NUMERICAL SIMULATION OF A POSSIBLE FREEZING AND SHEET FORMATION MECHANISM FOR BARIUM CLOUD STRIATIONS

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We present a possible mechanism for the freezing, or apparent freezing, of barium cloud striations and for the formation of long thin sheets of barium on the leading edge of the cloud. The essence of the model is that the finite Pedersen mobility of the barium ions allows them to separate from the electron cloud. The barium is replaced in the electron cloud by the ions constituting the ambient ionosphere, which are compressed up
to the required densities by their own Pedersen mobility. In the process of existing the electron cloud, the barium is expanded and hence attains a density lower than when it coexisted with the electron cloud. For many ionospheric parameters the barium will have in effect left the region of further structuring, which will still be proceeding the electron cloud. As the barium leaves the electron cloud, it forms a long thin sheet of lower density barium, one side of which is considerably steeper than the other, which extends from the leading (nonstructuring) edge of the cloud. An observer watching only the barium would conclude that the cloud had frozen. Further, the electron cloud itself may decay if the ions coexisting with it are subject to a fast recombination chemistry.
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1. Introduction

Although many features of barium cloud striations are understood (Oseakow 1979; Ossakow et al., 1982), there exist several aspects of the phenomenology which as of yet lack a satisfactory explanation. This paper addresses two of these phenomena: 1) the persistence of striations with scale sizes in the kilometer range for times on the order of hours [Prettie et al., 1977, J.A. Fedder and W. Chestnut, private communication, 1980]; and 2) the formation and persistence of long thin "sheets" of barium on the leading (non-structuring) side of the cloud. In what follows we shall attempt to show that both of these phenomena can be explained by a proper understanding of the role played by the finite Pedersen mobility both of the barium ions and of the ions constituting the ambient ionosphere. In essence we find that the finite Pedersen mobility of the ions vis à vis the electrons, which essentially have no Pedersen mobility at all, allows the barium to "leak" out of the electron cloud. Quasineutrality in the electron cloud is maintained by incoming ambient ionosphere ions (denoted here by O⁺, but these could also be NO⁺ or other ionospheric ion constituents), which compress up to the electron density as they replace the barium in the cloud. For many ionospheric parameters, once the barium has left the electron cloud, it is essentially free of the polarization fields associated with the electron cloud and hence immune to further structuring. An observer watching only the barium might conclude that it had "frozen". Further, the electron cloud itself may decay if the ions coexisting with it are subject to fast recombination chemistry.

The above ideas were arrived at independently by Prettie [1981] and by Fedder (private communication, 1981). The conclusions of the present paper differ from those in Prettie [1981] primarily in the fact that we find that the formation of sheets is directly related to the leaking of barium from the electron cloud, as just discussed, and is totally unrelated to the formation of "quadrupole" potential fields, as postulated by Prettie [1981]. Rather, the sheet forms as the barium leaks out of one side of the electron cloud. In the process it also leaves the regions of shielding provided by the enhanced Pedersen conductivity associated with the enhanced electron density (for many ionospheric parameters), and over a fairly short distance enters the unshielded region where it takes on the ambient c E₀ x B/B² drift velocity. From the reference frame of the center of the electron cloud, we are slowly

injecting barium into a flow field transverse to the injection direction which starts at zero at the electron cloud center and rapidly becomes large over a very short distance at the "edge" of the electron cloud. The result is a long, low density string of barium which we perceive as a "sheet". This concept, as well as the meaning of the term "many ionospheric parameters" used above, will be given in more detail later in this paper.

In section 2 of this paper we briefly review the theory and relevant equations for the multilevel/multispecies ionospheric barium cloud problem. In section 3 we describe in more detail the basic Pedersen drift mechanism proposed by Prettie (1981) and by Fedder (private communication 1981), and also present our proposed mechanism for barium sheet formation. In section 4 we discuss the effect of the Pedersen drift mechanism under various ionospheric parameter regimes within the context of a two level (two species) model. In section 5 we present preliminary numerical simulation results which seem to bear out the analysis, although one aspect of the simulations, the steep density gradients on one side of the sheet, is not well understood by us at this time. Finally in section 6 we present our conclusions and plans for future work.

2. Theory

Consider a fluid plasma species \( \alpha \) imbedded in a neutral gas and a magnetic field \( B \), which we assume is aligned along the cartesian axis \( z \). Under the time scales and ionospheric parameters of interest for the ionospheric barium cloud striations problem, this plasma will respond to an external force perpendicular to the magnetic field, \( F \), in two ways: 1) by drifting in the direction of the external force (Pedersen mobility); and 2) by drifting in a direction perpendicular to both the external force and to the magnetic field (Hall mobility). Specifically, if the momentum equation for species \( \alpha \) is

\[
\left( \frac{\partial}{\partial t} + v_\alpha \cdot \nabla \right) v_\alpha = \frac{q_\alpha}{m_\alpha} \left( \frac{v_\alpha \times B}{c} \right) - v_{an}(v_\alpha - U_n) + \frac{1}{m_\alpha} G_\alpha \tag{2.1}
\]

and we define the external force to be

\[
F_\alpha = q_\alpha E + v_{an} m_\alpha U_n + G_\alpha \tag{2.2}
\]
then

\[ v_{a_1} = k_{1a} F_{a_1} + k_{2a} F_{a_1} x z \]  

(2.3)

where

\[ k_{1a} = \frac{v_{an} c}{q_a B} \left[ 1 - \frac{(v_{an}/\Omega_a)^2}{1 + (v_{an}/\Omega_a)^2} \right] \]  

(2.4)

\[ k_{2a} = \frac{c}{q_a B} \left[ 1 - \frac{(v_{an}/\Omega_a)^2}{1 + (v_{an}/\Omega_a)^2} \right] \]  

(2.5)

Here we have neglected the left hand side of (2.1) since we are interested only in average drift velocities over time scales that are long compared to either the mean time between collisions or the gyroperiod. Also we have ignored all collisions except those between species \( a \) and the neutral gas. The notation in the above equations is as follows: \( v_a \), \( q_a \) and \( m_a \) are the fluid velocity, charge per particle, and mass per particle respectively of species \( a \); \( E \) is the electric field, \( t \) is time, \( c \) is the speed of light, \( v_{an} \) is the collision frequency of species \( a \) with the neutral gas, \( U_n \) is the neutral gas velocity, \( \Omega_a \) is the cyclotron frequency of species \( a \), \( \Omega_a = |q_a B/(m_a c)| \), and \( G_a \) is a term meant to represent all of the other external forces on species \( a \) (gravity, pressure gradients, etc). The subscript 1 refers to the components of the respective vector which are perpendicular to \( B \).

Starting with the above, one can derive the set of partial differential equations describing the barium cloud striations problem by demanding that each species \( a \) obey a continuity equation and that \( \nabla \cdot J = 0 \) where \( J \) is the electric current, which is to say that we demand electrical quasi-neutrality globally for all time. The quantity \( J \) is defined by multiplying (2.3) by \( q_a n_a \), where \( n_a \) is the number density of species \( a \), and summing over \( a \). If one further assumes that \( E = -\nabla \phi \) where \( \phi \) is the scalar electrostatic potential, then \( \nabla \cdot J = 0 \) yields an elliptic equation for \( \phi \). This derivation is given in several papers [Lloyd and Haerendel, 1973; Goldman et al., 1974; Perkins et al., 1973 for example] and will not be reproduced here. Rather, we simply describe the other simplifying assumptions and give the final equations.
We find that for ionospheric barium clouds it is a very good approximation to assume that the electrostatic potential \( \phi \) is constant along magnetic field lines, i.e., \( \phi = \phi(x,y) \) where the magnetic field \( \mathbf{B} \) is aligned along \( z \) in an \( x-y-z \) cartesian coordinate system. Further we find that since the currents parallel to \( \mathbf{B} \) are carried primarily by electrons and since the motion of ions parallel to \( \mathbf{B} \) consists primarily of a slow diffusion plus a bulk "falling" of the cloud, since \( \mathbf{g} \cdot \mathbf{B} \neq 0 \) where \( \mathbf{g} \) is the gravitational acceleration, it is sufficient to represent the ions as an array of two-dimensional planes of plasma perpendicular to \( \mathbf{B} \), each moving with the bulk "falling" velocity along \( \mathbf{B} \), and hence to treat numerically only the transport of ions perpendicular to \( \mathbf{B} \) within each layer. For cases where multiple ion species coexist in one layer, this layer is simply treated as multiple layers with the same location in space. This treatment is consistent with our neglect of all collisions except ions with neutrals and electrons with neutrals. Thus we use the phrase "multilevel/multispecies" to describe the model.

We draw the readers attention to the following remarks, whose importance shall become clear as we progress. The right hand side of (2.3) gives explicitly the Pedersen and Hall components, respectively, of the drift velocity of species \( a \). Looking at (2.4) we find that \( k_{la} = 0 \) for electrons, since \( v_{e}/Q_e = 0 \) (where the subscript \( e \) denotes electrons). Thus ions, for which \( v_{an}/Q_a \) ranges from 0.01 to greater than 1.0, are endowed with a Pedersen mobility which is lacking for the electrons: ions and electrons do not move together. Equally important, under the conditions that \( \mathbf{B} \) is uniform and that the external force \( F_{al} \) can be written as the gradient of a scalar, conditions which are satisfied in most of the barium cloud problems we shall consider, the Hall component of \( v_{al} \) is divergence free while the Pedersen component is not. Thus individual fluid elements of ions may compress or expand, while the electron fluid satisfies the constraint of incompressibility (in the \( x-y \) plane).

In this paper we shall limit our discussion to a model consisting of only two layers or species of ions, as this will considerably simplify the understanding of the ideas we shall put forth without, we believe, compromising the ideas themselves. We shall also confine our external force to an externally imposed electric field \( E_0 \) which is perpendicular to \( \mathbf{B} \), i.e.,
We denote by \( n_1 \) and \( n_2 \) the integrated (along \( B \)) number density of the ions (and by quasi-neutrality, the electrons) in levels 1 and 2 respectively. The equations describing this system are then:

\[
\begin{align*}
\frac{\partial n_\alpha}{\partial t} + \mathbf{v}_1 \cdot (n_\alpha \mathbf{v}_\alpha) &= 0 \quad \alpha = 1, 2 \\
\mathbf{v}_1 \cdot [(\mathbf{E}_{p1} + \mathbf{E}_{p2})\mathbf{v}_1] + \mathbf{H} &= \mathbf{E}_0 \cdot \mathbf{v}_1 (\mathbf{E}_{p1} + \mathbf{E}_{p2}) \\
H &= -\frac{\partial}{\partial x} [(E_{h1} + E_{h2})(\frac{\partial \phi}{\partial y} - E_{oy})] + \frac{\partial}{\partial y} [(E_{h1} + E_{h2})(\frac{\partial \phi}{\partial x} - E_{ox})]
\end{align*}
\]

\[\frac{\mathbf{E}_{pa}}{n_\alpha} = (1 + (v_{an}/\Omega_\alpha)^2)^{-1} n_{ac} q_\alpha / |B|, \alpha = 1, 2 \]

\[\frac{\mathbf{E}_{ha}}{n_\alpha} = \frac{v_{an}}{n_\alpha} \mathbf{E}_{pa}, \quad \alpha = 1, 2 \]

The terms comprising \( H \) in (2.7) are referred to as the "Hall terms," and for the purposes of the ideas to be presented in the present paper can be considered to be small. The quantity \( v_{al} \) in (2.6) is defined by (2.3)-(2.5) where

\[ F_{al} = q_\alpha (E_0 - \nabla \phi) \]

3. The Pedersen Leakage Mechanism: The Simplest Case

As mentioned in the previous section we shall consider only a model consisting of two ion species. Since we are neglecting all collisions except those between ions and neutrals (recall we have set \( v_{en}/\Omega_e = 0 \)), and since we are assuming that the only external forces acting are electric fields perpendicular to \( B \), which are constant along any given magnetic field line, one can conceptually think of these layers of plasma as coexisting at the same plane in space, as long as we enforce the caveat that the two ion species \( \alpha \) may "see" different neutral densities, which will be reflected in the value of \( v_{an} \). Hence all of our diagrams of the physical processes can be
of a single two-dimensional plane.

Furthermore, we shall specifically take the two ion species to be \( \text{Ba}^+ \) and the ambient ionosphere ion \( \text{O}^+ \), with the understanding that by "\( \text{O}^+ \)" we might also mean \( \text{NO}^+ \) or any of the other ambient ionospheric ions. Thus we shall henceforth use the subscripts \( b \) and \( o \) to denote \( \text{Ba}^+ \) and \( \text{O}^+ \), rather than the subscripts 1 and 2 used previously. Within the context of this model, it is clear that

\[
n_b + n_o = n_e
\]  

(3.1)

where \( n_e \) denotes the integrated electron density. Hence one can substitute a continuity equation for \( n_e \)

\[
\frac{\partial n_e}{\partial t} + V \cdot (n_e v_e) = 0
\]  

(3.2)

for one of the two equations in (2.6), since the substituted density can always be recovered from (3.1). For the remainder of this paper we shall use this fact and display profiles of barium density and electron density. For the purposes of this section only, we impose one last assumption:

\[
\frac{v_{bn}}{Q_b} = \frac{v_{on}}{Q_o}
\]  

(3.3)

Thus the two ion species are identical in terms of their electrical properties vis-à-vis (2.7) and in terms of their response to an external force (2.3). In fact, if we were interested only in electron densities and not in the actual identities of the ion species making up the "electron cloud", it would suffice to sum the two ion layers and treat them as simply one layer. This is precisely the implicit assumption made in the "one level model" [McDonald et al., 1980]. Instead we shall choose to follow each ion species separately, as this will have interesting consequences.

In Fig. 1 we show a simplified picture of an ionized barium cloud which has been placed in a uniform ionosphere consisting of \( \text{O}^+ \) ions. Iso-density contours of electrons are shown as solid lines, while those for barium are shown as dashed lines. At \( t = 0 \) the two sets of contours are identical. We impose an ambient electric field \( E_o \) directed to the right. The
electron/barium cloud will partially shield itself from this field, and experience an average reduced field $E'$, still directed to the right in Fig. 1. Referring to (2.3)-(2.5) and recalling that $\nu_{en}/Q_e = 0$ we find that the electron cloud simply drifts downward at velocity $cE'/B$. If we assume that $\nu_{bn}/Q_b$ is $\sim 10^{-2}$, then we see that the barium cloud too drifts downward at a velocity very close to $cE'/B$. However the barium also experiences a Pedersen drift to the right of value approximately $(\nu_{bn}/Q_b) cE'/B$. If the clouds initially have diameter $D$ and remain undeformed the barium cloud will completely exit the electron cloud in a time $t = (Q_b/\nu_{bn})DB/(cE')$, as depicted on the left side of Fig. 1. Of course by quasi-neutrality the ions in the electron cloud would then consist of $0^+$ ions which had been compressed up to the required density (the electron density) as they entered the electron cloud from the left. By the same principle, the barium must expand as it leaves the electron cloud to the right, showing that this motion cannot truly take place without deformation. Also one might expect to see the normal electron cloud steepening and structuring during this time interval. In addition as the barium exits the electron cloud to the right it leaves a region where the electric field has been reduced by shielding to $E'$ and enters a region where $E$ takes on the larger value $E_0$. Accordingly as it leaves it gets caught up in this higher velocity field and forms a sheet of lower density barium which leads the cloud. This somewhat more realistic picture is depicted on the right side of Fig. 1.

The above picture describes in qualitative terms the essence of this paper. We will now expand upon this idea in more quantitative terms by simplifying the picture even further so that quantitative results can be obtained and that a more precise picture may be obtained of the compression and expansion processes proceeding in the ion fluids.

In Fig. 2a we again show a profile, $n_0$ vs $x$, of an ionized barium cloud which has been placed in a uniform ionosphere consisting of $0^+$ ions, again subject to an externally imposed electric field $E_0$ pointing to the right. This time, however, we choose the cloud and ionosphere to be infinite in the $y$ direction, the so-called slab geometry. We denote the cloud thickness by $L$ and assume for the sake of example that its integrated density is exactly twice that of the background $0^+$ ionosphere. The barium fluid is shown cross-hatched, while the $0^+$ fluid is shown without cross-hatching. Again we are
Fig. 1 — Schematic depiction of the evolution of an ionized barium cloud subject to an externally imposed rightward directed electric field \( E_0 \). The cloud will partially shield itself, yielding a net field \( E' \) inside the cloud. Rightward Pedersen drifts then allow the barium to separate from the electron cloud in a time \( t_0 \). A simplified picture is shown on the left, while a more realistic evolution is depicted on the right.
assuming that $\nu_{bn}/\Omega_b = \nu_{on}/\Omega_o$ and that both quantities are $\sim 10^{-2}$. Then the potential equation (2.7) has the simple solution

$$E = E_o \quad \text{outside the electron cloud} \quad (3.4)$$

$$= E_o/3 \quad \text{inside the electron cloud}$$

If $B$ is taken to be upward in Fig. 2 then both ion species and the electrons are all drifting toward the reader with velocity very close to $cE/B$. However both the ions have in addition a Pedersen drift to the right given by

$$v_p = (\nu_{bn}/\Omega_b)cE/B \quad (3.5)$$

where $v_p$ is the rightward Pedersen velocity in this example. Noting the discontinuous changes in $E$ that occur at the edges of the electron cloud, we see that ion compression is taking place at the left edge of the electron cloud while expansion is taking place at the right edge. In fact in this example, the ions experience a compression of exactly a factor of 3 as they cross the left edge of the electron cloud, and an expansion of exactly a factor of 3 as they exit the right edge. Of course this is exactly what is required by the condition of quasi-neutrality ($\nabla \cdot J = 0$). One can also calculate the time $t_o$ it takes for the barium to completely exit the electron cloud in a manner analogous to that used previously for Fig. 1. One obtains

$$t_o = \frac{L}{(v_{bn}/\Omega_b)cE_o/3B} \quad (3.6)$$

where $L$ is the width of the cloud. In Fig. 2b we show the time evolution of the profile depicted in Fig. 2a for various units of time in terms of $t_o$. Note that after time $t_o$: 1) the "cloud" consists totally of $O^+$ ions; 2) the barium extends over a region three times its original extent but down by a factor of three in density; 3) the barium is completely removed from gradients in $n_e$ and hence $\Sigma_p$ and therefore would be removed from any further structuring taking place in the analogous situation for a two-dimensional cloud, as in Fig. 1. It is postulated that this may very well be what is known as "freezing". It should also be noted that if the $O^+$ cloud is subject
Fig. 2(a) — Schematic of a barium cloud in an ionosphere consisting of $0^+$ ions, similar to Fig. 1 except that now we are in a "slab" geometry (plasma has infinite extent in y direction). The barium ions are shown cross-hatched while the oxygen ions are shown without cross-hatching. The shielding by the cloud causes a compression of ions to take place at the left edge and an expansion to take place at the right edge. The envelope $n_o + n_b$ depicts the electron density $n_e$, which cannot change in time.

$$t_o = \text{TIME FOR } Ba^+ \text{ IONS TO LEAVE SLAB}$$

$$= \frac{L}{n_i \frac{cE_o}{\Omega_i} \frac{3B}}$$
Fig. 2(b) - Time evolution of this cloud in units of $t_0$. 

- $t = 0$
- $t = \frac{1}{3}t_0$
- $t = \frac{2}{3}t_0$
- $t > t_0$

Low density barium no longer in the "action" (frozen)

$O^+$ striation now subject to fast recombination chemistry (striation decay)
to short chemical recombination times, this mechanism may very well result in the decay of the electron density cloud as well as the freezing of the barium cloud. This recombination effect is discussed in more detail in Prettie [1981].

We are now in a position to understand how it is that the barium that leaks from the electron cloud via the above mechanism appears in the form of sheets of barium leading the electron density cloud. Referring to Fig. 2a, we note that as the barium exits the right edge of the electron cloud, it experiences a factor of three increase in the rightward-directed electric field. It is the change in Pedersen drift associated with this which is responsible for the expansion of the ions as they exit the electron cloud. However it is precisely this same change in \( \mathbf{E} \) which will produce a three-fold increase in the \( cE/B \) velocity of the ions toward the reader. Now consider the slice of barium located at \( y = 0 \) at \( t = 0 \). The right hand edge of this slice will immediately start moving toward the viewer at a velocity \( cE_0/B \) and to the right at velocity \( (\nu_0/\Omega) cE_0/B \). The left hand edge of this slice will move at precisely these same velocities but reduced by a factor of three. At time \( t_0 \) when the barium has completely exited the electron cloud of width \( L \), this slice will have its right hand and left hand coordinates located at

\[
(x = 3L, y = -3 \Omega_b \nu_b^{-1} L) \quad \text{and} \quad (x = 0, y = -\Omega_b \nu_b^{-1} L)
\]

respectively, where we have defined \( x = 0 \) to be the right edge of the electron cloud. The barium slice is now \( 3L \) wide in the \( x \) direction and \( 2L \Omega_b \nu_b^{-1} \) long in the \( y \) direction. For \( \Omega_b \nu_b^{-1} = 20 \), we get a barium slice which is 13 times larger in \( y \) than it is in \( x \). Referring to Fig. 1 we see that our postulated slice in Fig. 2 is analogous to the barium cloud itself in Fig. 1, so that again we expect the barium which has leaked out of the electron cloud to be much longer in the \( y \) direction than in the \( x \) direction, a geometry we could easily term a "sheet". The process is quite similar to what would happen if one were to eject a column of smoke sidewards from a moving automobile: the trail of smoke would be much larger along the direction of travel than transverse to it. From the frame of reference of the electron cloud in Fig. 1, we are injecting a column of barium "smoke" transverse to our direction of travel relative to the ambient plasma. Hence the "trails" of barium.

Prettie [1981] has suggested that the barium sheets are the result of image formation and the resultant "quadrupole" electric fields. While we
would be the last to suggest that image formation is not important in barium cloud striations [Scannapieco et al., 1976; Ossakow et al., 1980], we believe the above shows the explanation to be quite a bit simpler. Numerical simulations to be presented in section 5 also bear this out.

4. The Pedersen Leakage Mechanism: More General Ionospheric Parameters

In the last section we confined ourselves to the case of two ion species, each of which had the same value of $V_{\alpha}/\mu_{\alpha}$. In this section we relax this last constraint for the purposes of a short qualitative discussion. A more detailed discussion of this topic, as well as a treatment of the case of more than two levels will be given in a future paper.

To illustrate the principles involved, we shall again use the one-dimensional slab depicted in Fig. 2, with the understanding that there is no longer a one-to-one correspondence between electron density $n_e$ and total Pedersen conductivity $\Sigma_p = \Sigma_{pb} + \Sigma_{po}$. Hence we shall interpret Fig. 2 to depict the $n_e$ profile, with the $\Sigma_p$ profile depending on the ion species involved, and which is, as we shall see, time dependent.

The principle to have in mind when analyzing the general case is that the electrons cannot move to the left or right, since they have no Pedersen mobility. Hence the electron density profile $n_e$ is invariant in time as depicted in Fig. 2b. Since $n_b + n_o = n_e$, the $Ba^+$ and $O^+$ ions are constrained to move in such a way as to maintain this invariance in $n_e$. In addition, the velocity of each ion species must be proportional to the electric field and to the appropriate $V_{\alpha}/\mu_{\alpha}$. Furthermore, the electric field must be inversely proportional to $\Sigma_p$.

The problem posed above for the general case $V_{bn}/\mu_b \neq V_{on}/\mu_o$ can be solved exactly. We shall not give the proof here, but simply state the remarkably simple result: The time evolution depicted in Fig. 2b for the special case $V_{bn}/\mu_b = V_{on}/\mu_o$ is in fact the general solution for $V_{bn}/\mu_b \neq V_{on}/\mu_o$, except for a change in temporal scaling. Specifically,

$$n_b' = f n_b^0 \quad (4.1)$$

$$n_o' = f n_o^0 \quad (4.2)$$

13
\[ n_o'' = f^{-1} n_o'^o = n_o^o + n_b^o \]  
\[ n_b'' = 0 \]  
\[ t_o = \frac{L \Omega_b}{v_{bn} \Omega} \]  
\[ E'' = E_o \frac{M^o}{M^o} \]  
\[ f \equiv \frac{n_o^o}{n_o^o + n_b^o} \]  
\[ M^o = \frac{(n_o^o v_{\Omega o}^{-1} + n_b^o v_{\Omega b}^{-1})}{(n_o^o v_{\Omega o}^{-1})} \]

where the prime superscript on \( n \) denotes quantities evaluated within the expanded barium cloud at time \( t > t_o \) shown in Fig. 2b, the double prime superscript denotes quantities within the electron cloud at \( t > t_o \) in the same figure, the zero superscript denotes quantities at time \( t = 0 \), and \( t_o \) is, as before, the time taken for the barium cloud to leave the electron cloud. Note that this says that the barium will always leave the electron cloud, but since \( t_o \) is proportional to \( M^o/(v_{bn} / \Omega_b) \) the process may take an extremely long time if \( M^o \) is large or \( v_{bn} / \Omega_b \) is small. Just as important, it says that the Pedersen conductivity ratio \( M \) of the barium and electron clouds to the ambient ionosphere will change in the general case:

\[ M' = f M^o \]  
\[ M'' = f^{-1} \]

where the superscripts have the same meaning as above. Since \( 0 < f < 1 \), and in some cases may be close to zero, the barium cloud will always suffer a decrease in \( M \) value, and in fact it is possible to have \( M' < 1 \), in which case the barium cloud would actually represent a Pedersen conductivity depletion. As for the electron cloud, its \( M \) value may decrease or increase or stay the same, depending on \( f \). Sufficiently small \( f \) will result in large increases in \( M \) for the electron cloud, and is the basis for the freezing mechanism for the electron cloud proposed by Ossakow et al. [1981], since a large \( M \) has been shown to be associated with a resistance to further structuring on the part of the electron cloud [McDonald et al., 1981].
The above examples, and the parameter study suggested by (4.1) to (4.10) provide us with a vivid indication of the richness of the physics in the general case of $v_{bn}/\Omega_b \neq v_{on}/\Omega_o$. This richness is further compounded when we model the physics using more than two layers or species of ions. We are presently undertaking these parameter studies to determine these multilayer effects on the morphology of barium cloud striations. However, the point of this section is not to show how different and interesting this regime is, but rather to show that there are certain aspects of the physics, namely the leakage of barium from the electron cloud and its subsequent reduction in density and Pedersen conductivity by a factor $f$ (Eq. (4.1) and (4.10)), that are independent of whether $v_{bn}/\Omega_b = v_{on}/\Omega_o$.

5. Numerical Simulations

In this section we return to the case considered in section 3, that of $v_{bn}/\Omega_b = v_{on}/\Omega_o$. There are several reasons for this: First, we have shown in the previous section that the basic Pedersen mobility leakage mechanism proposed here is active independent of this assumption. Second, this is the implicit assumption made in the "one-level approximation", and it will be interesting to see what effects are missed when one only follows electrons. Finally this is the case for which "image effects" are absent, and it becomes impossible to argue, as does Prettie [1981], that image-induced quadrupole fields affect the formation of barium sheets. As the numerical simulations will show, well-defined sheets of barium are produced without any evidence of quadrupole fields.

The simulation was performed on a stretched grid with 43 and 120 grid points in the $x$ and $y$ directions respectively. The central 31 by 100 portion of the mesh was uniformly gridded with $\Delta x = \Delta y = 0.258$ km. The four boundaries of the grid were placed well away from the cloud (utilizing the stretched mesh) to ensure no influence of the boundaries on the evolution of the cloud. Boundary conditions on all plasma species were that the normal derivative vanished. Boundary conditions on $\phi$ were Dirichlet ($\phi = 0$) on the left and right boundaries, and Neumann ($\partial \phi / \partial y = 0$) at the top and bottom boundaries. Two ion species were used, $Ba^+$ and $O^+$, with a value of $v_{an}/\Omega_a = 0.06$ assigned to each. The quantity $n_0$ was initially assigned a normalized value of 1.0, and the initial barium cloud is given by
\[ n_b = 4.0 \exp(-\frac{r^4}{L^4})(1+P(x)) \] (5.1)

\[ r^2 = (x-x_0)^2 + (y-y_0)^2 \] (5.2)

\[ L = 2.5 \text{ km} \] (5.3)

\[ P(x) = 0.05 \cos(20(x-x_0)/L) \] (5.4)

where the point \((x_0, y_0)\) specifies the initial center of the cloud. The magnetic field \(B\) is taken to be 0.5 gauss directed along the positive \(z\) axis. The ambient electric field \(E_o\) is directed along the positive \(x\) axis with a magnitude such that \(cE_o \times B/B^2\) is 100 m/sec directed along the negative \(y\) axis. The numerical resolution of this cloud is intentionally low, in order to allow us to run the calculation to late times at moderate cost. The inherent numerical dissipation of this low resolution has the effect of inhibiting bifurcation of the cloud but, as we shall see, this is actually an advantage since it allows us to see the leakage of the barium from the electron cloud in a "clean" calculation, without the complicating effects of cloud bifurcation.

Figure 3 shows isodensity contours of \(n_e\) (cross-hatched, denoted "Density 1") and \(n_b\) (denoted "Density 2") at various times during the calculation. The slight asymmetry in the electron density plots is due to the Hall terms in the potential equation. All calculations are performed in a reference frame moving with the electron cloud center of mass. Note that at time \(t = 0\) sec the barium and electron contours are centered on one another, but that as time progresses the barium shows its inevitable drift to the right, "leaking" out of the electron cloud. At the latest time shown, 1096 sec, virtually all of the barium has left the electron cloud and formed a long, low density sheet of barium plasma to the right of and leading the electron cloud. One feature of the sheet that is quite striking is the much larger transverse gradients of \(n_b\) on the left side of the sheet than on the right side. There exists the possibility of optical or radar measurements of this barium cloud interpreting this steep gradient as "frozen structure".
Fig. 3 — Evolution of the barium cloud described in the text computed via numerical simulation. Electron density contours (cross-hatched) are shown on the left, while contours of barium density are shown on the right. (a) 0 seconds (initial conditions) and (b) 330 seconds.
Fig. 3 (Cont'd) — Evolution of the barium cloud described in the text computed via numerical simulation. Electron density contours (cross-hatched) are shown on the left, while contours of barium density are shown on the right. (c) 658 seconds and (d) 1096 seconds.
A possible explanation of this feature is as follows. The barium comprising the right edge of the sheet originated near the right edge of the original barium cloud and thus was subject to some expansion as it left the electron cloud. Hence the right side of the sheet can be expected to have gradients as shallow or shallower than that of the original cloud. The barium comprising the left edge of the sheet originated on the left side of the original barium cloud and was therefore compressed in the x direction at first, resulting in a steepening of contours in the x direction. Subsequently the barium was expanded but this time in the y direction, a process which has no effect at all on the relative steepness of gradients in the x direction. Thus the steepness of the contours in the x direction is preserved. Fig. 4 we show contours of constant electrostatic potential \( \phi \) at the same times as those used in Fig. 3. Notice the absence of any structure which might look even remotely like a "quadrupole".

It should be pointed out that while the density of the barium comprising the sheets may be somewhat low, it will appear bright optically to any observer whose line of sight is approximately parallel to the sheet orientation since the barium is optically thin and the integration path long. For a barium cloud driven by neutral winds, this will be the case for an observer anywhere near the initial cloud release site watching the cloud drift away at late times.

6. Conclusions and Future Work

We believe that we have shown the Pedersen drift mechanism proposed by Prettie [1981] and by Fedder (private communication, 1981) to be a viable candidate for barium cloud freezing. In addition we have shown that this mechanism can produce sheets of barium at late times which lead and are displaced from the electron cloud. We do not suggest, however, that this is the only possible mechanism. Indeed, we have proposed alternative mechanisms ourselves, and are presently considering still others. Nonetheless the mechanism looks sufficiently promising to merit further study. Certainly higher resolution numerical studies are in order, along with parameter studies of the effects of the relative Pedersen mobilities of the barium and ionospheric constituents. We hope to report on these in the near future.
Fig. 4 — Contours of electrostatic potential $\phi$ for the calculation shown in Fig. 3. Positive values of $\phi$ are contoured with solid lines, while negative values are contoured as dashed lines. Note the absence of any quadrupolar components. (a) Contours at 0 and 330 seconds and (b) contours at 658 and 1096 seconds.
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