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GRID GENERATION ABOUT A FIN-CYLINDER COMBINATION

G. H. Hoffman
GRID GENERATION ABOUT A FIN-CYLINDER COMBINATION

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An algebraic grid generation procedure is presented which produces a three-dimensional, body fitted coordinate system about a right circular cylinder with four symmetric fins attached. Special features of the grid are an initial value plane normal to the cylinder axis and the ability to cluster lines near the fin and cylinder surfaces for viscous/turbulent flow calculations. The method used is a modification of the
Jameson-Caughey procedure developed originally for inviscid transonic flow calculations about wing-fuselage combinations. In this procedure, a sequence of conformal transformations followed by a shearing transformation is used to map the irregular flow domain in physical space into a rectangular shaped computational domain. A three-dimensional grid is produced by stacking two-dimensional mappings. The method is therefore extremely fast. The main features of the procedure are discussed and two numerical examples of grids are presented for a fin composed of a symmetric Joukowsky airfoil.
Subject: Grid Generation about a Fin-Cylinder Combination

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I. INTRODUCTION

This report addresses the problem of generating a surface-fitted grid in a model fin-body problem consisting of a circular cylinder with four identical symmetric fins attached. This grid is to be used in the calculation of incompressible, laminar flow at moderate-to-high Reynolds numbers. The aim of the calculation is to resolve the details of the separated zone at the leading edge of the fin-cylinder juncture and the subsequent vortex that forms downstream. Thus, the grid must have proper clustering so as to resolve the regions of high flow gradients.

The approach used here is to generate the grid analytically but to determine the metric coefficients numerically. Such an approach has been pursued successfully by Jameson [1] and Caughey and Jameson [2-4] in solving three-dimensional inviscid transonic flows about wing-body combinations. The basic idea is to map the physical geometry to a strip of almost constant width using a sequence of conformal transformations. Then boundary fitted coordinates are generated by the application of a shearing transformation. The result of the latter transformation is a nonorthogonal coordinate system in the physical plane but one in which the nonorthogonality can be controlled.

The present work is an extension of the Jameson-Caughey technique for what is called the wind tunnel problem to the case of an initial value plane ahead of the airfoil. In order to treat viscous flow, clustering transformations are used so that the computational grid is uniform in all three directions.

One advantage of the present technique is that, owing to the simple cylindrical body geometry, a three-dimensional grid is generated by stacking a series of two-dimensional grids. Another advantage of the analytical approach over the numerical solution of elliptic partial differential equations as a
means of grid generation is its much greater speed which is especially important for three-dimensional applications.

II. ANALYSIS

2.1 Geometry of Computational Domain

We start the grid generation analysis by defining the geometry about which a surface fitted grid is to be generated and the extent of the computational domain.

1. The body is an infinitely long, hollow circular cylinder of radius $R_c$ with its centerline parallel to the free-stream velocity vector.

2. Four identical fins of constant unit chord and infinite span, consisting of symmetric airfoil sections, are mounted on the cylinder 90 degrees apart with their chord planes passing through the cylinder axis.

3. The computational domain consists of the region interior to an outer cylinder of radius $R_t$ which encases the inner cylinder and fins, bounded upstream and downstream by planes normal to the cylinder axis.

A schematic of one fourth of the geometry and computational domain is shown in Fig. 1 and a head-on view showing the coordinate system in the crossflow plane appears in Fig. 2. Since the fins are identical and equally spaced, we have four planes of symmetry, namely, at $\theta = 0, \pi/4, \pi/2$ and $3\pi/4$. Thus, in the flow field calculation for this model problem only the segment $0 < \theta < \pi/4$ needs to be considered.
2.2 Sequence of Transformations

Four transformations applied in sequence are required to map the fin-cylinder and surrounding computational domain into a rectangular parallelepiped. Then a fifth stretching transformation is applied to adjust the grid line spacings for proper flow field resolution in physical space and to allow a uniform step size in all three computational coordinates.

We start by defining polar coordinates \((r, \theta)\) in the crossflow plane, as shown in Fig. 2, according to

\[
r = \sqrt{y^2 + z^2},
\]

\[
\theta = \tan^{-1}\left(\frac{y}{z}\right).
\]

Thus, points in physical space are defined by standard cylindrical coordinates \((x, r, \theta)\).

Following Caughey and Jameson [2], the first transformation normalizes \((x, r, \theta)\) according to (all lengths are referred to the airfoil chord):

\[
\bar{x} = x - d_s + \ln 2,
\]

\[
\bar{r} = r - \frac{R_c}{R_t - R_c},
\]

\[
\bar{\theta} = 4\theta,
\]

where \(d_s\) is the location of the singular point of the unwrapping transformation and is just inside the leading edge of the airfoil. Note that in the above definitions, \(0 < \bar{r} < 1\) and \(0 < \bar{\theta} < \pi\) in the computational domain. The upper limit on \(\bar{\theta}\) is convenient in the next transformation.
Because $\bar{r} =$ constant is a surface fitted coordinate we need only generate a surface fitted grid in the $(\bar{x}, \bar{\theta})$ plane. The geometry of an $\bar{r} =$ constant surface in the computational domain is sketched in Fig. 3.

The conformal transformation

$$\bar{x} - i\bar{\theta} = \&n[1 - \cosh(\xi + i\eta)]$$, \hspace{1cm} (6)

applied to an $\bar{r} =$ constant surface unwraps the geometry in Fig. 3 to produce the domain shown in Fig. 4. The minus sign has been used on the left in Eq. (6) so that the upper symmetry plane maps to the positive $\xi$ axis.

In the present problem initial conditions from an axis metric boundary layer-potential flow composite solution are specified on the plane $\bar{x} = \bar{a}$. This initial value line in an $\bar{r} =$ constant surface (IVL) is shown as line segment ABC in Fig. 3. Under transformation (6), the IVL maps to a near semi-circle in the $(\xi, \eta)$ plane, as shown in Fig. 4. The airfoil image in this plane is the arc DEF.

We next apply another conformal transformation to nearly straighten out the IVL in Fig. 4. This transformation is

$$\bar{\xi} + i\bar{\eta} = \xi + i\eta + \frac{\xi_0^2}{\xi + i\eta}$$, \hspace{1cm} (7)

where $\xi_0$ is the intersection of the IVL with the $\xi$ axis (Point A in Fig. 4). The conformal transformation (7) maps the upper and lower boundaries in the $(\xi, \eta)$ plane into slowly varying functions of $\bar{\xi}$ in the $(\bar{\xi}, \bar{\eta})$ plane, as shown in Fig. 5. We note that near Points A and C the IVL is now cusp-like.
The fourth transformation is a shearing transformation which straightens out the upper and lower boundaries in the \((\vec{\xi}, \vec{\eta})\) plane. This transformation is

\[
\begin{align*}
X &= \vec{\xi}, \\
Y &= \frac{\vec{\eta} - \vec{\eta}_L}{D}, \\
Z &= \vec{r},
\end{align*}
\]  

(8) (9) (10)

where

\[
D = D(\vec{\xi}, \vec{r}) = \vec{\eta}_U - \vec{\eta}_L, 
\]  

(11)

and \(\vec{\eta}_U\) and \(\vec{\eta}_L\) are the ordinates, at a given \(\vec{\xi}\), of the upper and lower boundaries in the \((\vec{\xi}, \vec{\eta})\) plane.

Finally, to provide for clustering the grid lines near the fin and cylinder surfaces to resolve the viscous layers there and to space lines around the airfoil and in the wake as desired, we introduce one-dimensional stretching functions as follows:

\[
\begin{align*}
X_c &= F_1(X), \\
Y_c &= F_2(Y), \\
Z_c &= F_3(Z),
\end{align*}
\]

(12) (13) (14)

For the time being we leave \(F_1\), \(F_2\) and \(F_3\) unspecified. Thus \((X_c, Y_c, Z_c)\) are the computational coordinates devised so that the step sizes \(\Delta X_c\), \(\Delta Y_c\) and \(\Delta Z_c\) are constants.
2.3 Conformal Mapping Relations

Since the FORTRAN code is written in terms of real variables, the real and imaginary parts of the conformal mappings must be determined. In addition, the inverses of both mappings are needed because the grid generation procedure requires being able to proceed from the \((x,\theta)\) plane to the \((X_c, Y_c)\) plane and then back to the \((x,\theta)\) plane.

The real and imaginary parts of Eq. (6) yield the two relations:

\[
\cosh \xi \cos \eta = 1 - e^x \cos \theta, \tag{15}
\]

\[
\sinh \xi \sin \eta = e^x \sin \theta. \tag{16}
\]

The solutions for \(x\) and \(\theta\) are obtained by squaring (15) and (16), then adding and making use of the ordinary and hyperbolic trigonometric identities. The result for \(x\), choosing the proper sign, is

\[
\bar{x} = \ln(\cosh \xi - \cos \eta), \tag{17}
\]

and \(\bar{\theta}\) is obtained from Eq. (15), viz.,

\[
\bar{\theta} = \cos^{-1} \left( \frac{1 - \cosh \xi \cos \eta}{\cosh \xi - \cos \eta} \right). \tag{18}
\]

To obtain the solutions for \(\xi\) and \(\eta\) we first define,

\[
\bar{p} = 1 - e^x \cos \bar{\theta}, \tag{19}
\]

\[
\bar{q} = e^x \sin \bar{\theta}. \tag{20}
\]

Following the same procedures as above, we eliminate \(\eta\) to obtain a quadratic
equation for $\sinh^2 \xi$ which has the solution

$$\sinh^2 \xi = \frac{1}{2} \left[ (a^2 + 4q^2)^{1/2} - b \right]$$  \hspace{1cm} (21)

where

$$b = 1 - \frac{-p^2 - q^2}{2}. \hspace{1cm} (22)$$

In the right half plane $\xi$ is the positive root of Eq. (21). The expression for $\eta$ with the proper behavior ($0 < \eta < \pi$) is obtained from Eq. (15), viz.

$$\eta = \cos^{-1} \left( \frac{-p}{\cosh \xi} \right). \hspace{1cm} (23)$$

Next, the real and imaginary parts of Eq. (7) yield

$$\xi = \xi \left( 1 + \frac{\xi_0^2}{\xi^2 + n^2} \right), \hspace{1cm} (24)$$

$$\eta = \eta \left( 1 - \frac{\xi_0^2}{\xi^2 + n^2} \right). \hspace{1cm} (25)$$

We determine $\xi_0$ from Eq. (17) by setting $\xi = -\xi \Rightarrow \xi = -a + \ln 2$ and $n = 0$. The result is

$$\xi_0 = \cosh^{-1}(1 + 2e^{-d}). \hspace{1cm} (26)$$

where $a = d_g + d_{IVL}$.

To solve for $\xi$ and $n$ in terms $\xi$ and $\eta$ we return to the complex form which is written as,

$$w = z + \frac{\xi_0^2}{z}. \hspace{1cm} (27)$$
where
\[ w = \xi + i\eta , \quad (28) \]
\[ z = \xi + i\eta . \quad (29) \]

Solving Eq. (27) for \( z \) yields,
\[ \phi^2 = \frac{1}{4} \omega^2 - \xi_0^2 , \quad (30) \]

where
\[ \phi = z - \frac{1}{2} \omega . \quad (31) \]

Let us now define
\[ \phi = u + iv . \quad (32) \]

Then, combining Eqs. (28), (29) and (31) gives:
\[ \xi = u + \frac{1}{2} \xi , \quad (33) \]
\[ \eta = v + \frac{1}{2} \eta . \quad (34) \]

Now Eq. (30) leads to the following relations:
\[ u^2 - v^2 = \hat{p} , \quad (35) \]
\[ uv = \hat{q} , \quad (36) \]

where
\[ \hat{p} = \frac{1}{4} (\xi^2 - \eta^2) - \xi_0^2 , \quad (37) \]
\[ \hat{q} = \frac{1}{4} \xi \eta . \quad (38) \]

Equations (35) and (36) can be solved for \( u \) and \( v \) with the result:
where

\( \hat{u} = (p^2 + 4q^2)^{1/2} \). (41)

Then the final result for \( \xi \) and \( \eta \), combining Eqs. (33), (34), (39) and (40), is

\[
\xi = \frac{1}{2} \xi + \left[ \frac{1}{2} (\hat{u} + \hat{p}) \right]^{1/2},
\]

\[
\eta = \frac{1}{2} \eta + \left[ \frac{1}{2} (\hat{u} - \hat{p}) \right]^{1/2}.
\]

2.4 Calculation of Shearing Boundaries

The shearing boundaries, which are straightened out by the shearing transformation Eq. (9), are defined as \( \tilde{\eta}_U(\xi) \) and \( \tilde{\eta}_L(\xi) \). Thus \( \tilde{\eta}_U \) is the image of the upper airfoil surface and the line \( \bar{\theta} = 0 \) downstream of the trailing edge while \( \tilde{\eta}_L \) is the image of the upper half of the initial value line \( (x = -a) \) and the line \( \bar{\theta} = \pi \) for \( x > -a \).

We start by determining the image of the upper half of the airfoil in the \((\xi, \eta)\) plane. The airfoil will be given as a set of points \((x_F, y_F)\) where for convenience we take the origin at the leading edge. Then the scaled airfoil coordinates in the \((\xi, \eta)\) plane, for a given \( r \), are:

\[
\bar{x}_F = x_F + \xi n 2 - d_s,
\]

(44)
\[
\theta_F = 4 \sin^{-1} \left( \frac{y_F}{r} \right) .
\]

(45)

Next, the image in the \((\xi, \eta)\) plane is computed from

\[
\xi_F = \sinh^{-1} \left[ \frac{1}{2} (\alpha - \beta) \right]^{1/2} ,
\]

(46)

\[
\eta_F = \cos^{-1} \left( \frac{-\rho}{\cosh \xi_F} \right) ,
\]

(47)

where

\[
\alpha = (\beta^2 + 4q_{\lambda}^{-2})^{1/2} ,
\]

(48)

and \(\rho, q\) and \(\beta\) are given by Eqs. (19), (20) and (22). Then the image in the \((\xi, \eta)\) plane is

\[
\xi_F = \xi_F (1 + u) ,
\]

(49)

\[
\eta_U = \eta_F (1 - u) ,
\]

(50)

and

\[
\mu = \frac{\xi_{\lambda}^2}{\xi_F^2 + \eta_F^2} .
\]

(51)

The upper boundary beyond the airfoil trailing edge is the image of \(\theta = 0\) which maps to \(\eta = \pi\). To calculate \(\eta_U\) in this region we first compute a uniform point distribution of \(\xi\) on the interval \((\xi_{\text{TE}}, \xi_{\text{max}})\). Then
corresponding values of $\xi$ are computed by iteration from

$$\bar{\xi}(n+1) = \frac{\bar{\xi}}{1 + \mu_n}, \quad (52)$$

where superscript $n$ denotes the iteration number, and

$$\mu_n = \frac{\xi_0^2}{\pi_n^2 + \xi(n)}. \quad (53)$$

We note that Eq. (52) converges quite rapidly. With a value of $\xi$ known, $\bar{\eta}_U$ is computed from

$$\bar{\eta}_U = \pi \cdot (1 - u). \quad (54)$$

In the calculation of the lower $\bar{n}$ boundary the shearing transformation requires that the same $\tilde{\xi}$ distribution be used as was determined for $\bar{\eta}_U$. The lower boundary is computed in two segments, the first on the interval $(0, \xi_0)$, where $\xi_0$ is the image of $\xi_0$, and the second on the remaining interval $(\xi_0, \xi_{\text{max}})$.

On the interval $(0, \xi_0)$ we calculate $\xi$ and $n$ by iteration from the rapidly convergent formula:

$$\bar{\xi}(n+1) = \frac{\bar{\xi}}{1 + \mu}, \quad (55)$$

where in this case

$$\mu = \frac{\xi_0^2}{(\xi^2 + n^2)(n)}, \quad (56)$$

$$n = \cos^{-1}(\cosh \xi(n) - 2e^{-a}). \quad (57)$$

To start the iteration we set $u = 1$ in Eq. (55) which from Eq. (56) is seen
to be exact at \( \xi = \xi_0 \). With \( \xi \) and \( n \) known, \( \eta_L \) is calculated from

\[
\eta_L = n \cdot (1 - \mu).
\]  

(58)

On the interval \((\xi_0, \xi_{\text{max}})\) we know from Eq. (58) that the image of \( \bar{\theta} = \pi \) is

\[
\eta_L = 0.
\]  

(59)

Thus knowing the distribution of \( \eta_U \) and \( \eta_L \) on \((0, \xi_{\text{max}})\) we can obtain the distribution of the shearing distance \( D \) from Eq. (11).

2.5 Stretching Functions

The approach taken here, as already mentioned, is to use one-dimensional stretching functions, as indicated by Eqs. (12), (13), and (14). In the present application the location and length scales of regions of rapid variation of the solution are known beforehand. In a \( Z = \text{constant} \) plane of the computational domain, as shown in Fig. 6, clustering of \( Y = \text{constant} \) lines is needed near \( Y = 1 \) and 0 to resolve the boundary layer developing on the airfoil and the region around the corner singularity, \( \bar{x} = -\bar{a}, \bar{\theta} = \pi \), in the physical plane. Thus, for the variable \( Y \) a two-sided stretching function is required. Because of the primary viscous layer on the cylinder clustering is needed near \( Z = 0 \) which requires a one-sided stretching function for \( Z \). The stretching function for \( X \) depends on criteria related to the flow field and the mapping geometry which will be discussed later.

Vinokur [5] has determined approximate criteria for the development of one- and two-sided stretching functions of one variable which give a uniform truncation error independent of the governing differential equation or
difference algorithm. He investigates several analytic functions but finds that only \( \tan z \), where \( z \) is real or pure imaginary, satisfies all of his criteria.

We start with the stretching function for \( Y \) and note that both \( Y \) and \( Y_c \) are normalized variables as required in Vinokur's functions. In the present case, \( z \) is taken to be pure imaginary which leads to

\[
Y = \frac{\tanh(Y_c \Delta \phi)}{A \sinh \Delta \phi + (1 - A \cosh \Delta \phi) \tanh(Y_c \Delta \phi)},
\]

where

\[
A = \left( \frac{S_0}{S_1} \right)^{1/2},
\]

\[
B = \left( \frac{S_0 S_1}{S_1} \right)^{1/2},
\]

and \( S_0 \) and \( S_1 \) are dimensionless slopes defined as

\[
S_0 = \frac{dY_c}{dY}(0),
\]

\[
S_1 = \frac{dY_c}{dY}(1),
\]

which control the clustering at \( Y = 0 \) and \( Y = 1 \), and \( \Delta \phi \) is the solution of the following transcendental equation:

\[
B = \frac{\sinh \Delta \phi}{\Delta \phi}.
\]

To avoid solving Eq. (63) by iteration, Vinokur determines the following extremely accurate approximate solutions for small and large \( B \):
For $B < 2.7829681$

$$\Delta \phi = (6\bar{B})^{1/2} \left( 1 - 0.15\bar{B} + 0.057321429\bar{B}^2 - 0.024907295\bar{B}^3 + 0.0077424461\bar{B}^4 - 0.00107941238\bar{B}^5 \right),$$

where

$$\bar{B} = B - 1.$$  \hspace{1cm} (64)

For $B > 2.7829681$

$$\Delta \phi = V + (1 + 1/V)\ln(2V) - 0.02041793$$

$$+ 0.24902722W + 1.9496443W^2 - 2.6294547W^3$$

$$+ 8.5679591W^4,$$

where

$$V = \ln B,$$  \hspace{1cm} (67)

and

$$W = 1/B - 0.028527431.$$  \hspace{1cm} (68)

An example of this two-sided stretching function for $S_0 = 100$ and $S_1 = 10$ is shown in Fig. 7. For this case, $\Delta \phi$ computed from Eq. (66) is 5.926.

The one-sided counterpart of Eq. (6) is antisymmetric about the mid-point and, in terms of $Z$ and $Z_c$, is given by

$$Z = 1 + \frac{\tanh \left[ \frac{1}{2} \Delta \phi(Z_c - 1) \right]}{\tanh \frac{\Delta \phi}{2}}, \quad 0 < Z < 1,$$

where

$$Z = \frac{\ln(1 + B)}{\ln B}, \quad 1 < B,$$

and

$$Z_c = \frac{\ln(1 - B)}{\ln B}, \quad -1 < B < 0.$$
where $\Delta \phi$ is the solution of

$$S_0 = \frac{\sinh \Delta \phi}{\Delta \phi}, \quad (70)$$

and

$$S_0 = \frac{dZ_c}{dz}(0).$$

Two examples of this one-sided stretching function, $S_0 = 10$ and 100, are shown in Fig. 8.

The stretching function in $x$ is required to have the following properties:

1. It must have the ability to cluster points near the nose of the airfoil to resolve rapid flow field variations in that region.
2. Control points, where grid lines are required, are the corner, $X = X_o$, and the airfoil trailing edge, $X = X_{TE}$.
3. Downstream of the airfoil trailing edge where flow gradients are decreasing the step size should gradually increase.
4. The stretching function should have continuous first derivatives.
5. For proper flow field resolution, the number of steps on the intervals $(0, X_o)$ and $(X_o, X_{TE})$ are to be parameters.

The above requirements dictate the stretching function be made up of three piecewise continuous segments on $(0, X_o)$, on $(X_o, X_{TE})$ and on $(X_{TE}, X_{max})$.

We start by defining variables normalized on the corner location,

$$\hat{X} = \frac{X}{X_o}, \quad \hat{X}_c = \frac{X_c}{X_o}.\quad$$

An appropriate stretching function on the first segment is given by Eq. (61) of Vinokur, viz.
\[ \hat{X} = \hat{X}_c \left[ 1 + \frac{1}{2} (S_0 - 1)(1 - \hat{X}_c)(2 - \hat{X}_c) \right], \quad 0 < \hat{X}_c < 1, \quad (72) \]

where \( S_0 \) is the slope at the origin and is used to control clustering of points in that region. The uniform step size on Segment 1 is given by

\[ \Delta \hat{X}_c = \frac{1}{N_1}, \quad (73) \]

where \( N_1 \) is the number of intervals on Segment 1. We note that \( \Delta \hat{X}_c \), as given by Eq. (73), is also the step size on Segments 2 and 3.

On Segment 2, the scaled trailing edge coordinate is given by

\[ (\hat{X}_c)_{TE} = 1 + N_2 \Delta \hat{X}_c, \quad (74) \]

where \( N_2 \) is the number of intervals on Segment 2. We note that \( (\hat{X}_c)_{TE} \neq \hat{X}_{TE} \).

The constraints to be satisfied by the stretching function of Segment 2 are:

\[ \begin{aligned} \hat{X} &= 1, \quad \hat{X}' = \hat{X}'_1 \text{ on } \hat{X}_c = 1 \\ \hat{X} &= \hat{X}_{TE} \text{ on } \hat{X}_c = (\hat{X}_c)_{TE} \end{aligned} \]

where

\[ \hat{X}'_1 = \frac{d\hat{X}}{d\hat{X}_c} \bigg|_{\hat{X}_c=1} \]

which from Eq. (72) is

\[ \hat{X}'_1 = \frac{1}{2}(3 - S_0). \quad (73) \]

With three constraints a parabola is appropriate. The resulting stretching function is
\[
\hat{X} = 1 + \left[ X_1 + A(\hat{X}_c - 1)(\hat{X}_c - 1) \right],
\]
where
\[
A = \frac{\hat{X}_{TE} - 1 - \hat{X}_1[(\hat{X}_c)_{TE} - 1]}{[(\hat{X}_c)_{TE} - 1]^2}.
\]

On Segment 3 a geometric progression is used to increase the step size in \(\hat{X}\). Requiring continuity of \(\hat{X}\) at the junction with Segment 2, we have
\[
\hat{X}_k = \hat{X}_{TE} + \hat{X}_1 \left( \frac{1 - \hat{C}^{k-1}}{1 - \hat{C}} \right), \quad k > 2,
\]
where \(\hat{C}\) is the constant step size ratio defined by,
\[
\hat{C} = \frac{\Delta \hat{X}_k}{\Delta \hat{X}_{k-1}} > 1.
\]
Continuity of the first derivative at the junction is ensured by choosing \(\hat{X}_1\) equal to the last \(\hat{X}\) on Segment 2. No attempt is made to match \(\hat{X}_{max}\) exactly.

The stretching function for \(\hat{X}\) is seen to have four parameters, \(S_0, N_1, N_2\) and \(\hat{C}\), which provide considerable flexibility in the point distribution of \(\hat{X}\). A typical example is shown in Fig. 9.
3. RESULTS AND DISCUSSION

3.1 Generation of the Grid

The step-by-step procedure to generate a grid in the physical plane for a given airfoil shape and initial value plane location is as follows:

(1) The uniform computational grid \((X_c, Y_c, Z_c)\) is first established and then \((X_i, Y_j, Z_k)\) are calculated via the stretching functions described in Section 2.5.

(2) With \((X_i, Y_j, Z_k)\) known, \(r_k\) is determined from

\[
\bar{r}_k = Z_k. 
\]  

Then for \(\bar{r}\) fixed, the points in the \((X, Y)\) plane are transformed to the \((\xi, \eta)\) plane by

\[
\xi_{ijk} = X_{i}, \quad (81)
\]

\[
\eta_{ijk} = Y_{j}D_{ik} + (\eta_{L})_{ik}, \quad (82)
\]

where

\[
D_{ik} = (\eta_{U})_{ik} - (\eta_{L})_{ik}. 
\]  

By Eq. (45), \(\delta_F\) depends on \(r\) and hence \(\bar{r}\) and therefore \(\eta_{L}\) and \(\eta_{U}\) must be computed anew for each value of \(r\). The procedure used here is to calculate more points than needed on the shearing boundaries for a given \(\bar{r}\) and then to use Lagrange cubic interpolation to determine \(\eta_{L}\) and \(\eta_{U}\) for a given \(\bar{r}\).
(3) With $(\xi, \eta)$ known, the transformation to the $(\xi, \eta)$ plane is

\[
\xi_{ijk} = \frac{1}{2} \xi_{ijk} + \frac{1}{2} (\hat{u} + p) \sqrt{\xi_{ijk}},
\]

(84)

\[
\eta_{ijk} = \frac{1}{2} \eta_{ijk} + \frac{1}{2} (\hat{u} - p) \sqrt{\eta_{ijk}},
\]

(85)

where

\[
\hat{u} = (\hat{p}^2 + 4\hat{q}^2)^{1/2},
\]

(86)

\[
\hat{p} = \frac{1}{4} (\xi^2 - \eta^2) - \xi_0^2,
\]

(87)

\[
\hat{q} = \frac{1}{4} \xi \eta.
\]

(88)

(4) Next, the points in the $(\xi, \eta)$ plane are transformed to the $(\bar{x}, \bar{\theta})$ plane by

\[
\bar{x}_{ijk} = \xi_n (\cosh \xi_{ijk} - \cos \eta_{ijk}),
\]

(89)

\[
\bar{\theta}_{ijk} = \cos^{-1} \left( \frac{1 - \cosh \xi_{ijk} \cos \eta_{ijk}}{\cosh \xi_{ijk} - \cos \eta_{ijk}} \right).
\]

(90)

(5) The final step is to compute the cylindrical coordinates of each grid point from:

\[
x_{ijk} = \bar{x}_{ijk} + d_x - \ln 2,
\]

(91)

\[
\theta_{ijk} = \frac{1}{4} \bar{\theta}_{ijk},
\]

(92)

\[
r_k = R_c + (R_t - R_c) \bar{r}_k.
\]

(93)
3.2 Features of the Grid

The shearing transformation applied at the fourth stage necessarily produces a nonorthogonal grid in the \((\bar{x}, \bar{\theta})\) plane. The nonorthogonality is smallest on the lower shearing boundary, under most conditions, and largest at the airfoil surface on the upper shearing boundary, as can be seen from Fig. 5. On the upper (airfoil) boundary the nonorthogonality near the leading edge \((\bar{\xi} = 0)\) can be controlled by proper location of the singularity of the unwrapping transformation, Eq. (6). Away from the leading edge the only control over nonorthogonality is to keep the airfoil reasonably thin, say eight percent or less, which will maintain \(\bar{n}_y\) as close to the image of \(n = \pi\) as possible.

The parameter which controls grid orthogonality near the airfoil leading edge is \(d_s\) in Eq. (3). The most nearly orthogonal system in this region is produced when the leading edge maps into an \(n = \) constant line. In the \((x, \theta)\) plane such a line is closely approximated by a parabola centered about \(\theta = 0\) and is effectively characterized by its radius of curvature at the origin, given by

\[
\rho_0 = \frac{1}{\left(\frac{d^2 x}{d\theta^2}\right)_{\theta=0}} .
\]  

We determine \(\rho_0\) by setting \(n = n_{LE} = \) constant in Eqs. (15) and (16), differentiating the result twice with respect to \(\theta\) to find \(d^2x/d\theta^2\), plus noting that \(dx/d\theta = 0\) at \(\theta = 0\) and by virtue of Eqs. (3) and (5) that

\[
\frac{d^2x}{d\theta^2} = 16 \left(\frac{d^2 \bar{x}}{d\bar{\theta}^2}\right) .
\]
The result is

\[ \rho_o = -\frac{1}{16} \sin^2 \eta_{LE} \cos \eta_{LE} e^{-x_{LE}} \]  

(95)

From Eq. (3) evaluated at the airfoil leading edge \((x = 0)\) we have

\[-x_{LE} = \ln 2 - d_s \]  

(96)

and from Eq. (17) with \(\xi = 0\) and \(\eta = \eta_{LE}\) we find that

\[ \cos \eta_{LE} = 1 - 2e^{-d_s} \]  

(97)

from which it follows that

\[ \sin \eta_{LE} = 2(e^{-d_s}(1 - e^{-d_s}))^{1/2} \]  

(98)

Hence, Eq. (95) for \(\rho_o\) becomes

\[ \rho_o = \frac{1}{8} \left( \frac{1 - e^{-d_s}}{2e^{-d_s} - 1} \right) \]  

(99)

which can be solved for \(d_s\) to yield,

\[ d_s = \ln \left( \frac{1 + 16 \rho_o}{1 + 8 \rho_o} \right) \]  

(100)

Next, we fit the airfoil leading edge by an osculating parabola, viz.

\[ x = K\theta^2 \]  

(101)

where \(K = x_1/\theta_1^2\) and \((x_1, \theta_1)\) are appropriate airfoil coordinates near the leading edge. The radius of curvature of the airfoil at the leading edge is, from Eq. (101),

\[ \rho_{LE} = \frac{1}{\rho_o} = \frac{\theta_1^2}{2x_1} \]  

(102)
The optimum value of $d_s$ (which produces the most nearly orthogonal grid near $\xi = 0$) is obtained by equating $\rho_{LE}$ and $\rho_o$. Thus, $d_s$ can then be determined from Eq. (100). Figure 10 shows the variation of $\eta_U$ with $\xi$ for a six percent thick Joukowsky airfoil for three values of $d_s$, one of which was determined by Eqs. (100) and (102). In these three cases, we have $d_s \ll d_L$ which has the effect of limiting the influence of $d_s$ on $\eta_U$ to the region $0 < \xi < \xi_o$ where here $\xi_o \approx 0.87$. As $r$ increases from $R_c$ to $R_L$ the leading edge radius of curvature of the airfoil decreases because $\theta_F$ decreases—see Eq. (45). Thus $d_s$ must be decreased accordingly.

On the lower shearing boundary the nonorthogonality arises from the mapping of the initial value line (IVL) by Eq. (7). In the $(\xi, n)$ plane the IVL is very nearly half of an ellipse with the ratio of the semi-major to semi-minor axes lengths, defined as $\lambda = n_0/\xi_o$ ($n_0$ is the value of $n$ on the IVL at $\xi = 0$) given by

$$\lambda = \frac{\cos^{-1}(1 - 2e^{-a})}{\cosh^{-1}(1 + 2e^{-a})}. \quad (103)$$

Figure 11, in which $\lambda$ is plotted versus "a", shows that as "a" becomes large $\lambda$ approaches unity and therefore the IVL approaches a semi-circle in the $(\xi, n)$ plane. Thus for $\eta_L$ to have the smallest maximum (at $\xi = 0$) and hence for $\xi = \text{constant}$ lines at $\eta = \eta_L$ to be as nearly orthogonal as possible, "a" should be large, say 3 or 4, a circumstance desirable on physical grounds anyway.
At the image of the airfoil trailing edge in the \((\xi, \eta)\) plane (Points D and E in Fig. 5) when the trailing edge angle is finite the derivative of \(\eta_\xi\) with respect to \(\xi\) will be discontinuous. At the ends of the IVL (Points A and C in Fig. 5) the behavior of \(\eta_\xi\) is cusp-like which means that the second derivative of \(\eta_\xi\) with respect to \(\xi\) is discontinuous. These discontinuities produce similar type discontinuities in \(Y = \text{constant}\) lines via the shearing transformation. This behavior is one of the disadvantages of algebraic mappings involving shearing transformation which is absent in grids generated by solving elliptic partial differential equations. The discontinuous behavior of derivatives of \(Y = \text{constant}\) lines in the physical plane should therefore be accounted for in the calculation of affected metric coefficients and in the numerical method of solution of the viscous flow equations.

3.3 Numerical Examples

For simplicity a symmetric Joukowsky airfoil was used in the numerical examples of the grid. The ordinates of this airfoil (for unit chord) are given by,

\[
y_F = \frac{4T}{3\sqrt{3}} \left(1 - x_F\right) \left[4 x_F (1 - x_F)\right]^{1/2},
\]

where \(x_F\) is measured from the airfoil leading edge and \(T\) is the maximum thickness to chord ratio. Two example grids in the \((\xi, \eta)\) plane are presented with parameters listed in Table 1 below. The parameter \(J\) is the number of points in the \(Y\) direction.
Table 1. Grid parameters for Numerical Examples

Case 1 is shown in Fig. 12 and Case 2 in Fig. 13. Case 1 has no stretching function in X and no X = constant line through the corner. The non-orthogonality of the grid in Case 1 (12% thick) is seen to be more pronounced at the airfoil surface than in Case 2 (6% thick) which bears out the remark made earlier. Notice that both examples are for the grid in the (x, \bar{\theta}) plane on the cylinder surface (r = R_c) which corresponds to the intersection of the fin with the cylinder. Hence in these examples, by Eq. (45), the airfoil thickness in terms of \bar{\theta} is a maximum and thus the nonorthogonality is most pronounced.

The computer code listing is given in the appendix.
References


Figure 2. Coordinate System in Crossflow Plane.
Figure 3. Computational Domain in (x, y) Plane.
Figure 4. Boundary Images in $(\xi, \eta)$ Plane.
Figure 5. Boundary Images in $(\bar{\xi}, \bar{\eta})$ Plane.
Figure 6. Schematic of Computational Plane.

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Figure 7. Two-Sided Stretching Function for $Y$. 

$S_0 = 100 \quad S_1 = 10$
Figure 8. One-Sided Stretching Function for Z.
Figure 9. Three Segment Stretching Function for X.
Figure 10. Effect of Singularity Location on Upper Shearing Boundary.

6% THICK JUKOVSKY AIRFOIL

\( \frac{d_s}{R_C} = 0.020, 0.033, 0.050 \)

\( \frac{d_{VL}}{R_C} = 3.0 \)
Figure 11. Ellipticity of Initial Value Line in ($\xi, \eta$) Plane.
Figure 12. Grid for a Twelve Percent Thick Joukowski Airfoil.
Figure 13. Grid for a Six Percent Thick Joukowski Airfoil.
Appendix: Grid Generation Computer Code Listing
C PROGRAM NAME: CGNID3

THIS PROGRAM COMPUTES A SURFACE FITTED C-GRID FOR A FIN

THE FIN IN THIS VERSION IS A SYMMETRIC JOUKOWSKY AIRFOIL.

THIS IS THE 3-D VERSION.

C*****************************************************************************

IMPLICIT REAL*8 (A-H,O-Z)

COMMON /BLK01/ IMAX,JMAX,ITE,ITEM,ILAST,ISEG1,ISEG2

COMMON /BLK02/ XI0,XI10,XI00

COMMON /BLK03/ CI,C2,C3,C4,C5,PI,PISW

COMMON /BLK04/ XF(101),YF(101)

COMMON /BLK05/ XI1(151),SBAR(151),ETARL(151)

COMMON /BLK06/ SY0,SY1,SZ0,SX0,SSR

COMMON /BLK07/ ZC(151),B1GZ(151)

FORMAT(5I4)

FORMAT(5F10,4)

FORMAT(IH,6X,'INPUT PARAMETERS FOR C-GRID')

FORMAT(IH,9X,'ISEG1 =',I6/10X,'ISEG2 =',I6/10X,'JMAX =',I6/10X,'KMAX =',I6/10X,'ITE =',I6)

FORMAT(10X,'DIVL =',F10.4/10X,'DUB =',F10.4/10X,'FC =',F10.4/10X,'RT =',F10.4/10X,'SY0 =',F10.4/10X,'SY1 =',F10.4/10X,'SZ0 =',F10.4/10X,'SX0 =',F10.4/10X,'SFU =',F10.4/10X,'F1U =',F10.4/10X)

FORMAT(IH0)

FORMAT(5X,'STACKED C-GRID FOR FIN-CYLINDER GEOMETRY')

FORMAT(10X,'US =',F14.4)

C INPUT REQUIREMENTS

ISEG1 = NO. INTERVALS ON FIRST X-SEGMENT.

ISEG2 = NO. INTERVALS ON SECOND X-SEGMENT.

IMAX = NO. POINTS IN X-DIRECTION.

JMAX = NO. POINTS IN Y-DIRECTION.

KMAX = NO. POINTS IN Z-DIRECTION.

ITE = NO. POINTS ON INITIAL VALUE LINE.

DIVL = DISTANCE FROM AIRFOIL L.E. TO INITIAL VALUE LINE.

DS = DISTANCE FROM AIRFOIL L.E. TO SINGULARITY OF COORDINATE SYSTEM.

DUB = DISTANCE FROM AIRFOIL L.E. TO OUTFLOW BOUNDARY.

TAU = AIRFOIL MAX. THICKNESS TO CHORD RATIO.

RC = INNER CYLINDER RADIUS, IN TERMS OF AIRFOIL CHORD.

RT = OUTER CYLINDER RADIUS, IN TERMS OF AIRFOIL CHORD.

SY0 = Y-STRETCHING PARAMETER AT AIRFOIL SURFACE.

SY1 = Y-STRETCHING PARAMETER AT INITIAL SURFACE.

SZ0 = Z-STRETCHING PARAMETER AT INNER CYLINDER.

SX0 = INITIAL X-STRETCHING PARAMETER, SEGMENT 1.

SSR = X-GEOMETRIC PROGRESSION RATIO, SEGMENT 3.

READ(5,1) ISEG1,ISEG2,JMAX,KMAX,ITE

READ(5,2) DIVL,DUB

READ(5,2) TAU,RC,RT

READ(5,2) SY0,SY1,SZ0,SX0,SSR

ITEM=ITE+1

WRITE(6,10)

WRITE(6,11) ISEG1,ISEG2,JMAX,KMAX,ITE

WRITE(6,12) DIVL,DUB,TAU,RC,RT,SY0,SY1,SZ0,SX0,SSR

WRITE(6,13)

WRITE(6,14)

C C3=2.0D0*TAU/DSQRT(27.0D0)
C  CALCULATE AIRFOIL COORDINATES.
680  CALL FOIL
700  C  CALCULATE ZC AND BIGZ.
720  CALL STRFZ(ZC,BIGZ,KMAX,SZ0)
730  DELR=RT=RC
750  C  BEGIN CALCULATION OF STACKED GRID.
770  DO 50 K=1,KMAX
790  RAD=RC+DELR*BIGZ(K)
800  C  CALCULATE DS - DISTANCE FROM AIRFOIL LEADING EDGE TO
810  SINGULARITY OF UNWRAPPING TRANSFORMATION.
820  THF=DSIN(YF(4)/RAD)
830  RHO=0.5D0*THF*THF/XF(4)
850  DS=DLOG((1.0D0+15.0D0*RHO)/(1.0D0+8.0D0*RHO))
860  WRITE(6,13)
870  WRITE(6,13) DS
880  WRITE(6,13)
890  C1=DEXP(-((DIVL+DS))
900  C2=2.0D0*C1
910  RHS=DSQRT(4.0D0*C1*(1.0D0+C1))
920  CALL ASINH(XIO,RHS)
930  C4=DSQRT(2.0D0)-DS
940  C5=XIO*XIO
950  XIB0=2.0D0*XIO
960  C  CALCULATE XIBM - COORDINATE OF DOWNSTREAM BOUNDARY IN XI BAR -
980  ETA BAR PLANE.
990  XBM=XE+C4
1000  TERM=DEXP(XBL)=1.0D0
1020  RHS=DQRT(TERM*TERM-1.0D0)
1030  CALL ASINH(XIE,RHS)
1040  XIBM=XIE*(1.0D0+C5/(PI0+XIE*XIE))
1050  C  CALL SHEAR(RAD)
1060  KK=K
1070  CALL XGRID(KK,RAD)
1090  50 CONTINUE
1100  STOP
1110  END
1120  SUBROUTINE SHEAR(RAD)
1130  C******************************************************************************
1140  C  THIS SUBROUTINE CALCULATES SBAR VS. XI BAR, TO BE USED IN THE
1150  C  SHEARING TRANSFORMATION.
1160  C******************************************************************************
1170  IMPLICIT REAL*8 (A-H,O-Z)
1180  COMMON /BLK01/ IMAX,JMAX,ITE,ITEM,ILAST,ISEG1,ISEG2
1190  COMMON /BLK02/ XIBM,XIO,XIB0
1200  COMMON /BLK03/ C1,C2,C3,C4,C5,PI,PISQ
1210  COMMON /BLK04/ XF(101),YF(101)
1220  COMMON /BLK05/ XIB(151),SBAR(151),ETABL(151)
1230 C    DIMENSION ETABU(151)
1240 C
1250 C    10 FORMAT(5X,'SHEARING BOUNDARY IN XIBAR - ETABAR PLANE')
1260 C    11 FORMAT(1HO)
1270 C    12 FORMAT(5X,'i',8X,'XIBAR',9X,'ETABL',9X,'ETABU',9X,'SBAR')
1280 C    13 FORMAT(6D14.4)
1290 C    14 FORMAT(1H10,4X,'UNABLE TO CONVERGE XI IN 50 ITERATIONS',/5X,
1300 C    '1'XIBAR = ',D14.4)
1310 C
1320 C    COMPUTE NORMALIZED AIRFOIL COORDINATES FOR GIVEN CYLINDRICAL
1330 C    RADIUS AND TRANSFORM TO XI BAR - ETA BAR PLANE. THIS STEP
1340 C    GIVES THE FIRST PORTION OF THE UPPER BOUNDARY.
1350 C
1360 C
1370 C    WRITE(6,11)
1380 C    WRITE(6,10)
1390 C    WRITE(6,11)
1400 C    TI=RAD*RAD
1410 C    DO 50 I=1,ITE
1420 C    YFI=YF(I)
1430 C    THF=DASIN(YFI/RAD)
1440 C    XBFI=XF(I)+C4
1450 C    TBBFI=4.0D0*THF
1460 C
1470 C    T2=DEXP(X3FI)
1480 C    PBAR=1.0D0=T2*DCOS(TBBFI)
1490 C    GQW=QBAR*GQW
1500 C    BETA=1.0D0=PBAR*PBAR=GQW
1510 C    ALPHA=USQR(TBETA*QBAR+4.0D0*QSQ)
1520 C    RHS=USQR(0.5D0*(ALPHA-BETA))
1530 C    CALL ASINH(XIF,RHS)
1540 C    ARG=PBAR/DCOSH(XIF)
1550 C    IF((ARG+1.0D0).LT.0.0D0) ARG=-1.0D0
1560 C    ETA1=USQR((T2=0.0DO)*XBAR)
1570 C
1580 C
1590 C    XMU=C5/(XIF*XIF+LBETA*ETAF)
1600 C    XIB(I)=XIF*(1.0D0+XMU)
1610 C    ETABU(I)=ETAF*(1.0D0-XMU)
1620 C    50 CONTINUE
1630 C
1640 C    CONTINUE UPPER BOUNDARY CALCULATION BEYOND AIRFOIL T.E.
1650 C
1660 C
1670 C    DXIB=0.2DO
1680 C    ILAST=ITE+(XIBM=XIB(ITE)/DXIB
1690 C    IF(ILAST,GT,151) ILAST=151
1700 C    ITEP=ITE+1
1710 C    WRITE(6,12)
1720 C    DO 100 I=ITEP,ILAST
1730 C    XIBAR=XIB(I-1)+DXIB
1740 C    XIB(I)=XIBAR
1750 C    XIL=XIBAH
1760 C    DO 70 IT=1,50
1770 C    XMU=C5/(PI*D+XIL*XIL)
1780 C    XI=XIBAH/(1.0D0+XMU)
1790 C    IF(DASIN(XI-XIL),LT.1.0D0+08) GO TO 80
1800 C
1810 C
1820 C
1830 C    ETABU(I)=PI*(1.0D0-XMU)
CONTINUE
CALCULATE LOWER BOUNDARY IN XI BAR - ETA BAR PLANE AND
SBAR.

DO 200 I=1,ILAST
XIBAR=XIB(I)
IF(XIBAR.GE.XI0) GO TO 140
XIL=XIBAR
XMU=1.0D0
DO 120 II=1,SOAR
XI=XIBAR/(1.0D0+XMU)
ARG=UCOSH(XI)-C2
ETA=ACOS(ARG)
XM=XMU/(XI*XI+ETA*ETA)
IF(ABS(XI-XIL).LT.1.0D-08) GO TO 130
120 XI=XIL
WRITE(6,14) XIHAR
STOP
130 ETABL(I)=ETA*(1.0D0-XMU)
GO TO 150
140 ETABL(I)=0.0D0
150 SBAR(I)=ETABL(I)-ETABL(I)
160 CONTINUE
RETURN
END

SUBROUTINE XGRID(K,RAD)
THIS SUBROUTINE CALCULATES THE GRID IN THE XI BAR - THETA BAR PLANE.

DIMENSION BIGX(151),BIGH(151)
DIMENSION XC(151),YC(151)

11 FORMAT(1IH0)
12 FORMAT(5X,'I',5X,'J',5X,'K',5X,'X',5X,'Y',5X,'Z',5X,'THETA')
13 FORMAT(316b6V14.4)

SET UP GRID IN COMPUTATIONAL PLANE.

XBE=XIB(I)
NPT=ISEG1+ISEG2+1
CALL STRFX(XC,BIGX,ISEG1,ISEG2,IMAX,SX0,SSR,XIB,XBTE,XIBM)
CALL SIRFY(YC,BIGY,IMAX,SY0,SY1)

DETERMINE GRID IN PHYSICAL PLANE.

I=1
XIBAR=0.0D0
SI=SBAR(1)
ETABL=ETABL(1)
WRITE(6,11)
WRITE(6,12)
WRITE(6,11)
DO 70 J=1,JMAX
ETABAR=ETHLI+SBI*BIGY(J)
P=U,2500*ETABAR*ETABAR+C5
XI=0,000
ETA=0,500*ETABAR+DSQRT(P)
XBAR=LOG(1,000-UCOS(ETA))
THBAR=0,000
XX=XBAR-C4
THETA=0,000
WRITE(6,13) I,J,K,XC(I) ,YC(J) ,ZC(K) ,RAD,XX,THETA
70 CONTINUE
IBEG=1
IEND=ITE
DO 100 I=2,IMAX
XBAR=BIGX(I)
IF(I.LE.NPTE) GO TO 80
IBEG=ITE
IEND=ILAST
C INTERPOLATE TO FIND SBAR AND ETABL CORRESPONDING TO XIBAR.
C
CALL INTERP(XIB,SBAR,XIBAR,SBI,IBEG,IEND,INT,0)
CALL INTERP(XIB,ETABL,XIBAR,ETABL,IBEG,IEND,INT,1)
C WRITE(6,11)
WRITE(6,12)
WRITE(6,11)
DO 70 J=1,JMAX
ETABAR=ETHLI+SBI*BIGY(J)
P=U,2500*ETABAR*ETABAR+C5
XI=0,500*ETABAR+DSQRT(0,500*(XMU+P))
ETA=0,500*ETABAR+DSQRT(0,500*(XMU+P))
C
T1=DCOSH(XI)
T2=DCUS(ETA)
ARG1=T1-T2
XBAR=LOG(ARG1)
THBAR=UCOS((1,000-T1*T2)/ARG1)
THETA=0,2500*THBAR
XX=XBAR-C4
WRITE(6,13) I,J,K,XC(I) ,YC(J) ,ZC(K) ,RAD,XX,THETA
100 CONTINUE
RETURN
END
SUBROUTINE ASINH(ARG,RHS)
C*******************************************************************************
C THIS SUBROUTINE COMPUTES THE INVERSE HYPERBOLIC SINE USING
C NEWTON'S METHOD.
C*******************************************************************************
IMPLICIT REAL*8 (A-H,O-Z)
10 FORMAT(1H0,4X,'INVERSE HYPERBOLIC SINE CALCULATION FAILED FOR
1ISINH(X) =',D14.7)
C TEST=DABS(KHS)
IF(TES,T.GT.1,000) GO TO 30
3060      ARG=RHS
3070      GO TO 40
3080      30 ARG=DLGC(2,0D0*TEST)*DSGN(1.0D0,RHS)
3090       40 CONTINUE
3100      DO 50 K=1,50
3110      FA=DSINH(ARG)=RHS
3120      FPA=UCOSH(ARG)
3130      DARG=FA/FPA
3140      IF(DABS(DARG).LT.1.0D-14) RETURN
3150      ARG=ARG+DARG
3160       50 CONTINUE
3170      WRITE(6,10) RHS
3180      RETURN
3190      END
3200
3210 SUBROUTINE FOIL
3220 C************************************************************************
3230 C THIS SUBROUTINE GENERATES (X,Y) COORDINATES FOR A SYMMETRIC
3240 C JOKUIKSY AIRFOIL.
3250 C************************************************************************
3260 IMPLICIT REAL*8 (A-H,O-Z)
3270 COMMON /BLK01/ IMAX,JMAX,ITEM,ILAST,ISEG1,ISEG2
3280 COMMON /BLK03/ C1,C2,C3,C4,C5,PI,PISQ
3290 COMMON /BLK04/ XF(101),YF(101)
3300 C
3310   10 FORMAT(5A,'AIRFOIL COORDINATES')
3320   11 FORMAT(1H0)
3330   12 FORMAT(5A,'1',6X,'XF',13X,'YF')
3340   13 FORMAT(16,2D14.4)
3350 C
3360   DTH=PI/ITEM
3370   XF(I)=0.0D0
3380   YF(I)=0.0D0
3390   DO 50 I=2,ITEM
3400   TH=(I-1)*DTH
3410   T1=DCOS(TH)
3420   XF(I)=0.5D0*(1.0D0-T1)
3430       50 CONTINUE
3440   XF(ITEM)=1.0D0
3450   YF(ITEM)=0.0D0
3460   WRITE(6,11)
3470   WRITE(6,10)
3480   WRITE(6,11)
3490   WRITE(6,12)
3500   DO 60 I=1,ITEM
3510       60 WRITE(6,13) I,XF(I),YF(I)
3520      RETURN
3530      END
3540 SUBROUTINE INTERP(XX,YY,XINT,YINT,IBEG,IEND,INT,ISW)
3550 C************************************************************************
3560 C THIS SUBROUTINE USES LAGUANGE CUBIC INTERPOLATION TO
3570 C DETERMINE YINT FOR A GIVEN XINT.
3580 C
3590 C XX = INDEPENDENT VARIABLE.
3600 C YY = DEPENDENT VARIABLE.
3610 C IBEG = INITIAL INDEX FOR INTERPOLATION RANGE.
3620 C IEND = FINAL INDEX FOR INTERPOLATION RANGE.
3630 C INT = UPPER INDEX OF INTERPOLATION INTERVAL.
3640 C ISW = INTERPOLATION INTERVAL SEARCH SWITCH.
3650 C   0 PERFORM SEARCH.
3660 C   1 OMIT SEARCH.
C***                       IMPLICIT REAL*8 (A-H,0-Z)  
C                   DIMENSION XX(151),YY(151)  
C                   IF(IS*,GT,0) GO TO 75  
60 DO 70 I=IBEG,IEND  
   INT=I  
   IF(XX(I).GT.XINT) GO TO 75  
    70 CONTINUE  
   IF(INT.EQ,(IBEG+1)) GO TO 90  
    90 I1=IEND-3  
   I2=IEN+2  
   I3=IEND  
   I4=IEND  
  100 CONTINUE  
X1=XX(I1)  
X2=XX(I2)  
X3=XX(I3)  
X4=XX(I4)  
CF1=(X1-T-X2)*(X1 INT-X3)*(XINT-X4)/((X1-X2)*(X1-X3)*(X1-X4))  
CF2=(XINT-X1)*(XINT-X3)*(XINT-X4)/((X2-X1)*(X2-X3)*(X2-X4))  
CF3=(XINT-X1)*(XINT-X2)*(XINT-X4)/((X3-X1)*(X3-X2)*(X3-X4))  
CF4=(XINT-X1)*(XINT-X2)*(XINT-X3)/((X4-X1)*(X4-X2)*(X4-X3))  
YINT=CF1*YY(I1)+CF2*YY(I2)+CF3*YY(I3)+CF4*YY(I4)  
RETURN  
END  
SUBROUTINE SIMFY(XI,T,NPT,SYO,SY1)  
C***                       IMPLICIT REAL*8 (A-H,0-Z)  
C                   DIMENSION XI(151),T(151)  
C                   COMPUTE XI.  
DXI=1.00D0/(NPT-1)  
DO 40 J=1,NPT  
XI(J)=(J-1)*DXI  
40 XI(J)=(J-1)*DXI  
C                   COMPUTE DELTA Y.  
A=DSQRT(SYO/SY1)  
B=DQSRT(SYO*SY1)  
TEST=2.782968100  
IF(A.GT.TEST) GO TO 50  
YBAR=B=1.00D0
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GO TO 60
50 V=DLLOG(T)
   W=1.0D0/V=0.028527431D0
   DELY=(((8.567951100)**2.6294547D0)**1.9496443D0)**
   1.0+0.24902722D0)*W=0.02041793D0+V+(1.0D0+1.0D0/V)*
   2DLLOG(2.0D0/V)
60 CONTINUE

60 C
C COMPUTE T.
C
C1=A*DSINH(DELY)
C2=1.0D0-A*DCOSH(DELY)
DO 70 I=1,NPT
   T(I)=FNY/(C1+C2*FN)
70 CONTINUE
RETURN
END

SUBROUTINE STFXX(XI,T,NSEG1,NSEG2,NMAX,SX0,SSR,XIB0,XBTE,XIBM)

C THIS SUBROUTINE GENERATES A NONUNIFORM POINT DISTRIBUTION
C SPECIALIZED TO THE COORDINATE WRAPPED AROUND THE AIRFOIL.

C IMPLICIT REAL*4 (A-H,O-Z)
DIMENSION XI(151),T(151)
C SEGMENT NUMBER 1.
TTE=XBTE/XIB0
TMAX=XIDM/XIBO
DXI=1.0D0/NSEG1
NP1=NSEG1+1
SI=0.500*(SX0-1.0D0)
DO 50 I=1,NP1
   XX=(I-1)*DXI
   XI(I)=XX
50 T(I)=XX*(1.0D0+SI*(1.0D0-XX)*(2.0D0-XX))
C SEGMENT NUMBER 2.
AA=0.500*(3.0D0-SX0)
XWTE=NSEG2*DXI
BB=(TTE-1.0D0+AA*XWTE)/(XWTE*XWTE)
NP2=NSEG2+1
DO 60 K=2,NP2
   I=ISEG1+K
   X=(K-1)*DXI
   XI(I)=1.0D0+X
60 T(I)=1.0D0+X*(AA*X*X*BB)
C SEGMENT NUMBER 3.
N3=NSEG1+NSEG2
NP3=N3+1
XITE=XI(NP3)
DTI=T(NP3)-T(N3)
S1=DT1/(SSR-1.0D0)
KMAX=151=NPT
DO 70 K=2,KMAX
I=N3+K
XI(I)=XITE+(K-1)*DXI
TI=TITE+S1*(SSP**(K-1)-1.0D0)
T(I)=TI
IF(TI.GE.TMAX) GO TO 80
70 CONTINUE
80 NMAX=I
C RESCALE VARIABLES.
SCALE=XUTE/XITE
DO 90 I=1,NMAX
XI(I)=SCALE*XI(I)
90 T(I)=XITE*T(I)
RETURN
END

SUBROUTINE STRFZCXI,TNPT,SO)
C****
C THIS SUBROUTINE GENERATES A NONUNIFORM POINT DISTRIBUTION USING
C VINOKURS ONE-SIDED STRETCHING FUNCTION.
C
IMPLICIT REAL*8 (A-H,O-Z)
DIMENSION XI(151),T(151)
C COMPUTE XI.
DXI=1.0D0/(NPT-1)
DO 40 K=1,NPT
XI(K)=(K-1)*DXI
40 CONTINUE
C COMPUTE DELTA Y.
TEST=2.7829681D0
IF(SO.GT.TEST) GO TO 50
YBAR=SO-1.0D0
YBAR=YBAR*((8.840795911D0*w-2.6294547D0)*w+1.9496443D0)*w*
(1.0D0+1.0D0/V)*
2DLOG(2.0D0/V)
50 CONTINUE
V=DLOG(SO)
W=1.0D0/SO-0.02852741D0
DELY=(((0.010794123D0*YBAR+0.0077424461D0)*YBAR-
1.0D0)+0.024907295D0)*YBAR+0.057321429D0)*YBAR*
2+1.0D0)*USQF(6.0D0*YBAR)
5300 GO TO 60
5310 50 V=DLOG(SO)
5320 W=1.0D0/SO-0.02852741D0
5330 DELY=((8.56795911D0*w-2.6294547D0)*w+1.9496443D0)*w*
1+0.24902722D0)*w-0.0241793D0*V+(1.0D0+1.0D0/V)*
2DLOG(2.0D0/V)
5360 60 CONTINUE
C COMPUTE T.
C1=0.5D0*DELY
C2=1.0D0/DTANH(C1)
DO 70 K=1,NPT
T(K)=1.0D0+C2*DTANH(C1*(XI(K)-1.0D0))
70 CONTINUE
RETURN
END
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by G. H. Hoffman, dated 30 March 1983

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