THE MORPHOLOGY OF A MULTI-BUBBLE SYSTEM IN THE IONOSPHERE

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A multi-bubble model is developed to study the morphology of a finite array of plasma density depletions (bubbles) in the context of equatorial F-region irregularities during spread F. The Pedersen current conservation equation with quasi-neutrality is solved analytically using an electrostatic analogy. The solution is exact with no a priori assumption regarding the separation distance. A two-bubble system with a piecewise constant density profile is first analyzed and the technique is then applied to multi-bubble systems to calculate the polarization electric field and the rise velocities. It is shown that the... (Continues)
influence of the neighboring bubbles is relatively short-ranged and that a small number of bubbles can adequately model the essential physics in a large array of bubbles. For moderately short separation distances, it is found that the $E \times B$ rise velocity is substantially reduced in comparison with the single-bubble case and that the rise velocity is strongly sheared resulting in the deformation of the contours. The implications of the new morphological results on the stability and dynamical behavior of the bubbles are discussed. The analysis can also be applied to a multi-plasma density enhancement (striation fingers and plasma clouds) system such as one might encounter in plasma cloud striation fingers.
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I. INTRODUCTION

The behavior of ionospheric plasma has been extensively studied with the aim of understanding various ionospheric phenomena including equatorial spread F (ESF) and plasma cloud striations (see the following reviews: Ossakow, 1979; Fejer and Kelley, 1980; Ossakow, 1981; Kelley and McClure, 1981; Ossakow et al., 1982). In particular, ESF is thought to be initiated by the Rayleigh-Taylor instability, first proposed by Dungey (1956). In the context of this idea, the plasma density depletions resulting from the instability acquire upward polarization induced $E \times B$ drift velocities. A large body of literature has since been developed to describe the linear and nonlinear properties of Rayleigh-Taylor plasma density irregularities under various assumptions [Haerendel, 1975; Balsley et al., 1972; Chaturvedi and Kaw, 1975a,b; Hudson and Kennel, 1975]. In particular, considerable attention has been given to the morphology and motion [Scannapieco and Ossakow, 1976; Ossakow and Chaturvedi, 1978; Ott, 1978; Hudson, 1978; Anderson and Haerendel, 1979; Ossakow et al., 1979; Zalesak and Ossakow, 1980; Zalesak et al., 1982] of plasma depletions ("bubbles") in the equatorial ionosphere. Moreover, a number of observations [Woodman and LaHoz, 1976; Kelley et al., 1976; Hanson and Sanatani, 1971; McClure et al., 1977; Szuszczewicz et al., 1980, 1981] have indicated the presence of rising plasma bubbles.

Another phenomenon of interest is that of the striations in (artificial) plasma "clouds" (density enhancements). This effect has been attributed to the $E \times B$ gradient drift instability [Linson and Workman, 1970; Volk and Haerendel, 1971] and appears to be amenable to treatments similar to ESF [Scannapieco and Ossakow, 1976; Scannapieco et al., 1976; Ossakow and Chaturvedi, 1978]. The Rayleigh-Taylor instability and the $E \times B$ gradient drift instability are both interchange modes that may occur in the leading edge of plasma bubbles and backside of clouds respectively. In both cases, the resulting density depletions and enhancements are thought to drift by the polarization induced $E \times B$ drift. The electric field is produced by the polarization of the plasma across the earth's magnetic field. The essential ingredient is the small but nonzero ion-neutral collision frequency.

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As a result of the initial instability such as the Rayleigh-Taylor instability, an array of density depletions are formed with the wave vector perpendicular to the earth’s magnetic field. Moreover, the leading edge of a bubble itself is Rayleigh-Taylor unstable resulting in further "bifurcation" [Ossakow and Chaturvedi, 1978]. Similarly, the $E \times B$ instability causes the backside of an initial cloud to striate, forming "finger-like" structures. Thus, the plasma depletions and enhancements typically occur in multitudes. Indeed, McClure et al., (1977) and Szuszczewicz et al., (1980;1981) have given evidence for multiple bubbles. In the previous theoretical and numerical simulation works [Ossakow and Chaturvedi, 1978; Anderson and Haerendel, 1979; Zalesak and Ossakow, 1980; Overman et al., 1982], the morphology and evolution have been studied using one-bubble models or a uniformly distributed array of bubbles (clouds). As a result, the intrinsic influence of neighboring bubbles on each other has not been well quantified. Because the bubbles rise due to the polarization induced $E \times B$ drift, the electric field configuration in and around the bubbles is of central importance.

In the present paper, we study the structure of electrically interacting multi-bubble systems and seek to identify the nature and effects of the mutual interaction. For this purpose, we start with a simple two-bubble configuration in the context of a fluid description and with the emphasis on identifying the basic physics. A simple piecewise constant density profile is used and the electric field inside and outside the bubbles is obtained analytically by the method of images. The method is then applied to multi-bubble configurations. In a sense, the present work is a non-trivial extension of the single bubble work of Ossakow and Chaturvedi (1978).

The scope of the present paper is limited to discussing the morphology of multi-bubble systems and the time-dependent evolution is not explicitly considered. However, the implications of the results will be discussed in the context of the behavior of bubbles consistent with the approximations used in the analysis. Although we primarily discuss the ESF plasma density depletions (bubbles), the technique developed here can be extended straightforwardly to the treatment of plasma density enhancements (clouds).
In Section II, we develop a two-bubble model based on an electrostatic (dielectric) analogy and solve the current conservation equation with quasi-neutrality for the electric field. Section III describes a multi-bubble model and section IV contains the discussion.

II. A TWO-BUBBLE MODEL

A. Formulation

In the present paper, we consider the electric field configuration of electrically interacting multi-bubble systems imbedded in a uniform background plasma and neutral gas. In order to illustrate the basic physics and the theoretical method, we first develop a simple two-bubble model. For this purpose, we adopt a sharp-boundary density profile in which the plasma density is piecewise constant, being uniform inside ($n_1$) and outside ($n_2$) the bubbles and having a discontinuity at the bubble boundaries [Haerendel, 1973; Ossakow and Chaturvedi, 1978; Overman et al., 1982]. The bubbles are modelled by two-dimensional cylinders at the same altitude with circular cross-sections and the axis of the cylinders are aligned with the earth's magnetic field which is assumed to be uniform along the positive z-axis. No neutral wind is included. Figure 1 shows schematically the geometry and the coordinate system. The bubbles are located at $x = -x_0$ and $x = x_0$ so that the inter-bubble separation distance is $2x_0$. For reference purposes, we denote the cylinders by $C1$ and $C2$, respectively. The radius of each bubble is $a$. In the equatorial and low latitude regions, the $x$-axis is along the east-west direction and the gravitational force is along the negative $y$-axis.

The basic equations in our model are the particle conservation, momentum conservation and current conservation equations [see, for example, Ossakow, 1981]. In addition, quasi-neutrality can generally be assumed on the time scales of interest. In the present paper, we study the morphology of two- and multi-bubble systems by solving the current conservation equation to obtain the instantaneous electric field perpendicular to the magnetic field.
The current conservation equation with quasi-neutrality is

\[ \nabla \cdot \mathbf{J} = 0 \]  

(1)

where \( \mathbf{J} \) is the plasma current density due to ion and electron drifts. Neglecting the inertia terms in the momentum equations for cold ion and electron fluids, the current density in the lab frame can be expressed as (see, for example, Ossakow et al., 1979)

\[ \mathbf{J} = e \mathbf{n} \left[ \frac{1}{\Omega_i} \mathbf{g} \times \mathbf{z} + \frac{v_{in}}{\Omega_i} \left( \frac{1}{n_i} \mathbf{g} + \frac{c}{B} \mathbf{E} \right) \right], \]  

(2)

where \( \Omega_i = eB/m_i c \) is the ion cyclotron frequency, \( \mathbf{z} \) is the unit vector along the earth’s magnetic field, \( \mathbf{g} \) is the gravitational acceleration, \( n \) is the plasma density, and \( v_{in} \) is the ion-neutral collision frequency. In (2), we have used the fact that \( m_e/m_i \ll 1 \) and have neglected the electron \( \mathbf{g} \times \mathbf{z} \) contribution. In addition, in arriving at (2), we have made the approximation \( v_{in}/\Omega_i \ll 1 \). In the F region, \( v_{in}/\Omega_i \) is typically of the order of \( 10^{-2} \) or less.

The second term in the square brackets in (2) gives the force-field \((\mathbf{g} \text{ and } \mathbf{E})\) aligned drift currents due to the finite ion-neutral collisions. It is convenient to separate the electric field \( \mathbf{E} \) according to

\[ \mathbf{E} = \tilde{\mathbf{E}} - \frac{m_i}{e} \mathbf{g}, \]

The term \( -m_i/e \) \( \mathbf{g} \) is the component of \( \mathbf{E} \) cancelling the drift along the gravitational field so that the net drift perpendicular to the magnetic field is described by \( \tilde{\mathbf{E}} \). Then, the current \( \mathbf{J} \) can be written as

\[ \mathbf{J} = \sigma \tilde{\mathbf{E}} \mathbf{j}, \]  

(3)

where

\[ \sigma \equiv v_{in} \frac{n_{ec}}{B \Omega_i}, \]

and

\[ \tilde{\mathbf{E}} \mathbf{j} = \tilde{\mathbf{E}} + \frac{B}{e v_{in}} \mathbf{g} \times \mathbf{z}. \]  

(4)
Here, B and \( \mathbf{g} = -v \mathbf{y} \) are assumed to be uniform. Physically, \( E_J \) can be thought of as the electric field driving the current \( J \) perpendicular to the magnetic field in the frame moving with the velocity \( V_d = -v \mathbf{y} \times \mathbf{z} \) relative to the lab frame and \( \sigma \) may be identified as the Pedersen conductivity due to \( v_{in} \neq 0 \).

Equation (1) can now be written in the equivalent form

\[
\mathbf{V} \cdot (n\mathbf{E}) = -\frac{B}{cv_{in}} (g \times \mathbf{z}) \cdot \mathbf{V}n. \tag{5}
\]

Perkins et al. (1973) obtained this expression and noted that (5) describes a dielectric immersed in a uniform electric field \( E_0 \) where

\[
E_0 = \frac{B}{cv_{in}} g \times \mathbf{z}. \tag{6}
\]

In an earlier work, Longmire (1970) utilized a similar magnetostatic analogy to treat the motion of isolated ion clouds. Ossakow and Chaturvedi (1978) used (5) to analytically study a single bubble system. In this dielectric analogy, \( \mathbf{E} \) is the polarization (self) electric field of the bubbles in the uniform field \( E_0 \) and \( E_J \) corresponds to the total electric field \( (E_0 + \mathbf{E}) \) satisfying the boundary conditions across the bubble boundaries,

\[
(\sigma E_J) \parallel = \text{continuous} \tag{7}
\]

\[
(E_J) \perp = \text{continuous}
\]

and at infinity \((x,y \to \infty)\)

\[
E_J + E_0. \tag{8}
\]

Note that we have implicitly chosen a reference frame in which the electric field of the distant undisturbed ionosphere is \( E_0 \). The symbols \( \parallel \) and \( \perp \) refer to the directions parallel and perpendicular to the boundary surfaces, respectively. In the present paper, we also adopt the dielectric analogy and solve the current conservation equation (1) subject to the above boundary conditions (7) and (8). For this purpose, it is illuminating to
rewrite equation (1) as

$$\nabla \cdot (n \mathbf{E}_2) = 0$$  \hspace{1cm} (9)

In the following section, we describe the method of image dipoles used to solve this "dielectric equation". As a matter of notation, in the remainder of the paper, we use $\mathbf{E}$ without the subscript $J$ to denote the solution of (9).

B. The Method of Image Dipoles

The problem of solving Poisson's equation (9) with multiple disconnected boundaries is generally difficult. However, in the case treated here with circular cross-sections, the dielectric analogy allows us to construct an exact solution. Consider first a single dielectric cylinder of radius $a$ centered at $x = 0$ and a line charge density $q$ located at $x = b$ ($|b| > a$). It is well known (Smythe, 1968) that the induced electric field outside the cylinder is that of a line charge $q' = -q(1-K)/(1+K)$ located at $x = a^2/b$ and a line charge $-q''$ located at $x = 0$. The quantity $K$ is the ratio of the dielectric constant ($\varepsilon_1$) inside the cylinder to that outside the cylinder ($\varepsilon_2$)

$$K \equiv \frac{\varepsilon_1}{\varepsilon_2}.$$  \hspace{1cm} (10)

The induced electric field inside the cylinder is that of a single line charge $q'' = 2q/(1+K)$ located at $x = b$. If we replace the line charge $q$ by a line dipole moment $P = -q \mathbf{x}$ which is equivalent to two equal and opposite line charges separated by a vanishingly small distance, then we find that the electric field due to polarization of the cylinder is that of a single image dipole $P$ given by

$$P = -\left(\frac{1-K}{1+K}\right) \frac{a^2}{b^2} P_0,$$  \hspace{1cm} (11)

located at $x = a^2/b$. Note that no image dipole is present on the axis (to be contrasted with the line charge case) and that $P_0$ and $P$ are colinear, pointing in the opposite directions. The induced electric field inside the cylinder is that due to a dipole moment $P^*$ given by
\[ p^* = \frac{2}{1+K} p_0 \]  

located at \( x = b \).

We now consider two identical dielectric cylinders of radius \( a \) centered at \( x = -x_0 \) and \( x = x_0 \) (see figure 1) immersed in a uniform electric field \( E_0 \). Suppose, for the moment, that the two cylinders are non-interacting. Then, for the purpose of calculating the polarization electric field outside the cylinders, each cylinder may be replaced by a dipole moment \( p_0 \) located at \( x = -x_0 \) and \( x = x_0 \), where

\[ p_0 = \frac{1}{2} \left( \frac{1-K}{1+K} \right) a^2 E_0 \hat{x}, \]  

and \( K \) is defined by equation (10). The components of the self electric field are

\[ E_x = 2p_0 [f(x+x_0, y) + f(x-x_0, y)], \]  

and

\[ E_y = 2p_0 [h(x+x_0, y) + h(x-x_0, y)], \]  

where

\[ f(x,y) = \frac{x^2 - y^2}{(x^2+y^2)^2}, \]  

and

\[ h(x,y) = \frac{2xy}{(x^2+y^2)^2}. \]

Here the \( x \)- and \( y \)-components of the electric field due to a dipole \( p_0 \) at \( x = x_0 \) is \( 2p_0 f(x-x_0, y) \) and \( 2p_0 h(x-x_0, y) \), respectively. The electric field \( E^* \) inside the bubbles is

\[ E^* = \frac{2}{1+K} E_0. \]  

In the remainder of the paper, asterisks will be used to denote the electric fields inside the bubbles and dipole moments producing the fields.
Fig. 1 A schematic drawing of two plasma density depletions and the coordinate system. The depletions have circular cross-sections and are infinite in extent along the z-direction.
We now allow the two cylinders to interact with each other. In addition to the uniform external field $E_0$, each cylinder experiences the dipole field of the other. As a result, each cylinder is further polarized giving rise to image dipoles as given by (11) and (12). Iterating the method of image dipoles described above, it is straightforward to show that the total field outside the bubbles is given by

$$E_x = -E_0 + \sum_{n=0}^{\infty} 2P_n \left[ f(x+x_n, y) + f(x-x_n, y) \right],$$

(19)

and

$$E_y = \sum_{n=0}^{\infty} 2P_n \left[ h(x+x_n, y) + h(x-x_n, y) \right],$$

(20)

where $f$ and $h$ are defined by (16) and (17). Here, for $n \neq 0$,

$$P_n = -\left( \frac{1-K}{1+K} \right) \frac{a^2}{b_n} P_{n-1},$$

(21)

$$b_n = x_0 + x_{n-1},$$

(22)

and

$$x_n = x_0 - \frac{a^2}{x_0 + x_{n-1}}.$$

(23)

For $n = 0$, we have $x_0 = x_{n-1}$ and $P_0$ is given by (10).

Similarly, the total electric field inside each cylinder, say, (C2) located at $x = x_0$, is found to be

$$E^*_x = -\frac{2}{1+K} E_0 + \frac{2}{1+K} \sum_{n=0}^{\infty} 2P_n f(x+x_n, y),$$

(24)

and

$$E^*_y = \frac{2}{1+K} \sum_{n=0}^{\infty} 2P_n h(x+x_n, y),$$

(25)

where $P_n$ and $x_n$ are defined above. For the other bubble (C1), the field is obtained by replacing $x_0$ with $-x_0$ in the functions $f$ and $h$. 

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It can be seen from (19)-(23) that the convergence properties of these series depend on the parameter \( s \) defined by

\[
\frac{1-K}{1+K} \frac{a^2}{(2x_0)^2}.
\]

Each successive line dipole moment is reduced by a factor of \( s \) and a geometrical factor of order unity from the preceding one. Since \( x_0 > a \), we have \(|s| < 1/4\) for any \( K \). As a result, the series are generally rapidly convergent except for very small center-to-center separation distances \( (x_0 \sim a) \).

C. Applications

In order to apply the above dielectric results to the ionospheric bubble and cloud problems in accordance with the dielectric analogy (equations (5) and (6)), we identify the dielectric constant \( \sigma \) with the Pedersen conductivity defined in Section II and recall that \( E_o \) is given by equation (6). Note that the Pedersen conductivity is proportional to the plasma density so that

\[
K = \frac{n_1}{n_2},
\]

where \( n_1 \) and \( n_2 \) are the plasma densities inside and outside the bubbles. With these identifications, equations (19), (20), (24) and (25) describe the electric field of a two-bubble system in the frame moving with \( V_d = -\bar{\omega} \frac{1}{\bar{u}} \bar{\mathbf{B}} \times \bar{\mathbf{E}} \) relative to the earth. Note that our formalism guarantees that the field components obtained above exactly satisfy equation (9) and the boundary conditions (7) and (8), as can be verified easily. Thus, the solution is unique. It is also worth noting that all the image line dipole moments are aligned with the \( x \)-axis and no higher multipoles such as quadrupole moment arise. If the bubbles are at different heights, the interaction can still be expressed as a series of image dipole moments that are parallel to the \( x \)-axis but the images are not induced on the \( x \)-axis itself.

As a result of the polarization electric field, the plasma bubbles (density depletions) \( \mathbf{E} \times \mathbf{B} \) drift. The drift velocity relative to the distant undisturbed ionosphere where the electric field is \( \mathbf{E}_o \) is given
by $V = c \left( E_x - E_0 \right) \times \frac{B}{B^2}$. In particular, the single-bubble rise velocity $V_1$ is (using equation 18) 

$$V_1 = -\left( \frac{1-K}{1+K} \right) \frac{1}{v_{in}} \mathbf{g}.$$ 

(27)

For bubbles, $K < 1$ and $V_1$ is upward. This result has been obtained by Ossakow and Chaturvedi [1978]. Because $\mathbf{g}$ is assumed to be uniform, a single isolated bubble maintains its circular cross-section as it rises with the constant velocity $V_1$. The presence of a second bubble, however, modifies the rise velocity significantly. In particular, it is no longer uniform and the cross-sections do not remain unchanged. We have numerically carried out the summations indicated in (19), (20), (24), and (25). The results are depicted in figure 2. The solid lines represent the self electric field lines or equivalently the Pedersen current lines while the dashed lines correspond to the instantaneous $E \times B$ drift velocity ($V_2$) for a two-bubble system ($2x_0 = 2.5a$ and $K = 0$ for 100% depletion). This figure shows only one quadrant; the actual system is spatially uniform in the $z$-direction and is symmetric about the $y$-$z$ plane and the $x$-$z$ plane. The solid electric field lines are such that the line density is proportional to the field strength. This figure clearly shows that the electric field inside the bubble is significantly modified from the uniform field of an isolated bubble (see (18)). As a general remark, the interaction vanishes as $K + 1$ and the separation distance increases to infinity.

In this example, the electric field strength at the point $A$ ($x = x_0 - a$, $y = 0$) is approximately 1/3 of that at $B$ ($x = x_0 + a$, $y = 0$). The prominently nonuniform electric field inside the bubbles has a number of important implications for the dynamic behavior of the two-bubble system. The drift lines (dashed) in figure 2 show that different regions of a bubble undergo drift in the east-west ($\hat{x}$) direction with respect to the undisturbed ionosphere. This horizontal drift can be a significant fraction of the vertical rise velocity at some points inside the bubble. For example, $\left| \frac{E_y}{E_x} \right| \approx 0.55$ at $r = a_\ast$, and $\theta = \pi/8$ where $r$ is the radial distance from $x = x_0$, $a_\ast$ is just inside the bubble surface and $\theta$ is measured from the point $A$ ($\theta = 0$). Here, $x_0 = 1.25$ and $K = 0$ have been used (figure 2). The reason for the strong divergence in the field lines
Fig. 2  A drawing of the electric field lines (solid lines) and the $E \times B$ drift lines (dashed lines) showing one quadrant of a two-bubble system. The system is symmetric about the $x$-$z$ plane and $y$-$z$ plane. The separation distance is $2x_0 = 2.5a$ and $K = 0$ (100% depletion). The points A and B are inside the bubbles at $x = x_0 - a$ and $x = x_0 + a$, respectively.
is that the dipole field due to one bubble opposes the internal field of the other bubble.

Figure 3 shows the two-bubble rise velocity \( (V_y) \) at a number of points inside the bubble \( (C2) \) relative to the uniform rise velocity \( V_1 \) of a single bubble given by (27). The ratio \( R_2 \equiv V_y/V_1 \) is plotted for the interior points A and B (figure 2) as a function of \( x_0/a \). We note that the rise velocity \( V_y \) is strongly affected by the neighboring bubble for center-to-center separation distances \((2x_0)\) smaller than \( 5a \) to \( 6a \). Moreover, comparing \( V_y \) at A and at B, we see that the \( E \times B \) drift velocity is sheared. This, together with the conclusions of the preceding paragraph, implies that an initially circular cross-section would not remain circular so that the two-bubble system described here does not correspond to a steady-state system. This should be contrasted with single-bubble models in which steady-state solutions are possible for piecewise constant density profiles. Because a neighboring bubble generally introduces nonuniformity in the electric field, it seems difficult to construct a steady-state two-bubble system unless \( x_0/a \) is large.

In interpreting the results, we note that our results do not include the time-dependence. The above field configuration exists if the two-bubble system as described is created at some time (say, \( t = 0 \)). Thus, it would be appropriate as a consistent initial condition for the purpose of studying the subsequent time-evolution. In practice, the drift lines shown in figure 2 are expected to closely approximate the actual evolution for some period of time after \( t = 0 \) until the distortion changes the field topology significantly. However, the general feature of the field with the weakest \( x \)-component and hence the slowest rise velocity in the region nearest to the neighboring bubble should remain unchanged in time.

III. A MULTI-BUBBLE MODEL

In the preceding section, we have discussed in detail a two-bubble model. In considering a multi-bubble system that may be applicable to the ESF phenomenon, the basic physics and the theoretical treatment remain unchanged. However, as the number of bubbles increases, so does the number of image dipoles. For an \( N \)-bubble system, the \( n^{th} \) order expression must
Fig. 3 The ratio $R_2$ of the vertical rise velocity $V_y$ of a two-bubble system to the single-bubble rise velocity $V_1$ (equation (27)) as a function of $x_0/a$. The separation distance is $2x_0 = 2.5a$ and $K = 0$ (100% depletion). The curve A corresponds to the point $A (x = x_0 - a)$ and the curve B to the point $B (x = x_0 + a)$ in figure 2.
include \( N(N-1)^N \) image dipoles. Fortunately, the influence of a bubble decreases as the inverse square of the separation distance. Thus, the nearest and the second nearest neighbors are expected to have the dominant effects. A consideration of equations (11) and (12) shows that the second nearest neighbors have effects of the order of \( 4^{-2}E_0 = 0.06 E_0 \) and that the third nearest neighbors have effects of the order of \( 6^{-2}E_0 = 0.03 E_0 \) for a given value of \( a/x \). This means that the inter-bubble (and also inter-cloud) interaction is short-ranged and that the third nearest neighbors and beyond have no significant influence. Thus, only a small number of bubbles are necessary to model an \( N \)-bubble system with \( N \gg 1 \). Note that the influence of the bubbles still vanishes at infinity, differing from systems satisfying periodic boundary conditions.

The theoretical treatment described is exactly applicable to any \( N \). However, in the remainder of this section, we include up to a total of 5 mutually interacting bubbles. The mathematical manipulations involved are analogous to those of the preceding section, resulting in series expressions similar to (19), (20), (24), and (25). As before, only dipole moments, not higher multipoles, are induced. Since no new insight is to be gained by examining the actual expressions, we give below only the results. In figure 4, we show the electric field and drift configurations of a system with three plasma depletions. The three depletions are again modelled by cylinders of radius \( a \), located at \( x = -2x_0, 0, \) and \( 2x_0 \). Only one quadrant is shown. The neighboring bubbles are separated by a distance \( 2x_0 = 2.5a \), and are 100\% depleted \( (K = 0) \) as before. The "external" electric field is \( E_0 \) given by (6), and the density profile is piecewise constant. The solid lines represent the polarization electric field without \( E_0 \). The quantity \( c(\mathbf{E} - E_0) \times \mathbf{B}/B^2 \) is then the instantaneous drift velocity relative to the distant undisturbed ionosphere and is represented by the dashed lines. Although the three-bubble system is different from a two-bubble system in that the former has a central bubble about which the system is symmetric, the general features of the field and drift configurations are similar as can be seen by comparing figures 2 and 4. That is, the field lines and drift lines inside and around the end bubbles of the three-bubble system (figure 4) are similar to those of the two-bubble system (figure 2) because the dominant influence arises from the nearest neighbors. The central bubble in the three-bubble
Fig. 4 A drawing of the electric field lines (solid) and the \( E \times B \) drift lines (dashed), showing one quadrant of a three-bubble system. The bubbles are placed at \( x = \pm 2x_0 \) and \( x = 0 \). The points A, B, and C are inside the bubbles at \( x = 0 \), \( x = a \), and \( x = 2x_0 + a \). The separation distance is \( 2x_0 = 2.5a \) and \( K = 0 \) (100% depletion).
system is affected by two neighboring bubbles with comparable effects. The field is markedly reduced in the region nearest a neighboring bubble (point B in figure 4).

Figure 4 also shows the distortion in the electric field which renders the system non-steady-state, as in the two-bubble case. The drift velocity (dashed lines) has an east-west (horizontal) component that may be a significant fraction of the vertical velocity. For example, 

$$\left| \frac{E_y}{(E_x - E_0)} \right| \approx 0.48$$ at $\theta = \pi/8$ from the point B just inside the boundary of the central bubble and 

$$\left| \frac{E_y}{(E_x - E_0)} \right| \approx 0.55$$ at $\theta = \pi/8$ from the point $x = x_0 - a$ and $y = 0$ just inside the boundary of the side bubble. Thus, in the neighborhood of these points, the bubble elements should have significant horizontal drifts.

In addition to the distortion of bubble contours resulting from the non-uniform electric field, the figure also shows that the electric field is substantially reduced from that of an isolated single bubble. This fact is illustrated in figure 5 which gives $R_3 = \left| \frac{V_y}{V_1} \right|$ for the points A, B and C corresponding to $x = 0$, $x = a$ and $x = 2x_0 + a$, all just inside the bubble surfaces. Here, $V_y$ is the vertical drift velocity of the three-bubble system relative to the distant ionosphere. Comparing figure 5 with figure 3, we note that the qualitative behaviour of the rise velocity is similar in both systems, exhibiting significant reduction from that of a single bubble system. However, in the three bubble configuration, the electric field inside the central bubble is substantially weaker than the two neighboring bubbles. Thus, the central bubble has the lowest rise velocity. The line D in figure 5 gives the relative vertical rise velocity of a five-bubble system calculated at $x = a$. Curve D, to be compared with curve B, shows that the influence of the additional bubble on the field inside the central bubble is small. The reduction in rise velocities is increased as the number of bubbles is increased. However, the influence of the bubbles beyond the third nearest neighbor is small. In figure 6, we show the relative vertical velocity $R = \frac{V_y}{V_1}$ as a function of $N$, the number of bubbles, for several values of $x_0$. For a given separation distance, the rise velocity decreases with increasing $N$ and levels off for $N > 3$. Thus, a three-bubble system describes well the basic morphology of an $N$-bubble system.
The ratio $R_3$ of the vertical velocity $V_y$ of a three-bubble system to the single-bubble rise velocity $V_1$, plotted versus $x_0/a$. The curves A, B, and C correspond to the points A, B, and C in figure 4. The curve D (dashed) corresponds to $R_5 = V_y/V_1$ at $x = a$ in a five-bubble system, to be compared with the curve B.
Fig. 6  $R = V_y/V_1$ versus $N$, the number of bubbles, evaluated inside at $x = x_0 - a$ for the two-bubble case and at $x = a$ for all others. $K = 0$. The separation distances ($2x_0$) are (a) $2x_0 = 2.5a$, (b) $2x_0 = 3.2a$, (c) $2x_0 = 4.0a$, and (d) $2x_0 = 10a$. 
Figures 3 and 5 show that the vertical drift velocity of a bubble is a sensitive function of the separation distance \((2x_o)\) except for relatively large values \((2x_o/a \geq 6)\). In the context of the Rayleigh-Taylor instability, this implies that the electric field configuration and the drift velocities may depend sensitively on wavelengths \((2x_o)\).

As a general remark, we point out that the preceding results derived for plasma density depletions for which \(K < 1\) are also applicable to plasma density enhancements (clouds) for which \(K > 1\). Calculations for the cloud case show that the electric field configurations are qualitatively similar to that of the bubble case (figures 2 and 4). In particular, the electric field inside the clouds experiences the greatest reduction in the regions facing the neighboring clouds. However, the boundary condition \(K(E_{in}) = (E_{out})\) implies that \((E_{in})\) is smaller than \((E_{out})\) by a factor of \(K^{-1}\) for clouds. Thus, for a given separation distance \(2x_o\), the relative distortion in the electric field lines inside a cloud is less pronounced than in a bubble.

Finally, in figure 7, we have plotted the vertical drift velocity versus \(K = n_1/n_2\) for a two-bubble (\(K < 1\)) and two-cloud (\(K > 1\)) system. The velocity is calculated at \(x = \pm (x_o - a)\) (point A in figure 2) and is upward for bubbles and downward for clouds. The velocity is normalized to \(V_1\) (equation (27)) for each value of \(K\). Note that the point with \(K = 1\) does not exist for each line. For \(K = 1\), the ionosphere is not disturbed and there is no bubble or cloud drifting vertically. This is born out by the fact that the drift velocity vanishes for a single-bubble (cloud) and any multi-bubble (cloud) system. Clearly, the velocities vanish differently for different separation distances. Mathematically, the ratio of the vertical drift velocity \(V_y = c(E_x^* - E_o)/B_o\) to the single-bubble (cloud) drift velocity \(V_1\) has the limit

\[
\frac{V_y}{V_1} = - \left[ 1 - \frac{2}{1+K} \right] \delta f(x + x_o, y)
\]

as \(K \to 1\). The circled points in figure 7 correspond to the absolute value of the quantity which has no physical meaning.
Fig. 7  \( R_2 = V_y/V_1 \) versus \( K \) for two-bubble (\( K < 1 \)) and two-cloud (\( K > 1 \)) systems. The separation distances (\( 2x_0 \)) are (a) \( 2x_0 = 2.5a \), (b) \( 2x_0 = 4a \), and (c) \( 2x_0 = 10a \).
IV. SUMMARY AND DISCUSSION

In the preceding sections, we have solved the current conservation equation (1) with quasi-neutrality for two-bubble and three-bubble systems using a dielectric analogy. These two configurations include the dominant near-neighbor interaction and can model the essential morphology of the multi-bubble systems described. This is demonstrated by actually calculating the field including up to five bubbles. The solutions are exact, satisfying the specified boundary conditions on all the multiple disconnected boundary surfaces and at infinity.

Equations (19), (20), (24) and (25) give the solution of the two-bubble system as a superposition of image line dipole moments. Similar expressions are obtained for three- and N-bubble (cloud) systems \((N > 3)\). In all cases, the interaction is dominated by the nearest neighbors and is sensitive to the separation distance \((2x_0)\) between bubbles. An important result is that the electric field inside the bubbles is generally highly non-uniform so that the multi-bubble and multi-cloud systems are not steady-state configurations even with piecewise constant density profiles (figures 2 and 4). Moreover, the electric field inside the bubbles is significantly weaker than that in a one-bubble system so that the vertical drift velocity relative to the undisturbed ionosphere is slower than the one-bubble case (equation (27) and figures 3 and 5). For moderately small center-to-center separation distances, \(2x_0 \leq 6a\), the reduction in the rise velocity is substantial. This implies that the electric field configuration and the \(E \times B\) drift velocity may depend sensitively on the wavelengths of instabilities causing the initial density fluctuations (e.g., the Rayleigh-Taylor instability). In particular, the bubbles with smaller wavelength-to-radius ratio would rise more slowly and be distorted more strongly. This conclusion may be particularly applicable to the initial linear or early-time non-linear stages of the evolution. It has also been shown that in some regions inside bubbles, the horizontal drift velocity may be comparable to the vertical drift velocity with \(|E^y/(E^x + E_0^y)|\) as large as 1/2 (Sections II and III).

A corollary that follows from the non-uniform electric field in the bubbles is that the polarization induced \(E \times B\) drift velocity is sheared
and that a component of the electric field parallel to the local density gradient is developed in the leading edges of the bubbles. For the simple piecewise constant density profiles used for our analysis, the density gradient is not well defined. In a more realistic density profile, however, some previous model calculations indicate that the electric field component parallel to the density gradient has stabilizing influences on the Rayleigh-Taylor instability [Guzdar et al., 1982] and the $E \times B$ drift instability [Perkins and Doles, 1975; Huba et al., 1982]. More specifically, these calculations show that velocity shear preferentially stabilizes the short wavelength modes. In light of these results, we suggest that the bifurcation behavior of bubbles and clouds may be inhibited by the presence of nearby bubbles (clouds).

The emphasis of our analysis has primarily been on plasma density depletions (bubbles). However, plasma density enhancements (clouds and striation fingers) can also be treated in a similar fashion [Scannapieco and Ossakow, 1976; Scannapieco et al., 1976; Ossakow and Chaturvedi, 1978]. If a cloud exists in the equatorial or low latitude region, the preceding results are all applicable with the replacement of $K > 1$ ($\lambda < 1$ for bubbles) where $K = n_1/n_2$. In particular, as the backside of an initial cloud begins to bifurcate, the small $\lambda_0/a$ and small $N$ results may be applicable. One point to note is that the electric field inside a two- and three-cloud system with a piecewise constant density profile is similar to that shown in figure 2 and 4 with weaker field strength in the regions facing the neighboring clouds. However, the boundary condition $K(E_{in}^l = (E_{out}^l)$ implies that $(E_{in}^l)$ is smaller than $(E_{out}^l)$ by a factor of $K^{-1}$ for clouds. As a result, the distortion in the electric field lines inside a cloud is less pronounced than in a bubble.

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