AN APPLICATION OF RESOURCE ALLOCATION METHODOLOGY TO ARMY R&D PROJECT MAN. (U) GEORGIA INST OF TECH ATLANTA SCHOOL OF INDUSTRIAL AND SYSTEMS. L G CALLAHAN ET AL.

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AN APPLICATION OF RESOURCE ALLOCATION METHODOLOGY TO ARMY R&D PROJECT MANAGEMENT

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Redstone Arsenal, Alabama 35809

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**Abstract:**

This research was directed at developing an improved procedure for the allocation of financial resources among competing research and development projects under the constraints of decrement funding, and the requirement for providing minimum support in two functional areas. A new methodology was developed that transformed a prioritized project list of ordinal ranks from the currently used zero-base budgeting procedures to scaled utility values. This methodology used a binary integer computer program which maximizes...
20. ABSTRACT (cont'd)

the investment return of projects selected, and maintains the viability of
functional laboratory areas. The report contains the complete computer code
that was developed to demonstrate the procedure using FY '81 data related to
the R&D Laboratories at the U.S. Army Missile Command.
# TABLE OF CONTENTS

<table>
<thead>
<tr>
<th>Chapter</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>ACKNOWLEDGEMENTS</td>
<td>ii</td>
</tr>
<tr>
<td>LIST OF TABLES</td>
<td>v</td>
</tr>
<tr>
<td>LIST OF ILLUSTRATIONS</td>
<td>vi</td>
</tr>
<tr>
<td>SUMMARY</td>
<td>viii</td>
</tr>
<tr>
<td>Chapter</td>
<td></td>
</tr>
<tr>
<td>I. INTRODUCTION</td>
<td>1</td>
</tr>
<tr>
<td>Description of the Problem</td>
<td></td>
</tr>
<tr>
<td>Research Objectives</td>
<td></td>
</tr>
<tr>
<td>Summary</td>
<td></td>
</tr>
<tr>
<td>II. RESEARCH AND DEVELOPMENT BACKGROUND</td>
<td>5</td>
</tr>
<tr>
<td>General</td>
<td></td>
</tr>
<tr>
<td>Organization of DARCOM</td>
<td></td>
</tr>
<tr>
<td>R&amp;D Budgetary Process</td>
<td></td>
</tr>
<tr>
<td>Description of Current Procedure</td>
<td></td>
</tr>
<tr>
<td>III. LITERATURE REVIEW</td>
<td>17</td>
</tr>
<tr>
<td>General</td>
<td></td>
</tr>
<tr>
<td>Scoring Models</td>
<td></td>
</tr>
<tr>
<td>Economic Models</td>
<td></td>
</tr>
<tr>
<td>Risk Analysis Models</td>
<td></td>
</tr>
<tr>
<td>Mathematical Programming Models</td>
<td></td>
</tr>
<tr>
<td>IV. DEVELOPMENT OF A METHODOLOGY</td>
<td>26</td>
</tr>
<tr>
<td>General</td>
<td></td>
</tr>
<tr>
<td>Assumptions</td>
<td></td>
</tr>
<tr>
<td>Solution Procedure</td>
<td></td>
</tr>
<tr>
<td>V. DEMONSTRATION OF METHODOLOGY</td>
<td>34</td>
</tr>
<tr>
<td>General</td>
<td></td>
</tr>
<tr>
<td>Statement of the Problem</td>
<td></td>
</tr>
<tr>
<td>Problem Formulation</td>
<td></td>
</tr>
<tr>
<td>Problem Results</td>
<td></td>
</tr>
</tbody>
</table>
VI. CONCLUSIONS AND RECOMMENDATIONS

Conclusions
Limitations of Research
Recommendations for Future Research

APPENDIX A. COMPUTER CODE

BIBLIOGRAPHY
# LIST OF TABLES

<table>
<thead>
<tr>
<th>Table</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>2-1. R&amp;D Expenditures 1969-1979</td>
<td>6</td>
</tr>
<tr>
<td>2-2. Project Priority Ranking &amp; Associated Funding for FY 1981</td>
<td>16</td>
</tr>
<tr>
<td>5-1. Laboratory Program FY1981</td>
<td>35</td>
</tr>
<tr>
<td>5-2. Project Priority Ranking &amp; Associated Funding for FY 1981</td>
<td>37</td>
</tr>
<tr>
<td>5-3. Case Study Results</td>
<td>42</td>
</tr>
<tr>
<td>5-4. Comparison of Solutions Generated by Methodologies To Attain Required Budget of $25.422 Million</td>
<td>46</td>
</tr>
<tr>
<td>5-5. Comparison of Solutions Generated by Methodologies To Attain Required Budget of $24.422 Million</td>
<td>47</td>
</tr>
<tr>
<td>5-6. Comparison of Solutions Generated by Methodologies To Attain Required Budget of $23.422 Million</td>
<td>48</td>
</tr>
</tbody>
</table>
# LIST OF ILLUSTRATIONS

<table>
<thead>
<tr>
<th>Figure</th>
<th>Description</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>2-1.</td>
<td>Organizational Chart DARCOM</td>
<td>8</td>
</tr>
<tr>
<td>2-2.</td>
<td>US Army Missile Command Organization</td>
<td>9</td>
</tr>
<tr>
<td>4-1.</td>
<td>Conversion of Ordinal Ranking to Scaled Value</td>
<td>28</td>
</tr>
<tr>
<td>5-1.</td>
<td>Conversion of Ordinal Ranking to Scaled Value</td>
<td>38</td>
</tr>
<tr>
<td>5-2.</td>
<td>Plot of Computation Time</td>
<td>45</td>
</tr>
</tbody>
</table>
SUMMARY

An application study of the US Army Missile Command (MICOM) exploratory research and development resource allocation and project selection problem is conducted. Research focused on four areas: 1) description of the problem environment; 2) development of a methodology; 3) demonstration of the methodology; and 4) comparison of MICOM's and the proposed technique.

The problem environment is developed by describing the Army's Research and Development organization and current budgetary process in allocating financial resources among exploratory research and development projects. A discussion is presented on zero-base budgeting procedures employed by MICOM to meet budget limits and decrements.

After a review of several categories of models, a methodology was developed that featured transforming a prioritized project list of ordinal rank to scaled utility values. A binary integer programming model was developed that maximized the investment return of projects selected and that maintained the viability of the MICOM Laboratory.

A case study was presented using data furnished by the Missile Command to illustrate the application of the proposed methodological approach. A corresponding computer code capable of handling this problem size was discussed.

The comparison of solutions generated by the US Army Missile Command and the proposed technique was made to show the advantages and limitations of both methodological approaches. The Laboratory Director
was presented with an improved solution technique for allocating resources and selecting projects to meet budget limits and further budget decrements.
CHAPTER I

INTRODUCTION

Description of the Problem

In May, 1980 Dobbins [16] in his Ph.D. dissertation developed and demonstrated a methodology that transformed several individual multi-criteria rank-ordered lists of Research and Development (R&D) projects or products* into a single, aggregated, prioritized rank-ordered list. Inherent also in his work was the introduction of a weighting methodology to perform this conversion of single ranked lists from various formats. The decision-maker and others who aided in the subjective judgemental analysis were assigned various weights to aid in the prioritization of projects. An actual example cited showed that 13 sublists with 95 projects and 44 requirements were successfully aggregated. This priority listing of ordinally ranked projects that resulted subsequently provided the decision-maker with a management tool in the investment of R&D resources.

The allocation of R&D resources was accomplished in a strictly "top down" approach. Directed to provide maximum return for invested funds in projects, the decision-maker allocated resources in a manner consistent with the priority listing of ordinally ranked projects until

*The words "project" and "product" can be used interchangeably to denote a "... specifically defined unit of R&D effort or group of closely related R&D effort which is established to fulfill a stated or anticipated requirement or objective" [22].
the available budget was exhausted. When the funds ran out, those projects remaining below the "budget limit" were not funded. If the fund limit partitioned a project, development of the project was either curtailed or simply not funded. So it was either a case of a project being funded at its projected resource level or not at all.

Dobbins' model provided a valuable management tool to the decision-maker faced with operating an R&D organization constrained by zero-base budgeting regulations. While the model did provide a lexicographic ordering of projects, it was not capable of translating an ordinal ranking into a cardinal or weighted measure. A methodology was developed for determining that one project was preferred to another, but it did not provide for determining a weighted measure for a project to distinguish the degree one project was preferred in relation to others.

While the method of allocating constrained resources according to the priority listing of ordinally ranked projects is easily accomplished once the listing is firmly established, this approach might not provide the optimal investment return to the organization. For example, the goals of the R&D organization might be better attained by eliminating a high priority project in favor of several lower ranked ones to meet budget limits or further imposed budget decrements.

In view of the above, it appears clear that a solution technique should be developed that will provide an alternative management tool to the decision-maker in the allocation of limited resources. The technique must also be capable of handling various in-house constraints so that not only is high return on investment generated but assures
the continuity of technological base essential to the future functioning of the R&D organization is maintained.

Research Objectives

There are four objectives to be accomplished within this research:

1. To describe the research and exploratory development process of a Department of Defense laboratory in terms of resource allocation procedures.

2. To develop a methodology that will provide the decision-maker with an alternative management technique capable of allocating discrete financial resources among competing projects.

3. To demonstrate the methodological technique utilizing fiscal year 1981 R&D project data from the US Army Missile Command Laboratory (MICOM) at Redstone Arsenal, Alabama.

4. To compare the author's resource allocation solution against one generated by employing MICOM's current allocation procedures.

Summary

Chapter II provides the background against which this investigation is set by describing the military R&D organization and R&D budgetary process. Further elaboration is made concerning the description of the current procedures being used by the US Missile Command in allocating financial resources among exploratory research and development projects.

There are many models and mathematical programming techniques that have been developed during the last several decades whose objective is to handle project selection and resource allocation
problems similar to the one described earlier. A discussion of some of these approaches is presented in Chapter III. The chapter concludes with providing the rationale for selecting integer programming as an appropriate method for handling the problem.

Chapter IV develops the recommended methodology. In addition, the problem assumptions and mathematical formulation are presented along with its corresponding computer model.

In Chapter V an actual problem is provided to demonstrate the proposed methodological approach discussed in the preceding chapter.

Chapter VI presents the conclusions from this thesis, limitations of this research and recommends several areas for further research and investigation. Basically, this research suggests an improved technique of allocating financial resources among selected exploratory development products within MICOM's (Missile Command) Laboratory structure. The recommended methodology provides the decision-maker another feasible alternative in reaching a final solution.
CHAPTER II

RESEARCH AND DEVELOPMENT BACKGROUND

General

Under the Department of Defense Budgeting System, the number 6 identifies the Army's Research and Development program. There are a total of 6 categories under this main program. They are:

6.1 Research
6.2 Exploratory Development
6.3 Advanced Development
6.4 Engineering Development
6.5 Management and Support
6.7 Operational Systems Development

Categories 6.1 and 6.2 represent the Army's applied Research and Basic Development efforts while the remaining ones are related to those development activities associated with the actual fielding of systems to support the Army [22].

During the period 1969-1979, the Army received approximately 10% of the total Department of Defense (DOD) R&D funding [18]. Of the Army R&D funding, 51% was targeted for applied research and exploratory development (categories 6.1 and 6.2). On a dollar basis for fiscal year 1979 this translates to 526.0 million dollars. Table 2-1 shows the dollar amounts in millions by year spent by DOD and the US Army for total R&D activities [21]. Also included are the funds
Table 2-1. R&D Expenditures 1969-1979.

<table>
<thead>
<tr>
<th>Year</th>
<th>TOTAL DOD R&amp;D SPENDING (Millions)</th>
<th>TOTAL ARMY R&amp;D SPENDING (Millions)</th>
<th>TOTAL ARMY 6.1, 6.2 SPENDING (Millions)</th>
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</thead>
<tbody>
<tr>
<td>FY69</td>
<td>$7,672.0</td>
<td>$695.0</td>
<td>$391.0</td>
</tr>
<tr>
<td>FY70</td>
<td>7,338.0</td>
<td>634.6</td>
<td>372.5</td>
</tr>
<tr>
<td>FY71</td>
<td>7,423.0</td>
<td>613.5</td>
<td>397.6</td>
</tr>
<tr>
<td>FY72</td>
<td>8,294.0</td>
<td>803.2</td>
<td>453.5</td>
</tr>
<tr>
<td>FY73</td>
<td>8,382.0</td>
<td>763.4</td>
<td>422.5</td>
</tr>
<tr>
<td>FY74</td>
<td>8,396.0</td>
<td>743.6</td>
<td>426.5</td>
</tr>
<tr>
<td>FY75</td>
<td>8,833.0</td>
<td>780.5</td>
<td>426.5</td>
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<tr>
<td>FY76</td>
<td>9,592.0</td>
<td>829.1</td>
<td>462.1</td>
</tr>
<tr>
<td>FY77</td>
<td>10,439.0</td>
<td>875.4</td>
<td>469.0</td>
</tr>
<tr>
<td>FY78</td>
<td>11,371.0</td>
<td>867.8</td>
<td>462.8</td>
</tr>
<tr>
<td>FY79</td>
<td>12,437.0</td>
<td>1,030.7</td>
<td>526.0</td>
</tr>
</tbody>
</table>
expended by the Army for just applied research and exploratory development activities.

**Organization of DARCOM**

The Army Materiel Development and Readiness Command (DARCOM) is responsible for performing assigned materiel functions of the Department of the Army to include Research and Development and related activities, for developing and providing managerial and related logistics management and for commanding over fifty laboratories responsible for performing the research and development required for the various materiel systems required by the Army. Figure 2-1 is a simplified organizational chart that shows the Army's Materiel Development and Readiness Command and its major subordinate commands.

Of particular interest to this research is MICOM's structure. Figure 2-2 shows the Missile Command's organization.

The MICOM Laboratory Director is responsible for handling two major program elements, missile technology and high energy lasers. These major elements, in turn, are divided into technical areas. While the high energy laser element is made up of nine technical areas, the missile technology element consists of 13 technical areas. This research addresses the allocating of funds to the 13 missile technology technical areas. The same allocation techniques could be applied to the LASER technical areas. These technical areas describe best the laboratory's operational or mission functions. For fiscal year 1981, 78 projects are distributed among 12 technical areas whose directors account for in excess of $25 million in R&D funds.
Figure 2-1. Organizational Chart DARCOM.
Figure 2-2. US Army Missile Command Organization.
**R&D Budgetary Process**

The budget cycle is a lengthy interactive process between the several levels of DOD hierarchy plus the Office of Management and Budget, President and Congress. The cycle is divided into several phases in which the budget is developed, refined, apportioned, and reallocated among research projects. The program and budget development phase is initiated six years before the beginning of the budget year. On an annual basis, interactions are involved between the major headquarters, subordinate headquarters, the laboratory director and their staffs. From the time the President submits his budget until Congress appropriates funds, another 6-9 months of interaction result in additional refinement and eventual apportionment of the budget to the research organizational elements [18]. Figure 2-3 depicts a schematic of the budget process.

The MICOM Laboratory Director must operate within this federal budgetary framework. More specifically, the director is concerned with Single Project Funding (SPF) which is 6.1 research and with Single Program Element Funding (SPEF) which is 6.2 exploratory research. Under this funding concept, the Laboratory Director is given reprogramming authority in an effort to improve the mission relevancy and efficiency of the laboratory by avoiding the financial fragmentation of programs and by permitting the laboratory director a high degree of operational flexibility [22].

The objectives of research and development as outlined in [22] are:
Laboratory Director's Recommendation

Review Board, DARCOM

Commanding General DARCOM

Department of the Army

Department of Defense

Office Management & Budget

President

Congress

Apportions Funds

Figure 2-3. Schematic of the Budget Process.
(1) Insure the flow of fundamental knowledge needed by the Army as a prime user of scientific facts related to military technologies.

(2) Insure awareness by the Army of new scientific developments and keep scientists aware of the Army needs.

(3) Maintain a broad base in basic and applied research with which to provide the requisite state-of-the-art and technological base for supporting systems development, and to provide a sound basis for determining the technical feasibility, times required, and cost of proposed development efforts.

(4) Minimize the need for state-of-the-art breakthroughs as a part of engineering or operational system developments.

(5) Provide major technological advances needed to gain and maintain qualitative superiority in military technologies and materiel.

To attain these objectives and further requirements imposed by higher management officials, the Laboratory Director is responsible for converting a selected portfolio of projects and allocating resources into a laboratory program. The laboratory program enters into the normal budgetary cycle and is subject at all levels to elaborate discussion and modification before the tentative approval is granted. The Laboratory Director is a key figure in the whole process in that he must compile, present, defend and later adjust and terminate programs, shift resources or initiate new research investigations to meet the laboratory objectives and to maintain its state-of-the-art technical capability. According to the Army policy of Single Program Element Funding, the Laboratory Director possesses the final discretionary authority to establish the most productive and best balanced R&D program.

It is apparent that the allocation of resources is a complex and seemingly endless process that continually confronts the Laboratory Director. The problem is magnified by the abstract nature of R&D projects at this early developmental stage. Determining a value or
associating a benefit to a R&D project is difficult. As the project progresses in the R&D cycle, efforts to obtain realistic objective and constraint parameters tend to become more readily available allowing the use of mathematical techniques to support allocation decisions.

**Description of Current Procedure**

Currently, Department of Defense agencies employ zero-base budgeting procedures to allocate resources among competing projects or programs. The zero-base budgeting method is a technique for relating action plans to dollar plans. This is done in such a way that upper management can evaluate action plans and determine the appropriate funding allocation for each activity. Zero-Base Budgeting is a procedure of assembling and reporting planned activities to top management for budgetary decision-making. The budget is built up from the smallest activity, based on the assumption that anything could be zeroed-out. This approach begins with zero activities and zero benefits and proceeds upward by first selecting the most cost-effective activity, then the second and so forth until the available budget is exhausted [49].

To assist the MICOM Laboratory Director in the investment of R&D resources, Dobbins [16] developed a majority-rule methodology. His technique transformed individual multi-criteria rank-ordered lists submitted by the directors in charge of the various technical areas into a single, aggregated, prioritized rank-ordered list. The resultant list was ordinal, without feedback. To evaluate the rank orders, Dobbins used Kendall's coefficients of consistency and concordance. The developed model was capable of computing the aggregation of up to
100 full or partial length individual rank orders with a maximum of 100 different alternatives.

Given a prioritized, rank-ordered list of projects, the Laboratory Director allocated resources among the selected 78 projects for fiscal year 1981 in the missile technology element program consistent with the zero-base budgeting concepts. The rank-ordered projects were funded in the "top-down" approach. If the available budget was exhausted before it reached the bottom of the list, the ordinally ranked projects falling below the budget limit were not funded. If budget decrements were introduced by higher management and government officials, those projects which escaped the budget axe earlier were subject to it now.

While zero-base budgeting insures an annual review of projects and demanded resources, this method is arbitrary, time-consuming and arduous to implement. In the R&D environment, important projects are subject to being "zeroed" under this concept. Projects are basically prone to being accepted or rejected. Those projects rejected may have significant impact on the remaining ones in an unforecasted manner, especially if the project involves developing state-of-the-art technology semi-related to a vital weapons system. Further, the prioritized list features an ordinal not cardinal ranking. The list depicts a preference relationship among the projects saying that project A is preferred to project B and so forth, but Dobbins' methodology does not further elaborate on the degree of preference one project is preferred to another.

For example, consider the prioritized list of projects (see Table 2-2) developed for MICOM's fiscal year 1981 6.2 exploratory
development program and their corresponding costs. The Laboratory Director has selected 78 projects from the proposed portfolio of projects and requested a budget of $26.422 million. After the review of this proposed R&D program and others submitted by the DARCOM Laboratories, the MICOM Laboratory Director is informed by higher headquarters that the requested budget of $26.422 million has been reduced to $25.422 million. As a result, projects PC12, PJ3, PG4, PI7, PI9, and PJ5 will be eliminated to comply with the budget decrement. These projects, as can be seen from Table 2-2, were chosen from the bottom of the priority listing until the budget decrement was met.
Table 2-2. Project Priority Ranking & Associated Funding for FY 1981.

<table>
<thead>
<tr>
<th>Ordinal Ranking</th>
<th>Project</th>
<th>Associated Funding</th>
<th>Ordinal Ranking</th>
<th>Project</th>
<th>Associated Funding</th>
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<td>PK1</td>
<td>693</td>
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<td>2</td>
<td>PK2</td>
<td>107</td>
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<td>PL2</td>
<td>450</td>
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<td>978</td>
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<td>7</td>
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<td>312</td>
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<td>PI4</td>
<td>141</td>
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<td>9</td>
<td>PA7</td>
<td>750</td>
<td>48</td>
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</table>
CHAPTER III

LITERATURE REVIEW

General

Over the past two decades, especially in the sixties, many models were proposed for optimizing the R&D project selection and resource allocation problem [2], [4], [8]. However, indications were that most R&D laboratories either did not use such models at all or did not employ them for any significant period of time as an aid in decision-making [4]. Consequently, the effectiveness of these models as tools for the decision-maker in the R&D process has not been fully realized [46].

In a review of the literature, proposed models or techniques dealing with the project selection and resource allocation problem may be generally classified into one of four categories [4], [8], [33]: (1) scoring models, (2) economic models, (3) risk analysis models, and (4) mathematical programming models. However, as mentioned above, there has been limited practical application whatever the type of model proposed.

Scoring Models

In 1959 at the Fifteenth National Meeting of the Operations Research Society of America in Washington, D.C., Mottley and Newton [37] proposed a technique that evaluated applied research proposals based on the use of numerical scores to quantify pertinent project attributes.
Considered criteria were of a technical, administrative, strategic and marketing nature. Based on the composite project scores, a rating was derived to determine the best portfolio of projects for a given allocation of resources. However, project evaluations were subjective in nature and the project scores were arbitrarily obtained by multiplying the criteria scores together. No rationale was provided for the multiplicative procedure other than "... spreading out the values over a wide range" [37]. It was also pointed out that this method is not applicable to all R&D projects and was intended principally for use in the area of applied research. In any event, this early decision theory approach provided the decision-maker with a managerial tool to evaluate projects and compile a portfolio of projects that would maximize a company's investments in R&D projects given limited resources.

Moore and Baker [36] extended the work of Mottley-Newton on the scoring model by investigating more fully their structure and output in regard to R&D project selection. While Mottley-Newton proposed the use of a multiplicative index to generate a wide range of project scores, an additive index was found to consistently produce a higher degree of rank-order consistency. However, Moore and Baker found that the chief shortcoming of the scoring model was the relatively arbitrary fashion in which the models were constructed and the failure of the model builders to recognize the impact of certain structural considerations like interval width and their number on resulting project scores. Overall, their research indicated that the scoring models were not totally practical and advocated more research in application studies.
Research interest in scoring models has continued. Souder used the scoring methodology for assessing the suitability of management science models [45]. Later, Souder expounded upon this particular methodological approach for R&D project evaluation [48]. While Souder cited the scoring model's appealing nature, their practical use has been mainly limited to academic discussion.

Economic Models

Economic models are characterized with employing calculations such as net present value, internal rate of return or economic equations. Significant research efforts performed in this area were made by Dean-Sengupto (1962), Disman (1962) and Cramer-Smith (1964) [14], [15], [9].

Dean-Sengupto's model featured determining first an optimal research budget then estimating the discounted net value and the probability of technical and commercial success of each project. Using linear programming techniques, selection was made by maximizing the expected discounted net value subject to budgetary constraint(s) [14].

Disman's approach employed similar parameters. However, in this method the ratio of the maximum expenditure justified to an estimated project cost provided an index relating the desirability of the project. The maximum expenditure justified was obtained by estimating the discounted net value modified by a probability factor of technical and/or commercial success. Project selection was accomplished by optimizing the index, subject to budgeting constraint(s). Disman recommended that this method be used on new project and process improvement R&D projects [15].
In 1964 Cramer-Smith proposed an economic analysis and operations research approach [9]. Included also was the use of utility theory. Projects were ranked on the basis of expected value or expected utility. A shortcoming noted by Baker and Pound [4] was the lack of project independence.

Souder reported in 1973 that notable applications were being made utilizing expected value project selection models [47]. However, they were not routinely used. Extending this research effort Souder investigated the usefulness of three simple expected value model forms as aids to development R&D investment planning within five on-going R&D organizations. He concluded that while expected value models were promising and indicated a potential for their use during early stage research efforts, additional field applications and evaluations were required.

Risk Analysis Models

Risk analysis models are based on a simulation analysis of input data in distribution form that provide output in the form of distributions of benefit factors such as rate of return or market share. Few efforts other than those made by Hertz (1964), Hespos and Strassman (1965), Pessemier (1966) and Odom (1976) have been pursued in risk analysis regarding the project selection and resource allocation problem [23], [24], [42], [41]. Baker and Freeland reported [3] that there were few applications of risk analysis models due to the excessive cost and time required by management and the research staff to initialize and update the data set.
However, Odom's proposed methodology [40] in multirisk programming possesses much potential for application to the R&D project selection problem. It is an innovative, new technique for analysis of management decision problems involving fuzzy multiple goals and constraints, and uncertainties in input data. His analysis concept is based on determination of the decision alternatives which maximize probabilities that the decision maker's goals and constraints will be satisfied. Due to the selection of another technique in this research, it is suggested that Odom's method is another area for extending the results of this thesis.

Mathematical Programming Models

Mathematical programming models feature optimizing an economic objective function subject to specific resource constraints. While much research has been accomplished in this area, studies of industrial and government applications are few [33].

Clark [8], Baker and Pound [4], Baker and Freeland [3], Centron, et al [5] and Lockette and Gear [33] have compared the features of several mathematical models that are designed as an aid in R&D project selection and resource allocation. While Asher [1] employed a linear programming approach toward the solution of the allocation of R&D resources for a pharmaceutical company, Hess [25] proposed using a dynamic programming approach to R&D budgeting and project selection.

Nutt [38], [39] applied the work of Weingartner [53] in designing a modified linear programming model for allocating exploratory development funds at the Air Force's Flight Dynamics Laboratory in 1969.
transportation problem analysis [32].

Limitations associated with goal programming are [30]:

1. The goal relationships must be linear.

2. The activities must be additive in the objective function and constraints.

3. The decision environment is assumed to be static; that is, all of the model coefficients must be constant.

Another limitation not cited by Lee but derived from practical experience is that the goal programming technique has been found not to be capable of handling more than four goal priority levels. Current work in goal programming is being directed toward the development and application of integer goal programming [31].

While linear programming and goal programming methodologies present fractional decision variables in the solution, integer programming algorithms yield integer ones. Several algorithms using the integer programming technique feature binary (0,1) decision variables. Among those that are pertinent to this research is an algorithm developed and investigated by Balas (1965) for solving the zero-one linear problem. Important modifications in Balas' ideas were later given by Glover (1965), whose work was the basis for other developments by Geoffrion (1967, 1969) and later by Balas again (1967) and others. Balas' original enumeration scheme was later refined by Glover (1965). Geoffrion (1967) then showed how Balas' algorithm could be super-imposed on Glover's enumeration scheme [15]. Weingartner [54] has also made significant contributions to integer programming theory and application.

The concept of implicit enumeration assumes that the solution
Nutt's model featured maximizing an objective function of projects at different resource levels subject to manpower, contract costs and budgetary constraints. The decision variable was allowed to assume values between zero and one and fractional values were rounded off to the nearest integer. To cite Weingartner [53]

This model will select among independent alternatives those task resource levels whose total measure of effectiveness is maximum, but whose total resource consumption is within the budget limitation. The problem of indivisibilities is solved in the sense that the linear programming solution implicitly looks at all combinations of resource levels of tasks, not just one resource level of one task at a time, to select that set whose total measure of effectiveness is maximized. Furthermore, the upper limit of unity on each $x_n$ guarantees that no more than one of any resource level of any task will be included in the final program. The omission of such a limitation would clearly lead to allocating the entire budget to multiples of the "best" resource levels.

However, due to the nature of this research problem in which the project is either accepted or rejected at a discrete resource level, Weingartner's model is not appropriate.

A current technique for dealing with this problem is goal programming. This technique was developed in concept by Charnes and Cooper, and introduced in their linear programming book published in 1961 [6]. Goal programming is essentially a modification and extension of linear programming which allows simultaneous solution of a system of prioritized goals based on minimizing an objective function of deviations from established goal levels. Lee [30], and Ignizio [27] have published books concerning the underlying concepts, solution methods and applications. Example applications include advertising media planning [7], academic planning, financial planning, economic planning and hospital administration [8], capital budgeting [20], and
space of an integer program possesses a finite number of possible feasible points. A technique for solving these type problems is to exhaustively (or explicitly) enumerate all such points. The optimal solution is determined by the point(s) that yields the best (maximum or minimum) value of the objective function.

A limitation on this technique occurs when the number of enumerated points \(2^n\) becomes extremely large driving the computation time required for obtaining a feasible solution to increase at an exponential rate. The idea of implicit (or partial) enumeration calls for considering only a portion of all possible points while automatically discarding the remaining ones as nonpromising (fathoming).

More efficient algorithms have been developed (Geoffrion, 1967) that utilize the surrogate (or substitute) constraint which is developed by solving the dual of the continuous correspondent of the present partial solution [19]. The surrogate constraint combines all the original constraints of the problem into one constraint and does not eliminate any of the original feasible points of the problem [51]. Use of the surrogate improves the computation time; however, problems with 100 variables seem to present an upper limit on the problem size based on reported computational experiences.

Based on the information presented above and considering the structure of this decision problem under investigation, an integer programming approach utilizing binary decision variables is appropriate. Further, a computer code employing the surrogate constraint is desirable to improve the computation time.
It is noteworthy to mention that research efforts are being pursued in solving large scale zero-one programming problems other than by enumeration processes. Senju and Toyoda (1968) developed a simple approach to obtain approximate solutions for this type problem which features a significant improvement in computational efficiency [43]. Toyoda (1975) improved upon this 0,1 approximation algorithm and reported a capability of handling large problems very efficiently. For example, Toyoda cited a problem with 1000 variables and 100 constraints which was solved in 208 seconds using an IBM 360/195 computer [52]. It appears that Toyoda's algorithm may be applied to the present decision problem; however, since another method has been selected, it is recommended that Toyoda's algorithm may be another area for a further extension of the results of this thesis.
CHAPTER IV

DEVELOPMENT OF A METHODOLOGY

General

The Laboratory Director is responsible for allocating discrete funds and selecting projects from among an available set of projects that maximizes the investment return to the US Army, subject to the following budgetary constraints:

1. There is a designated upper funding level for the MICOM Laboratory.
2. There is a minimum funding level for each technical area.
3. Projects must either be selected at the discrete funding level or rejected.

Assumptions

In formulating this decision problem, the following assumptions are made:

1. Projects are ordinally ranked.
2. The projects are assumed to be independent. That is, the completion of one project is not dependent on others or a project doesn't have to be completed before another begins.
3. The discrete funding level for each project to be considered has already been selected by management.
4. Initially, the number of projects to be considered for funding has been selected from a set of available projects.
5. The availability of technical skills is considered during the project selection and resource allocation process so that the Laboratory Director has planned for the availability of the required technical skills, either through in-house capability or by contract.

6. Maintaining critical skill capability for each technical area has been considered in selecting projects and providing adequate funding for each technical area. A critical skill capability is defined as that item which is necessary to maintain state-of-the-art technology or that skill which influence directly the development of a project, without which the project development would be seriously impaired.

7. The scaled utility value of ordinally ranked projects is dependent upon the number of selected projects from the initial set.

Solution Procedure

Conversion of Ranks

The ordinal ranking was translated to a utility value by a procedure suggested by MacCrimmon [34]. A graph was constructed associating the ordinal ranking of available projects to the percentage of projects selected initially. Percentages considered representative were 30, 50 and 70. Figure 4-1 has reflected this linear translation by Lines A, B, C, respectively. As a result of this method, a utility value, \( b_i \), is derived for each project using linear regression techniques [26] for use in the objective function of the problem.

A method available to translate the ordinal ranking to a cardinal measure is a technique suggested by Kendall [29] and later developed by Wood and Wilson [55]. The suggested procedure requires not only an
Figure 4-1. Conversion of Ordinal Ranking to Scaled Value.
ordinal ranking, but the measured differences between each item. An additional approach is the use of a scoring model which evaluates projects and assigns them a relative worth factor [4], [37] and [49]. While this technique is highly judgemental, it is particularly adaptable to evaluating R&D projects across many attributes in the early stages of exploratory development. Unfortunately, government and industrial applications are few [4].

While the particular type of method selected could be of prime importance in the final solution, the principal interest of this research is not in the method but rather in the overall solution procedure. It is then assumed that a comparable method has been selected to determine the relative value and is used in a consistent manner throughout.

Mathematical Formulation

This project selection and resource allocation problem is formulated as a binary (0,1) integer programming problem. $C_i$ represents the discrete cost of project $i$, $b_i$ represents the relative measure of project $i$ determined by a technique described in the preceding section, $M$ is the overall budget of the laboratory, $m$ with an associated subscript of A, B, K, L, C, D, E, F, G, H, I or J indicates the minimum restrictive funding level of the technical area and $x_i$ represents the decision variable, that is, each project is selected at its "discrete funding level or rejected. The summation subscripts indicate the number of projects to be considered in each technical area.
The integer programming formulation is as follows:

Maximize: \[ \sum_{i=1}^{78} b_i x_i \]

Subject to:

1. \[ \sum_{i=1}^{78} c_i x_i \leq M \]
2. \[ \sum_{i=1}^{A=14} c_i x_i \geq m_A \]
3. \[ \sum_{i=1}^{B=11} c_i x_i \geq m_B \]
4. \[ \sum_{i=1}^{K=2} c_i x_i \geq m_K \]
5. \[ \sum_{i=1}^{L=1} c_i x_i \geq m_L \]
6. \[ \sum_{i=1}^{C=12} c_i x_i \geq m_C \]
7. \[ \sum_{i=1}^{D=4} c_i x_i \geq m_D \]
8. \[ \sum_{i=1}^{E=3} c_i x_i \geq m_E \]
Computer Model

The integer programming problem as stated in the preceding section is solved utilizing the integer programming program XINP on the Georgia Institute of Technology Cyber 74 Computer System. The computer program XINP is an integer programming algorithm based on the branch-and-bound technique utilizing the surrogate constraint. Further, it is an interactive program that requires a minimization format as follows:

Minimize: \( Cx \)
Subject to: \( Ax \geq b \)
\[ x = 0,1 \]
where all \( c_i \geq 0 \). If any \( c_i < 0 \), a transformation is made where
\[ x_i = 1 - x'_i. \]
Further, when the program requests the "original rhs", the negative value of the original right hand side is entered.

This particular computer code was selected because of its availability, capability, ease of use and relative efficiency for an integer programming algorithm. The program XINP is readily available for use in the Industrial and Systems Engineering Department computer library. Since this program featured an ability to handle up to 150 variables with 31 constraints, it was capable of handling this research problem which deals with 78 variables and 13 constraints.

The program's implicit enumeration procedure investigated implicitly and explicitly all \( 2^n \) binary points or \( 2^{78}(3.022 \times 10^{23}) \) binary points for this problem. Since the specific ordering of the variables and constraints may have an adverse impact on the efficiency of the algorithm, the most restrictive constraint was listed first while the variables were arranged according to an ascending order. Both conditions were favorable to producing "faster" fathoming of partial solutions [51]. The computer code was written in Fortran IV and is listed in Appendix A.

**Summary**

In summary, the steps of the proposed methodology are as follows:

1. Convert the ordinal ranking to utility values.
2. Transform the problem from maximization to minimization form for computer input.
3. Run the interactive computer program XINP.
4. Transform the computer solution to the appropriate decision.
variable values.

5. Select the appropriate projects that allocate the discrete resources according to the required budget limit.

6. If the projects chosen are unacceptable for deletion, run the computer model again after deleting those projects from computation consideration.

7. Calculate the recommended budget from selected projects and compare to required budget.
CHAPTER V

DEMONSTRATION OF METHODOLOGY

General

This case study is presented to illustrate the application of the methodology discussed in the preceding chapter to the resource allocation and project selection problem. This problem is typical of one confronting a Department of Defense Laboratory and that of industrial laboratories as well. However, this case study pertains exclusively to the Missile Command Laboratory located at Redstone Arsenal, Alabama. The decision problem is to allocate financial resources among an available group of R&D projects in a feasible manner consistent with budgetary limits and laboratory constraints. The objective of the Laboratory Director is "to allocate his discretionary funds to the R&D technology projects that will produce the most return for its investment cost to the Army and will maintain the viability of the Laboratory" [16].

Statement of the Problem

The Laboratory Director has been given the responsibility of allocating discrete financial resources among an available set of R&D projects subject to a variety of constraints. The tentatively selected 78 projects for fiscal year 1981 are distributed among 12 technical areas that must be maintained to preserve the viability of the laboratory. Table 5-1 shows the MICOM technical areas with associated projects and minimum funding levels that each technical area must not
Table 5-1. Laboratory Program FY 1981.

<table>
<thead>
<tr>
<th>Technical Areas</th>
<th>Selected Projects</th>
<th>Minimum Funding Level</th>
</tr>
</thead>
<tbody>
<tr>
<td>Sensors</td>
<td>PA1,PA2,PA3,PA4,PA6,PA7,PA8,PA9,PA10,PA11,PA12,PA13,PA14,PA15</td>
<td>$1.0M</td>
</tr>
<tr>
<td>Guidance &amp; Control</td>
<td>PB1,PB2,PB3,PB4,PB5,PB6,PB7,PB8,PB9,PB10,PB12</td>
<td>.8M</td>
</tr>
<tr>
<td>Technology Integration</td>
<td>PK1,PK2</td>
<td>.6M</td>
</tr>
<tr>
<td>Applications &amp; Analysis</td>
<td>PL2</td>
<td>.1M</td>
</tr>
<tr>
<td>Terminal Guidance</td>
<td>PC1,PC2,PC3,PC4,PC5,PC6,PC7,PC8,PC9,PC11,PC12,PC15</td>
<td>.8M</td>
</tr>
<tr>
<td>Digital Technology</td>
<td>PD1,PD2,PD3,PD4</td>
<td>.1M</td>
</tr>
<tr>
<td>Simulation</td>
<td>PE1,PE2,PE3</td>
<td>1.0M</td>
</tr>
<tr>
<td>Technology Demonstration</td>
<td>PF1,PF2,PF4</td>
<td>.8M</td>
</tr>
<tr>
<td>Airoballistics</td>
<td>PG1,PG2,PG3,PG4,PG5,PG7</td>
<td>.3M</td>
</tr>
<tr>
<td>Propulsion</td>
<td>PH1,PH2,PH6,PH7,PH8,PH9,PH10,PH11,PH12</td>
<td>.5M</td>
</tr>
<tr>
<td>Ground Support Equipment</td>
<td>PI1,PI2,PI4,PI6,PI7,PI8,PI9</td>
<td>.5M</td>
</tr>
<tr>
<td>Structures</td>
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</tr>
<tr>
<td>High G Terminal Guidance</td>
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<td></td>
</tr>
</tbody>
</table>

35
fall below. Table 5-2 depicts the project priority ranking and corresponding discrete costs for each project. The project priority ranking was developed by Dobbins' methodology [16]. It reflects an appropriate one to be considered by the Laboratory Director for allocating resources. In addition, the Laboratory Director is required to submit a list of selected projects and allocated resources that will meet budget levels of $26.422 million, $25.422 million, $24.422 million and $23.422 million. Further, the Laboratory Director considers that the number of selected projects can represent either 30, 50 or 70 percent of the total number of projects considered for possible exploratory development.

Problem Formulation

The resource allocation and project selection problem has been formulated as a maximization 0,1 integer programming problem. The value coefficients in the objective function reflect the conversion of the ordinal ranking to a scaled value representative of 30, 50 and 70 percent of the available projects selected initially. These values have been computed in accordance with the methodology introduced in the preceding chapter. Figure 5-1 shows the conversion scale utilized. There are 13 constraints corresponding to the overall budget limit and each of the minimum funding levels for the 12 technical areas. The zero-one variable $x_i$ represents a project that is either selected at the recommended discrete resource funding level or not selected at that particular funding level. The integer programming formulation representing the 50 percent level is as follows:
### Table 5-2. Project Priority Ranking & Associated Funding for FY 1981.

<table>
<thead>
<tr>
<th>Ordinal Ranking</th>
<th>Project</th>
<th>Associated Funding</th>
<th>Ordinal Ranking</th>
<th>Project</th>
<th>Associated Funding</th>
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<td>107</td>
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<td>PA11</td>
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<tr>
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<td>65</td>
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</tr>
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<td>67</td>
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<td>PD3</td>
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</tr>
<tr>
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<td>71</td>
<td>PD4</td>
<td>330</td>
</tr>
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<td>PC11</td>
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<td>250</td>
<td>73</td>
<td>PC12</td>
<td>240</td>
</tr>
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<td>35</td>
<td>PB4</td>
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<td>74</td>
<td>PJ3</td>
<td>100</td>
</tr>
<tr>
<td>36</td>
<td>PB6</td>
<td>300</td>
<td>75</td>
<td>PG4</td>
<td>500</td>
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<tr>
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<td>154</td>
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<tr>
<td>38</td>
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<td>PI9</td>
<td>135</td>
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<td>39</td>
<td>PG1</td>
<td>375</td>
<td>78</td>
<td>PJ5</td>
<td>100</td>
</tr>
</tbody>
</table>
Figure 5-1. Conversion of Ordinal Ranking to Scaled Value.
Maximize:

\[ \begin{align*}
100x_1 + 99.35x_2 + 98.7x_3 + 98.05x_4 + 97.40x_5 + 96.75x_6 + 96.10x_7 \\
95.45x_8 + 94.80x_9 + 94.15x_{10} + 93.50x_{11} + 92.85x_{12} + 92.2x_{13} + \\
91.55x_{14} + 90.9x_{15} + 90.25x_{16} + 89.6x_{17} + 88.95x_{18} + 88.30x_{19} + \\
87.65x_{20} + 87.0x_{21} + 86.35x_{22} + 85.70x_{23} + 85.05x_{24} + 84.40x_{25} + \\
83.75x_{26} + 83.1x_{27} + 82.45x_{28} + 81.80x_{29} + 81.15x_{30} + 80.50x_{31} + \\
79.85x_{32} + 79.20x_{33} + 78.55x_{34} + 77.90x_{35} + 77.25x_{36} + 76.60x_{37} + \\
75.95x_{38} + 75.30x_{39} + 74.65x_{40} + 74.0x_{41} + 73.35x_{42} + 72.7x_{43} + \\
72.05x_{44} + 71.4x_{45} + 70.75x_{46} + 70.1x_{47} + 69.45x_{48} + 68.8x_{49} + \\
68.15x_{50} + 67.5x_{51} + 66.85x_{52} + 66.2x_{53} + 65.55x_{54} + 64.9x_{55} + \\
64.25x_{56} + 63.6x_{57} + 62.95x_{58} + 62.3x_{59} + 61.65x_{60} + 61.0x_{61} + \\
60.35x_{62} + 59.7x_{63} + 59.05x_{64} + 58.4x_{65} + 57.75x_{66} + 57.1x_{67} + \\
56.45x_{68} + 55.8x_{69} + 55.15x_{70} + 54.5x_{71} + 53.85x_{72} + 53.2x_{73} + \\
52.55x_{74} + 51.9x_{75} + 51.25x_{76} + 50.6x_{77} + 49.95x_{78}
\end{align*} \]

Subject to:

(1) \[ \begin{align*}
693x_1 + 107x_2 + 450x_3 + 602x_4 + 835x_5 + 885x_6 + 260x_7 + 312x_8 + \\
750x_9 + 302x_{10} + 250x_{11} + 100x_{12} + 523x_{13} + 311x_{14} + 412x_{15} + \\
100x_{16} + 330x_{17} + 279x_{18} + 400x_{19} + 325x_{20} + 198x_{21} + 275x_{22} + \\
153x_{23} + 200x_{24} + 776x_{25} + 81x_{26} + 680x_{27} + 268x_{28} + 125x_{29} + \\
520x_{30} + 345x_{31} + 358x_{32} + 425x_{33} + 250x_{34} + 250x_{35} + 300x_{36} + \\
80x_{37} + 270x_{38} + 375x_{39} + 200x_{40} + 200x_{41} + 978x_{42} + 150x_{43} + \\
270x_{44} + 195x_{45} + 350x_{46} + 141x_{47} + 337x_{48} + 395x_{49} + 875x_{50} + \\
175x_{51} + 180x_{52} + 400x_{53} + 155x_{54} + 360x_{55} + 750x_{56} + 320x_{57} + \\
400x_{58} + 200x_{59} + 125x_{60} + 225x_{61} + 925x_{62} + 205x_{63} + 170x_{64} + \\
297x_{65} + 150x_{66} + 300x_{67} + 270x_{68} + 365x_{69} + 500x_{70} + 330x_{71} + \\
\end{align*} \]
\[145x_{72} + 240x_{73} + 100x_{74} + 500x_{75} + 154x_{76} + 135x_{77} + \]
\[100x_{78} \leq 26422.0\]

(2) \[312x_8 + 750x_9 + 302x_{10} + 250x_{11} + 100x_{12} + 523x_{13} + 311x_{14} + \]
\[100x_{15} + 400x_{16} + 198x_{21} + 275x_{22} + 153x_{23} + 978x_{42} + \]
\[150x_{43} \geq 1000.0\]

(3) \[200x_{24} + 425x_{33} + 250x_{35} + 300x_{36} + 270x_{38} + 350x_{46} + 395x_{49} + \]
\[175x_{51} \geq 800.0\]

(4) \[639x_1 + 107x_2 \geq 600.0\]

(5) \[450x_3 \geq 100.0\]

(6) \[776x_{25} + 81x_{26} + 268x_{28} + 125x_{29} + 520x_{30} + 345x_{31} + 358x_{32} + \]
\[250x_{34} + 80x_{37} + 337x_{48} + 145x_{72} + 240x_{73} \geq 800.0\]

(7) \[300x_{67} + 270x_{68} + 500x_{70} + 330x_{71} \geq 100.0\]

(8) \[750x_{56} + 320x_{57} + 925x_{62} \geq 1000.0\]

(9) \[835x_5 + 885x_6 + 680x_{27} \geq 800.0\]

(10) \[375x_{39} + 875x_{50} + 400x_{53} + 400x_{58} + 125x_{60} + 500x_{75} \geq 300.0\]

(11) \[602x_4 + 260x_7 + 412x_{15} + 300x_{17} + 279x_{18} + 325x_{20} + 155x_{54} + \]
\[225x_{61} + 297x_{65} \geq 500.0\]

(12) \[195x_{45} + 141x_{47} + 180x_{52} + 360x_{55} + 200x_{59} + 154x_{76} + \]
\[135x_{77} \geq 500.0\]

40
\[(13) \quad 205x_{63} + 170x_{64} + 150x_{66} + 365x_{69} + 100x_{74} + 100x_{78} \geq 200.0\]

\[x_i = 0, 1\]

The formulation of the objective function indicating the utility value of those projects at the 30 and 70 percent level are obtained in a similar manner.

The first constraint ensures that the overall budget limit is either attained or met as closely as possible, but in any event not exceeded. The second through thirteenth constraint, inclusive, states that the minimum funding level of each technical area will be met or exceeded.

The maximization problem was next converted into the minimization form to provide the appropriate data input for the computer model. The computer code is listed in Appendix A.

**Problem Results**

The resource allocation problem was run utilizing the computer model with each of three different objective functions at each of the imposed overall budget limits of $26.422$ million, $25.422$ million, $24.422$ million and $23.422$ million for a total of 12 computer runs.

Table 5-3 shows the results of the computer runs.

The integer programming solution provided by the computer model indicates that:

1. Maximizing the project measures generated by the scaling of the ordinal ranking at the 30 percent joint as shown by Line A in Figure 5-1 demonstrated that as the budget was reduced by one million
<table>
<thead>
<tr>
<th>Budget Limit*</th>
<th>No. of Projects Funded</th>
<th>Projects Deleted</th>
<th>Deleted Projects Cost</th>
<th>Recommended Budget</th>
<th>Budget Surplus</th>
<th>Computation Time**</th>
<th>No. of Iterations to Reach Optimum</th>
</tr>
</thead>
<tbody>
<tr>
<td>$26.422</td>
<td>A 78</td>
<td>0</td>
<td>0</td>
<td>$26.422</td>
<td>0</td>
<td>.673</td>
<td>1</td>
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<tr>
<td></td>
<td>B 78</td>
<td>0</td>
<td>0</td>
<td>$26.422</td>
<td>0</td>
<td>.673</td>
<td>1</td>
</tr>
<tr>
<td></td>
<td>C 78</td>
<td>0</td>
<td>0</td>
<td>$26.422</td>
<td>0</td>
<td>.673</td>
<td>1</td>
</tr>
<tr>
<td>$25.422</td>
<td>A 76</td>
<td>PE1,PG4</td>
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<td>$24.997</td>
<td>$.425</td>
<td>6.725</td>
<td>238</td>
</tr>
<tr>
<td></td>
<td>B 76</td>
<td>PE1,PG4</td>
<td>$1.425</td>
<td>$24.997</td>
<td>$.425</td>
<td>8.522</td>
<td>379</td>
</tr>
<tr>
<td></td>
<td>C 76</td>
<td>PE1,PG4</td>
<td>$1.425</td>
<td>$24.997</td>
<td>$.425</td>
<td>8.282</td>
<td>338</td>
</tr>
<tr>
<td>$24.422</td>
<td>A 75</td>
<td>PA1,PG3,PE1</td>
<td>$2.788</td>
<td>$23.644</td>
<td>$.788</td>
<td>17.791</td>
<td>830</td>
</tr>
<tr>
<td></td>
<td>B 75</td>
<td>PA1,PG3,PE1</td>
<td>$2.788</td>
<td>$23.644</td>
<td>$.788</td>
<td>25.115</td>
<td>1312</td>
</tr>
<tr>
<td></td>
<td>C 75</td>
<td>PA1,PG3,PE1</td>
<td>$2.788</td>
<td>$23.644</td>
<td>$.788</td>
<td>47.614</td>
<td>1940</td>
</tr>
<tr>
<td>$23.422</td>
<td>A 73</td>
<td>PA1,PG3,PE1,PG4,PI7</td>
<td>$3.432</td>
<td>$22.990</td>
<td>$.432</td>
<td>28.875</td>
<td>1414</td>
</tr>
<tr>
<td></td>
<td>B 74</td>
<td>PFI,PA1,PG3,PE1</td>
<td>$3.458</td>
<td>$22.964</td>
<td>$.458</td>
<td>74.815</td>
<td>3513</td>
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<tr>
<td></td>
<td>C 74</td>
<td>PFI,PA1,PG3,PE1</td>
<td>$3.458</td>
<td>$22.964</td>
<td>$.458</td>
<td>121.737</td>
<td>5881</td>
</tr>
</tbody>
</table>

*In millions  
**In CP seconds
dollars from $26.422 to $25.422 million, two projects PEI and PG4 were deleted at a cost savings of $1.425 million. Also, a management reserve or budget surplus of $.425 million was generated. A two million dollar cut from $26.422 to $24.422 million deleted projects PA1, PG3 and PEI for a cost savings of $2.788 million. The management reserve or budget surplus showed $.788 million. An extreme budget cut of $3 million dropped the number of selected projects to 73 reflecting the deletion of projects PA1, PG3, PEI, PG4 and PI7 for a cost savings of $3.432 million and a budget surplus of $.432 million. As the budget decrements increased the computation time increased as well as the number of iterations to reach the optimum solution.

2. Maximizing the project measures generated by the scaling of the original ranking at the 50 percent point as shown by Line B in Figure 5-1 demonstrated that as the budget was reduced by one million dollars from $26.422 million to $25.422 million the same two projects described above were deleted at the same cost savings providing a similar budget surplus. A two million dollar budget cut recommended the same three projects deleted as indicated above. However, a further budget decrement of one million dollars recommended that projects PF1, PA1, PG3 and PEI should be deleted as compared to projects PA1, PG3, PEI, PG4 and PI7 as indicated previously at a similar budget cut. The cost savings were different too; one shows a cut of $3.458 million compared to $3.432 million and a budget surplus of $.458 million as compared to $.432 million. Further, there was a significant increase in both computation time and the number of iterations required to reach the optimum solution.
3. Maximizing the project measures generated by the scaling of the ordinal ranking at the 70 percent point as shown by Line C in Figure 5-1 produced similar results for project values generated by value Line B, except that the computation time and the number of iterations required to reach the optimum solution increased.

4. Overall, the model produces a solution that recommends selecting high cost projects for deletion to meet budget decrements rather than deleting projects from the bottom of the ordinally ranked priority listing until the budget decrement is met.

5. As the amount of budget cuts increased so did the computation time and the number of iterations to obtain the optimal, feasible solution, regardless of the project measures indicated in the objective function.

6. Also, as the project measures generated by the conversion of the ordinal ranking to scaled measures reflected by Line A, B, and C and indicative of increasing the number of projects selected initially, the computation time and number of iterations required to reach an optimal solution increased at an exponential rate. However, this phenomenon cannot be generalized to further budget decrements and is pertinent only to this particular problem and budget decrements. Figure 5-2 shows the plot of budget decrements versus computation time required to reach the optimum solution.

As indicated in Tables 5-4, 5-5 and 5-6 the comparison between MICOM's methodology and the author's proposed methodology for allocating resources and selecting projects to attain required budget decrements indicated different solutions. At a budget decrement of one million
Figure 5-2. Plot of Computation Time.
Table 5-4. Comparison of Solutions Generated by Methodologies To Attain Required Budget of $25.422 Million.

<table>
<thead>
<tr>
<th>Present MICOM Procedure</th>
<th>Recommended Solution at Incremental Line A</th>
<th>Recommended Solution at Incremental Line B</th>
<th>Recommended Solution at Incremental Line C</th>
</tr>
</thead>
<tbody>
<tr>
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<td>$25.422</td>
<td>$25.422</td>
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<tr>
<td>Recommended Budget</td>
<td>$25.193</td>
<td>$24.997</td>
<td>$24.997</td>
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<tr>
<td>Deleted Projects Cost</td>
<td>$1.229</td>
<td>$1.425</td>
<td>$1.425</td>
</tr>
<tr>
<td>Budget Surplus</td>
<td>$.229</td>
<td>$.425</td>
<td>$.425</td>
</tr>
<tr>
<td>Deleted Projects</td>
<td>PC12,PJ3, PG4, PI7, PI9, PJ5</td>
<td>PE1, PG4</td>
<td>PE1, PG4</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
*In millions
Table 5-5. Comparison of Solutions Generated by Methodologies To Attain Required Budget of $24.422 Million.

<table>
<thead>
<tr>
<th></th>
<th>Present MICOM Procedure</th>
<th>Recommended Solution at Incremental Line A</th>
<th>Recommended Solution at Incremental Line B</th>
<th>Recommended Solution at Incremental Line C</th>
</tr>
</thead>
<tbody>
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<td>$24.422</td>
<td>$24.422</td>
<td>$24.422</td>
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<tr>
<td>Recommended Budget</td>
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<td>$23.644</td>
<td>$23.644</td>
<td>$23.644</td>
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<td>Deleted Projects Cost</td>
<td>$ 2.204</td>
<td>$ 2.788</td>
<td>$ .788</td>
<td>$ .788</td>
</tr>
<tr>
<td>Budget Surplus</td>
<td>$ .204</td>
<td>$ .788</td>
<td>$ .788</td>
<td>$ .788</td>
</tr>
<tr>
<td>Deleted Projects</td>
<td>PD3,PD4,PC11,PC12,PJ3,PG4,PJ7,PI9,PJ5</td>
<td>PA1,PG3,PE1</td>
<td>PA1,PG3,PE1</td>
<td>PA1,PG3,PE1</td>
</tr>
</tbody>
</table>

*In millions
Table 5-6. Comparison of Solutions Generated by Methodologies To Attain Required Budget of $23,422 Million.

<table>
<thead>
<tr>
<th>Present MICOM Procedure</th>
<th>Recommended Solution at Line A</th>
<th>Recommended Solution at Line B</th>
<th>Recommended Solution at Line C</th>
</tr>
</thead>
<tbody>
<tr>
<td>Required Budget*</td>
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<td>$23.422</td>
</tr>
<tr>
<td>Recommended Budget</td>
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<td>$22.990</td>
<td>$22.964</td>
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<tr>
<td>Deleted Projects Cost</td>
<td>$3.109</td>
<td>$3.432</td>
<td>$3.458</td>
</tr>
<tr>
<td>Budget Surplus</td>
<td>$0.109</td>
<td>$0.432</td>
<td>$0.458</td>
</tr>
<tr>
<td>Deleted Projects</td>
<td>PD2, PJ1, PD3, PD4, PC11, PC12, PJ3, PG4, PI7, PI9, PJ5</td>
<td>PAI, PG3, PEI, PG4, PI7</td>
<td>PF1, PA1, PG3, PE1</td>
</tr>
</tbody>
</table>

*In millions
dollars according to zero-base budgeting procedures, projects PC12, PJ3, PG4, PI7, PI9 and PJ5 were deleted at a cost of $1,229 million with a budget surplus of $.229 million while according to the proposed methodology two projects PE1 and PG4 were recommended for deletion at a cost of $1.425 million with a budget surplus of $.425 million. It is noteworthy that project PG4 appeared in both solutions.

A two million dollar budget decrement from $26.422 million to $24.422 million indicated that projects PD3, PD4, PC11, PC12, PJ3, PG4, PI7, PI9 and PJ5 were deleted utilizing MICOM's procedure compared to PA1, PG3 and PE1 under the author's proposed methodology. The budget surplus of $.204 million under MICOM's methodology compared very favorably against the proposed approach which showed a surplus of $.788 million. Table 5-5 lists all the pertinent comparison data.

An additional decrement of one million dollars to $23.422 million showed that under MICOM's procedures projects PD2, PJ1, PD3, PD4, PC11, PC12, PJ3, PG4, PI7, PI9 and PJ5 were deleted at a cost of $3.109 million producing a small surplus of $.109 million in comparison with deleted projects PA1, PG3, PE1, PG4 and PI7 and a budget surplus of $.432 million with project measures generated by Line A. Other projects were recommended for deletion for those determined by Lines B and C. Deleted projects recommended were PFL, PA1, PG3 and PE1 at a cost savings of $3.458 million producing a budget surplus of $.458 million. Table 5-6 indicates this data with a budget decrement to $23.422 million.

In order to meet the required budget, the proposed methodology recommends retaining projects that produce the largest value-cost ratio which, in effect, generally retains the least costing projects, regard-
less of scaled utility value. Projects associated with a high cost and low priority rank are recommended for deletion to meet the required budget decrement. Mathematically, the equation describing the scaled utility value associated with the ordinally ranked project is given by:

$$\text{utility}(i) = u_0 - m_i,$$

where $i$ is the project considered, $u_0$ is the utility value of project one given slope $m$. Dividing the equation by $a_i$, the cost associated with project $i$, and driving the slope $m$ to zero yields the value-cost ratio of project $i$. This value-cost ratio obtained indicates that the least costing projects will be selected ahead of the high cost ones.

The proposed technique as applied to this problem features several advantages over MICOM's present budgeting procedures. They are:

1. Provides for maintaining critical skills and state-of-the-art technology capability instead of subjecting these elements to loss or obsolescence.

2. Generates adequate financial management reserve of over ten percent.

3. Provides the Laboratory Director with an alternative technique in allocating discrete resources instead of deleting projects from the bottom of a prioritized, ordinally ranked list to meet resource limits.
4. Provides an additional insight into the resource allocation and project selection procedure to the decision-maker.

Several limitations in utilizing the proposed methodology are that exact solutions are not entirely possible and that solutions will produce budget surpluses if required budget limits are not to be exceeded. However, in actual practice a budget surplus can be very desirable to management officials in that it allows them a budget reserve to meet emergencies, contract overruns, unanticipated project costs, shifts in program priorities, etc.
CHAPTER VI

CONCLUSIONS AND RECOMMENDATIONS

Conclusions

This research develops and demonstrates a methodology which features improvement over MICOM's zero-base budgeting procedures in allocating financial resources among competing projects. The current procedures allocate resources in a "top down" approach until the budget limit is exhausted; whereas, the new methodology will permit the additional consideration of cost constraints and utility value.

This research concluded that:

1. Methodology produced a static solution that provided an acceptable annual budget consistent with budgetary and laboratory constraints in addition to providing a sufficient management reserve.

2. Methodology produced a solution that recommended deleting projects associated with a low priority rank and with a high value-cost ratio rather than deleting projects from the bottom of a priority rank-ordered list to meet budgetary requirements.

3. As the budget decrement increased, the computation time and number of iterations required to reach the optimum solution increase for this problem.

4. Methodology provided an improved solution to be considered by the decision-maker in allocating discrete resources and in selecting a portfolio of projects for the US Missile Command Laboratory Annual
Limitations of Research

This research considered that the projects were initially, ordinally ranked using some prioritization scheme without consideration of some specific project cost, funded for one year, funded at discrete levels and selected from a set of available projects.

Recommendations For Future Research

In the course of this research, additional areas of investigation have been opened. To this end, it is recommended that additional research be pursued in: obtaining approximate solutions to the proposed problem utilizing Toyoda's methodology cited in [52]; using multi-risk programming techniques developed and demonstrated by Odom in [40]; and extending the proposed methodology to incorporate multi-year funding requirements for projects.
APPENDIX A

COMPUTER CODE
PROGRAM INP(OUTPUT,TAPE6=OUTPUT,INPUT,TAPE5=INPUT)
INTEGR IM8, ITER, JNO, NC, IFQ, NCP, ICTR1, ICTR2, ICTR3, ICTR4,
ICTR5, ICTR6, ICTR7, ICTR8, ITOPT, I, IAUQ, ISO, NSC, IPCT, IO, N
IFQ, NS, N, H, J, IS2, K, M1, NF, IS1, IU1, IJ, ILPC, IJJ, NF1, IX
3X, JP, IC, JJ, HF, IND, JPF, IP, NCH, JPP, IFLAG, L, ITEST
INTEGR I900002, I90001, I90000
REAL ZS, ZBAR, XLB, X, A, BS, C, DJ, DJP, PC1, ZGAP, DEL, XSTAR,
1DT, DJT, VAL, ZNEG, BIP, CJP2, CJP3, BINV, CJP1, CC, ZLB0, ZCOL
2, PCOL, RATIO, RNEM, PIVOT
DIMENSION A(31,150), BS(31), XSTAR(150), DJ(150), DJP(150), XLB(1
150), C(150), IS1(150), IS2(150), X(150), IU1(150), BIP(150), BINV(2
150, 150), CJP1(150), CJP2(150), CJP3(150), PCOL(150), IXX(150), II
3J(150), NCH(50)
COMMON /STOR/ NF, IND, ICTR8, IJJ, XLB, DIP, CJP3, BINV

Incorrect line: **PROGRAM ENUMERA**

C
C TO ALTER PROGRAM CAPACITY, CHANGE ALL DIMENSIONED VARIABLES ACCORDINGLY
C EXCEPT NCH(50). ALSO CHANGE DO-LOOP INDEX IN 100-LOOP.
C
C**************************DEFINITION OF PROGRAM VARIABLES******************************
C
C ITER --- ITERATION COUNT. ***
C ITOPT --- ITERATION COUNT OF DISCOVERY OF CURRENT OPTIMUM. ***
C M --- NUMBER OF ROWS IN ORIGINAL PROBLEM. ***
C N --- NUMBER OF COLUMNS IN ORIGINAL PROBLEM. ***
C NS --- NUMBER OF ELEMENTS IN THE SET S.(NO. FIXED VARIABLES) ***
C NFQ --- ITERATION FREQ. FOR SOLVING IMBEDDED L. P. ***
C JNO --- SUBSCRIPT OF THE MOST RECENTLY FIXED VARIABLE. ***
C NCP --- POINTS TO A-MATRIX ROW SUBSCRIPT OF MOST RECENT SURROGATE CONSTRAINT ***
C ICTR --- constraint added. ***
C ICTR1 --- NO. FATHOMS BY ZBAR LESS THAN OR EQUAL TO ZS. ***
C ICTR2 --- NO. OF FATHOMS BY FEASIBLE BEST POSSIBLE COMPLETION. ***
C ICTR3 --- NO. FATHOMS BY SINGLE CONSTRAINT INFEASIBILITY. ***
C ICTR4 --- NO. FATHOMS BY PERCENT ERROR FACTOR. ***
C ICTR5 --- NO. FATHOMS BY INTEGER DUALS TO I.L.P. ***
C ICTR6 --- NO. FATHOMS BY GENERATION OF INFEASIBLE SURROGATE CONSTRAINT. ***
C NC --- COUNTS THE NO. OF ADDITIONS AND DELETIONS SINCE LAST EXIT FROM I.L.P.
C NCH(I) --- SUBSCRIPT OF I-TH VARIABLE FIXED OR FREED SINCE LAST EXIT.
C IMB --- =1 ON INITIAL ENTRY TO I.L.P.
C =2 WHEN ENTRY TO I.L.P. ONLY INVOLVES OBJ. FCN. CHANGE.
C =3 WHEN ENTRY TO I.L.P. REQUIRES DELETION AND/OR ADDITION OF NEW PROGRAM VARIABLES.
C ZNEG --- VALUE OF OBJ. FCN. OF I.L.P. FROM MOST RECENT ITERATION.
C ILPCT --- NO. L.P. ITERATIONS ON CURRENT ENTRY TO I.L.P.
C BIP(I) --- I-TH ELEMENT OF MOST RECENT R.H.S. OF I.L.P.
C BINV(I,J) --- I-TH ROW, J-TH COL. ELEMENT OF BASIS INVERSE MATRIX OF MOST RECENT I.L.P. ITERATION.
C CJP1(J) --- J-TH ELEMENT OF COST COEFFICIENTS FOR MULTIPLIER ACTIVITIES IN MOST RECENT TABLEAU OF I.L.P.
C CJP2(J) --- SAME FOR BOUNDS ACTIVITIES OF I.L.P.
C CJP3(J) --- SAME FOR SLACK ACTIVITIES (I.E., DUAL VARIABLES) OF I.L.P.
C PCGL(I) --- I-TH ELEMENT OF CURRENT PIVOT COL. IN I.L.P.
C IXX(K) --- K-TH BASIC VARIABLE IN I.L.P.
C IIJ(K) --- SUBSCRIPT OF FREE VARIABLE TO WHICH K-TH SLACK ACTIVITY OF I.L.P. CORRESPONDS (ALSO GIVES BOUND ACTIVITY CORRESPONDENCE).
C JP --- SUBSCRIPT OF PIVOT COL. IN CURRENT I.L.P. ITERATION.
C IP --- SUBSCRIPT OF PIVOT ROW IN CURRENT I.L.P. ITERATION.
C PIVOT --- BINV(IP,JP).
C ZLBD --- LOWER BOUND ON FEASIBLE COMPLETIONS TO CURRENT PARTIAL SOLUTION.

C**********************************************
C**********************************************

C*******PART I --- STORAGE INITIALIZATION.
C
```
WRITE(6,001)
4001 FORMAT (1H1)
IMB = 1
ITER = 0
JND = 0
NC = 0
IFQ = 0
NGP = 0
ICTR1 = 0
ICTR2 = 0
ICTR3 = 0
ICTR4 = 0
ICTR5 = 0
ICTR6 = 0
ICTR7 = 0
ICTR8 = 0
ITUPT = 0
ZS = 0.
ZBAR = 0.
DO 90000 I = 1,150,1
XLB(I) = 0.

C**********PART II --- DATA INPUT.
C
100 X(I) = 0.
90000 CONTINUE
WRITE(6,10000)
10000 FORMAT (5X,"INPUT CONTROL PARAMETERS-FREE FIELD-X",/,
10X,"M-NUMBER 1 OF CONSTRAINTS (LE 31 PLUS SURROGATE)",/,
10X,"N-NUMBER OF VARIABLE EXCLUDING SLACKS (LE 150)",/,
10X,"NS-NUMBER OF VARIABLES FIXED INITIAL",/,
10X,"NFO-FREQUENCY OF SOLUTION OF IMBEDDED LP",/,
10X,"IPGT-PER CENT ERROR ALL 50ED IN FINAL SOLUTION",/,
10X,"IBD-O-BOUND REDefinition, O-NO REDefinition",/,
7,"IAUG-O-BALAS RULE, 1-MINIMUM BRANCH",/)
```
READ(5,*) M,N,NS,NNQ,IO,IPCT,NSC,ISD,IAUG
WRITE(6,10001)
10001 FORMAT(10X,"INPUT ORIGINAL RHS-FREE FIELD WITH DECIMAL",/10X,"M 1 VALUES",/)
READ(5,*) (BS(I),I=1,M)
WRITE(6,10002)
10002 FORMAT(10X,"INPUT OBJECTIVE FUNCTION COEFFICIENTS, N VALUES",/)
READ(5,*) (C(I),I=1,N)
WRITE(6,10003)
10003 FORMAT(10X,"INPUT UPPER BOUNDS, N VALUES",/)
READ(5,*) (DJ(I),I=1,N)
WRITE(6,10004)
10004 FORMAT(10X,"INPUT CONSTRAINT MATRIX, ROW BY ROW",/10X,"WITH DECIM 1AL",/)
READ(5,*) ((A(I,J),J=1,N),I=1,M)
IF (NS .EQ. 0) GO TO 255
WRITE(6,10005)
10005 FORMAT(10X,"INPUT INITIAL PARTIAL SOLUTION",/15X,"0-XJ FREE",/15 X,"1-XJ FIXED, INCREASING",/15X,"-1-XJ FIXED, DECREASING",/10X,"N 2 VALUES IN ORDER",/)
READ(5,*) (IS1(I),I=1,N)
WRITE(6,10006)
10006 FORMAT(10X,"INPUT NS VALUES, -J FOR XJ FIXED",/)
READ(5,*) (IS2(I),I=1,NS)
WRITE(6,10007)
10007 FORMAT(10X,"UNDERLINE INFORMATION",/15X,"1-XJ FIXED UNDERLINED, 10-OTHERWISE",/15X,"N VALUES",/)
READ(5,*) (IUL(I),I=1,N)
WRITE(6,10008)
10008 FORMAT(10X,"INITIAL SOLUTION=0-XJ FREE, K-XJ=K",/)
READ(5,*) (X(I),I=1,N)

C
C********** PART III --- SOLUTION INITIALIZATION.
C
JND = IS2(NS)
DO 90001 J = 1,NS,1
\[ K = IS2(J) \]
\[ DO 90002 I = 1, M, 1 \]
\[ 240 BS(I) = BS(I) + A(I, K) \times X(K) \]
\[ 90002 CONTINUE \]
\[ 250 ZS = ZS + C(K) \times X(K) \]
\[ 90001 CONTINUE \]
\[ 255 DO 90033 I = 1, N, 1 \]
\[ ZBAR = ZBAR + DJ(I) \times C(I) \]
\[ 260 OJP(I) = DJ(I) \]
\[ 90003 CONTINUE \]
\[ PCT = IPCT \]
\[ M1 = 4 \]
\[ NSG = M + NSG \]
\[ NCP = M \]
\[ 300 ITER = ITER + 1 \]
\[ IC = 0 \]
\[ IF ((ITER / 1000) \times 1000 \neq \text{ITER}) IO = 1 \]
\[ NF = N - NS \]

C**********PART IV --- SIMPLE FATHOMING TEST.
C
\[ ZGAP = ZBAR - ZS \]
\[ IF (IO .NE. 1) \text{GO TO 305} \]
\[ WRITE(6, 6050) \text{ITER, ZBAR, ZS, (IS1(I), I=1, N)} \]
\[ WRITE(6, 6052) (X(I), I=1, N) \]
\[ IC = 0 \]
\[ 305 \text{IF (ZBAR .GT. ZS) GO TO 400} \]
\[ ICTR1 = ICTR1 + 1 \]
\[ IF (IO .EQ. 1) \text{WRITE(6, 6020)} \]
\[ IF (IS1(JND) .EQ. 1) \text{GO TO 350} \]
\[ IF (C(JND) .EQ. 0.) \text{GO TO 350} \]
\[ DEL = \text{AINT}(-ZGAP / C(JND)) \]
\[ IF (DEL .LE. 0.) \text{GO TO 1000} \]
\[ X(JND) = X(JND) - DEL \]
\[ DO 90034 I = 1, M1, 1 \]
310 BS(I) = BS(I) - DEL * A(I,JND)
90004 CONTINUE
   ZS = ZS - DEL * C(JND)
   IF (X(JND) .GE. 0.) GO TO 1000
350 IUL(NS) = 1
C
C**********PART V --- FATHOMING TEST FOR FEASIBILITY OF BEST POSSIBLE COMPLETION.
C
GO TO 1000
400 DO 90005 I = 1, M, 1
   IF (BS(I) .LT. 0.0) GO TO 500
410 CONTINUE
90005 CONTINUE
   ICTR2 = ICTR2 + 1
   ITOPT = ITEK
   IF (IO .EQ. 1) WRITE(6,6021)
   DO 90006 I = 1, K, 1
   420 XSTAR(I) = X(I)
90006 CONTINUE
   ZBAR = ZS
   WRITE(6,4000) ITER, ZBAR, (XSTAR(I), I=1,N)
4000 FCRMAT (6H ITER=,I6,5X,5HZBAR=,F10.2,6HXSTAR=/ (1X,20F6.1))
   IF (IO .EQ. 1) WRITE(6,5011) ZBAR, (XSTAR(I), I=1,N)
   IF (IS1(JND) .EQ. 1) GO TO 350
   IF (X(JND) .LE. 1.) GO TO 1000
   DEL = X(JND)
   DO 90007 IJ = 1, M, 1
   IF (A(IJ,JND) .LE. 0.) GO TO 430
   DT = AINT(BS(IJ) / A(IJ,JND))
   IF (CT .GE. DEL) GO TO 430
   DEL = DT
   IF (DEL .EQ. 0.) GO TO 1000
430 CONTINUE
90007 CONTINUE
   X(JND) = X(JND) - DEL
   DO 90008 I = 1, M1, 1
440 BS(I) = BS(I) - DEL * A(I,JND)
90008 CONTINUE
   ZS = ZS - DEL * C(JND)
   XSTAR(JND) = X(JND)
   ZBAR = ZS
   WRITE(6,4000)ITER,ZBAR,(XSTAR(I),I=1,N)
   IF (IJ .EQ. 1) WRITE(6,5011)ZBAR,(XSTAR(I),I=1,N)
   IF (X(JND) .LE. 0.) IUL(NS) = 1
C
C********PART VI --- REDEFINITION OF UPPER BOUNDS.
C
   GO TO 1000
500 DO 90009 I = 1,N,1
   IF (IS1(I) .NE. 0) GO TO 550
   IF (C(I) .EQ. 0.) GO TO 550
   DJTEMP = AINT(ZGAP / C(I) - 1.E-7)
   IF (CJTEMP .LT. DJP(I)) DJP(I) = DJTEMP
C
C********PART VII --- TEST FOR FATHOMING BY CONSTRAINT INFEASIBILITY.
C
550 CONTINUE
90009 CONTINUE
   DO 90010 I = 1,M1+1
   IF (ES(I) .GE. 0.) GO TO 680
   VAL = BS(I)
   DO 90011 J = 1,N,1
   IF (IS1(J) .NE. 0) GO TO 610
   IF (A(I,J) .LE. 0.) GO TO 610
   VAL = VAL + A(I,J) * DJP(J)
610 CONTINUE
90011 CONTINUE
   IF (VAL .GE. 0.) GO TO 680
   ICTR3 = ICTR3 + 1
   IF (IJ .EQ. 1) WRITE(6,6051)I
   GO TO 1000
C
C*********PART VIII --- SOLUTION OF IMBEDDED LINEAR PROGRAM.
C
680 CONTINUE
90010 CONTINUE
   IF (AF .GT. 0) GO TO 699
   WRITE(6,5017)
   STOP
699 IFQ = IFQ + 1
   IF ((IFQ / NFQ) * NFQ .NE. IFQ) GO TO 900
   GO TO (700,750,800), IMB
C
C*****IMB = 1 --- INITIALIZE AND SOLVE THE IMBEDDED LINEAR PROGRAM.
C
700 ZNEG = - ZGAP
   ILPC1 = 0
   K = 0
   DO 9012 I = 1,N,1
      IF (IS1(I) .NE. 0) GO TO 703
      K = K + 1
      IIJ(K) = I
      BJP(K) = C(I)
      CJP2(K) = DJP(I)
      CJP3(K) = 0.
9012 CONTINUE
703 CONTINUE
90012 CONTINUE
   NF1 = NF - 1
   DO 9013 I = 1,NF1,1
      K = I + 1
      I90000 = IABS(K - NF) + 1
      I90001 = 1 - K
      DJ 9014 I90002 = I,I90000,1
         J = I90002 - I90001
         BINV(I,J) = 0.
      704 BINV(J,I) = 0.
90014 CONTINUE
   IXX(I) = 600 + IIJ(I)
705 BINV(I, I) = 1.
90013 CONTINUE
   IXX(NF) = 600 + IIJ(NF)

   BINV(NF,NF) = 1.
90015 I = 1:M,1
706 CJP1(I) = BS(I)
90015 CONTINUE
707 JP = 0
   ILPCT = ILPCT + 1
   IF (IO .EQ. 1) WRITE(6,5004)ILPCT,ZNEG
   IC = 1
   GC = - 1.E-5
90016 I = 1:M,1
   IF (CJP1(I) .GE. GC) GO TO 708
   JP = I
   GC = CJP1(I)
708 CONTINUE
90016 CONTINUE
90017 I = 1:NF,1
   IF (CJP2(I) .GE. GC) GO TO 709
   IC = 2
   JP = I
   GC = CJP2(I)
709 IF (CJP3(I) .GE. GC) GO TO 710
   IC = 3
   JP = I
   GC = CJP3(I)
710 CONTINUE
90017 CONTINUE
   IF (JP .NE. 0) GO TO 720
   IF (IO .EQ. 1) WRITE(6,5000)
   IF (NSC .EQ. M) GO TO 714
NCP = NCP + 1
IF (NCP .GT. NSC) NCP = M + 1
IF (M1 .LT. NSC) M1 = NCP
BS(NCP) = ZGAP - 1.E-7
DO 90018 I = 1,NF,1
JJ = IXX(I)
IF (JJ .GT. 300) GO TO 711
BS(NCP) = BS(NCP) + BIP(I) * BS(JJ)
711 CONTINUE
90018 CONTINUE
DO 90019 I = 1,N,1
A(NCP,I) = C(I)
DO 90020 J = 1,NF,1
JJ = IXX(J)
IF (JJ .GT. 300) GO TO 712
A(NCP,I) = A(NCP,I) + BIP(J) * A(JJ,I)
712 CONTINUE
90020 CONTINUE
713 CONTINUE
90019 CONTINUE
90020 CONTINUE
714 ZLBD = ZNEG + ZBAR
NFP = NF
C
C*****TEST FOR INTEGER CUAL VARIABLES FOR THE IMBEDDED L. P.
C
NG = 0
DO 91021 I = 1,NF,1
CC = ABS(CJP3(I)) - AINT(CJP3(I))
IF (CC .LT. .0001) GO TO 716
IF (CC .GT. .9999) GO TO 716
IPB = 2
IF (-100. .LE. ZNEG .LE. ZLBD .GT. PCT) GO TO 715
IF (IBD .GT. 0) CALL BOUND
IF (IND .EQ. 0) GO TO 500
IF (I) .EQ. 1) WRITE(6,5014)
GO TO 1000
715 ICTRL4 = ICTRL4 + 1
IF (IO .EQ. 1) WRITE(6,5012)
GO TO 1000
716 CONTINUE
90021 CONTINUE
ICTR5 = ICTRL5 + 1
ITOPT = ITER
IF (IO .EQ. 1) WRITE(6,5005)(CJP3(I),I=1,NF)
ZBAR = 0.
DO 90022 I = 1,NF,1
J = IIJ(I)
717 XSTAR(J) = AINT(CJP3(I) + .5)
90022 CONTINUE
DO 90023 I = 1,N,1
IF (IS1(I) .EQ. 0) GO TO 718
XSTAR(I) = X(I)
718 ZBAR = ZBAR + G(I) * XSTAR(I)
90023 CONTINUE
WRITE(6,4000)ITER,ZBAR,(XSTAR(I),I=1,N)
IF (IO .EQ. 1) WRITE(6,5011)ZBAR,(XSTAR(I),I=1,N)
IMB = 2
GO TO 1000
720 ZCOL = CC
IF (IC .EQ. 3) GO TO 726
IF (IC .EQ. 2) GO TO 724
DO 90024 I = 1,NF,1
721 PCOL(I) = 0.
90024 CONTINUE
DO 90025 J = 1,NF,1
K = IIJ(J)
DO 90026 I = 1,NF,1
722 PCOL(I) = PCOL(I) + BINV(I,J) * A(JP,K)
90026 CONTINUE
723 CONTINUE
90625 CONTINUE
  JPP = JP
  GO TO 728
724 DO 90027 J = 1,NF,1
725 PCOL(J) = - BINV(J,JP)
90027 CONTINUE
  JPP = 300 + I1J(JFP)
  GO TO 728
726 DO 90028 J = 1,NF,1
727 PCOL(J) = BINV(J,JP)
90028 CONTINUE
  JPP = 600 + I1J(JP)
728 IP = 0
  RATIC = 1,E20
  DO 90029 I = 1,NF,1
  IF (PCOL(I) .LE. 1,E-5) GO TO 729
  RNEW = BIP(I) / PCOL(I)
  IF (RNEW .GE. RATIO) GO TO 729
  RATIO = RNEW
  IP = I
729 CONTINUE
90029 CONTINUE
C
C****MIN Z = -(INFINITY) --- BACKTRACK.
C
  IF (IP .NE. 0) GO TO 736
  IF (I) .EQ. 1 WRITE(6,5001)
730 IMB = 2
  NC = 0

  NFP = NF
  ICTRE = ICTR6 + 1
  IF (NSC .EQ. M) GO TO 1000
  NCP = NCP + 1
  IF (NCP .GT. NSC) NCP = M + 1
IF (M1 .LT. MSG) M1 = NCP
BS(NCP) = ZGAP - 1.E-7
DO 90030 I = 1, NF, 1
JJ = IXX(I)
IF (JJ .GT. 300) GO TO 731
BS(NCP) = BS(NCP) + BIP(I) + BS(JJ)
731 CONTINUE
90030 CONTINUE
DO 90031 I = 1, N, 1
A(NCF,I) = - G(I)
DO 90032 J = 1, NF, 1
JJ = IXX(J)
IF (JJ .GT. 300) GO TO 732
A(NCP,I) = A(NCP,I) + BIP(J) + A(JJ,I)
732 CONTINUE
90032 CONTINUE
733 CONTINUE
90031 CONTINUE
GO TO 1000
736 PIVOT = PCOL(IP)
ZNEG = ZNEG - ZCOL * BIP(IP) / PIVOT
IXX(IP) = JPP
DO 90033 I = 1, NF, 1
IF (I .EQ. IP) GO TO 738
RATIC = PCOL(I) / PIVOT
DO 90034 J = 1, NF, 1
737 BINV(I,J) = BINV(I,J) - RATIO * BINV(IP,J)
90034 CONTINUE
BIP(J) = BIP(I) - RATIO * BIP(IP)
738 CJP3(I) = CJP3(I) - ZCOL * BINV(IP,I) / PIVOT
90033 CONTINUE
DO 90035 I = 1, NF, 1
739 BINV(IP,I) = BINV(IP,I) / PIVCT
90035 CONTINUE
BIP(IP) = BIP(IP) / PIVCT
740 DO 91036 I = 1, K, 1
741 CJPl(I) = BS(I)
90036 CONTINUE
   DO 90037 I = 1,NF,1
   K = IIJ(I)
   DO 90038 J = 1,M,1
   742 CJPl(J) = CJPl(J) + CJP3(I) * A(J,K)
90038 CONTINUE
743 CONTINUE
90037 CONTINUE
   DO 90039 I = 1,NF,1
   J = IIJ(I)
   744 CJP2(I) = DJP(J) - CJP3(I)
90039 CONTINUE
   IF (2NEG .LT. -1.E-5) GO TO 707
   IF (IO .EQ. 1) WRITE(6,5002)
   GO TO TC 730
C
C**** IMB=2 --- RE-ENTRY TO I.L.P. -- ONLY OBJ. FCN. HAS BEEN ALTERED.
C
750 CONTINUE
   IF (IO .EQ. 1) WRITE(6,5003)
   ILPCT = 0
   DO 90040 I = 1,NF,1

751 CJP3(I) = 0.
90040 CONTINUE
   DO 90041 I = 1,NF,1
   IF (IXX(I) .GT. 300) GO TO 752
   J = IXX(I)
   CC = BS(J)
   GO TO 754
752 IF (IXX(I) .GT. 600) GO TO 757
   J = IXX(I) - 300
   CC = DJP(J)
754 DO 90042 J = 1,NF,1
755 CJP3(I,J) = CJP3(I,J) - GC * BINV(I,J)
90042 CONTINUE
757 CONTINUE
90041 CONTINUE
   ZNEG = - ZGAP
   DO 90043 I = 1, NF, 1
        K = IIJ(I)
   760 ZNEG = ZNEG + CJP3(I) * C(K)
90043 CONTINUE
C
G*****IMB=3 --- RE-ENTRY TO I.L.P. -- AUGMENTATION OR DELETION REQUIRED.
C
   GO TO 740
800 IF (NG .GT. 50 .OR. (IFQ / 200) * 200 .EQ. IFQ) GO TO 700
   IF (IO .EQ. 1) WRITE(6,5006)
   DO 90044 K = 1, NC, 1
        IF (NGH(K) .EQ. 0) GO TO 825
        IF (K .EQ. NC) GO TO 802
        I90000 = IABS(K - NC) + 1
        I90011 = 1 - K
        DO 90045 I90002 = 1, I90000, 1
                J = I90002 - I90001
        IF (NGH(K) + NGH(J) .NE. 0) GO TO 801
        NGH(K) = 0
        NGH(J) = 0
   GO TO 825
801 CONTINUE
90045 CONTINUE
C
G*****PERFORM NECESSARY DELETIONS.
C
802 IF (NGH(K) .LT. 0) GO TO 825
   JJ = NGH(K)
   DO 90046 I = 1, NFP, 1
        J = IIJ(I)
   IF (J .EQ. JJ) GO TO 804
803 CONTINUE
90046 CONTINUE
  WRITE(6,5009) JJ
  WRITE(6,5010)
  GO TO 780
804 JPP = 600 + JJ
  JPPP = 300 + JJ
  JP = I
  DO 90047 J = 1,NFF,1
  IF (IXX(J) .NE. JPP) GO TO 807
  IF (J
  GO TO 817
807 IF (IXX(J) .NE. JPPP) GO TO 808
  IP = J
  ZNEG = ZNEG + BIP(IP) * CJP3(JP)
  GO TO 817
808 CONTINUE

90047 CONTINUE
  IFLAG = 0
  809 ICOL = CJP3(JP)
  IF = J
  RATIO = 1.E20
  DO 90048 I = 1,NFF,1
  IF (BINV(I,JP) .LE. 1.E-5) GO TO 810
  RNEW = BIP(I) / BINV(I,JP)
  IF (RNEW .GE. RATIO) GO TO 810
  RATIC = RNEW
  IP = I
810 CONTINUE
90048 CONTINUE
  IF (IP .NE. 0) GO TO 813
  IFLAG = IFLAG + 1
  IF (IFLAG .EQ. 2) GO TO 812
  DO 90049 I = 1,NFF,1
811  BINV(I,JP) = - BINV(I,JP)
90049  CONTINUE
  CJP3(JP) = DJP(JJ) - CJP3(JP)
  GO TO 809
812  WRITE(6,5010)
  GO TO 700
813  PIVOT = BINV(IP,JP)
  ZNEG = ZNEG - ZCOL * BIP(IP) / PIVOT
  DO 90050  I = 1,NFP,1
    IF (I .EQ. IP) GO TO 815
    RATIO = BINV(I,JP) / PIVOT
  DO 90051  J = 1,NFP,1
814  BINV(I,J) = BINV(I,J) - RATIO * BINV(IP,J)
90051  CONTINUE
  BIP(I) = BIP(I) - RATIO * BIP(IP)
815  CJP3(I) = CJP3(I) - ZCOL * BINV(IF,I) / PIVOT
90050  CONTINUE
  DO 90052  I = 1,NFP,1
816  BINV(IP,I) = BINV(IP,I) / PIVOT
90052  CONTINUE
  BIP(IP) = BIP(IP) / PIVOT
817  IF (JP .EQ. NFP) GO TO 820
   IIJ(JP) = IIJ(NFP)
   CJP3(JP) = CJP3(NFP)
   DO 90053  J = 1,NFP,1
818  BINV(J,JP) = BINV(J,NFP)
90053  CONTINUE
820  IF (IP .EQ. NFP) GO TO 823
   IXX(IP) = IXX(NFP)
   BIP(IP) = BIP(NFP)
   DO 90054  J = 1,NFP,1
821  BINV(IP,J) = BINV(NFP,J)
90054  CONTINUE
823  NFP = NFP - 1
   IF (IO .EQ. 1) WRITE(6,5007) JJ
   IF (NFP .NE. 0) GO TO 825
WRITE(6,5018)
GO TO 700
825 CONTINUE
90044 CONTINUE
  DO 90055 I = 1,NC,1
C
C*****PERFORM NECESSARY ADDITIONS.
C
  IF (NCH(I) .GE. 0) GO TO 850
  JJ = - NCH(I)
  K = NFP + 1

  BINV(K,K) = 1.
  DO 90056 J = 1,NFP,1
  BINV(K,J) = 0.
  826 BINV(J,K) = 0.
  90056 CONTINUE
  DO 90057 J = 1,NFF,1
  IF (IXX(J) .GT. 300) GO TO 832
  L = IXX(J)
  CC = A(L,JJ)
  DO 90059 L = 1,NFF,1
  830 BINV(K,L) = BINV(K,L) - CC * BINV(J,L)
  90058 CONTINUE
  832 CONTINUE
  90057 CONTINUE
  GJP3(K) = 0.
  BIP(K) = G(JJ)
  DO 90059 J = 1,NFF,1
  L = IIJ(J)
  834 BIP(K) = BIP(K) + G(L) * BINV(K,J)
  90059 CONTINUE
  IIJ(K) = JJ
  IXX(K) = 600 + JJ
  IF (BIP(K) .GE. 0.) GO TO 840
IX(K) = 300 + JJ
BIP(K) = - BIP(K)
ZNEG = ZNEG - BIP(K) * DJP(JJ)
DO 90060 J = 1, K, 1
BINV(K, J) = - BINV(K, J)
836 CJP3(J) = CJP3(J) - BINV(K, J) * DJP(JJ)
90060 CONTINUE
840 NFP = K
IF (IO .EQ. 1) WRITE(6, 5000) JJ
850 CONTINUE
90055 CONTINUE
ILPCT = 0
C
C**********PART IX --- AUGMENTATION.
C
GO TO 750
900 IF (IAUG .EQ. 0) GO TO 932
ITEST = 0
DO 90061 J = 1, N, 1
IF (IS1(J) .NE. 0) GO TO 930
IF (ITEST .EQ. 1) JND = J
ITEST = 1
IF (CJP(J) - XLB(J) .GE. DJP(JND) - XLB(JND)) GO TO 936
JND = J
930 CONTINUE
90061 CONTINUE
X(JND) = XLB(JND)
IS1(JND) = 1
GO TO 938
932 VAL = - 1.E20
DO 90062 I = 1, N, 1
IF (IS1(I) .NE. 0) GO TO 935
CC = 0.
DO 90063 J = 1, M, 1
DJTEMP = BS(J) + DJP(I) * A(J, I)
IF (DJTEMP .LT. 0.) CC = CC + DJTEMP
90062 CONTINUE
90063 CONTINUE
90065 CONTINUE
90070 CONTINUE
CONTINUE
IF (CG .LE. VAL) GO TO 935
VAL = CC
JNU = I

CONTINUE
935 CONTINUE
90061 CONTINUE
X(JNO) = DJP(JND)
IS1(JNO) = -1
938 IF (IO .EQ. 1) WRITE(6,6025)JNO
ICTR7 = ICTR7 + 1
NS = NS + 1
IS2(NS) = JND
ZS = ZS + C(JNO) * X(JNO)
DO 90064 I = 1,M1,1
940 BS(I) = BS(I) + X(JNO) * A(I,JND)
90064 CONTINUE
IUL(NS) = 0
IF (DJP(JND) .EQ. XLB(JND)) IUL(NS) = 1
IF (IMB .NE. 1) IMB = 3
NC = NC + 1
IF (NC .LT. 50) NCH(NC) = JND

C
C**********PART X --- BACKTRACK
C
GO TO 300
1000 IF (NS .EQ. 0) GO TO 2000
1005 IF (IUL(NS) .NE. 1) GO TO 1010
IF (IMB .NE. 1) IMB = 3
NC = NC + 1
IF (NC .LT. 50) NCH(NC) = - JND
ZS = ZS - C(JND) * X(JND)
DO 90065 I = 1,M1,1
1007 BS(I) = BS(I) - X(JND) * A(I,JND)
90065 CONTINUE
   X(JND) = 0.
1069 NS = NS - 1
   IS1(JND) = 0
   JND = IS2(NS)
   IF (NS .EQ. 0) GO TO 2000
   GO TO 1005
1010 DO 90066 I = 1,N,1
   IF (IS1(I) .NE. 0) GO TO 1015
   XLB(I) = 0.
   DJP(I) = DJ(I)
1015 CONTINUE
90066 CONTINUE
   IF (IS1(JND) .EQ. 1) GO TO 1050
   X(JND) = X(JND) - 1.
   DO 90067 I = 1,M1,1
1020 BS(I) = BS(I) - A(I,JND)
90067 CONTINUE
   ZS = ZS - C(JND)
   IF (X(JND) .GT. XLB(JND)) GO TO 300
   IUL(NS) = 1
   GO TO 300
1050 X(JND) = X(JND) + 1.
   DO 90068 I = 1,M1,1
1060 BS(I) = BS(I) + A(I,JND)
90068 CONTINUE
   ZS = ZS + C(JND)
   IF (X(JND) .LT. DJP(JND)) GO TO 300
   IUL(NS) = 1

C
C**********PART XI --- TERMINATION OF ENUMERATION PROCESS.
C
GO TO 300
2000 IF (ICTR2 + ICTR5 .NE. 0) GO TO 2005
   WRITE(6,5013)
GO TO 2007
2005 WRITE(6,3000)(XSTAR(I),I=1,N)
WRITE(6,3001)ZBAR,ITER
2007 CC = 1.
DO 90069 I = 1,N,1
2010 CC = CC * (DG(I) + 1.)
90069 CONTINUE
VAL = ITER
PCT = (CC - VAL) * 100. / CC
WRITE(6,5050)CC,VAL,PCT,ICTR1,ICTR2,ICTR3,ICTR4,ICTR5,ICTR6,ICTR8
1,ICTR7,ICTPT
STOP
5000 FORMAT (41H OPTIMAL SOLUTION OF IMBEDDED L. P. FOUND)
5001 FORMAT (16H MIN Z = -(INF.))
5002 FORMAT (32H FATHOM BY SURROGATE CONSTRAINT.)
5003 FORMAT (45H ENTERED OBJ. FCN. UPDATE PORTION OF I. L. P.)
5004 FORMAT (20H ILP ITERATION NO. =,I4,5X,7H ZNEG =,E20.8)
5005 FORMAT (24H INTEGER QUALS TO I.L.F.(/1X,22F6.2))
5006 FORMAT (28H ENTERED I.L.P. MODIFICATION)
5007 FORMAT (17H DELETED VARIABLE ,I5)
5006 FORMAT (16H ADDED VARIABLE ,I5)
5009 FORMAT (24H UNABLE TO FIND VARIABLE ,I4,32H FOR DELETION -- RE-INITIALIZE.)
5010 FORMAT (58H UNABLE TO RE-ENTER I.L.P. SUCCESSFULLY -- RE-INITIALIZE 12E.)
5011 FORMAT (41H CURRENT BEST FEASIBLE SOLUTION --- ZBAR=,F10.3/5X,2HY=1.12F10.3/(7X,12F10.3))
5012 FORMAT (48H FATHOKED BY PERCENT POSSIBLE IMPROVEMENT FACTOR)
5013 FORMAT (29H NO FEASIBLE SOLUTION EXISTS.)
5014 FORMAT (35H FATHOKED BY BOUND INCONSISTENCIES.)
5017 FORMAT (70H0***APPARENT ERROR IN PROGRAM --- ATTEMPT TO ENTER I.L.
1P. WITH NF=0***)
5018 FORMAT (54H0***APPARENT PROGRAM ERROR --- ATTEMPT TO SET NFP=0***)
5050 FORMAT (///36H ADDITIONAL SOLUTION INFORMATION ---/7X,21HTOTAL NU
1MBER OF NODES, 22X, E15.6/7X, 37HNUMBER OF NODES EXPLICITLY ENUMERATE
20, 6X, E15.6/7X, 41HPERCENTAGE OF NODES IMPLICITLY ENUMERATED, 2X, E15.
38/7X, 27HNUMBER OF TIMES FATHOMED BY/10X, 17HZBAR LESS THAN ZS, 33X,
410/10X, 39HFEASIBILITY OF BEST POSSIBLE COMPLETION, 11X, I10/10X, 24H
SAN INFEASIBLE CONSTRAINT, 26X, I10/10X, 29HPERCENTAGE IMPROVEMENT FAC
6TOR, 21X, I10/10X, 34HINTEGER DUALS OF THE IMBEDDED L.P., 16X, I10/10X,
73BHIMBEDDED L.P. SURROGATE CONSTRAINT, 16X, I10/10X, 36HUPPER OR LOWE
8R BOUND INCONSISTENCIES, 14X, I10/7X, 32HNUMBER OF AUGMENTATIONS REQU
9IRED, 21X, I10/7X, 49HITERATION NUMBER ON WHICH OPTIMUM WAS FOUND, 10X
+, I10)
6020 FORMAT (25H FATHOMED BY ZBAR .LE. ZS)
6021 FORMAT (44H FATHOMED BY FAS. BEST POSSIBLE COMPLETION.)
6025 FCRMAT (38H AUGMENTATION STEP --- VARIABLE FIXED=-15)
6050 FCRMAT (10H ITERATION, 15, 3X, 5HZBAR=.E15.8, 3X, 3HZS=.E15.8/5X, 4HIS1=
1.3X, 3.I3/(12X, 3013))
6051 FCRMAT (38H FATHOMED BY INFEASIBLE CONSTRAINT NO., 15)
6052 FCRMAT (5X, 2MX=.E15.8, 12F10.3/(10X, 12F10.3))
6000 FORMAT (21H1OPTIMAL SOLUTION ---/(25X, E20.8))
8001 FORMAT (28H1OPTIMAL VALUE OF OBJECTIVE=.E20.8, 5X, 17HITERATION COUN
1IT =, I10)
9000 FORMAT (10I5)
9001 FORMAT (3E10.0)
END
BIBLIOGRAPHY


22. Headquarters, Department of the Army, "Research and Development", Army Regulation 70-1, May 1975.


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