**Human Individual Differences in Military Systems**

This report is based on presentations concerning human individual differences and their implications at the Joint National Meeting of the Operations Research Society of America and The Institute of Management Sciences, held in San Diego in October 1982.
HUMAN INDIVIDUAL DIFFERENCES IN MILITARY SYSTEMS

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FOREWORD

This report, which was prepared as part of work unit ZR000-01-042.005 (Models and Measures of Human Performance), deals with the pervasiveness of human individual differences (IDs) and discusses the use of IDs in selecting and assigning people to jobs. The subject of this report was the theme of a session of the October 1982 Joint National Meeting of the Operations Research Society of America (ORSA) and The Institute of Management Sciences. The session was sponsored by the ORSA Technical Section on Military Applications and was chaired by Dr. Richard C. Sorenson. The presentations in that session by Dr. Bernard Rimland (NAVPERSRANDCEN), Dr. Paul Horst (consultant to NAVPERSRANDCEN) and Dr. Joe Ward (Air Force Human Resources Laboratory, Brooks Air Force Base, Texas) form the basis of this report.

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INTRODUCTION

During the last 40 years, two concepts have become increasingly important to those considering military matters as well as to society in general. The first concept, termed "Systems Thinking" by Ackoff (1981), deals with sets of elements in which (1) the behavior of each element affects the behavior of the whole, (2) the behavior of each element and its effects on the whole are influenced by the behavior of other elements, and (3) elements in the set are so connected that independent subgroups cannot be formed. The key to systems thinking is synthesis, the putting of elements together and considering their connections and interactions. Many times, elements interact such that optimal behavior on the part of certain elements results in suboptimal behavior of the system. We have become much more aware of "systems considerations" in recent years.

The second concept is that of the role of human beings. Humans are not simply the owner-operators of the system, somehow standing aloof and manipulating it to suit their ends. Further, they are not homogeneous, noninteracting elements as they were considered to be during the early stages of the industrial revolution. Rather, humans are elements in the set of elements forming the system. Therefore, in systems design, human beings must be considered not only as interactive elements to be integrated into the system, but also as elements varying on many dimensions that contribute significantly to system variance. Furthermore, in the operation of systems, advantage must be taken of human individual differences in assigning people to jobs. Some jobs may be much more appropriate than other jobs for individuals within the population. Wide variations in performance may be associated with assigning particular individuals to particular jobs. A complex technology has been developed to predict and take advantage of individual differences in performance.

This report deals with the pervasiveness of human individual differences (ID) and discusses the characteristics of systems to take advantage of IDs in the military.
HUMAN INDIVIDUAL DIFFERENCES
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Few remarks are as commonplace as "Everyone is different" and "No two people are alike." Despite the fact that human individual differences (IDs) are so widely acknowledged and exist in every imaginable dimension (e.g., height, weight, intelligence, musical talent, motivation, age, experience, athletic ability), they are, surprisingly, often overlooked and underemphasized in such supposedly scientific fields as psychology, human factors, and operations research. Like the weather, it is far easier to discuss IDs than to deal with them. For this reason, and others, they tend to be ignored. Statisticians have expended considerable effort in devising tests of statistical significance that will obscure the effects of IDs. For example, when an experiment comparing two groups is conducted, there is usually an immense range of IDs in the outcome measure and only a relatively small difference between the means of the two groups being compared. The statistician is trained to use various sophisticated devices that systematically obscure the large IDs. Such differences are referred to--sometimes disparagingly--as "within-group variance" or, worse yet, as "error variances," in an attempt to emphasize the smaller and possibly less important between-group differences.

There are relatively few books on IDs, and few colleges offer courses on the topic. Thus, the few college instructors who wish to teach a course on IDs have perhaps four textbooks to choose from, compared to literally dozens for those who wish to teach such standard courses as abnormal psychology, educational psychology, or developmental psychology. This is further evidence of our embarrassment about the existence of IDs and of our wish to dismiss them, rather than to cherish them and employ them constructively.

Let us discuss some of the biological roots of IDs before considering their effects on human performance. Professor Roger Williams, at the University of Texas, one of the world's foremost authorities on IDs, presents an enormous array of information on the topic in his book entitled Biochemical Individuality (1956). Part of the book is devoted to anatomical differences between humans, and the differences are indeed astounding. For example, Williams points out that, whereas anatomy textbooks typically show an exemplary drawing of a human heart, human hearts actually vary enormously in size and in structure. He presents a series of drawings (Figure 1, page 29), showing 12 variations of the right atrium. The forms vary so widely that, as Williams states, one might doubt they are from the same species. The same is true of aortic arches. Some hearts have several very large blood vessels entering them. Others have perhaps an equal amount of blood entering and leaving, but the ports of entry and departure are composed of a large number of small vessels. A clot can block one of these small vessels very easily. In view of these massive structural differences between hearts, it is understandable why some individuals are much more susceptible to heart attack than others.

Similarly, Williams discusses (page 36) the differences in the tendon structure of the human hand. Very few hands look like the textbook drawings. People have different numbers of tendons and the tendons are attached at different points. All in all, it is truly remarkable that hands that superficially look so similar can differ so much from each other in terms of their internal structure.

The student of IDs will also be interested in the extraordinary range of biochemical differences. Williams discusses a study in which the amount of alcohol needed to cause
drunkeness in a group of subjects was determined. Some subjects became drunk on as little as one-fourth of an ounce of alcohol. Others required ten times as much alcohol to bring about the same effect. In another study, it was found that 10 percent of the subjects became drunk when their blood alcohol level reached .05 percent while, at the other end of the distribution, about 7 percent remained sober after their blood levels had reached a point eight times as high.

The differences between people are immense in qualitative as well as in quantitative terms. For example, in studying the effects of morphine on 29 normal subjects, a huge variety of symptoms was seen. Eighteen of the subjects became nauseous, 16 became very sleepy, 13 became dizzy, 9 became drunk, 9 suffered severe itching, and 7 suffered severe speech impairment.

Humans also differ markedly in their need for various nutrients. For example, before vitamin C's ability to prevent scurvy was discovered, hundreds of thousands of sailors died of this disease. However, while half of a ship's crew might die because the shortage of fresh fruits and vegetables led to a lack of vitamin C, many other crew members seemed to thrive and show little or no effects from vitamin C deprivation. In Williams' own studies of B-vitamin needs, normal healthy individuals differed in their requirements by over 2,000 percent, even in groups as small as ten.

The primary interest of operations research analysts and psychologists is, of course, not anatomy or biology but, rather, human behavior and performance. Here too, the range of differences between people is striking. Rather than the 10 to 20 or 30 percent difference that most people seem to feel characterizes the range of performance differences, the range in even small groups rarely falls below 200 percent, and differences in the thousands of percents are not uncommon. Let us consider human performance differences in a few fields.

The most obvious field in which IDs may be studied is sports, since the "name of the game" involves keeping and comparing records of what individuals do. The differences in the performance of trained competitive athletes and people in general are so great as to virtually defy calculation. For example, while very few randomly selected people can run even a mile, there are marathoners who run over 26 miles at high speed. In the most recently completed New York City marathon, with approximately 16,000 entrants, Alberto Salazar won for the third straight year, demonstrating not only remarkable ability but also remarkable consistency in performance. In the discus throw, another area of athletic ability, Al Oerter won at the Olympics in 1956, 1960, 1964, and 1968, thereby dominating his field for a span of 12 years. Whereas the typical college male can do 16 push-ups (some can do none), certain individuals can do over 1,000. Similarly, while some individuals can do only a few sit-ups, the record is over 14,000. While, among the general population, some can only hold their breath for a minute or less, there have been instances where divers have held their breath under water for over 6 minutes.

You may feel that it is unfair to compare highly trained, specialized athletes with amateurs or people in general. Perhaps it is. However, there are enormous IDs of several hundred percent, even when one is comparing professional athletes in the same sport. In baseball, for example, some pitchers consistently win 20 games a season, while other starting pitchers win only 4 or 5 games. Similarly, some batters hit around 350 for a season, whereas others hit 150 or less. Basketball players average from over 30 points to 10 or less, even when position is controlled for.

As with other types of endeavors, striking differences between individuals have been found in studies of academic performance. In one such study, 10 percent of high school
seniors surpassed 50 percent of college seniors in terms of scholastic achievement. In a similar study on younger children, it was found that 10 percent of fifth graders surpassed 50 percent of eighth graders in terms of reading ability. In terms of test items correct, differences of several hundred percent are commonly found (e.g., scores ranging from 30 to 90+) in classrooms of rather homogeneous college students.

While IQ scores cover a wide range of perhaps 80 to 150 in a large high school, they do not reflect the actual range of differences in intelligence in the population. In terms of actual test items correct (e.g., vocabulary, reasoning, general information, etc.), the individual with an IQ of 150 will be able to outscores the individual with an IQ of 80 by perhaps 50 to 1—not the mere 2:1 ratio implied by comparing the IQ scores.

Wechsler, in his book, The Range of Human Capacities (1952), mentions the occasion on which mathematician George Willis amused himself by extracting the square root of a number containing 53 digits. A month later, he was able to reproduce both the number and the root.

Although it is common knowledge that some individuals differ in their productivity, such differences are usually vastly underestimated. When an industrial supervisor is asked to estimate the difference between the most and least productive employee, he or she will typically guess 20 to 30 percent. However, when actual production figures are available, it turns out that the range of differences between the most and least productive employees in a group is often a factor of 10. If the job is a simple one, such as keypunch operating, typing, or operating a sewing machine, differences in the range of 200 to 300 percent are found (Wechsler, 1952). However, when the job is more complex, such as that of a computer programmer, electronics troubleshooter, etc., differences of several thousand percent are frequently found (Williams & Rimland, 1977).

A good example of IDs in programmer facility was reported to us recently by a rather glum research assistant. This man had just returned from the 1982 Southern California Programming Competition, where he and several teammates from his college had competed against 14 other teams from UCLA, USC, Cal Poly, and elsewhere. One member of the UCLA team outperformed each of the other teams, each of which had at least three competitive programmers!

A more formal demonstration of the same phenomenon appears in studies of programmer performance and training (Meyer, 1965; Grant & Sachman, 1966). Meyer compared two methods of training programmers and found that the means of the two groups differed by only one point, 49.5 errors vs. 50.5 errors. However, the individual error scores of the 25 subjects ranged from 30 to 70, leading Meyer to conclude that selecting good students to begin with was perhaps more important than improving the training methods. Grant and Sachman also found very large differences in programmer performance in their 12 professional programmers. Examples are given below:

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<th>Measure</th>
<th>Poorest Score</th>
<th>Best Score</th>
<th>Ratio</th>
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<tr>
<td>Debug hours</td>
<td>170</td>
<td>6</td>
<td>28:1</td>
</tr>
<tr>
<td>CPU time</td>
<td>541</td>
<td>50</td>
<td>11:1</td>
</tr>
<tr>
<td>Code hours</td>
<td>50</td>
<td>2</td>
<td>25:1</td>
</tr>
<tr>
<td>Program size</td>
<td>6,137</td>
<td>1,050</td>
<td>6:1</td>
</tr>
<tr>
<td>Run time</td>
<td>8.0</td>
<td>0.6</td>
<td>13:1</td>
</tr>
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With striking regularity, students of warfare have commented that the most important determinate of military success is not numbers of soldiers or superiority in weapons but, instead, the quality of fighting personnel. Brackney (1959) has stated that man's success in war has "depended more on how he has used his weapons than upon any superiority of weapon design and performance." The history of warfare shows many examples where the tide of battle and, ultimately, the tide of war, has hinged on the spectacularly effective performance of just a few individuals. The story of Audie Murphy is an interesting case in point. Murphy, who was only 5' 5" tall and 110 pounds, was rejected by both the Navy and Air Force and was finally accepted for duty by the Army. Recounting even part of his extraordinary performance in the field would take more space than this article. He became the most decorated soldier of World War II, having over 33 military awards, citations, and decorations, including every possible American medal for valor as well as three French medals and one Belgian model. Similarly, the exploits of certain pilots in World War I are almost legendary. Baron von Richtofen alone was responsible for having downed 80 enemy aircraft. In that same war, Samuel Woodfill, an American, single-handedly overcame three German machine gun nests in one afternoon, dispatching 21 enemy soldiers with only his rifle and pistol.

The findings of Brigadier General S. L. A. Marshall indicate that these are not isolated findings. As Toomepuu (1980) notes:

 Marshall has published important findings on combat performance of soldiers in World War II, Korea, Viet Nam, and the Arab/Israeli wars. While gathering historical data from front-line infantry units during World War II, Marshall made the startling discovery that only about 15 percent of the soldiers in battle actually fired their weapons, and that the fighters were observably different from the other soldiers. In a bitterly fought battle for Omaha Beach, he found that, on a two-division front, only six rifle companies could be considered effective as units, and only 47 men, at widely scattered intervals along the beach, saved the day from disaster. Marshall concluded that the outcome of battles is decided by a relatively few effective participants, a conclusion supported by other astute observers of the performance of soldiers in battle.

By contrast, a single ineffective performer in a key position can cripple a mission. A crew on a bomber is only as good as its weakest link, since incompetence on the part of the aircraft commander, radar observer, or bombardier can lead to failure. It has been said of the recent Falkland Islands conflict that, if the final Argentine Exocet missile hadn't been mistakenly wasted on a cargo ship, the war may have had a decidedly different outcome. It becomes clear, then, that the differences between individuals in contributing to success on the battle field, in sports, and in industry are enormous.

Individual differences are not only an important factor in combat, but also in the development of weapons. The fact that military hardware often fails to live up to its potential is well known. Kaplan (1981) attributes this "performance gap" to a tendency to ignore the human component of the system and provides an example of a high-performance jet aircraft that was inadequately designed. As a result, a number of pilots would have had both of their legs broken upon trying to eject. The design had apparently been made without considering the variability of actual human configurations. Since roughly 200 new material systems will be added to the Army's inventory alone in the next
decade (Baker, 1980), the potential costs of inefficient design due to a failure to consider individual differences are substantial.

It is sometimes thought that, even though people may differ initially in their level of performance, they will tend to become more similar as they gain experience. This appears to be untrue. To illustrate, Maier (1963) cites a study by Peterson and Darley (1936) who studied three groups of typists: The first group had less than 1 year of experience; the second, from 1 to 5 years of experience; and the third, 5 or more years of experience. The range of ability in each group was immense, ranging from approximately 5 to 65 words per minute (corrected for typing errors). For the first group, who had less than 1 year of experience, the average was 39 words per minute, compared to 42 words per minute for the two experienced groups. Experience, therefore, had little effect on the range of individual differences.

Similarly, Vineberg, Sticht, Taylor, and Caylor (1971), in a study of the performance of military technicians, found that the original differences between technicians in terms of aptitude were reflected in their performance 5 years later. Tiffin (1952), in discussing workers in industry, suggests that training tends to increase individual differences in proportion to the complexity of the task in question. It becomes clear, then, that the difference in individual performance levels is enormous, especially in an increasingly technological world, when the situation requires originality and analytical ability from the performers. Yet, operations research analysts, along with most psychologists and human factors specialists, find it much more convenient to deal only with the hypothetical average individual than to take full cognizance of the enormous gains that would be possible if only the best suited individuals were selected and assigned to critical tasks. Similarly, lack of adequate concern for IDs results in the enormous disasters that can (and do) occur when performers at the low end of the suitability spectrum are permitted to function in critical situations.

Having by this time been convinced of the importance of IDs and of the undoubted fact that they are not considered adequately, the question arises, "Well, what can we do about it?" The following suggestions may be helpful as a beginning:

1. First, be aware of the magnitude and importance of IDs. Grievious effects are much less likely to occur if the researcher is, at a minimum, aware of the dangers and opportunities posed by the vast array of IDs among humans.

2. Develop methodologies that will produce an envelope of predictions to allow policy makers to understand the implications of picking superior people, or inferior people, to perform the operations. Perhaps some relative of fuzzy set theory can be developed to permit us to deal with the knowledge that working with human beings involves great uncertainties (Newman, 1982).

3. Conduct research to determine the effects of IDs on the models and procedures that you develop. Try to find which dimensions are important for your purpose, how those dimensions may be measured, and how you can take advantage of them, or at least take them into account in your work. Efforts should be increased to develop behavioral taxonomies that can facilitate the selection of applicants on the basis of job-relevant abilities. While this is, of course, the nominal strategy of most current selection programs, we have yet to derive a "behavioral taxonomy midway between the world of work and the world of human attributes" (Dunnette, 1976). Peterson and Bownas (1982) have made a start in this direction. They propose a taxonomy of 51 cognitive,
psychomotor, and personality characteristics, and suggest a process for empirically linking it to a job/task taxonomy.

Why have IDs been so steadfastly ignored? At least three possible reasons come to mind:

1. **Ignorance**—Most people are simply unaware of the vast extent of human difference.

2. **Convenience**—One can often simplify matters, albeit sometimes at greater expense, by the self-deception that IDs are of little consequence.

3. **Idealogy**—Particularly in the social sciences, where human perfectability and egalitarianism have long been articles of faith, the facts of IDs tend to be perceived as threatening.

Whatever the reason for ignoring IDs in the past, there seems little justification for failing to acknowledge them now and in the future.
TRIANGULAR PEGS IN OBLONG HOLES: TAKING ADVANTAGE
OF INDIVIDUAL DIFFERENCES

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Many who want to utilize our human resources effectively believe in the old proverb about not putting square pegs in round holes. We might also add triangular and oblong pegs and holes to those that are square and round. However, the analogy is somewhat misleading because it includes two major false assumptions: (1) that there are only two kinds of holes and pegs or at most four, and (2) that there is at least one square hole for every square peg and at least one round hole for every round peg, and the same for triangular and oblong shapes.

The human elements of our social systems, whether civil or military, are not that simple. The number of different kinds of people and the number of different kinds of jobs to be done by them in the real world are obviously much greater than two, or even four. Furthermore, there is no reason to assume that for every person there is an ideal job and for every job there is an ideal person. Even assuming that we have achieved an adequate technology for evaluating the abilities of people and the requirements of jobs in terms of these abilities, we still have the problem of how to channel people to jobs so that the overall efficiency of the system or institution will be maximized.

We begin with the facts of life that follow from the pervasiveness of human individual differences already discussed: (1) some people can do many things well, (2) some people can do very few things well, and (3) most people can do some things better than they can do other things. How do we assign them to jobs?

By now, the available technology for allocating persons to activities so as to optimize the expected proficiency of an organization, whether civilian or military, is enormous and very complex. Even yet, however, some of the technical and mathematical problems are not perfectly resolved, and sizable administrative, operational, and political issues impede the implementation of available technology.

The optimal assignment procedures, as they have been developed since World War II, have been based largely on a batching model, as distinguished from a sequential model. The batching model assumes that a group of persons is available for assignment to a number of activity categories. In the military services, these can be training schools preparatory to operational activities or they can be the activities themselves. A linear programming approach based on the transportation model has been traditionally used with the batching strategy. The sequential model considers only one person at a time and attempts to recommend one or more appropriate assignments.

The objective of both models is to make assignments to maximize the overall expected proficiency of the organization. Both models assume that a number of predictor measures, such as test scores, are available for each of the candidates. Also, they assume that, by appropriate statistical procedures, it is possible to obtain from the predictor scores estimates of proficiency for each of the activities. The last assumption implies that a vast amount of previous research has been conducted with large numbers of persons already in job activities for whom both test scores and measures of success in the job activities are available. We refer, of course, to the traditional validation programs.
and procedures necessary to justify the use of psychological tests and other measures for guidance, counseling, selection, and assignment.

Unfortunately, the technological, administrative, and other impediments to large-scale comprehensive multiple-criterion validation programs have been formidable in both civilian and military settings. As a consequence, even yet, the capability of providing valid estimates of proficiencies in a wide range of activity categories, whether civilian or military, is far from adequate. Nevertheless, no amount of armchair or crystal ball manipulation of test scores and other pre-activity predictor data can ever remotely substitute for scientific validation programs. Only the implementation of such programs can hope to yield mathematical formulas for converting test score and other pre-activity data into credible estimates of proficiency in activity categories, such as military occupational specialties. Furthermore, comprehensive validation programs are essential not only for developing credible conversion formulas for test and other predictor variables but also for the generation and selection of those predictor measures that can result in the most valid optimal differential assignment procedures.

A necessary condition for an efficient differential assignment program is the development of a set of high quality measuring instruments and the best state-of-the-art procedures for their administration. However, unless a program of development for tests and testing procedures is accompanied by a comprehensive differential validation program, such development efforts cannot realize their maximum contribution to the optimal utilization of the human components of any operational system, whether civilian or military.

An adequate validation program implies that the human abilities required in the different kinds of jobs to be done within an operating system are known. It also assumes that instruments and procedures are available for evaluating the proficiencies with which the various aspects of the jobs are being performed by the job incumbents, and that these instruments and procedures are being adequately utilized. These assumptions imply extensive job analysis and proficiency evaluation programs within the operating system. Furthermore, they imply continuous updating and implementation of such programs.

Neither the importance nor the magnitude of an adequate validation program in a large operational system can be overemphasized. Much of the work on optimal differential assignment begins with the assumption that acceptable estimates of differential proficiencies for the various job categories are available for the candidates who are to be assigned. Only to the extent that this assumption is satisfied is it even worthwhile considering alternate models of optimal differential assignment. However, since estimated proficiency evaluations for various possible assignments are so crucial for optimal utilization of personnel resources, it might be useful to review briefly the procedures underlying the conduct of an optimal differential assignment program.

We begin with the concept of two fundamental types of variables. These are, of course, the predictor and the criterion variables, or what are sometimes called the dependent and the independent variables, respectively. An essential difference between the two is that, for a specified sample of persons, measures on the predictor variables are typically available prior to measures on the criterion variables. The predictor variables should measure a large number of statistically independent traits, or abilities, so as to increase the possibility of covering the abilities required for succeeding in the various kinds of assignments within an operational system.

This principle of independence or multidimensionality is directly opposed to the concept of generalizability. Generalizability theory, which has attracted considerable
attention in recent times, has only a limited role in the development of measuring instruments for optimal differential assignment.

Each predictor variable should have a large dispersion of validity coefficients, or correlations with proficiency measures in the various criterion or assignment categories, and the intercorrelations of these validities among the assignment categories should be low. The more closely these two conditions are satisfied, the greater the possibilities for optimal differential assignment and maximum human resource utilization. These requirements of maximum dispersion and minimum intercorrelation are contrary to the concept of validity generalization that, like that of generalizability theory, has recently attracted considerable interest.

Neither of the recently popular concepts of generalizability theory nor validity generalization has much to contribute to optimal differential assignment and the maximum utilization of human resources that seek to exploit as fully as possible not only interindividual differences but also intra-individual trait or ability differences.

To continue with our review of the procedures for the development of an optimal differential assignment program, we have a set of predictor variables, a set of criterion variables, and a sample of persons on which we have measures of these variables. From these three, we may construct two matrices: (1) a matrix of predictor measures and (2) a matrix of criterion measures. In both matrices, the rows are the persons in the sample and the columns are the predictor and criterion variables, respectively. Having given these data, the multiple regression methods are available for finding a transformation matrix that converts the predictor measures to estimated proficiency measures. There are, of course, various options for deriving the transformation matrix. These include reduced rank solutions, predictor accretion and elimination techniques, optimal test length models, cross validation models, and others.

The traditional multiple regression method, however, assumes complete data for all cases in the sample, including both predictor and criterion variables. For most real-life situations, such complete data matrices are not available. For example, in the military setting, we may have a complete matrix of predictor data for a specified sample of enlistees but, subsequently, each of the persons in the sample will participate in only one, or perhaps a few, of the military occupational specialties. This means that, even though predictor measures will typically be available for all or most of the persons in the sample on most of the predictor variables, proficiency measures for each person will be available for only one or several of the possible operational assignments.

Typically then, in any large-scale validation program where we wish to develop or derive a transformation matrix for converting predictor measures into estimated multiple criterion measures, the criterion submatrix for the validation sample will, of necessity, be sparse. This is true whether the setting be military, industrial, or educational. The theory and technology for maximally utilizing the available data in the derivation of transformation matrices from incomplete data sets of this type have even yet not been entirely satisfactory. However, a considerable body of theory and technology on the problems of sample selection and bias in sampling has evolved over the years. This can be profitably utilized in the derivation of transformation matrices in large-scale multiple prediction programs.

Let us assume now that all of the technical, administrative, operational, and political problems involved in a validation program have been adequately addressed and resolved. Assume further that we have developed a transformation matrix for converting predictor measures into estimated proficiency measures for each of the job categories. Several
important issues still remain. One of the most important and troublesome of these is the problem of origin and scale. A great deal of attention has been devoted in the military services over the past decade to scaling ASVAB test scores and to equating alternate and successive forms of this instrument. The issue of scale and origin of estimated proficiency evaluations for a wide range of criterion categories, however, has not been seriously addressed.

Before the problem of scale and origin of estimated proficiency evaluations can be adequately solved, satisfactory solutions to the absolute origin of actual proficiency evaluations for the various job categories must be implemented. In most defense department settings, recruits are assigned to various training schools before going into ongoing operational activities. The problems of determining and assigning numerical indices of success on a single scale for all trainees of a given category are well known. This is only a special case of the problem of equating school grades for students coming from various regions and schools and assigned by instructors whose grading standards vary widely. The use of standardized objective proficiency and performance tests can reduce disparity of standards for schools purporting to train for the same specialties. However, the problems of providing meaningful and useful performance ratings in subsequent operational training or actual on-the-job performance are much more difficult than that for the pre-activity training agencies.

However difficult the problems for achieving comparability of proficiency indices for both training and operational performance within a specified military occupational specialty, it is even more difficult to equate scales of performance indices from one specialty to another. For example, is a proficiency scale value of 50 for an airplane pilot as good as a scale value of 50 for a cook and baker in the Air Force? The issue raised by this question is but a special case of the equal-pay-for-equal-work issue that has plagued the equal rights movements of modern times. Models do exist, and doubtless improved ones can be developed, that could provide adequate and practical solutions to the problem. It is highly probable that satisfactory solutions to the comparable-proficiency measures problem will be forthcoming, once the importance of the problem for the optimal utilization of human resources is sufficiently recognized by those responsible for authorizing the resources necessary for more adequate solutions.

It must be clear by now that the preceding discussion concerns an issue that for more than half a century has been and still continues to be the cause of much handwringing among psychometricians. This is, of course, the problem of the criterion, and the related concept of statistical validity. More specifically, it is the problem of predictive validity. Some have tried to circumvent the fundamental validation issues by inventing other concepts and including the word "validity" in their labels, such as construct validity, content validity, face validity, and convergent validity, among others. The concepts included in these semantic variations make far less demands on technological, administrative, and operational resources than do the kind of predictive validity research programs that must provide the basis for credible optimal differential assignment programs. However, they cannot be expected to contribute substantially to the maximum utilization of human resources.

One concept of recent origin is called "criterion-referenced tests." The idea here is that, for a given instrument, a score is established below which persons are designated as disqualified for success in a particular criterion activity. Also, weighting strategies may be established for combining the scores from several tests and then designating a composite score below which persons are excluded from a specified activity. Such procedures may create the illusion of precision and gain acceptance by operational personnel. However, unless both the weighting and cutting values have been determined
by sound validational procedures, the classification and assignment of personnel by them can be far from optimal. Unfortunately, since armchair and subjective methods are, in general, much less demanding of time, effort, and other resources than adequate validational programs, the temptation to use them in practice can be very great.

A serious objection to the criterion-referenced concept is the assumption of a catastrophic single-step functional relationship between test performance and job performance. In general, the functional relationship between these variables can be expected to be continuous and at least monotonic. It is, of course, conceivable that the function could have a maximum at some point within a finite interval. For example, persons with very low scores might be unsuccessful in a simple, highly repetitive, monotonous job. Persons with medium scores might in general be adequately successful, while persons with high scores might find the monotony and lack of challenge of the job intolerable. As a result, because of low morale and motivation, they are unproductive on the job. Such a score-proficiency relationship could exhibit a single nonasymptotic maximum. However, in any case, it is difficult to conceive of score-proficiency functions with more than one nonasymptotic maximum or with critical values that could trigger step-function behavior. The concept of "criterion-referenced" test scores or combinations of test scores, while used extensively in the military services, probably contributes little to optimal differential assignment in the maximum utilization of military personnel resources.

Perhaps it is evident by now that selection and classification instruments can be effective only to the extent that their development and use are based on a sound and comprehensive validation program. Such a program should be continuous and integrated with operational selection and classification procedures. Many technical, administrative, and organizational considerations and problems are obviously involved in the implementation and conduct of such a program. They are not, however, insurmountable, as evidenced by the air crew selection and classification program during World War II in the Flying Training Command of the U.S. Army Air Forces.

Assume then that we do have the capability of obtaining acceptable estimates of proficiency in each of the activity categories of an operational system for job candidates from their predictor measures. Assume that these estimates yield measures that are comparable from one job category to another. These assumptions imply that a transformation matrix has been generated that, when applied to a matrix of predictor measures on a sample of candidates, will yield a matrix of estimated proficiency measures for each of the job categories. These assumptions therefore imply that, for each candidate, there is a vector of predictor measures that can be multiplied by the transformation matrix to yield a vector of expected job category proficiency measures.

How do we use these vectors of expected proficiency in assigning a candidate to a job category? Obviously, we cannot merely assign him or her to the job for which his or her prediction is highest. If we do this, a number of undesirable consequences may result:

1. The candidate may be unhappy with the assignment.
2. The candidate may be assigned to a job for which there is no opening.
3. Not enough recruits may be assigned to fill the needs of some job categories.
4. Not enough candidates from various minority or other special groups may be assigned to satisfy political or policy constraints.
5. The overall number of recruits assigned during a specified period may be more than the system can absorb at the time.

6. The overall number of recruits assigned during a specified period may not be enough to satisfy the overall requirements of the system at that time.

It would seem highly desirable to have a second-stage transformation matrix that would convert the expected proficiency vectors to constrained expectancy proficiency vectors that satisfy two conditions: (1) The constrained vectors for all recruits within a specified time period should be maximally congruent with the unconstrained vectors in some mathematically rigorous sense, and (2) the conditions that need to be satisfied will be closely approximated if each recruit is assigned to the job category for which his or her constrained expected proficiency has the highest value.

It is obviously true that the conditions to be satisfied may vary over time. The distributional and correlational parameters of the characteristics of recruits may also vary over time. However, such changes could be continuously monitored and the transformational vectors could be modified accordingly.

We have now indicated two kinds of expected proficiency vectors: (1) the unconstrained vector and (2) the constrained vector. The unconstrained vector gives the expected proficiencies for each of the job categories under consideration, irrespective of the operational and policy constraints that must be satisfied in the actual differential assignments. The constrained vector is maximally congruent with the unconstrained but so modified that, if each recruit is assigned to the job category corresponding to the highest element in his or her vector, the operational and policy constraints of the system will be approximated.

We have indicated that the unconstrained expected proficiency vector is obtained from predictor measures by a transformation matrix derived on the basis of an adequately implemented validation program. We have suggested that the constrained expected proficiency vector could be obtained by multiplying the unconstrained vector by a transformation matrix. This second transformation is determined so as to satisfy both the conditions of maximal congruency and the operational and policy constraints (for a general discussion of this approach, see Horst and Sorenson, 1976). However, it should be obvious that, mathematically, we can multiply the first transformation matrix by the second to yield a single product matrix that can be applied directly to the predictor vectors. We can then call the first matrix the unconstrained transformation matrix and the product matrix the constrained transformation matrix.

We see now that the foregoing discussion implies three distinct kinds of matrices with their constituent entity, or person, vectors: (1) the predictor matrix, (2) the unconstrained expected proficiency matrix, and (3) the constrained expected proficiency matrix. It is important to note that the matrix of dependent, or criterion, variables does not appear directly among these three. It has been involved in the generation of the unconstrained transformation matrix during the validation program. To simplify further discussion, we shall refer to these three types of matrices respectively as the predictor, unconstrained, and constrained matrices.

A great amount of effort has gone into activities that can generate predictor matrices by the major testing organizations in the country, such as the Educational Testing Service, the American College Testing Program, the U.S. Office of Personnel Management, the U.S. Employment Service, and the U.S. Department of Defense. Relatively little effort by any of these agencies has gone into activities that can
generate proficiency matrices. This is not for want of technological resources. It is, in part, due to the formidable problems of administration, coordination, and cooperation required in the implementation of comprehensive validation programs. It is also due to a general lack of understanding and appreciation of the importance of estimated proficiency matrices and how these are generated from predictor measures. Much of the political difficulties that have surfaced in the testing movement over the recent past can be laid to the fact that far too much effort has been devoted to the development, implementation, and promotion of testing instruments and far too little to large-scale programs for the validation of these instruments.

Our chief concern in optimal differential assignment is with the unconstrained and the constrained matrices. The predictor matrix is simply the starting point for the development of the former two. Of these, the unconstrained matrix has traditionally played the major role in efforts to implement optimal differential assignments. It is the unconstrained matrix representing a sample of job candidates with which models of differential assignment have traditionally been concerned. The batching model assumes a sample of recruits or candidates, each member of which is to be allocated to one of the activity categories included in the matrix. Linear programming techniques have been used to assure that the overall expected proficiency will in some sense be optimal. Some form of the transportation model is typically involved.

A batching procedure may be explicitly designed to yield results in allocation that will satisfy prespecified quota designations. Perhaps the simplest criterion of optimality is a sum or average of the expected proficiencies of the assigned candidates. A precise mathematical formulation of this criterion begins with an orthogonal binary classification matrix whose order is the same as the batching matrix of expected proficiencies. Assuming rows for persons and columns for job categories, unit elements of this matrix are assigned so that the vector of column sums conforms to the quota vector, and the trace of the product of this unconstrained matrix with the classification matrix is a maximum. In many situations, there may be no unique solution.

For a given time period, it is, of course, possible that there are more applicants for the system than are required at the time. A further complication of the procedure is required, viz., a rejection category. A rejection parameter can be incorporated into the allocation model. Such a parameter can be varied as the supply and demand for recruits change over time.

However, in many real-life settings and in the military during a peacetime volunteer system, candidates may not arrive in convenient batches at classification centers. A more general and flexible model of assignment would appear to be one that is sequential in that it considers candidates one at a time. Such a model would yield a modified expected proficiency vector directly from a predictor vector. On the basis of this vector, a single or several recommended assignments can be made. If these recommendations are followed, the operational and policy constraints within specified time periods should be approximated and the overall expected proficiency of assignees optimized.

These objectives can be approximated by the use of a constrained transformation matrix that is continually updated to satisfy changing operational and policy constraints and changing distributions and intercorrelations of predictor variables. Such a transformation matrix would yield the vectors of a constrained expected proficiency matrix. Therefore, it would provide the basis of a sequential, or one-at-a-time, assignment program, as well as the vectors for expected proficiency matrices implied by batching procedures.
On the other hand, suppose only the unconstrained expected proficiency matrix were available at a specified time. Then, presumably, a linear programming procedure could be applied as a basis for optimizing personnel assignments. In general, however, such unconstrained matrices could not be used without modification for sequential assignment procedures that would satisfy operational and policy constraints. Therefore, a sequential assignment model can be used for batching assignment procedures but batching assignment models cannot, in general, be used for sequential assignment procedures.

We have indicated that one criterion of optimal differential assignment is the sum of expected proficiencies in their job assignments for all assignees. However, this may well be a poor criterion for an overall personnel system. Such a simple criterion could result in a very high expected proficiency for some activity categories and a very low one for others. Although the expected average proficiencies could be adequate, components of operational systems typically must interact with one another and the effectiveness of one component could be greatly influenced by that of another. To give an obsolete example from World War II, the pilots in an operational Air Force unit might have superior flying skills but the bombardiers may be incompetent in the manipulation of bombsights. The average skills of these two components of an air crew may be adequate but the effectiveness of a mission could be no better than that of the bombardiers.

This is all by way of suggesting that the criterion of optimal assignment might well be more than a simple summation of proficiencies for all of the activity categories in the system. We might require, for example, that this summation be a maximum, with the constraints that the average expected proficiencies for all categories be equal or that the dispersion of these be a minimum. Such constraints can be included in the derivation of the constrained transformation matrix.

Obviously, the foregoing discussion implies a large amount of mathematical theory and modeling that cannot be explicated in this presentation. However, several rather fundamental issues should be clarified. Some of these have to do with the number of distinct activities that can be validly distinguished in an optimal differential assignment system. One of these concerns the systematic dimensionality of the predictor matrix. It is important to distinguish between the number of variables in the predictor set and the latent dimensionality. Although a matrix of measures may have, let us say, 20 variables and 1,000 cases, it may be possible to reproduce these variables with linear functions of a much smaller number of variables, say 6. The 20 variables are called the manifest variables of the system; and the 6, the latent variables. Thus, we say that the predictor set has a latent dimensionality of 6.

The systematic latent dimensionality of a predictor matrix can be estimated from its corresponding correlation matrix. The distribution of the eigenvalues of the matrix, together with estimates of the reliabilities of the manifest variables, can be used to estimate the latent dimensionality of the matrix. For example, analyses of the ASVAB test batteries show that, while their manifest dimensionality may be twelve, the latent dimensionality cannot possibly be greater than six or seven. It might be as low as four. Some have maintained that it has a systematic rank of only two but this estimate is probably too low. These estimates can vary, depending on the particular version of the battery, the sample analyzed, and the loss and scaling functions used in the factoring analysis.

In any case, however, one cannot expect to have genuine optimal differential assignment unless the latent dimensionality is at least as great as the number of activity
categories. Stated mathematically, the systematic rank of the predictor matrix must equal or exceed the number of activity categories.

It is well known that the rank of the product of two matrices cannot be greater than the rank of the factor of lower rank. Therefore, the rank or latent dimensionality of the expected proficiency matrix cannot be greater than the systematic rank of the predictor matrix, since the former matrix is the product of the predictor by the transformation matrix.

Actually, an estimated proficiency matrix can have a systematic rank even lower than that of the predictor matrix, even though its manifest dimensionality, or number of activity categories, is much greater. This can happen if the rank of the transformation matrix is smaller than that of the predictor matrix. This can occur, for example, when the different activity categories are represented by service schools that train for various operational activities. If criterion measures are based on school grades, achievement in these grades may have a large common component for most of the schools. This can result in transformation matrices whose effective rank is less than that of the predictor matrix. Therefore, it would result in expected proficiency matrices with rank less than that of the predictor matrix.

Even operational activity categories, such as designated military occupational specialties, may exhibit a great deal of overlapping in the traits and abilities called for. The greater such overlapping, the smaller in general will be the latent dimensionality of a proficiency matrix.

In general, the latent dimensionality of a set of manifest operational or administrative categories must be regarded as the base from which the development of an optimal differential assignment system proceeds. If it turns out that its latent dimensionality is much less than its manifest dimensionality, job clustering procedures are called for. The number of clusters should approximate this latent dimensionality. The jobs within a cluster should, in general, be homogeneous and the clusters should be mutually heterogeneous. A large body of theory and technology is available for clustering procedures. In general, however, optimal differential assignments can be made effectively for only clusters of jobs.

There are, of course, many ways in which the number of predictors may be increased. Among these is the use of smaller units of test elements, such as individual item scores themselves. Other methods include the development or addition of different types of material. Even the use of higher order product variables generated from original and supplemental test variables should be considered. These are sometimes called configural variables. Actually, the potential number of predictor variables can be greatly increased without exacting more time and effort from examinees. A great deal more processing and computational work might be required but such activities are by now enormously facilitated by contemporary computer hardware.

We have indicated that optimal differential assignment can function only for a number of assignment categories equal to or less than the latent dimensionality of the predictor or expected proficiency matrix, whichever of the two is the smaller. Special cases may be constructed where this principle could appear to be contradicted. For example, suppose we had only a single predictor variable, such as a single test. Then the transformation matrix for converting the corresponding test score vector for a sample of cases to an expected proficiency matrix would obviously be a vector and the product matrix would, by definition, be of rank one. Suppose the criterion of optimal differential assignment were that of a maximum sum of expected proficiencies. Assume that the
elements of the transformation vector were all distinct. Each element would correspond to a job category and, for each job, we specify what percentage of the batch shall be assigned to it.

Suppose now we arrange the elements in the transformation vector in order of magnitude from high to low and the persons in the batch from high to low in order of test score. It can readily be shown that the following procedure satisfies the maximum sum criterion. We fill the quota for the first job with the persons whose estimate of proficiency is highest. We then fill the quota for the second job with those not assigned to the first job whose estimates are highest. Similarly, we fill successively the remaining quotas. Hence, we have at least the illusion of optimal differential assignment from a single predictor with no restriction on the number of different activity categories.

However, there are at least two obvious flaws in this demonstration. The first is due to the fact that we have dispersion in the elements in the transformation vector, primarily if we have dispersion in the degree of relationships of the predictor with the various criteria. We thus infer that, to have adequate optimal differential assignment efficiency with a single predictor, this predictor must show a wide range of validity among the criteria. In other words, we depend heavily on a test that is of little value in estimating success for some of the assignments if we hope to achieve optimal differential assignment. Even if we assume that this situation is acceptable, we still are forced into an unacceptable position with reference to the criterion of assignment efficiency. It can readily be shown that, with the rank-one case, the maximum sum criterion yields a maximum dispersion of average group expected proficiencies that, as we have seen, can be a very poor outcome for the overall functioning of a system.

Other examples can be constructed that appear to achieve optimal differential assignment from rank-one assignment matrices. However, whenever we have succeeded in constructing one, it can be shown that it rests on unacceptable conditions or implicitly effects proliferations of predictors due to nonlinear relationships between predictor and criterion variables.

We have said little about the mathematical aspects of constrained expected proficiency evaluation models. In general, these models include the function to be optimized, together with the constraining functions and their associated Lagrangian matrices, vectors, and scalers. Solutions are typically nonlinear in some of the parameters to be solved for and iterative solutions are usually required. A great advantage of models that introduce constraints is that political or policy pressures can be absorbed. Optimal constrained solutions can be achieved while, at the same time, the cost of these constraints to the efficiency of the system can be demonstrated.

In conclusion, we note that, if there were enough of those relatively few people who can do almost anything well, there would be no need to bother about optimal differential assignment procedures. Those persons could do the work of the world and the rest of us could take it easy. However, since there are not many such people, we should utilize that large middle group who can do some things better than they can do other things. To them we should apply the best methodologies available, or to be developed, for optimal differential assignment and the maximum utilization of our human resources.
STRATEGIES FOR CAPITALIZING ON INDIVIDUAL DIFFERENCES IN MILITARY PERSONNEL SYSTEMS

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A major responsibility of personnel systems developers is to provide personnel managers with techniques to select, classify, assign, and reassign people to appropriate jobs that tend to maximize overall system effectiveness. These techniques involve both the definition and measurement of relevant attributes of people (referred to as person characteristics) and the definition and measurement of relevant attributes of jobs (referred to as job properties). The various items of information about people and jobs are then combined to yield an indicator of expected future performance (referred to as "payoff," "value," "worth," or "utility") of each person on every job (Ward, 1977). Using these indicators of payoff, personnel are assigned to jobs in a way that will tend to maximize the overall system effectiveness (usually by maximizing the sum of the person-job payoff values associated with the designated assignments). While techniques might be available for designating without choice which person will go into which job (Langley, Kennington, & Shetty, 1979), most personnel assignment systems allow for some but not total freedom of choice by both personnel (for jobs) and job managers (for personnel). This leads to a requirement for techniques that provide for each person-job assignment combination an indicator of the desirability of that particular assignment for overall maximization of personnel assignment effectiveness. Such an indicator has been referred to by the general term "allocation index" (Ward, Haney, Hendrix, & Pina, 1978). However, the allocation index may be called by other more appropriate names such as decision index, differential assignment index, differential classification index, optimality index, or personnel optimality index. An allocation index is dependent on the particular group of people and particular set of jobs being considered in the assignment system. However, the payoff value for a specific person-job combination is generally independent of other people and other jobs being considered for assignments. The term optimality index will be used in this discussion to refer to an indicator of overall personnel systems effectiveness.

An optimality index that has been used by the Air Force is a decision index (Ward, 1959). This index was developed to provide helpful information to Air Force personnel specialists to assist in making good personnel classifications.

Recently, an important relation has been recognized between the problem of optimal personnel assignment (and its related optimality indicator) and the concept of interaction among people and jobs. This recognition opens up a wide range of new opportunities to improve the effectiveness of personnel assignment systems, including the following:

1. A measure is available for the extent to which it is possible to make any improvement in personnel systems effectiveness.

2. Assuming that there is a big potential for assignment effectiveness (as indicated by a large amount of interaction), it is possible to measure for each job the importance of making a good personnel assignment to that job.

3. It is possible to measure for each person the importance of making a good job assignment for that person.

4. It might be possible to use a set of person measures or job measures in a noninteracting form in association with other predictor information used in interactive
ways on only a limited sample of data to obtain a more accurate prediction system. Then the noninteractive measures need not be collected for future operational predictions. This feature has potential for reducing the amount of operational testing required or may eliminate the need for operational use of controversial predictor variables.

Before discussing in detail the four implications mentioned above, a general overview of personnel assignments and the equivalence of the assignment problem and interaction will be presented.

Figure 1 provides a general view of an array of predicted payoff values. There are $N_P$ "real" people to be considered for assignments to a total of $J$ jobs consisting of a total of $M_1$ Type 1 jobs plus $M_2$ Type 2 jobs through $M_L$ Type L jobs. To allow for the possibility of all jobs being unfilled (i.e., occupied by a shadow person) and to allow for all persons being rejected from the personnel system (i.e., assigned to external jobs), we allow for a total of $N_J$ shadow persons and $M_P$ external jobs, with $N_J = M_J$ and $N_P = M_P$. Therefore, there is a total of $N (=N_P \text{ (real)} + N_J \text{ (shadow)})$ persons to be considered for $M (=M_J \text{ (internal)} + M_P \text{ (external)})$ jobs. For the remainder of the discussion, it is assumed that the predicted payoff array is of dimension $N$ by $M$, with $M = N$. We will speak of an $N \times N$ payoff array and $N!$ possible, but not unique, allocations. For each allocation, a total payoff can be calculated from the $N$ values associated with each person-job assignment. Many of these different assignments might produce the same sum. These values are referred to as the objective function, $Z$.

<table>
<thead>
<tr>
<th>NUMBERS</th>
<th>JOB 1</th>
<th>JOB 2</th>
<th>JOB L</th>
<th>EXTERNAL JOBS</th>
</tr>
</thead>
<tbody>
<tr>
<td>REQUIRED AVAILABLE</td>
<td>$M_1$</td>
<td>$M_2$</td>
<td>$\cdots$</td>
<td>$M_L$</td>
</tr>
<tr>
<td>PERSON 1</td>
<td>1</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>PERSON 2</td>
<td></td>
<td>1</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>PERSON $N_P$</td>
<td></td>
<td></td>
<td></td>
<td>1</td>
</tr>
<tr>
<td>SHADOW PERSONS</td>
<td></td>
<td></td>
<td>$N_J$</td>
<td></td>
</tr>
</tbody>
</table>

Figure 1. Predicted payoff array.
The problem of optimal assignment is to assign persons so that the sum of the payoff values (objective function $Z$) is maximized. If the elements of the predicted payoff array are designated $P$, then the concern is to compare alternative assignments as shown in Figure 2.

One alternative is for person $I$ to be assigned to job $J$ and person $K$ to be assigned to job $L$. Another alternative is for $I$ to be assigned to $L$ and $K$ to $J$. The difference in predicted payoff is given by:

$$\left( P_{IJ} + P_{KL} \right) - \left( P_{IL} + P_{KJ} \right) = (CS)_{IJKL}. \tag{1}$$

If the comparisons, $(CS)_{IJKL}$, for all possible persons and jobs are all equal to zero, then all of the assignments will give the same objective value. When the absolute values of $(CS)_{IJKL}$ are large, then different assignments will potentially make a bigger difference in the value of the objective function.

Now notice Figure 3 and a conceptually different view. The interest here is in applying analysis of variance or a general linear model approach to the study of interaction among people and jobs.
The degree of interaction is determined by:

\[(P_{IJ} - P_{IL}) - (P_{KJ} - P_{KL}) = (CD)_{IJKL}.\] (2)

In this case, if all comparisons, \((CD)_{IJKL}\), are equal to zero, then it is said that no interaction exists. And, as above, the larger absolute values of \((CD)_{IJKL}\) indicate that more interaction is present.

Equations 3 and 4 summarize the fact that the basic concerns of the investigation of optimum assignments are equivalent to the concerns about interaction among people and jobs.

\[\begin{align*}
(P_{IJ} + P_{KL}) - (P_{IL} + P_{KJ}) &= (P_{IJ} - P_{IL}) - (P_{KJ} - P_{KL}) \\
(CS)_{IJKL} &= (CD)_{IJKL}
\end{align*}\] (3)

To obtain an indicator of the potential for improved assignments, we examine the relation between the variance of the objective function, the variance of the optimality index, and the interaction.

The interaction sum of squares for the \(N\) by \(N\) payoff array is shown in Equation 5:

\[
S = \sum_{I=1}^{N} \sum_{J=1}^{N} (P_{IJ})^2 - \frac{1}{N} \left[ \sum_{I=1}^{N} \left( \sum_{J=1}^{N} P_{IJ} \right)^2 + \sum_{J=1}^{N} \left( \sum_{I=1}^{N} P_{IJ} \right)^2 \right] \\
+ \frac{1}{N^2} \left[ \sum_{I=1}^{N} \sum_{J=1}^{N} P_{IJ} \right]^2
\] (5)

Equation 6 shows the mean and variance of the \(N!\) possible values of the objective function, \(Z\).

\[
\bar{Z} = \frac{1}{N} \sum_{I=1}^{N} \sum_{J=1}^{N} P_{IJ}; \quad \sigma^2_Z = \frac{S}{(N-1)}.
\] (6)

Equation 7 defines an optimality index (OPI) for person I on job J. The mean and variance of the \(N^2\) values of OPI are 0 and \(S/N^2\), respectively.

\[\text{(OPI)}_{IJ} = P_{IJ} - \frac{1}{N} \sum_{I=1}^{N} P_{IJ} - \frac{1}{N} \sum_{J=1}^{N} P_{IJ} + \frac{1}{N^2} \sum_{I=1}^{N} \sum_{J=1}^{N} P_{IJ}.
\] (7)

The value of \((\text{OPI})_{IJ}\) is a direct indicator of the extent to which the assignment of person I to job J can be expected to increase (if positive) or decrease (if negative) the overall objective function. OPI values can be computed and used to determine an ordered job list from which a person may choose a job. In addition, the OPI values can be used to determine an ordered personnel list from which a job manager may select.
Equations 8 and 9 summarize the relation between the variance of the objective function, the variance of the optimality index, and the interaction sum of squares:

\[
\sigma_Z^2 = \frac{S}{N-1}, \tag{8}
\]

\[
\sigma_{OPI}^2 = \frac{S}{N^2}. \tag{9}
\]

It is now apparent that, as the interaction approaches zero, the variances of the objective function (Z) and the optimality index (OPI) approach zero. These three indicators measure the potential improvement for making better personnel assignments. Furthermore, they are related to differential prediction efficiency, \( \phi \) (Horst, 1954):

\[
\phi = (N-1)(\bar{\sigma}^2 - \bar{c}) \tag{10}
\]

where \( \bar{\sigma}^2 \) is the average predicted variance, \( \bar{c} \) is the average predicted covariance, and

\[
\phi = \frac{S}{N}. \tag{11}
\]

If the variance of the OPI values is large (i.e., there is a large amount of interaction among people and jobs), it is of interest to determine that part of total interaction variance that is associated with each job. The variance of the OPI values for the job C gives this measure as shown in Equation 12:

\[
\sigma_{OPI}^2(C) = \frac{1}{N} \sum_{I} [P_{IC}]^2 - \frac{2}{N^2} \sum_{I} P_{IC} \sum_{J} P_{IJ}^2 + \frac{1}{N^3} \sum_{I,J} P_{IJ}^2 \tag{12}
\]

\[
- \frac{1}{N^2} \sum_{I} [P_{IC}]^2 + \frac{2}{N^3} \sum_{I} P_{IC} \sum_{J} P_{IJ}^2 - \frac{1}{N^4} \sum_{I,J} P_{IJ}^2.
\]

If a job has a very large \( \sigma_{OPI}^2(C) \) compared with other jobs' variances, then it is very important to attend to the task of making a "better assignment" for that job. Otherwise, that job has a good chance of receiving a bad personnel assignment. A proposed name for this measure is the job-potential index.

Having considered the variance of the OPI for each job, it is also of interest to compute a similar measure for each person. The variance of the OPI for person R is shown in Equation 13:

\[
\sigma_{OPI}^2(R) = \frac{1}{N} \sum_{J} [P_{RJ}]^2 - \frac{2}{N^2} \sum_{J} P_{RJ} \sum_{I} P_{IJ}^2 + \frac{1}{N^3} \sum_{I,J} P_{IJ}^2 \tag{13}
\]

\[
- \frac{1}{N^2} \sum_{J} [P_{RJ}]^2 + \frac{2}{N^3} \sum_{J} P_{RJ} \sum_{I,J} P_{IJ}^2 - \frac{1}{N^4} \sum_{I,J} P_{IJ}^2.
\]

Similar to the discussion of the job OPI variance, if a person has a very large \( \sigma_{OPI}^2(R) \) compared with other persons' variances, then it is very important to attend to
the task of making a better assignment for that person. A proposed name for this measure is the person-potential index.

The relative quality of an allocation of persons to jobs is not affected by adding (or subtracting) a constant to an entire row (or column) of the predicted payoff array. This opens the interesting prospect of improving the accuracy of prediction of the payoff values by including among the predictor variables a subset that does not involve any person-job interaction. These predictor variables are referred to as the noninteractive variables. The other predictor variables are called interactive variables. The prediction system is developed using both the interactive and the noninteractive variables, thereby obtaining the regression weights for the interactive variables in the presence of the noninteractive ones. Then the operational assignment system applies only the regression weights for the interactive variables (computed in the model with the noninteractive variables) to the values of the interactive variables—the noninteractive variables are not required.

If improved prediction can be achieved in this manner, then fewer predictor variables are required. This reduction in variables could reduce testing time and/or allow for the elimination of controversial variables.

Recognition of the relation between optimal assignment procedures and interaction among persons and jobs suggests consideration of the following recommendations:

1. Use an optimality index (OPI) in operational assignment systems.

2. For a predicted payoff array, compute one of the following: (a) variance of the objective, (b) variance of the optimality index, or (c) interaction sum of squares, and use one or more of these as a measure of potential for assignment improvement.

3. Compute for each job the variance of the optimality index and use this variance as a measure of the importance of making a good assignment to that job.

4. Compute for each person the variance of the optimality index and use this variance as a measure of the importance of assigning the person to a good job.

5. Explore the possibility of reducing the number of predictors and improving predicted payoffs required in personnel assignment systems.
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Chief of Naval Technical Training (015)
Commander Naval Military Personnel Command (NMPC-013C)
Commanding Officer, Naval Aerospace Medical Institute (Library Code 12) (2)
Commanding Officer, Naval Underwater Systems Center
Commanding Officer, Naval Education and Training Program Development Center (Technical Library) (2)
Commanding Officer, Naval Regional Medical Center, Portsmouth (ATTN: Medical Library)
Commanding Officer, Naval Regional Medical Center, San Diego (Alcohol Rehabilitation Service D36-1)
Commanding Officer, Naval Training Equipment Center (Technical Library) (5)
Commanding Officer, Office of Naval Research Branch Office, Chicago (Coordinator for Psychological Sciences)
Commanding Officer, Service School Command, San Diego (Code 3200)
Director, Career Information and Counseling School (Code 3W34)
Director, Training Analysis and Evaluation Group (TAEG)
Officer in Charge, Naval Occupational Development and Analysis Center
President, Naval War College (Code E114)
Superintendent, Naval Postgraduate School
Secretary Treasurer, U.S. Naval Institute
Commander, U.S. Army Soldier Support Center, Fort Benjamin Harrison (Human Dimensions Division)
Commander, Army Research Institute for the Behavioral and Social Sciences, Alexandria (PERI-ASL)
Commander, Army Research Institute for the Behavioral and Social Sciences, Alexandria (PERI-ZT)
Headquarters Commandant, Military Enlistment Processing Command, Fort Sheridan
Director, Systems Research Laboratory, Army Research Institute for the Behavioral and Social Sciences, Alexandria (PERI-SZ)
Director, U.S. Army TRADOC Systems Analysis Activity, White Sands Missile Range (Library)
Chief, Army Research Institute Field Unit--USAREUR (Library)
Chief, Army Research Institute Field Unit, Fort Harrison
Commander, Air Force Human Resources Laboratory, Brooks Air Force Base (Manpower and Personnel Division)
Commander, Air Force Human Resources Laboratory, Lowry Air Force Base (Technical Training Branch)
Commander, Air Force Human Resources Laboratory, Williams Air Force Base (AFHRL/OT)
Commander, Air Force Human Resources Laboratory, Wright-Patterson Air Force Base (AFHRL/LR)
Commander, 314 Combat Support Group, Little Rock Air Force Base (Career Progression Section)
Program Manager, Life Sciences Directorate, Bolling Air Force Base
Commanding Officer U.S. Coast Guard Institute
Commanding Officer, U.S. Coast Guard Research and Development Center, Avery Point
Superintendent, U.S. Coast Guard Academy
President, National Defense University (3)
Director, Science and Technology, Library of Congress
Defense Technical Information Center (DDA) (12)