Systems, Inc., Air Force Contract #F33615-82-C-5001. This work was performed during the period of Oct. 82 to Dec. 82.

The writer is grateful to Dr. S.W. Tsai for supporting this work and for valuable discussions.
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I. INTRODUCTION

The behavior of a composite laminate depends on variety of characteristics including stiffness, strength and behavior under environmental changes. The large number of parameters and the extensive amount of calculations involved in the characterization of composite laminates suggest the use of electronic computers. Algorithms for solutions of laminate problems in various computing facilities are given in [1-3]*. In the present work an algorithm for the solution of the general laminate shown in Figure 1 is provided using an Apple computer.

A review of relevant equations is provided in Section II which includes modulus and compliance analysis, hygrothermal effects, strain computation, and strength analysis. This material is based on a book by S. W. Tsai and H. T. Hahn, [4].

Instruction for program running and control is given in Section III. This includes data input procedure and printout control.

---


II. REVIEW OF EQUATIONS

A short review of relevant equations is given in this section. For a detailed derivation the reader is referred to reference 4.

1. Modulus and Compliance Analysis

With deformation prescribed, the effective loads are found from

\[
\begin{bmatrix}
N_1 \\
N_2 \\
N_3 \\
M_1 \\
M_2 \\
M_3
\end{bmatrix} =
\begin{bmatrix}
A_{11} & A_{12} & A_{13} & B_{11} & B_{12} & B_{13} \\
A_{22} & A_{23} & B_{21} & B_{22} & B_{23} & 0 \\
A_{33} & B_{31} & B_{32} & B_{33} & 0 & 0 \\
0 & D_{11} & D_{12} & D_{13} & 0 \\
0 & D_{22} & D_{23} & 0 & 0 \\
0 & 0 & D_{33} & 0 & 0
\end{bmatrix}
\begin{bmatrix}
\varepsilon_1^0 \\
\varepsilon_2^0 \\
\varepsilon_3^0 \\
k_1 \\
k_2 \\
k_3
\end{bmatrix}
\]

or

\[
\begin{bmatrix}
N \\
M
\end{bmatrix} =
\begin{bmatrix}
A & B \\
B & D
\end{bmatrix}
\begin{bmatrix}
\varepsilon^0 \\
k
\end{bmatrix}
\]

\( (1.1) \)

where \( \varepsilon_i^0 \) and \( k_i \), \( i = 1-3 \), are mid-surface strain and curvature components, respectively, and \( N_i, M_i, i = 1-3 \), are average forces per unit length and average moments per unit length, respectively. Referring to Figure 1 for notation, the stiffness matrices in (1.1) are given by

\[
\begin{array}{|c|c|c|c|}
\hline
 & V_{0x} & U_2 & U_3 \\
\hline
x_{11} & U_1 & V_{1x} & V_{2x} \\
x_{22} & U_1 & -V_{1x} & V_{2x} \\
x_{12} & U_4 & -V_{2x} & -V_{2x} \\
x_{33} & U_5 & -V_{2x} & -V_{2x} \\
x_{13} & V_{3x/2} & V_{4x} & -V_{4x} \\
x_{23} & V_{3x/2} & -V_{4x} & -V_{4x} \\
\hline
\end{array}
\]

\( (1.2) \)

* For convenience the axis "6" in reference 4 is replaced in this work by "3"
where
\[ [V_{iA}, V_{iB}, V_{iD}] = \int_{-h/2}^{h/2} \phi \cdot f_i[z, z^2]dz, \ i = 0-4 \]

\[ f_0 = 1, \ f_1 = \cos 2\theta, \ f_2 = \cos 4\theta, \ f_3 = \sin 2\theta, \ f_4 = \sin 4\theta \]

\[ \phi = 0 \ for -h/2 + M_0 \leq z \leq -h/2 + M_0 + h_c, \ \phi = 1 \ otherwise \ (1.3) \]

\( h_0, h_c, h = \) ply, core, and laminate thickness, respectively

\( M = \) number of plies below core

\[ U_1 = (3Q + 2Q_{xy} + 4Q_{ss}) / 8 \]
\[ U_2 = (Q_{xx} - Q_{yy}) / 2 \]
\[ U_3 = (Q - 2Q_{xy} - 4Q_{ss}) / 8 \]  
\( Q = Q_{xx} + Q_{yy} \)

\( Q_{xx} = m_0 E_x, \ E_x = \) longitudinal Young's modulus

\( Q_{yy} = m_0 E_y, \ E_y = \) transverse Young's modulus

\( Q_{xy} = Q_{yx} = m_0 E_y \nu_x, \ \nu_x = \) longitudinal Poisson's ratio \ (1.5)

\( Q_{ss} = E_s, \ E_s = \) longitudinal shear modulus

\[ m_0 = 1/(1 - \nu_x^2 E_y/E_x) \]

With the aid of (1.1), the deformation can be expressed in terms of effective loads

\[ \begin{bmatrix} \varepsilon^0 \\ \kappa \end{bmatrix} = \begin{bmatrix} \alpha & \beta \\ \beta^T & \delta \end{bmatrix} \begin{bmatrix} N \\ M \end{bmatrix} \]  
\( \alpha = A^{-1} - \beta B A^{-1}, \ \beta = -A^{-1} B S, \ S = (Q - BA^{-1} B)^{-1} \)  
(1.6)

It is possible to normalize (1.1) and (1.6) with respect to the total laminate thickness, h. The results are:
\[
\begin{bmatrix}
N_* \\
M_*
\end{bmatrix} =
\begin{bmatrix}
A^* & B^* \\
3B^* & D^*
\end{bmatrix}
\begin{bmatrix}
\varepsilon_1^0 \\
\kappa^*
\end{bmatrix}
\] (1.8)

\[
\begin{bmatrix}
\varepsilon_1^0 \\
\kappa^*
\end{bmatrix} =
\begin{bmatrix}
\alpha^* & \varepsilon^*/3 \\
\beta^* & \delta^*
\end{bmatrix}
\begin{bmatrix}
N_* \\
M_*
\end{bmatrix}
\] (1.9)

where

\[
A^* = \frac{A}{h}, \quad B^* = \frac{2B}{h^2}, \quad D^* = \frac{12D}{h^3}
\]

\[
\alpha^* = \alpha h, \quad \varepsilon^* = \frac{\varepsilon}{h^2}, \quad \delta^* = \frac{\delta}{h^3}/12
\]

\[
N^* = \frac{N}{h}, \quad M^* = \frac{6M}{h^2}
\]

\[
\varepsilon_1^0 = \varepsilon^0, \quad \kappa^* = \kappa h/2
\]

2. Hygrothermal Analysis

The effective loads generated by temperature change, $\Delta T$, and moisture content change, $C$, are determined using the following procedure:

(i) The nonmechanical strain components, $\varepsilon_1$, are given by

\[
\varepsilon_1 = \alpha_i \Delta T + \beta_i C, \quad i = x, y, \quad \varepsilon_5 = 0
\] (2.1)

where $\alpha_i$ and $\beta_i$ are coefficients of thermal expansion and swelling, respectively.

(ii) The stresses required to produce these strains, $\sigma_j^N$, are found from

\[
\sigma_j^N = Q_{jk}^N e_k, \quad J, k = x, y, \quad \sigma_5^N = 0
\] (2.2)

where the superscript "N" has been assigned to indicate nonmechanical stresses.
(iii) The on-axis stresses in (2.2) can be transformed to off-axis stresses using (2.3)

\[
\begin{array}{c|c|c}
\sigma_1^N & 1 & \cos 2\theta \\
\sigma_2^N & 1 & -\cos 2\theta \\
\sigma_3^N & \sin 2\theta & \\
\end{array}
\]

where \( p^N = (\sigma_x^N + \sigma_y^N)/2 \), \( q^N = (\sigma_x^N - \sigma_y^N)/2 \)

(iv) The effective nonmechanical forces and moments are given by

\[
\frac{h}{2} \left[ N_i^N, M_i^N \right] = \int_{-h/2}^{h/2} \phi \cdot \sigma_i^N [1,z] \, dz, \quad i = 1 - 3
\]

or

\[
\begin{array}{c|c|c}
N_1^N, M_1^N & V_{OA}, V_{OB} & V_{1A}, V_{1B} \\
N_2^N, M_2^N & V_{OA}, V_{OB} & -V_{1A}, -V_{1B} \\
N_3^N, M_3^N & V_{3A}, V_{3B} & \\
\end{array}
\]

where the \( V^{\phi} \) and \( \phi \) are defined in (1.3)

3. Strain Analysis

The object here is to determine on-axis and off-axis interlaminar strains from prescribed loadings (mechanical or nonmechanical).

Assuming a linear strain variation across the laminate thickness, i.e.

\[
\varepsilon = \varepsilon^0 + zk
\]
and using (1.6) in (3.1), the off-axis strains at z is given by

\[ \varepsilon = qN + \mathbf{M} + z \left( \mathbf{N}^T + \mathbf{M} \right) \]  

(3.2)

Next, the on-axis strains are found using the transformation in (3.3)

\[
\begin{array}{c|ccc}
   & p & q & r \\
\hline
\epsilon_x & 1 & \cos 2\theta & \sin 2\theta \\
\epsilon_y & 1 & -\cos 2\theta & -\sin 2\theta \\
\epsilon_s & -2\sin 2\theta & 2\cos 2\theta \\
\end{array}
\]

(3.3)

where \( p = (\epsilon_1 + \epsilon_2)/2, \ q = (\epsilon_1 - \epsilon_2)/2, \ r = \epsilon_3/2 \)

4. Strength Analysis

In this work laminate strength is examined using two failure criteria, i.e. the Tsai-Hill and the Maximum Strain.

In the maximum strain criterion failure is assumed when one of the six conditions below met first

\[
\epsilon_x, \epsilon_y, \epsilon_s > 0: (\epsilon_x, \epsilon_y, \epsilon_s) \mid \text{allowed} = \left( \frac{X}{E_x}, \frac{Y}{E_y}, \frac{S}{E_s} \right) \\
\epsilon_x, \epsilon_y, \epsilon_s < 0: - " - = \left( \frac{-\epsilon'}{E_x}, \frac{-\epsilon'}{E_y}, \frac{-S}{E_s} \right)
\]

(4.1)

where \( X \) and \( X' \) are longitudinal tensile and compressive strength, respectively, \( Y \) and \( Y' \) are transverse tensile and compressive strength, respectively, and \( S \) is the shear strength.
Defining strength ratio $R$ as
\[ R = \frac{\epsilon_i|\text{allowed}}{\epsilon_i|\text{imposed}}, \quad i = x, y, s \quad (4.2) \]
and assuming nonmechanical strain as well as mechanical strain exist, then, with superscript "M" assigned for mechanical strain
\[ \epsilon_i|\text{allowed} = R \epsilon_i^M + \epsilon_i^N - \epsilon_i \quad (4.3) \]
using (4.3) in (4.1), one has
\[ R = \min \left[ \left( \frac{X}{E_x} - \frac{\epsilon^N}{\epsilon_x} \right)/\epsilon_x^M, \left( \frac{Y}{E_y} - \frac{\epsilon^N}{\epsilon_y} \right)/\epsilon_y^M, \left( \frac{S}{E_s} - \frac{\epsilon^N}{\epsilon_s} \right)/\epsilon_s^M \right] \quad (4.4) \]
where $X, Y, S = X, Y, S$ for positive $\epsilon_i^M$, $i = x, y, s$
and $X, Y, S = -X', -Y', -S$ for negative $\epsilon_i^M$, $i = x, y, s$

In the Tsai-Hill criterion failure occurs when
\[ G_{ij} \epsilon_i|\text{allowed} \epsilon_j|\text{allowed} + G_i \epsilon_i|\text{allowed} - 1 = 0, \quad i, j = x, y, s \quad (4.5) \]
where the nonvanishing terms in (4.5) are
\[ G_i = F_j Q_{ij} \]
\[ G_{kl} = F_{ij} Q_{ik} Q_{j1}, \quad i, j, k, l = x, y \quad (4.6) \]
\[ G_{ss} = (Q_{ss}/S)^2 \]
\[ F_x = 1/X - 1/X', \quad F_y = 1/Y - 1/Y' \]
\[ F_{xx} = 1/(XX'), \quad F_{yy} = 1/(YY'), \quad F_{xy} = F_{xy} (F_{xx} F_{yy})^{1/2} \quad (4.7) \]
Introducing (4.3) in (4.5), one finds two roots for $R$, one positive and the other negative. Only the positive solution is given (the negative root corresponds to a reverse straining).
III PROGRAM CONTROL

The program language is in "Applesoft" ("BASIC" with some additions) and it is described in the Apple instruction manual. The program flow diagram is shown in Table I. Terminology for input and output data is given in Table II and computer memory allocation in Table III. The program listing and illustrative examples are shown in page 15 and 21, respectively. Program control and data input procedure are summarized below.

1. Running the program

With the disk inserted into the disk drive, the program "composite" is loaded automatically into the computer memory once the computer is turned on. Note that the disk contains a subprogram used for printing data in scientific format. This program is also automatically loaded into the computer memory.

2. Data Input Procedure

Data are inputted through both program line editing (before running the program) and computer keyboard during program run, according to the procedure outlined in Table I.

For convenience, material properties for five composites and aluminum are stored in the program according to the following scheme:

<table>
<thead>
<tr>
<th>Program Line</th>
<th>Material Type</th>
<th>Material Identification</th>
</tr>
</thead>
<tbody>
<tr>
<td>40</td>
<td>40</td>
<td>T300/5208 (graphite/epoxy)</td>
</tr>
<tr>
<td>50</td>
<td>50</td>
<td>B(4)/5505 (boron/epoxy)</td>
</tr>
<tr>
<td>60</td>
<td>60</td>
<td>AS/3501 (graphite/epoxy)</td>
</tr>
<tr>
<td>70</td>
<td>70</td>
<td>Scotchply 1002 (glass/epoxy)</td>
</tr>
<tr>
<td>80</td>
<td>80</td>
<td>Kevlar 49/epoxy (aramid/epoxy)</td>
</tr>
<tr>
<td>90</td>
<td>90</td>
<td>Aluminum</td>
</tr>
</tbody>
</table>
Material selection is achieved through keyboard by inputting the material type number in the table above. Other materials can be analyzed by introducing appropriate material properties in either program line 40 to 90.

Mechanical forces and moments on a per unit length basis are inputted in program lines 940 and 950, respectively. The current values are \( N_1 = 1/10^9 \, \text{M-GPa}, N_2 = N_3 = M_1 = M_2 = M_3 = 0 \). The strength parameter \( F^* \) is inputted in line 1580. Its current value is -0.5.

3. Printout Control

If a "hard copy" printout is desired the printer should be activated prior to running the program. The display and printing format requires that both the CRT screen and the printer page width should be set to at least 80 character.

For some applications a printout of all output data blocks indicated in Table I may be excessive. A selective output printout is possible using the CN(I), I = 1-8, array in program line 20, as described in Table I. For instance, if in program line 20 we have "CN(1)=0: CN(2)=0: CN(3)=0: CN(4)=0: CN(5)=1: CN(6)=0: CN(7)=0: CN(8)=0", then only on-axis strains will be printed. The current values are CN(I)=1, I = 1-8.

4. Program Pause

As indicated in Table I, after each block printout the program pauses to give the user an ample time to observe the output on the CRT screen. This is accompanied by cursor flashing. To resume computer operation press the "RETURN" key. To eliminate program pause the user should delete the "Get G$" statements in program lines 2130 and 2190.
Figure 1. Notation for General Laminate with Core.
TABLE I - FLOW DIAGRAM

NOTATION

- PROGRAM LINE DATA INPUT
- KEYBOARD DATA INPUT (TYPE THE INPUT VALUE AND THEN "RETURN")
- DATA PRINTOUT CONTROL PARAMETERS GIVEN IN PROGRAM LINE 20. CN[1]=1 FOR PRINT, CN[1]=0 FOR NO PRINT
- DATA BLOCK PRINTOUT
- PROGRAM PAUSES. PRESS "RETURN" TO CONTINUE

NOTE: SEE FIGURE 1 AND TABLE II FOR OTHER NOTATION
TABLE II
TERMINOLOGY FOR DATA INPUT AND OUTPUT

Data Input

C (C)* - moisture content

\( h_0, h_c \) (HO, HC) - ply and core thickness, respectively, (M)**

\( N, M \) (N, M) - number of plies in laminate and number of plies below core, respectively

\( N_i, M_i \) (N(I, O), M(I, O), I = 1-3) - effective mechanical force and effective mechanical moment components (on a per-unit length basis), respectively, (M·Gpa, M²·Gpa)

Material Type (MT) - see table in page 8

\( E_x, E_y \) (EX, EY) - longitudinal and transverse Young's modulus, respectively, (Gpa)

\( E_s \) (ES) - longitudinal shear modulus, (Gpa)

\( S \) (S) - longitudinal shear strength, (Gpa)

\( X, X' \) (X, XC) - longitudinal tensile and compressive strength, respectively, (Gpa)

\( Y, Y' \) (Y, YC) - transverse tensile and compressive strength, respectively, (Gpa)

\( F_{xy} \) (FXY STAR) - Parameter related to material strength (See (4.7))

\( \alpha_x, \alpha_y \) (AX, AY) - coefficient of thermal expansion along x and y direction, respectively, (1/K)

\( \beta_x, \beta_y \) (BX, BY) - swelling coefficient in x and y direction, respectively

\( \nu_x \) (PX) - longitudinal Poisson's ratio = \(-\epsilon_y/\epsilon_x\)

\( \Delta T \) (DT) - temperature difference, (K⁰)

\( \theta_i \) (O(I)) - orientation of \( i^{th} \) ply (Deg.)

Data Output

\( A, B, D \) (A, B, D) - stiffness matrices, (M·Gpa, M²·Gpa, M³·Gpa)

\( A^*, B^*, D^* \) (A*, B*, D*) - normalized stiffness matrices, (Gpa)

*Quantities in parenthesis indicate program variables

**Dimension : M - Meter, pa - paschall, Gpa - 10⁹pa, K⁰ - deg. Kelvin
<table>
<thead>
<tr>
<th>Term</th>
<th>Description</th>
<th>Unit(s)</th>
</tr>
</thead>
<tbody>
<tr>
<td>(N^M(N)), (N^N(NN))</td>
<td>Mechanical and nonmechanical force/unit length Vector, respectively, ((\text{M} \cdot \text{Gpa}))</td>
<td></td>
</tr>
<tr>
<td>(M^M(M)), (M^N(MN))</td>
<td>Mechanical and nonmechanical moment/unit length Vector, respectively, ((\text{M} \cdot \text{Gpa}))</td>
<td></td>
</tr>
<tr>
<td>(N^<em>M(N^</em>)), (N^<em>N(NN^</em>))</td>
<td>Normalized mechanical and nonmechanical force/unit length Vector, respectively, ((\text{Gpa}))</td>
<td></td>
</tr>
<tr>
<td>(M^<em>M(M^</em>)), (M^<em>N(MN^</em>))</td>
<td>Normalized mechanical and nonmechanical moment/unit length Vector, respectively, ((\text{Gpa}))</td>
<td></td>
</tr>
<tr>
<td>(\alpha, \beta, \beta^T, \delta) ((\text{ALPHA, BETA, TRBETA, DELTA}))</td>
<td>Compliance matrices, ((1/\text{M} \cdot \text{Gpa}, 1/\text{M}^2 \cdot \text{Gpa}, 1/\text{M}^2 \cdot \text{Gpa}, 1/\text{M}^3 \cdot \text{Gpa}))</td>
<td></td>
</tr>
<tr>
<td>(\alpha^<em>, \beta^</em>, \beta^{<em>T}, \delta^</em>) ((\text{ALPHA^<em>, BETA^</em>, TRBETA^<em>, DELTA^</em>}))</td>
<td>Normalized compliance matrices, ((1/\text{Gpa}))</td>
<td></td>
</tr>
<tr>
<td>(R) ((R))</td>
<td>Strength ratio</td>
<td></td>
</tr>
</tbody>
</table>
### TABLE III

COMPUTER MEMORY STORAGE

**Modulus and Compliance Components**

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>$X(I, J, 0)$</td>
<td>$A^*(I, J, 0)$, $I, J = 1-3$</td>
</tr>
<tr>
<td>$X(I, J, 1)$</td>
<td>$B^*$</td>
</tr>
<tr>
<td>$X(I, J, 2)$</td>
<td>$D^*$</td>
</tr>
<tr>
<td>$X(I, J, 3)$</td>
<td>$\alpha^*$</td>
</tr>
<tr>
<td>$X(I, J, 4)$</td>
<td>$\beta^*$</td>
</tr>
<tr>
<td>$X(I, J, 5)$</td>
<td>$\beta^T$</td>
</tr>
<tr>
<td>$X(I, J, 6)$</td>
<td>$\delta^*$</td>
</tr>
</tbody>
</table>

**Strain Components**

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>$E(k, I, 0)$</td>
<td>$\epsilon^M(I)$ in the $k^{th}$ ply</td>
</tr>
<tr>
<td>$E(k, I, 1)$</td>
<td>$\epsilon^N(I)$ in the $k^{th}$ ply</td>
</tr>
</tbody>
</table>

$I = 1-3$: on-axis strain components at lower ply surface

$I = 4-6$: on-axis strain components at upper ply surface

$I = 7-9$: off-axis strain components at lower ply surface

$I = 10-12$: off-axis strain components at upper ply surface

**Strength Ratio**

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
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</thead>
<tbody>
<tr>
<td>$R_l(k, o)$</td>
<td>$k^{th}$ ply, $R_{Tsai-Hill}$, lower surface</td>
</tr>
<tr>
<td>$R_l(k, 1)$</td>
<td>$k^{th}$ ply, $R_{Tsai-Hill}$, upper surface</td>
</tr>
<tr>
<td>$R_m(k, o)$</td>
<td>$k^{th}$ ply, $R_{Max}$ strain, lower surface</td>
</tr>
<tr>
<td>$R_m(k, 1)$</td>
<td>$k^{th}$ ply, $R_{Max}$ strain, upper surface</td>
</tr>
</tbody>
</table>
IV. PROGRAM LISTING

10 DIM CN(10)
20 CN(1) = 1;CN(2) = 1;CN(3) = 1;CN(4) = 1;CN(5) = 1;CN(6) = 1;CN(7) = 1;
    N(0) = 1
30 INPUT "MATERIAL TYPE, MT=";MT
40 EX = 181;EY = 10.2;FX = .29;ES = 7.17;HO = .000125;X = 1.5;XC = 1.5;Y = 
    .04;YC = .246;S = .068;AX = .02 / 1E6;AY = 22.5 / 1E6;BX = 0;BY = .0:
    IF MT = 40 THEN GOTO 100
50 EX = 294;EY = 18.5;FX = .29;ES = 7.17;HO = .000125;X = 1.26;YC = 2.5;Y = 
    .065;YC = .102;S = .067;AX = 6.1 / 1E6;AY = 30.3 / 1E6;BX = 0;BY = .0:
    IF MT = 50 THEN GOTO 100
60 EX = 138;EY = 9.95;FX = .29;ES = 7.17;HO = .000125;X = 1.447;YC = 1.447;Y = 
    .0517;YC = .205;S = .093;AX = -.3 / 1E6;AY = 28.1 / 1E6;BX = 0;BY = .0:
    IF MT = 60 THEN GOTO 100
70 EX = 30.6;EY = 0.27;FX = .29;ES = 4.14;HO = .000125;X = 1.02;YC = .21;
    Y = .031;YC = .119;S = .072;AX = 8.6 / 1E6;AY = 22.1 / 1E6;BX = 0;BY = .0:
    IF MT = 70 THEN GOTO 100
80 EX = 76;EY = 5.5;FX = .34;ES = 2.3;HO = .000125;X = 1.4;YC = .138;Y = 
    .034;S = .053;AX = -4.9 / 1E6;AY = .79 / 1E6;BX = 0;BY = .0:
    IF MT = 80 THEN GOTO 100
90 EX = 69;EY = 8.9;FX = .33;ES = 26.9;HO = .000125;X = .4;YC = .16;
    Y = .4;S = .33;AX = .22.4 / 1E6;AY = AX;BX = 0;BY = .0:
    IF MT = 90 THEN GOTO 100
100 MP = 1 / (1 - FX * PX * EY / EX)
110 DIM O(3,3)
120 A# = "####-####-####-
130 O(1,1) = MP * EX / O(2,2) = MP * EX / O(2,1) = MP * PX * EY
140 O(1,2) = O(1,1) * O(1,1) = ES
150 D = O(1,1) + O(1,2)
160 U1 = (3 * D + 2 * D(1,2) + 4 * D(3,3) / 8
170 U2 = (D(1,1) - D(2,2)) / 2
180 U3 = U1 - 2 * U1 + 4 * D(3,3) / 8
190 U4 = U2 + 6 * D(1,2) - 4 * D(3,3) / 8
200 U5 = U3 + 2 * D(1,2) + 4 * D(3,3) / 8
210 INPUT "NUMBER OF LISTS, N=";N
220 INPUT "NUMBER OF LISTS BETWEEN 2-N/2 AND CORE, M=";M
230 PRINT: PRINT "PLY ORIENTATION (FROM 2-N/2 TO 2-H/2)"
240 DIM O(60),P(60,4),X(3,3,7),V(4,2).M:IO0)
380 V = U5F2 = 1
390 FOR I = 1 TO N
400 INPUT "PLY ANGLE = ": O(I)
410 IF N = M THEN GOTO 330
420 IF I = M THEN PRINT "CORE, "; INT ((-N + 1 / HN) * 100 + .5) / 100
430 IF M THEN PRINT "PLY THICK"
440 O(I) = O(I) * 7.1415926535 / 180
450 P(I, 0) = 1
460 P(I, 1) = COS (2 * O(I))
470 P(I, 2) = COS (4 * O(I))
480 P(I, 3) = SIN (2 * O(I))
490 P(I, 4) = SIN (4 * O(I))
500 IF I > M THEN D = 1
510 W1(I) = - .5 + D * HC + (I - 1) * HN
520 NEXT I
530 FOR K = 0 TO 2: K1 = K + 1: IF K = 2 THEN FK = 4
540 FOR I = 1 TO M
550 IF K = 0 THEN TT = HN
560 IF K = 1 THEN TT = HN * (HN + 2 * W1(I))
570 IF K = 2 THEN TT = (W1(I) + HN) ^ 2 - W1(I) ^ 2
580 FOR J = 0 TO 4
590 V(J, I) = V(J, K) + P(I, J) * TT * FK
600 NEXT J
610 NEXT I
620 C1 = V(0, K) * U1: C2 = V(1, K) * U2: C3 = V(2, K) * U3: C4 = V(3, K) * U4 / 2: C5 = V(4, K) * U5
630 X(0, 1, K) = C1 + C2 + C3
640 X(1, 2, K) = V(0, K) * U4 - C3
650 X(1, 0, K) = C4 + C5
660 X(2, 1, K) = X(1, 2, K)
670 X(3, 2, K) = C1 - C2 + C3
680 X(3, 1, K) = X(1, 3, K)
690 X(1, 1, K) = X(2, 3, K)
700 X(2, 3, K) = V(0, K) * U5 - C3
710 NEXT K
720 LI = 0: LD = 9: GOSUB 2310
730 F = 1: LA = 9: LB = 1: LC = 7: GOSUB 2420
740 F = 1: LA = 1: LB = 9: LC = 8: GOSUB 2420
750 F = - 3: LA = 1: LB = 7: LC = 6: GOSUB 2420
760 LA = 2: LB = 6: LC = 6: GOSUB 2520
770 LI = 6: LD = 6: GOSUB 2510
780 F = - 3: LA = 7: LB = 6: LC = 4: GOSUB 2420
790 F = - 1: LA = 4: LB = 8: LC = 3: GOSUB 2420
800 LA = 9: LB = 3: LC = 3: GOSUB 2520
810 FOR I = 1 TO 3
820 FOR J = 1 TO 3
830 X(J, I, 5) = X(I, J, 4)
840 NEXT J
850 NEXT I
860 IF CN(1) = 0 THEN GOTO 900
790 X1 = 1: X2 = 1: H = 0: GOSUB 2220
795 X1 = 2: X2 = 1: H = 1: GOSUB 2220
800 IF CN(2) = 0 THEN GOTO 640
810 X$ = "ALPHA": Y$ = "BETA": Z$ = "(1/GPA)": W$ = "TRBETA": N$ = "DELTA": GOSUB 2150
820 X1 = 1: X2 = 1: H = 0: GOSUB 2220
830 X1 = 1: X2 = 1: H = 0: GOSUB 2220
840 IF CN(3) = 0 THEN GOTO 660
850 X$ = "A": Y$ = "B": Z$ = "C": W$ = "D": GOSUB 2150
860 X1 = H: X2 = H: H / 2: H = 0: GOSUB 2220
870 X1 = 0.5 * H: X2 = H: 3 / 12: H = 1: GOSUB 2220
880 IF CN(4) = 0 THEN GOTO 920
890 X$ = "ALPHA": Y$ = "BETA": Z$ = "TRBETA": W$ = "DELTA": GOSUB 2150
900 X1 = 1 / H: X2 = 2 / H: 2: K = 3: GOSUB 2220
910 X1 = 2 / H: 2: X2 = 12 / H: 3: K = 5: GOSUB 2220
920 X$ = "STRAIN ANALYSIS": Y$ = "Z": W$ = "": GOSUB 2150
930 DIM M(0, 1), M(1, 1), E(0, 12), S13(3, 1), S23(3, 1), ET(3)
940 N(1, 0) = 1 / E: N(2, 0) = 0: N(3, 0) = 0
950 M(1, 0) = v*M(2, 0) = 0: M(3, 0) = 0
960 FOR I = 1 TO 7
970 N(I, 0) = N(I, 0) / H
980 M(I, 0) = M(I, 0) / (H * H / 6)
990 NEXT I
1000 INPUT "TEMP. DIFFERENCE (IN K), DT=": DT; INPUT "MOISTURE CONTENTS, T =": C: PRINT
1010 ET(1) = AX * DT + BX * C: ET(2) = AY * DT + BY * C
1020 JJ = 1: IF ABS(DT) + ABS (C) = 0 THEN JJ = 0
1025 IF JJ = 0 THEN GOTO 1130
1040 SX = G(1, 1) * ET(1) + G(1, 2) * ET(2)
1050 SY = G(2, 1) * ET(1) + G(2, 2) * ET(2)
1060 FN = .5 * (SX + SY): ON = .5 * (SX - SY)
1070 N(1, 1) = FN * V(0, 0) + ON * V(1, 0)
1080 N(2, 1) = FN * V(0, 0) - ON * V(1, 0)
1090 N(2, 1) = ON * V(0, 0)
1100 M(1, 1) = T * (FN * V(0, 1) + ON * V(1, 1))
1110 M(2, 1) = T * (FN * V(0, 1) - ON * V(1, 1))
1120 M(3, 1) = T * ON * V(3, 1)
1130 PRINT FMB(8): "EFFECTIVE STRESSES": "EFFECTIVE MOMENTS"
1140 EJ(1) = "MECHANICAL": EJ(1) = "NON-MECHANICAL"
1150 A$ (1, 0) = "N": A$ (2, 0) = "M": A$ (3, 0) = "N": A$ (4, 0) = "M"
1160 A$ (1, 1) = "MN": A$ (2, 1) = "MN": A$ (3, 1) = "MN": A$ (4, 1) = "MN"
1170 FOR J = 0 TO JJ: X1 = 1: X2 = 1
1180 PRINT TAB(27): EJ(J)
1190 FOR L = 0 TO 1: PRINT A$ (1 + 2 * L, J)
1200 FOR L = 1 TO J: PRINT USEA$: X1 * N(I, J)
1210 NEXT I: PRINT "": A$ (2 + 2 * L, J) = X1 * H
1220 FOR I = 1 TO J: PRINT USEA$: X1 * H: M(I, J)
1230 NEXT I: X2 = X1 * H * H / 6: PRINT
1240 NEXT L
1250 NEXT J
17
1600 FOR P = 0 TO JJ
1610 GOSUB 2580: NEXT P
1620 FOR K = 1 TO N
1630 FOR L = 0 TO 1
1640 FOR J = 0 TO JJ
1650 FOR I = 1 TO J
1660 E(K, I + 3 * L, J) = S1(I, J) + 2 * (W1(K) + L * HN) * S2(I, J)
1670 E(K, I + 3 * L + 6, J) = E(K, I + 3 * L, J)
1680 NEXT I
1690 NEXT J
1700 FOR J = 0 TO JJ
1710 GOSUB 2660
1720 NEXT J
1730 NEXT L
1740 NEXT I
1750 PRINT: PRINT "PLY": TAB( 0 ); "LOWER PLY SURFACE": NEXT I: "UPPER PLY SURFACE"
1760 IF CN(5) + CN(6) = 0 THEN GOTO 1560
1770 Z%0,0) = "ON AXIS MECHANICAL STRAIN":Z%0,1) = "OFF AXIS MECHANICAL STRAIN":Z%1,0) = "ON AXIS NON-MECHANICAL STRAIN":Z%1,1) = "OFF AXIS NON-MECHANICAL STRAIN"
1780 FOR J = 0 TO JJ
1790 PRINT: FOR M6 = 1 - CN(6) TO CN(6): PRINT TAB( 20 ); Z%0, M6): NEXT I
1800 IF J = 0 TO 1
1810 FOR I = 1 TO 3
1820 PRINT USEA():E(K, I + 3 * L + M6 * 6, J): NEXT J: NEXT I: NEXT M6
1840 PRINT
1850 NEXT K
1860 NEXT J
1870 IF CN(7) + CN(8) = 0 THEN GOTO 2140
1880 DIM F(I,1):G(I, 3):EX(6):U(J):C(M60,1),R1(60,1),RM(60,1),F(3)
1890 EYSTART = -0.5
1900 F(I, 1) = 1 / (X * XC);F(2, 2) = 1 / (Y * YC)
1910 EY = 1 / X * Y / YC;F(1, 1) = F(2, 2);F(2, 1) = F(1, 2)
1920 EX = X / X;EX=EX(3) = Y / Y;EX=EX(3) = S / ES;EX=EX(4) = - X;EX=EX(5) = - Y;EX=EX(6) = - S / ES
1930 EX = F(I, 1) + FY * O(1, 2)
1940 GY = F(I, 2) + FY * O(2, 2)
1950 FOR K = 1 TO 2
1960 FOR J = 1 TO 2
1970 C(K,F) = 0
1980 FOR I = 1 TO 2
1990 FOR J = 1 TO 2
2000 C(K,F) = G(K,F) + F(I, J) * O(I,K) * O(J,F)
2010 NEXT J
2020 NEXT I
2030 NEXT F
2040 NEXT K
2050 C(3,3) = (O(3,3) / S) + 2
1760 FOR I = 1 TO N
1770 FOR L = 1 TO I
1780 A = 0: B = 0
1790 FOR I = 1 TO L: I = I + 3 * L: BB = 0
1800 IF E(I, L, O) = 0 THEN BE = 0
1810 UI(J) = E(K, IL, O) - IT(I): IF E(K, IL, O) = 0 THEN E(K, IL, O) = E(K, IL, O) + 1
1820 R(I) = R(I) + 80: U(I) = E(K, IL, O): IF I = 1 THEN GOTO 1840
1830 IF R(I) = R(0, L) THEN GOTO 1850
1840 R(I, L) = R(I) + CM(K, L) = I
1850 NEXT I
1860 FOR I = 1 TO 3: IL = I + 3 * L
1870 FOR J = 1 TO 3: JL = J + 3 * L
1880 A = A + G(I, J) * E(K, IL, O) * E(K, JL, O): IF JL = 0 THEN GOTO 1910
1890 B = B + G(I, J) * (E(K, IL, O) * U(J) + E(K, JL, O) * U(J))
1900 C = C + G(I, J) * U(I) * U(J)
1910 NEXT J
1920 NEXT L
1930 NEXT K
1940 X* = "STRENGTH ANALYSIS": Y* = "": Z* = "": W* = "": GOSUB 150
1950 FOR I = 1 TO 2
1960 PRINT " ": "TSAI-HILL": ";"MAX. STR.": ";"ST. COMP.":
1970 NEXT I
1980 PRINT " ": STR: 
1990 X* = H:K:(I) = "STRENGTH RATIO, R":K*(O) = "NORMALIZED STRENGTH RATIO", R"
2000 FOR K = 1 TO CH(7): ID CH(8)
2010 PRINT TAB(10)**:K*(O): PRINT
2020 FOR I = 1 TO N: PRINT I, TAB(5)
2030 FOR L = 0 TO 1
2040 * PRINT USEA*:R1(1, L) / X*: PRINT "; ": & PRINT USEA*:RM(1, L) / X*: PRINT " "; & CH(1, L) = A + 1; ; " ":
2050 NEXT L
2060 PRINT
2070 PRINT
2080 NEXT I
2090 M = 1: GET G*: NEXT K
2100 END
2110 PRINT
2120 PRINT "****************************************
2130 PRINT TAB(15):X; " "; Y; " "; Z * W *
2140 PRINT TAB(15):Z*; " "; W*
2150 GO TO 1
2160 PRINT
2170 RETURN
FOR I = 1 TO 3: XM = X1
FOR L = K TO K + 1: IF L = K + 1 THEN XM = X2
FOR J = 1 TO 3
PRINT USEA$: XM * X(1, J, L):
NEXT J
NEXT L
PRINT
NEXT I
RETURN
A = X(1, 1, L1): B = X(1, 2, L1): C = X(1, 3, L1): D = X(2, 2, L1): E = X(2, 1, L1)
F = X(3, 3, L1)
DET = A * (D * F - E * E) + B * (2 * E * C - F * B) - D * C * C
X(1, 1, LO) = (D * F - E * E) / DET
X(1, 2, LO) = (C * E - B * F) / DET
X(2, 1, LO) = X(1, 2, LO)
X(1, 3, LO) = (B * E - D * C) / DET
X(2, 2, LO) = (A * F - C * E) / DET
X(2, 3, LO) = X(3, 3, LO)
X(3, 1, LO) = X(1, 3, LO)
X(3, 2, LO) = (A * F - C * E) / DET
X(3, 3, LO) = X(3, 2, LO)
RETURN
FOR I = 1 TO 3
FOR J = 1 TO 3
SU = 0
FOR K = 1 TO 3
SU = SU + X(I, K, LA) * X(K, J, LB)
NEXT K
NEXT I
RETURN
X(I, J, LC) = F * SU
RETURN
V. ILLUSTRATIVE EXAMPLES

Problem #1

\[ t = t' - F \cdot \sin \theta \]

\[ \alpha = \beta \]

\[ \delta = \Delta \]

\[ G = 1 \]

\[ B = b \]

\[ \gamma, \lambda, \mu, \nu \]

\[ 0 = 0 \]
### THERMAL ANALYSIS

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<tr>
<th>Sample</th>
<th>Thermal</th>
<th>Mechanical</th>
</tr>
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<td>2.98E+08</td>
</tr>
<tr>
<td>2</td>
<td>5.78E+08</td>
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</tr>
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<th>Strength</th>
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22
Problem #2

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<tr>
<th>Ply Orientation (from Z = H/2 to Z = H/2)</th>
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<tr>
<td>Ply Angle 0°</td>
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<td>Ply Angle 45°</td>
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<td>Ply Angle 90°</td>
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23
**D**

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<tr>
<th>Alpha</th>
<th>Beta</th>
<th>Gamma</th>
<th>Delta</th>
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<tbody>
<tr>
<td>1.42E+04</td>
<td>1.09E+06</td>
<td>0.99E+00</td>
<td>3.92E+02</td>
</tr>
<tr>
<td>1.09E+00</td>
<td>0.99E+00</td>
<td>1.09E+06</td>
<td>1.42E+04</td>
</tr>
<tr>
<td>3.92E+02</td>
<td>1.42E+04</td>
<td>0.99E+00</td>
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**MEAN DIFFERENCE IN %, STD\
MOISTURE CONTENTS, Etc.**

**EFFECTIVE STRESSES**

**MECHANICAL**

| 1.33E-07 | 1.00E+00 | 0.00E+00 | 0.00E+00 |
| 1.09E+00 | 1.00E+00 | 0.00E+00 | 0.00E+00 |

**FLY**

| LOWER PLY SURFACE | UPPER PLY SURFACE |

**ON AXIS MECHANICAL STRAIN**

| 1.77E-09 | 5.89E-09 | -5.89E-09 | -1.77E-09 |
| 1.09E-09 | 1.9E-09 | -1.9E-09 | 1.09E-09 |
| 1.38E-09 | 4.06E-09 | 4.06E-09 | 1.38E-09 |
| 2.87E-09 | 5.73E-09 | 5.73E-09 | 2.87E-09 |
| 1.15E-09 | 1.15E-09 | 1.15E-09 | 1.15E-09 |
| 2.06E-09 | 1.03E-10 | 1.03E-10 | 2.06E-09 |
| 1.03E-09 | 1.03E-09 | 1.03E-09 | 1.03E-09 |

**OFF AXIS MECHANICAL STRAIN**

| 1.77E-09 | 5.89E-09 | -5.89E-09 | -1.77E-09 |
| 1.09E-09 | 1.9E-09 | -1.9E-09 | 1.09E-09 |
| 1.38E-09 | 4.06E-09 | 4.06E-09 | 1.38E-09 |
| 2.87E-09 | 5.73E-09 | 5.73E-09 | 2.87E-09 |
| 1.15E-09 | 1.15E-09 | 1.15E-09 | 1.15E-09 |
| 2.06E-09 | 1.03E-10 | 1.03E-10 | 2.06E-09 |
| 1.03E-09 | 1.03E-09 | 1.03E-09 | 1.03E-09 |

| 2.51E-09 | 2.51E-09 | 2.51E-09 | 2.51E-09 |

24
Problem #3

Problem #3

**STRENGTH ANALYSIS**

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| 1 | -1.4E-06 | -3E-09 | 0 | 9E-10 | 1.2E-06 | 7E-09 | | 
| 2 | -1.2E-09 | -3E-09 | 0 | 9E-10 | 1.5E-09 | 7E-09 | | 
| 3 | -1.5E-12 | -3E-09 | 0 | 9E-10 | 2.1E-09 | 7E-09 | | 
| 4 | -2.1E-09 | -3E-09 | 0 | 9E-10 | 4.4E-09 | 7E-09 | | 
| 5 | -4.1E-09 | -3E-09 | 0 | 9E-10 | 4.4E-09 | 7E-09 | | 
| 6 | 8.4E-09 | -3E-09 | 0 | 9E-10 | 8.4E-09 | 7E-09 | | 
| 7 | 9.4E-09 | -3E-09 | 0 | 9E-10 | 1.1E-09 | 7E-09 | | 
| 8 | 1.1E-08 | -3E-09 | 0 | 9E-10 | 1.2E-09 | 7E-09 | | 
| 9 | 2.3E-08 | -3E-09 | 0 | 9E-10 | 2.3E-09 | 7E-09 | | 
| 10 | 3.5E-08 | -3E-09 | 0 | 9E-10 | 1.7E-09 | 7E-09 | | 
| 11 | 9.2E-09 | -3E-09 | 0 | 9E-10 | 1.9E-09 | 7E-09 | | 
| 12 | 1.7E-09 | -3E-09 | 0 | 9E-10 | 2.1E-09 | 7E-09 | | 
| 13 | 4.5E-09 | -3E-09 | 0 | 9E-10 | 4.4E-09 | 7E-09 | | 
| 14 | 8.6E-09 | -3E-09 | 0 | 9E-10 | 8.6E-09 | 7E-09 | | 
| 15 | 1.3E-08 | -3E-09 | 0 | 9E-10 | 1.3E-09 | 7E-09 | | 
| 16 | 2.1E-08 | -3E-09 | 0 | 9E-10 | 2.1E-09 | 7E-09 | | 

ON AXIS MECHANICAL STRAIN | | | 
| 1 | 5.1E-05 | -3E-09 | 0 | 9E-10 | 5.4E-05 | 2E-09 | | 
| 2 | 5.4E-05 | -3E-09 | 0 | 9E-10 | 5.7E-05 | 2E-09 | | 
| 3 | -1.7E-05 | -3E-09 | 0 | 9E-10 | -1.9E-05 | 2E-09 | | 
| 4 | -1.3E-09 | -3E-09 | 0 | 9E-10 | -1.5E-09 | 2E-09 | | 
| 5 | -1.3E-09 | -3E-09 | 0 | 9E-10 | -1.5E-09 | 2E-09 | | 
| 6 | 1.8E-09 | -3E-09 | 0 | 9E-10 | 1.8E-09 | 2E-09 | | 
| 7 | 1.8E-09 | -3E-09 | 0 | 9E-10 | 1.8E-09 | 2E-09 | | 
| 8 | 2.8E-09 | -3E-09 | 0 | 9E-10 | 2.8E-09 | 2E-09 | | 
| 9 | 1.7E-09 | -3E-09 | 0 | 9E-10 | 1.7E-09 | 2E-09 | | 
| 10 | 1.7E-09 | -3E-09 | 0 | 9E-10 | 1.7E-09 | 2E-09 | | 
| 11 | 1.7E-09 | -3E-09 | 0 | 9E-10 | 1.7E-09 | 2E-09 | | 
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| 14 | 1.7E-09 | -3E-09 | 0 | 9E-10 | 1.7E-09 | 2E-09 | | 
| 15 | 1.7E-09 | -3E-09 | 0 | 9E-10 | 1.7E-09 | 2E-09 | | 
| 16 | 1.7E-09 | -3E-09 | 0 | 9E-10 | 1.7E-09 | 2E-09 | | 

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**STRENGTH RATIO R**

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