(1)

REPORT DOCUMENTATION PAGE

READ INSTRUCTIONS BEFORE COMPLETING FORM

1. REPORT NUMBER	2. GOVT ACCESSION NO.	3. RECIPIENT'S CATALOG NUMBER
N/A	AD-A129664 N/A	N/A

4. TITLE (and Subtitle)

Measuring the Fractal Dimensions of Surfaces

5. TYPE OF REPORT & PERIOD COVERED

N/A

6. PERFORMING ORG. REPORT NUMBER

7. AUTHOR(s)

Mark C. Shelberg
Nina Lam
Harold Moellering

8. CONTRACT OR GRANT NUMBER(s)

N/A

9. PERFORMING ORGANIZATION NAME AND ADDRESS

Defense Mapping Agency Aerospace Center/CD
St. Louis AFS, MO 63118

10. PROGRAM ELEMENT, PROJECT, TASK AREA & WORK UNIT NUMBERS

N/A

11. CONTROLLING OFFICE NAME AND ADDRESS

DMAAC
2nd & Arsenal Sts.
St. Louis AFS, MO 63118

12. REPORT DATE

Oct. 16, 1983

13. NUMBER OF PAGES

9

14. MONITORING AGENCY NAME & ADDRESS *(if different from Controlling Office)*

N/A

15. SECURITY CLASS. *(of this report)*

UNCLASSIFIED

15a. DECLASSIFICATION/DOWNGRADING SCHEDULE

N/A

16. DISTRIBUTION STATEMENT *(of this Report)*

Distribution Unlimited approved for Public Release

17. DISTRIBUTION STATEMENT *(of the abstract entered in Block 20, if different from Report)*

N/A

18. SUPPLEMENTARY NOTES

To be presented at Auto-Carto 6, Ottawa, Canada, 16-21 October 1983

19. KEY WORDS *(Continue on reverse side if necessary and identify by block number)*

Fractal dimension, Linear regression, Isarithm line, Isarithm interval, Digital Elevation Model

20. ABSTRACT *(Continue on reverse side if necessary and identify by block number)*

The fractal dimension of a surface is a measure of its geometric complexity and can take on any non-integer value between 2 and 3. Normally, the topological dimension of surfaces is 2; however, their fractal dimensions increase with greater amounts of complexity or roughness. For example, a fractal dimension of 2.3 is found to be a common value in describing the relief on the earth.

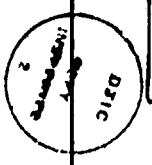
(continued

This paper discusses and presents examples of an algorithm designed (next

20. ABSTRACT (continued)

to measure the fracticality of surfaces. The algorithm was developed at the Ohio State University and is shown to be reliable and robust. It is placed in an interactive setting and is based on the premise that the complexity of isarithm lines may be used to approximate the complexity of a surface. The algorithm operates with the following scenario: Starting with a matrix of Z-heights, an isarithm interval is selected and isarithm lines are constructed on the surface. A fractal dimension is computed for each isarithm line by calculating their lengths over a number of sampling intervals. The surface's fractal dimension is the result of averaging the fractal dimensions of all the isarithm lines and adding 1.

Potential applications for this technique include a new means for data compression, a quantitative measure of surface roughness, and be used for generalization and filtering.



MEASURING THE FRACTAL DIMENSIONS OF SURFACES

Mark C. Shelberg
Cartographer, Techniques Office
Aerospace Cartography Department
Defense Mapping Agency Aerospace Center
St. Louis AFS, Missouri 63118

Nina Lam
Assistant Professor
Department of Geography
Ohio State University
Columbus, Ohio 43210

Harold Moellering
Associate Professor
Department of Geography
Ohio State University
Columbus, Ohio 43210

ABSTRACT

The fractal dimension of a surface is a measure of its geometric complexity and can take on any non-integer value between 2 and 3. Normally, the topological dimension of surfaces is 2; however, their fractal dimensions increase with greater amounts of complexity or roughness. For example, a fractal dimension of 2.3 is found to be a common value in describing the relief on the earth.

This paper discusses and presents examples of an algorithm designed to measure the fracticality of surfaces. The algorithm was developed at The Ohio State University and is shown to be reliable and robust. It is placed in an interactive setting and is based on the premise that the complexity of isarithm lines may be used to approximate the complexity of a surface. The algorithm operates with the following scenario: Starting with a matrix of Z-heights, an isarithm interval is selected and isarithm lines are constructed on the surface. A fractal dimension is computed for each isarithm line by calculating their lengths over a number of sampling intervals. The surface's fractal dimension is the result of averaging the fractal dimensions of all the isarithm lines and adding 1.

Potential applications for this technique include a new means for data compression, a quantitative measure of surface roughness, and be used for generalization and filtering.

INTRODUCTION

The problem of describing the forms of curves and surfaces has vexed researchers over the years. For example, a coastline is neither straight, nor circular, nor elliptic and therefore Euclidean lines cannot adequately describe most real world features. Imagine attempting to describe very rough or bumpy terrain in terms of classical geometry. An intriguing concept proposed by Mandelbrot (1967, 1977, 1982) is to use fractals to fill the void caused by the absence of suitable geometric representations. A fractal characterizes curves and surfaces in terms of their complexity by treating dimension as a continuum. Normally, dimension is an integer number (1 for curves, 2 for areas, and 3 for volumes); however, fractal dimensions may vary anywhere between 1 and 2 for a curve and 2 and 3 for a surface depending upon the irregularity of the form. Although individual fractals have been around since the 1900's, Mandelbrot was the first to recognize their applications outside of mathematics.

This paper discusses an algorithm, developed at Ohio State and implemented in an interactive setting, designed to measure the fracticality of a surface. It also presents results from examining three surfaces.

DEFINITION OF FRACTALS AND SELF-SIMILARITY

In Euclidean geometry every curve has a dimension of 1 and every plane has a dimension of 2. This is generally referred to as the topological dimension (Dt). These dimensions remain constant no matter how complex or irregular a curve or plane may be. For example, the Rocky Mountain area contains many irregularities, but the topological dimension remains 2.

In the fractal domain a curve's dimension may be between 1 and 2 according to its complexity. The more contorted a straight line becomes, the higher its fractal dimension. Similarly, a plane's dimension may be a non-integer value between 2 and 3. The fractal dimension for any curve or surface is denoted by (D) and within this framework: $D \geq Dt$. Mandelbrot (1977) proposes the following definition for a fractal: "A fractal will be defined as a set for which the Hausdorff-Besicovitch dimension strictly exceeds the topological dimension."

Central to the concept of fractals is the notion of self-similarity. Self-similarity means that for any curve or surface a portion of the curve or surface can be considered a reduced image of the whole. However, seldom in nature (crystals are one exception) does self-similarity occur and therefore a statistical form of self-similarity is often encountered. In other words, if a curve or surface is examined at any scale it will resemble the whole in a statistical sense; therefore, D will remain constant. Brownian motion is an excellent example of statistical self-similarity. Because of this principle, a curve can be decomposed into $N=r$ nonoverlapping parts and each subsegment has a length of $1/r=1/N$. Similarly, a unit square can be divided into $N=r^2$ squares, where the similarity ratio is $r(N) = 1/r = 1/N^2$. In either case the following equation applies:

$$D=\log N/\log (1/r) \qquad (1)$$

and could be called the shape's similarity dimension. D can also be expressed as:

$$D=\log (N/N_0)/\log(\lambda_0/\lambda) \qquad (2)$$

where λ_0 and λ are two sampling intervals and N_0 and N are the number of such intervals contained. If a curve resembles a straight line then when the sampling interval is halved, N doubles and the proportion equals 1. The majority of cartographic curves are not straight lines and therefore N will more than double causing D to be greater than 1. The principle of self-similarity is dismissed by Goodchild (1980), Hakanson (1978), and Scheidegger (1970). Hakanson, for example, points out the absurdity of postulating the validity of self-similarity down to the size of the pebbles on the coastline and at the molecular interstices of those pebbles. Goodchild demonstrates that although Richardson (1961) found the west coast of Britain to have a constant D of 1.25 over sampling intervals between 10 and 1000km., he found the east coast to vary between 1.15 and 1.31 for a similar sampling interval. This suggests that whatever created the irregularities on the coastline acted at specific scales. Goodchild states that since self-similarity is only one aspect of the fractal approach, it would be unwise to reject the entire concept.

DEVELOPMENT OF THE FRACTAL SURFACE ALGORITHM

The following algorithm is based upon the research performed by Goodchild (1980). He noted the earlier empirical work performed by Richardson (1961) and later

extended by Mandelbrot (1967). Richardson measured the lengths of several frontiers by manually walking a pair of dividers along the outline so as to count the number of steps. The opening of the dividers (n) was fixed in advance and a fractional side was estimated at the end of the walk. The main purpose in this section of Richardson's research was to study the broad variation of Σn with n.

Richardson produced a scatterplot in which he plotted log total length against log step size for five land frontiers and a circle. Mandelbrot (1967) discovered a relationship between the slope (β) of the lines and fractal dimension (D). To Richardson the slope had no theoretical meaning, but to Mandelbrot it could be used as an estimate of 1-D, which leads to:

$$D=1-\beta \qquad (3)$$

Goodchild computed the fractal dimensions of surfaces by constructing contour lines and calculating their lengths. He extended Hakanson's (1978) analysis in order to calculate D. In computing length, Hakanson counted the number of intersections between a grid and a coastline, and for a complex curve, it is possible that any even number of intersections could exist. Goodchild points out the scale of the map, which determines the size of the grid, is not strictly related to a sampling interval. In terms of D, it would be more appropriate to count the number of cells intersected instead of the number of intersections.

Goodchild estimated D for several self-similar surfaces generated by shear displacement (Mandelbrot 1975). He selected a contour line at a height equal to the mean of the minimum and maximum heights. Cells were aggregated into larger aggregates of 5x5, 7x7, 9x9, and up to 19x19, and classified them as above (black) or below (white) the contour line. To count the number of boundary cells or, in other words, compute the length of the contour line, as a function of aggregate size, a count was made of the number of black aggregates containing at least one white cell. This is where the cells are cut by the boundary. The log of the average number of boundary cells was then regressed against the log of the aggregate size. Goodchild calculated D for the contour line using Equation 3 and extending that same principle to a surface, the following equation is used

$$D=2-\beta \qquad (4)$$

The following algorithm developed by Shelberg (1982) extends Goodchild's work in that D can be approximated for nonself-similar surfaces and is placed in an interactive setting. Because Goodchild's study only dealt with self-similar surfaces, he could select a contour line at a height equal to the mean of the minimum and maximum heights. A self-similar surface possesses the same statistical properties at all scales and at all locations; therefore, choosing the average contour line is a valid method. However, a surface may not be entirely self-similar and selecting the mean contour line may produce a false fractal dimension. For example, if a surface consisted of a plain and a mountain range, then selecting the average contour line would neglect both points on the plain and mountain range and bias the overall fracticality. Although two of the surfaces reported in this paper are self-similar, the algorithm is designed to analyze nonself-similar surfaces. Instead of choosing only the mean contour line, any number of contour lines, up to a maximum of 200 can be used to examine a surface.

All surfaces are in the form of a Digital Elevation Model (DEM) and the algorithm begins with the user inputting whether the fractal dimension will be determined by rows of columns. This option is provided so that a trend in the surface may be captured. The contour interval is then entered and the number of contour lines determined. Next, the maximum cell size, which is comparable to Richardson's opening of the pair of dividers, is input. The minimum allowable maximum cell

size is 5 because any number less than 5 would leave the linear regression open to question.

For each contour line, cells are aggregated into larger aggregates, up to the maximum cell size, and classified as above (black) or below (white) the contour line. To count the number of boundary cells, as a function of aggregate size, a count is made of the number of black aggregates containing at least one white cell. Figure 1 represents a 6x6 surface in which the contour line of 35 is denoted by the darker line on the surface.

The algorithm starts with the lowest contour line, in this case 35, and classifies each cell as white (1) or black (2); see Figure 2. Always beginning with a cell size of 1, it then compares each neighboring cell, along the columns, for boundary cells. For example, the comparison between (1,1) and (1,2); and (1,5) and (1,6) indicates a boundary cell is not present. Conversely, the comparison between cells (1,4) and (1,5); and (6,2) and (6,3) indicates boundary cells exist and in these cases the number of boundary cells is incremented by 1. After the entire surface is examined using 1 cell size, the search begins again using a cell size of 2. Now cells (1,1) and (1,3); and (6,2) and (6,4) are compared and so on. This continues until the maximum cell size is reached. For each contour line the same process is repeated.

It is possible, for a given cell size, that no boundary cells are encountered. In Figure 3, this occurs when the cell size equals 3, 4 or 5. For example, the only possibility of encountering a boundary cell is in row 6 when the contour line equals 19. If the cell size equals 5, cells (1,1) and (1,6) are compared and no boundary cells are reported present. The results remain the same for the remaining comparisons within that row. For any given cell size, if no boundary cells are encountered, the contour line is eliminated from the calculations. Since the algorithm regresses log (average number of boundary cells) against log (cell size), without eliminating the contour line, but just deleting the cell size from the regression, could result in 1, 2 or 3 points used in defining the regression line and immediately poses obvious difficulties. Although, by eliminating some contour lines, the method neglects certain small portions of the surface, it accounts for greater variations than if only the mean contour line is selected. The strategy is to choose the lowest contour interval with the fewest contour lines being eliminated.

After counting the number of boundary cells per cell size for each contour line, a linear regression is performed. For every contour line, log (average number of boundary cells) is regressed against log (cell size). A D is computed for each contour line and extended to a surface by using Equation 4. A surface's fracticality is the average of all the D's for the included contour lines.

EXAMPLES AND RESULTS

Two of the three surfaces used as examples were generated by the shear displacement method and are statistically self-similar. Each is in the form of a 50x50 matrix and, on the CRT, are represented by a perspective view with hidden lines eliminated.

The first surface, in Figure 4, has a theoretical D of 2.2. Table 1 shows a rather large array of D values and the selection of numerous contour intervals is intended to display the stability of the algorithm. In an effort to preserve consistency among the results and not fall below the minimum acceptable maximum cell size, in the following examples, the maximum cell size is 5. In all cases, D is computed along the rows.

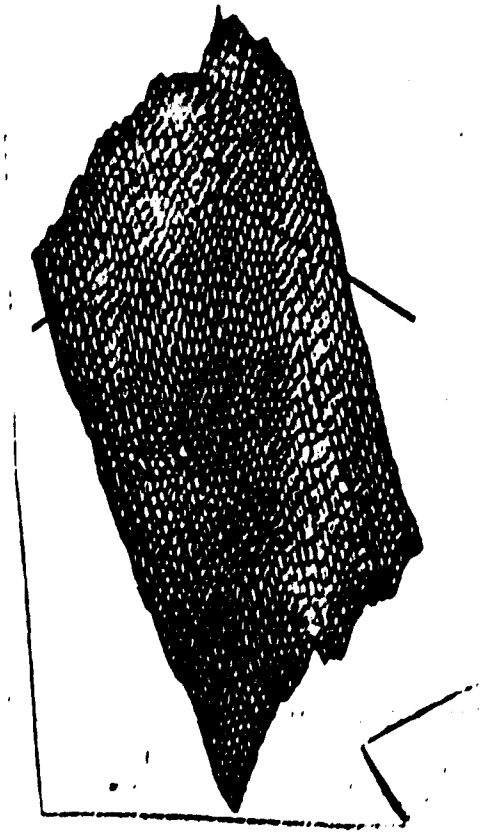


Figure 4. 50 x 50 surface with a theoretical D of 2.2. Minimum and maximum elevations are -503 and 269.

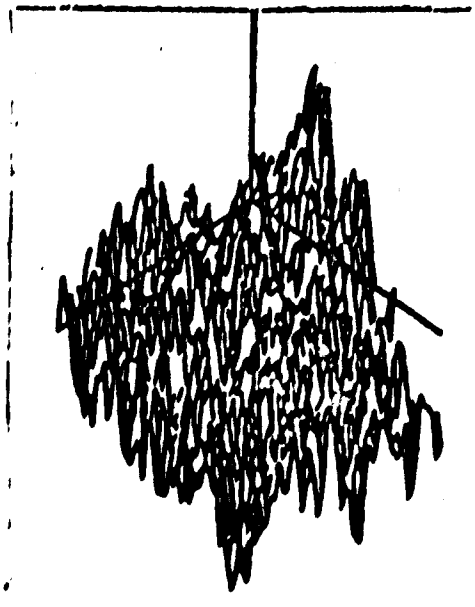


Figure 5. 50 x 50 surface with a theoretical D of 2.6. Minimum and maximum elevations are -42 and 41.

NOTE: Heights are unitless.

Contour Interval	Maximum Cell Size	D	No. of included lines	No. of lines not included
5	5	2.1915	146	8
10	5	2.1869	72	4
20	5	2.1732	36	2
22	5	2.2071	33	1
24	5	2.2000	30	1
30	5	2.1893	24	1
50	5	2.2365	14	0
100	5	2.2143	7	0
200	5	2.2268	3	0
385	5	2.1289	1	0

Table 1. Results from examining a 50x50 surface with a theoretical D of 2.2

A D of 2.1915 would be selected to represent the surface because of the large number of contour lines (146) used in the calculations versus only 8 contour lines being eliminated.

Figure 5 shows a 50x50 surface with a theoretical D of 2.6. Table 2 displays the results over a number of contour intervals.

Contour Interval	Maximum Cell Size	D	No. of included lines	No. of lines not included
2	5	2.6720	34	7
5	5	2.6671	14	2
7	5	2.6679	10	1
10	5	2.6290	7	0
20	5	2.6590	3	0
41	5	2.5985	1	0

Table 2. Results from examining a 50x50 surface with a theoretical D of 2.6

A D of 2.6720 would be selected because 34 contour lines are included in the calculations versus only 7 being eliminated.

The third surface is also 50x50, and is a Defense Mapping Agency's standard elevation matrix over a small portion of Southern Nevada. The minimum and maximum heights, in meters, are 2011 and 2376. Although the degree of self-similarity is not known, the D values, in Table 3, are extremely stable. A D of 2.1893 would be selected to best approximate the complexity of the surface. The actual surface is not shown because the hardcopy is illegible.

Contour Interval	Maximum Cell Size	D	No. of included lines	No. of lines not included
5	5	2.1893	66	6
15	5	2.1905	22	1
20	5	2.1707	16	1
150	5	2.1701	1	0

Table 3. Results from examining a "real-world" 50x50 surface.

35

6	14	24	39	40	48	60
5	17	27	30	42	47	59
4	16	26	28	36	52	65
3	14	25	32	43	50	62
2	15	27	30	34	48	61
1	12	22	31	33	44	58
	1	2	3	4	5	6

Figure 1. 6 x 6 surface with a contour line of 35.

6	1	1	2	2	2	2
5	1	1	1	2	2	2
4	1	1	1	2	2	2
3	1	1	1	2	2	2
2	1	1	1	1	2	2
1	1	1	1	1	2	2
	1	2	3	4	5	6

Figure 2. 6 x 6 surface where below contour line = 1 and above contour line = 2.

19

6	5	10	20	18	16	13
5	2	11	12	18	17	14
4	9	12	13	17	16	15
3	4	8	9	16	14	13
2	7	11	14	15	13	13
1	4	10	11	14	15	17
	1	2	3	4	5	6

Figure 3. 6 x 6 surface with a contour line of 19. A case where no boundary cells are encountered for cell sizes 3, 4 and 5.

SUMMARY AND CONCLUSIONS

The results demonstrate the stability of the algorithm in which, over a number of contour or sampling intervals, the fractal dimension remains reasonably constant. Other 50x50 self-similar surfaces which were examined, ranging in D from 2.1 to 2.9, produced similar results. By using a number of contour lines, the maximum variation in the complexity of the surface can be captured and therefore D can be closely approximated.

Potential applications of this algorithm include a new means to compute surface roughness, a means to measure the amount of surface generalization or filtering attained, and a method to store a compressed surface so that greater complexity can be introduced to match a target surface with a specific fractal dimension.

REFERENCES

Agterberg, F. P. (1982), "Recent Developments in Geomathematics," Geo-Processing, Vol 2, pp. 1-32.

Carpenter, L.C. (1981), "Computer Rendering of Fractal Curves and Surfaces," Unpublished Research Paper, Boeing Computer Services, Seattle, Washington.

Dutton, G. (1981), "Fractal Enhancement of Cartographic Line Detail," The American Cartographer, Vol 8, No. 1, pp. 23-40.

Goodchild, M. F. (1980), "Fractals and the Accuracy of Geographical Measures," Mathematical Geology, Vol. 12, No. 2, pp. 85-98.

Hakanson, L. (1978), "The Length of Closed Geomorphic Lines," Mathematical Geology, Vol. 10, No. 2, pp. 141-167.

Lam, N. S. (1980), Methods and Problems of Areal Interpolation, Ph.D. dissertation, The University of Western Ontario, London, Ontario.

Mandelbrot, B. B. (1967), "How Long is the Coast of Britain? Statistical Self-Similarity and Fractal Dimension." Science, Vol. 156, pp. 636-638.

Mandelbrot, B. B. (1975), "Stochastic Models of the Earth's Relief, the Shape and the Fractal Dimension of the Coastlines, and the Number-area Rule for Islands," Proceedings of the National Academy of Sciences, Vol. 72, pp. 3825-3828.

Mandelbrot, B. B. (1977), Fractals: Form, Chance and Dimension, Freeman, San Francisco, 365 pps.

Mandelbrot, B. B. (1982), The Fractal Geometry of Nature, Freeman, San Francisco, 460 pps.

Nordbeck, S. and B. Bystedt (1972), Computer Cartography, Studentlitteratur, Lund, Sweden, p. 39.

Richardson, L. F. (1961), "The Problem of Contiquity," General Systems Yearbook, Vol. 6, pp. 139-187.

Scheidegger, A. E. (1970), Theoretical Geomorphology (2nd ed.), Springer Verlag, New York, 333 pps.

Shelberg, M. C. (1982), The Development of a Curve and Surface Algorithm to Measure Fractal Dimensions and Presentation of Results, Unpublished Master's Research Paper, The Ohio State University.

Shelberg, M. C., Moellering H., Lam, N. S. (1982), "Measuring the Fractal Dimensions of Empirical Cartographic Curves," Proc. AUTO-CARTO V/ISPRS IV Symposium, pp. 481-490.

Shelberg, M. C. and H. Moellering (1983), "IFAS: A Program to Measure Fractal Dimensions of Curves and Surfaces," Proceedings ACSM, 43, pp. 483-492.