MICROCOPY RESOLUTION TEST CHART
NATIONAL BUREAU OF STANDARDS-1963-A
A STUDY OF COMPUTER-AIDED
GEOMETRIC OPTICAL DESIGN

THESIS

31/10/29/95/10624 Michael J. Wahlstedt
Sept. 1983

Approved for public release; distribution unlimited.
A STUDY OF COMPUTER-AIDED
GEOMETRIC OPTICAL DESIGN

THESIS

Presented to the faculty of the School of Engineering
of the Air Force Institute of Technology
Air University
in Partial Fulfillment of the
Requirements for the Degree of
Master of Science

by

Michael J. Wahlstedt, B.S.
Capt USAF
Graduate-Engineering Physics
October 1982

Approved for public release; distribution unlimited
PREFACE

The purpose of this study was to learn about computer-aided optical design and to demonstrate what was learned, by designing an optical system for the Air Force Weapons Laboratory.

I am indebted to many persons, who gave valuable assistance in the accomplishment of this study. I wish to thank John Loomis of the University of Dayton for making the FALCON computer program available to me. I also wish to thank the Questar Corporation of New Hope, Pennsylvania for making available detailed design information on their three and one-half inch telescope. Finally, I especially appreciate the guidance of John Erkkila, my faculty advisor, and the other members of my thesis advisory committee.

Michael J. Wahlstedt
CONTENTS

<table>
<thead>
<tr>
<th>Section</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>Preface</td>
<td>ii.</td>
</tr>
<tr>
<td>List of figures</td>
<td>iv.</td>
</tr>
<tr>
<td>Abstract</td>
<td>v.</td>
</tr>
<tr>
<td>I. Introduction</td>
<td>1</td>
</tr>
<tr>
<td>Background</td>
<td>1</td>
</tr>
<tr>
<td>Problem</td>
<td>2</td>
</tr>
<tr>
<td>Scope</td>
<td>3</td>
</tr>
<tr>
<td>Assumptions</td>
<td>3</td>
</tr>
<tr>
<td>Approach and Presentation</td>
<td>4</td>
</tr>
<tr>
<td>II. Computer Hardware Setup and Software Selection</td>
<td>5</td>
</tr>
<tr>
<td>III. Raytracing and Aberrations</td>
<td>9</td>
</tr>
<tr>
<td>Raytracing</td>
<td>9</td>
</tr>
<tr>
<td>Aberrations</td>
<td>14</td>
</tr>
<tr>
<td>IV. FALCON and Optical Design</td>
<td>25</td>
</tr>
<tr>
<td>FALCON</td>
<td>25</td>
</tr>
<tr>
<td>Optical System Design</td>
<td>32</td>
</tr>
<tr>
<td>V. Results and Discussion</td>
<td>36</td>
</tr>
<tr>
<td>VI. Conclusions</td>
<td>52</td>
</tr>
<tr>
<td>Bibliography</td>
<td>53</td>
</tr>
<tr>
<td>Appendix A: Aberration Coefficient Equations</td>
<td>55</td>
</tr>
<tr>
<td>Appendix B: Diagram of Lens Design</td>
<td>56</td>
</tr>
<tr>
<td>Vita</td>
<td>57</td>
</tr>
<tr>
<td>Figure</td>
<td>Description</td>
</tr>
<tr>
<td>--------</td>
<td>--------------------------------------------------</td>
</tr>
<tr>
<td>1</td>
<td>Contour Plot of Wavefront</td>
</tr>
<tr>
<td>2</td>
<td>Three Dimensional Plot of Wavefront</td>
</tr>
<tr>
<td>3</td>
<td>Field Curvature and Distortion Plot</td>
</tr>
<tr>
<td>4</td>
<td>Spot Diagram</td>
</tr>
<tr>
<td>5</td>
<td>Radial Energy Distribution</td>
</tr>
<tr>
<td>6</td>
<td>Geometric Optical Transfer Function</td>
</tr>
<tr>
<td>7</td>
<td>Knife Edge Energy Distribution</td>
</tr>
<tr>
<td>8</td>
<td>Diagram of Lens Design</td>
</tr>
</tbody>
</table>
ABSTRACT

Computer-aided geometric optical design was investigated. The definition of terms and optical design procedures were studied. Available computer programs were compared and FALCON was selected. Test problems were evaluated with FALCON to aid in understanding computerized optical design procedures.

An 18 inch long optical system with a 50 meter effective focal length was designed for the Air Force Weapons Laboratory. This optical system would image, on a solid state detector, an interference pattern created by two laser beam samples, to allow the two lasers to be locked in phase with each other.

The starting point of the design was a Questar 3.5 inch telescope. Additional optical elements were added and the design improved until the desired tolerances were met or exceeded.
I. INTRODUCTION

BACKGROUND

To design an optical system, it is necessary to specify the radii of curvature of the surfaces, the thickness of the optical elements, the air spaces, the diameters of the various components and the types of glasses to be used. The ideal design will result in all rays of all wavelengths originating at a given object point passing through a single image point. The image of a plane object should be a plane, without any appearance of distortion or curvature in the images of straight lines (3:1).

Prior to about 1930, all optical systems were designed manually using logarithm tables (3:xi). There is no greater test of someone's dedication to his work than to design an optical system by logarithms, yet this is how it was done for over a hundred years, and all the well-known classical lenses were designed in this incredibly laborious manner. These lenses include the Dagor, the Protar, the Tessar, the Biotar, the Ernostar, and many others. After about 1930, electrically driven desk calculators appeared, but these still required endless references to sine and arc-sine tables (1:58).

Up until about 1950, even the celebrated IBM Card Programmed Calculator used by Donald P. Feder at the National Bureau of Standards, required about 34 seconds to trace a ray through an optical surface (2:630). By 1957
only about 1.5 seconds were needed to trace a more difficult skew ray through an optical surface (1:58).

Current computers are capable of tracing tens of thousands of rays per surface per second. Several computer programs take advantage of this speed and contain automatic optimization routines (18:203). These routines modify parameters in order to find the optimum design (5:46).

As we can see, computers have increased in speed and capability. This increases their utility to the optical system designer. Learning to use these new and powerful tools to design an optical system is a formidable challenge. The Wright-Patterson Air Force Cyber computers will be used both as a learning tool and as a design computer for an Air Force Weapons Laboratory project.

**PROBLEM**

The problem is to learn about computerized geometric optical design and to apply what was learned by designing an optical system for the Air Force Weapons Laboratory. The Weapons Laboratory desires to couple two Argon-Krypton lasers in phase. Ultimately, many lasers can be coupled and a large, powerful and constant phase beam can be produced.

The optical system to be designed will be used to image an interference pattern created by two laser beam samples on a solid-state detector connected to a feedback loop to drive cavity mirrors through suitable electronics. Solenoids connected to the mirror will change the surface of the mirror, and drive the interference pattern to the
desired configuration, and lock the two lasers in phase.

In order to create the desired interference pattern, the Air Force Weapons Laboratory specified the optical system should have a 50 meter effective focal length. Physical space limits the maximum system length to 18 inches, so a compact 3.5 inch Questar telescope will be used as the starting point with additional optical elements added to provide the required effective focal length.

SCOPE

For this investigation, only geometric optics techniques will be used. The wave nature of light and the wave interference pattern will not be examined. The wavelengths of interest are .5145, .5017, .4965, and .4880 microns.

The literature search includes Defense Technical Information Center, the Miami Valley Interlibrary Loan System and the Wright-Patterson Air Force Base libraries.

ASSUMPTIONS

The laser beam will be represented as perfect plane waves. That is, the object will be a point source an infinite distance away.

Third order aberration theory will be a primary indicator and is assumed to be mostly sufficient to characterize the quality of the optical system.

The additional lenses to be added to the Questar telescope will be of simple spherical figure and will be made of easily obtainable Schott glasses. The minimum radii
of curvature of the additional lenses will be a centimeter or greater in order to insure a practical and easily constructed system, if the final design allows.

APPROACH AND PRESENTATION

The Tektronix 4025 computer terminal on loan from the Air Force Weapons Laboratory must be connected and the communication parameters adjusted to interface properly with the Cyber computer system.

A computer program must be selected from those available. ACCOS V, CODE V, COOL-GENII, and FALCON will be examined. The best available program will be used as a learning tool to design the optical system for the Air Force Weapons Laboratory. The selected program must be verified to insure the raytracing and aberration sections perform as expected and as desired.

The physical meanings of aberrations will be studied and desired tolerances will be determined. Lens design and aberration control techniques will be examined. The 3.5 inch Questar telescope will be computer modeled, and the initial additional lens design will be determined and refined to meet the desired tolerances.
II. COMPUTER HARDWARE SETUP AND SOFTWARE SELECTION

The Tektronix 4025 computer terminal, on loan from the Air Force Weapons Laboratory, was connected to an extension phone line. The communications parameters were determined to be 7 bits with even parity, terminal self-echo, and one stop bit. The terminal was set to these parameters to properly interface with the Wright-Patterson Air Force Base Cyber computers. The tape drive unit included with the terminal was used to save both pre-programmed keys and short programs on tape.

A computer account number and Cyber computer manuals were obtained. A familiarity with the use and maintenance of computer files was a necessity inorder to complete the project.

A computer program must be selected from those available. ACCOS V, CODE V, COOL-GENII, and FALCON documentation was requested and examined. A thorough study of the documentation was necessary so that an understanding of the differences and similarities of the programs could be accomplished. It was also necessary to gather from the documentation exactly what the capabilities of this generation of computer codes were.

ACCOS V, the property of Scientific Calculations Incorporated, is a large computer program that will run in either interactive or batch mode. The interactive mode allows the design to be evaluated in real-time and modifications readily seen. The design input is organized
into a series of subfiles describing the lens specifications, ray information, merit function definition, optimization variables, and target values (5:46). It is an automatic program that is capable of optimizing a user-controlled merit function. A merit function measures the state of correction of an optical system. It could be more properly called a defect function since a larger number indicates a more defective optical system. The merit function specifies the relative weights of several aberrations (17:1210). The most important or critical aberrations will have greater relative weights. The merit function is a sum of the squares of the product of the aberration and weight factor. ACCOS V has a graphic plotting capability for use with its associated analysis features (5:46). It is available at the Air Force Weapons Laboratory but not at Wright-Patterson at the present time.

CODE V, the property of Optical Research Associates, is a large (60,000 FORTRAN and Assembly Language Statements) optical design package that will run in either batch or time-sharing mode. The design input is organized into predefined subtasks or options. The lens description is input in detail while other options are available with program-supplied default values. The default values often work well enough to get preliminary results, or they can be easily over-ridden by the user. The predefined options of the CODE V system usually allow the optical system designer to proceed fairly rapidly into the design aspects of a
problem although some versatility in the approach to a problem may be sacrificed. CODE V has a graphic plotting capability for use with its analysis options. CODE V uses a damped least squares optimization of a user controlled merit function (5:47). It is not available at Wright-Patterson at this time.

COOL-GENII, the property of Genesee Computer Center, is an integrated optical design system incorporating the complete capabilities of the David Grey Computer Optics Package (COOP) and the Genesee GENII lens design system. Access from both batch and time-sharing terminals is available. The design input for the COOL-GENII program is broken into sections that include the lens data input, the ray set data, the merit function definition, target inputs, and the optimization and analysis control inputs. Optimization can be accomplished either by completely developing a complete merit function in terms of user-defined operands and target values or by using default parameters that may provide adequate optimization and analysis of the design problem. Some of the default values may be over-ridden by the user quite easily. It also uses damped least squares optimization of a default or user defined merit function. The only graphics available for the COOL-GENII program system are for the various ray fans and the lens layout, which are printed by the Grey portion of the system on line printer plots (5:47). It is also not available at Wright-Patterson at this time.
FALCON is a medium size computer program written by John Loomis, formerly at the University of Arizona Optical Sciences Center. He is now at the nearby University of Dayton and he was willing to make FALCON available to me on the Wright-Patterson Air Force Base Cyber computers. FALCON will run in either batch or interactive mode. The program will evaluate an optical system but does not include automatic optimization.

All the programs examined will raytrace and calculate aberration coefficients. Plots and tables are output to aid in the interpretation of the information. Ray fans, spot diagrams, wavefront variance, Strehl ratio, vignetting diagrams and optical transfer functions are provided by most of the programs.

FALCON was selected as the best available program. It has the flexibility to be run in both interactive or batch modes. The optical systems can be put in a library file and recalled and modified. Line printer plots can be generated in batch mode, while Tektronix compatible plots can be generated in the interactive mode. The optical system can be displayed on the terminal's screen and modifications are easily seen.

FALCON will be examined in detail later, but first some fundamental material needs to be covered. A study of ray tracing is necessary to properly input parameters into the optical design program.
III. RAYTRACING AND ABERRATIONS

RAYTRACING

Raytracing is a technique to analyze an optical system by mathematically simulating the passage of several individual rays of light through the optical system. At each surface, the light ray will be bent by Snell's law. A paraxial approximation can be used to simplify the mathematics where required. Michael Lea's GEOMETRICAL OPTICS (Ref 14) contains a very clear discussion of general raytracing techniques and forms the foundation of this material.

We will assume that the optical system has rotational symmetry about the optical axis to simplify this discussion. An axial ray begins at the foot of the object (on the axis) and enters the optical system off the axis. The point at which the axial ray again crosses the optical axis is where the image is formed. If the optical system is perfect, all the rays that leave the foot of the object will cross the axis at the same point. A virtual image that results when the axial rays are diverging, can be located by extending the rays back until they cross the optical axis (14:15).

The coordinate system should be specified at this point. The optical axis is called the z-axis. The object is most commonly placed to the left of the optical system or towards the negative end of the z-axis. The y-axis helps determine the vertical or TANGENTIAL plane. The x-axis
helps determine the horizontal or SAGITTAL plane. For simplicity in a symmetrical optical system, it is customary to consider only object points in the tangential plane. Rays which start in the tangential plane, stay in that plane and are called TANGENTIAL RAYS. Rays with some x-component (usually drawn perpendicular to and in or out of the paper) are called SKEW RAYS. Most of the design of an optical system can be done by considering only the tangential plane (14:15).

If we trace an axial ray from infinity and find the position of the image, we are faced with the problem of determining from where in a complex lens we measure the focal length. If we extend the incoming ray forward and the emerging ray backwards until they cross, the point of intersection determines the location of the SECOND PRINCIPLE PLANE. The EFFECTIVE FOCAL LENGTH is the distance from the second principle plane to the image location. It is as though the whole lens assembly could be replaced by a single surface at the second principle plane which does all the ray bending (14:18).

The FIRST FOCAL POINT and PRINCIPLE PLANE are found by bringing in a ray from right to left. The FIRST FOCAL LENGTH is defined as the distance from the first focal point to the first principle plane. The entire lens assembly can now be approximated by a "black box" characterized by the focal and principle points. One very nice result of this approximation is that all rays are
simply shifted horizontally (along the z-axis) between the two principle planes (14:19).

The **power** of the lens is defined as the index of refraction at either focal point divided by the respective focal length. The power of the lens assembly is a constant, that is, either the first or second focal point and the respective index of refraction can be used to get the same result. If the lens assembly focuses in air (we assume a refractive index of exactly one), then the power of the lens assembly is equal to the inverse of the focal length (14:19).

A ray that passes through a focus will hit the principle plane and emerge from the other principle plane parallel to the optical axis. Conversely, a ray parallel to the optical axis will continue parallel between the principle planes and then pass through the appropriate focus. By constructing two rays from the head of the object, one parallel to the optical axis and one passing through the nearest focus, we can determine the location and size of the image (14:20).

The **aperture stop** is the component in the system which limits the size of the imaging ray bundle. It is usually a diaphragm included in the system for this very purpose. To find the aperture stop, trace an axial ray and scale it away from the optical axis until it hits the edge of something. That something is the **aperture stop**. The cone of rays which forms an image point has a semi-angle of $\theta$. The
NUMERICAL APERTURE and F-NUMBER are essentially measures of this angle. The numerical aperture is equal to the refractive index times the \( \sin \theta \), while the f-number is equal to the index of refraction divided by twice the numerical aperture (14:23).

A very useful tool is the LAGRANGE INVARIANT. It was found that the product of the height of the object or image, the refractive index, and the angle of the axial ray at the respective object or image was a constant for that optical system. So the angle of the axial ray helps determine the size of the image (14:25).

The aperture stop limits the size of the imaging bundle. However, since it is usually buried in the middle of the system, the light arriving at the lens sees the image of the aperture stop in all the components preceding it. So the size of the incoming bundle is limited by this image known as the ENTRANCE PUPIL. Similarly, the EXIT PUPIL is the image of the aperture stop in all components which follow it (14:27).

To find the position of the pupils, we trace a ray through the center of the aperture stop. This is called the CHIEF RAY. By extending the object and image rays until they cross the optical axis, we find the position of the pupils. The size of the pupils can be determined by using the angles of the chief ray (14:31).

The FIELD STOP determines the field of view of the optical system. By scaling up the angle of the chief ray
until it hits something, we can determine the limit of the field of view of the system. Often, the field stop is simply the edge of the image plane similar to the edge of the film or the TV face plate (14:31).

VIGNETTING occurs when, as our object rays go off axis, some other surface begins to cut off these rays (20:113). This is characterized by a loss of intensity at the outside of the image.

A known manual raytracing example from MODERN OPTICAL ENGINEERING by Warren J. Smith was input into FALCON. The raytracing information was confirmed within numerical accuracy (14:140). We will now examine image defects.
ABERRATIONS AND ABERRATION CONTROL

Defects in the image can be quantified in terms of their aberrations. A study of these aberrations, their definitions and methods of control, is necessary to get the best possible image quality. The total values of the aberration coefficients permit an analysis of the correction state of the system and hence the quality of the final image, while a study of the partial coefficients at the surfaces show how the aberration of each type and order build in the system. This leads to an understanding of how a system achieves its performance, for an analysis of the balance of the different orders of each type of aberration may be made. In addition, it becomes clear where modifications must be made in the system to alter its final performance (4:379).

After some familiarity with the general image effects of the aberration coefficients has been developed, one comes to regard the image forming quality of a lens system as specified by the values of these coefficients. With a little experience, it is possible to infer the performance to be expected of a system almost from a glance at the coefficients and to say why the system achieves a particular correction state. This makes the task of comparing alternative designs simpler and makes it easier to evaluate the relative merits of different lines of development in investigations in initial design (4:379).

To study aberrations, we restrict ourselves to the
back-end of the system, from the exit pupil to the image plane. We can define any ray with three coordinates, two specify the position in the exit pupil plane and one specifies the image height. For a symmetrical optical system, we can simplify further and by convention, consider only object and image points in the tangential plane.

A definition of the term "zone" is helpful at this point. For some aberrations, it is convenient to consider cones of rays from rings or "zones" in the exit pupil to the image. A zone is a region specified by the relative fraction of the radius of the exit pupil (14:85).

SPHERICAL ABBERRATION is the variation of focus position with zone or a longitudinal difference of focus between rays from successive zones of aperture (14:97). In general, in a system of finite positive focal length, the effect of spherical aberration is that the optical system reproduces a point object as a small circular patch of light, instead of the image point demanded by geometrical theory. In extreme cases, the light patch may consist of a bright ring with a fainter center, or a small bright nucleus and a rather tenuous halo. In less extreme cases there may be a central rather hard disk and a faint concentric fringe. Since the image of an object of finite size lying on or near the axis has then to be formed by the superposition of light patches of finite size, its outlines are softened and details may not be visible in the image (7:69). Spherical aberration is the only aberration, other
than chromatic aberration that will be discussed later, to affect definition on the axis of the system. As good central definition is one of the most important features of the performance of an instrument, it is essential that spherical aberration be reduced (7:68). Positive spherical aberration is most commonly associated with diverging optical elements (8:51). Positive spherical aberration causes the wavefront to curl outward at the edges (8:64). Spherical aberration is positive when the paraxial focus lies to the right of the marginal focus, and it grows as the square of the aperture (6:303).

COMA is the variation of magnification with zone (14:98). Coma causes an unsymmetrical deformation of individual off-axis image points to a comet-like appearance. Coma softens the definition of the image (7:73). It is positive when the chief ray passes above the dislocated outer rays, and it grows as the distance of an image point and as the square of the aperture (6:303). In contrast to spherical aberration, coma depends on the position of the aperture stop and on the related position of the entrance and exit pupils. One standard method of eliminating coma in a system, is to introduce a high degree of symmetry. The system is made symmetrical about a center aperture stop, and the object and image sizes are made the same size. Under these conditions the coma is exactly balanced. In other circumstances, coma is eliminated in much the same way as spherical aberration. Coma of opposite
sign is introduced to bring the sum to as near zero as possible (7:74).

FIELD CURVATURE or the PETZVAL SUM, is characterized by a curved focal surface (14:99). That is, the locus of image points produced from a planar object is not planar. The field curvature has no effect of any kind upon the sharpness or definition of individual image points. When astigmatism is also present, then the sagittal and tangential focus of any one oblique pencil are formed always on the same side of the Petzval surface and the distances from the Petzval surface are in a one to three ratio. If both astigmatism and distortion are of the same sign, the curvature of the field is aggravated; but if the astigmatism is under control, then a judicious amount of astigmatism with the opposite sign of distortion affords the only means of minimizing the Petzval curvature in a great majority of optical systems in which the Petzval sum must be accepted at whatever value it assumes. Correction for flatness of the tangential field is usually the best compromise, and calls for the astigmatism coefficient to be one-sixth the value of the distortion and of opposite sign (6:304). Positive lenses introduce an inward curvature of the Petzval surface and negative lenses introduce a backward curvature. The Petzval curvature of a thin simple lens element is equal to one-half of the square of the image height divided by the product of the focal length and index of refraction of the lens element (8:56).
ASTIGMATISM occurs when the tangential and sagittal rays have different field curvatures (14:100). Another way of looking at astigmatism is that tangential and sagittal ray bundles do not focus at the same point. Between the astigmatic focii, the image is an elliptical or circular blur (8:54). Astigmatism and coma are both aberrations that appear when the object is off-axis (3:191). Astigmatism is positive when the sagittal focus lies to the right of the tangential focus. The astigmatic difference of focus is independent of the aperture, but grows as the square of the distance of an image point from the optical axis (6:304). Astigmatism is corrected or balanced in the same way, in general, as spherical aberration and coma. At some surfaces negative sagittal and tangential astigmatism are introduced, at others there is positive astigmatism, both of first and higher orders. The curves of the refracting surfaces are to be chosen so that the first order astigmatism balances the higher orders (7:79).

DISTORTION is the variation of magnification with image height or the departure from strict geometrical similarity of any configuration in the object plane with its image located in the image plane (14:102). The magnification in the central part of the field being fixed by the theorem of Lagrange, we measure the distortion by the distance from the actual off-axis image point to its ideal position. One effect of distortion is that if we take a short line in the object plane, lying along a radius from
the axis of the system, and move it out along this radius, the length of its image will vary. This provides an alternate method of measuring the distortion present in an optical system (7:83). We give a positive sign to the distortion when the ideal image point falls above the actual one. Thus defined, the distortion grows as the cube of the distance of the image point from the optical axis. Negative distortion causes the outer parts of the image to be on too small a scale, and is called barrel distortion; in the case of positive distortion the outer parts of the image are on too large a scale, and we describe it as pincushion distortion (14:103). Distortion has no effect upon the sharpness of the individual image points; it only dislocates them towards or away from the optical axis (6:304). The correction of distortion is carried out in the same general way as the correction of the other aberrations. At some surfaces, pincushion distortion is introduced, at others barrel distortion. The contribution of distortion for any surface depends on its position relative to the object and image, and relative to the aperture stop and pupils (7:83). A little study of the matter will show that a system which produces distortion of one sign, will produce distortion of the opposite sign when object and image are interchanged (8:56).

TRANSVERSE CHROMATIC ABERRATIONS are a variation of magnification with wavelength or a chromatic variation of the image height, which affects only the off-axis part of
the field, and which causes white object points to be depicted in the form of a linear rainbow disposed radially with reference to the center of the field and proportional in length to the distance of the individual object points from the optical axis (6.305). Chromatic aberrations occur because the refractive index of a glass is not a constant but is a function of wavelength (10.12). By using more than one type of glass it is possible to balance the chromatic aberration for a specified range of wavelengths (10.180).

The OPTICAL TRANSFER FUNCTION is the image contrast as a function of detail (spatial frequency). The finer the detail in an image, the less the contrast between dark and light sections, until eventually the contrast fades out and detail is unresolved. This is a more complicated but complete indicator (13.11). The optical modulation transfer function is the ratio of the modulation in the image to that in the object as a function of spatial frequency in inverse length units for a sine wave pattern (10.212). A plot of the optical modulation transfer function against frequency is an almost universally applicable measure of the performance of an image forming system. It can be applied not only to lenses, but to films, phosphors, image tubes, the eye, and even to complete systems. One particular advantage of the optical modulation transfer function is that it can be cascaded by simply multiplying the optical modulation transfer functions of two or more components to obtain the optical modulation transfer function.
function of the combination (8:311).

The STREHL RATIO is the ratio of the light intensity at the peak of the diffraction pattern of an aberrated image to that at the peak of an aberration free image, and is one of the many criteria that have been proposed for image evaluation. It can be computed by calculating the volume under the three-dimensional optical modular transfer function and dividing by the volume under the curve for an aberration free lens. A similar criterion for quick general evaluation of image quality is the normalized area under the optical modulation transfer curve (8:311).

The RADIAL ENERGY DISTRIBUTION is the integral of the point spread function and gives the image spot size for a point object (13:11). The radial energy distribution curve is a graph of the encircled fraction of the energy (that is the fraction of the ray intersection points in the spot diagram) plotted against the radius (semi-diameter) of the aperture (8:16).

The control of spherical aberration and coma can be accomplished through a process known as "lens bending". The total power, and hence the focal length of the lens, is kept constant but the curvature of one surface is changed while the constant focal length drives the shape of the other surface (15:15). In a simple positive lens, coma varies linearly with lens shape, taking a large positive value when the lens is a meniscus with both surfaces concave toward the object. As the lens is bent through
plano-convex, convex-plano and convex meniscus shapes, the amount of coma becomes more negative, assuming a zero value near the convex-plano form of a simple lens. The spherical aberration of this lens is always undercorrected (8:60). By experimenting with FALCON, this was quickly verified. The spherical aberration reaches a minimum value at approximately the same shape for which the coma is zero. This is the shape one would select if the lens were to be used as a telescope objective to cover a rather small field of view (8:59). The shape which produces minimum spherical aberration also produces the maximum field curvature, so that this shape, which gives the best image near the axis, is not suitable for wide field coverage. The meniscus shapes at either side if the figure represent a much better choice for this purpose, for although the spherical aberration is much larger at these bendings, the field is much more nearly flat. This is the type of lens used in inexpensive box cameras (8:62). Graphing the aberrations can assist in selecting the best configuration.

Field curvature, distortion, transverse chromatic aberration and coma can be varied by moving the position of the aperture stop. Distortion and lateral curvature are zero when the aperture stop is at the symmetrical point of the optical system (8:59). The results of the aberrations can be plotted. Since each aberration usually has a different optimum aperture stop, a best compromise can be selected.
Aberration control can usually best be accomplished by finding a solution with small, evenly distributed aberration contributions. It is seldom advisable to introduce a large contribution on one surface to cancel out several small aberrations created by other surfaces (11: 8-15).

Another guideline is to try to avoid large angles of incidence. The angle of incidence strongly affects the magnitude of higher order aberration contributions. Usually, the closer to normal incidence at a surface, the better (11: 8-15).

If a surface introduces a large amount of aberration, try to correct this by another surface as nearby as possible. This tends to better cancel out the higher order aberrations (11: 8-15).

The shape of the lens has to be carefully chosen so that the first order aberration has a suitable value in comparison with the higher order aberration. The shape is governed by the angles of incidence and refraction of the light rays transmitted. If the object point is moved relative to the lens these angles are changed, and there can be no guarantee that the balance of first and higher order aberrations is maintained under such changed conditions. In general, it is not maintained, and for this reason to get the best results even such a simple system as a doublet objective must be used with the distances of object and image points for which it was designed (7:68).
The balance of negative and positive corrected spherical aberration in more complex systems is established in the same general way. At some surfaces negative correction is introduced, at others, not necessarily contact surfaces, positive correction is introduced.

When a lens is stopped down, or the passage of rays through the entrance pupil is restricted by the leaves of an iris diaphragm, the definition afforded is usually improved owing to the removal of the patch of light produced by the marginal rays and their positive spherical aberration. There is a limit to the improvement of definition that may be produced by stopping down a system, set by diffraction (7:68).
IV. FALCON AND OPTICAL DESIGN

FALCON

FALCON is a computer program to assist in the design and evaluation of optical systems. The program is intended to be used in either time-sharing or batch environments. Input consists of identifier words and numeric parameters entered in free format from a computer terminal or by punched cards. Printed output is limited to 80 columns to fit on standard terminals. Because of this abbreviated line width, line-printer output may be cut to fit into standard notebooks and letter-sized file folders. Plots are written on a general plot file, which may be diverted to a Tektronix terminal for on-line display or disposed to an off-line plotting device at the end of the job. Line-printer plots are also available.

Optical system data can be saved in a library file. Library commands are available to add or delete systems on this file. Text editor commands are provided to modify the description of the optical system in active storage. The program can handle a wide variety of optical systems including spherical surfaces, toric shapes, diffraction gratings, and specialized irregular surfaces. Surfaces may be decentered or tilted to any orientation.

Evaluation techniques include paraxial and real ray traces, calculation of classic third and fifth order aberrations, ray fans, spot diagrams, wavefront variance and Strehl ratio calculations, vignetting diagrams, and
optical transfer functions.

FALCON consists of a number of program segments called by the user through commands. Some commands, such as LENS, cause the program to read additional lines of input until an END statement is encountered. Other commands are executed without any additional input. Command lines may contain additional identifiers along with the command word to further control the operation of the program. The command word and its modifiers are restricted to one input line of 80 characters.

Input to the program is a succession of records, each of which consists of a string of up to 80 characters (including blanks). A record may be a punched card or a line on a computer terminal. The words "line" and "card" are therefore interchangeable. The end of a line is denoted in one of several ways. It may be the physical end of a punched card, the carriage return on a terminal, or a="/". The use of the slash character allows the user to pack several records on a single line.

The LENS task is used to define the properties of a lens or general optical system consisting of several lenses and other optical components, such as mirrors and gratings. In general, one input record is used to describe each surface of the optical system. This information is stored in a data base divided into blocks of memory. Each block contains a description of one optical surface. The LENS task is completed by typing the word END at the end of the
final input record, which is usually the image surface.

The logic and notation used by the LENS task is closely patterned on that of ACCOS V, a proprietary product of Scientific Calculations Incorporated. The intent is to promote a universal language for the specification of optical surfaces and alleviate much of the burden of using different lens design and analysis programs (16:6). By adding the word ACCOSV to the lens record, an input deck can be entered in standard ACCOS V format. Almost any input deck for ACCOS V can be interpreted correctly by FALCON. However, the input form for FALCON is somewhat more general than that of ACCOS V, and there are some important differences between the two programs, so care must be taken in using complicated ACCOS V input decks on FALCON.

For single configuration optical systems, surfaces are numbered in the order in which they are intersected by a ray of light traveling from the object surface to the image surface. The object surface is assumed to be number 0. If a general surface is denoted by j, the properties of the space between j and j+1 are associated with the surface j. All properties of a surface are defined on a local coordinate system associated with that surface. A conventional right-handed rectangular coordinate system is used. The z-axis is assumed to be the optical axis, and light is assumed to propagate in the positive z-direction. The YZ plane is used as the tangential plane of a symmetrical optical system. The XZ plane is then the
FALCON provides a variety of ways to define the properties of a lens system. The example below, from the FALCON Version Two User's Manual (Ref 9) by John Loomis, is one way an optical system may be defined.

```plaintext
LENS 1
LI SINGLE THIN LENS EXAMPLE 2
SAY 1.5 SAY -5.0 TH 20 UNITS CM 3
CV .05 TH .5 SCHOTT BK7 4
PHY -.15 PY 5
END 6
```

In this example, the first line is the command record, which identifies the following records as a lens data set. The numbers on the far right are not part of the input records but are put there to label the lines for reference. Line 2 is a lens identification record, denoted by the LI at the beginning of the record. Informative comments identifying the lens follow the word LI. The object surface is entered on line 3. The linear units of the optical system are given as centimeters. The axial ray is defined to intersect the lens at 1.5 centimeters (SAY entry), and the chief ray is defined to start at a distance of -5 centimeters along the y-axis in the local coordinate system of the object plane. The distance from the object plane to the first surface of the lens is given as 20 centimeters. By default, the object surface is assumed to be a plane.
and the medium following the object surface is assumed to be air. Line 4 describes the first surface of the lens. The CV entry identifies the shape of the surface as a sphere with a curvature of 0.05 inverse centimeters. Curvature is the inverse of the radius of the spherical surface. By convention, positive numbers represent a sphere whose center is to the right of the vertex (where the surface intersects the optical axis), that is, a vector drawn from the vertex to the center of the sphere is in the positive Z direction. The axial thickness of the lens is given as 0.5 centimeters. Finally, the glass from which the lens is made is given as BK7 from the SCHOTT catalog. FALCON contains an internal catalog of the Schott glasses. This data base is used to specify the index of refraction of the glass at various wavelengths. The default operating wavelength is 0.58756 microns.

Line 5 defines the second surface of the lens by means of paraxial solves. The curvature of the surface is set so that the axial ray leaves the surface at an angle of -0.15. Since the ray is paraxial, the angle and its tangent or sine are assumed to be indistinguishable. The separation of the following surface is calculated so that the axial ray intersects the optical axis, which makes this next surface an image plane. The absence of a numeric parameter following the paraxial thickness solve (PY) indicates that a ray height of zero is to be used. Since no other material is specified, the image is assumed to lie in air. Line 6
defines the image surface of the optical system. In this case, it is denoted only by the END statement.
FALCON has many commands, some of the major ones are:

<table>
<thead>
<tr>
<th>Command</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>DISPLA</td>
<td>Draws optical system and rays</td>
</tr>
<tr>
<td>FANS</td>
<td>Plots set of ray fans</td>
</tr>
<tr>
<td>FLDCV</td>
<td>List or plot field sags and distortion verses field</td>
</tr>
<tr>
<td>FOB</td>
<td>Sets object point and traces central ray</td>
</tr>
<tr>
<td>FORD</td>
<td>Lists third and fifth order aberrations</td>
</tr>
<tr>
<td>GOTF</td>
<td>Geometric optical transfer function</td>
</tr>
<tr>
<td>LEPRT</td>
<td>LEns PRint, extended listing of optical system</td>
</tr>
<tr>
<td>LIST</td>
<td>Short text listing of lens assembly</td>
</tr>
<tr>
<td>OCON</td>
<td>Short list of first order operating conditions</td>
</tr>
<tr>
<td>PARAX</td>
<td>Paraxial ray trace</td>
</tr>
<tr>
<td>RAY</td>
<td>Prints or draws a ray through the optical system</td>
</tr>
<tr>
<td>SPOT</td>
<td>Geometric spot analysis</td>
</tr>
<tr>
<td>STOP</td>
<td>Stops program execution</td>
</tr>
<tr>
<td>WAMAP</td>
<td>Plots contour map of wavefront</td>
</tr>
</tbody>
</table>
OPTICAL SYSTEM DESIGN

The 3.5 inch Questar telescope was computer modeled using proprietary information graciously provided by the Questar Corporation. NOTE: Proprietary information will not be included in accordance with an agreement with the Questar Corporation.

The Questar 3.5 inch telescope is a Maksutov Cassegrain catadioptric system. It consists of a corrector plate and two mirrors. This allows a light path to be folded into a compact package. The diameter of the clear aperture is 3.5 inches.

The 3.5 inch Questar telescope has a variable focal length between 50.5 and 56 inches, this is attained by moving the primary mirror. A position between the two focal length limits, 54.2 inches, was selected to insure a reasonable starting focal length.

To increase the effective focal length, it is necessary to add a negative lens assembly to make the converging beam slightly less convergent. Since each optical surface adds aberrations, one additional lens was first examined. FALCON's curvature solves are a tremendous help in this task. By using the PUY command, the axial slope angle solve adjusts the curvature of the last surface so the desired effective focal length can be set. Then only one variable, the first surface curvature, needs to be modified while the total lens power is automatically compensated and kept constant.
Using the FORD ALL command, the ray aberration contributions of each surface are individually listed. Since the aberrations of the Questar telescope are small, we are basically interested in adjusting the additional lens or lenses so that the aberration contributions are as small as possible. Some balancing is also useful. If the total aberrations of the system have a specific ratio and one of the lens surfaces has a similar ratio, by changing the curvature of that surface we can properly affect the total aberrations of the system.

Because these changes are made manually, a written record of the parameter you are changing and the resulting aberration is invaluable. It is best to change only one parameter at a time. Optimizing first one, then the other parameters in turn. The interactive mode allows dozens of changes to be made in a short time. Caution should be used as it is easy to focus on only one parameter and drive to an impractical design. At reasonable intervals, the entire design should be examined to insure reasonable values.

Since, in this design, the object is allowed to be a point source at infinity on the axis of an axially symmetric optical system, many of the aberrations become zero. These include coma, astigmatism, distortion and the Petzval sum or field curvature. The assumption that the object is a point source at infinity was tested by bringing the object towards the optical system and giving it a finite size. At distances on the order of 50 meters and
greater; coma, astigmatism, distortion and the Petzval sum are small.

Spherical aberration is important and needs to be reduced first. FALCON displays the third, fifth, and seventh order spherical aberrations. By the process of lens bending, the spherical aberration was reduced. Using the FORD ALL command, a surface with third, fifth, and seventh order ratios similar to the total spherical aberrations, was bent and the effect on the total spherical aberration was noted. The process would be repeated until the spherical aberration was essentially zero and resulted in a flat waveform.

Chromatic aberrations were next examined. The use of a diagram of optical glasses is a great help, but a glass table, with the refractive index and Abbe number, is sufficient (19:180). The Abbe number gives us a measure of the dispersion of the glass. By selecting glass with a different Abbe number, we can affect the chromatic aberration. By selecting a different glass, the chromatic aberration will either increase or decrease. Glass changes are made until the aberration reaches the desired level.

It was determined that a single lens could not do the job. The desired effective focal length could be obtained, but the aberrations could not be reduced as low as desired. The spherical aberration approach described above was repeated, but resulted in many dead ends from which further improvement was not possible. If the third, fifth, and
seventh order ratios are not similar to the total system spherical aberration, lens bending can improve one or two orders of aberration while increasing the other order of aberration. More surfaces were required in order to introduce sufficient degrees of freedom, this would permit the aberrations to be properly handled.

Two lenses were then examined. A proper balance of lens power was found. The first lens is mildly diverging, with the normal of both surfaces essentially parallel to the rays. This is to especially reduce the spherical aberration which is one of the largest problems. It was found that by carefully reducing the spherical aberration, many of the other problems were also reduced. To have very small aberrations, too much power had to be concentrated in the last lens. The curvatures, although mathematically allowed, were not physically possible.

The final lens was then split up into two lenses. The design process was then repeated. By working with three lenses, the aberrations were controlled and the curvatures were made reasonable.
V. RESULTS AND DISCUSSION

The final design has three negative lenses, each of half a centimeter thickness. The first is convex meniscus toward the Questar telescope, the second and third are both bi-concave lenses. The first lens is constructed from Schott SF7 glass and has radii of 3.030946 and 2.790622 centimeters. This lens is located 5.28 centimeters behind the first surface of the primary mirror. The exact placement of this lens is not critical because the location of the next lenses has considerable adjustment ability.

The second and third lenses should be mounted in a rack-and-pinion or comparable adjustable focusing tube. The second lens has radii of -1.282051 and 1.282051 centimeters and is constructed of Schott BK7 glass. This lens should be located about 7.57 centimeters from the first lens. The third lens is also constructed of Schott BK7 glass and has radii of -1.176471 and 1.053613 centimeters and is located 2.0 centimeters from the second lens. These lenses result in an image formed 8.73 centimeters from the last lens.

The final design (see APPENDIX B) results in small aberration values, but to determine if the values are sufficiently small, an aberration tolerance must be determined. The Rayleigh criterion allows no more than one-quarter wavelength of deviation over the wavefront with respect to a reference sphere. The Rayleigh criterion represents a high standard of quality essentially
indistinguishable from a perfect image (8:298).

In a perfect optical system, all rays converge on the paraxial image point and the wavefront emerging from the exit pupil is a sphere centered on the paraxial image point. We measure the deviation of the wavefront from this perfect sphere (8:311). The WAMAP command generates a contour plot of the wavefront. The default contour step, in waves, is chosen by the program so the final plot is not cluttered.

A three-dimensional plot option also allows the wavefront to be examined in greater detail. The next two pages reveal the final design is flat to better than two-tenths of a wavelength.
LI QUESTAR PLUS

Figure 2. Three-Dimensional Plot of Wavefront.
The FLDCV command generates a combined plot of field curvature and distortion as a function of relative field height. The fractional field height is plotted on the vertical axis and either the focal shift or the distortion is plotted on the horizontal axis.

Field curvature and distortion plots are calculated by taking a family of central rays from different object points along the y-axis and looking at the intersection of the ray with the image surface. Differential ray tracing provides the focal shift to the tangential and sagittal foci. These focal shifts are plotted as a function of field in the field curvature plot. Distortion is calculated as the difference between the actual image height and the paraxial image height for each field, and is plotted as a percentage change as a function of field (9:81).

Because we are assuming an object point at infinity and on the axis, both the field curvature and the distortion should be non-existent, as is displayed on the next page.
The SPOT PLOT command generates a spot diagram. Each point represents the intersection of a ray with the image surface. The total spread of the diagram and the distribution of the spots provide a measure, first of the optical quality of the system and second, of the appearance of the image. Many spot diagrams can be characterized by a central zone of high spot density and an outer zone or flare region of much lower density. The shape of the diagram is indicative of the principle aberrations in the optical system. Multi-wavelength spot diagrams may reveal distinct color separation that may not be desirable (9:90).

The spot diagram is extremely useful to a designer in evaluating an optical system. The diagram indicates how well the optical system concentrates the energy from the object point into an image point (11:8-1).

The spot diagram on the following page shows a majority of the rays striking near the paraxial focus point. A few rays at the edge of the field (further out than .8) show some chromatic deviation, but the weighed average is .005 centimeters. Since the laser beam sample centers will be separated by 7.5 centimeters and the sources will be 50 meters away (from the effective focal length) and for a .5002 micron wavelength, the fringe separation, imaged on the detector, is .033 centimeters. We desire the aberrations to be a small fraction of the fringe separation to avoid a blurring of the interference pattern.
seen by the detector. The result is a chromatic ray deviation of less than two-tenths of the fringe separation.
The SPOT RED command calculates the fraction of rays that fall within a circle of given radius and displays the results as a function of radius. The output is called the radial energy distribution or the encircled energy, or sometimes, the power-in-a-bucket. It corresponds to a measurement of energy transmitted through a variable aperture in the image plane. Default output is a plot of the energy transmitted (in percent) through an aperture of a given radius. The horizontal scale is chosen automatically.

The radial energy distribution on this next page shows a sharp image in that the horizontal scale is small and the plot rapidly rises at the smaller encircled radii.
The GUTF command evaluates the geometric optical approximation to the optical modulation transfer function. This function has been previously discussed and is a good general indicator of the performance of an optical system.

The geometric transfer function plot on the next page shows a very good result for such a complicated optical system, since the final result is the product of all the individual optical transfer functions. It is my experience that the total optical transfer function is a very sensitive indicator of optical performance. Each individual optical transfer function has a profound effect on the total optical transfer function. For the total optical transfer function to be this good, all the individual optical transfer functions must be very good.
The SPOT KED command calculates the fraction of rays that are uncovered by a knife edge moving in a specified direction, as a function of the distance moved. The output is a plot of energy transmitted past the knife edge (in percent) as a function of knife edge position (9:94).

For the knife edge distribution, a perfect optical system would be indicated by a step function at the zero point. In practice, the edges of the step function are rounded off. The knife edge distribution on the next page shows little edge rounding. This indicates good image quality.
The aberrations of the final design were small. For the average wavelength of .5002 microns, the third, fifth, and seventh order spherical aberrations were .000000, -.000005, and -.000003 centimeters respectively. The primary first order axial color and secondary first order axial color aberrations were .000039 and -.000005 centimeters respectively. These are extremely small values, very much less than the fringe separation of .033 centimeters.

Another measure of the quality of an optical system is the Strehl ratio. This is the ratio of the light intensity at the peak of the diffraction pattern of an aberrated image to that of an aberration free image. The final system design had a Strehl ratio of .983.
VII. CONCLUSIONS

1. The goal of this project was to learn about computerized optical design and to design an optical system for the Air Force Weapons Laboratory.

2. The necessary information was gathered from the literature and is included in this report for the benefit of those who may need it in the future.

3. FALCON, the selected computer program, was examined using test problems to learn how to use an optical design program.

4. An initial design of the additional lenses was completed and design procedures established to improve the design until the aberrations were judged sufficiently low when compared to the desired final image.

5. A computer and the proper computer program form a powerful and useful tool that can be of great assistance to the optical designer.

2. Feder, Donald P., "Optical Calculations with Automatic Computing Machinery", Journal of the Optical Society of America, 41: 630 (September 1951)


17. Feder, Donald P., "Automatic Optical Design", Applied Optics, 2, 1209 (December 1963)


APPENDIX A

Aberration Coefficient Equations

\[
\begin{align*}
K &= -\frac{1}{2 n' u_a^2} \\
Spherical \text{ Aberration} &= \sum S_a \frac{i_a^2}{v} \\
\text{Coma} &= k \sum S_a i_a i_b \\
\text{Astigmatism} &= k \sum S_a i_a^2 \\
\text{Field Curvature} &= k \sum \frac{c}{n} \left( \frac{1}{n'} - 1 \right) H \\
\text{Distortion} &= k \sum S_a i_a i_b - \frac{H (u_b^2 - u_a^2)}{
where
n &= \text{index of refraction} \\
u_a &= \text{angle of the axial ray} \\
u_b &= \text{angle of the chief ray} \\
\text{Primed values are final values through the optical system.}}
\end{align*}
\]

\[
\begin{align*}
S_a &= n \left( \frac{n}{n'} - 1 \right) y_a (i_a + u_a) \\
S_b &= n \left( \frac{n}{n'} - 1 \right) y_b (i_b + u_b) \\
y_a &= \text{height of axial ray} \\
y_b &= \text{height of chief ray} \\
c &= \text{curvature of lens surface} \\
i &= \text{angle of incidence on a surface for the axial or chief ray} \\
i &= y c + u \\
H &= n (u_a y_b - u_b y_a)
\end{align*}
\]

\[
\begin{align*}
\text{Transverse Axial Chromatic Aberration} &= k \sum 2 y_a n i_a \left( \frac{dn}{n} - \frac{dn'}{n'} \right) \\
\text{Transverse Chromatic Aberration} &= k \sum 2 y_a n i_b \left( \frac{dn}{n} - \frac{dn'}{n'} \right)
\end{align*}
\]
Diagram of Lens Design

Incorpor Ray from Questar

Image is 8.73 cm from last lens

56
Michael J. Wahlstedt was born on 15 August 1950 in Chicago, Illinois. He graduated from high school in Addison, Illinois in 1968 and attended Northern Illinois University, DeKalb, Illinois from which he received the degree of Bachelor of Science in Physics in June 1972. After graduation, he received a commission in the USAF through the OTS program, and completed navigator training in 1973. He served as a KC-135 navigator and instructor navigator with the 906th Air Refueling Squadron at Minot Air Force Base, North Dakota and as an instructor navigator and wing command post controller in the 416th Bomb Wing at Griffiss Air Force Base, New York. He entered the Air Force Institute of Technology, School of Engineering in June 1981.

Permanent Address: 115 Rose Avenue
Addison, Illinois 60101
**Title:** A Study of Computer-Aided Geometric Optical Design

**Authors:** Michael J. Wahlstedt

**Performing Organization Name and Address:**
Air Force Institute of Technology

**Report Date:**

**Number of Pages:** 57

**Distribution Statement:** Approved for public release; distribution unlimited

**Supplementary Notes:**
Approved for public release: IAW AFR 100-77.

**Keywords:** Optical Design, Computer

**Abstract:** Computer-aided geometric optical design was investigated. The definition of these and optical design procedures were studied. Available computer programs were compared and Falcon was selected. Test problems were evaluated with Falcon to aid in understanding.
BLOCK 20 CONTINUED

COMPUTERIZED OPTICAL DESIGN PROCEDURES.

AN 18 INCH LONG OPTICAL SYSTEM WITH A 50 METER EFFECTIVE
FOCAL LENGTH WAS DESIGNED FOR THE AIR FORCE WEAPONS LABORATORY.
THIS OPTICAL SYSTEM WOULD IMAGE, ON A SOLID STATE DETECTOR, AN
INTERFERENCE PATTERN CREATED BY TWO LASER BEAM SAMPLES, TO ALLOW
THE TWO LASERS TO BE LOCKED IN PHASE WITH EACH OTHER.

THE STARTING POINT OF THE DESIGN WAS A QUESTAR 3.5 INCH
TELESCOPE. ADDITIONAL OPTICAL ELEMENTS WERE ADDED AND THE DESIGN
IMPROVED UNTIL THE DESIRED TOLERANCES WERE MET OR EXCEEDED.