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EFFECT OF HF HEATING ARRAY DIRECTIVITY PATTERN
ON THE FREQUENCY RESPONSE
OF GENERATED ELF/VLF

by
Kenneth J. Carroll, A. J. Ferraro
H. S. Lee, Roger Allshouse
Bruce Long, Ray J. Lumen
January 1983

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IONOSPHERE RESEARCH LABORATORY

University Park, Pennsylvania
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ABSTRACT

Directivity patterns at 3.17 MHz and 5.1 MHz are calculated for the HF antenna array at the high power HF heating facility at the Arecibo Observatory in Puerto Rico. The pattern was calculated using pattern multiplication and method of moment techniques. The calculated pattern is shown to be a good approximation to an experimentally measured pattern in one plane of the array. A simple model was used to approximate the effect of the pattern on the frequency response of ELF/VLF signals generated by the HF heating. The frequency response was determined at two ELF/VLF receiver sites. Results show that ELF/VLF generated by side lobes of the HF pattern have sufficient strength to create an ELF/VLF interference pattern at receiving locations.
EFFECT OF HF HEATING ARRAY DIRECTIVITY PATTERN ON THE FREQUENCY RESPONSE OF GENERATED ELF/VLF

INTRODUCTION

ELF/VLF generation experiments were conducted at the Arecibo Observatory (A.O.) in Puerto Rico. The HF heating facility for A.O. is located at 18° 29' N and 66° 40' W geographic latitude and longitude respectively. The ELF/VLF receiving site was located 7.7 km from the heating facility and 238° to the east.

The motion of the ionospheric plasma, in the presence of the earth's magnetic field, causes natural ionospheric currents to flow. By changing the conductivity of a small portion of the ionosphere, the natural currents within that portion can be modulated. Modulating the currents in the ELF/VLF range causes a ELF/VLF signal to be radiated by the ionosphere.

The conductivity in the ionosphere is dependent upon the electron collision frequency, which is in turn dependent upon the electron temperature. An HF electromagnetic wave is absorbed by the ionosphere. The EM wave adds kinetic energy to the electrons, which in effect increases the electron temperature. Thus, by modulating the HF transmission at a ELF/VLF rate, the ionospheric conductivities will be modulated at the same rate, and a ELF/VLF signal will be radiated from the ionosphere.

The antenna, radiating the HF signal heating the ionosphere, has a pattern consisting of a main beam, side lobes, and possibly grating lobes. Heating occurs where each of these penetrates the ionosphere. By determining the pattern of the HF antenna, the spatial distribution
of the heating in the ionosphere can be determined. Thus, the
coloration in the ionosphere and the intensity of each of the ELF/VLF
radiating sources can be determined. From this the characteristics of
the ELF/VLF radiation from the ionosphere can be calculated.

This section will describe the calculation of an approximation to
the Arecibo HF heating array directive gain pattern and apply the
results to find a zero order approximation to an ELF/VLF radiating
array. The technique employed to calculate the HF array pattern is
one which uses the combination of pattern multiplication techniques
and computer numerical analysis. The numerical program used was the
Antenna Modeling Program (AMP)(1). The AMP output was then used with
analytical equations in a program written at the Ionosphere Research
Laboratory at Penn State University to carry out the pattern
multiplication.

PATTERN MULTIPLICATION THEORY

The pattern multiplication technique is based upon the
calculation of the total pattern of an array by taking the product of
an array factor (AF) with the elemental pattern. The array is made up
of identical elements. The elemental pattern is the pattern of an
individual element of the array. The AF is obtained by replacing each
of the elements of the array with an isotropic radiator and
calculating the pattern for the array of isotropic radiators. A
detailed discussion of pattern multiplication can be found in
reference (2). A description of the theory used in this analysis
follows.
For example, assume there are an even number "N" of colinear isotropic radiators separated by a distance "d" as shown in figure (1-1). The far field, at a point "P", due to the n<sup>th</sup> radiator, is proportional to a complex current amplitude, I<sub>n</sub>, a phase factor, e<sup>-jβR<sub>n</sub></sup>, and is inversely proportional to the distance from the radiator to "P", equation (1-1).

\[ E_n \propto I_n \left[ e^{-j\beta R_n/(4\pi R_n)} \right] \]  

(1-1)

The far field approximation states that "P" is far enough away that "R<sub>n</sub>" can be assumed to be parallel to "R", and the length of "R<sub>n</sub>" is approximately equal to "R". Under these conditions the approximations in equation (1-2) can be made.

\[ 1/R \approx 1/R_n \]  

(1-2a)

\[ R_n \approx (2|m| - 1)(d/2) \cos \alpha + R \quad ; \quad n < 0 \]  

(1-2b)

\[ R_n \approx R - (2|m| - 1)(d/2) \cos \alpha \quad ; \quad n > 0 \]  

(1-2c)

While the small difference in length of "R<sub>n</sub>" can be neglected in the "1/R<sub>n</sub>" term, these differences can have a significant effect on the phase term, e<sup>-jβR<sub>n</sub></sup>. Incorporating the far field approximation equation (1-2) with equation (1-1), the total field at "P" can be expressed (1-3).

\[ E = \sum_{n=-N/2}^{N/2} E_n + \sum_{n=1}^{N/2} I_n \left[ \sum_{m=1}^{N/2} I_m e^{-j\beta((2|m| - 1)(d/2)\cos \alpha + R)} \right] \\
+ \sum_{m=1}^{N/2} I_m e^{-j\beta[R - (2|m| - 1)(d/2)\cos \alpha]} \left[ 1/(4\pi R) \right] \]  

(1-3)

Assume that "I<sub>n</sub>" is equal to a constant "I<sub>0</sub>" and collect all the like terms. Equation (1-3) reduces to equation (1-4).
Figure 1-1  N colinear isotropic radiators
\[ E = \left( \frac{I_0}{4\pi R} \right) e^{-jBR} \left[ \sum_{n=\{-N/2\}}^{N/2} e^{-j\gamma_n} + \sum_{n=1}^{N/2} e^{j\gamma_n} \right] \quad (1-4) \]

\[ \gamma_n = \beta(2 n - 1)(d/2)\cos \alpha \]

The first term is a constant for a fixed value of "R". The second term is dependent on alpha. It defines the antenna pattern and is the AF. The AF is given in equation (1-5).

\[ AF = \sum_{n=\{-N/2\}}^{N/2} e^{-j\gamma_n} + \sum_{n=1}^{N/2} e^{-j\gamma_n} \quad (1-5) \]

Noting that \( \gamma_{-n} \) is equal to \( \gamma_n \), equation (1-5) can be simplified to equation (1-6).

\[ AF = \sum_{n=1}^{N/2} (e^{j\gamma_n} + e^{-j\gamma_n}) = 2 \sum_{n=1}^{N/2} \cos \gamma_n = 2 \sum_{n=1}^{N/2} \cos \beta(2n-1)(d/2)\cos \alpha \]

\[ (1-6) \]

Since the 2 in equation (1-6) is a constant, it may be dropped from the array factor. In addition, in equation (1-6) 2n-1 can be replaced with n, and the result simplified to equation (1-7).

\[ AF = \sum_{n=1}^{N/2} \cos[n\beta(d/2)\cos \alpha] \quad (1-7) \]

Equation (1-7) is the array factor for a colinear array of an even number of N isotropic radiators with equal current amplitudes. The angle \( \alpha \) is measured from the line of the array to the observation point.
ANTENNA MODELING PROGRAM (AMP) THEORY

The Antenna Modeling Program (AMP) was used to analyze the array element. This program was developed by MBA/Information Systems. (1)

The computer program applies the method of moments to the thin wire approximation of the integral equation for the electric field due to a volume current distribution, equation (1-8), (1)

\[
E(\mathbf{r}_o) = \iiint \mathbf{i} \mu_0 \omega \mathbf{J}(\mathbf{r}) \cdot (\mathbf{G}(\mathbf{r}, \mathbf{r}_o)) d\mathbf{r}
\]  \hspace{1cm} (1-8)

\[
\mathbf{G}(\mathbf{r}, \mathbf{r}_o) = -\frac{1}{4\pi} \mathbf{I}^* (1/k^2) \nabla g
\]

\[
g = (e^{-ik|\mathbf{r} - \mathbf{r}_o|} / |\mathbf{r} - \mathbf{r}_o|)
\]

\[
k = \omega \sqrt{\mu_0 \varepsilon_0}, \quad \mathbf{I} = \text{unit } 2\text{nd rank tensor}
\]

\[
|\mathbf{r} - \mathbf{r}_o| = \text{distance measured from wire axis (source point) to observation point on the surface.}
\]

The thin wire approximation requires that the diameter of the wire be small compared with the wavelength. Thus azimuthal current flow around the wire can be neglected and the volume integral in equation (1-8) can be changed to a line integral, equation (1-9), (1)

\[
-\mathbf{s}_o \cdot \mathbf{E}(\mathbf{r}_o) = \frac{-i \omega \mu_0}{4\pi} \int_{L} \mathbf{I}(s) [\mathbf{s} \cdot \mathbf{s}_o - (1/k^2) (\partial^2 / \partial s \partial s) g(\mathbf{r}, \mathbf{r}_o)] ds
\]  \hspace{1cm} (1-9)

\[
\mathbf{s} = \text{unit tangent at source point}
\]

\[
\mathbf{s}_o = \text{unit tangent at observation point}
\]

\[
I = (na^2 J) / 2na
\]

\[
a = \text{wire radius}
\]
Included in equation (1-9) is also the boundary condition for a metal surface, equation (1-10).

\[ E_{\text{tan}}^I + E_{\text{tan}}^S = 0 \]  

(1-10)

\( E_{\text{tan}}^I \) = Tangential component of incident electric field

\( E_{\text{tan}}^S \) = Tangential component of scattered electric field

AMP solves equation (1-9) numerically by converting it into matrix form. This is accomplished by expanding the unknown currents, \( I_n \), in terms of a set of basis functions, \( I_n \), and taking the inner product of both sides of equation (1-9) with a set of weighting functions \( w_m \). A general discussion on this method of solution can be found in reference (3).

Equation (1-11) is obtained by expressing equation (1-9) in operational format, where the operator \( L_\text{op} \) denotes the integral and \( "<A,B>" \) denotes the inner product of quantities \( A \) and \( B \).

\[ \sum_{n=1}^{N} A_n \langle w_m, L_\text{op} I_n \rangle = \langle w_m, E^I \rangle \]  

(1-11)

where \( I = \sum_{n=1}^{N} A_n I_n \)

Equation (1-11) must be true for each \( w_m \), and thus may be written in matrix form as expressed in equation (1-12).

\[
\begin{bmatrix}
\langle w_1, L_\text{op} I_1 \rangle & \langle w_2, L_\text{op} I_1 \rangle & \ldots & \langle w_N, L_\text{op} I_1 \rangle \\
\langle w_1, L_\text{op} I_2 \rangle & \langle w_2, L_\text{op} I_2 \rangle & \ldots & \langle w_N, L_\text{op} I_2 \rangle \\
\vdots & \vdots & \ddots & \vdots \\
\langle w_1, L_\text{op} I_N \rangle & \langle w_2, L_\text{op} I_N \rangle & \ldots & \langle w_N, L_\text{op} I_N \rangle \\
\end{bmatrix}
\begin{bmatrix}
A_1 \\
A_2 \\
\vdots \\
A_N \\
\end{bmatrix}
= 
\begin{bmatrix}
\langle w_1, E^I \rangle \\
\langle w_2, E^I \rangle \\
\vdots \\
\langle w_N, E^I \rangle \\
\end{bmatrix}
\]  

(1-12)
Since $E_I$, $I_n$ and $w_m$ are known, by matrix inversion the values of $A_n$ can be calculated.

Specifically, AMP uses sine and cosine functions as basis functions and employs a method of collocation, or point matching, by choosing the weighting functions as $\delta$ functions.

**THEORY APPLIED TO A.O. ARRAY**

To apply the techniques of AMP and pattern multiplication to the A.O. heating array, the array's physical characteristics must be known.* The HF antenna consists of a 4x8 array of radiating elements. This array is oriented as shown in figure (1-2). Each of the elements in the array is constructed in the shape of an inverted pyramid with four sides. The faces of the pyramid are at an angle of $45^\circ$ with the ground and contain two nonplanar log-periodic antennas (NLPA). One NLPA is contained in the north and south faces, and the other is contained in the east and west faces. A sketch of an array element is shown in figure (1-3). Note that the elements in the south and west faces have been rotated $180^\circ$ about the corresponding face's NLPA elements feed lines. A diagram looking down at the top elements of the pyramid is shown in figure (1-4).

Both NLPA are designed with a $\tau$ of .88. The dimensions of the array elements in the north and south faces are shown in figure (1-5). The dimensions of the east and west faces are scaled to $\tau^{1/4}$ of the north and south faces. This will result in right hand circular polarization radiation when the north and south faces are fed $180^\circ$ out

*Note: See appendix III for additional information on HF antenna geometry.
Figure 1-2 HF heating array

Figure 1-3 HF heating array element

Figure 1-4 View of top elements looking down at pyramid
of phase with the east and west faces. When fed in phase, left hand circular polarization radiation will be transmitted.\(^{(4)}\) The north and south and the east and west faces are fed against ground by separate transmitters. This gives the capability to adjust the phase between different faces and thus change the radiation polarization.

The array element was analyzed using the AMP computer program. Then the pattern multiplication technique was used to calculate the total array pattern.

The array element was analyzed using AMP for frequencies of 3.17 MHz and 5.1 MHz. The data file used is listed in Appendix I program 1. The GW and GM cards generate the antenna structure depicted in figure (1-3) and orient it with respect to the coordinate axis shown in figure (1-4). The GN card specified the conductivity (.03 \(\Omega/m\)) and relative permittivity (20) of the ground below the HF array. The actual conductivity and permittivity for the heater site was not known, but based on the fact that maps indicate the heater array is located on marshy ground, a conductivity and a relative permittivity for "good" earth\(^{(5)}\) were used. As shown in the RP card, the power gain was computed for 2.5 degree steps in "theta" and 5 degree steps in "phi."

In order to use the output of AMP in the pattern multiplication, it was necessary to develop an elemental pattern function. Given a "theta" and "phi", the function returns a value of the power gain in that direction. To accomplish this, the power gains form the AMP output for selected constant "phi" surfaces were combined with a linear interpolation scheme. The selected values of "phi" are listed in table I.
Figures (1-6) and (1-7) show the comparison of the interpolated values (shown by X) with the AMP results (shown by •). The maximum error is approximately one db. Figure (1-6) is a plot of power gain as a function of "phi" for a constant "theta." The power gain decreases as the radius of the polar plot increases. The unsymmetrical nature of the plot is due to the unsymmetrical nature of the array element. Figure (1-7) is a plot of power gain as a function of "theta" for a constant "phi."

<table>
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<th>3.17 MHz</th>
<th>5.1 MHz</th>
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<tr>
<td>Phi (deg)</td>
<td>Phi (deg)</td>
</tr>
<tr>
<td>0</td>
<td>10</td>
</tr>
<tr>
<td>90</td>
<td>60</td>
</tr>
<tr>
<td>130</td>
<td>90</td>
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<tr>
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<td>150</td>
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<td>180</td>
</tr>
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<td>320</td>
<td>215</td>
</tr>
<tr>
<td>360</td>
<td>270</td>
</tr>
<tr>
<td>320</td>
<td>320</td>
</tr>
</tbody>
</table>

Table I. Selected Values of Phi for Interpolation Routines

The total array pattern for the A.O. array shown in figure (1-1) was calculated by taking the product of two separate array factors. In essence this is a pattern multiplication. The array pattern of one array becomes the elemental pattern of the other array. One array is the 4-element array on the north and south line. The other array is an 8-element array on an east and west line.

The expansion of equation (1-7) for the 4-element and 8-element arrays can be simplified by using trigonometric identities. These trigonometric identities were obtained from Chebyshev polynomials, as shown in Appendix II. The simplified equation for the antenna factors are given in equations (1-13) and (1-14). The 4-element array factor
Figure 1-6a  Power gain vs. phi for a constant theta, 3.17 MHz
Figure 1-6b  Power gain vs. phi for a constant theta, 5.1 MHz
Figure 1-7a.1 Power gain vs. theta for phi = 0°, 3.17 MHz
Figure 1-7a.3  Power gain vs. theta for \( \phi = 150^\circ \), 3.17 MHz
Figure 1-7a.4  Power gain vs. theta for phi=240°, 3.17 MHz
Figure 1-7b.1  Power gain vs. theta for phi=0°, 5.1 MHz
Figure 1-7b.2 Power gain vs. theta for phi=130°, 5.1 MHz
Figure 1-7b.3 Power gain vs. theta for phi=250°, 5.1 MHz
is equation (1-13), and the 8-element array factor is equation (1-14). Figure (1-8) shows the orientation of the 4- and 8-element arrays on an X,Y,Z coordinate system. The Y-axis was taken to be north and the X-axis as east.

\[ AF_1 = \frac{\sin(4\theta(d/2)\cos\xi_1)}{\sin(\theta(d/2)\cos\xi_1)} \quad (1-13) \]
\[ AF_2 = \frac{\sin(8\theta(d/2)\cos\xi_2)}{\sin(\theta(d/2)\cos\xi_2)} \quad (1-14) \]

The array pattern of the 8-element array is solid of revolution about the X-axis, and the 4-element array pattern is a solid of revolution about the Y-axis.

Using the transformation in equation (1-15), \( AF_1 \) and \( AF_2 \) can be transformed to spherical coordinates. The total array factor \( AF \), equation (1-16), is the product of \( AF_1 \) and \( AF_2 \).

\[ \cos \xi_1 = \sin \theta \sin \phi \]
\[ \cos \xi_2 = \sin \phi \]

\[ AF = AF_1 \times AF_2 = \frac{\sin(4\theta(d/2)\sin\phi)}{\sin(\theta(d/2)\sin\phi)} \quad (1-15) \]
\[ = \frac{\sin(8\theta(d/2)\sin\phi\cos\phi)}{\sin(\theta(d/2)\sin\phi\cos\phi)} \quad (1-16) \]

The total array power pattern can be calculated by taking the square of the total array factor, \( AF \), and multiplying it by the elemental pattern function, which is an interpolation of the AMP results. To achieve the goal of the calculation of a directive gain pattern, a correction factor must be determined. This factor is a result of neglecting the constants in calculating the array factor.

The directive gain "in a given direction is defined as the ratio of the radiation intensity in that direction to the average radiated power."\(^5\) Since the constants which were neglected in calculating \( AF \) are also contained in the calculation of the average radiated power.
Figure 1-8  Orientation of 4- and 8-element arrays
and the directive gain is a ratio, these same constants must also be neglected in calculation of the average radiated power. Thus the calculation of the average radiated power becomes the correction factor necessary to convert the array power pattern to an array directivity pattern.

Equation (1-17) was used to calculate the average radiated power, \( W_r \).

\[
W_r = \int_0^{2\pi} \int_0^{\pi/2} \left( AF(\Theta, \phi) \right)^2 \times 10 \left( ELF(\Theta, \phi) / 10 \right) \sin \Theta d\Theta d\phi
\]  

(1-17)

\[
ELF(\Theta, \phi) = \text{Elemental Power gain from interpolated AMP output}
\]

\[
AF(\Theta, \phi) = \text{Total Array Factor}
\]

The integration was performed numerically using Simpson integration, equation 6 (1-18). The programs are given in Appendix I, programs 2 and 3.

\[
\int_a^b f(x) dx = \frac{\Delta x}{3} [f(x_0) + 4f(x_1) + 2f(x_2) + 4f(x_3) + 2f(x_4) + \ldots] + 2f(x_{2n-2}) + 4f(x_{2n-1}) + f(x_{2n})
\]

\[
\Delta x = \frac{(b-a)}{(2n)}
\]

The first quadrant integration was carried out for two cases. One case was with a "phi" step size of 2.5 degrees; the other case was with a "phi" step size of 1 degree. A "theta" step size of 1 degree was used in both cases. No significant difference was found in the result of the integrations. Based on this result, the step sizes chosen for the total integral were 1 degree and 2.5 degrees for "theta" and "phi" respectively. The correction factors determined were 9.82 db and 8.62 db for 5.1 MHz and 3.17 MHz respectively.
Equation (1-19) combines the total array factor AF, the elemental pattern ELF, and the correction factor to calculate the directive gain pattern for the A.O. heating array.

\[ D(\theta, \phi) = 20 \log[AF(\theta, \phi)] + ELF(\theta, \phi) - \text{Correction factor} \quad (1-19) \]

Figure (1-9) is a plot of the pattern in the "phi" equal zero plane (north-south plane). The "x's" are experimentally measured values. The values were measured from a Boeing 707 aircraft at 2900 ft (8.84 km). The plane was flown over the A.O. array on a north-south line, while the heater was operating at 5.1 MHz. The plot shows that the array pattern obtained by the combination of pattern multiplication and numerical techniques is a good approximation of the A.O. array pattern.

Figures (1-10) and (1-11) are plots of the directivity pattern of the A.O. heater array for 3.17 MHz and 5.1 MHz respectively. Each plot is the variation of the directive gain with "theta" in a constant "phi" plane. "Phi" is varied in 5 degree steps from 0 degrees to 180 degrees. Two additional "phi" plane patterns have been plotted in figures (1-12) and (1-13). These two patterns are the directivity patterns in the "phi" equal 121.5° and 146° planes respectively. These patterns are in planes corresponding to the direction of Los Canos and the A.O., respectively. The programs used to make the directivity patterns are given in Appendix I, programs 4 and 5 for 3.17 MHz and 5.1 MHz respectively.
Figure 1-9 Comparison of experimental and theoretical patterns
Figure 1-10  Directive gain pattern for Arecibo Observatory HF heating array. Frequency = 3.17 MHz.
PHI= 25.0 DEG. W

HEATER GAIN (DB)

0 (DEGREES)
\text{PHI} = 45.0 \text{ DEG. W}

\text{HEATER GAIN (DB)}

\text{\text{THETA}} 0 (\text{DEGREES})

-30 -20 -10 0 10 20 30

-90 -60 -30 0 30 60 90
Figure 1-11  Directive gain pattern for Arecibo Observatory HF heating array. Frequency = 5.1 MHz.
PHI = 10.0 DEG. W

HEATER GAIN (DB)

THETA 0 (DEGREES)
EFFECT OF HF HEATING ARRAY DIRECTIVITY PATTERN ON THE FREQUENCY RESPONSE 0...(U) PENNSYLVANIA STATE UNIV UNIVERSITY PARK IONOSPHERE RESEARCH L.
UNCLASSIFIED K J CARROLL ET AL. JAN 83 PSU-IRL-SCI-475 F/G 20/34 NL
PHI = 135.0 DEG. W

HEATER GAIN (DB)
PHI = 170.0 DEG. W

HEATER GAIN (DB)
Figure 1-12.a Directive gain pattern in direction of Los Canos (3.17 MHz)
Figure 1-12.b Directive gain pattern in direction of Los Canos (5.1MHz)
Figure 1-13.a: Directive gain pattern in direction of Arecibo Observatory (3.17 MHz)
Figure 1-13.b Directive gain pattern in direction of Arecibo Observatory (5.1 MHz)
Grating lobes (lobes which have the same intensity as the main beam\(^2\)) in the antenna factor will occur when both the numerator and denominator of both terms of equation (1-15) are zero. This will occur when equation (1-20) is satisfied.

\[
\frac{\beta}{d/2} \sin \theta \sin \phi = 0 \text{ or } \pi \\
\frac{\beta}{d/2} \sin \theta \cos \phi = 0 \text{ or } \pi
\]  

(1-20)

For the A.O. array \(d/2\) is equal to 2.82 and 4.54 for 3.17 and 5.1 MHz respectively. Since \(\sin \theta \sin \phi\) and \(\sin \theta \cos \phi\) are never larger than 1, the 3.17 MHz pattern does not and should not have grating lobes. However, grating lobes will be present in the 5.1 MHz pattern because 4.54 is larger than \(\pi\).

These grating lobes will occur for the angles given in table II. As can be seen in the plots of figure (1-11) major lobes do occur at these angles. They are attenuated when they are multiplied by the elemental pattern during the calculations of the total array pattern.

<table>
<thead>
<tr>
<th>(\theta (\text{deg}))</th>
<th>(\phi (\text{deg}))</th>
</tr>
</thead>
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<tr>
<td>43.8</td>
<td>0</td>
</tr>
<tr>
<td>43.8</td>
<td>90</td>
</tr>
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<td>43.8</td>
<td>180</td>
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<tr>
<td>43.8</td>
<td>270</td>
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</tr>
<tr>
<td>78.1</td>
<td>225</td>
</tr>
<tr>
<td>78.1</td>
<td>315</td>
</tr>
</tbody>
</table>

Table II. Location of Grating Lobes in 5.1 MHz pattern

ELF/VLF ARRAY MODEL

Having established a directive gain pattern, it remains to relate the pattern to the heating of the ionosphere. Richardson\(^8\) has shown that the largest change in conductivity caused by the heating occurs
at approximately a 70 km altitude. As a zero order approximation, the heating pattern can be projected on a plane located at a 70 km altitude. The location and relative intensity of the major heated regions can be found. By placing elementary dipoles with the same relative amplitude of current at the respective heated regions, a field intensity at a receiving site on the ground can be calculated.

A correction factor is needed to project the pattern onto a 70 km altitude plane. The pattern shows the relative distribution of the power on a spherical surface of radius "R." Since the distance to a plane increases when "theta" is greater than zero, the power density on the plane will decrease from that indicated by the pattern. The power is being spread over a larger spherical surface as "R" is increased. The projection of the pattern on a plane surface requires multiplying the pattern by an attenuation factor of $\cos^2 \theta$. This attenuation is plotted as a function of "theta" in figure (1-14).

Programs 6 and 7 in Appendix I were used to compute a pattern on a square section of a plane (122 km north and south by 122 km east and west of the main beam) at a 70 km altitude. The data were then plotted using Statistical Analysis System (SAS). (9,10) Figures (1-15) and (1-16) show the relative power density on a 70 km altitude plane for 3.17 MHz and 5.1 MHz respectively. Only levels above that of an isotropic radiator (0 db) were plotted.

The 122 km dimension of the plotted data corresponds to an angle of "theta" equal to 60.2 degrees. The major lobes created by the antenna factor grating lobes at 78.1 degrees are not seen in figure (1-16). These lobes are for the most part attenuated to the zero reference level by the long propagation path. Table III gives the
Figure 1-14  Propagation loss due to power spreading
RELATIVE POWER AT 70 KM

Figure 1-15 Relative HF power above isotropic on a 70 km altitude plane (3.17 MHz)
RELATIVE POWER AT 70 KM

Figure 1-16 Relative HF power above isotropic on a 70 km altitude plane (5.1 MHz)
<table>
<thead>
<tr>
<th>(deg)</th>
<th>(deg)</th>
<th>Directive Gain (db)</th>
<th>Path Attenuation (db)</th>
<th>Relative Power Density at 70 km altitude (db)</th>
<th>Location Center x (km)</th>
<th>y (km)</th>
</tr>
</thead>
<tbody>
<tr>
<td>77</td>
<td>45</td>
<td>6.251</td>
<td>-12.96</td>
<td>-6.71</td>
<td>214.4</td>
<td>214.4</td>
</tr>
<tr>
<td>71</td>
<td>135</td>
<td>7.568</td>
<td>-9.75</td>
<td>-2.18</td>
<td>143.8</td>
<td>143.8</td>
</tr>
<tr>
<td>72</td>
<td>225</td>
<td>12.486</td>
<td>-10.20</td>
<td>2.29</td>
<td>152.3</td>
<td>152.3</td>
</tr>
<tr>
<td>73</td>
<td>315</td>
<td>11.399</td>
<td>-10.68</td>
<td>.72</td>
<td>161.9</td>
<td>161.9</td>
</tr>
</tbody>
</table>

Table III. Grating lobes location and power density on 70 km altitude plane.
location and relative power density of these lobes on the 70 km altitude plane.

Data on the lobes shown in figures (1-15) and (1-16) are given in tables IV and V. The current was assumed to be proportional to the square root of the power density. To find the average current per unit length for each lobe, the current was integrated over the area of the plane disturbed by the lobe and divided by the length of the region. The current distribution on the plane was approximated using a pyramid with a quadrilateral as a base. The length and width of the heated region are the diagonals of the quadrilateral base; the peak current is the altitude. The volume of the pyramid is equal to one third of the area of the base times the altitude.\(^{10}\) The area of the quadrilateral base is one half of the product of the diagonals times the sine of the angle between the diagonals.\(^{11}\) So the formula for calculating the average current per unit length becomes equation (1-21).

\[
I_{av} = \frac{1}{3} \frac{(1/2)a \cdot b \sin 90\degree (I_p/a) = (1/6) b I_p}{a = \text{length (diagonal)} \quad b = \text{width (diagonal)} \quad I_p = \text{peak current (altitude)}}
\]

In this zero order approximation each of the lobes is represented as an elementary current element source. The source has a current equal to the average current calculated in equation (1-21), a length equal to the length of the lobe, and is located at the \(x, y\) coordinates of the peak power density for the lobe on the 70 km altitude plane. The location, length, and average current are summarized in tables IV and V.
<table>
<thead>
<tr>
<th>Location x(km) y(km)</th>
<th>$P_d$ Density (db)</th>
<th>$a$ Length (km)</th>
<th>$b$ Width (km)</th>
<th>$I_p$ Peak Current (10$P_d/20$)</th>
<th>$I_{av}$ Average Current ($1/6 \cdot a \cdot b \cdot I_p$)</th>
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</thead>
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<tr>
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<td>7</td>
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</tr>
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<td>26</td>
<td>9</td>
<td>1.967</td>
<td>2.951</td>
</tr>
<tr>
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<td>32</td>
<td>9</td>
<td>3.647</td>
<td>5.471</td>
</tr>
<tr>
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<td>3.647</td>
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</tr>
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<td>9</td>
<td>1.967</td>
<td>2.951</td>
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<td>16</td>
<td>7</td>
<td>1.206</td>
<td>1.407</td>
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Table IV. Lobe Statistics for Frequency = 3.17 MHz
<table>
<thead>
<tr>
<th>Location x(km)</th>
<th>Location y(km)</th>
<th>Density Pd (db)</th>
<th>a Length (km)</th>
<th>b Width (km)</th>
<th>Ip Peak Current (10^4/20)</th>
<th>Iav Average Current (1/6 a·b·I_p)/a</th>
</tr>
</thead>
<tbody>
<tr>
<td>-60.5</td>
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<td>54</td>
<td>14</td>
<td>5.970</td>
<td>13.93</td>
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<td>14</td>
<td>3.214</td>
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<td>10</td>
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<td>4.217</td>
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<td>65</td>
<td>14.301</td>
<td>28</td>
<td>28</td>
<td>5.189</td>
<td>24.213</td>
</tr>
</tbody>
</table>

Table V. Lobe Statistics for Frequency = 5.1 MHz.
The magnetic field due to a current element is given in equation (1-22). The source is located at the origin and along the Z' axis. \( H_\phi' \) is the magnetic field intensity in the source coordinate system \( X', Y', Z' \). Since \( H_\phi' \) is a vector and there are a number of sources at different locations whose fields need to be superimposed at the observation point, it would be beneficial to translate the fields to the observation frame of reference, \( X, Y, Z \).

\[
H_\phi' = \left( \frac{I dl}{4\pi r'} \right) \sin \theta' e^{-j\beta r'} \left[ j\beta + \left( \frac{1}{r'} \right) \right]
\]

The source and observation coordinate systems are shown in figure (1-17). The source is located at point \( O(X_0, Y_0, Z_0) \) and along the \( Z' \)-axis, which is parallel to the \( X \)-axis in the observation point coordinate system. The \( Y' \)- and \( Y' \)-axis are also parallel and the \( X' \)-axis is in the negative z direction. From the geometry of the figure the relationships given in equation (1-23) can be found. In addition, using the transformation from spherical coordinates to Cartesian coordinates, equation (1-24), the expression for the magnetic field, equation (1-22), can be transformed to the observation point coordinates.

\[
X' = X - X_0
Y' = Y - Y_0
Z' = Z - Z_0
r' = \sqrt{(X - X_0)^2 + (Y - Y_0)^2 + (Z - Z_0)^2}
\]

\[
\sin \theta' = \frac{\sqrt{(Z - Z_0)^2 + (Y - Y_0)^2}}{r'}
\]

\[
H_y = H_y' = \frac{H_\phi'(x')}{\sqrt{x'^2 + y'^2}}
- H_z = H_z' = \frac{H_\phi'(y')}{\sqrt{y'^2 + z'^2}}
H_x = H_x' = 0
\]

The phase term \( e^{-j\beta r'} \) is important when performing the superposition of the fields from all of the sources at the observation point.
Figure 1-17 Relative orientation of observation and source coordinates
point. Small differences in the value of $r'$ can be magnified by causing a significant change in the term's value. Thus it would affect the summation of the real and imaginary parts of all the sources.

An additional term must be added to $Br'$ to maintain the proper relationship between the sources. This is because the HF heating pulse must travel different path lengths to the source region. The difference in path lengths causes a time delay between the sources, equation (1-25). This delay can be expressed in terms of an additional path length $\Delta R$, equation (1-26). This allows the phase term, $e^{-j\beta r'}$, in equation (1-22) to be expressed as $e^{-j\beta (r'+\Delta R)}$. By combining the phase delay, equations (1-23) and (1-24) into equation (1-22), an expression for the magnetic field intensity at the point of observation and in terms of the observation point coordinate system can be found, equation (1-27).

Time delay = \( \frac{(Z_0^2 + Y_0^2 + X_0^2)^{1/2} - 70}{c} \)  

(1-25)

Phase delay = \( \frac{(Z_0^2 + Y_0^2 + X_0^2)^{1/2} - 70}{c} = \frac{(2\pi/\lambda)\left((Z_0^2 + Y_0^2 + X_0^2)^{1/2} - 70\right)}{c} \)  

(1-26)

\[
\bar{H} = \frac{I_1[(ZP-ZO)^2+(YP-YO)^2]^{1/2}}{4\pi (XP-XO)^2+(YP-YO)^2+(ZP-ZO)^2} e^{-j\beta [(XP-XO)^2+(YP-YO)^2]^{1/2}} \\
\left(\frac{(ZP-ZO)^2}{(XP-XO)^2+(YP-YO)^2}\right)^{1/2} \frac{\hat{a}_x}{\bar{a}_y} + \left(\frac{(YP-YO)^2}{(XP-XO)^2+(YP-YO)^2}\right)^{1/2} \frac{\hat{a}_z}{\bar{a}_y} \right] \\
\times \left[ \frac{j\beta + \frac{1}{[(XP-XO)^2+(YP-YO)^2+(ZP-ZO)^2]^{1/2}}}{} \right] 
\]

(1-27)
Programs 8 and 9 Appendix I were written to calculate the strength of the magnetic field at an observation point due to the two source patterns of figures (1-15) and (1-16). For each of the two cases a current element was placed at the location of the lobes, and a total magnetic field was calculated at an observation point corresponding to Los Canos. The calculations were carried out for the frequencies listed in table VI. These frequencies correspond to the frequencies used during the ELF/VLF experiments conducted in Puerto Rico. Figure (1-18) shows the result of the calculation. It is a plot of relative magnetic field strength as a function of frequency.

<table>
<thead>
<tr>
<th>Frequency (kHz)</th>
<th>Frequency (kHz)</th>
</tr>
</thead>
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</tr>
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</tr>
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</tr>
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<tr>
<td></td>
<td>5.0</td>
</tr>
</tbody>
</table>

Table VI. Experimental Frequencies

In order to determine the effect of the lobes on the value of the magnetic field at the observation point, a calculation was done with only the main lobe acting as the source. The plot of the relative field strength as a function of frequency is given in figure (1-19). A comparison of figures (1-18) and (1-19) shows that the lobes have a significant effect on determining the frequency response of the ELF/VLF radiating source. The location of the relative maximums and minimums in both 5.1 MHz and 3.17 MHz generated ELF/VLF response have shifted in frequency. A significant reduction in the field strength occurs between 2.5 to 4.5 kHz for the 5.1 MHz generated Y component.
Figure 1-18 Current element array VLF/ELF response
Figure 1-19 Main beam current element VLF/ELF response
In addition to computing the relative field strengths for the test frequencies, a calculation was made in 100 Hz frequency steps from 500 Hz to 5 kHz. The result is plotted in figure (1-20). As can readily be seen from the figure, two deep minimums occur for the 5.1 MHz generated ELF/VLF. One occurs between 600 Hz and 700 Hz, and the other occurs between 3400 Hz and 3600 Hz. The 3.17 MHz generated ELF/VLF has only one deep minimum, which is located between 700 Hz and 900 Hz.

A calculation was made for an alternate receiving site. This site is located at Salinas, Puerto Rico, 17.98° N and 66.30° W geographic latitude and longitude respectively. The frequency response results for the two HF heating patterns are shown in figure (1-21). Comparison of figures (1-20) and (1-21) shows the changes in the response due to the different propagation paths. The first striking difference is the disappearance of the null in between 3 and 4 KHz in the 5.1 MHz pattern generated VLF. Second, the relative maximums and minimums for the Salinas location have been shifted to a lower frequency. In general, the relative field strengths are lower for the Salinas site due to the increase in propagation distance.

CONCLUSION

This completes the description of the zero order approximation. In summary, a pattern for the Arecibo Observatory has been calculated using the technique of pattern multiplication and AMP. The calculated pattern for the "PHI" equal "0" plane compares with the experimentally measured pattern. The pattern shows that there is enough power in the side lobes and grating lobes to cause significant heating at a 70 km
Figure 1-20 Current element array VLF/ELF response. Y component of magnetic field
Figure 1-21 Current element array VLF/ELF response at Salinas. Y component of magnetic field
altitude. Using the HF antenna pattern, a zero order approximation of the model for the ELF/VLF radiating system was determined. The frequency responses of the ELF/VLF radiation system for observation points corresponding to Los Canos and Salinas were calculated. The calculations determined that the regions heated by the side lobes and grating lobes have a significant effect on the strength of the received signal.

REFERENCES


7. Preliminary Comparison of Theoretical and Observed Antenna Patterns for the Arecibo HF Heating Facility, Rice University, Houston, Texas, Feb. 18, 1982.


Appendix I

Computer Programs
Program I

AMP data file
Program II

Total Array Pattern Simpson Integration, 3.17 MHz
Program III

Total Array Pattern Simpson Integration, 5.1 MHz
// EXEC FMCG
// *PJ SERVICE=OFFER
// SYSIN PP *

DIMENSION A(144)
AX=0.
AY=1.570796
AY=0.
AY=2.93185
C NHT MUSF BE EVEN AND GREATER THAN 4. NHT+1 IS # OF THEA ANGLES.
C NHT=M0
C NHP MUSF BE EVEN AND GREATER THAN 4. NHP+1 IS # OF PHI ANGLES
NHP=144
T0=0.
HT=(RX-AX)/NHT
HFR(H=AY)/NHP
X=AX
Y=AY
NO 300 J=1.144
300 CONTINUE
C NHP=0. ANGLES THETA-4. (FIRST AND LAST TWO OF SERIES PLUS SECOND IN LOOP)
C NHP=144.
C NHP=0. ANGLES PHI AT WHICH INTEGRALS IN THETA ARE EVALUATED.
NHP=1
H 100 2J=1,144
C=FLUAT(1)
A(J)=PI(X,Y)
H 100 1+1=1.144
C=FLUAT(1)
X=X*CHR
A(J)=A(J)*PDR(X,Y)
A(J)=A(J)+1.144
100 CONTINUE
X=X+HT*HT
A(J)=A(J)*PDR(X,Y)
X=X+HT*HT
A(J)=A(J)*PDR(X,Y)
X=X
X=X+HT
200 CONTINUE
300 CONTINUE
T=0.
T=0.
NHP-2
H 400 J=1,2NP+2
T=0.
T=0.
1=1+1=1.
A(J)
400 CONTINUE
A(J)=A(J)*A(J)
A(J)=A(J)*A(J)
A(J)=A(J)XNP+1
WRITE(6,400) ANS(A(J),J),1.144
400 PRINT THE TOTAL INTEGRAL. CE = A.PAR[1,114.1,115.1,111,1,146,10,11,13,111]
CSTOP
CINIT
CINIT PDR(X,Y)
H 10.1 PDR(X,Y)
H 10.1 PDR(X,Y)
X=X
X=X
X=X
Program IV

Directive Gain Pattern Calculation and Plotting, 3.17MHz
CALL ANGLE(r, R, 0, 1) F300
   R = 2
   X = 4.00
   CALL ANGLE(X, X, 0, 1)
   X = 4.00
   CALL ANGLE(X, X, 0, 1)
400 CONTINUE
400 CALL FINISH
   CONTINUE
   STOP
FUNCTION ANGLE(THETA, PHII)
THETA = THETA
PHII = PHII
RETURN
END
COMMON ZMCEK7,XVAL(11),YVAL(11),YCH(11),XCH(11)
DIMENSION XV(11),YVN(11)
INTEGER YSCALE,AXIS
C NMIN = # OF DIVISIONS ON X AXIS
C MIN = VALUE OF X AT ORIGIN
C XMAX = VALUE OF X AT END OF AXIS
C YMIN = VALUE OF Y AT ORIGIN
C YMAX = VALUE OF Y AT END OF AXIS
C NPTS = # OF POINTS TO BE PLATED, MUST BE LESS THAN 101
CALL PLOY(662,0,1)
CALL START
CALL PLUT(0.0,0.0,0)
CALL PLUT(0.0,1.0,0)
CALL PLUT(0.0,0.0,0)
CALL PLUT(11.0,0.0,0)
CALL PLUT(11.0,0.0,0)
CALL PLUT(11.0,0.0,0)
CALL PLUT(0.0,0.0,0)
CALL PLUT(0.0,0.0,0)
CALL NEWPEN(11)
C DEFINE NEW ORIGIN FOR PLOT AXIS
CALL PLUT(2.0,0.0,0.0)
CALL FACTIM(1,0)
C DRAW AXIS
ENTRY MXPT(INIM,NUMN,XMIN,XMAX,YMIN,YMAX,NPNTS)
CALL RECT(0.0,0.0,10.0,0.0)
C DRAW MARKS ON AXIS. SIZE DIVISION SIZE IN INCHES X AXIS.
C SIZE DIVISION SIZE IN INCHES Y AXIS
C SIZE = FLOAT(NUM)
DSIZE=5.0/FLOAT(NUM)
X1=0.0
Y1=0.0
Y2=1.0
I1=4.0
I2=4.0
I3=4.0
B=200.0/IM
XHASE=FLOAT(IM-1)*C SIZE
X=XYHASE
CALL PLUT(X,Y1,1)
CALL PLUT(X,Y2,2)
CALL PLTEN(I1,NUMN)
YHASE=FLOAT(NUMN-1)*C SIZE
CALL PLUT(X1,YHASE,1)
CALL PLUT(X2,YHASE,2)
CALL CONTINUE
X1=7.0
X2=1.0
Y2=1.0
Y2=1.0
CALL CONTINUE
X=1.0
Y=0.0
CALL CONTINUE
C PUT SCALE ON Y AXIS
C UNIT = UNIT FOR UNIT IN Y AXIS
C UNIT = (YMAX-YMIN)/FLOAT(NUMN)
YCALC=KAT((YMIN-YCALC)/KAT+1.0)
YCALC=KAT((YMIN-YCALC)/KAT+1.0)
YCALC=KAT((YMIN-YCALC)/KAT+1.0)
YCALC=KAT((YMIN-YCALC)/KAT+1.0)
RETURN
END
ACTIONS
DATA INIT,IN,INHH,INH,INH
DATA EXIT,EXIT,EXIT,EXIT
/.
END

//DATA, F37Foot on WIN,ref=NEW,JRC,LC,IN,DISP=NEW,KEV)
// INH,NRN,JRC,LC,INIT,INIT,SPACE=CYL,(1,1),LSF)
// INH,(RECH,FR,RECL,RL,RL,FL,FL,FL)
//DATA, INHIT on 1
Program V

Directive Gain Pattern Calculation and Plotting, 5.1 MHz
C. 4 CHARACTERS PER INTEGER IN INTEGER ARRAYS. SEE BLOCK DATA SUFFIX:

C   DIMENSION ATHA1(I),ATH1(I),CHECK1,CHKP1

C   CHECK1(I) = 0, 1
C   CHECK2(I) = 0, 1
C   CHECK3(I) = 0, 1
C   CHECK4(I) = 0, 1
C   CHECK5(I) = 0, 1
C   CHECK6(I) = 0, 1
C   CHECK7(I) = 0, 1
C   CHECK8(I) = 0, 1
C   CHECK9(I) = 0, 1
C   CHECK10(I) = 0, 1
C   CHECK11(I) = 0, 1
C   CHECK12(I) = 0, 1

C   PHI = 1.0*PI + 40.0
C   PHI = 0.0*PI + 40.0
C   PHI = 2.0*PI + 40.0
C   PHI = 3.0*PI + 40.0
C   PHI = 4.0*PI + 40.0
C   PHI = 5.0*PI + 40.0
C   PHI = 6.0*PI + 40.0

C   ATHA1(I) = 0, 1
C   ATH1(I) = 0, 1
C   THR = 0.0*PI + 40.0

C   500 CONTINUE
C   NI = 2000 JP = 1, 2
C   THE1 = 0.0*PI + 40.0
C   THETA = 0.0*PI + 40.0
C   THETACH1
C   C FIND DIAL FIFTH, 9.82 IS FACTOR TO NORMALIZE TO GAIN IN OVER INTEGRING
C   PHI = 1.0*PI + 40.0
C   PHI = 2.0*PI + 40.0
C   PHI = 3.0*PI + 40.0
C   PHI = 4.0*PI + 40.0
C   PHI = 5.0*PI + 40.0
C   PHI = 6.0*PI + 40.0

C   ATHA1(I) = 0, 1
C   ATH1(I) = 0, 1
C   THR = 0.0*PI + 40.0

C   100 CONTINUE
C   WRITE*(4,200)(ATHA1,PHI,ATH1,JP,THETA,THR) = 1000
C   WRITE*(4,1000)(ATHA1,ATH1,JP,THETA,THR)
C   WRITE*(4,200)
C   WRITE*(4,1000)

C   200 FORMAT(5E18.12,F18.6)
C   100 FORMAT(1X,F20.6)
C   300 FORMAT(1X)
C   400 FORMAT(1X,F20.6)
C   500 FORMAT(1X,F20.6)
C   600 FORMAT(1X,F20.6)
C   700 FORMAT(1X,F20.6)
C   800 FORMAT(1X,F20.6)
C   900 FORMAT(1X,F20.6)
C   1000 FORMAT(1X,F20.6)
C   1100 FORMAT(1X,F20.6)
C   1200 FORMAT(1X,F20.6)
C   1300 FORMAT(1X,F20.6)
C   1400 FORMAT(1X,F20.6)
C   1500 FORMAT(1X,F20.6)
C   1600 FORMAT(1X,F20.6)
IFJ (J1 + J1) CALL MTFPL ((N.A.-40.,-40.-30.,-30.,40.,40.),
IFJ (J1 + J1) GO TO 2100

2100 CALL MTFPL ((N.A.-40.,-40.,-30.,-30.,30.,30.),

CALL LETFM (X.Y.,15.,PHI(1.0,0,0)),
CALL MTFPL ((X.Y.,PHI(1.0,0,0)),

Y=X,310
CALL LETFM (X.Y.,15.,PHI(1.0,0,0)),
CALL NEWFIN(1)

400 CONTINUE

CALL FINISH

CONTINUE

STOP

END

FUNCTION AFT (THETA, PHI)
THETA=THETA/180.,180.,360.
PHI=PHI/180.,180.,360.

C AFT=2*PHI/(C)*((/2., AResistance BETWEEN RADIATORS,)

C AFT.THEP*PHI,
C
C AFT=180.(PHI)*COS(THETA)
C
C AFT=(SIN(AFT4*STHFTA*SPHI)/SIN(AFT4*STHFTA*SPHI)) *SIN(RFTHITA*STHFTA*SPHI)

C IF (ARS(THTA-0.), L.F., 2, H.R.,ARS(THETA-180.), L.F., 2) GO TO 100

GO TO 200

200 AFT=ARS(SIN(AFT4*STHFTA*SPHI)/SIN(AFT4*STHFTA*SPHI))

300 IF (ARS(THETA-0.), L.F., 2, H.R.,ARS(THETA-180.), L.F., 2) GO TO 600

GO TO 200

AFT=ARS(SIN(AFT4*STHFTA*SPHI)/SIN(AFT4*STHFTA*SPHI))

C IF (ARS(AFT), L.F., 10) GO TO 10

AFT=20.,(ARSHA(AFT))

RETURN

C 10 AFT=90.
C RETURN

FUNCTION ELF (THETA, PHI)

C IF (ARS(AFT), L.F., 10) GO TO 10

C RETURN

FUNCTION PFF (THETA, PHI)

C IF (ARS(AFT), L.F., 10) GO TO 10

C RETURN

FUNCTION
CALL PLT(3,6.21)
CALL PLT(3,6.21)
CALL PLT(3,6.21)
CALL PLT(3,6.21)

_740_ CONTINUE
CALL PLT(3,6.21)
CALL PLT(3,6.21)
CALL PLT(3,6.21)

YN = 300 = 1, NMIN)
YN = FLAT(NMIN-1), NMIN) + 1
CALL PLT(3,6.21)
CALL PLT(3,6.21)

_400_ CONTINUE
X1 = 7.0
Y1 = 5.0

CALL PLT(3,6.21)

C. PLOT SCALF ON Y AXIS
C. UNITS = UNITS PER DIV ON Y AXIS
UNITX = (YMAX-YMIN)/FLOAT(NMIN))
YSCLAF = INT(XMAX/XMIN)*.5
IF (YMIN, LT, 0) YSCLAF=YSCLAF*(0-1)

_724_ CONTINUE
X1 = 1.0
Y1 = 1.0

CALL LETTER(XT, YT, 1.5, YMCH, 90.0, 1.0)
C. PLOT SCALF ON X AXIS
C. UNITS = UNITS PER DIV ON X AXIS
UNITX = (XMAX-XMIN)/FLOAT(NMIN))
XSCALF = INT(XMAX/XMIN)*.5

_724_ CONTINUE
X1 = 1.0
Y1 = 1.0

CALL LETTER(XT, YT, 1.5, YMCH, 90.0, 1.0)
C. PLOT DATA
C. UNITS = UNITS PER DIV ON Y AXIS
UNITX = (YMAX-YMIN)/FLOAT(NMIN))
UNITX = (YMAX-YMIN)/FLOAT(NMIN))
UNITX = (YMAX-YMIN)/FLOAT(NMIN))
UNITX = (YMAX-YMIN)/FLOAT(NMIN))
UNITX = (YMAX-YMIN)/FLOAT(NMIN))
UNITX = (YMAX-YMIN)/FLOAT(NMIN))
UNITX = (YMAX-YMIN)/FLOAT(NMIN))
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UNITX = (YMAX-YMIN)/FLOAT(NMIN))
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UNITX = (YMAX-YMIN)/FLOAT(NMIN))
UNITX = (YMAX-YMIN)/FLOAT(NMIN))
UNITX = (YMAX-YMIN)/FLOAT(NMIN))
UNITX = (YMAX-YMIN)/FLOAT(NMIN))
UNITX = (YMAX-YMIN)/FLOAT(NMIN))
UNITX = (YMAX-YMIN)/FLOAT(NMIN))
Program VI

*Program to compute relative HF power above isotropic on a 70km altitude plane. Frequency=3.17 MHz

*note: Function subprograms AF (Theta, Phi) and ELF (Theta, Phi) are not shown. They are the same as in program IV.
C-4

r.
N. V.

Ca
Program VII

*Program to compute relative HF power above isotropic on a 70km altitude plane. Frequency=5.1MHz.

*note: Function subprograms AF (Theta,Phi) and ELF (Theta, Phi) are not shown. They are the same as those in program V.
Program VIII

Program to compute relative magnetic field strength at observation point for ionospheric ELF/VLF current element array.

3.17 HF pattern.
Program IX

Program to compute relative magnetic field strength of an observation point for an ionspheric ELF/VLF current element array. 5.1 MHz HF pattern.
APPENDIX II

USE OF CHEBYSHEV'S POLYNOMIALS TO SIMPLIFY ANTENNA FACTORS

The Chebyshev's polynomials are the solution to the Chebyshev differential equation (A-1). The solution has the form of equation (A-2)

\[(1 - x^2) \frac{d^2 y}{dx^2} - x \frac{dy}{dx} + n^2 y = 0 \tag{A-1}\]

With a recursive formula given in equation (A-3)

\[T_m(x) = \cos (m \cos^{-1}x) \quad m \geq 0, \lvert x \rvert < 1 \tag{A-2}\]

\[T_{m+1}(x) = 2x T_m(x) - T_{m-1}(x) \tag{A-3}\]

Equation (A-4) and (A-5) follow from substitution of 0 and 1 for \(m\) in (A-2)

\[T_0 = 1 \tag{A-4}\]

\[T_1 = x \tag{A-5}\]

By using the recursion relationship (A-3), the following polynomials are obtained:

\[T_2(x) = 2x^2 - 1\]
\[T_3(x) = 4x^3 - 3x\]
\[T_4(x) = 8x^4 - 8x^2 + 1\]
\[T_5(x) = 16x^5 - 20x^3 + 5x\]
\[T_6(x) = 32x^6 - 48x^4 + 18x^2 - 1\]
\[T_7(x) = 64x^7 - 112x^5 + 56x^3 - 7x\]
\[T_8(x) = 128x^8 - 256x^6 + 160x^4 - 32x^2 + 1\]

Let "w" equal "\cos^{-1}x" then "x" is equal to "\cos w."

Substitution for "x" in equation (A-2) gives (A-6).

\[T_m(\cos w) = \cos (m w) \tag{A-6}\]
By using equation (A-6) with the polynomials, trigonometric identities can be found for expressing "\( \cos (m \, w) \)" or "\( \sin (m \, w) \)" in terms of "\( \sin w \)" and "\( \cos w \)." For example, using "\( T_2(x) \)" and letting "\( x \)" equal "\( \cos w \)," equation (A-7) is obtained.

\[
T_2(\cos w) = \cos (2w) = 2 (\cos^2 w) - 1 \tag{A-7}
\]

Expressions for \( \sin(m \, w) \) can be obtained by taking the derivative of the "\( \cos (m \, w) \)" identity. Equation (A-8) was obtained by taking the derivative of (A-7).

\[
\sin (2w) = 2 (\cos w) \sin w \tag{A-8}
\]

To obtain the antenna factors in the form of equation (1-13) and (1-14), equation (1-7) must be expanded. Equation (A-9) is the expansion for the 8-element case. Let \( w = \beta (d/2) \sin \theta \).

\[
AF = \sum_{m=1,3,5...}^{8-1} \cos (m \beta d/2 \sin \theta) = \cos(w) + \cos (3w) + \cos (5w) + \cos (7w) \tag{A-9}
\]

Use the Chebyshev's polynomials to find trigonometric identities to reduce (A-9) into an equation in terms of powers of \( \cos w \). These identities are given in equation (A-10).

\[
\begin{align*}
T_1 &= \cos w = \cos w \\
T_3 &= \cos 3w = 4\cos^3 w - 3 \cos w \\
T_5 &= \cos 5w = 16\cos^5 w - 20\cos^3 w + 5\cos w \\
T_7 &= \cos 7w = 64\cos^7 w - 112\cos^5 w + 56\cos^3 w - 7\cos w
\end{align*} \tag{A-10}
\]

Substituting (A-10) into (A-9), the expression for the antenna factor becomes (A-11).

\[
AF = 64\cos^7 w - 96\cos^5 w + 40\cos^3 w - 4 \cos w \tag{A-11}
\]
An identity for "cos(8w)" can be found from polynomial "T_8." By taking the derivative of "cos (8w)" , an identity for "sin (8w)" can be found, (A-12).

\[
\sin 8w = 1024 \cos^7w \sin w - 1536 \cos^5w \sin w + 640 \cos^3w \sin w - 64 \cos w \sin w = 16 \sin w (64\cos^7w - 96 \cos^5w + 40 \cos^3w - 4\cos w)
\]

Equation (A-12) can be rearranged into (A-13).

\[
\frac{1}{16} \frac{\sin 8w}{\sin w} = 64 \cos^7w - 96 \cos^5w + 40 \cos^3w - 4 \cos w
\]

Equation (A-14) can be obtained by substituting equation (A-13) into (A-11).

\[
AF = \frac{1}{16} \frac{\sin 8w}{\sin w}
\]

This is identical with equation (1-14) for the 8-element array antenna factor with the exception of the 1/16 constant in (A-14). The constant can be neglected at this point, due to the fact that when the directive gain is calculated, the AF will be normalized.

A similar calculation can be performed to obtain the 4-element array factor, (1-13).
APPENDIX III
REVISED ARECIBO HF ANTENNA ARRAY GEOMETRY

After completion of the work described in the main body of this report, a preliminary copy was reviewed by the Arecibo Observatory. At this time the Arecibo Observatory made available additional information about the HF array. The following differences were reported between the model used in the main text and the actual A.O. array:

1. The $\tau$ for the antenna is .774.

2. The pyramid structure is elevated five feet above ground.

3. The feed point of each face is capacitively loaded.

This appendix presents the information provided by the A.O. and discusses its possible effects on the work presented in the main body of this report.

The $\tau$ of the NPLA is .774. It is obtained by taking the ratio of the lengths of two consecutive elements of a face on the same side of the feed line. Table III-1 provides a list of the element lengths and the length of the feed line to that element. The even numbered elements represent the dimensions for the scaled faces of the pyramid structure.

Figure (III-1) provides a view of the structure at the vertex of the pyramid. It was concluded from this figure that height of the vertex of the pyramid was 1.524 m (60 inches).
<table>
<thead>
<tr>
<th>Element Number</th>
<th>Element Length (m)</th>
<th>Feed Line Length (m)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>25.000</td>
<td>35.704</td>
</tr>
<tr>
<td>2</td>
<td>23.454</td>
<td>33.49</td>
</tr>
<tr>
<td>3</td>
<td>22.001</td>
<td>31.42</td>
</tr>
<tr>
<td>4</td>
<td>20.638</td>
<td>29.47</td>
</tr>
<tr>
<td>5</td>
<td>19.361</td>
<td>27.65</td>
</tr>
<tr>
<td>6</td>
<td>18.163</td>
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<td>7.70</td>
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<td>26</td>
<td>5.060</td>
<td>7.22</td>
</tr>
<tr>
<td>27</td>
<td>4.746</td>
<td>6.78</td>
</tr>
<tr>
<td>28</td>
<td>4.450</td>
<td>6.36</td>
</tr>
<tr>
<td>29</td>
<td>4.176</td>
<td>5.96</td>
</tr>
<tr>
<td>30</td>
<td>3.917</td>
<td>5.59</td>
</tr>
</tbody>
</table>

Table III-1. NPLA Element and Feed Line Lengths Provided by A.O. Even element numbers are for the scaled faces. \( \tau = .774 \).
Figure III-1  Vertex of pyramid for Arecibo Observatory HF non-planar log-periodic array
Figure (III-2) shows a sketch of a feed point of one of the faces of the pyramid. The wires detailed by "A" are in addition to previously used geometry. All of the conducting elements of the antenna consist of three #12 twisted steel wires with 10% aluminum coating.

The additional information on the array elements was combined into a new geometry description for AMP. Discrepancies exist within the new information provided. Figure (III-1) depicts the height of the vertex at five feet, while figure (III-2) shows a six-foot height. Table III-1 and figure (III-2) give different distances between the feed point and the bottom element. It is unclear whether figure (III-2) represents a scaled face or unscaled face of the pyramid and whether the dimensions of the additional wires are also scaled between the two sets of faces. For the new geometry description to be used with AMP the following geometry was decided upon:

1. The height of the vertex is 1.524 m.
2. The lengths of all the feed lines to the elements are all taken from table III-1 (including the length to the bottom element).
3. Figure (III-2) was taken to be an unscaled face of a pyramid. The 144" dimension was taken to be correct and the other dimensions were adjusted to correspond to table III-1.
4. All wires and dimensions from the unscaled face were scaled by the fourth root of $\tau$ ($\tau = 0.774$) for the scaled face. This includes the additional wires shown in figure (III-2).

The final AMP geometry deck incorporating these changes is given in figure (III-3). The computer results of the power gain
Figure III-2 Capacitively loaded feed region for one face of pyramid element in Arecibo Observatory HF heating non-planar log-periodic array.
Figure III-3 AMP geometry deck containing structure modifications: $r = 0.774$, capacitively loaded feed, and elevation of 1.524 m
Figure III-3 (c.jn.) AMP geometry deck containing structure modifications: t = .774, capacitively loaded feed, and elevation of 1.524 m
for the two cases of 3.17 MHz and 5.1 MHz are shown in figures (III-4) and (III-5). In these figures the "x" denotes the results of the new modified geometry given in this appendix, and the "." denotes the results of the "old" geometry used in the main body of this report. Figure (III-4) shows the power gain as a function of "phi" for selected constant values of "theta." These are the same values of "theta" used in figure (1-6).

Figure (III-5) is a plot of power gain as a function of "theta" for selected values of "phi." The selected values of "phi" are the same values of "phi" used in figure (1-7) plus additional values corresponding to the "x" and "y" axis (i.e., φ = 0°, 90°, 180°, 270°).

Examination of figures (III-4) and (III-5) leads to the conclusion that the changes in geometries between the two cases result in only a small difference in power gain for small values of "theta" and large differences for large values of "theta."

It is necessary to determine what effect the new geometry will have on the results of the main body of this report. The elemental power gain for θ < 50° is approximately equal for the two geometries. Since a large portion of the radiated power is contained in this region, it is expected that the directivity of the total array would remain approximately the same. The results shown in figures (1-10) and (1-11) should be approximately the same for θ < 50° but significant differences could occur for θ > 50°.
Figure III-4a  Power gain vs. phi for constant theta. Comparison of "new" and "old" heating array element geometry. Frequency = 3.17 MHz
Figure III-4b  Power gain vs. phi for constant theta. Comparison of "new" and "old" heating array element geometry. Frequency= 5.1 MHz
Figure III-5a.1  Power gain vs. 'theta' for constant 'phi'. Comparison of 'new' and 'old' heating array element geometry. Phi = 0°
Frequency = 3.17 MHz
Figure III-5a.2 Power gain vs. 'theta' for constant 'phi'. Comparison of 'new' and 'old' heating array element geometry. Phi = 90°
Frequency = 3.17 MHz
Figure III-5a.3  Power gain vs. 'theta' for constant 'phi'. Comparison of 'new' and 'old' heating array element geometry. Phi = 180°
Frequency = 3.17 MHz
Figure III-5a.4 Power gain vs. 'theta' for constant 'phi'. Comparison of 'new' and 'old' heating array element geometry. Phi = 270°
Frequency = 3.17 MHz
Figure III-5a.5  Power gain vs. 'theta' for constant 'phi'. Comparison of 'new' and 'old' heating array element geometry. Phi = 40°
Frequency = 3.17 MHz
Figure III-5a.6 Power gain vs. 'theta' for constant 'phi'. Comparison of 'new' and 'old' heating array element geometry. Phi = 150°
Frequency = 3.17 MHz
Figure III-5a.7 Power gain vs. 'theta' for constant 'phi'. Comparison of 'new' and 'old' heating array element geometry. Phi = 240°
Frequency = 3.17 MHz
Figure III-5b.1 Power gain vs. \( \theta \) for constant \( \phi \). Comparison of 'new'
and 'old' heating array element geometry. \( \phi_1 = 0 \)
and frequency = 3.1 MHz.
Figure III-5b.2 Power gain vs. 'theta' for constant 'phi'. Comparison of 'new' and 'old' heating array element geometry. Phi = 90°
Frequency = 5.1 MHz
Figure III-5b.4 Power gain vs. 'theta' for constant 'phi'. Comparison of 'new' and 'old' heating array element geometry. Phi = 270°
Frequency = 5.1 MHz
Figure III-5b.5  Power gain vs. 'theta' for constant 'phi'. Comparison of 'new' and 'old' heating array element geometry. Phi = 130°
Frequency = 5.1 MHz
Figure III-5b.6  Power gain vs. 'theta' for constant 'phi'.  Comparison of 'new' and 'old' heating array element geometry.  Phi = 250°
Frequency = 5.1 MHz
Of particular interest is the differences in directive gain along the "x" and "y" axis. The ELF/VLF source array originates from the HF antenna response along these lines. Table III-2 shows the value of "theta" to the center and furthest edge of the ELF/VLF source region (shown in figures (1-15) and (1-16)) which is furthest from the origin.

<table>
<thead>
<tr>
<th>Frequency</th>
<th>Source Location</th>
<th>Length</th>
<th>Width</th>
<th>Center</th>
<th>Edge</th>
</tr>
</thead>
<tbody>
<tr>
<td>3.17 MHz</td>
<td>63 km 0</td>
<td>24 km</td>
<td>-----</td>
<td>42°</td>
<td>47°</td>
</tr>
<tr>
<td>3.17 MHz</td>
<td>0 38 km</td>
<td></td>
<td>7 km</td>
<td>29°</td>
<td>31°</td>
</tr>
<tr>
<td>5.1 MHz</td>
<td>60 km 0</td>
<td>54 km</td>
<td></td>
<td>41°</td>
<td>51°</td>
</tr>
<tr>
<td>5.1 MHz</td>
<td>0 65 km</td>
<td></td>
<td>28 km</td>
<td>43°</td>
<td>49°</td>
</tr>
</tbody>
</table>

Table III-2. Value for θ to the Source Regions Furthest from Origin.

Since all the ELF/VLF sources are located in a region where "theta" is less than 50°, it is concluded that the new geometry will not significantly affect the zero approximation ELF/VLF array model. The plots of relative field intensity versus ELF/VLF frequency should remain essentially the same. The fundamental conclusion of the main body of the report that the ELF/VLF frequency response is affected by the geometry of the HF heating antenna pattern remains intact.
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Directivity patterns of 0.17 and 0.1 are calculated for the HF antenna array at the high power HF heating facility on Narcissus Rock. The pattern was calculated using pattern multiplication and method of moment techniques. The calculated pattern is shown to be a good approximation of an experimentally measured pattern in the plane of the array. A simple model was used to approximate the effect of the pattern on the frequency response of EDF/SEP signals generated by the VHF heating. The frequency response was determined for two EDF/SEP receiver sites. Results show that EDF/SEP generated by the sites of the \( \phi \) pattern have sufficient strength to create a EDF/SEP interference pattern at receiving locations.