IMPLEMENTATION OF A RELIABILITY SHORTHAND ON THE TI-59
HANDHELD CALCULATOR(U) NAVAL POSTGRADUATE SCHOOL
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Implementation of a Reliability Shorthand on the TI-59 Handheld Calculator

by

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Two TI-59 programs are provided as a computational aid.
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# TABLE OF CONTENTS

I. INTRODUCTION  
II. THE CONCEPT OF A RELIABILITY SHORTHAND  
   A. BASIC NOTATION  
   B. CONVOLUTION OF DISTRIBUTIONS  
   C. MIXTURE OF DISTRIBUTIONS  
      1. MIX-Notation  
      2. Distributive Law  
      3. Degeneracy at the Origin  
III. APPLYING A RELIABILITY SHORTHAND  
   A. SUMS OF EXPONENTIALS WITH WEIGHT ONE  
      1. Simple Series System  
      2. Simple Parallel System  
      3. Standby-System with Dissimilar Components  
   B. SUMS OF EXPONENTIALS WITH WEIGHT BETWEEN ZERO AND ONE  
      1. Parallel System with Dissimilar Failure Rates  
      2. Series System with One Spare  
      3. Two-out-of-Three System  
IV. IMPLEMENTING THE SHORTHAND ON THE TI-59  
V. SUMMARY
LIST OF FIGURES

1. Distributive Property of the MIX-Notation - - - - 14
2. Two-Component Series System - - - - - - - - - - - - 17
3. Two-Component Parallel System - - - - - - - - - - - - 18
4. Standby System - - - - - - - - - - - - - - - - - - - - 20
5. Series System with one Spare - - - - - - - - - - - - - 24
6. Two-out-of-Three System - - - - - - - - - - - - - - - - 27
7. Two-out-of-Three System - - - - - - - - - - - - - - - - 27
I. INTRODUCTION

Systems and components can be in either of two states: either they are functioning or they have failed. The ability, that a system stays functioning over a predetermined time interval is called its reliability. It is generally not realistic to assume that a system, say a lightbulb, will fail at a specified time, but rather that $T$, the time to failure, is a random variable which has a probability distribution that can be specified. The probability distribution for a time to failure is called its life distribution.

In this paper we will solely be concerned with one specific type of life distribution which is especially important in reliability theory and practice, the exponential distribution. It has the property that the remaining life of a used component is independent of its age (the "memoryless" property), i.e. a functioning component is always as good as new, the failure rate is constant. The memoryless property is the basis for a reliability shorthand, one that can be implemented on a handheld calculator.

Depending on the size, structure and life distribution of a system, probability statements about its time to
failure are in general not easily achieved. Forming the sum of independent life lengths (i.e. convolving the corresponding life distributions) requires knowledge of integral calculus and computations can become rather tedious.

In the case of the exponential distribution, though, computations can be simplified by translating the problem into a simple shorthand notation and using this shorthand as input for some computing device.

In this paper we will show how a reliability shorthand can be implemented on a handheld calculator. Basic structures are used to show how the shorthand can be applied. Two TI-59 programs are provided as a computational aid. Formulas for the convolution of up to four exponential random variables can be found in Appendix A. Appendix B contains a user guide to the TI-59 programs.
II. THE CONCEPT OF A RELIABILITY SHORTHAND

A. BASIC NOTATION

The survival function of a life length can be derived from the distribution function.

Let

\[ T \text{ : life length} \]
\[ F(t) = P(T \leq t) \text{ be the distribution function of} \]

Then

\[ \bar{F}(t) = P(T > t) = 1 - F(t) \]

is the survival function of \( T \).

In the case of the exponential distribution, \( \bar{F}(t) = e^{-\lambda t} \), where \( \lambda \) is the failure rate. Translated into shorthand, the life distribution is denoted \( \text{EXP}(\lambda) \).

B. CONVOLUTION OF DISTRIBUTIONS

When independent random lives are summed up, the corresponding life distributions have to be convolved to determine the probability that the sum of the lives will exceed a specified time \( t \).

Let

\[ T_1, T_2 \text{ : independent life lengths} \]
\( \overline{F}_1(t), \overline{F}_2(t) \) : the corresponding survival functions

\( f_1(t), f_2(t) \) : the corresponding density functions

\( T = T_1 + T_2 \) : the total life length

Then

\[
\overline{F}(t) = P(T > t) = P(T_1 + T_2 > t) = \overline{F}_1(t) + \int_0^t \overline{F}_2(t-s) f_1(s) ds.
\]

This means that \( T \) will exceed a specified time \( t \) when

- either \( T_1 \) exceeds \( t \)
- or \( T_1 \) is smaller than \( t \), say equal to \( s \), and \( T_2 \) exceeds \( t-s \).

Integration with respect to \( s \) (i.e. summing over all possible values of \( s \)) is called the convolution of \( T_1 \) and \( T_2 \).

When \( T_1 \) and \( T_2 \) are both exponentially distributed with failure rates \( \lambda_1 \) and \( \lambda_2 \), i.e.

\[
\overline{F}_1(t) = e^{-\lambda_1 t}, \\
\overline{F}_2(t) = e^{-\lambda_2 t},
\]

then the survival function of \( T \) is

\[
\overline{F}(t) = e^{-\lambda_1 t} + \int_0^t e^{-\lambda_2 (t-s)} \lambda_1 \lambda_2 e^{-\lambda_1 s} ds.
\]

Translated into shorthand, the survival function is denoted

\[
\text{EXP}(\lambda_1) + \text{EXP}(\lambda_2).
\]
This shorthand notation is heuristically apparent. We can visualize a 1 component / 1 spare system with $\text{Exp}(\lambda_1)$ and $\text{Exp}(\lambda_2)$ lives respectively. From component 1 the system has an $\text{Exp}(\lambda_1)$ life to begin with. When component 1 fails, the system has an extra $\text{Exp}(\lambda_2)$ life.

C. Mixture of Distributions

1. MIX-Notation

In the previous chapter, we formed the sum of independent random lives, which each had weight one, i.e.

$$T = T_1 + T_2.$$ 

Now consider

$$T = \begin{cases} 
T_1 & \text{with probability } p_1 \\
T_2 & \text{with probability } p_2 
\end{cases}$$

where $p_1 + p_2 = 1$.

Let $D_1$ and $D_2$ be the probability distributions of the random variables $T_1$ and $T_2$ respectively. The corresponding survival functions are $F_1(t)$ and $F_2(t)$.

Then

$$F(t) = p_1 F_1(t) + p_2 F_2(t).$$

In shorthand, the mixture of distributions $D_1$ and $D_2$ with respect to the mixing probabilities $p_1$ and $p_2$ is denoted

$$\text{MIX} [p_1 D_1, p_2 D_2].$$
2. **Distributive Law**

Now let

\[ T = T_3 + T' \]

where

\[
T' = \begin{cases} 
T_4 \text{ with probability } p \\
T_2 \text{ with probability } 1-p.
\end{cases}
\]

Then

\[
T = \begin{cases} 
T_4 \text{ with probability } p \\
T_2 \text{ with probability } 1-p. \\
T_3 + T_4 \text{ with probability } p \\
T_3 + T_2 \text{ with probability } 1-p.
\end{cases}
\]

The distributive law holds due to the fact that the sum of the mixing probabilities for \( T_4 \) and \( T_2 \) is equal to one.

The survival function of \( T \) can be found by convolution:

\[
F(t) = F_3(t) + \int_0^t (pF_4(t-s) + (1-p)F_2(t-s))f_3(s)\, ds.
\]

With \( D_4, D_2, D_3 \) being the probability distributions for \( T_4, T_2, T_3 \), the distributive law can be applied to the shorthand notation:

\[
D_3 \overset{\text{MIX}}{\rightarrow} [pD_4, (1-p)D_2] = \text{MIX} [p(D_4 + D_2), (1-p)(D_2 + D_3)].
\]
Graphically this can be represented as follows:

\[ D_3 + (1-p) D_2 = p D_4 + (1-p) D_2 + D_3 \]

Figure 1: Distributive Property of the MIX-Notation

3. Degeneracy at the Origin

Let

\[ P(T=0) = 1. \]

Then the distribution of \( T \) is degenerate at zero.

In shorthand notation, such a distribution is called the ZERO-distribution.

Now let \( T = T_4 + T_0 \)

where \( T_4 \) and \( T_0 \) have probability distributions \( D_4 \) and ZERO and survival functions \( F_4(t) \) and \( F_0(t) \) respectively.

Then

\[
\bar{F}(t) = \bar{F}_4(t) + \int_0^t \bar{F}_0(t-s) f_4(s) \, ds \\
= \bar{F}_4(t).
\]
The ZERO-distribution doesn’t add anything to another distribution, so for instance

\[ D_4 \ast \text{ZERO} = D_4 \]

\[ D_2 \ast \text{MIX}[pD_4, (1-p)\text{ZERO}] = \text{MIX}[p(D_4 + D_2), (1-p)D_2]. \]
III. APPLYING A RELIABILITY SHORTHAND

After this brief survey over the concept of a reliability shorthand we will now show how the shorthand can be applied. To do so we will use basic structures. Part A of this chapter will give examples whose representation in shorthand requires only basic notation described in Chapter II, Parts A and B, whereas Part B of this chapter will give examples whose representation in shorthand makes use of the MIX-notation and the ZERO-distribution.

A. SUMS OF EXPONENTIALS WITH WEIGHT ONE

1. Simple Series System

A series system is a system which is functioning, when all its components are functioning. A two-component series system can be graphically represented as shown in Fig. 2.

Let

\[ T : \text{life of the system} \]
\[ T_i : \text{life of component } i \]
\[ T_2 : \text{life of component } 2 \]
\[ F_i(t) = \text{survival function of component } i \]
\[ = e^{-λ_i t} \]
Figure 2: Two-Component Series System

\[ F_2(t) = \text{survival function of component 2} \]

\[ = e^{-\lambda_2 t} . \]

Then

\[ T = \min(T_1, T_2) \]

\[ \bar{F}(t) = \text{survival function of the system} \]

\[ = P(T_1 > t, T_2 > t) \]

Assuming independence of the two components

\[ \bar{F}(t) = P(T_1 > t) P(T_2 > t) \]

\[ = \bar{F}_1(t) \bar{F}_2(t) \]

\[ = e^{-\lambda_1 t} e^{-\lambda_2 t} \]

\[ = e^{-(\lambda_1 + \lambda_2) t} . \]

The shorthand notation for this system is

\[ \text{EXP} (\lambda_1 + \lambda_2) . \]
This is intuitively apparent, as the system has an exponential survival function with failure rate $\lambda_1 + \lambda_2$.

2. Simple Parallel System

A parallel system is a system which is functioning, when at least one of its components is functioning. A two-component parallel system can be graphically represented as follows:

![Diagram of a two-component parallel system]

Figure 3: Two-Component Parallel System

Let

$T_1 \sim \text{EXP}(\lambda), T_2 \sim \text{EXP}(\lambda)$.

Then

$T = \max(T_1, T_2)$

$\bar{F}(t) = P(\max(T_1, T_2) > t)$

$= 1 - P(\max(T_1, T_2) \leq t)$

$= 1 - P(T_1 \leq t, T_2 \leq t)$
Assuming independence of the two components,

\[ \bar{F}(t) = 1 - P(T_1 \leq t) P(T_2 \leq t) \]
\[ = 1 - P_1(t) P_2(t) \]
\[ = 1 - (1-e^{-\lambda t})(1-e^{-\lambda t}) \]
\[ = 1 - (1-2e^{-\lambda t} + e^{-2\lambda t}) \]
\[ = 2e^{-\lambda t} - e^{-2\lambda t}. \]

The shorthand notation for the system is

\[ \text{EXP}(2\lambda) + \text{EXP}(\lambda). \]

This follows intuition as the system has an \( \text{EXP}(2\lambda) \) life to begin with and when one component fails it has an extra \( \text{EXP}(\lambda) \) life due to the memoryless property of the exponential distribution.

3. **Standby-System with Dissimilar Components**

Suppose a system consists of two components, one active and one spare. The active component stays in service until it fails and then immediately is replaced by the spare.

Let the time to failure of the two components be

\( T_1 \sim \text{EXP}(\lambda_1) \) and \( T_2 \sim \text{EXP}(\lambda_2) \) respectively.

Then the system time to failure is

\[ T = T_1 + T_2 \]

and the survival function of the system is

\[ \bar{F}(t) = P(T > t) \]
Figure 4: Standby System

\[
F_4(t) = \bar{F}_4(t) + \int_0^t \bar{F}_2(t-s) f_4(s) \, ds \\
= e^{-\lambda_4 t} + \int_0^t e^{-\lambda_2(t-s)} \lambda_1 e^{-\lambda_4 s} \, ds \\
= \frac{\lambda_4}{\lambda_4 - \lambda_2} e^{-\lambda_4 t} - \frac{\lambda_2}{\lambda_4 - \lambda_2} e^{-\lambda_4 t}
\]

The shorthand notation for the system's survival function should be obvious. The system has an EXP(\(\lambda_4\)) life from the active component and an additional EXP(\(\lambda_2\)) life from the spare. So the shorthand notation is

\[\text{EXP}(\lambda_4) + \text{EXP}(\lambda_2)\]
B. Sums of Exponentials with Weight Between Zero and One

The examples given in the previous chapter only involved exponential lives with weight one. Now we will look at some structures, whose survival function has a shorthand notation which includes the MIX-notation and/or the ZERO-distribution.

1. Parallel System with Dissimilar Failure Rates

The notion of a parallel system has been introduced in Chapter III.A.2. We now look at the case where

\[ T_1 \sim \text{Exp}(\lambda_1) \text{ and } T_2 \sim \text{Exp}(\lambda_2). \]

Then

\[ T = \max(T_1, T_2) \]

\[ \bar{F}(t) = P(\max(T_1, T_2) > t) \]

\[ = 1 - P(\max(T_1, T_2) \leq t) \]

\[ = 1 - P(T_1 \leq t, T_2 \leq t) \]

Assuming independence of the two components

\[ \bar{F}(t) = 1 - P(T_1 \leq t) P(T_2 \leq t) \]

\[ = 1 - F_1(t) F_2(t) \]

\[ = 1 - (1 - e^{-\lambda_1 t})(1 - e^{-\lambda_2 t}) \]

\[ = 1 - (1 - e^{-\lambda_1 t} - e^{-\lambda_2 t} + e^{-\lambda_1 t} + e^{-\lambda_2 t} - e^{-\lambda_1 t} - e^{-\lambda_2 t}) \]

\[ = e^{-\lambda_1 t} + e^{-\lambda_2 t} - e^{-(\lambda_1 + \lambda_2) t}. \]
To find the shorthand notation of the system consider all the ways which lead to the survival of the system:

- either both components survive
- or component 1 fails and component 2 survives
- or component 2 fails and component 1 survives.

If one component fails and one survives, in \( \frac{\lambda_2}{\lambda_1 + \lambda_2} \) fraction of the cases the survivor will be component 1 and in \( \frac{\lambda_1}{\lambda_1 + \lambda_2} \) fraction of the cases it will be component 2.

This can graphically be represented as

\[
\frac{\lambda_2}{\lambda_1 + \lambda_2} \quad \exp(\lambda_1) \quad \frac{\lambda_1}{\lambda_1 + \lambda_2} \quad \exp(\lambda_2)
\]

\[
\exp(\lambda_1 + \lambda_2) + \frac{\lambda_2}{\lambda_1 + \lambda_2} \quad \exp(\lambda_1) \quad \frac{\lambda_1}{\lambda_1 + \lambda_2} \quad \exp(\lambda_2)
\]

Making use of the MIX-notation the shorthand notation then is

\[
\exp(\lambda_1 + \lambda_2) + \text{MIX}[\frac{\lambda_2}{\lambda_1 + \lambda_2} \exp(\lambda_1), \frac{\lambda_1}{\lambda_1 + \lambda_2} \exp(\lambda_2)]
\]

and using the distributive property it becomes

\[
\text{MIX}[\frac{\lambda_2}{\lambda_1 + \lambda_2}(\exp(\lambda_1) + \exp(\lambda_2 + \lambda_2)) + \frac{\lambda_1}{\lambda_1 + \lambda_2}(\exp(\lambda_2) + \exp(\lambda_1 + \lambda_2))].
\]
As a check to see that this shorthand notation represents the survival function of the system, we derive the survival function from the shorthand notation:

\[
\overline{F}(t) = \frac{\lambda_1}{\lambda_1 + \lambda_2} \left( e^{-\lambda_1 t} + \int_0^t e^{-(\lambda_1 + \lambda_2)(t-s)} \lambda_2 e^{-\lambda_2 s} ds \right) \\
+ \frac{\lambda_2}{\lambda_1 + \lambda_2} \left( e^{-\lambda_2 t} + \int_0^t e^{-(\lambda_1 + \lambda_2)(t-s)} \lambda_1 e^{-\lambda_1 s} ds \right) \\
= e^{-\lambda_1 t} + e^{-\lambda_2 t} - e^{-(\lambda_1 + \lambda_2)t}.
\]

This verifies that the shorthand notation indeed represents the system's survival function.

2. **Series System with One Spare**

Let us now look at a two-component series system, whose components have dissimilar failure rates with one component having a spare:

Component 1 has the constant failure rate \( \lambda_1 \), and component 2 and the spare have the constant failure rate \( \lambda_2 \).

The spare can only replace component 2.

Let

- \( \overline{F}_1(t) \) : the survival function of component 1
- \( \overline{F}_2(t) \) : the survival function of the standby system component 2 with its spare.
The survival function for a standby system was derived in Chapter II.B. Therefore

\[
F_2(t) = e^{-\lambda_2 t} + \int_0^t e^{-\lambda_2 (t-s)} \lambda_2 e^{-\lambda_2 s} \, ds
\]

\[
= e^{-\lambda_2 t} + \lambda_2 e^{-\lambda_2 t} \int_0^t \lambda_2 e^{s} \, ds
\]

\[
= (1 + \lambda_2 t) e^{-\lambda_2 t}.
\]

Now \( F_4(t) = e^{-\lambda_4 t} \)

Then \( F(t) = F_4(t) F_2(t) \)

\[
= (1 + \lambda_2 t) e^{-(\lambda_4 + \lambda_2) t}.
\]

To translate the survival function into shorthand notation, let us consider the ways in which the system can survive:

- either both components survive
- or component 2 fails and its spare survives.
If one component fails, in \( \frac{\lambda_2}{\lambda_1 + \lambda_2} \) fraction of the time it will be component 1, which means that the system will not survive; in \( \frac{\lambda_1}{\lambda_1 + \lambda_2} \) fraction of the time the failing component will be component 2.

This can graphically be represented as

\[
\frac{\lambda_1}{\lambda_1 + \lambda_2} \quad \text{ZERO} \\
\frac{\lambda_2}{\lambda_1 + \lambda_2} \quad \text{EXP}(\lambda_1 + \lambda_2)
\]

Using the MIX-notation the survival function then is

\[
\text{EXP}(\lambda_1 + \lambda_2) + \text{MIX} \left[ \frac{\lambda_1}{\lambda_1 + \lambda_2} \text{ZERO}, \frac{\lambda_2}{\lambda_1 + \lambda_2} \text{EXP}(\lambda_1 + \lambda_2) \right]
\]

\[
= \text{MIX} \left[ \frac{\lambda_1}{\lambda_1 + \lambda_2} (\text{ZERO} + \text{EXP}(\lambda_1 + \lambda_2)), \frac{\lambda_2}{\lambda_1 + \lambda_2} \text{EXP}(\lambda_1 + \lambda_2) \right]
\]

\[
= \text{MIX} \left[ \frac{\lambda_1}{\lambda_1 + \lambda_2} \text{EXP}(\lambda_1 + \lambda_2)), \frac{\lambda_2}{\lambda_1 + \lambda_2} \text{EXP}(\lambda_1 + \lambda_2) \right].
\]

To prove, that the shorthand notation does represent the survival function, we derive the latter from the shorthand:

\[
\bar{F}(t) = \frac{\lambda_1}{\lambda_1 + \lambda_2} e^{-(\lambda_1+\lambda_2)t} + \frac{\lambda_2}{\lambda_1 + \lambda_2} \left( e^{-(\lambda_1+\lambda_2)t} + \right)
\]

25
This is the previously found result and this verifies, that the shorthand notation does represent the system's survival function.

3. **Two-out-of-Three System**

As a last example in this chapter, we will look at a Two-out-of-Three system.

Consider a three component system, whose components have constant failure rates $\lambda_1$, $\lambda_2$, and $\lambda_3$ respectively. The system is functioning, as long as two out of three components are functioning (see Fig. 6).

In other words, the system is functioning as long as there is a path through the system.

Alternatively, the system can be visualized as a parallel-series system (compare Fig. 7).

The survival function of the system is

$$
F(t) = P(T_1 \geq t \land T_2 \geq t) + P(T_1 \geq t \land T_3 \geq t) \\
+ P(T_2 \geq t \land T_3 \geq t) \\
- P((T_1 \geq t \land T_2 \geq t) \land (T_1 \geq t \land T_3 \geq t)) \\
- P((T_1 \geq t \land T_2 \geq t) \land (T_2 \geq t \land T_3 \geq t)) \\
- P((T_1 \geq t \land T_3 \geq t) \land (T_2 \geq t \land T_3 \geq t))
$$
Figure 6: Two-out-of-Three System

\[ P((T_1 > t \land T_2 > t) \land (T_4 > t \land T_5 > t)) \]

Figure 7: Two-out-of-Three System

\[ P(T_2 > t \land T_3 > t). \]
Thus

\[ \bar{F}(t) = P(T_1 > t \land T_2 > t) + P(T_1 > t \land T_3 > t) + P(T_2 > t \land T_3 > t) - 3P(T_1 > t \land T_2 > t \land T_3 > t) + P(T_4 > t \land T_2 > t \land T_3 > t) \]

Therefore, and assuming independence of the components,

\[ \bar{F}(t) = P(T_1 > t) P(T_2 > t) + P(T_1 > t) P(T_3 > t) + P(T_2 > t) P(T_3 > t) - 3P(T_1 > t) P(T_2 > t) P(T_3 > t) + P(T_4 > t) P(T_2 > t) P(T_3 > t) \]

\[ = e^{-(\lambda_1 + \lambda_2)t} + e^{-(\lambda_1 + \lambda_3)t} + e^{-(\lambda_2 + \lambda_3)t} - 2e^{-(\lambda_1 + \lambda_2 + \lambda_3)t} \]

Now let us consider all the possible ways, in which the system can survive:

- either all components survive

- or component 1 fails and component 2 and 3 survive

- or component 2 fails and component 1 and 3 survive
- or component 3 fails and component 1 and 2 survive.

If a component fails and the other two survive, in \( \frac{\lambda_i}{\lambda_1 + \lambda_2 + \lambda_3} \) fraction of the time it will be component \( i = 1, 2, 3 \).

This can graphically be represented as

\[
\frac{\lambda_1}{\lambda_1 + \lambda_2 + \lambda_3} \quad \text{EXP}(\lambda_1 + \lambda_3) \\
\frac{\lambda_2}{\lambda_1 + \lambda_2 + \lambda_3} \quad \text{EXP}(\lambda_1 + \lambda_2) \\
\frac{\lambda_3}{\lambda_1 + \lambda_2 + \lambda_3} \quad \text{EXP}(\lambda_1 + \lambda_2 + \lambda_3)
\]

The shorthand notation then is

\[
\text{EXP}(\lambda_1 + \lambda_2 + \lambda_3) + \text{MIX}[\frac{\lambda_1}{\lambda_1 + \lambda_2 + \lambda_3} \text{EXP}(\lambda_1 + \lambda_3) , \frac{\lambda_2}{\lambda_1 + \lambda_2 + \lambda_3} \text{EXP}(\lambda_1 + \lambda_2) , \frac{\lambda_3}{\lambda_1 + \lambda_2 + \lambda_3} \text{EXP}(\lambda_1 + \lambda_2 + \lambda_3) ],
\]

\[
= \text{MIX}[\frac{\lambda_1}{\lambda_1 + \lambda_2 + \lambda_3} (\text{EXP}(\lambda_1 + \lambda_3) + \text{EXP}(\lambda_1 + \lambda_2 + \lambda_3)) , \frac{\lambda_2}{\lambda_1 + \lambda_2 + \lambda_3} (\text{EXP}(\lambda_1 + \lambda_2) + \text{EXP}(\lambda_1 + \lambda_2 + \lambda_3)) , \frac{\lambda_3}{\lambda_1 + \lambda_2 + \lambda_3} (\text{EXP}(\lambda_1 + \lambda_2 + \lambda_3) )].
\]
Again, as a check that the shorthand notation represents the survival function, let us derive the survival function from the shorthand notation:

\[
P(t) = \frac{\lambda_1}{\lambda_1 + \lambda_2 + \lambda_3} \left[ e^{-(\lambda_1+\lambda_2)t} + \int_0^t e^{-(\lambda_1+\lambda_2+\lambda_3)(t-s)}(\lambda_2+\lambda_3)e^{-(\lambda_1+\lambda_2)s} \, ds \right]
\]

\[
+ \frac{\lambda_2}{\lambda_1 + \lambda_2 + \lambda_3} \left[ e^{-(\lambda_1+\lambda_3)t} + \int_0^t e^{-(\lambda_1+\lambda_2+\lambda_3)(t-s)}(\lambda_1+\lambda_3)e^{-(\lambda_1+\lambda_3)s} \, ds \right]
\]

\[
+ \frac{\lambda_3}{\lambda_1 + \lambda_2 + \lambda_3} \left[ e^{-(\lambda_1+\lambda_2)t} + \int_0^t e^{-(\lambda_1+\lambda_2+\lambda_3)(t-s)}(\lambda_1+\lambda_2)e^{-(\lambda_1+\lambda_2)s} \, ds \right]
\]

\[
= e^{-(\lambda_1+\lambda_2)t} + e^{-(\lambda_1+\lambda_3)t} + e^{-(\lambda_1+\lambda_2)t} - e^{-(\lambda_1+\lambda_2+\lambda_3)t}
\]

The result again proves that the shorthand notation indeed represents the survival function of the system.
IV. IMPLEMENTING THE SHORTHAND ON THE TI-59

The concept of a reliability shorthand is introduced in the course "Reliability and Weapons System Effectiveness Measurements", GA 4302, at the Naval Postgraduate School, Monterey. Most students taking the course are in the Operations Research (OR) curriculum.

The choice of the TI-59 as the computing device, on which the shorthand was to be implemented, was based on the fact that each student in the OR-Curriculum is issued a TI-59 for use in basic probability and statistics courses. Thus, almost every student at the Naval Postgraduate School, who is introduced to the shorthand, is familiar with the TI-59 and has access to such a calculator.

A program, that uses the shorthand notation, times to failure and failure rates as input, should

- calculate the survival probability of basic structures / small systems and
- require moderate computation time.

To achieve these requirements it was decided to incorporate all solutions for the convolution of up to four exponential random variables in the program. The formulas that were used are given in Appendix A.
Two programs are provided in this paper.

Program 1 can be used when all rates are dissimilar or all are the same. It uses the formulas on pages 37 and 38 only.

Program 2 can be used for the general case. It makes use of all the formulas given in Appendix A. The program includes a sorting routine that determines the applicable formula from the entered failure rates.

A user guide to the two programs is provided in Appendix B.
V. SUMMARY

There is a reliability shorthand that denotes the survival function of a system, assuming that the failure rates of all components are constant.

This shorthand can be implemented on the TI-59 handheld calculator. With failure rates, time to failure and shorthand as input the TI-59 calculates the survival probability of the system.

Knowledge of calculus is not necessary to use this method, whereas the standard procedure, finding the survival probability by convolution, requires knowledge of integral calculus.

The choice of the TI-59 as the computing device for the implementation of the shorthand, though, implied limitations; the number of failure rates is limited due to the limited storage capacity of the TI-59, and computing times are comparatively long. The TI-59 can therefore only be used for smaller systems, preferably for the solution of classroom problems.

For the solution of larger problems, the shorthand should be implemented on a state-of-the-art personal
computer using a general algorithm for the convolution of
any number of exponential random variables.
APPENDIX A

CONVOLUTION FORMULAS

Appendix A contains formulas for the convolution of up to four exponential random variables.

For the two special cases, when all random variables have the same failure rate and all have different failure rates, general formulas for the convolution of any number of exponential random variables are given.

These formulas are used in the two TI-59 programs provided in Appendix B.
System:

Shorthand: \( \text{EXP}(\lambda) \)

Survival Function: \( F(t) = e^{-\lambda t} \)

Description:

A single active component with constant failure rate \( \lambda \).
System:

\[
\text{Survival Function: } F(t) = \frac{(\lambda t)^0}{0!} + \frac{(\lambda t)^1}{1!} + \cdots + \frac{(\lambda t)^{n-1}}{(n-1)!} e^{-\lambda t}
\]

\[
= \sum_{i=1}^{n} \frac{(\lambda t)^{i-1}}{(i-1)!} \frac{-\lambda t}{e}
\]

Description:

A single active component with constant failure rate is supported by \( n-1 \) identical spares.
System:

\[ \text{Shorthand: } \exp(\lambda_1) + \exp(\lambda_2) + \ldots + \exp(\lambda_n) \]

Survival Function: \( \bar{F}(t) = \sum_{i=1}^{n} \left( \prod_{d \neq i} \frac{-\lambda_d}{\lambda_d - \lambda_i} \right) \frac{\lambda_i}{e^{\lambda_i t}} \)

Description:

A single active component with constant failure rate is supported by \( n-1 \) spares. The active component and the spares have all constant, but dissimilar failure rates.
System:

\[ A_1 \]

\[ S_2 \]

\[ S_2 \]

Shorthand: \( \exp(\lambda_4) + \exp(\lambda_2) + \exp(\lambda_2) \)

Survival Function: \( \overline{F}(t) = A e^{-\lambda_4 t} + (B + C e^{-\lambda_2 t}) \)

where \( A = \frac{\lambda_2^2}{(\lambda_2 - \lambda_4)^2} \)
\( B = 1 - A \)
\( C = \frac{\lambda_4 \lambda_2}{\lambda_4 - \lambda_2} \)

Description:

A single active component with constant failure rate \( \lambda_4 \)
is supported by two spares with identical constant failure rate \( \lambda_2 \).
System:

\[
\begin{array}{c}
\text{A}_1 \\
\downarrow \\
\text{S}_2 \\
\downarrow \\
\text{S}_2 \\
\downarrow \\
\text{S}_2
\end{array}
\]

Shorthand: \( \exp(\lambda_1) + \exp(\lambda_2) + \exp(\lambda_2) + \exp(\lambda_2) \)

Survival Function: \( \bar{F}(t) = A e^{-\lambda_1 t} + (B + Ct + Dt^2) e^{-\lambda_2 t} \)

where \( A = \frac{\lambda_2^3}{(\lambda_2 - \lambda_1)^3} \)

\( B = 1 - A \)

\( C = \lambda_2 - \frac{\lambda_2^3}{(\lambda_1 - \lambda_2)^2} \)

\( D = \frac{\lambda_1 \lambda_2^2}{2 (\lambda_1 - \lambda_2)} \)

Description:

A single active component with constant failure rate \( \lambda_1 \)
is supported by three spares with identical constant failure rate \( \lambda_2 \).
System:

![Diagram of system with components and failure rates](image)

Shorthand: $\exp(\lambda_1) + \exp(\lambda_1) + \exp(\lambda_2) + \exp(\lambda_2)$

Survival Function: $F(t) = (A + Bt)e^{-\lambda_1 t} + (C + D^2)e^{-\lambda_2 t}$

where $A = \frac{\lambda_2^3 - 3 \lambda_2^2 \lambda_1}{(\lambda_2 - \lambda_1)^3}$

$B = \frac{\lambda_1 \lambda_2}{(\lambda_2 - \lambda_1)^2}$

$C = 1 - A$

$D = \frac{\lambda_2^2}{(\lambda_1 - \lambda_2)^2}$

Description:

A single active component with constant failure rate $\lambda_1$ is supported by one identical spare and two spares with dissimilar, constant failure rate $\lambda_2$. 
System:

\[ \lambda_1 \]

Shorthand: \( \exp(\lambda_1) + \exp(\lambda_2) + \exp(\lambda_3) + \exp(\lambda_4) \)

Survival Function: \( \bar{F}(t) = A e^{-\lambda_1 t} + B e^{-\lambda_2 t} + (C + D t) e^{-\lambda_1 t} \)

where

\[ A = \frac{\lambda_2 \lambda_3^2}{(\lambda_2 - \lambda_4)(\lambda_3 - \lambda_4)^2} \]

\[ B = \frac{\lambda_1 \lambda_3^2}{(\lambda_1 - \lambda_2)(\lambda_3 - \lambda_2)^2} \]

\[ C = \frac{\lambda_1 \lambda_2}{(\lambda_1 - \lambda_3)(\lambda_2 - \lambda_3)} + \frac{\lambda_1 \lambda_2 \lambda_3}{(\lambda_1 - \lambda_3)(\lambda_2 - \lambda_3)^2} - \frac{\lambda_1}{(\lambda_2 - \lambda_3)^2} \]

\[ D = \frac{\lambda_1 \lambda_2 \lambda_3}{(\lambda_1 - \lambda_3)(\lambda_2 - \lambda_3)} \]

Description:

A single active component with constant failure rate \( \lambda_1 \) has three spares. One spare has constant failure rate \( \lambda_2 \), two spares are identical with constant failure rate \( \lambda_3 \).
APPENDIX B

USER GUIDE TO TI-59 PROGRAMS

Appendix B contains a user guide to two TI-59 programs, which use reliability shorthand and failure rates as input to compute the survival probability of a system.

PROGRAM 1 is designed for the two special cases where the reliability shorthand is of the form

\[ \text{EXP}(\lambda) + \text{EXP}(\lambda) + \ldots + \text{EXP}(\lambda) \]

or

\[ \text{EXP}(\lambda_1) + \text{EXP}(\lambda_2) + \ldots + \text{EXP}(\lambda_n) \]

In the first case the number of terms is not limited, whereas in the second case the number of terms is limited to 40 due to limited storage capacity of the TI-59. In this case the number of terms can be increased to 70 by entering 9 in the display and pressing 2nd Op 1 7.

PROGRAM 2 is designed to solve problems of the kind, that were introduced in Chapter III.8. Due to limited memory of the TI-59 the number of exponential terms under one weight in shorthand notation is limited to four.
All results will be printed, if the TI-59 is connected
to a TI PC-100A or TI PC-100C printer.
PROGRAM 1 : Procedure

1. Use any library module. Read in program 1 (side 1 of the magnetic card)
2. Press 2nd C' to initialize.
3. Enter $n$, the number of exponential terms to be convolved, in the display and press A.
4. Enter time $t$ and press B.
5. Enter $\lambda_i$ and press C. When all failure rates are the same, enter $\lambda$ only once.
6. a) To find the survival probability of the system, when all failure rates are the same, press 2nd A'.
   b) To find the survival probability of the system, when all failure rates are dissimilar, press 2nd B'.

   .
PROGRAM 1: Sample Problems

1. Find the survival probability of a parallel system
   (compare Chapter III.A.2)

a) \( \lambda = 0.3 \), \( t = 7 \), \( n = 2 \)

b) Shorthand notation:

\[ \text{EXP}(0.6) + \text{EXP}(0.3) \]

Enter Comment Press Display

- \( \text{Initialize} \) \( \lambda' \) 0
- 2 \( n \) A 0
- 7 \( t \) B 7
- 0.6 \( 2 \lambda \) C 0.3
- 0.3 \( \lambda \) C 0.3
- \( \bar{F}(t) \) B' \( 0.2299172797 \)

Calculation takes 13 seconds
2. Find the survival probability of a standby-system with dissimilar components (compare Chapter III.A.3).

a) \( \lambda_1 = .4 \), \( \lambda_2 = .5 \), \( t = 6 \), \( n = 2 \)

b) Shorthand notation:

\[
\text{EXP(.4)} + \text{EXP(.5)}
\]

c) Enter Comment Press Display

<table>
<thead>
<tr>
<th>Initialize</th>
<th>( r )</th>
<th>0</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td>( n )</td>
<td>A</td>
</tr>
<tr>
<td>6</td>
<td>( t )</td>
<td>B</td>
</tr>
<tr>
<td>.4</td>
<td>( \lambda_1 )</td>
<td>C</td>
</tr>
<tr>
<td>.5</td>
<td>( \lambda_2 )</td>
<td>C</td>
</tr>
<tr>
<td>( \overline{F}(t) )</td>
<td>B'</td>
<td>.254441493</td>
</tr>
</tbody>
</table>

calculation takes 13 seconds
3. Find the survival probability of a standby-system with one active component and four similar spares.

a) \( \lambda = .3 \), \( t = 7 \), \( n = 5 \)

b) Shorthand notation:

\[ \exp(.3) + \exp(.3) + \exp(.3) + \exp(.3) + \exp(.3) \]

c) Enter Comment Press Display

<table>
<thead>
<tr>
<th>Enter</th>
<th>Comment</th>
<th>Press</th>
<th>Display</th>
</tr>
</thead>
<tbody>
<tr>
<td>Initialize</td>
<td>( c' )</td>
<td></td>
<td>0</td>
</tr>
<tr>
<td>5</td>
<td>( n )</td>
<td>( A )</td>
<td>0</td>
</tr>
<tr>
<td>7</td>
<td>( t )</td>
<td>( B )</td>
<td>7</td>
</tr>
<tr>
<td>.3</td>
<td>( \lambda )</td>
<td>( C )</td>
<td>.3</td>
</tr>
</tbody>
</table>

\[ \overline{F}(t) = A' = .9378738848 \]

calculation takes 9 seconds
PROGRAM 2 : Procedure

CASE I : To find the convolution of up to four exponential random variables.

1. Use any library module.
   Re-Partition ( enter 2 in the display, press 2nd Op 17 ).
   Read in all four sides of the magnetic card.

2. Press 2nd C' to initialize.

3. Enter n, the number of exponential terms to be convolved, in the display and press A.

4. Enter time t and press B.

5. Enter $\lambda_j$ and press C ( n entries ).

   REMARK: Failure rates, which appear only once in the expression, have to be entered before failure rates, that appear several times.

6. To find the survival probability of the system press E.
PROGRAM 2, CASE I: Sample Problems

(1) Shorthand notation

\[ \exp(h_1) + \exp(h_2) + \exp(h_3) \]

Sample values: \( h_1 = .3, h_2 = .4, t = 7 \)

Procedure:

<table>
<thead>
<tr>
<th>Enter</th>
<th>Comment</th>
<th>Press</th>
<th>Display</th>
</tr>
</thead>
<tbody>
<tr>
<td>Initialize</td>
<td>( c' )</td>
<td>( 0 )</td>
<td></td>
</tr>
<tr>
<td>3</td>
<td>n</td>
<td>A</td>
<td>0</td>
</tr>
<tr>
<td>7</td>
<td>t</td>
<td>B</td>
<td>7</td>
</tr>
<tr>
<td>.3</td>
<td>( h_1 )</td>
<td>c</td>
<td>.3</td>
</tr>
<tr>
<td>.4</td>
<td>( h_2 )</td>
<td>c</td>
<td>.4</td>
</tr>
<tr>
<td>.4</td>
<td>( h_2 )</td>
<td>c</td>
<td>.4</td>
</tr>
<tr>
<td>( F(t) )</td>
<td>E</td>
<td>.5363473866</td>
<td></td>
</tr>
</tbody>
</table>

calculation takes 14 seconds
(2) Shorthand notation

\[ \exp(\lambda_1) + \exp(\lambda_2) + \exp(\lambda_3) + \exp(\lambda_4) \]

Sample values: \( \lambda_1 = .2, \lambda_2 = .4, t = 3 \)

Procedure:

<table>
<thead>
<tr>
<th>Enter</th>
<th>Comment</th>
<th>Press</th>
<th>Display</th>
</tr>
</thead>
<tbody>
<tr>
<td>n</td>
<td>A</td>
<td>4</td>
<td>0</td>
</tr>
<tr>
<td>t</td>
<td>B</td>
<td>3</td>
<td></td>
</tr>
<tr>
<td>.2</td>
<td>( \lambda_1 )</td>
<td>C</td>
<td>.2</td>
</tr>
<tr>
<td>.4</td>
<td>( \lambda_2 )</td>
<td>C</td>
<td>.4</td>
</tr>
<tr>
<td>.4</td>
<td>( \lambda_3 )</td>
<td>C</td>
<td>.4</td>
</tr>
<tr>
<td>.4</td>
<td>( \lambda_4 )</td>
<td>C</td>
<td>.4</td>
</tr>
</tbody>
</table>
| \( \bar{F}(t) \) | E       | \( \exp(0.9809746099) \)

Calculation takes 20 seconds
(3) **Shorthand notation**

\[ \exp(\lambda_4) + \exp(\lambda_1) + \exp(\lambda_2) + \exp(\lambda_3) \]

Sample values: \( \lambda_4 = .4 \), \( \lambda_2 = .3 \), \( t = 5 \)

**Procedure:**

<table>
<thead>
<tr>
<th>Enter</th>
<th>Comment</th>
<th>Press</th>
<th>Display</th>
</tr>
</thead>
<tbody>
<tr>
<td>Initialize</td>
<td>( \tau )</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>4</td>
<td>( n )</td>
<td>A</td>
<td>0</td>
</tr>
<tr>
<td>5</td>
<td>( t )</td>
<td>B</td>
<td>5</td>
</tr>
<tr>
<td>.4</td>
<td>( \lambda_4 )</td>
<td>C</td>
<td>.4</td>
</tr>
<tr>
<td>.4</td>
<td>( \lambda_4 )</td>
<td>C</td>
<td>.4</td>
</tr>
<tr>
<td>.3</td>
<td>( \lambda_2 )</td>
<td>C</td>
<td>.3</td>
</tr>
<tr>
<td>.3</td>
<td>( \lambda_2 )</td>
<td>C</td>
<td>.3</td>
</tr>
<tr>
<td>( \bar{F}(t) )</td>
<td>E</td>
<td>( .9029040721 )</td>
<td></td>
</tr>
</tbody>
</table>

calculation takes 20 seconds
(4) Shorthand notation

\[ \exp(\lambda_4) + \exp(\lambda_2) + \exp(\lambda_3) + \exp(\lambda_3) \]

Sample values: \( \lambda_1 = .1 \), \( \lambda_2 = .3 \), \( \lambda_3 = .5 \),

\( t = 10 \)

Procedure:

<table>
<thead>
<tr>
<th>Enter</th>
<th>Comment</th>
<th>Press</th>
<th>Display</th>
</tr>
</thead>
<tbody>
<tr>
<td>Initialize</td>
<td>( c' )</td>
<td>( 0 )</td>
<td></td>
</tr>
<tr>
<td>4</td>
<td>( n )</td>
<td>( A )</td>
<td>( 0 )</td>
</tr>
<tr>
<td>10</td>
<td>( t )</td>
<td>( B )</td>
<td>( 10 )</td>
</tr>
<tr>
<td>.1</td>
<td>( \lambda_1 )</td>
<td>( \gamma )</td>
<td>( .1 )</td>
</tr>
<tr>
<td>.3</td>
<td>( \lambda_2 )</td>
<td>( \gamma )</td>
<td>( .3 )</td>
</tr>
<tr>
<td>.5</td>
<td>( \lambda_3 )</td>
<td>( \gamma )</td>
<td>( .5 )</td>
</tr>
<tr>
<td>.5</td>
<td>( \lambda_3 )</td>
<td>( \gamma )</td>
<td>( .5 )</td>
</tr>
<tr>
<td>( F(t) )</td>
<td>( E )</td>
<td>( .7312684703 )</td>
<td></td>
</tr>
</tbody>
</table>

calculation takes 25 seconds
PROGRAM 2: Procedure

CASE II: to solve problems of the kind, that were introduced in Chapter III.B.

1. Derive the system's shorthand notation. Find either the
   - graphical representation or
   - the MIX-notation.

2. Use any library module.
   
   Re-Partition (enter 2 in the display, press 2nd Op 17).
   Read in all four sides of the magnetic card.

3. Press 2nd C' to initialize.

4. Enter time t and press B.

5. Repeat the following steps for each path of the graphical representation, i.e. for each convolution in the MIX-notation.
   
   a) Enter n, the number of exponential terms to be convolved, in the display and press A.
   
   b) Enter \( \lambda_i \) and press C.

   REMARK: Failure rates, which appear only once in the expression, have to be entered before failure rates, that appear several times.

   c) Enter \( p_i \), the weight in the ith path, and press D.

   d) To find the part of the system's survival probability, that is contributed by the ith path, press E.
6. To find the survival probability of the system

press 2nd E'.
PROGRAM 2, CASE II : Sample Problems

1. Find the survival probability of a parallel system with dissimilar failure rates (compare Chapter III.B.1).
   a) $\lambda_1 = .1 , \lambda_2 = .2 , \tau = 2$
   b) Shorthand notation

   \[
   F(t) = \text{MIX}(\frac{.2}{.3})(\text{EXP}(.1) + \text{EXP}(.3), \frac{.1}{.3})(\text{EXP}(.2) + \text{EXP}(.3)).
   \]
Procedure:

<table>
<thead>
<tr>
<th>Enter</th>
<th>Comment</th>
<th>Press</th>
<th>Display</th>
</tr>
</thead>
<tbody>
<tr>
<td>Initialize</td>
<td>c'</td>
<td>0</td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>t</td>
<td>B</td>
<td>2</td>
</tr>
<tr>
<td>2</td>
<td>n_1</td>
<td>A</td>
<td>0</td>
</tr>
<tr>
<td>.1</td>
<td>λ_1</td>
<td>C</td>
<td>.1</td>
</tr>
<tr>
<td>.3</td>
<td>λ_1 + λ_2</td>
<td>C</td>
<td>.3</td>
</tr>
<tr>
<td>(.2/.3)</td>
<td>p_1</td>
<td>D</td>
<td>.6666666667</td>
</tr>
<tr>
<td></td>
<td></td>
<td>E</td>
<td>.635793541</td>
</tr>
<tr>
<td>2</td>
<td>n_2</td>
<td>A</td>
<td>0</td>
</tr>
<tr>
<td>.2</td>
<td>λ_2</td>
<td>C</td>
<td>.2</td>
</tr>
<tr>
<td>.3</td>
<td>λ_1 + λ_2</td>
<td>C</td>
<td>.3</td>
</tr>
<tr>
<td>(.1/.3)</td>
<td>p_2</td>
<td>D</td>
<td>.3333333333</td>
</tr>
<tr>
<td></td>
<td></td>
<td>E</td>
<td>.30445622</td>
</tr>
<tr>
<td>F(t)</td>
<td>E'</td>
<td>.940239163</td>
<td></td>
</tr>
</tbody>
</table>

57
2. Find the survival probability of a series system with one spare as introduced in Chapter III.B.2.

a) \( \lambda_1 = 0.3 \), \( \lambda_2 = 0.5 \), \( t = 7 \)

b) Shorthand notation

\[
\bar{F}(t) = \text{MIX}[\text{EXP}(0.8), \text{EXP}(0.8) + \text{EXP}(0.8)]
\]
c)

**Procedure:**

<table>
<thead>
<tr>
<th>Enter</th>
<th>Comment</th>
<th>Press</th>
<th>Display</th>
</tr>
</thead>
<tbody>
<tr>
<td>Initialize</td>
<td>Z'</td>
<td>0</td>
<td></td>
</tr>
<tr>
<td>7</td>
<td>t</td>
<td>B</td>
<td>7</td>
</tr>
<tr>
<td>1</td>
<td>n₁</td>
<td>A</td>
<td>0</td>
</tr>
<tr>
<td>.8</td>
<td>λ₁ + λ₂</td>
<td>C</td>
<td>.8</td>
</tr>
<tr>
<td>(.3/.8)</td>
<td>p₄</td>
<td>D</td>
<td>.375</td>
</tr>
<tr>
<td></td>
<td></td>
<td>E</td>
<td>.0013866989</td>
</tr>
<tr>
<td>2</td>
<td>n₂</td>
<td>A</td>
<td>0</td>
</tr>
<tr>
<td>.8</td>
<td>λ₁ + λ₂</td>
<td>C</td>
<td>.8</td>
</tr>
<tr>
<td>.8</td>
<td>λ₁ + λ₂</td>
<td>C</td>
<td>.8</td>
</tr>
<tr>
<td>(.5/.8)</td>
<td>p₂</td>
<td>D</td>
<td>.625</td>
</tr>
<tr>
<td></td>
<td></td>
<td>E</td>
<td>.0152536878</td>
</tr>
<tr>
<td></td>
<td>F</td>
<td>.0166403867</td>
<td></td>
</tr>
</tbody>
</table>

F(t) | E' | .0166403867 |
3. Find the survival probability of a Two-out-of-Three System as introduced in Chapter III.B.3.

a) \( \lambda_1 = 0.2, \lambda_2 = 0.4, \lambda_3 = 0.5, \tau = 9 \)

b) Shorthand notation

\[
\bar{F}(t) = \text{MIX}\left[ \begin{array}{c}
(0.2/1.1) (\exp(0.9) + \exp(1.1)), \\
(0.4/1.1) (\exp(0.7) + \exp(1.1)), \\
(0.5/1.1) (\exp(0.6) + \exp(1.1)) \end{array} \right].
\]
c) Procedure:

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