SEISMIC COMMUNICATION IN BASIN AND RANGE PROVINCE VALLEYS

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Seismic Communication in Basin and Range Province Valleys

It has long been recognized that the Earth provides a virtually indestructible path of last resort for communicating with hard buried facilities. There is a need to establish the characteristics for communicating seismically in a landform suitable for MX deployment. We calculate the information rates for the seismic noise and response values measured at representative valley sites within the Basin and Range Province under the optimizing condition of constant reception spectra.
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SEISMIC COMMUNICATION IN BASIN AND RANGE PROVINCE VALLEYS

1. INTRODUCTION

1.1 Statement of Need

It has long been recognized that the Earth provides a virtually indestructible full duplex path for communicating with hard buried facilities (1). Work on the subject has highlighted estimates of channel capacity for communicating in competent rock (2), as well as the performance of specific hardware and coding at short and long ranges (3,4).

There is a need to determine the information capacity for communicating seismically in a landform suitable for basing MX (5). Of paramount interest is the information rate for communicating seismically in deep, dry alluvial valleys of the Basin and Range Province, the leading landform for MX basing. To this end we establish the maximum information rate for sites in Railroad Valley, Steptoe Valley and Jackass Flats, Nevada over a range of source strengths, bandwidths, and distances.

1.2 Approach

We treat the Earth to be a linear, time invariant transmission path defined by its Green's Function. Seismic signals observed at a distant receiver point are delayed, colored versions of those generated by the source masked partly by additive noise. For surface sources and surface observers, seismic transmission at a distance is dominated by waves attenuated by Q losses and cylindrical spreading.

Seismic noise at the candidate sites is well represented as stationary, independent, Gaussian processes convolved with the appropriate site sensitive shaping operators.

Maximum capacity seismic channels are computed after Hartley-Shannon
for the seismic signals and noise terms found in each area. At high signal-
to-noise ratios the optimum transmission channel is tantamount to matched
filtering. The effect of source and receiver arrays on communication is also
considered for the simple case of coherent signals and incoherent additive
noise.

2. FINDINGS

Communication by seismic means is as commonplace as a "knock at the
door"; its feasibility is not in question. What is in question is the capaci-
ty of seismic data links operated at specific source strengths and distances
in the Basin and Range Province Valleys.

Information capacity for a ground path is readily calculated once the
signal and noise spectra are specified. Data links can be optimized by
weighting at the source and/or receiver. In any event, signal distortion due
to seismic propagation can be nullified by a linear operator that is the
inverse of the seismic impulse response, $G(r,t)$.

Seismic waves excited by surface impacts are well represented by
unimodal $Q$ damped surface waves. Under this representation signal spectra can
be generated over any desired range of distances, source strengths and
bandwidths.

The effect of range, bandwidth and source strength on channel capacity
is established for the seismic signals and noise determined at each site. We
find that seismic data links of 2 bits/second are quite realizable for
distances of a few kilometers using sources of less than 100 Newtons rms in
the band $2 \leq f \leq 20$ Hz.

The major uncertainty encountered when estimating capacity is caused by
the uncertainty in $Q$ and the presence of lateral inhomogeneities even in these
seemingly simple valleys.
3. **SEISMIC NOISE IN BASIN VALLEYS**

3.1 **Statistical Classification**

Seismic noise is sensitive both to local ground structure and the kinds and distribution of noise sources. For sites excited by a large number of independent sources, we can anticipate Gaussian attributes. If the noise sources are also well distributed about an observer, the seismic noise field has well defined spacial properties for uniformly layered structures (6).

Figure 1 shows the distribution of the ground's particle velocity during quiet intervals at a site in Steptoe Valley near Ely, Nevada. The plot is arranged to give a straight line relation for Gaussian variates. The observed distribution is well represented by a zero mean, Gaussian process $N(0, s^2), s^2 = 0.02$ (microns/sec)$^2$ (7). In remote areas, seismic noise tends to exhibit Gaussian attributes (8). The noise in the Basin and Range Province sites is not exceptional. Seismic noise in these rural valley areas also tests stationary (7,9). In contrast, noise in urban areas more often is non-stationary, reflecting the rhythms of the work cycle (8).

3.2 **Spectral Estimates**

Being both stationary and Gaussian, seismic noise at a point can be fully characterized by spectra (10). Figure 2 is a large average, spectral estimate of base level seismic noise measured in Railroad Valley. For frequencies much above 1 Hz, the spectra are relatively flat when cast into terms of particle velocity. The modest bulge in the spectra just above 10 Hz coincides with a loss of impedance of the ground to acoustic loads. The location of the frequency for generating air-coupled seismics is determined by the velocity and density structure neighboring the site.
For the sites considered, Railroad Valley is much the quietest. The seismic level here compares favorably to the level forecast for quality seismic detection sites in the southwestern United States (11). In turn, the seismic noise spectra at depth at these sites can be expected to be red versions of those measured at the surface in that the shorter wavelength, higher frequency components are almost always proportionately more attenuated at depth (12,13).

4. SEISMIC SIGNALS

4.1. Elastic Response

For an elastic body with homogeneous boundaries, the displacement at time, $T'$, for a point located at $X'(x',y',z')$ relates to forces within a source volume located at $X(x,y,z)$ acting at time, $T$, through the temporal convolution of the force with the appropriate Green function (14,15).

$$u_k(X',T') = \int G_{ki}(X',T';X,T) * f_i(X,T) \, dv \, dt$$

In elastodynamics, the convention is to let a displacement at a field point caused by forces applied within the source region define response, Figure 3. The elastic response is causal, linear and time invariant. Also, when the source region is slowly moving and small with respect to the radiated seismic velocities and wavelengths, the source time history can be recovered by linear deconvolution (16). Further, source and field points can be exchanged in accordance with the reciprocity rule.

$$G_{ki}(X',T';X,T) = G_{ik}(X,-T;X',-T')$$

For uniformly layered areas and vertical surface loads, ground response
is insensitive to azimuth. Placing the source at the origin, we then express response solely as a function of distance, \( r \) and retardation time, \( t \).

\[
G(r,t)
\]

### 4.2. Normal Mode Representation

We represent the Fourier transform of \( G_{33}(r,t) \), the vertical displacement of the ground surface at a "large" distance, \( r \), due to a vertical impulse at the origin, to be the sum of the normal mode contributions attenuated by a material loss factor, \( Q \),

\[
G_{33}(r,w) = \sum_{j=1}^{N} A_j(w) \cdot r^j \exp \left( -k_j(w) \cdot r \right) / 2Q(w) \cdot \exp \text{i} \left( k_j(w) \cdot r + \phi_j(w) \right)
\]

Here \( j \) is the mode number and \( A, \phi \) are the apparent amplitude and phase associated with the source. As given here, attenuation is governed by cylindrical spreading and a spacial \( Q \) loss defined by the fractional loss of amplitude over one wavelength. The spacial \( Q \) term used here relates to a temporal determination of \( Q \) through the relation,

\[
Q^{-1} \text{(temporal)} = \frac{U}{c} \cdot Q^{-1} \text{(spacial)}
\]

with \( c, U \) the phase and group velocities of the mode in question.

For our test sites experimental values of \( G_{33}(r,w) \) are fitted to a fundamental mode representation by determining \( A, k, \phi, Q \) over a discrete distance set, \( r_\ell, \ell = 1, 2, 3... \).
4.2.1. Propagation

The average value for unit amplitude, $k'$ weighted surface impacts over the distance, $r \in \{1, 2, 3, \ldots, L\}$ is

$$F_{33}(k', \omega) = \frac{1}{L} \sum_{\ell=1}^{L} G_{33}(r_{\ell}, \omega) \cdot \exp(-ik' r_{\ell}) \cdot G_{33}(r_{\ell}, \omega)^{-1}$$

$F_{33}(k', \omega)$ has a maximum value of unity for unimodal, low noise measurements when $k' = (\omega/c)$. Figure 4 is a sample computation of $F_{33}(k', \omega)$ based on 50 response wavelets measured over the range $120 \leq r \leq 200$ meters. The calculated maxima are quite close to unity over the pass band $3.7 \leq f \leq 37$ Hz; the seismic response at the site can be well represented by a unimodal surface wave for these frequencies. Figure 5 locates the $(k', \omega)$ pairs that generate absolute maxima for the data; the relation establishes the propagation characteristics at the test site. The phase and group delay times are determined from $c = \omega/k$ and $U = d\omega/dk$

$$t_p = r/c \quad t_g = r/U$$

4.2.2. Source Phase

Once $k'$ is determined, source phase is calculated directly from the residual phase at $r=0$.

$$\text{ARG} \left( G_{33}(0, \omega) \right) = \text{ARG} \left( G_{33}(r, \omega) \right) \cdot \exp(ik'r)$$

In Figure 6 the average phase residuals over reliable $(k, \omega)$ pair values are independent of frequency. The theoretical value for cylindrically spreading surface waves due to a surface impact on a uniformly layered elastic half space at large distances, $r$, is $\pi/4$. The calculated source term is consistent with the normal mode representation used here.
4.2.3. **Attenuation**

For unimodal propagation in uniformly layered, lossy media, surface wave attenuation at a distance can be obtained from

\[ |G(r, \omega)| = A(\omega) \cdot \frac{1}{r^\frac{1}{2}} \cdot \exp(-kr/2Q) \]

In Figure 7 we plot \( \ln |G(r, 50) r^{\frac{1}{2}}| \) for each of 63 impact wavelets measured in Jackass Flats as they relate to distance. \( Q \) is then calculated from the slope of the best fitting (least square) slope of the data. A measure of the goodness of fit, \( E \), is also calculated from the residuals.

Figure 8 shows our estimate for \( Q \) and \( E \) for Jackass Flats as they relate to frequency. \( Q \) estimates tend to be decreasing functions of frequency. A \( Q \) of 35 in surficial sediments is modestly larger than \( Q \) values used elsewhere (1).

4.2.4. **Prediction Errors**

The dispersion relation and \( Q \) values found for Railroad Valley are used to adjust individual impulse responses over 100\( \leq r \leq 200 \) meters to a common reference distance, \( r=150 \). The average amplitude response for 150 meters, \( [G(150, f)] \) ave is then obtained, see Figure 9. The ground is found to be a band pass transmission element. The low limb is controlled by the nature of the elastic response. The high limb roll off is dominated by the material quality factor, \( Q \).

The ground disturbance predicted in Railroad Valley for a standard impulse as it relates to distance is given in Figure 10. The synthetic responses shown here obey the \( A, k, \phi, Q \) parameters calculated at the site for unimodal surface waves in the range, 100\( \leq r \leq 200 \) meters.

The synthetic waveforms are compared to the actual wavelets. Error
wavelets, the difference between predicted and measured values, are quite small, Figure 11. The error is ascribed to additive seismic noise, higher mode responses and lateral inhomogenities.

The scatter in the individual amplitude and phase responses about an average value for Jackass Flats is given in Figure 12. Scatter diagrams such as these are used to support the proposition that the original measurements are heavily corrupted by additive noise for frequencies greater than 35 Hz. (16).

It is worth noting that the predicted seismics at distances greater than a few hundred meters are quite sensitive to our estimate of Q. A unit difference in Q causes about a 6 db difference in the predicted midband amplitude response at 1 km. Indeed, a major impetus for this study has been the lack of reliable attenuation data in Basin and Range Valley materials. Recent interest in mantle Q permits an upgrading of earlier work concerning communicating seismically at long range (4); it bears little on data rate estimates at modest distances (10km) for the deep alluvial valleys in the Basin and Range Province.

Even in these supposedly simple valley sites, attenuation is due to lateral structural changes as well as Q losses. To show this we determine the phase velocity for a number of headings in Railroad Valley, Figure 13. The seismic response at this site is found to be azimuth sensitive. Ground structure is not laterally uniform. Attenuation is not solely a function of distance and Q loss. As can be seen, the effect of heading is somewhat less severe at lower frequencies, suggesting the valley is more uniform at depth.
5. **CHANNEL INFORMATION RATES**

The capacity of a communication channel in bits/sec over the band \( a \leq f \leq b \) is computed after Hartley-Shannon as

\[
C = \int_{a}^{b} \log_2 \left( \frac{P_{ss} + P_{nn}}{P_{nn}} \right) df
\]

where \( P_{ss} \) and \( P_{nn} \) are the power spectra of the signal and noise processes, respectively (18). Channel capacity is a maximum when \( P_{ss} + P_{nn} \) is constant. Constant reception spectra can be obtained either by weighting at the source or receiver. In either case we can negate distortions due to the seismic path by application of a linear convolution operator that is the inverse of the ground response, \( G(r,t) \).

5.1. **Narrowband Sources**

The center frequency for minimum strength sources and optimum transmission is illustrated for the transmission and noise characteristics in Railroad Valley. Figure 14 is the \( P_{ss}(f)/P_{nn}(f) \) ratio for a white source of strength, \( S = 6.25 \) Newtons rms measured at a range of 150 meters. \( P_{ss}/P_{nn} \) is a maximum near 18 Hz. In this case a channel centered at 18 Hz is optimum in the sense that it maximizes channel capacity for a given source level, bandwidth and range.

In order that the maximum channel capacity for broadband sources be realized, the inverse of the path response must be known. Even the detection of signals with a peak \( G_{ss}/G_{nn} \) substantially less than unity is not without difficulty. The capacity of channels, using one or more transmitter-receiver pairs uncompensated for seismic dispersion, passes a sub optimally coded sequence.
The maximum information capacity for a source of 31.25 Newtons rms and 1 Hz bandwidth is determined for Railroad valley. From Figure 15 we find we can transmit and receive in excess of 2 bits/second over a 1 Hz band at a distance of 1 km using a source of only 31.25 Newtons rms.

After a range of a few hundred meters, narrowband sources are relatively efficient seismic transmitters. In Figure 16 the effect of source bandwidth and range on information capacity is shown for a source strength of 9.375 Newtons rms. For this source level and locality, a bandwidth of 3 Hz is optimum at a range of 0.50 km.

In Figure 17 we determine the source strength needed to maintain a channel capacity of 2 bits/second over a bandwidth of 1 Hz. A source level of at least 25 Newtons rms is needed to insure a channel capacity of 2 bits/second at a range of 1 km. Under these constraints an approximately linear relationship exists between source strength and distance.

5.2 Broadband Sources

The information rate for a channel consisting of a broadband signal source and a receiver whose response is the inverse of that of the ground is depicted in Figure 18 for a range of 150 meters and source strength of 4.419 Newtons in the band of 50 Hz. The plot shows seismic communication to be a low pass phenomenon in that 95% of the channel capacity is obtained from frequencies less than 35 Hz. The bandwidth for efficient seismic communication is also range dependent. To show this we compute the cutoff frequency needed to attain 98% of channel capacity for white sources, Figure 19. The useful information bandwidth for weak sources at distances greater than 1 km is less than 20 Hz in Railroad Valley.

To further show the impact of range on broadband communications, we
estimate channel capacities in Railroad Valley as they relate to bandwidth over a suite of distances, Figure 20. Broadband seismic communications require relatively strong sources to excite frequencies much above 25 Hz for distances greater than 200 meters.

6. TRANSMITTER AND RECEIVER ARRAYS

Coherent signals embedded in additive incoherent noise provide a conservative basis to estimate the effect of both transmitter and receiver arrays on channel capacity. For such a construction $P_{ss}/P_{nn}$ increases directly as the sum of the number of receiver and transmitter elements. Channel capacity for systems employing multiple transmitter and receiver elements is shown in Figure 21 for narrowband transmissions at a range of 300 meters.

The prospects of surface arrays separating signals from noise are considerably better for buried sources than for surface sources. The reason for this is that both the noise and signals excited by surface sources propagate largely as low order surface waves. Arrays cannot separate seismic waves traveling in the same direction with the same mode. In contrast, low mode surface seismics are only poorly excited by buried sources. Arrays using velocity filtering should be quite effective for separating seismic signals from noise in the case of buried sources (19).

7. SITE EFFECTS

In Figure 22 the seismic wavelets produced by a standard impact at a distance of 150 meters are shown for Jackass Flats and Railroad valley. Seismic response is clearly site sensitive. It immediately follows that channel data rates are also site sensitive. Optimum capacity channels for a broadband transmitter in Jackass Flats is given in Figure 23. For the ranges
and frequencies considered, Railroad Valley has the greater capacity. Low noise in Railroad Valley is the controlling factor.

All the estimates given in this study are for point surface sources and surface receivers. For a surface source and a buried receiver at large offset distances both the noise and signal are attenuated in roughly similar fashion. $P_{ss}/P_{nn}$ ratios at depth should be much the same as those found for surface sources and receivers. The results of this study should apply. In contrast, $P_{ss}/P_{nn}$ should be significantly smaller for a source at depth and a surface receiver. Reliable assessments for communicating to the surface from a buried facility requires seismic measurements at depth in the areas of interest.
8. REFERENCES


16. The Determination of Source Properties, Stump, B., 1977, BSSA.


FIGURES

1. Seismic Noise Distribution - Steptoe Valley
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17. Effect of Source Strength
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21. Effect of Arrays
22. Effect of Site
23. Channel Capacities: Wide Band Source

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SEISMIC NOISE DISTRIBUTION - STEPTOE VALLEY

Figure 1
SEISMIC NOISE SPECTRA: RAILROAD VALLEY

Figure 2
\[ u_k \sim \text{DISPLACEMENT} \quad f_i \sim \text{FORCE} \]

\[ u_k(x', t') = \int \int \int \int \int G_{ki}(x'; t'; x, t) * f_i(x, t) \, dv \, dt \]

* TEMPORAL CONVOLUTION

GREEN'S FUNCTION REPRESENTATION

Figure 3
Figure 4

North: RRV

ANGULAR FREQUENCY (2\pi f)

QUALITY OF UNIMODAL MODEL
Figure 5
DELTA PHASE (Corrected for $+ \pi/4$)

SOURCE PHASE

Figure 6
Figure 7

ATTENUATION: ETB

Frequency = 25 Hz

\[ \ln(|A| \cdot r^2) \]

DISTANCE (FT)
Q ESTIMATES

Figure 8
AVERAGE AMPLITUDE RESPONSE

Figure 9
GROUND DISPLACEMENT

**DISTANCE (meters)**

<table>
<thead>
<tr>
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<th>Angstroms (per Newton)</th>
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<tr>
<td>150</td>
<td>0.0108</td>
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<td></td>
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<td>0.00148</td>
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<tr>
<td>900</td>
<td>-0.00104</td>
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</table>

**Time (secs.)**

0.000 1.024 2.048 3.072 4.096 5.120

PREDICTED RESPONSE—RAILROAD VALLEY (270°)

Figure 10
Railroad Valley 270°
Response to a hammer blow
Distance 150 meters

ERROR WAVELET

Figure 11
DELTA PHASE: DEGREES

AMPLITUDE AND PHASE SCATTER: 25 Hz

JACKASS FLATS

Figure 12
LATERAL INHOMOGENEITIES IN RAILROAD VALLEY

(a) \(c(f=15 \text{ Hz})\)

(b) \(c(f=25 \text{ Hz})\)

Figure 13
Bandwidth 50 Hz
Source 6.25 Newtons rms

\[ \frac{P_{ss}}{P_{nn}} \text{ RATIO; RRV 150 METERS} \]

Figure 14
Bandwidth 1.00 Hz
31.25 Newtons rms

CHANNEL CAPACITY: RRV (270°) NARROW BAND SOURCE

Figure 15
Source 9.375 Newtons rms

150 meters

300 meters

450 meters

600 meters

CAPACITY BITS/SEC

BANDWIDTH Hz

EFFECT OF BANDWIDTH

Figure 16
Bandwidth 1 Hz
Capacity 2 Bits/Sec

EFFECT OF SOURCE STRENGTH

Figure 17
SOURCE 4.419 Newtons rms
Bandwidth 50 Hz

GROUND BANDPASS CHARACTERISTIC: RRV 150METERS

Figure 18
Useful Bandwidth (98%)
Source 31.6 Newtons rms
Bandwidth 0-50 Hz

Figure 19
Source 31.6 Newtons rms

Distance meters
150

300

600

CHANNEL CAPACITIES: WIDE BAND SOURCE
RAILROAD VALLEY

Figure 20
Source Strength 1.5625 Newtons rms
Range 300 meters
Bandwidth 1.0 Hz

EFFECT OF ARRAYS

Figure 21
ANGSTROMS/NEWTON

EFFECT OF SITE

Figure 22
Source 31.6 Newtons rms

Distance meters

CHANNEL CAPACITIES: WIDE BAND SOURCE
JACKASS FLATS

Figure 23
GLOSSARY OF TERMS

<table>
<thead>
<tr>
<th>Term</th>
<th>Description</th>
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<tr>
<td>A</td>
<td>Source Amplitude</td>
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<tr>
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<td>Phase Velocities</td>
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<td>C</td>
<td>Capacity</td>
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<td>Decibel</td>
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<td>E</td>
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<td>tg</td>
<td>Group Delay</td>
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<td>Phase Delay</td>
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<tr>
<td>G</td>
<td>Green's Function</td>
</tr>
<tr>
<td>G_{33}</td>
<td>Vertical Displacement of Ground Surface at a &quot;Large&quot; Distance</td>
</tr>
<tr>
<td>i</td>
<td>$\sqrt{-1}$</td>
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<tr>
<td>j</td>
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<td>P_{nn}</td>
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