A. SUBHARMONIC GENERATION/BIFURCATION IN LIQUID HELIUM

Measurements of the thresholds for the generation of first-sound sub-
harmonics show a classical liquid response above the superfluid transition
\( T = 2.17 \) K and bifurcation following the Feigenbaum universal convergence
ratio below the superfluid transition. Ion-trapping techniques reveal that
the threshold for the generation of the first subharmonic response corresponds
to the threshold for acoustic generation of quantum vortex line.
**B. ACOUSTIC STREAMING IN LIQUID HELIUM**

Quantitative measurements of acoustic streaming shows a classical response above $T = 2.17$ K. Below the superfluid transition we have shown that only the normal fluid component streams and that the temperature and frequency dependences are in quantitative agreement with the Khalatnikov two-fluid hydrodynamic equations if the coefficient of second viscosity is properly taken into account.

**C. HEAT TRANSFER BETWEEN COPPER AND LIQUID HELIUM**

A conventional Kapitza resistance experiment with an acoustic streaming field directed transverse to the surface normal shows 1) no enhanced heat transfer below the superfluid transition and 2) above the superfluid transition the thermal conductance increases linearly with acoustic velocity amplitude reaching a value of 2.5 times the no sound level for a sound velocity amplitude of 0.8 centimeters/second.
SUMMARY OF RESEARCH FOR GRANT PERIOD - 10/1/80-9/30/81 and 10/1/81-3/31/81

The work summarized in this report has centered on four discoveries made in our laboratory over the past five years. First, we find that ultrasonic transducers operating in the low megahertz range are efficient generators of quantum turbulence (quantum vortex-line tangle) in superfluid helium. We mean by efficient that one can generate about a factor of 50 more vortex-line per unit volume per input power acoustically than by the usual thermal counter-flow techniques. Second, we find that the threshold for the generation of the first acoustic subharmonic \( (f_0/2) \) below the superfluid transition corresponds to the threshold for the acoustic generation of quantum turbulence and that this is related to bifurcation of the Feigenbaum type. Third, acoustic streaming in liquid helium follows classical predictions above the superfluid transition but below 2.17 K only the normal component streams. The rather complicated temperature and frequency dependences observed are in quantitative agreement with the predictions of the second order Khalatnikov two-fluid hydrodynamic equations if the coefficient of second viscosity is properly taken into account. Fourth, optical detection of the first subharmonic in bulk liquid helium and observations of the asymmetries of the propagation of sound waves using laser light diffraction quantitatively reveal bulk liquid nonlinearities and the necessity to go to second and third order terms in the equation of state of liquid helium to explain the response of the liquid to finite amplitude sound.

A. SUBHARMONIC GENERATION/BIFURCATION IN LIQUID HELIUM

Measurements of the finite-amplitude first-sound response of liquid helium-4 show a complicated acoustic subharmonic spectrum\(^1\). Above the superfluid transition, \( T_s = 2.17 \) K, liquid helium behaves macroscopically as a classical liquid...
response, dominated by vapor bubble dynamics of the type recently carefully documented for the case of water\(^2\). This includes a subharmonic route to chaos (acoustic cavitation), but does not show bifurcation of the Feigenbaum type. However, below the superfluid transition where the existence of conventional vapor bubbles is excluded by quantum-fluid properties, one observes not only a sharper subharmonic spectral response but a well defined bifurcation sequence which quantitatively exhibits the Feigenbaum convergence ratio, \(\delta^3\).

The first subharmonic, \(f_o/2\), appears for a drive voltage on the ultrasonic transducer which corresponds to a sound displacement velocity of 0.15 cm/s. As the sound intensity is increased, other subharmonics as well as ultraharmonics appear. The convergence ratio, \(\delta\) was calculated by using linear regression analysis to estimate the onset threshold of each subharmonic of the first three components of the first subharmonic-bifurcation sequence \(f_o/n\) where \(n = 2, 4,\) and 8. Four independent sets of data at 1.60 K yield \(\delta = 4.83 \pm 0.6\) in quantitative agreement with the predicted value. Determinations at two other temperatures and at \(f_o = 9.8751\) MHz fall within the above range. Attempts to measure the rescaling factor \(\mu\) for fully developed adjacent members of the bifurcation sequence have met with less success. Analysis limitations arising from the combined effects of the \(f_o/8\) peak falling on a low-frequency noise shoulder and of a tendency for the fully developed \(f_o/4\) peak to phase lock to nearby subharmonics permit us only to place a range of 6-10 dB on the scaling parameter. However, this result is not in conflict with the predicted value of 8.2 dB.

Above the superfluid transition one still observes subharmonics. The threshold for the generation of \(f_o/2\) is larger. Bifurcation of the Feigenbaum type is not present: \(f_o/4\) is difficult to generate and all peaks appear broader and set upon a noisy background.
Using a gated tritium source, an appropriate drift space, and a guarded collector, one can produce a current of negative ions (electron microbubbles) with which to probe the sound field. If quantum vortex line is present, ions can be trapped on the line and the current to the collector will decrease. We observe that as the sound-pressure amplitude is increased from zero, no ion trapping occurs until the threshold for the production of the first subharmonic-bifurcation sequence is exceeded. This effect is large and has the unmistakable lifetime edge and polarity dependence for ion trapping on vortex line. In this manner, for the case of acoustic subharmonic bifurcation in superfluid helium, we identify the physical aspect associated with the universal convergence as the threshold for the production of quantum turbulence (vortex-line generation), not the threshold for the production of classical turbulence by sound (acoustic cavitation), which lies 2 orders of magnitude higher in sound-pressure amplitude.

A publication on this work is attached which resulted from this important discovery and provides further detail.

B. ACOUSTIC STREAMING IN LIQUID HELIUM

The propagation of sound in an absorptive medium gives rise to mass flow. This phenomenon is known as acoustic streaming. It is a second order nonlinear effect.

The experimental cell consists of a tritium source and grid arranged to inject a well-defined beam of negative ions (electron microbubbles) onto a pair of guarded collectors 2.0 centimeters away, spaced 0.03 centimeters apart. A plane wave first sound beam, generated with a PZT4 thickness mode transducer resonant at 9.87 megahertz, was directed orthogonal to the ion beam and filled the drift space region. In the presence of acoustic streaming, the negative ion beam was observed to translate from the first collector to
the second collector in the direction of sound propagation. Knowing the geometry and the negative ion mobility, streaming velocities as small as 0.01 centimeters/second could easily be measured.

We observed the acoustic streaming velocity as a function of sound displacement velocity frequency and temperature. The streaming velocity is proportional to the square of the displacement velocity up to 0.053 \((\text{cm/s})^2\). This corresponds to the intensity threshold for the ultrasonic generation of vortex line. For increased sound intensity, after a transition region, the streaming velocity is again proportional to the square of the displacement velocity. The slope of this second straight line region is less than the first since mutual friction can now couple the normal fluid component to the superfluid component and both are expected to stream.

The temperature dependence of the streaming velocity for a given sound intensity is determined by the temperature dependence of the first sound absorption coefficient via the coefficient of second viscosity in detailed agreement with the theory by Khalatnikov and Boguslavskii, et.al. By repeating the above types of measurements at 1 MHz and 3 MHz (in addition to the 10 MHz results discussed) we have also verified the frequency squared dependence of the streaming velocity which comes into the problem via the first sound absorption coefficient, \(a\).

A publication is attached which resulted from this work and provides further details concerning this very involved study.

C. HEAT TRANSFER BETWEEN COPPER AND LIQUID HELIUM

In a conventional Kapitza resistance experiment involving heat transfer across a copper surface into liquid helium, an acoustic streaming velocity field (at 10 MHz) was directed transverse to the surface normal. Ultrasound
had no observable effect on the heat transfer to the superfluid phase (He-II) but in the normal fluid phase (He-I) the thermal conductance increased linearly with acoustic velocity amplitude, reaching a value 2.5 times the zero sound conductance for a sound velocity amplitude of 0.8 centimeters/second. We feel these factors are important in the design of cryogenic heat exchanges, heat sinks and superconducting machinery.

A publication resulting from this work is attached.

D. USE OF ACOUSTO-OPTICAL TECHNIQUES TO MEASURE NONLINEAR PROPERTIES OF CRYOGENIC LIQUIDS

We have carried out the first continuous quantitative measurements of the intensity asymmetries of the diffraction patterns to fifth order of a clean laser beam as it traverses a region of ultrasonic waves. These asymmetries are related to the non-sinusoidal wave form which develops progressively away from the sound source as a result of nonlinearities of the bulk liquid. We have completed our results for methonal and liquid nitrogen and find good agreement with the B/A nonlinearity ratios for these liquids as measured by other techniques. We hope to be able to extend this work in detail to liquid nitrogen-oxygen mixtures and liquid helium.

Preliminary results look very interesting for liquid helium below the superfluid transition. This work will eventually result in a publication and a thesis if support for continuation can be found. It appears that it is necessary to include third order effects (C/A ratios) in the case of liquid helium, even relatively far away from the superfluid transition.

E. REFERENCES


F. PUBLICATIONS AND PRESENTATIONS

a.) Publications in Refereed Journals

b.) Presentations at Professional Society Meetings


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Bifurcation Universality for First-Sound Subharmonic Generation in Superfluid Helium-4

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Measurements are presented which show that below the superfluid transition, the generation of first-sound subharmonics in the low-megahertz range quantitatively follows the Feigenbaum universal convergence. In addition, by using ion-trapping techniques the physical nature of the onset of the first bifurcation sequence is identified as the threshold for the generation of quantum vortex line, not the threshold for the production of macroscopic classical turbulence, i.e., acoustic cavitation.

PACS numbers: 67.40.Mj, 43.25.+y, 47.25.-c, 67.40.Vs

The response of driven nonlinear systems has been investigated by an enormous number of experimental techniques. Recently such studies have included Rayleigh-Benard experiments,\(^1\) couette flow,\(^2\) optically bistable laser cavities,\(^3\) acoustic cavitation noise,\(^4\) charge-density waves,\(^5\) pinning dynamics of dislocation lines,\(^6\) and nonlinear discrete electronic circuits.\(^7\) These systems generate output signals rich in spectral detail. Generally one is interested in how the frequency content of the output signals varies as some driving parameter is changed. Typically, driven nonlinear systems exhibit (1) harmonic generation, (2) subharmonic generation (which displays onset thresholds but may or may not show period doubling), (3) ultraharmonic generation (harmonics of the subharmonics), and at a sufficiently large value of the driving parameter, (4) a transition to a noisy, chaotic or turbulent regime.

The recent theory by Feigenbaum\(^8\) concerning the discovery of certain universal properties in period-doubling bifurcations of iterated one-dimensional maps has catalyzed a search for analogous universal behaviors in experimental nonlinear systems. He has shown that for systems which lead to a transition to chaotic behavior via a sequence of period-doubling bifurcations, an ordered set of values of the driving parameter, \(\lambda_n\), for which bifurcations occur, converges to a universal number \(\delta\), where \(\delta=(\lambda_{n+1}-\lambda_n)/(\lambda_{n+2}-\lambda_n)=4.669\ldots\). In addition, the ratio of the amplitudes of the Fourier components of adjacent fully developed bifurcated subharmonics scales by \(\mu=6.57\) or \(10\log_{10}\mu=8.2\) dB, again a universal number, independent of the details of the nonlinear system. As a result of this expected commonality, a rather large literature is emerging which describes many nonlinear systems exhibiting subharmonic routes to chaotic behavior. However, it must be emphasized that to date only three types of experiments quantitatively show the Feigenbaum period-doubling bifurcation universalities. These are Rayleigh-Benard experiments, couette flow experiments, and nonlinear discrete electronic circuits.

This paper presents the results of measurements of the finite-amplitude first-sound response of liquid helium-4, a system known to exhibit an acoustic subharmonic spectrum.\(^9\) Above the superfluid transition, \(T_s=2.17\) K, liquid helium behaves macroscopically as a classical liquid. One observes a very rich nonlinear response, dominated by vapor bubble dynamics of the type recently carefully documented for the case of water.\(^4\) This includes a subharmonic route to chaos (acoustic cavitation), but does not show bifurcation of the Feigenbaum type. However, below the superfluid transition where the existence of conventional vapor bubbles is excluded by
quantum-fluid properties, one observes not only a sharper subharmonic spectral response but a well defined bifurcation sequence which quantitatively exhibits the Feigenbaum convergence ratio, $\delta$. The experiment consists of radiating liquid helium with first sound and measuring the frequency response of the transport of energy through the liquid. A pair of piezoelectric (PZT4) thickness mode transducers are positioned parallel and centered on a common axis in an open geometry in the liquid helium, that is, not in a closed resonant cavity. One transducer, employed as a first-sound source, is driven at its electromechanical resonant frequency, $f_0$. (For the data presented herein, $f_0=2.6883$ MHz.) The other transducer, carefully chosen to have a relatively flat response in the frequency range of interest, receives the signal, which is frequency analyzed and digitally signal averaged. At low acoustic-pressure amplitudes harmonic generation, typical of a driven nonlinear dynamic system, is observed. We emphasize that the nonlinearity in this case is the response of the liquid helium to finite-amplitude first sound. It is simple to confirm, as we have done, that the sound source and analysis instrumentation do not constitute a nonlinear electronic network in the range of the drive voltages and frequencies employed.

The first subharmonic, $f_0/2$, appears for a drive voltage on the ultrasonic transducer which corresponds to a sound displacement velocity of 0.15 cm/s. As the sound intensity is increased, other subharmonics as well as ultraharmonics appear. Figure 1 shows 256 digitally signal-averaged spectra at 1.60 K. Figure 2 shows the peak amplitude in decibels, corresponding to the power contained in each of the first three components of the first subharmonic-bifurcation sequence $f_0/n$, where $n=2$, 4, and 8. The convergence ratio, $\delta$, was calculated by using linear regression analysis to estimate the onset threshold for each subharmonic. Four independent sets of data at 1.60 K similar to those presented in Fig. 2 yield $\delta=4.83 \pm 0.6$ in quantitative agreement with the predicted value. Determinations at two other temperatures and at $f_0=9.8751$ MHz within the above range. Attempts to measure the rescaling factor $\mu$ for fully developed adja members of the bifurcation sequence have with less success. Analysis limitations are from the combined effects of the $f_0/8$ peak on a low-frequency noise shoulder and of a tendency for the fully developed $f_0/4$ peak to phase lock to nearby subharmonics permit us only to place a range of 6–10 dB on the scaling param-

**FIG. 1.** The acoustic response of superfluid helium-4 (256 digitally signal-averaged spectral) clearly exhibiting harmonics, subharmonics, and ultraharmonics. These spectra were recorded at a sound level 6.25 times greater than the threshold for the production of the first subharmonic and approximately twice the sound level for the onset of acoustic cavitation.

**FIG. 2.** The upper graph shows the peak amplitude in decibels corresponding to the power in each of the first three components of the first bifurcation sequence $f_0/n$, where $n=2$, 4, and 8, as a function of the rms voltage on the drive transducer (proportional to the sound-pressure amplitude). Linear regression analysis yields $\lambda_1=0.32$, $\lambda_2=0.61$, and $\lambda_3=0.67$. The lower curve shows the onset of ion trapping due to vortex line generation occurring at the same drive voltage as the onset of the first bifurcation.
Above the superfluid transition one still observes subharmonics. The threshold for the generation of $f_0/2$ is larger. Bifurcation of the Feigenbaum type is not present: $f_0/4$ is difficult to generate and all peaks appear broader and set upon a noisy background.

Using a gated tritium source, an appropriate drift space, and a guarded collector, one can produce a current of negative ions (electron microbubbles) with which to probe the sound field. If quantum vortex line is present, ions can be trapped on the line and the current to the collector will decrease. We observe that as the sound-pressure amplitude is increased from zero, no ion trapping occurs until the threshold for the production of the first subharmonic-bifurcation sequence is exceeded; see Fig. 2. This effect is large and has the unmistakable lifetime edge and polarity dependence for ion trapping on vortex line. In this manner, for the case of acoustic subharmonic bifurcation in superfluid helium, we identify the physical aspect associated with the universal convergence as the threshold for the production of quantum turbulence (vortex-line generation), not the threshold for the production of classical turbulence by sound (acoustic cavitation), which lies 2 orders of magnitude lower.

There are several other features in our data which show systematic behaviors. (1) In additional to the first subharmonic-bifurcation sequence, $f_0/n$, where $n = 2, 4, 8$, we observe the "prime subharmonics" at $n = 3, 5, 7$. The thresholds for the generation of the prime subharmonics increase monotonically as $n$ increases. (2) At a fixed sound intensity, sufficient to produce a spectrum rich in subharmonics, a slight departure from thermal equilibrium, for example a temperature increase of 10 mK for 2 s, will result in slow amplitude oscillations of the subharmonic peaks. Peaks of smaller amplitude may momentarily grow at the expense of larger peaks. Some subharmonics may temporarily disappear. Such transients last a few minutes before the system settles down. (3) At sound intensities well above the threshold for subharmonic generation occasionally neighboring subharmonic peaks (particularly $f_0/4$ with $f_0/3$ or with $f_0/5$) move toward each other, merge into a band, and then move back to their correct positions. This banding is relatively unstable and can easily be disturbed by a very small change in temperature, sound amplitude, or sound frequency.

We conclude that liquid helium-4 presents an interesting system in which to study the production of acoustic subharmonics in that below the superfluid transition a bifurcation sequence of the Feigenbaum type is quantitatively observed and above the superfluid transition it is not. Furthermore, below the superfluid transition, we are able to identify the physical nature of the onset of the first subharmonic-bifurcation sequence not to be the threshold for the acoustic production of macroscopic classical turbulence (acoustic cavitation) but to be the threshold for the ultrasonic generation of microscopic quantum turbulence (vortex-line generation).

We are indebted to James A. Rooney for helpful discussions concerning nonlinear acoustic phenomena. This work was supported in part by the U. S. Air Force Office of Scientific Research, Grant No. 76-3113.

Quantative measurements of acoustic streaming velocity in liquid helium as a function of sound intensity (up to the cavitation threshold), frequency (1, 3 and 10 MHz) and temperature (1.43 K < T < 2.19 K) are reported. A transition to superfluid turbulence, several flow regions and flow fluctuations are observed. Comparison with the predictions of the second order Khalatnikov two-fluid hydrodynamic equations indicates good functional and quantitative agreement.

I. Introduction.

The propagation of sound in an absorptive fluid results in a flow of the fluid known as acoustic streaming. To understand this flow, the hydrodynamic equations for the fluid must be solved beyond the linear approximation and effects of dissipation taken into account. Measurement of this nonlinear effect can yield information concerning the equation of state and the dissipative coefficients of the fluid. In particular, a viscous fluid will respond, in second order, with a time independent flow in proportion to the ratio $\alpha/n$, where $\alpha$ is the coefficient of sound attenuation and $n$ is the shear viscosity. An ideal Eulerian fluid will not show a time independent flow. Based upon the two-fluid model for liquid helium below the lambda point (2.17 K), one expects streaming of the normal fluid component but no streaming of the superfluid component, provided there is no vortex line present coupling the two-fluid components via mutual friction.
In our studies of nonlinear acoustic phenomena in liquid helium, we have shown that ultrasound in the low megahertz frequency range is an effective generator of superfluid turbulence and that the threshold for superfluid turbulence is identical to the threshold for the generation of subharmonics (2). Other researchers have also observed subharmonic responses (3) in liquid helium, as well as cavitation (4, 5). The purpose of this work is to measure acoustic streaming in superfluid helium, to determine its relationship to other nonlinear effects and to test certain aspects of the Khalatnikov (6) two-fluid hydrodynamic equations. Measurements at three frequencies, 1 MHz (0.969), 3 MHz (2.93) and 10 MHz (9.87) for sound intensities up to the cavitation threshold were carried out in the temperature range 1.43 K < T < 2.19 K.

The momentum and mass flow associated with sound waves in liquid helium can be calculated by solving the two-fluid hydrodynamics equations to second order. For first sound, the fluid velocities in the wave are given to first order by

\[ v_{s,1} = v_{n,1} = A \exp(i(kx - \omega t) \exp(-\alpha x) \tag{1} \]

where \( \omega/k \) is the sound velocity, \( c \), and \( A \) is the velocity amplitude evaluated at the coordinate \( x = 0 \). The two-fluid hydrodynamic equations are solved to second order and the time independent components, \( v_2 \), are found by time averaging. The solutions have been carried out by Boguslavskii, et.al. (7) and by Puttermann and Garrett (8). They show that the time independent velocity of the superfluid component is zero

\[ v_{s,2} = 0 \tag{2} \]

and the time independent velocity of the normal fluid component is given by

\[ v_{n,2} = \frac{\rho \alpha A^2}{2n} a^2 f(r) \tag{3} \]
where $\rho$ is the total liquid density, $a$, the radius of the plane disk transducer, and $f(r)$ is a geometric factor determined by the boundary conditions imposed by the size and shape of the sound beam and flow return path. The expression in Eq. (3) is the classic relation for acoustic streaming in a viscous liquid, and has been shown to predict reliable values for the attenuation coefficient for a wide range of fluids. For liquid helium, however, the quantity $\rho a/n$ is quite large for frequencies in the megahertz range. In particular, we note that Eq. (3) predicts $v_{n,2} > A$ for all $A > 10^{-3}$ cm/s. For this reason, at measurable streaming velocities, Eq. (3) will be beyond its region of validity but may still be capable of providing insight into the functional dependence of the observed flows.

II. The Experiment.

A. Cell Design

The usual tracer techniques employed to observe acoustic streaming patterns in conventional fluids are difficult to use in liquid helium due to, among other reasons, the scarcity of particulate matter of comparable density. It is well known, however, that both the positive and negative ion structures in liquid helium move with the normal fluid component. We have utilized this effect to measure streaming in superfluid helium.

A schematic diagram of the experimental cell is shown in Fig. 1. A PZT4 thickness mode transducer is located in the grid-collector region of a triode. Ion structures are produced using 1 Ci tritium source on a copper disk 2.5 cm in diameter. The grid plate consists of a copper disk with a rectangular hole 0.25 cm by 0.64 cm. The hole is covered with an 85% open nickel mesh. Four isolated collectors, each 0.14 cm by 0.64 cm and separated by 0.03 cm, are used to measure the ion current. For all data reported in this paper,
a beam of negative ions (electron microbubbles) is employed. The beam is initially positioned such that \( I_1 = I_2 \) and \( I_3 = I_4 = 0 \). Since the inward transducer face is at ground potential, the grid plate and a surrounding grid are also held at ground to establish an axially symmetric although a somewhat nonuniform electric field between the grid and collector plates. For a collector voltage of +100 volts and a source voltage of -10 volts with respect to the grounded grid, beam current densities on the order of \( 10^{-12} \) amps are observed. Beam spreading is expected to be small. This is verified by the observation that \( I_3 \ll I_2 \) unless large acoustic streaming velocities are produced.

B. Flow Configuration

Streaming velocities or at least their functional dependence can be estimated from the theoretical results shown in Eqs. (2) and (3). Assuming a nonslip boundary condition at the walls, a viscous liquid is expected to flow away from the transducer with a bowed velocity profile. A return flow region is also expected as described by Nyborg\(^1\). Our transducer and ion beam configuration was designed so that the return flow does not traverse the ion path. As shown in Fig. 1, the width of the four collectors taken together is less than the transducer diameter. The return flow is primarily on either side of the sound beam. The success of this design was confirmed by direct qualitative observation, above the superfluid transition, of the motion of small vapor bubbles within the streaming and return flow regions.

C. Estimate of the Transducer Response

In order to quantitatively test the theoretical description presented in Section I, a quantitative estimate of the transducer response was carried out. A radiation force technique\(^{10}\) was employed. The irradiated intensity, \( I \), from the transducer is given by \( I = \rho c A^2 / 2 \). The radiation force is measured by the deflection (apparent weight change) of a sound absorbing material.
suspended from a microbalance. The PZT4 transducer is one arm of an r.f. capacitance bridge which is brought to null far from the transducer resonant frequency. The voltage across the bridge on resonance is proportional to the motion of the transducer\(^{11}\). We caution the reader that the above procedure was carried out in degassed water at room temperature and it is expected that the transducer response will be different at liquid helium temperatures\(^{12}\). For example, the ratio of the electromechanical coupling constant at 2K to that at 300K is 0.80, the mechanical quality factor increases by 20 percent and the resonant frequency shifts down by 4 percent. Taking these factors into consideration, together with the known values of the acoustic impedances of the two media, our estimation of the transducer response is probably no better than ±20 percent. However, for the purpose of the work presented here, we feel this is sufficient, since absolute accuracy is not the main issue.

Measurement of the heat produced by the transducer was carried out using a calometric technique for the ranges of r.f. voltage and temperature employed. For the data presented in this paper, the thermal power output by the transducer was at most 2 milliwatts and nominally under 1 milliwatt. This assures that the sound source was not producing significant thermal counterflow of the normal fluid component to complicate interpretation of the flow data.

III. Data.

Figure 2 shows data for the ion current measured at collectors labeled 1 and 2 for a range of bridge volts. As the bridge voltage, and hence the sound amplitude, increases, we observe \(I_1\) decrease with a corresponding increase in \(I_2\). The sum of the currents is very nearly constant over the range of sound amplitudes shown. When the ion current measured at collector
3 is small, we may deduce the position of the beam from the ratio $I_2/I_1$. This assures that the beam position determination is not affected by changes in the total current due to electrometer drift, temperature drift, or due to superfluid hydrodynamic complications like trapping of the electron bubbles on vortex lines.

The magnitude of the electric field, $E$, in the experimental cell is just the collector voltage, $V$, divided by the height of the cell, $h$. The average ion velocity in the cell is given by $\mu E$ where $\mu$ is the temperature dependent ion mobility. Because of space charge effects and the presence of the transducer, the field is not strictly uniform. However, the field is only linearly dependent on the distance between the grid and collector and the average value for the ion velocity will still be $\mu E$. The average streaming velocity is just the distance the ions travel in the direction of streaming, $x$, divided by the time of flight from the grid to the collector. Therefore, the average streaming velocity, $v_{n,2}$, can be written as

$$v_{n,2} = \mu E \frac{x}{h}.$$  \hspace{1cm} (4)

The distance $x$ is determined from knowledge of the geometry of the cell and the changes in currents $I_1$ and $I_2$. Using Eq. (4), experimental cell dimensions and the transducer calibration, the data from Fig. 2 is replotted and shown in Fig. 3. At low sound amplitudes the acoustic streaming velocity is quadratic in sound amplitude and the data is quite smooth. At higher sound amplitudes the streaming velocity falls below the original quadratic behavior and shows greater fluctuation. We illustrate these points in Fig. 4 by replotting the streaming velocity data from Fig. 3 versus sound amplitude squared. In Fig. 5 we plot similar data at two other temperatures and the data from Fig. 4. For $T < T_\lambda$, (1.43 K and 1.55 K) there is a well-defined region where $v_{n,2}$ is proportional to $A^2$. There exists, too, a clear break from that behavior at $A = 0.22$ cm/s. For sound amplitudes 0.22 cm/s
\( A \leq 0.4 \text{ cm/s}, \) and after a transition region the streaming velocity, \( v_{n,2} \), is again proportional to \( A^2 \) but with smaller slope than in the low amplitude region. For \( A > 0.6 \text{ cm/s} \) the fluctuation in \( I_1 \) and in \( I_2 \) and \( I_3 \) increasing current \( I_3 \) make further computation of beam deflection difficult. For \( T > T_\lambda \) (2.19 K) the streaming velocity is observed to be proportional to \( A^2 \) but we observe no transitions in the streaming velocity similar to those for \( T < T_\lambda \).

IV. Discussion.

A. Laminar Flow Region

For \( 0 \leq A \leq 0.22 \text{ cm/s} \) we find the normal fluid streaming velocity in front of the transducer is given by

\[
v_{n,2} = B(T) A^2 \tag{5}
\]

where \( B(T) \) is a temperature dependent coefficient. This result, predicted in Eq. (3), extends the observed \( A^2 \) dependence of the acoustic streaming velocity to higher values of \( A \) than reported in other liquids\(^1\). The proportionality shown in Eq. (5) is good evidence that the assumptions of Sec. II B are appropriate.

We also find that the temperature dependence of \( B(T) \) is principally that of the first sound absorption coefficient, \( \alpha \) is given by

\[
\alpha = \frac{\omega^2}{2 \rho \frac{4}{3} \frac{\zeta_2}{\eta} + \frac{\zeta_2}{n}} \tag{6}
\]

where \( \zeta_2 \) is the coefficient of second viscosity. The temperature dependence of the coefficient \( B(T) \) in Eq. (5) was tested by determining the initial slope of streaming velocity for plots of the type shown in Fig. 5 for 5 different temperatures below the superfluid transition. A graph of the quantity \( (4/3 + \zeta_2/\eta) \) as a function of that
initial slope is shown as Fig. 6. The magnitude of $\zeta_2/\eta$ was calculated from values reported in the literature(16). The linear relationship clearly demonstrates that the temperature dependence of $B(T)$ is controlled by the temperature dependence of $\alpha$ via the functional dependence of Eqs. (3) and (6)(13).

Measurement of the acoustic streaming velocity as a function of frequency provides a further test of Eqs. (3) and (6). Results for streaming velocity as a function of the square of the sound amplitude are shown for 1, 3 and 10 MHz in Fig. 7. The data is in reasonable agreement with the theoretical prediction of Eq. (6). Streaming at 3 MHz was complicated by transducer heating but shows the expected functionality at low sound amplitudes (the slope of the 10 MHz curve is approximately 11.1 times larger than that of the 3 MHz curve). Measureable streaming occurs only at higher sound amplitudes at 1 MHz.

B. Transition to the Turbulent Regime

In addition to the functional dependences discussed above for low sound amplitudes, information can be gained about turbulence by examining the data presented in Fig. 5 for intermediate sound amplitudes. One notes that below the superfluid transition and above a critical sound amplitude, the streaming velocity departs from its dependence on the square of the sound amplitude. The value of the sound amplitude at which this departure is observed is identical to the threshold sound amplitude for the onset of turbulence (quantum vortex line production) as evidence by ion trapping(14) and the onset of the subharmonic signal as evidenced by spectral analysis and light diffraction(2,15). These different experiments have all been carried out in our laboratory using the same electronic instrumentation, calibration techniques and dewar system. For this reason, we feel strongly that these three thresholds are indeed identical.
As the sound amplitude is increased further, additional regions of stabilized flow apparently are established. For these regions the streaming velocity is again dependent on the square of the sound amplitude but the slopes are less than those of the laminar state. It is of interest to note that the ratio of the pre- to post-transition slopes of the curves is independent of temperature. For these experiments, this ratio has a mean value of 3.6. As shown in Fig. 5, the slope of the streaming velocity versus amplitude squared decreases with the initial onset of vortex line production and may in fact become negative. From Eqs. (2) and (3) and the coefficient shown in Fig. 6, we can see that if the shear viscosity increases than the streaming velocity will decrease. The shear viscosity coefficient in the turbulent state can be viewed as consisting of two additive components, the normal shear viscosity and an eddy (or vorticity) shear viscosity. The eddy viscosity is evidently large at the threshold for turbulence and then decreases to a lower value when the next stable regime is reached. We can estimate the magnitude of the eddy viscosity by using the ratio of the slopes from Fig. 5 and values for $\zeta_2$ and $n$ from the literature. For a typical temperature value (1.83K) we find that the eddy viscosity in the stable region is about 100 $\mu P$, which is at the high end of the range (10 $\mu P$ to 100 $\mu P$) determined by several different techniques \cite{16}. We believe that this agreement provides corroborative evidence that the onset of production of vortices and therefore, mutual friction between the normal and superfluid components is associated with the initial slope change in Fig. 5.

With further increase in sound amplitude, there apparently exists additional stable regions within the turbulent state. Supplemental evidence for these states was obtained by summing the currents $I_1$ and $I_2$. Results for 1.55 K are shown in Fig. 8. As can be seen, the total current is
relatively constant for low amplitudes. For amplitudes just above the threshold for turbulence, there is a sudden decrease in the total current, presumably the result of ion trapping on the vortex lines associated with the turbulence and a corresponding increase in space charge. For still larger amplitudes, the turbulent state evidently establishes a stable region and the streaming and current stabilize. Further increase in sound amplitude results in a series of unstable and stable regimes with related changes in the total current.

V. Conclusions.

This quantitative study of acoustic streaming in liquid helium demonstrates that the theory developed for acoustic streaming of normal liquids, when combined with the Khalatnikov two-fluid hydrodynamic equations, can be adapted to describe acoustic streaming in the superfluid state. The normal fluid component streams with the functional dependences identical to that of conventional fluids. The temperature dependence of the streaming velocity is determined by the temperature dependence of the ratio of the coefficient of second viscosity to the coefficient of first viscosity. The transition of the fluid flow to a turbulent state and several relatively stable flow patterns within that state have been observed. Measurement of the slopes of the streaming velocity versus the square of the sound amplitude provide a technique to estimate the eddy viscosity of the turbulence in superfluid helium.

ACKNOWLEDGEMENTS

The authors would like to thank Manu J. Tejwani for helpful comments during the preparation of this paper. This work is supported in part by the Air Force Office of Scientific Research, NP 80-151 and the National Science Foundation, DMR 8005358.
FOOTNOTES


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FIGURE LEGENDS

Fig. 1 Schematic view of experimental cell showing array of collectors, grid, ion source and transducer. (a) top view (b) side view. Arrows in top view illustrate flow pattern.

Fig. 2 Ion current at collectors numbered 1 and 2 as a function of bridge voltage (sound velocity amplitude).

Fig. 3 Magnitude of acoustic streaming velocity as a function of sound velocity amplitude. Frequency is 9.87 MHz and temperature is 1.55 K.

Fig. 4 Magnitude of acoustic streaming velocity as a function of the square of sound velocity amplitude. Frequency is 9.87 MHz and temperature is 1.55 K.

Fig. 5 Magnitude of acoustic streaming velocity as a function of square of sound velocity amplitude for three temperatures. Frequency is 9.87 MHz. Data for 2.19 K is for He I, the normal phase. The arrows indicate various flow regimes and correspond to arrows in Fig. 8.

Fig. 6 Plot of the coefficient \((4/3 + \zeta_2/\eta)\) as a function of the slope of the magnitude of the acoustic streaming velocity versus sound velocity amplitude squared below the onset of turbulence.

Fig. 7 Magnitude of acoustic streaming velocity as a function of the square of sound velocity amplitude for 1, 3 and 10 MHz at 1.43 K. The ratio of the slope of the 10 MHz curve to that of 3 MHz curve is approximately 11 as would be expected for the frequency squared dependence predicted by Eqs. (3) and (6).
Fig. 8  Sum of collector currents $I_1$ and $I_2$ as a function of the square of sound velocity amplitude. Arrows correspond to those shown in Fig. 5.
(a)

(b)

FIG. 1
$\frac{4}{3} + \xi_2/\eta$

9.87 MHz

1.43 K

1.55 K

1.63 K

1.74 K

1.83 K

SLOPE S/CM
Figure 7: Graph showing streaming (cm/s) vs. (sound amplitude)^2 (cm/s)^2. The graph includes data for 10 MHz, 3 MHz, and 1 MHz, with the temperature set at 1.43 K.
In a conventional Kapitza resistance experiment involving heat transfer across a copper surface into liquid helium, an acoustic streaming velocity field (at 10 MHz) was directed transverse to the surface normal. Ultrasound had no observable effect on the heat transfer to the superfluid phase (He-II) but in the normal fluid phase (He-I) the thermal conductance increased linearly with acoustic velocity amplitude, reaching a value 2.5 times the zero sound conductance for a sound velocity amplitude of 0.8 centimeters/second.

Introduction

In recent years liquid helium has become a widely used coolant in research and development for applications involving microcircuits to large scale machines\(^1\). Unlike conventional coolants, liquid helium has two very different phases\(^2\). The high temperature phase, called the normal fluid phase (He-I) exists for the temperature range 2.17K<T<4.2K and behaves macroscopically like a conventional classical liquid. Below 2.17K, the low temperature phase, called the superfluid phase (He-II) possesses two remarkable properties which recommend it as a coolant; a vanishingly small viscosity (at least 10^6 times smaller than He-I) and a very large thermal conductance (about 800 times that of room temperature copper). However, studies, beginning with Kapitza\(^3\) in 1941, have shown that a thermal boundary resistance is
associated with heat transfer from a solid into liquid helium. Measured values of this thermal boundary resistance, usually reported as the Kapitza conductance, \( h_K \), range by more than two orders of magnitude from a lower limit given by acoustic mismatch theory\(^{(4)}\) to an upper limit given by phonon radiation theory\(^{(5)}\). Experience has shown that surface type and preparation account for much of this variation and must be properly taken into account\(^{(6)}\).

In this paper we report results of a study of the influence of ultrasound on heat transfer from a copper surface into He-I and He-II. We were motivated to undertake this study to further understand the Kapitza conductance problem and to investigate the possibility of ultrasonic enhancement of heat transfer in liquid helium similar to that which has been observed for more conventional liquids\(^{(7)}\). In addition, if a thin solid layer of helium at the solid-liquid interface is responsible for the Kapitza conductance, then perhaps ultrasound could be used to modify that layer.

Experimental Procedure

A conventional thermal conductivity technique\(^{(8)}\) for measuring Kapitza conductance was employed with the modification that an ultrasonic transducer was added to produce an acoustic streaming velocity field. Specifically, a copper block, with a heater attached, was mounted in a cryostat-dewar system such that a clean smooth surface of the block was in contact with liquid helium. An acoustic streaming velocity field, generated by a 10 MHz PZT-4 thickness mode transducer, was directed transverse to the copper surface normal. The temperature of the block and the liquid helium was measured with several carbon resistance thermometers using standard lock-in amplifier techniques. A constant current supplied to the copper block heater produced heat fluxes from zero to 1.5 milliwatts/centimeter\(^2\). Temperature changes as
small as a quarter of a millikelvin over a temperature range of $1.3K < T < 4.2K$ could be observed. Acoustic amplitudes up to the threshold for collapse cavitation were employed. The ultrasonic transducer formed one arm of a capacitance bridge so that the sound velocity amplitude was directly measured$^9$. Calibration of the transducer was carried out using a radiation pressure technique$^{10}$.

Results and Conclusions

The effect of 10 MHz ultrasound on heat transfer from the copper block to He-I, both maintained near 2.64K, is shown in Fig. 1. The temperature difference between the block and the liquid helium is plotted as a function of heat flux for various sound velocity amplitudes. We observe a decrease in the temperature difference for a given heat flux as the sound amplitude is increased. The Kapitza conductance, defined as the temperature difference per heat flux, is the reciprocal of the slope of the lines shown in Fig. 1 and is plotted as a function of sound amplitude in Fig. 2. For He-I we note that the Kapitza conductance increases linearly with sound amplitude. Gould$^7$ has observed similar results for heat transfer at interfaces using conventional liquids. He has attributed the enhanced transfer to acoustic streaming. Based upon independent studies of acoustic streaming in liquid helium$^{11}$, we feel that streaming is the major mechanism in the present case as well.

In contrast, we find no effect by the ultrasound on heat transfer when the helium is in the superfluid state. Our values for the Kapitza conductance are in good agreement with those of other investigators$^8$, however, they are independent of sound amplitude up to the threshold for collapse cavitation. Acoustic streaming is still operative below the superfluid transition$^{11}$ but apparently has a substantially reduced influence on heat transfer. Furthermore,
we can conclude that if a thin film of solid helium exists at the metal surface, its adherent forces are sufficient to leave it undisturbed by the ultrasound.

Acknowledgement

This work was supported in part by the United States Air Force Office of Scientific Research.

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References


Figure Captions

Fig. 1: Temperature difference between the copper block and liquid helium (He-I) at 2.64K as a function of heat flux for 5 sound velocity amplitudes, \( \omega A \), where \( \bigcirc = 0.00 \) cm/s, \( \square = 0.18 \) cm/s, \( \triangle = 0.36 \) cm/s, \( \Theta = 0.54 \) cm/s, \( \bigcirc = 0.72 \) cm/s and \( \bigtriangleup = 1.08 \) cm/s.

Fig. 2: Thermal conductance at 2.64K as a function of sound velocity amplitude.
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FIG 1

HEAT FLUX (mW/K cm²)

TEMPERATURE DIFFERENCE (MK)

HeI 2.64K
The diagram shows the relationship between sound amplitude and Kapitza conductance for He II and He I. He II at 1.80 K and He I at 2.64 K. The y-axis represents Kapitza conductance in mW/K cm², and the x-axis represents sound amplitude in cm/s. The graph indicates a linear increase in conductance with increasing sound amplitude.