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NAVAL POSTGRADUATE SCHOOL
Monterey, California

THESIS

AIRCRAFT STATE ESTIMATION FOR A GROUND DIRECTED BOMBING SYSTEM

by

John A. Jauregui

December 1982

Thesis Advisor: H. A. Titus

Approved for public release, distribution unlimited
**Title:** Aircraft State Estimation for a Ground Directed Bombing System

**Author:** John A. Jauregui

**Abstract:**
The performance of a modified linear Kalman filter with adaptation is compared with that of a common adaptive alpha-beta filter for state estimation of a pilot controlled, ground directed bombing system. Of particular concern is the accuracy and response of the alternative filters when the aircraft conducts random maneuvers in the vicinity of the target. The desirability of including deterministic forcing in the filter model is discussed and a technique utilizing an adaptive Kalman identifier to establish the pilot response to ground control heading commands is presented.
Aircraft State Estimation for a Ground Directed Bombing System

by

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Captain, United States Marine Corps
B.S., United States Naval Academy, 1973

Submitted in partial fulfillment of the requirements for the degree of

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from the

NAVAL POSTGRADUATE SCHOOL
December 1982

Author: John A. Jauregui
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Thesis Advisor: Daniel J. Collins

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I. INTRODUCTION

The ground directed bombing system simulated is conceptually similar to the USMC AN/TPB-1D produced by Sierra Research Corporation. This system tracks the tactical aircraft with a conical scan radar, filters the noisy radar data, calculates heading commands based on the smoothed trajectory, and transmits this guidance information to the pilot via the Tacan navigation system located in the cockpit. This heading information directs the pilot to fly the aircraft so that its ground track vector passes through the calculated ordnance release point. Audio signals transmitted to the pilot designate the bomb release time.

In an operational environment such a system would possibly be required to track and guide aircraft conducting significant maneuvers enroute to the target. These maneuvers would most likely be dictated by tactical doctrine or by the threat environment.

With this operational model in mind an appropriate concern is the capability of a ground directed bombing system to track and guide an aircraft exhibiting random maneuvers until moments prior to bomb release. It is obvious that the smoothing filter should be able to respond
to maneuvers, yet settle quickly to an accurate solution as the pilot steadies the aircraft. These conflicting requirements are investigated utilizing both alpha-beta and Kalman filtering techniques.

Similar filtering techniques were utilized for a simulation of the USMC AN/TPQ-27, [1]. However in that ground directed bombing system, control signals were directly coupled to the aircraft aerodynamic controls, thus eliminating the uncertainty of pilot response in the control loop. In that study significant improvements in filter response and accuracy were realized by including deterministic forcing autopilot commands in the state estimation via the Kalman filters.
II. GROUND DIRECTED BOMBING SYSTEM (GDBS) SIMULATION

A. COORDINATE SYSTEMS

A Cartesian coordinate system was chosen for the aircraft dynamics model and a radar centered inertial reference frame. In this reference system the y-axis is directed toward true North and the x-axis toward the east. The z-axis is directed away from the center of the earth. All radar measurements of aircraft position, however, are obtained in spherical polar coordinates, i.e. slant range (R), azimuth angle from true North (A), and elevation angle (E) from the horizontal. Figure 2.1 shows these coordinate systems and their transformation relationship. Wind is modeled with a constant velocity in the x-y plane with no vertical component.

Curvature of the earth and the fact that pilot heading information is oriented to magnetic north, were not taken into account in the simulation. Also bomb ballistics and therefore coriolis forces were not included in the model. The aircraft is simply directed to a release point in space, which in a full simulation would be derived from the projected bomb trajectory, ballistic winds, coriolis forces, and a number of other factors, all of which are important to
Fig. 2.1. Model Simulation Coordinate Systems

\[ x = R \cos(E) \sin(A) \]
\[ y = R \cos(E) \cos(A) \]
\[ z = R \sin(E) \]
the problem as a whole but are not necessarily germane to
the objective of evaluating the response and accuracy of
alternative state estimating filters for a goal oriented
maneuvering bomber. Thus the simulation has been simplified
appropriately.

B. AIRCRAFT DYNAMICS MODEL

The dynamic model of the aircraft for the purpose of
simulation was assumed to be a free inertial \((1/s^2)\) plant
since the bombing profile dictates a constant aircraft
airspeed. The discrete realization of this plant is shown
in (2.1).

\[
X(k+1) = \phi(k+1/k)X(k) + \Delta(k+1/k)U(k) \tag{2.1}
\]

where

\[
\phi(k+1/k) =
\begin{bmatrix}
1 & T & 0 & 0 & 0 & 0 \\
0 & 1 & 0 & 0 & 0 & 0 \\
0 & 0 & 1 & T & 0 & 0 \\
0 & 0 & 0 & 1 & 0 & 0 \\
0 & 0 & 0 & 0 & 1 & T \\
0 & 0 & 0 & 0 & 0 & 1
\end{bmatrix} \tag{2.2}
\]
\[
\begin{bmatrix}
T^{\frac{3}{2}} & 0 & 0 \\
T & 0 & 0 \\
0 & T^{\frac{3}{2}} & 0 \\
0 & 0 & T \\
0 & 0 & 0
\end{bmatrix}
\]

\(\Delta(k+1/k) = \) \hspace{1em} (2.3)

In the controlled mode the aircraft model simulates the pilot responding to heading inputs transmitted from the radar site to the TACAN navigation system in the cockpit. The model is driven by a pilot/aircraft control function similar to that developed in [1] and shown in Figure 2.2. The input is bearing to the target and the output is a bank angle which generates a heading rate that can be transformed into \(x-y\) acceleration components for entry into the dynamic model. It is assumed that in the controlled mode heading changes are made with coordinated turns performed by the pilot in response to heading commands displayed by the TACAN. The pilot/aircraft controller induced \(x-y\) accelerations are depicted in Figure 2.3 and summarized by (2.4) and (2.5) below.

\[
\ddot{x}(k) = V(k) \cos(\psi(k)) \dot{\psi}(k) \] \hspace{1em} (2.4)

\[
\ddot{y}(k) = -V(k) \sin(\psi(k)) \dot{\psi}(k) \] \hspace{1em} (2.5)
$\psi(k)$ and $V(k)$ are aircraft heading and velocity respectively at time $k$, and $\dot{\psi}(k)$ is the heading rate which is derived in (2.6) through (2.9) from the free-body diagram shown in Figure 2.4. The aircraft weight is shown in (2.6) below.

$$W = mg = L \cos \theta$$  \hspace{1cm} (2.6)
Equation (2.7) depicts the centripetal force generated in a turn, where $R$ is the turn radius, $L$ is lift, $V$ is velocity, and $\theta$ is the bank angle.

$$P = m\frac{V^2}{R} = L \sin \theta \quad (2.7)$$

But $\frac{V}{R} = \dot{\psi}$, the turn rate, so

$$P = mV \dot{\psi} = L \sin \theta \quad (2.8)$$
Dividing (2.8) by (2.6) and rearranging terms yields (2.9), which defines $\dot{\psi}$, the aircraft turn rate, as a function of aircraft bank angle $\theta$, and velocity $V$.

$$\dot{\psi} = \frac{g}{V} \tan \theta$$  \hspace{1cm} (2.9)

From [1] the aircraft roll response is assumed to be of the form shown in (2.10) where $\tau$ is the roll response time constant.
\[
\frac{\theta}{\theta_4} = \frac{1}{(sT + 1)} \tag{2.10}
\]

No effort has been made to specifically model pilot delays or response to visual inputs from the TACAN.

In the maneuvering mode the maneuver model described in [2] was used and is shown in Figures 2.5 and 2.6.

![Acceleration Probability Density Model](image)

**Fig. 2.5. Acceleration Probability Density Model**

This was simulated for uncontrolled random flight since the aircraft is assumed to typically move at a constant velocity with turns, evasive maneuvers, and air turbulence interpreted as perturbations upon the constant velocity trajectory. These maneuver perturbations or accelerations
The acceleration in (2.11) is the maximum that can reasonably be expected from the pilot/aircraft in the environment described. \( P_1 \) is that probability assigned to the maximum acceleration \( \pm A \), \( P_0 \) is that probability assigned...
to no maneuver, and the assumed probability distribution between these values is uniform with amplitude \( P(a) \). Equation (2.12) is the correlation function which yields an acceleration time duration, \( R(a) \), which is based on the magnitude of the acceleration \( |a| \). \( M \) and \( t \) are simply correlation factors which determine how the \( R(a) \) varies over the range of possible acceleration amplitudes. From this model one can see how the duration of a high G maneuver for threat avoidance would be considerably less than for a low G clearing turn.

C. GDBS MODEL IMPLEMENTATION

The GDBS model was simulated on an IBM 370 in single precision Fortran. Figure 2.7 shows the basic flow diagram for the computer program, which implements this simulation model. The module labeled 'State Estimation Filter' represents those filter algorithms discussed in the next chapter.
Set Program Const. and Initialize Variables

Initialize A/C State Vector \((X,Y,Z)\)

Radar Measurements in Polar Coord. \((R,A,E)\)

Add White, 0-Mean Gaussian Noise to Polar Measurements

Transform Polar Coord. to Radar Cartesian Coord.

State Estimation Filter \((\hat{\dot{x}},\hat{\dot{y}},\hat{\dot{z}})\)

Target Bearing Provided to A/C Based on State Est.

Control Law Provides X-Y Acceleration to Dynamic Model

Is Maneuvering Permitted?

Yes: Maneuver Law Provides X-Y Acceleration to Dynamic Model

No

Fig. 2.7. GDBS Model Flow Diagram
III. AIRCRAFT STATE ESTIMATION

A. BACKGROUND

A great deal has been written on the theory and application of estimation filters. In particular, [2] provides a good overview of several such filters, including the alpha-beta and Kalman filters, and compares their relative performance not only in terms of accuracy and in response, but also in terms of computer implementation costs in computation time and storage overhead.

The general conclusion is that the Kalman filter out-performs an alpha-beta filter of comparable order by about 2 to 1. However, the cost for such performance is increased computer computation time and memory, of the same relative magnitude.

B. GENERAL DESCRIPTION OF THE ALPHA-BETA FILTER

The basic theory of the alpha-beta filter is derived from minimizing the mean square error of the filtered states. A classic analysis of the alpha-beta filter is provided by [3]. The filter recursive equations are summarized below.

\[ x(k/k-1) = x(k-1/k-1) + T \dot{x}(k-1/k-1) \]  
(3.1)
\begin{align*}
  x(k/k) &= x(k/k-1) + \alpha (z(k) - x(k/k-1)) \quad (3.2) \\
  \dot{x}(k/k) &= \dot{x}(k/k-1) + \beta/T (z(k) - x(k/k-1)) \quad (3.3)
\end{align*}

\( x(k/k-1) \) is the predicted position, \( x(k/k) \) is the updated position, \( \dot{x}(k/k) \) is the updated velocity, and \( z(k) \) is the noise contaminated measurement of position at the \( k \)-th interval. \( T \) is the sample rate of the measurement process, \( \alpha \) and \( \beta \) are usually fixed real constants. As pointed out in [4] these alpha-beta equations are analogous to a steady state Kalman filter. For typical parameter values the alpha-beta filter is simply low pass with a heavily damped time response. Thus the filter eliminates not only most high frequency measurement and process noise, but also most maneuver energy from the state estimate.

C. GENERAL DESCRIPTION OF THE KALMAN FILTER

The Kalman filter generates a minimum variance estimate of the plant (aircraft) state vector when the measurement and plant process noise statistics are known and conform to the criteria shown below.

\[ E(\mathbf{v}(k)\mathbf{v}(j)^T) = R(k) \delta(k,j) \quad (3.4) \]
\[ E(\Delta(u(k)w(j)^T)\Delta^T) = Q(k)\delta(k, j) \]  
\[ E(v(k)w(j)^T) = 0 \quad \text{for all } k, j \]  

where \[ \delta(k, j) = \begin{cases} 
0 & k \neq j \\
1 & k = j 
\end{cases} \]

A linear time-invariant system is assumed, as in the discrete model representation shown in Figure 3.1. \( X(k) \) represents the \((n \times 1)\) state vector, \( Z(k) \) the \((m \times 1)\) output vector; \( \phi(k) \) the state transition matrix, \( H(k) \) the \((m \times n)\) observation matrix, \( W(k) \) state excitation or process noise, and \( V(k) \) the measurement noise.

The Kalman filter recursion algorithm is summarized below.

\[ X(k+1) = \phi(k)X(k) + \Delta(k)W(k) \]  
\[ Z(k) = H(k)X(k) + V(k) \]  
\[ \hat{x}(k/k-1) = \phi(k, k-1)\hat{x}(k-1/k-1) + \Delta(k)U(k-1) \]
Fig. 3.1. Dynamic System Model and Discrete Kalman Filter

\[ P(k/k-1) = \phi(k, k-1) P(k-1/k-1) \phi(k, k-1)^T + Q(k) \quad (3.11) \]

\[ G(k) = P(k/k-1) H(k) \left[ H(k) P(k/k-1) H(k)^T + R(k) \right]^{-1} \quad (3.12) \]
\[ \hat{X}(k/k) = \hat{X}(k/k-1) + G(k) [Z(k) - H(k) \hat{X}(k/k-1)] \] (3.13)

\[ P(k/k) = (I - G(k)H(k))P(k/k-1) \] (3.14)

\( \hat{X}(k/k-1) \) denotes the estimate of the state vector \( X(k) \) based on \( (k-1) \) measurements, \( Z(1), Z(2), \ldots, Z(k-1) \).

D. FILTER ORDER CONSIDERATIONS

The filter order is chosen to match as closely as possible the expected plant dynamics of the system being modeled. A first order filter would be expected to estimate a constant velocity trajectory effectively and a second order filter would accordingly observe a trajectory exhibiting constant acceleration. The order is used here in the mathematical sense and refers to the order of the differential equation that defines the filter.

Since the aircraft is known to be constrained to a constant velocity profile as it approaches the release point, it would seem reasonable to select a first order filter for modeling. The aircraft dynamics are anticipated to depart from the first order model enroute to the target thus creating transient errors which must be dealt with by filter adaptation. The alternative to this strategy is to
increase the order of the filter to observe these high energy maneuvers for state estimation. However, the settling time of a first order filter is generally less than that of a second order filter as discussed in [5] and graphically illustrated by the first and second order Kalman gain schedules shown in Figures 3.2 and 3.3 respectively.

![First Order Kalman Gain Schedule](image)

Fig. 3.2. First Order Kalman Gain Schedule
E. FILTER ADAPTATION

1. Background

No matter what the order or the complexity of the filter type selected, it cannot be expected to fully model the aircraft dynamics and process noise covariance. The model is based on a linear time-invariant system and the
process noise is assumed to be stationary, white, and Gaussian. During portions of the flight, particularly as the aircraft approaches the release point, the system dynamics are expected to approximate very closely the assumed model. However, during other portions of the flight the aircraft dynamics are anticipated to depart significantly from the filter model. Certainly the pilot should not be constrained to behave in a manner consistent with the model, if the environment dictates otherwise.

Since the pilot/aircraft dynamics are not fully modeled, the suboptimal filter that results might be expected to diverge, e.g. the error covariance generated by the filter and the actual error covariance become inconsistent. The desire is for the filter to transition smoothly between accurate estimations, when the aircraft dynamics conform to those assumed for the model, and less accurate estimations, when the aircraft dynamics do not agree with the model. An adaptive filter realizes this smooth transition by adjusting filter parameters to vary the filter bandwidth to allow a more consistent match between the calculated and actual filter error covariances.
In the case of Kalman filter adaptation, the calculated error covariance becomes a function of the measured data indirectly by making the filter parameters dependent on the observed aircraft motion. The adaptive techniques for the alpha-beta filter are similar in concept. In either case the adaptive process is conceptually straightforward; first divergence is detected, then the filter parameters are modified.

In the case of a ground directed bombing system of the type considered here, the aircraft's behavior could depart from the filter model in a random fashion when the pilot maneuvers in response to a random event in the environment, or deterministically when he responds to target bearing inputs from the ground radar. These two situations may be treated separately or together for the purpose of filter adaptation. To treat them separately, as random and deterministic processes, requires knowledge of the pilot/aircraft response to target bearing inputs. In the case of the ground directed bombing system described in [1] this transfer function was known quite accurately since the input signals from the ground radar were directly coupled to the aircraft aerodynamic controls, with the uncertainties of
pilot response isolated from the control loop. With that information the deterministic forcing could simply be integrated into the filter model. Unfortunately such is not the case for this simulation. A unique approach to this problem will be discussed later. The alternative approach is to consider both processes to be random and proceed from that assumption.

2. Innovations Statistics and Maneuver Detection

As described in [5], the innovations or residual sequence of a filter can be observed in order to detect a bias that would indicate divergence of the state estimate from the true state. This is given by

\[ \nu(k/k-1) = Z(k) - H(k)\hat{X}(k/k-1) \]  

(3.15)

By substituting for \( Z(k) \) from (3.9), the measurement model, we see that

\[ \nu(k/k-1) = v(k) - H(k)\epsilon(k/k-1) \]  

(3.16)

where

\[ \epsilon(k/k-1) = \hat{X}(k/k-1) - X(k) \]  

(3.17)

Taking expected values, we find that

\[ E(\nu(k/k-1)) = 0 \]  

(3.18)
\[ E(\nu(k/k-1)\nu^T(k/k-1)) = R(k) + H(k)P(k/k-1)H^T(k) \quad (3.19) \]

Thus by referring to the model statistics for \( R(k) \) and \( P(k/k-1) \), it becomes clear that when the system conforms to the system model, i.e. the filter is operating optimally, the innovation sequence should be zero-mean Gaussian with variance

\[ \sigma^2_v = \sigma^2(k) + P(k/k-1) \quad (3.20) \]

One approach to adaptation considers the correlation of the innovation sequence, where the autocovariance

\[ Q_\nu(i) = E(\nu(k/k-1)\nu(k-i/k-1-i)^T) \quad (3.21) \]

should vanish for \( i \neq 0 \). Based on these statistics, maneuver detection can be realized by observing the signs of the innovation sequence. The probability that a given sequence is either positive or negative is

\[ P(0 > \nu > 0) = \frac{1}{1-N} \quad (3.22) \]

Another approach utilizes (3.23)

\[ |\tilde{\nu}_N(k)| > C \sigma_{\nu_N}(k) \quad (3.23) \]

to declare a maneuver when \( \tilde{\nu}_N \) exceeds a specified value, usually two or three standard deviations of \( \sigma_{\nu_N} \).
Subsequent to maneuver detection, the filter parameters must be modified to correct filter divergence. Reference [5] summarizes numerous techniques, some being quite complex and computation intensive. The strategy chosen for this simulation was simply to reset the error covariance in response to a detected maneuver. In the situation of a ground directed bombing system, the resulting cost of a false detection becomes high only as the aircraft approaches the release point. This cost can be reduced by disabling filter adaption within a specified time to go.

Still another approach attempts to adapt the filter bandwidth by adjusting $Q(k)$ in (3.11). This approach, investigated in [1] and [6] calculates $Q(k)$ by

$$Q(k) = a \Delta l(k) \Delta l(k) + b \Delta l(k-1) \Delta l(k-1) \quad (3.24)$$

where $a$ and $b$ are determined by data analysis, and

$$\Delta l(k) = \dot{x}(k/k) - \dot{x}(k/k-1) \quad (3.25)$$

A variation of this technique that looks at only the change in the highest order state is investigated in the Kalman filter simulation.

Most of the discussion thus far concerning filter adaptation has been directed toward Kalman filters. Most
approaches to adapting alpha-beta filter simply open the bandwith by switching to a different set of parameters when a maneuver has been detected. Reference [4] discusses an adaptation scheme that is more nearly optimal in the sense of covariance matching. However, for the sake of comparison the simple parameter switching technique is implemented in the alpha-beta filter simulation subroutine, since that is the approach used in the AN/TPB-1D.

F. ESTIMATION OF PILOT RESPONSE TO TARGET BEARING INPUTS

As discussed in the previous section, aircraft dynamics depart from the filter model randomly when the pilot responds to events in the environment and deterministically when he responds to target bearing inputs from the GDBS. If his response to these inputs were known with some degree of certainty, then deterministic forcing might be included in the filter model in a manner similar to that found in [1]. The importance of identifying parameters which define a system so that modern control strategies can be implemented is discussed in [7]. In this case the parameters would be those that describe the pilot/aircraft response to heading inputs.
By using an Autoregressive Moving Average (ARMA) representation for the pilot/aircraft system, and Kalman filtering to process the heading-in (actually heading error from the viewpoint of the pilot), bank angle out data, the coefficients associated with the ARMA equation could be identified. From [7] we know that the pilot/aircraft system can be represented by the ARMA equation

$$\theta(k) = \sum_{j=0}^{m} a_j \theta(k-j) - \sum_{j=1}^{n} b_j \theta(k-j)$$

(3.26)

where the present bank angle output, $\theta(k)$, is a linear combination of past outputs, $\theta(k-j)$, and of past and present heading error inputs, $\theta(k)$. Estimating the coefficients of this ARMA equation can be formulated as an adaptive Kalman identifier, where the heading error and the bank angle are simple functions of the velocity and acceleration state estimates generated by the Kalman state estimation filter previously discussed.

If the $a_j$ and $b_j$ coefficients of the ARMA equation are treated as states of the pilot/aircraft system, then the state vector becomes

$$\begin{bmatrix} a \\ b \end{bmatrix}$$

(3.27)
We assume that these coefficients experience random perturbations so that

\[ a_i(k+1) = a_i(k) + w_i(k) \]
\[ b_j(k+1) = b_j(k) + w_j(k) \]  

(3.28)

Equation (3.26) then becomes

\[ \theta(k) = \sum_{j=0}^{m} a_j \theta(k-j) - \sum_{j=1}^{n} b_j \theta(k-j) + v(k) \]  

(3.29)

where \( w_i(k), w_j(k), v(k) \) are noise processes that have the same statistics described for the Kalman filter earlier.

Combining (3.28) and (3.29) we have

\[
\begin{bmatrix}
    a_i(k+1) \\
    b_j(k+1)
\end{bmatrix} =
\begin{bmatrix}
    a_i(k) \\
    b_j(k)
\end{bmatrix} + w(k)
\]  

(3.30)

The measurement vector is defined,

\[ H(k) = \begin{bmatrix}
    \theta_1(k) & \theta_1(k-1) & \cdots & \theta_1(k-m) \\
    \theta_2(k) & \theta_2(k-1) & \cdots & \theta_2(k-m) \\
    \cdots & \cdots & \cdots & \cdots \\
    \theta_n(k) & \theta_n(k-1) & \cdots & \theta_n(k-m)
\end{bmatrix} \]

(3.31)

From [7] the solution is then formulated as

\[
\begin{bmatrix}
    \hat{\theta}_1(k+1/k) \\
    \hat{\theta}_2(k+1/k) \\
    \cdots \\
    \hat{\theta}_n(k+1/k)
\end{bmatrix} =
\begin{bmatrix}
    I - G(k)H(k)
\end{bmatrix}
\begin{bmatrix}
    \hat{\theta}_1(k/k-1) \\
    \hat{\theta}_2(k/k-1) \\
    \cdots \\
    \hat{\theta}_n(k/k-1)
\end{bmatrix} + G(k) \theta(k)
\]

(3.32)

where

\[ G(k) = P(k/k-1)H^T(k) \left[ H(k)P(k/k-1)H^T(k) + R \right]^{-1} \]

(3.33)
\[ P(k/k) = P(k/k-1) - G(k) H(k) P(k/k-1) + Q \]  

(3.34)

\[ Q = E \left\{ \begin{bmatrix} v_i(k) \\ v_j(k) \end{bmatrix} \begin{bmatrix} v_i(k) & v_j(k) \end{bmatrix} \right\} \]  

(3.35)

\[ R = E \{ v(k) v(k) \} \]  

(3.36)

and

\[ P(k/k-1) = E \left\{ \begin{bmatrix} \frac{a(k)}{b(k)} - \frac{\hat{a}(k/k)}{\hat{b}(k/k)} \\ \frac{\hat{a}(k/k)}{\hat{b}(k/k)} \end{bmatrix} \begin{bmatrix} \frac{a(k)}{b(k)} - \frac{\hat{a}(k/k)}{\hat{b}(k/k)} \end{bmatrix}^T \right\} \]  

(3.37)

 Initialization of the states(coefficients) and the error covariance would be similar to that discussed in the next section.

G. FILTER IMPLEMENTATIONS

Three separate filter subroutines were developed to simulate the filtering of raw radar data generated by the ground radar of the bombing system previously described. All three filter configurations are oriented in the three dimensional Cartesian coordinate system described in Chapter 2 since the aircraft dynamics are assumed to be more nearly linear and well behaved than in the polar coordinate system in which the measurements are generated. This disparity
between the measurement reference frame and the model dynamics reference frame results in a basic nonlinearity when transformations are required from one frame to the other. Probably a better coordinate frame for modeling aircraft motion would be one that translates with the aircraft and is oriented along the velocity vector. Such a coordinate system was found to be very awkward and difficult to implement, especially considering the problem of the transformation nonlinearity just mentioned.

The first of these filters, designated ALFBTA, is a simple sixth order alpha-beta filter with the parameter switching adaptation technique described in the previous section. Adaptation is initiated when a heading rate of 1 degree per second is observed for period of 5 seconds or more. The second and third subroutines implement sixth(KALMN1) and ninth(KALMN2) order Kalman filters respectively. Two separate adaptive techniques, which were described in the previous section, are included with each of these filters. The first of these adaptive algorithms, designated ADPTV1, adjusts the Q(k) matrix from changes computed in the highest order estimate. The second algorithm, designated ADPTV2, simply resets the covariance
of error matrix $P(k)$ when a bias is detected in the innovations sequence for more than one second. As mentioned before, the difference between the sixth and ninth order filters is that the former do not estimate the acceleration states of the aircraft. The cost of this additional information provided by the ninth order filter is more computation time and computer memory.

The aircraft model is formed by defining a three-dimensional Cartesian state vector

$$X_3 = [x \ y \ z]^T$$

(3.38)

where $x$, $y$, and $z$ are each one-dimensional two element state vectors (position and velocity) for ALFBTA and KALMN1, and three element state vectors (position, velocity, and acceleration) for KALMN2. The state prediction equations are given by (3.1) for the alpha-beta filter and (3.10) for both Kalman filters. For the Kalman filter configuration

$$\Phi(k) = \begin{bmatrix}
\phi & 0 & 0 \\
0 & \phi & 0 \\
0 & 0 & \phi 
\end{bmatrix}$$

(3.39)
\[ \Delta(k) = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \]  

(3.40)

where \( \phi(k) \) and \( \Delta(k) \) are defined by (3.41) and (3.42) for KALMN1 and (3.43) and (3.44) for KALMN2.

\[ \phi(k) = \begin{bmatrix} 1 & T \\ 0 & 1 \end{bmatrix} \]  

(3.41)

\[ \Delta(k) = \begin{bmatrix} T^2/2 \\ T \end{bmatrix} \]  

(3.42)

\[ \phi(k) = \begin{bmatrix} 1 & T & T^2/2 \\ 0 & 1 & T \\ 0 & 0 & 1 \end{bmatrix} \]  

(3.43)

\[ \Delta(k) = \begin{bmatrix} T^3/6 \\ T^2/2 \\ T \end{bmatrix} \]  

(3.44)

\( \phi \) and \( \Delta \) are in general functions of \( k \), however for this simulation they are not since a constant data rate is assumed and no extended predictions are required. The \( U(k) \) matrix would be utilized to include deterministic forcing in the model, if this information were available as in [1].
However, since an adaptive Kalman identifier was not implemented to estimate the pilot/aircraft response, no attempt was made to include deterministic forcing in the model.

1. **Kalman Filter Covariance of Measurement Noise, \( R(k) \)**

Kalman filter theory assumes linear relationships among measurements and states as can be seen from (3.9). Since aircraft motion is modeled in a Cartesian reference frame and measurements are generated in a polar reference frame, the resulting relationships among the states and measured values are nonlinear, as can be seen from the transformation equations shown below.

\[
\begin{align*}
  x &= R \cos(E) \sin(A) \quad (3.45) \\
  y &= R \cos(E) \cos(A) \quad (3.46) \\
  z &= R \sin(E) \quad (3.47)
\end{align*}
\]

Using these polar/Cartesian transformations to nonlinearly combine the polar observations, three-dimensional Cartesian measurements are generated from (3.3) to form (3.48) below.
\[
\begin{bmatrix}
    z_x(k) \\
    z_y(k) \\
    z_z(k)
\end{bmatrix} =
\begin{bmatrix}
    R \cos(E) \sin(A) \\
    R \cos(E) \cos(A) \\
    R \sin(E)
\end{bmatrix}
+ \begin{bmatrix}
    v_x(k) \\
    v_y(k) \\
    v_z(k)
\end{bmatrix}
\tag{3.48}
\]

where the observation matrix for KALN1 is
\[
H(k) =
\begin{bmatrix}
    1 & 0 & 0 & 0 & 0 \\
    0 & 1 & 0 & 0 & 0 \\
    0 & 0 & 0 & 1 & 0
\end{bmatrix}
\tag{3.49}
\]

and for KALN2 is
\[
H(k) =
\begin{bmatrix}
    1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
    0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\
    0 & 0 & 0 & 0 & 1 & 0 & 0 & 0
\end{bmatrix}
\tag{3.50}
\]

In order to compute the measurement error variance, it is necessary to first linearize the measurement error. Differentiating equation (3.48) with respect to each of the measurement variables yields (3.51), where \( s \) and \( c \) represent \( \sin \) and \( \cos \) respectively.
\[
J(x) =
\begin{bmatrix}
    sAEC & rCAE & rsASE & -rsASe & -rSASE & -rsASE & rCE \\
    cACE & -rsAEC & -rsASE & rSASE & -rSASE & rSASE & 0
\end{bmatrix}
\tag{3.51}
\]

Thus we find that the linearized Cartesian errors can be expressed as
\[
\begin{bmatrix}
    v_x \\
    v_y \\
    v_z
\end{bmatrix}
= J(x)
\begin{bmatrix}
    v_R \\
    v_A \\
    v_E
\end{bmatrix}
\tag{3.52}
\]
Therefore assuming no cross correlation of polar errors, the linearized Cartesian measurement error covariance matrix is

\[ R_c(k) = J(x) R_p(k) J(x)^T \]  

(3.53)

The diagonal terms of \( R(k) \) are

\[
R(1,1) = r^2 \left( \sigma_e^2 s^2 + \sigma_A^2 c^2 + \sigma_c^2 e^2 \right) \\
R(2,2) = r^2 \left( \sigma_e^2 c^2 + \sigma_A^2 s^2 + \sigma_c^2 e^2 \right) \\
R(3,3) = \sigma_E^2 e^2 + \sigma_R^2 s^2
\]

(3.54) \quad (3.55) \quad (3.56)

The off-diagonal elements are

\[
R(1,2) = r^2 \sigma_E^2 sE c^2 sA + (\sigma_R^2 - r^2 \sigma_A^2) \sigma_c^2 e^2 sA \\
R(1,3) = (\sigma_R^2 - r^2 \sigma_E^2) sE cEsA \\
R(2,3) = (\sigma_R^2 - r^2 \sigma_E^2) sE cE cA
\]

(3.57) \quad (3.58) \quad (3.59)

and due to the symmetry of \( R(k) \)

\[
R(2,1) = R(1,2) \\
R(3,1) = R(1,3) \\
R(3,2) = R(2,3)
\]

(3.60) \quad (3.61) \quad (3.62)

It should be noted that the \( R(k) \) matrix is not constant since it depends on range, azimuth, and elevation.

2. Filter Initialization

All three filters were initialized with reasonable state values for position, velocity, and acceleration on the first pass through the filter, since it is assumed that
the GDBS has maintained a good track for sometime prior to the final leg to the target. Consequently the covariance of error for the initial state prediction vector is not set to an arbitrarily large number such as $10^6$, as is often done when there is little confidence in the initial state values. Instead $10^3$ is used since it is more consistent with the covariance of error in good initial state values. This simulated pass from some other tracking filter to the filter of interest is realistic and reduces the settling time.

Program constants and constant array calculations for the Kalman $H$, $\phi$, $\Delta$, and $Q$ matrices are set up on this first iteration. The process noise $W$ is set at an arbitrarily small number to ensure that the gain matrix will not converge to zero and accentuate divergence problems.
IV. PRESENTATION OF RESULTS

A. QUALITATIVE OBSERVATIONS

Seven different filter configurations were ultimately implemented and evaluated with the GDBS simulation. These configurations are discussed below and assigned alpha-numeric symbols for ease of reference.

The sixth order alpha-beta filter, ALFETA(AB) was implemented with the simple parameter switching technique outlined in Chapter 3. However, the heading rate maneuver detection process proved to be too insensitive to slow maneuvers and was therefore augmented with a trigger that reset filter parameters when the heading error exceeded 3 degrees. The results of this change proved to be worthwhile, as will be shown later.

The next three filters are variations of the sixth order KALMN1 filter. (K10) is KALMN1 without adaptation, (K11) is KALMN1 with the process noise adaptation scheme for modifying Q(k), and (K12) is KALMN1 adapted by resetting P(k), the error covariance matrix, as discussed in Chapter 3. Filters (K20), (K21), and (K22) are variations of KALMN2 which correspond to the KALMN1 variants described above.
Since the objective was to evaluate the accuracy and response of each filter in the final phase of bomb delivery, the simulation was initialized with the aircraft within 90 seconds of the release point, at a speed of 480 knots, and on a heading within 10 degrees of the target bearing. For each run, after allowing 5 seconds for the filter to settle, Tacan target bearing information was provided to the pilot controller. At 15 seconds elapsed time, filter adaptation was enabled and random maneuvers were begun at 20 seconds. Five separate runs were evaluated for each filter where the random maneuvers were ceased at 80, 60, 50, 40, and 30 seconds prior to arrival at the release point.

The following plots were generated for the case where maneuvers were stopped with a time-to-go of 50 seconds. Figures 4.1 through 4.21 show true and estimated (connected symbols) position, velocity, and acceleration trajectories as functions of time for all filter configurations. Note that the target position is designated by a circle on the horizontal trajectory plot. Also representative filter gain schedules are included to show the effects of the particular adaptation process being utilized.
Figures 4.1 and 4.2 indicate the magnitude of the maneuver encountered and the good state estimation qualities of this simple sixth order alpha-beta filter. Notice there is no significant divergence of the estimated trajectory from the true trajectory throughout the run. Figure 4.3 shows the step gain adaptation at the beginning of the run in response to the controlled turn to the target heading. This gain is then reduced after the turn is completed and before the first random maneuver begins, when the gain is again increased.

The trajectory shown in Figure 4.4 contrasts sharply with that shown in 4.1, showing significant filter divergence for the nonadaptive sixth order Kalman filter. The significant lag in velocity state estimation shown in Figure 4.5 results from the convergent gain properties characterized in Figure 4.6.

The performance of K10 changes dramatically when it is made adaptive as shown in Figures 4.7, 4.9, and 4.9 for K11 and Figures 4.10, 4.11, and 4.12 for K12. Figure 4.9 shows continuous gain adjustment in response to perceived changes in the process noise. Figure 4.12 shows the effect of resetting the covariance of error in response to a maneuver.
Figures 4.13, and 4.14 show the improvement in state estimation of the K20 nonadaptive Kalman filter, when the order is increased over that of K10. It is interesting to note that we see significant overshoot in the velocity estimate for the first time. Unlike the filters discussed thus far, K20 provides an acceleration estimate which can be seen in Figure 4.15 and accounts for the sensitivity of the velocity estimate.

Adapting K20 through the noise process technique results in K21 which produces the position, velocity, and acceleration estimates shown in Figures 4.16, 4.17 and 4.18 respectively. Figures 4.19, 4.20, and 4.21 provide the same information for K22, which represents the covariance of error adaptation variant of K20. These last two adaptive filters show little, if any, apparent improvement over the nonadaptive version. This observation is supported in the following section.

B. QUANTITATIVE RESULTS

A single run, for each filter configuration evaluated for each maneuver termination time, is not sufficient to properly determine filter performance over the range of possible maneuver trajectories and measurement noise.
sequences. Therefore, 30 simulation trajectories per filter, per maneuver period, were conducted with different random maneuver and measurement noises sequences generated for each run. The bomb release signal to the pilot was assumed to occur at the closest point of approach (CPA) to the target release point. The average of the resulting CPA's for each 30 test runs are shown in Table I. CPA's greater than 250 feet are classified unsatisfactory and labeled 'U' appropriately.

At a glance it is apparent that the adaptive sixth order alpha-beta filter performs very well in such a dynamic environment, except when maneuvers are continued very close to the target. The nonadaptive sixth order Kalman filter is obviously unsuited by itself, but when made adaptive, performs very well, particularly for the process noise adaptation technique when maneuvers are terminated late in the target run. The ninth order Kalman filter performance, both adaptive and nonadaptive, is comparable to the alpha-beta and adaptive sixth order Kalman filters, but has problems in close due to its longer settling time. Notice that the adaptive variants of the ninth order Kalman have little effect on that filter's performance, as we surmised in the last section.
### TABLE I
Release Point Error Table

<table>
<thead>
<tr>
<th>TTG (sec)</th>
<th>Alpha-Beta Adptv</th>
<th>Kalman (sixth order)</th>
<th>Kalman (ninth order)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Non-Adptv</td>
<td>Adptv</td>
<td>Non-Adptv</td>
</tr>
<tr>
<td>80</td>
<td>AB</td>
<td>K10</td>
<td>K11</td>
</tr>
<tr>
<td>60</td>
<td>20'</td>
<td>46'</td>
<td>38'</td>
</tr>
<tr>
<td>50</td>
<td>43'</td>
<td>U</td>
<td>42'</td>
</tr>
<tr>
<td>40</td>
<td>41'</td>
<td>U</td>
<td>75'</td>
</tr>
<tr>
<td>30</td>
<td>55'</td>
<td>U</td>
<td>54'</td>
</tr>
</tbody>
</table>

Table II shows the relative cost, in computation time and memory, to implement KALMN1 and KALMN2 in relation to ALFBTA.
TABLE II
Relative Computation and Memory Costs

<table>
<thead>
<tr>
<th>Method</th>
<th>Cost</th>
</tr>
</thead>
<tbody>
<tr>
<td>Alpha-Beta (sixth ord)</td>
<td>1</td>
</tr>
<tr>
<td>Kalman (sixth ord)</td>
<td>2.9</td>
</tr>
<tr>
<td>Kalman (ninth ord)</td>
<td>3.4</td>
</tr>
</tbody>
</table>

The only advantage to implementing the most responsive variant of KALMN1 would be to reduce the probability of a GDBS generated abort, due to large predicted bomb impact errors, when maneuvers are carried very close to the release point. Lastly, if the adaptive Kalman identifier proved to be useful in providing deterministic forcing for the Kalman filter model and resulted in improved accuracy and response over the alternatives presented, the cost in computation and memory resources would be even greater than we have seen here.
Fig. 4.1. ALFBTA(AB) Horizontal Position Trajectory
Fig. 4.2. ALFBTA(AB) True and Estimated Y-Velocity

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Fig. 4.3. ALFBTA(AB) Y-Gain Schedule
Fig. 4.4. KALMN1(K10) Horizontal Position Trajectory
Fig. 4.5. KALM1(K10) True and Estimated Y-Velocity
Fig. 4.6. KALMN1(K10) Y-Gain Schedule
Fig. 4.7. KALMN1(K11) Horizontal Position Trajectory
Fig. 4.8. KALMN1(K11) True and Estimated Y-Velocity
Fig. 4.9. KALEM1(K11) Y-Gain Schedule
Fig. 4.10. KALR/H(K12) Horizontal Position Trajectory
Fig. 4.11. KALMN1(K12) True and Estimated Y-Velocity
Fig. 4.12. KALM1(K12) Y-Gain Schedule
Fig. 4.13. KALMN2(K20) Horizontal Position Trajectory
Fig. 4.14. KALMN2(K20) True and Estimated Y-Velocity
Fig. 4.15. KALMN2 (K20) True and Estimated Y-Acceleration
Fig. 4.16. KALMN2(21) Horizontal Position Trajectory
Fig. 4.17. KALMN2(K21) True and Estimated Y-Velocity
Fig. 4.18. KALMN2 (K21) True and Estimated Y-Acceleration
Fig. 4.19. KALMN2(K22) Horizontal Position Trajectory
Fig. 4.20. KALMN2(K22) True and Estimated Y-Velocity
Fig. 4.21. KALMN (K22) True and Estimated Y-Acceleration
**GROUND DIRECTED BOMBING SYSTEM SIMULATION**

**IMPORTANT VARIABLES DEFINED:**

<table>
<thead>
<tr>
<th>Variable</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>TACRXX</td>
<td>State of A/C in雷达坐标系，包括位置和速度。</td>
</tr>
<tr>
<td>TACRRY</td>
<td>State of A/C at time(n) in radar frame.</td>
</tr>
<tr>
<td>TACRRZ</td>
<td>True A/C state(1,2,3) at time(n) in A/C Cartesian coord reference frame.</td>
</tr>
<tr>
<td>TACACX</td>
<td>True A/C state(1,2,3) at time(n) in A/C Cartesian coord reference frame.</td>
</tr>
<tr>
<td>TACACZ</td>
<td>True A/C state(1,2,3) at time(n) in A/C Cartesian coord reference frame.</td>
</tr>
<tr>
<td>TACMXX</td>
<td>Reference frame.</td>
</tr>
<tr>
<td>TACMXY</td>
<td>Estimated A/C state(1,2,3) at time(n) in A/C Cartesian coord reference frame.</td>
</tr>
<tr>
<td>TACMXY</td>
<td>Estimated A/C state(1,2,3) at time(n) in A/C Cartesian coord reference frame.</td>
</tr>
<tr>
<td>TACREZ</td>
<td>Reference frame.</td>
</tr>
<tr>
<td>TACRXX</td>
<td>True A/C position(x) in radar frame.</td>
</tr>
<tr>
<td>TACRXY</td>
<td>True A/C position(y) in radar frame.</td>
</tr>
<tr>
<td>TACRZZ</td>
<td>True A/C position(z) in radar frame.</td>
</tr>
<tr>
<td>ROSFQ</td>
<td>Radar data sample frequency.</td>
</tr>
<tr>
<td>DT</td>
<td>Radar sample interval time.</td>
</tr>
<tr>
<td>N</td>
<td>N-th iteration at sample rate DT.</td>
</tr>
<tr>
<td>TACCHG</td>
<td>True aircraft(AC) heading in radar frame.</td>
</tr>
<tr>
<td>TACVEL</td>
<td>True aircraft velocity in radar frame.</td>
</tr>
<tr>
<td>TTIME</td>
<td>Time to go until next tgt.</td>
</tr>
<tr>
<td>HDGERR</td>
<td>Difference between observed and true heading.</td>
</tr>
<tr>
<td>ELPSTM</td>
<td>Elapsed time in seconds since beginning of simulation.</td>
</tr>
<tr>
<td>SRTMVR</td>
<td>Start maneuvers: A/C maneuvering can begin.</td>
</tr>
<tr>
<td>STPHVR</td>
<td>Stop maneuvers: A/C maneuvering must stop.</td>
</tr>
<tr>
<td>MNVRPIT</td>
<td>Maneuvering pilot.</td>
</tr>
<tr>
<td>CNTLPT</td>
<td>Controlled pilot.</td>
</tr>
<tr>
<td>MODEL</td>
<td>Free inertia model of A/C without wind effects.</td>
</tr>
<tr>
<td>BNKANG</td>
<td>Aircraft bank angle.</td>
</tr>
<tr>
<td>NTIMES</td>
<td>Number of times through simulation loop.</td>
</tr>
<tr>
<td>GTRNG</td>
<td>Range(RNG) from A/C to tgt along desired ground track.</td>
</tr>
<tr>
<td>TGTRXX</td>
<td>Target coord. in radar frame.</td>
</tr>
<tr>
<td>TGTRRX</td>
<td>Target coord. in radar frame.</td>
</tr>
<tr>
<td>TDWRR</td>
<td>True wind direction in radar frame.</td>
</tr>
<tr>
<td>TWRR</td>
<td>True wind velocity in radar frame.</td>
</tr>
<tr>
<td>TDWP</td>
<td>Estimated wind direction in radar frame.</td>
</tr>
<tr>
<td>TWVP</td>
<td>Estimated wind velocity in radar frame.</td>
</tr>
<tr>
<td>TWVXX</td>
<td>True wind X-Y velocity components.</td>
</tr>
<tr>
<td>TWVYY</td>
<td>Estimated wind X-Y velocity components.</td>
</tr>
<tr>
<td>X2SIGM</td>
<td>Process noise variance.</td>
</tr>
<tr>
<td>Y2SIGM</td>
<td>Process noise variance.</td>
</tr>
<tr>
<td>Z2SIGM</td>
<td>Process noise variance.</td>
</tr>
<tr>
<td>PINITL</td>
<td>Error covariance diagonal initial value.</td>
</tr>
<tr>
<td>GX</td>
<td>Filter X,Y,Z gain values at time n.</td>
</tr>
<tr>
<td>GY</td>
<td>Filter X,Y,Z gain values at time n.</td>
</tr>
<tr>
<td>GZ</td>
<td>Filter X,Y,Z gain values at time n.</td>
</tr>
<tr>
<td>XRES</td>
<td>Filter X,Y,Z residue values at time n.</td>
</tr>
<tr>
<td>YRES</td>
<td>Filter X,Y,Z residue values at time n.</td>
</tr>
<tr>
<td>ZRES</td>
<td>Filter X,Y,Z residue values at time n.</td>
</tr>
<tr>
<td>RADRES</td>
<td>Radial residue at time n.</td>
</tr>
</tbody>
</table>

**IMPLICIT REAL(A-H,C-Z), INTEGER(I-N)**
DOUBLE PRECISION DSEEED
COMMON/DOUB|E/DSEEED
COMMON/TMR../N, TACRRX(3,1000), TACC|Y(3,1000), TACCRZ(3,1000)
COMMON/TM4CST/I N,TACACX(3,1000), TACACY(3,1000), TACACZ(3,1000)
COMMON/TG|CST/NT, TACACX(3,1000), TACACY(3,1000), TACACZ(3,1000)
COMMON/TPNWRD/ND TACRRX, TGRRRY, TGRRRZ, TWRRR, TWRVR, EWRRR,
COMMON/PILOT/TA, SRTMV, STPMVR, DT
COMMON/NOISE/TACRMX(1000), TACRY(1000), TACRMZ(1000)
COMMON/FILTER/TACRX(3,1000), TACRY(3,1000), TACREZ(3,1000)
COMMON/PROCES/XX2S|GN, Y2S|GN, Z2S|GN, EMVRRX, EVMRRY, GX(1000), GY(1000), GZ(1000), RAR|E(1000), RXE(1000), YRE(1000), ZRE(1000), W11(1000), W22(1000), W33(1000),
+P(1000), Q, PINI TE
COMMON/PARAM/G, G2, G1, GK1, GK2
DIMENSION X(1000), Y(1000), TGRX(1), TGY(1), TRUACC(1000)
TIME(1000), X1(1000), Y1(1000), E|STACC(1000)

INITIALIZE PARAMETERS:
BNKANG=0.0
DEADZN=0.0
OLDHDE=0.0
K=0
K1=1
K2=2.0
PINI T=1000.
SRTMV=20.0
STPMVR=50.0
IOSSF=8
TTG=999.
N|TIMES=800
SE|C|ORD=3.0
HOCERR=0.0
G=32.174049
G2=16.087007
GK1=3.5
GK2=3.5
PI=3.141592654
OL|DHDE=0.0
DSEEED=1.0C0
RS|FGMA=100.
ASS|GMA=0.001
ES|GMA=0.001
KFILT=0
KADAP=0
X2S|GN=1.
Y2S|GN=1.
Z2S|GN=1.

READ INITIAL AIRCRAFT STATES IN AC REF. FRAME.
WRITE(6,6)
READ(5,1) TACACX(1,1), TACACY(1,1), TACACZ(1,1)
READ(5,1) TACACX(2,1), TACACY(2,1), TACACZ(2,1)
READ(5,1) TACACX(3,1), TACACY(3,1), TACACZ(3,1)

OBTAIN X & Y WIND COMPONENTS IN RADAR FRAME & ADD TO CORRESPONDING A/C INITIAL VELOCITY STATES TO OBTAIN INITIAL AIRCRAFT STATES IN RADAR REFERENCE FRAME.
READ(5,2) TGT,RRX, TGT,RRY, TGT,RRZ, TWD,RR, TWVR, R, EWD,RR, EWVR, R
TWR, RX=TWT,RR+5IN(TWD,RR+PI/180.0)1.68705556
TWR, RY=TWT,RR*CO, S(TWD,RR+PI/180.0)1.68705556
EWVR, R=EWVR*5IN(EWD,RR+PI/180.0)1.68705556
EWVR, R=EWVR*CO, S(EWD,RR+PI/180.0)1.68705556
DO 41 I=1,3
IF (I.NE. 2) GO TO 3
TACRRX(I,1)=TACACY(I,2)+TWR, RX
TACRRY(I,1)=TACACY(I,2)+TWR, RY
TACRRZ(I,1)=TACACZ(I,2)
GO TO 41
3 CONTINUE
TACRRX(I,1)=TACACY(I,1)
TACRRY(I,1)=TACACY(I,1)
TACRRZ(I,1)=TACACZ(I,1)
41 CONTINUE

WRITE(6,4)
WRITE(6,6) TACRRX(1,1),TACRRY(1,1),TACRRZ(1,1)
WRITE(6,10) TACRRX(2,1),TACRRY(2,1),TACRRZ(2,1)
WRITE(6,14) TWD,RR,TWVR, R
WRITE(6,16) R, SIGMA, A, SIGMA, E, SIGMA
WRITE(6,18) X, SIGMA, Y, SIGMA, Z, SIGMA
WRITE(6,20) PINITL
WRITE(6,21) KFILTR
WRITE(6,22) KADAPT
WRITE(6,24) STPMVR
WRITE(6,26) STPMVR
WRITE(6,30)
WRITE(6,53)
WRITE(6,60)
WRITE(6,63)
WRITE(6,65)

DETERMINE RADAR SAMPLE INTERVAL (CT).
DT=1.0/FLOAT(IRD,FSQ)

PERFORM SIMULATION LOOP N-TIMES.
DO 900 N=1,N,TIMES
ELPSM=(N-1)*DT
TIME(N)=ELPSM
NOW CALL PLRNOI TO TRANSFORM RADAR CARTESIAN COORDS
IN POLAR, ADD WHITE NOISE, AND RETRANSFORM INTO RDR
CARTESIAN COORDS IN PREPARATION FOR FILTERING.
CALL PLRNOI(RSIGMA,ASIGMA,ESIGMA,RANGE,AZMITH,
+ELEV,N)

IF (KFILTR.NE.0) GO TO 45
CALL ALF,RAD1,HOGERR,TTG1
GO TO 59
45 CONTINUE
IF (KFILTR.NE.1) GO TO 46
CALL KALMN1(KADAPT,DT,RSIGMA,ASIGMA,ESIGMA,RANGE,
+AZMITH,ELEV,N,TTG)
GO TO 59
46 CONTINUE
CALL KALMN2(KADAPT,DT,RSIGMA,ASIGMA,ESIGMA,RANGE,
+AZMITH,ELEV,N,TTG)
59 CONTINUE
75
NOW CALL TACAN SUBROUTINE TO OBTAIN HEADING TO FLY TO TGT.
CALL TACAN(TACHDG,DSROGT,TACVEL,TTG,GTRNG,HDGERR)

IF (N=.GT. 800) GO TO 580
IF (MOD(N-1,8).NE.0) GO TO 550
WRITE(6,75) ELPSMT,TACRX(1,N),TACRY(1,N),
+ TACRZ(1,N),TACRX(2,N),TACRY(2,N),TACRZ(2,N),
+ TACRX(3,N),TACRY(3,N),TACRZ(3,N),TACRX(1,N),TACRY(1,N),TACRZ(1,N),
+ TACRX(2,N),TACRY(2,N),TACRZ(2,N),TACRX(3,N),TACRY(3,N),TACRZ(3,N),
WRITE(6,78) TACREX(1,N),TACREY(1,N),TACREZ(1,N),
+ TACREX(2,N),TACREY(2,N),TACREZ(2,N),
+ TACREX(3,N),TACREY(3,N),TACREZ(3,N)
WRITE(6,76) TIG,TACRX(N),TACRY(N),TACRZ(N)
+,HDGERR,DSROGT,TACHDG,TACVEL,GTRNG
WRITE(6,78) GX(N),GY(N),GZ(N),XRES(N),YRES(N)
+,ZRES(N),W11(N),W22(N),W33(N)
WRITE(6,78) P(N,1),P(N,2),P(N,3),P(N,4),
+P(N,5),P(N,6),P(N,7),P(N,8),P(N,9)
550 CONTINUE
580 CONTINUE

NO CONTROL INPUT UNTIL FILTER SETTLES.
IF (ELPSMT .LT. 5.) GO TO 800
DETERMINE CONTROL INPUT TO AC.

NO CONTROL INPUT IF TTG IS LESS THAN 3 SECOND OR IF CLOSEST POINT OF APPROACH(CPA) HAS BEEN REACHED.
CALL TTGCPA(N,K,TTG,GTRNG,IFLAG)

IF (((IFLAG) .EQ. 1) .AND. (ABS(HDGERR) .GT. DEADZN
+)) GO TO 600
TACAX(3,N)=0.0
TACAY(3,N)=0.0
TACAZ(3,N)=0.0
GO TO 800

CHOOSE MNVRPT OR CNTLPT RESPONSE BASED ON MAGNITUDE OF HDGERR, AND SRTMVR AND STPMVR CRITERIA.
600 CONTINUE
IF (ABS(HDGERR)) .GT. 523598776) GO TO 700
IF ((ELPSMT .LT. SRTMVR).OR. (TTG .LT. STPMVR))
+GO TO 700
CALL MNVRPT(K1)
GO TO 800

CALL CNTLPT SUBROUTINE IF MANEUVER NOT PERMITTED.
700 CONTINUE
700 CALL CNTLPT(K2,HDGERR)

 Provide MODEL SUBROUTINE AC STATES AT TIME N.
800 CALL XMODEL

MODEL RETURNS STATES IN AC FRAME FOR TIME N+1.
ADD WIND EFFECTS OVER DT TO OBTAIN AC STATES IN RADAR FRAME.
CALL WDEFF(TWVRRX,TWVRRY,DT)
900 CONTINUE
STOP
1 FORMAT(3F10.0)
2 FORMAT(7F10.0)
4 FORMAT(5X,'GCBG SIMULATION')
6 FORMAT(///2X,'INITIAL CONDITIONS:')
8 FORMAT(///5X,'A/C INITIAL POSITION IN RADAR REFERENCE
 +SYSTEM(F) = *3(10X,F10.3)'/)
10 FORMAT(///5X,'A/C INITIAL VELOCITY IN RADAR REFERENCE
 +SYSTEM(F) = *3(10X,F10.3)'/)
12 FORMAT(///5X,'TRUE WIND = *10X,F10.3,///X,F9.3)'
14 FORMAT(///5X,'EST. WIND = *10X,F10.3,///X,F9.3)'
16 FORMAT(///5X,'MEASUREMENT NOISE VARIANCES(R(FT),A(RAD),
 +F(RAD)) = *10X,F10.3)'
18 FORMAT(5X,'PROCESS NOISE VARIANCES(X,Y,Z) = *,16X,
 +3(10X,F10.3)'/)
20 FORMAT(5X,'ERROR COVARIANCE DIAGONAL = *,10X,F10.3)'
21 FORMAT(5X,'FILTER DESIGNATION = *,14X,11)
22 FORMAT(5X,'FILTER ADAPTATION = *,5X,II)'
24 FORMAT(5X,'START MANEUVER = *,10X,F10.3)'
25 FORMAT(5X,'STOP MANEUVER = *,10X,F10.3)'/
53 FORMAT('0',18X,'TACRE1',6X,'TACRE2',6X,'TACRE3',6X,
 +TACRE4',6X,'TACRE5',6X,'TACRE6',6X,'TACRE7',6X,
 +TACRE8',6X,'TACRE9',6X,'TACRE10',6X,'TACRE11',6X,
 +TACRE12',6X,'TACRE13',6X,'TACRE14',6X,'TACRE15',6X,
 +TACRE16',6X,'TACRE17',6X,'TACRE18',6X,'TACRE19',6X,
 +TACRE20',6X,'TACRE21',6X,'TACRE22',6X,'TACRE23',6X)
55 FORMAT('0',18X,'TACRM1',6X,'TACRM2',6X,'TACRM3',6X,
 +TACRM4',6X,'TACRM5',6X,'TACRM6',6X,'TACRM7',6X,
 +TACRM8',6X,'TACRM9',6X,'TACRM10',6X,'TACRM11',6X,
 +TACRM12',6X,'TACRM13',6X,'TACRM14',6X,'TACRM15',6X,
 +TACRM16',6X,'TACRM17',6X,'TACRM18',6X,'TACRM19',6X,
 +TACRM20',6X,'TACRM21',6X,'TACRM22',6X,'TACRM23',6X)
75 FORMAT('0',18X,'TACRM1',6X,'TACRM2',6X,'TACRM3',6X,
 +TACRM4',6X,'TACRM5',6X,'TACRM6',6X,'TACRM7',6X,
 +TACRM8',6X,'TACRM9',6X,'TACRM10',6X,'TACRM11',6X,
 +TACRM12',6X,'TACRM13',6X,'TACRM14',6X,'TACRM15',6X,
 +TACRM16',6X,'TACRM17',6X,'TACRM18',6X,'TACRM19',6X,
 +TACRM20',6X,'TACRM21',6X,'TACRM22',6X,'TACRM23',6X)
77 CONTINUE STOP
SUBROUTINE TTGCPA(NKPT,G,TTG,RNG,IFLAG)
    IF (TTG.GT.3.0) OR (K.EQ.0) GO TO 600
GO TO 700

600 CONTINUE
    IF (NKPT.LT.100) OLDRNG=RNG
    IF (DIFFNC.OE.OLDRNG-RNG) GO TO 800
   IF (DIFFNC) 700,800,800
700 IFLAG=1
    K=1
GO TO 900
800 CONTINUE
    OLDRNG=RNG
RETURN

SUBROUTINE MNVRPT(K1)
    DOUBLE PRECISION DSEED
COMMON/DOUBLE/DSEED
COMMON/TMACST/N,TACACX(3,1000),TACACY(3,1000),
+TACACZ(3,1000)
COMMON/PILOT/TB,STPMVR,STPMVR,DT
COMMON/PARAM/G,G2,GK1,GK2
    IF (N.EQ.K1) GO TO 100
CMDACC=0.0
    N=0
PA=GGUBFS(DSEED)
AMXACC=14.0*(PA-.5)
ACCDUR=20.0*EXP(.25*ABS(AMXACC))
100 CONTINUE
    ACCCUR=ACCDUR-DT
    IF (ACCDUR.LE.0.0) GO TO 90
CMDACC=AMXACC*(1.-EXP(-DT/TB))+CMDACC*EXP(-DT/TB)
AACHDG=TRUEDG(TACACX(3,N),TACACY(3,N))
TACACX(3,N)=CMDACC*CCS(AACHDG)*G
TACACY(3,N)=-CMDACC*SIN(AACHDG)*G
TACACZ(3,1000)=0.0

SUBROUTINE CNTLPT(K2,HDGERR)
    GENERATE CONTROLLED PILOT BANK ANGLE.
    PHD1 IS COMPONENT BASED ON ANGLE ERROR HDE.
P HD2 IS COMPONENT BASED ON ANGLE ERROR RATE HDEDOT.
P HD IS DESIRED PILOT GENERATED BANK ANGLE.
    DOUBLE PRECISION DSEED
COMMON/DOUBLE/DSEED
COMMON/TMACST/N,TACACX(3,1000),TACACY(3,1000),
+TACACZ(3,1000)
COMMON/PARAM/G,G2,GK1,GK2
COMMON/PILOT/TB,STPMVR,STPMVR,DT
    IF (N.EQ.K2) GO TO 200
OLDHDE=0.0
VKANG=0.0
Y2=0

200 CONTINUE
SUBROUTINE XMODEL
DOUBLE PRECISION OSEED
COMMON/DDOUBLE/OSEED
COMMON/TMACST/N, TACACX(3,1000), TACACY(3,1000),
+ TACACZ(31000)
COMMON/PILCT/TB, STMVR, STMVR, DT
COMMON/PROCES/X2SIGM, Y2SIGM, Z2SIGM
COMMON/PARAMT/GG1, GG2, G1K, G2K
COMMON/PHI6, PHI6, PHI6, PHI6, PHI6, PHI6
+ XN1(6,1), X1(6,1)
T2=(CT*DT)/2
IF (N .NE. 1) GO TO 160
COMPUTE STATE TRANSITION MATRIX (PHI).
DO 130 I=16
DO 120 J=16
PHI(I,J)=0.0
IF (I .EQ. J) PHI(I,J)=1.0
CONTINUE
120 CONTINUE
PHI(1,2)=DT
PHI(1,4)=DT
PHI(5,6)=DT
COMPUTE DEL MATRIX (DEL)
INITIALIZE & 0.0.
DO 140 I=13,6
DO 130 J=1,3
DEL(I,J)=0.0
CONTINUE
130 CONTINUE
DEL(1,1)=T2
DEL(1,3)=T3
DEL(4,1)=DT
DEL(4,2)=DT
DEL(6,3)=DT
SET UP 'X' MATRIX.
CALL PRODUCT SUBROUTINE TO MULTIPLY PHI & X.
CALL PRCCCT(PHI, X, 6, 1, 6, XI)

CALL PRODUCT SUBROUTINE TO MULTIPLY DEL & U.
CALL PRCCCT(DEL, U, 3, 1, 6, F)

CALL ADD SUBROUTINE TO ADD X & F TO OBTAIN STATES AT TIME (N+1).
CALL ADD(XI, F, 1, 6, XN1)

RETURN
SUBROUTINE ADD(A,B,L,M,C)
DIMENSION A(L),B(M,L),C(M,L)
INITIALIZE "C" to 0.0.
DO 13 I=1,M
DO 13 J=1,L
C(I,J)=0.0
13 CONTINUE
CONTINUE
DO 213 L=1,L
213 CONTINUE
CONTINUE
NOW ADD "A* & "B*.
DO 220 J=1,L
220 CONTINUE
CONTINUE
RETURN
END

SUBROUTINE WDEFF(TWVRRX,TWVRRY,DT)
COMMON/TMACST/NT,TACAX(3,1000),TACACY(3,1000),
+TACAY(3,1000)
COMMON/TMRRST/N,TACRX(3,1000),TACRY(3,1000),
+TACRZ(3,1000)
DO 31 IP=1,N
TACRX(1,IP)=TACAX(1,IP)+DT*TWVRRX*IP
TACRX(2,IP)=TACAX(2,IP)+DT*TWVRRX*IP
TACRX(3,IP)=TACAX(3,IP)+DT*TWVRRX*IP
TACRY(1,IP)=TACAX(1,IP)+DT*TWVRRY*IP
TACRY(2,IP)=TACAX(2,IP)+DT*TWVRRY*IP
TACRY(3,IP)=TACAX(3,IP)+DT*TWVRRY*IP
TACRZ(1,IP)=TACAX(1,IP)
TACRZ(2,IP)=TACAX(2,IP)
TACRZ(3,IP)=TACAX(3,IP)
RETURN
END

FUNCTION TRUHDG(XVEL,YVEL)
PI=3.141592654
IF (XVEL) .GT. 31.41592654
31 IF (YVEL) .GT. 31.41592654
41 IF (YVEL) .LT. -31.41592654
51 IF (YVEL) .LT. -31.41592654
61 TRUHDG=PI/2.0 - ATN(YVEL/XVEL)
GO TO 131
71 TRUHDG=-PI/2.0 - ATN(YVEL/XVEL)
GO TO 131
91 TRUHDG=PI/2.0
GO TO 131
101 TRUHDG=PI
GO TO 131
111 TRUHDG=-PI/2.0
GO TO 131
121 TRUHDG=-PI
GO TO 131
131 RETURN
END

THIS SUBROUTINE TRANSFORMS CARTESIAN COORDS INTO
POLAR IN THE RCR REF. FRAME THEN ADDS WHITE NOISE OF
0 MEAN AND VARIANCE, RSIGMA, ASIGMA, AND ESIGMA TO THE
RANGE, AZIMUTH, AND ELEVATION RESPECTIVELY. THESE
CONSTITUTE THE MEASURED POSIT OF THE A/C. THESE NOISY POLAR MEASUREMENTS ARE THEN TRANSFORMED BACK INTO THE RADAR CARTESIAN SYSTEM IN PREPARATION FOR FILTERING.

SUBROUTINE PLRNOI(RSIGMA, ASIGMA, ESIGMA, RNGPNS, AZIPNS, *ELVPNS)
DOUBLE PRECISION DSEEED
COMMON/DDOUBLE/DSEEED
COMMON/TMRAS/N, TACRRX(3, 1000), TACRRY(3, 1000), *+TACRRZ(3, 1000)
COMMON/NOISE/TACRMX(1000), TACRMY(1000), TACRMZ(1000)
COMMON/PARMRT/G, 9G2, 9K1, 9K2

SLNTRG=SQR(TACRRX(1, N)**2 + TACRRY(1, N)**2 + TACRRZ(1, N)**2)
AZMITH=TRUECG(TACRRX(1, N), TACRRY(1, N))
GNDNNG=SQR(TACRRX(1, N)**2 + TACRRY(1, N)**2)
ELVPNS=ATAN(TACRRZ(1, N)/GNDNNG)
R=GGNQF(DSEEED)
RNGPNS=SLNTRG*R*R*RSIGMA
A=GGNQF(DSEEED)
AZIPNS=AZMITH*A*ASIGMA
E=GGNQF(DSEEED)
ELVPNS=ELVPNS+E*ESIGMA
TACRMX(N)=RNGPNS*COS(ELVPNS)*SIN(AZIPNS)
TACRMY(N)=RNGPNS*COS(ELVPNS)*COS(AZIPNS)
TACRMZ(N)=RNGPNS*SIN(ELVPNS)
RETURN

END

SUBROUTINE ALFBTA(CT, HDGERR, TTG)
COMMON/TMRAS/N, TACRRX(3, 1000), TACRRY(3, 1000), +TACRRZ(3, 1000)
COMMON/NOISE/TACRMX(1000), TACRMY(1000), TACRMZ(1000)
COMMON/FILTER/TACREX(3, 1000), TACREY(3, 1000), +TACREZ(3, 1000)
COMMON/PROCES/X2SIGM, Y2SIGMZ2SIGMEWVRRX, EWRRRY,*+GX(1000), GY(1000), GZ(1000), RDRRES(1000), XRES(1000), +YRES(1000), ZRES(1000), W11(1000), W22(1000), W33(1000), +P1(1000, 9), PINITL

DT2=DT*DT
IF (N .NE. 1) GC TO 10

INITIALIZE FILTER.
FPXKK=TACRRX(N)
FPYKK=TACRMY(N)
FPZKK=TACRMZ(N)
FVXXK=TACRRX(2, N) + EWRRX
FVYKK=TACRMY(2, N) + EWRRY
FVZKK=TACRRZ(2, N)
FPXKK1=FPXKK+DT*FVXKK
FPYKK1=FPYKK+DT*FVYKK
FPZKK1=FPZKK+DT*FVZKK
STAGRF=0.0
STAGRT=0.0
MVRFLG=0
WRCNT=0.0
SELECT PARAMETERS FOR FILTERING BASED ON MANEUVERING AND NOISE CRITERIA.

HIGH OR LOW NOISE ENVIRONMENT?

\( \text{RUFRNG} = \sqrt{TACRMX(N)^2 + TACRMY(N)^2 + TACRMZ(N)^2} \)

IF ( \( \text{RUFRNG} \) .GT. 1500.) GO TO 20

NOSFLG=0
GO TO 30

CONTINUE

NOSFLG=1
CONTINUE

MANEUVERING?

IF TTG .LT. 30 SEC ASSUME NO MANEUVERING AND GO ON.

IF (TTG .LT. 30.) GO TO 40

CALL STRG(TT, NOSFLG, HDGERR, STAGER, STRGRT)
CALL TMNVR(STRGRT, MVRCNT)
IF ( (DT*FLOAT(MVRCNT)) .GT. 5.0 ) MVRFLG=0
IF ( (DT*FLOAT(MVRCNT)) .LT. -5.0 ) MVRFLG=0
IF (ABS(HDGERR) .GT. .052359878) MVRFLG=1
GO TO 50

CONTINUE

MVRFLG=0
CONTINUE

FIND ALFA/BETA PARAMETERS
CALL CRSTRK(NOSFLG, MVRFLG, TTG, CALFA, CBETA)
GX(N)=CALFA
GY(N)=CALFA
GZ(N)=CALFA

FILTER X-COORD. DATA.
CURRENT STATES.

XRESDU=TACRMX(N)-FPXKK1
FPXKK=FPXKK1+CALFA*XRESDU
FVXKK=FVXKK+CBETA*XRESDU/DT

PREDICTED STATES.
FPXKK1=FPXKK+FVXKK*DT

FILTER Y-COORD. DATA.
CURRENT STATES.

YRESDU=TACRMY(N)-FPYKK1
FPYKK=FPYKK1+CALFA*YRESDU
FVYKK=FVYKK+CBETA*YRESDU/DT

PREDICTED STATES.
FPYKK1=FPYKK+FVYKK*DT

FILTER Z-COORD. DATA.
CURRENT STATES.
ZRESU=TACRMZ(N)-FPZKK
FPZKK=FPZKK1+CALFA*ZRESU
FVZKK=FVZKK+CBETA*ZRESU/DT

PREDICTED STATES.

FPZKK1=FPZKK+FVZKK*DT

XRES(N)=XRESU
YRES(N)=YRESU
ZRES(N)=ZRESU
RADRES(N)=SQRT(XRESU**2+YRESU**2+ZRESU**2)

CONTINUE
TACREX(1,N)=FPXKK
TACREY(1,N)=FYPKK
TACREZ(1,N)=FPZKK
TACREX(2,N)=FVXKK
TACREY(2,N)=FYKK
TACREZ(2,N)=FVZKK
RETURN
END

SUBROUTINE STRG(DT,NOSFLG,HDGERR,STRGER,STRGRT)
IF (NOSFLG .EQ. 1) GO TO 15
ALFA= .0625
BETA= .0078125
GO TO 15
ALFA= .03125
BETA= .01953125
GO TO 20
CONTINUE
STRGER= STRGER+ALFA*(HDGERR-STRGER)
STRGER= STRGER+BETA*(HDGERR-STRGER)
RETURN
END

SUBROUTINE TMNVR(STRGRT,MVRCNT)
IF (ABS(STRGRT) .GT. .00672665) GO TO 10
IF (MVRCNT.LT. 0) MVRCNT=0
MVRCNT=MVRCNT+1
GO TO 30
IF (ABS(STRGRT) .LT. .0174533) GO TO 20
IF (MVRCNT .GT. 0) MVRCNT=0
MVRCNT=MVRCNT-1
GO TO 30
MVRCNT=0
GO TO 30
RETURN
END

SUBROUTINE CRSTRK(NOSFLG,MVRFGLG,TTG,CALFA,CBETA)
IF (NOSFLG .EQ. 0) GO TO 50
IF (MVRFGLG .EQ. 0) GO TO 10

LOW PRECISION (MANEUVERING)
CALFA= .1875
CBETA= .0078125
GO TO 30

84
CONTINUE
HIGH PRECISION (NON-MANEUVERING)
IF (TTG .LT. 30.) GO TO 20
MANEUVERING STOPPED WITH .GT. 30 SEC TO GO.
CALFA=.0390625
CBETA=.002441406
GO TO 30

CONTINUE
MANEUVERING STOPPED WITH .LT. 30 SEC TO GO.
CALFA=.078125
CBETA=.009765625
GO TO 99

CONTINUE
LOW NOISE.
IF (MVRF LG .EQ. 0) GO TO 60
LOW PRECISION (MANEUVERING)
CALFA=.375
CBETA=.03125
GO TO 80

CONTINUE
HIGH PRECISION (NON-MANEUVERING)
IF (TTG .LT. 30.) GO TO 70
MVRFLG STOPPED WITH 30 OR MORE SEC TTG.
CALFA=.078125
CBETA=.009765625
GO TO 80

CONTINUE
MVRNG STOPPED WITH LESS THAN 30 SEC TTG.
CALFA=.15625
CBETA=.00390625
GO TO 111

CONTINUE
RETURN
END

SUBROUTINE KALWN1(KAD, DT, RSIGMA, ASIGMA, ESIGMA, RANGE,
+AZMITH, ELEVTN, TTG)

DOUBLE PRECISION CSEED
COMMON/DOUBLE/ CSEED
COMMON/TMRST/N, TACRX(3,1000), TACRY(3,1000),
+TACRZ(3,1000)
COMMON/NOISE/TACR MX(1000), TACRMY(1000), TACRMZ(1000)
COMMON/FILTER/TACRX(3,1000), TACR Y(3,1000),
+TAC RZ(3,1000)
COMMON/PROCES x2SIGMA, Y2SIGMA, Z2SIGMA, EWVRRX, EWVRRY,
+GX(1000), GY(1000), GZ(1000), RADRES(1000), XRES(1000),
+YRES(1000), ZRES(1000), W11(1000), W22(1000)
+W33(1000), W12(1000, 9), PINITL

DIMENSION G(6,6), PKK(6,6), R(6,6), M(6,6), G(6,6),
+PHI(6,6), DEL(6,6), DT(6,6), NULL(6,6), M(6,6), XKK(6),
+XKKE(6), EXTRAX(6,6), EXTRA2(6,6), PKK1(6,6),
+WKAREA(18), EXTRA3(3,3), EXTRA4(3,3), Z(6)

IF (N .NE. 1 ) GO TO 25
IDT=2
CT2=DT*DT/2.0
DO 20 I=1,6
20 Z(I)=0.0

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DO 10 J=1,6
   A(I,J)=0.0
   PKK(I,J)=0.0
   IF (I.EQ. J) PKK(I,J)=PINITL
   R(I,J)=0.0
   W(I,J)=0.0
   G(I,J)=0.0
   PHI(I,J)=0.0
   DEL(I,J)=0.0
   DT(I,J)=0.0
   IF (I.EQ. J) DT(I,J)=1.0
   NULL(I,J)=0.0
   N(I,J)=0.0
10 CONTINUE

CONTINUE

M1=X2SIGM*X2SIGM
M2=Y2SIGM*Y2SIGM
M3=Z2SIGM*Z2SIGM
WX=0.0
WY=WZ=0.0
IXFLG=0
IYFLG=0
IZFLG=0
DEL(1,1)=DT2
DEL(2,1)=DT
DEL(3,2)=DT2
DEL(4,2)=DT
DEL(5,3)=DT2
DEL(6,3)=DT
HI(1,1)=1.0
HI(2,3)=1.0
HI(3,5)=1.0
PHI(1,1)=0.0
PHI(1,2)=1.0
PHI(2,3)=1.0
PHI(3,4)=DT
PHI(4,5)=1.0
PHI(5,6)=DT
PHI(6,6)=1.0
RVAR=RSTSIGMA*RSIGMA
AVAR=ASIGMA*ASIGMA
EVAR=ESIGMA*ESIGMA
TACREX(1,1)=TACRMX(1)
XKK(1)=TACREX(1,1)
TACREY(1)=TACRMY(1)
TACREY(3)=TACRMZ(1)
TACREZ(1,1)=TACRMX(N)
XKK(5)=TACREZ(1,1)
TACREZ(2,1)=TACRMX(2,N)+EWVRRX
XKK(4)=TACREZ(2,1)
TACREZ(3,1)=TACRRZ(2,N)
XKK(6)=TACREZ(2,1)

IF (N.EQ. 1) GO TO 999

CONTINUE

Z(1)=TACRMX(N)
Z(3)=TACRMY(N)
Z(5)=TACRMZ(N)

END
R2 = RANGE * RANGE
CA = COS (AZMITH)
SA = SIN (AZMITH)
CE = COS (ELEVNT)
SE = SIN (ELEVNT)
CA2 = CA * CA
SA2 = SA * SA
CE2 = CE * CE
SE2 = SE * SE

R(1, 1) = R2 * (EVAR * SE2 * SA2 + AVAR * CE2 * CA2) + RVAR * CE2 * SA2
R(2, 2) = R2 * (EVAR * SE2 * CA2 + AVAR * CE2 * SA2) + RVAR * CE2 * SA2
R(1, 3) = R2 * (EVAR * SE2 * RVAR * CE2) + RVAR * CE2 * SA2
R(3, 1) = R(1, 3)
R(2, 3) = R(2, 3)

FK1(1) = 0.0
FK1(2) = 0.0
FK1(3) = 0.0
FK1(4) = 0.0
FK1(5) = 0.0
FK1(6) = 0.0

IF (KAD .NE. 0) GO TO 29
W(1, 1) = W1
W(2, 2) = W2
W(3, 3) = W3
GO TO 34

29 CONTINUE
IF (KAD .NE. 1) GO TO 31
CALL ADPTV1 (N, 6, 120, W1, W2, W3, TACREX, TACREY, TACREZ, + W)
GO TO 34

31 CONTINUE
IF (TIT .LT. 20.0) GO TO 33
XRS = XRES(N-1)
YRS = YRES(N-1)
ZRS = ZRES(N-1)
CALL ADPTV2 (6, XRS, IYFLG, YRS, IYFLG, ZRS, IZFLG, PKK)

33 CONTINUE
W(1, 1) = W1
W(2, 2) = W2
W(3, 3) = W3

34 CONTINUE
W11(N) = W1(1, 1)
W22(N) = W2(2, 2)
W33(N) = W3(3, 3)

GENERATE Q-MATRIX.
CALL QUADPS (DEL, W, NULL, 6, 6, Q)

X(K/K-1) = PHI(K, K-1) * X(K-1/K-1) + F(K-1)
GO TO 210
IF = 1 + 6
XKK1(I) = FK1(I)
DO 200 J = 1 + 6
XKK1(I) = XKK1(I) + PHI(I, J) * XKK(J)

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CONTINUE

P(K/K-1) = PHI(K/K-1)*P(K-1/K-1)*TRANSPOSE(PHI) + Q(K-1)
CALL QUADPS(PHI,PKK,Q,G,6,6,PKK)

G(K) = P(K/K-1)*H(K)*INV(H(K))*P(K/K-1)*TRANSPOSE(H) + R(K))
CALL QUADPS(H,PKK,R,6,6,EXTRA1)
DO 212 I = 1,3
DO 211 J = 1,3
EXTRA2(I,J) = EXTRA1(I,J)
CONTINUE
211 DO 230 I = 1,6
DO 225 J = 1,6
EXTRA2(I,J) = 0.0
DC 220 K = 1,6
EXTRA2(I,J) = EXTRA2(I,J) + PKK(I,K) * H(J,K)
CONTINUE
220 CONTINUE
230 CONTINUE

INVERT EXTRA1 AND MULTIPLY BY EXTRA2.
CALL LINV2F(EXTRA4,3,3,EXTRA3,IDGT,WKAREA,IER)
CALL PRODCT(EXTRA4,EXTRA3,3,3,3,6,G)
G(J) = G(1,3)
G(N) = G(3,1)
G(N) = G(5,3)
C X(K/K-1) = X(K/K-1) + G(K)*(Z(K) - H(K)*X(K/K-1))
DO 250 I = 1,6
EXTRA1(I,1) = Z(I)
DO 240 J = 1,6
EXTRA1(I,1) = EXTRA1(I,1) - H(I,J)*XKK1(J)
CONTINUE
240 CONTINUE
RADRES(N) = SQRT(EXTRA1(1,1)**2 + EXTRA1(2,1)**2 + EXTRA1(3,1)**2)
XRES(N) = EXTRA1(1,1)
YRES(N) = EXTRA1(2,1)
ZRES(N) = EXTRA1(3,1)
DO 370 I = 1,6
XXK(I) = XKK1(I)
DO 260 J = 1,3
XXK(I) = XXK1(I) + G(I,J) * EXTRA1(J,1)
260 CONTINUE
TA4RES(1,N) = XXK(1)
TA4RES(2,N) = XXK(2)
TA4RES(3,N) = XXK(3)
TA4RES(4,N) = XXK(4)
TA4RES(5,N) = XXK(5)
TA4RES(6,N) = XXK(6)

P(K/K) = P(K/K-1) * G(K) * H(K) * P(K/K-1)
CALL PRODCT(G,H,6,6,6,EXTRA1)
CALL PRODCT(EXTRA1,PKK,6,6,6,EXTRA2)
DO 290 I = 1,6
DO 280 J = 1,6
PKK(I,J) = PKK(I,J) - EXTRA2(I,J)
CONTINUE
280 CONTINUE
290 CONTINUE
999 CONTINUE

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C

SUBROUTINE KALMN2(KADOTRSIGMA, RSIGMA, ASIGMA, ESIGMA, RANGE,
+ AZMITH, ELEVTN, TTG)

DOUBLE PRECISION DSEED
COMMON/DDOUBLE/DSEED
COMMON/TMRRST/N,TACRRX(3,1000), TACRRY(3,1000),
+TACRRZ(3,1000)
COMMON/NOISE/TACRMX(1000), TACRMY(1000), TACRMZ(1000)
COMMON/FILTER/TACRFX(3,1000), TACRFY(3,1000),
+TACRFZ(3,1000)
COMMON/PROCES/X2SIGM, Y2SIGM, Z2SIGM, EWVRRX, EWVRYY,
+GXR(1000), GY(1000), GZ(1000), RADRES(1000), XRES(1000),
+YRES(1000), ZRES(1000), W1(1000), W2(1000)
*,X3(1000), P(1000,9), PINITL

DIMENSION Q(9,9), PKK(9,9), R(9,9), W(9,9), G(9,9),
+PHI(9,9), DEL(9,9), DI(9,9), NULL(9,9), H(9,9), XXK(9),
+XXK1(9), PK1(9), EXTRA(9,9), EXTRA2(9,9), PKK1(9,9)
*,WKAPEA(75), EXTRA3(3,3), EXTRA4(3,3), Z(9)

IF (N .NE. 1 ) GO TO 25
100 DT2=DT*DT/2.0
DT3=DT2*DT/3.0

DO 20 I=1,9
20    Z(I)=0.0
DO 10 J=1,9
10   G(I,J)=0.0
   PKK(I,J)=0.0
   IF (I .EQ. J) PKK(I,J)=PINITL
   R(I,J)=0.0
   W(I,J)=0.0
   G(I,J)=0.0
   G(I,J)=0.0
   D(I,J)=0.0
   DI(I,J)=0.0
   NULL(I,J)=0.0
   H(I,J)=0.0
10 CONTINUE

20 CONTINUE

W1=X2SIGM*X2SIGM
W2=Y2SIGM*Y2SIGM
W3=Z2SIGM*Z2SIGM
W4=0.0
W5=W1
W6=W4
IXFLG=0
IYFLG=0
IZFLG=0
DEL(1,1)=DT3
DEL(5,1)=DT2
DEL(4,4)=DT3
DEL(5,2)=DT2
DEL(6,2)=DT2
DEL(7,3)=DT3
DEL(9,3)=DT
M(1,1)=1.0
M(2,4)=1.0
M(3,7)=1.0
PHI(1,1)=0
PHI(1,4)=DT2
PHI(2,2)=1.0
PHI(2,3)=0
PHI(3,3)=1.0
PHI(4,4)=0
PHI(4,6)=DT2
PHI(5,5)=1.0
PHI(5,6)=0
PHI(6,6)=1.0
PHI(7,7)=1.0
PHI(7,9)=0
PHI(8,8)=1.0
PHI(8,9)=1.0
RVAR=R_SIGMA*R_SIGMA
AVAR=A_SIGMA*A_SIGMA
EVAR=E_SIGMA*E_SIGMA
TACREX(1,N)=TACRMX(1)
XKK(1)=TACREX(1,N)
TACREX(2,N)=TACRX(2,N)+EWVRRX
XKK(2)=TACRXX(2,N)
XKK(3)=TACRRZ(3,N)
TACREY(1,N)=TACRMY(1)
XKK(4)=TACREY(1,N)
TACREY(2,N)=TACRRY(2,N)+EWVRRY
XKK(5)=TACREY(2,N)
XKK(6)=TACRXY(3,N)
TACREZ(1,N)=TACMZ(1)
XKK(7)=TACREZ(1,N)
TACREZ(2,N)=TACRRZ(2,N)
XKK(8)=TACREZ(2,N)
XKK(9)=TACRRZ(3,N)

IF (N .EQ. 1) GO TO 999

CONTINUE

IZ(1)=TACRMX(N)
IZ(2)=TACRMY(N)
IZ(3)=TACRMZ(N)

R2=RANGE*RANGE
CA=COS(azimuth)
SA=SIN(azimuth)
CE=COS(azimuth)
SSASTNf

LEVITN

~E2:SE*Ss

C

C

R(1,1) = R2*(EVAR*SE2*SA2 + AVAR*CE2*CA2) + RVAR*CE2*SA2
R(2,2) = R2*(EVAR*SE2*CA2 + AVAR*CE2*SA2) + RVAR*CE2*CA2
R(3,3) = R2*(EVAR*CE2*SA2 + RVAR*SE2)
R(1,2) = R2*(EVAR*SE2*SA2 + RVAR-R2*AVAR)*(CE2*SA*CA)
R(2,1) = R(1,2)
R(1,3) = (RVAR-R2*EVAR)*SE2*CA
R(2,3) = R(1,3)
R(3,2) = R(2,3)

C

FK1(1) = 0.0
FK1(2) = 0.0
FK1(3) = 0.0
FK1(4) = 0.0
FK1(5) = 0.0
FK1(6) = 0.0
FK1(7) = 0.0
FK1(8) = 0.0
FK1(9) = 0.0

C

IF (KAD .NE. 0) GO TO 29
W(1,1) = W1
W(2,2) = W2
W(3,3) = W3
GO TO 34

29 CONTINUE
IF (KAD .NE. 1) GO TO 31
CALL ADPTV1(N, 9, 120, W1, W2, W3, TACREX, TACREY, TACREZ, + W)
GO TO 34

31 CONTINUE
IF (TTG .LT. 20.) GO TO 33
XRS = XRES(N-1)
YRS = YRES(N-1)
ZRS = ZRES(N-1)
CALL ADPTV2(9, XRS, IXFLG, YRS, IYFLG, ZRS, IZFLG, PKK)

33 CONTINUE
W(1,1) = W2
W(2,2) = W2
W(3,3) = W3

34 CONTINUE
W11(N) = W(1,1)
W22(N) = W(2,2)
W33(N) = W(3,3)

C GENERATE Q-MATRIX
CALL QUADPS(DEL, W, NULL, 9, 9, Q)

C

X(K/K-1) = PHI(K/K-1)*X(K-1/K-1) + F(K-1)
DO 213 I = 1, 9
XKK1(I) = FK1(I)
DO 200 J = 1, 9
XKK1(I) = XKK1(I) + PHI(I, J)*XKK(J)
200 CONTINUE

210 CONTINUE
```plaintext
C P(K/K-1) = PHI/(K/K-1) * P(K-1/K-1) * TRANSPOSE(PHI) + Q(K-1)
   CALL QUADPS(PHI,PKK,G,9,9,PKK1)
C G(K) = P(K/K-1) * PHI(K) * INV(H(K)) * P(K/K-1) * TRANSPOSE(H) + R(K)
   CALL QUADPS(H,PKK1,R,9,9,EXTRA1)
   DO 250 I = 1,3
   DO 211 J = 1,3
      EXTRA4(I,J) = EXTRA1(I,J)
   CONTINUE
   211 CONTINUE
   212 CONTINUE
   DO 230 I = 1,9
   DO 225 J = 1,9
      EXTRA2(I,J) = 0.0
   DO 220 K = 1,9
      EXTRA2(I,J) = EXTRA2(I,J) + PKK1(I,K) * H(J,K)
   CONTINUE
   220 CONTINUE
   225 CONTINUE
   230 CONTINUE
C INVERT EXTRA1 AND MULTIPLY BY EXTRA2.
   CALL INVF(EXTRA4,3,3,EXTRA3,IDGT,WKAREA,IER)
   CALL PRODCT(EXTRA2,EXTRA3,3,3,G)
   GX(N) = G(1,1)
   GY(N) = G(4,2)
   GZ(N) = G(7,3)
C X(K/K) = [ X(K/K-1) + G(K) * (Z(K) - H(K)*X(K/K-1) ) ]
   DO 240 I = 1,9
   EXTRA1(I,1) = Z(I)
   DO 260 J = 1,9
      EXTRA1(I,J) = EXTRA1(I,J) - H(I,J) * XKK1(J)
   CONTINUE
   240 CONTINUE
   250 CONTINUE
   RADRES(N) = SQRT(EXTRA1(1,1)**2 + EXTRA1(2,1)**2 + EXTRA1(3,1)**2)
   XRES(N) = EXTRA1(1,1)
   YRES(N) = EXTRA1(2,1)
   ZRES(N) = EXTRA1(3,1)
   DO 270 I = 1,9
      XKK(I) = XKK1(I)
   DO 280 J = 1,3
      XKK(I) = XKK(I) + G(I,J) * EXTRA1(J,1)
   CONTINUE
   260 CONTINUE
   270 CONTINUE
   TACRX(1,N) = XKK(1)
   TACRX(2,N) = XKK(2)
   TACRX(3,N) = XKK(3)
   TACRX(4,N) = XKK(4)
   TACRX(5,N) = XKK(5)
   TACRX(6,N) = XKK(6)
   TACRX(7,N) = XKK(7)
   TACRX(8,N) = XKK(8)
   TACRX(9,N) = XKK(9)
C C P(K/K) = P(K/K-1) * PHI(K) * H(K) * P(K/K-1)
   CALL PRODCT(G,H,9,9,9,EXTRA1)
   CALL PRODCT(EXTRA1,PKK1,9,9,9,EXTRA2)
   DO 290 I = 1,9
   DO 280 J = 1,9
      PKK1(I,J) = PKK1(I,J) - EXTRA2(I,J)
   CONTINUE
   280 CONTINUE
   290 CONTINUE
   999 CONTINUE
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```
```
P(N,1)=PKK(1,1)
P(N,2)=PKK(2,2)
P(N,3)=PKK(3,3)
P(N,4)=PKK(4,4)
P(N,5)=PKK(5,5)
P(N,6)=PKK(6,6)
P(N,7)=PKK(7,7)
P(N,8)=PKK(8,8)
P(N,9)=PKK(9,9)

C FORMAT(*'G(*,I*','*I*,')*=',F20.10)
RETURN
END

SUBROUTINE QUADPS(X,H,R,M,N,C)
DIMENSION X(M,N),R(M,N),R(M,M),C(M,M)
DO 400 I = 1,M
DO 300 J = 1,M
C(I,J)=R(I,J)
DO 100 K = 1,N
C(I,J)=C(I,J)+X(I,K)*H(K,L)*X(J,L)
100 CONTINUE
200 CONTINUE
300 CONTINUE
400 CONTINUE
RETURN
END

SUBROUTINE AOPTV1(NT13,JSTART,W1,W2,W3,TACRX,TACRY,
+TACREZ,W)
DIMENSION TACRX(3,1000),TACRY(3,1000),
+TACREZ(3,1000),W(13,13)
ISTATE=13/3
IF (N.LT. JSTART) GO TO 27
W=8.0*(TACRX(ISTATE,N-1)-TACRX(ISTATE,N-2))
WY=8.0*(TACRY(ISTATE,N-1)-TACRY(ISTATE,N-2))
WZ=8.0*(TACREZ(ISTATE,N-1)-TACREZ(ISTATE,N-2))
GO TO 35
27 CONTINUE
W=0.0
WY=0.0
WZ=0.0
35 CONTINUE
IF (ISTATE .EQ. 3) GO TO 37
W(1,1)=W+(W*W)/3
W(2,2)=W+(W*W)/3
W(3,3)=W+(W*W)/3
GO TO 41
37 CONTINUE
W(1,1)=W+(W*W)/81
W(2,2)=W+(W*W)/81
W(3,3)=W+(W*W)/81
41 CONTINUE
RETURN
END

SUBROUTINE AOPTV2(NORDER,XRES,IXFLG,YRES,IYFLG,ZRES,
+IZFLG,PKK)
DIMENSION PKK(NORDER,NORDER)
RESET=1000.
```
CALL BIAS(XRES,IXFLG)
IF (IABS(IXFLG) .LT. 8) GO TO 10
P KK(1,1)=RESET
P KK(2,2)=RESET
IF (NORDER .EQ. 9) P KK(3,3)=RESET
CONTINUE

CALL BIAS(YRES,IXFLG)
IF (IABS(YXFLG) .LT. 8) GO TO 14
IF (NORDER .EQ. 9) GO TO 12
P KK(3,3)=RESET
P KK(4,4)=RESET
GO TO 14
CONTINUE

CALL BIAS(ZRES,IXFLG)
IF (IABS(ZXFLG) .LT. 8) GO TO 20
IF (NORDER .EQ. 9) GO TO 16
P KK(5,5)=RESET
P KK(6,6)=RESET
GO TO 20
CONTINUE

SUBROUTINE BIAS(RESI,D,IFLAG)
IF (RESI) 10,20,30
10 IF (IFLAG .LE. 0) GO TO 12
IFLAG=0
GO TO 40
CONTINUE
12 IFLAG=IFLAG+1
GO TO 40
CONTINUE
14 IFLAG=0
GO TO 40
CONTINUE
20 IFLAG=IFLAG+1
GO TO 40
CONTINUE
30 IFLAG=0
GO TO 40
CONTINUE
32 IFLAG=IFLAG+1
CONTINUE
RETURN
END
LIST OF REFERENCES


AIRCRAFT STATE ESTIMATION FOR A GROUND DIRECTED BOMBING SYSTEM(U) NAVAL POSTGRADUATE SCHOOL MONTEREY CA
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