ANNUAL TECHNICAL REPORT

RESEARCH ON TOPICS IN TRANSONIC FLOW
THEORY AND ADAPTIVE
GRID GENERATION

This report covers the period
February 1, 1982 to January 31, 1983

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Bolling Air Force Base
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by

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This report summarizes the work performed during the period February 1, 1982 to January 31, 1983 under the sponsorship of AFOSR Contract F49620-79-C-0054. The work is concerned with some topics connected with a transonic flow theory and also some problems in adaptive mesh procedures.

The work on adaptive mesh procedures is concerned with the development of adaptive mesh strategies and solution
Block 20 Continued.

procedures for highly clustered adaptive meshes. It has been found that the strong conservation law form of the governing equations in computational variables cannot capture the shock waves correctly for arbitrary clustering. Methods for correcting this problem have been investigated.

The work on transonic flow theory is concerned with the existence of multiple solutions in full potential calculations. Since the full potential equation is difficult to analyze compared with small disturbance equation, multiple solutions have been found using transonic small disturbance theory. These results have been analyzed using the transonic integral equation theory and indicate that the transonic potential theory is not formulated uniquely.
RESEARCH ON TOPICS IN TRANSONIC FLOW THEORY
AND ADAPTIVE GRID GENERATION

RESEARCH OBJECTIVES

The research objectives for the reporting period were to investigate adaptive mesh generation techniques so that accurate finite difference solutions to a set of nonlinear partial differential equations can be obtained with the minimum number of mesh points. Some preliminary work on adaptive mesh procedures based on nonlinear truncation error analysis indicated four basic problems that need to be resolved: (1) clustering makes the numerical solution of the transformed equations more difficult due to the extra stiffness introduced in the partial differential equations; (2) proper clustering functions are necessary to minimize the truncation errors; (3) the truncation errors must be filtered and smoothed before they are suitable for use as clustering criteria; and (4) artificial dissipation (probably depending on the local mesh size) must be introduced to guarantee smooth and monotonic solutions at shock waves and other flow discontinuities. The resolution of these four problems are the research objectives for the present reporting period.

In the realm of transonic flow theory the work is concerned with an investigation into the occurrence of multiple solutions for a class of flow parameters. Specifically the occurrence of lifting solutions for symmetric airfoils at zero angle of attack.

STATUS OF RESEARCH EFFORT

The first research objective for this reporting period was to investigate means of overcoming the stiffness introduced by the adaptive meshing. The stiffness is a measure of the range
of the eigenvalues of the flux Jacobian. The greater the range, the stiffer the problem becomes and more difficult to solve numerically. The adaptive mesh increases the stiffness in two ways; by a reduction in the smallest mesh spacing and by the mesh velocity. For explicit schemes nothing can be done about the reduced mesh spacing. However, the increased stiffness due to the mesh velocity is more critical for explicit schemes and can be alleviated by matrix splitting. The splitting refers to the splitting of the flux Jacobian so that the effects of the convective velocities are decoupled from the effects of the mesh velocity. Since the two effects can be decoupled, each can be treated separately without loss of accuracy and hence arbitrary mesh velocities (and arbitrary mesh clustering) can be allowed in solution procedures with explicit schemes. This decoupling allows the mesh velocities to be sufficiently high so that the clustering function can keep up with the flow field features without violating the stability criteria of explicit schemes.

The most important accomplishment has been the discovery that the procedure introduced by Viviand, Reference 1, for deriving the governing differential equation in the strongly conservative form of the arbitrary curvilinear coordinate system is not valid. It is commonly thought that Viviand's form of the transformed equations are the proper conservation equations which yield the correct shock strengths and speeds for arbitrary mesh-clustering or mesh velocities. However, it has been shown in the present study that the shock strength and speed are modified by the mesh clustering function and mesh velocity through the shock transition region. To obtain the proper shock jumps and speed either the mesh clustering function or speed must be uniform through the shock transition region or the transition region must be of zero thickness. If the former condition is met then there is no need for the strongly conservative form of the transformed differential equation and the much simpler chain rule conservation law form is adequate. If the latter condition is to be met, then a shock fitting procedure is required and again the strong conservation law form is not needed.
Some numerical computations have been carried out to test the effect of mesh clustering and mesh velocity on the shock strength and speeds and the above conclusions have been verified. These results are directly applicable to the resolution of the above mentioned research objectives (2) and (3).

Due to the invalidity of Viviand's transformation for thick shock waves, the clustering function is no longer simply a function of the truncation errors. It must also satisfy certain restrictions so that the proper weak solution is recovered by the numerical scheme. The restriction is that the mesh should be nearly uniform throughout the shock (or contact surface) transition region. Since this is also the region where the truncation errors vary most rapidly there is no possibility of adapting the mesh so that the truncation error is uniform over the entire computational domain. The mesh induced truncation errors can be greatly reduced (actually completely eliminated) if the fine but uniform mesh occurs only in regions where the solution truncation errors are large and the coarse mesh is only in regions where the truncation errors are small. The transition between the fine and coarse mesh need not be smooth provided that they occur only in the regions where the solution is locally uniform and the mesh transformation metrics are computed according to Reference 2.

The third research objective was to obtain the proper filtering and smoothing functions for the truncation errors. The truncation errors can be considered to be wave packets moving along with the features of the flow field. The purpose of filtering and smoothing truncation error is to find the envelope of the packet. The details within the wave packets are not important. The nonlinear truncation error analysis provide not only the envelope but also the details within the wave packet. The envelope can be determined in many cases by the curvature of the numerical solution. Thus, it is more efficient in most cases simply to look at the curvature of the numerical
solution as an approximation of the lowest order harmonics of the truncation errors. The exact nonlinear truncation errors are not required. This is fortunate since it is quite expensive to compute the nonlinear truncation errors.

The final research objective was to investigate the mesh dependent dissipation required to obtain monotonic and smooth shock waves. It is known (Ref. 3) that to obtain monotonic solutions at shock waves that the mesh spacing must not exceed a critical value set by the local amount of dissipation (either artificial or numerical). If, however, the mesh spacing is very much less than the critical value then the shock becomes excessively diffused or smeared resulting in a loss of the effective use of the available number of mesh points. To utilize this trick in an adaptive mesh strategy it is necessary to know the numerical dissipation rate, which is difficult to determine since it is a nonlinear function of both the solution and the metrics. So far the mesh dependent artificial dissipation has not successfully produced monotonic and smooth shock waves.

The above results are for the explicit scheme and were done by Nielsen Engineering & Research, Inc. (NEAR). The work for implicit schemes is presently underway and is to be done by Professor D. S. McRae.

TRANSONIC MULTIPLE SOLUTIONS

In recent years multiple solutions to the numerical approximation to the full potential equations have appeared in the literature (Refs. 4 and 5). Initially the phenomena appeared in computations of the flow over a symmetric airfoil at zero angle of attack when two lifting solutions were present in addition to the expected nonlifting solution. In Reference 5 some results for a nonsymmetric airfoil, a RAE 2822 section, are also presented. Steinhoff and Jameson (Ref. 5) suggested that the
change from one of the solutions to another is discontinuous and noted a hysteresis effect indicating that the lift coefficient ($C_L$) depended on whether the angle of attack ($\alpha$) was increasing or decreasing. More recent work is by Salas (Ref. 6) who has extended the computations of the flows considered by Steinhoff and Jameson (Refs. 4 and 5) to show that it is possible to construct a smooth $C_L - \alpha$ curve connecting the three solutions for a symmetric airfoil.

The investigations noted above are meticulously performed and are essentially numerical experiments. There is a limited amount of understanding that can be gained from such experiments and consequently a more analytic technique may yield more information. Furthermore, although the numerical results are invaluable they do not exclude the possibility that the multiple solutions are due to the numerical approximation to the differential equation. The present investigation is based on the integral equation formulation (Ref. 7) which allows some degree of insight into the problem.

The transonic integral equation method of Reference 4 is only applicable to the transonic small disturbance (TSD) equation rather than the full potential equation (FPE) that is used in the earlier work. Consequently, the first step is to reproduce multiple solutions using the TSD equation. Once these solutions are obtained they can be analyzed using the ideas of the transonic integral equation theory. In this investigation these suggestions have been implemented and the conclusions are as follows. The study indicates that the formulation of the TSD equation (and by implication the FPE) is not unique even with the Kutta condition enforced. The formulation indicates that eigensolutions can exist which can be combined with the correct solution to give erroneous results. These eigensolutions introduce arbitrary constants into the solution and a preliminary examination indicates that there is no obvious means of determining these constants.
REFERENCES


PAPERS AND REPORTS


PERSONNEL

The Principal Investigator on the transonic theory is Dr. David Nixon. For the adaptive grid aspects, Dr. Goetz H. Klopfer is Co-Principal Investigator. Dr. David S. McRae of North Carolina State University is a subcontractor of this work.

PRESENTATIONS


TECHNICAL APPLICATIONS

The computer code developed to treat strong shock waves in potential theory and described in the previous annual report, has been requested by NASA/Ames Research Center to use with a boundary layer code.