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for performance. The first section discusses the psychological properties
of verbal rules, the information contained in them, and shows, via a computa-
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them operational. The second section reports 5 principles about verbal rules
that were derived from an empirical study of the use of verbal rules in
problem solving. The first principle is that correct problem solving with
verbal rules depends on an interaction between available parsing procedures and available verbal rules. Selection of rules can be affected by difficulty in executing parsing procedures and by verbal labels associated with the rule and the problem domain. Second, parsing difficulties can interfere with successful execution of verbal rules. Third, mental procedures constructed from verbal rules can be structurally different from the instructed verbal rules, and may include knowledge from related domains not included in the instruction. Fourth, verbal rules tend to be used optionally and drop out of performance when a person becomes skilled. The final principle shows that verbal rule use can reappear when appropriate mental procedures are unavailable. A brief closing comment is made about the relation between verbal rules and plans in general.
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ABSTRACT

This paper develops the idea of a verbal rule as a propositional plan for performance. The first section discusses the psychological properties of verbal rules, the information contained in them, and shows, via a computational model, that 4 additional kinds of knowledge are needed to make them operational. The second section reports 5 principles about verbal rules that were derived from an empirical study of the use of verbal rules in problem solving. The first principle is that correct problem solving with verbal rules depends on an interaction between available parsing procedures and available verbal rules. Selection of rules can be affected by difficulty in executing parsing procedures and by verbal labels associated with the rule and the problem domain. Second, parsing difficulties can interfere with successful execution of verbal rules. Third, mental procedures constructed from verbal rules can be structurally different from the instructed verbal rules, and may include knowledge from related domains not included in the instruction. Fourth, verbal rules tend to be used optionally and drop out of performance when a person becomes skilled. The final principle shows that verbal rule use can reappear when appropriate mental procedures are unavailable. A brief closing comment is made about the relation between verbal rules and plans in general.
On the Nature of Verbal Rules and Their Use in Problem Solving

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Constructing and implementing a plan is one approach to solving problems, especially when a prestored solution method is not available. Plans can also be acquired from such sources as another person or a book. These externally acquired or instructed plans can be called verbal rules. This paper develops the idea of a verbal rule and presents some results about the role and use of verbal rules in problem solving, some characteristics of the procedures constructed from verbal rules, and some factors that affect their application.

There are many motor and cognitive skills for which verbal rules can be formulated and used felicitously. For example, a golfer is advised to keep her left elbow straight and keep her eye on the ball. A student of spelling is told, "Put i before e, except after c." A contemporary calculus textbook recommends to its readers that "It is best to learn differentiation formulas ... in words rather than with particular letters such as u and v." For example, the text's verbal rendition for how to differentiate a product is, "The derivative of a product is the first times the derivative of the second, plus the second times the derivative of the first" (Fraleigh, 1980, p. 71).
The major purpose of this paper is to articulate the idea of a verbal rule as a succinct, linguistic plan for performance. Many kinds of verbal statements might arguably be called verbal rules. This paper focuses on verbal rules that describe procedures in an imperative form: Do this to accomplish that. More precisely, the paper is concerned with verbal rules that can be characterized as a description of actions, mental or physical, to be performed in order to accomplish a goal.

The discussion is organized into three main sections. The first section explicates the concept of a verbal rule by presenting a discussion of the psychological properties of verbal rules, the information contained in verbal rules, and the information needed to make verbal rules operational. The second section reports the results of an empirical investigation of the use of verbal rules in problem solving. These results are summarized in five principles that describe factors that affect rule selection, the role of parsing knowledge in the execution of verbal rules, some features of the content and structure of mental procedures that are derived from verbal rules, and two conditions under which verbal rules are typically used or not used. The third section is a general discussion that considers the relation of verbal rules to plans and planning, and summarizes the primary characteristics of the nature and use of verbal rules in problem solving.
Apparently the concept of a verbal rule has not been developed specifically. The analysis reported here contributes to the development of theories about the role of verbal statements in performance and about the process of converting declarative statements into actions (Neves & Anderson, 1981). In particular, it considers how a description of a procedure can be used to facilitate problem solving, and how it can be made operable. The analysis also contributes to a theory about the use of plans in problem solving by identifying a special kind of knowledge, the verbal rule, that can assist the planning process.

I.

Psychological properties of verbal rules. Theoretically, verbal rules have at least four psychological functions. First, they are plans for performance. Like other plans, verbal rules must be implemented with performable mental procedures. In practice, verbal rules tend to be high-level plans; therefore, their verbal statements are incompletely or generally specified. For example, some cookbook recipes contain such instructions as "Mix and bake as usual" or "Cook the sauce until thick" (Sheraton, 1981). Also, they usually contain minimal or no indication of when that plan should be applied—that is, the application conditions are not always given. Second, a verbal rule can serve in a verification process that may be part of implementing the plan contained in that rule into specific mental procedures. A problem solver can evaluate the constructed procedural implementation to see if it is consistent with the declarative
statement of the rule. In other words, the solver can check whether the implemented procedures satisfy or accomplish the specifications of the verbal rule. Third, verbal rules can be stated in a linguistic form, thus they are easy to communicate and memorize. This linguistic form permits the communication of general or abstract principles in a way that is not possible with procedural knowledge which must be communicated via specific examples. Fourth, in addition to being linguistic, verbal rules are usually stated concisely, which contributes as well to their facile communication. The concise form makes a verbal rule easier to memorize and retrieve. By way of example, the rule "i before e, except after c" is abstract in that it does not specify when or where this rule should be used; that it is easy to communicate, memorize and retrieve is probably verified by the reader's experience with this rule.

Information contained in a statement of a verbal rule. It may be useful for the following discussion to distinguish a statement of a verbal rule from a verbal rule. A "statement of a verbal rule" is the string of words that indicates a "verbal rule", which is a propositional representation (i.e., semantic interpretation) of the verbal statement. A statement of a verbal rule must contain descriptions of actions. It may also contain descriptions of application conditions; an implicit temporal sequence for carrying out the individual actions described in the verbal rule; and a marking of conditions from actions by adverbial conjunctions, such as "when", "if", and "unless".
Consider this statement of a verbal rule taken from the domain of signed-number arithmetic: When adding unlike-signed numbers, take the difference and keep the sign of the larger. The first part of this description is the application condition. It specifies when the actions that follow should be used. If application conditions are not stated explicitly, then clearly a person must formulate some. It happens that verbal rule statements for signed-number problems have explicit application conditions. This is necessary, since there is more than one verbal rule in this domain of arithmetic. Not all statements of verbal rules contain explicit application conditions. Notice that the examples given earlier did not provide explicit descriptions of application conditions. However, the context in which a description is given may provide adequate information from which a person could infer a set of conditions that determine when it is (and is not) appropriate to apply that description. Similarly, abstract or partially-specified verbal rules may be supplemented with additional application conditions to be satisfied. In the case of the signed-number rules, a person may specify what conditions constitute "adding" and "unlike-signed numbers".

The second part of this sample signed-number verbal rule is the action description. An action description is a partial specification of the actions that should be performed if the application conditions are satisfied. In this example, the intended interpretation of the instruction, "Take the difference", is that the absolute value of the numbers be used. Further specification of action descriptions into directly performable actions is part of the process of semantic
interpretation. Constructing a semantic interpretation of a verbal statement of a rule involves organizing appropriate knowledge in such a way that it would be adequate for initiating problem solving operations directly from this representation. This representation is comparable to the outcome of the process of understanding written problem instructions in which a representation of a problem space and a set of operators for changing problem states are constructed (Hayes & Simon, 1974; Simon & Hayes, 1976). Further discussion of the structure of this representation is contained in the next subsection.

Creating a semantic interpretation of a verbal rule statement can be facilitated by syntactic properties of the statement. The temporal sequence for applying a verbal rule is given implicitly by the linear order in which it is stated. In the example being discussed, a person must first check for adding unlike signs, then take the difference, then give the sign of the larger. The description of the conditions of a verbal rule are sometimes distinguished from the actions by an adverbial conjunction. In the example, "when" indicates the condition under which the action is applied. Notice also that this indication might contribute to the interpretation of "adding" as a condition to be satisfied and not as an action to perform.

Information needed to elaborate verbal statements of verbal rules into operable procedures. Verbal rules do not specify all the procedures needed for correctly solving a problem. In order to investigate the status of verbal rules in a problem solving system, a computational model was constructed to solve problems using verbal
rules (see Note 1). This model provides a theoretical description of the knowledge needed to make verbal rules operational so that they can solve problems.

As mentioned before, verbal rules tend to be high-level plans, so their action descriptions are stated in general terms. A semantic interpretation of the statement of the verbal rule was needed that interpreted the descriptions into a set of actions that could be performed directly. Actions described in the statement of a verbal rule were taken as goals. These actions were decomposed into a hierarchical tree with directly executable procedures at the terminal nodes. In most cases, directly executable procedures are probably obtained from existing procedural skills that a person had constructed prior to studying a suggested verbal rule. This representation of a verbal rule, which was given to the model, is similar to Sacerdoti's procedural net (Sacerdoti, 1977).

Four kinds of knowledge were needed for the model to use this rule representation to solve problems. First, a set of procedures—which can be called parsing knowledge—were needed to determine the objects in the problem statement referred to in the verbal description. Objects extracted by these parsing procedures were needed by a second kind of knowledge—rule-selection procedures that determined whether a problem satisfied the application conditions of a verbal rule. Parsing procedures also provided arguments required by the third kind of knowledge—procedures that performed the executable actions in the elaborated rule representation. Fourth,
executive control procedures were needed to organize and direct performance. The operations of these four kinds of knowledge correspond in a general way to the operations of the control schema in pattern-directed inference systems: selection, matching, execution, and scheduling (Waterman & Hayes-Roth, 1978, p. 11).

In short, this analysis shows that elaborating a verbal rule into an operable procedure involves adding a considerable amount of knowledge that is implicit in the rule statement. This result reinforces the general point that verbal descriptions are not adequate for immediate application in problem solving operations or skilled performance (Bott, 1979; Brown, Collins, & Harris, 1978; Lewis & Mack, 1982; Neves & Anderson, 1981).

The analysis also shows that rule selection and rule execution both depend upon proper parsing knowledge. To investigate the role of parsing knowledge in performance and to examine the conditions under which verbal rules might be used directly in problem solving, a study was conducted of persons who were taught some verbal rules as a method of problem solving.

II.
This section describes a study of the use of verbal rules in a specific domain that uses them frequently in instruction: addition and subtraction of positive and negative integers. The data collection will be described, followed by a presentation of five principles that summarize characteristics of the use of verbal rules.
in performance and the conditions under which verbal rules are normally used.

Method

Task domain. Thirty-six arithmetic problems—18 addition and 18 subtraction—were used as the problem set. They were constructed by manipulating three structural features: order of magnitude for the two numbers (larger first or second), sign on the first number (+, -, or no sign), and sign on the second number. All combinations were used which resulted in 18 \((2 \times 3 \times 3)\) possibilities. This generation procedure was used for both the addition problems and the subtraction problems. Numbers were selected randomly save for observing the relative magnitude constraint. Examples include: "-37 + 23" and "6 - -18".

Subjects. Five adult students who were enrolled in a remedial arithmetic course at a local community college were interviewed. These students had been instructed, as part of their regular coursework, to solve signed-number problems by applying a set of three verbal rules. Two or three weeks prior to the interviews, the students had taken an in-class exam on this material. If their instruction had been successful, they should have been able to solve signed-number problems competently at the time of the interviews.
Procedure. The students were tested individually in a private room. They were given the problem set typed on two sheets of paper. They were also given scratch paper and told that they were free to use it in their computations. The students were instructed to read aloud each problem before starting to solve, to describe orally both what they were thinking while solving each problem and how they were solving it, and to use the methods they normally use. Students were told to write the answer to each problem beside the typed problem on the sheet. No time limit on solution attempts was imposed. No feedback was given intentionally. Some students were occasionally reminded to think aloud. After solving the problem set, students were asked to describe any rules they might have used to solve these problems. They were also questioned informally about how they were solving particular problems, and about any difficulties they had. The entire interview was tape-recorded and transcribed.

Results

The first two results to be discussed show that parsing knowledge is crucial for proper selection and execution of verbal rules. The third result shows that although students are instructed in the use of verbal rules, they can acquire problem solving procedures that are not structured like the instructed rules. This contrasts with the computational model which always solves the problems using the control structure implicit in its rule representation. The fourth and fifth results to be reported describe conditions on when verbal rules are directly retrieved in the process of problem solving. These results
contrast with the model which always explicitly retrieves the instructions contained in its verbal rules as part of the problem solving process. These results help to clarify the characteristics of the use of verbal rules in performance. Because most of the results do not represent new theoretical principles, the evidence on which they are based will not be discussed in detail; rather, short illustrative examples will be given to convey the general idea of each principle. The evidence for these principles are discussed fully in Chaiklin (1982).

1. Rule selection depends on available parsing procedures and can be aided or hindered by verbal labels.

Prestored plans cannot be applied usefully unless there is some way to retrieve them and decide when their actions are appropriate for the stimulus conditions in the problem at hand. Without some rule selection mechanism, verbal rules would be unavailable for useful application. Stored verbal rules are applied usually once their conditions are satisfied. Excluding guessing, the common method of rule selection is a process that matches elements in a problem statement with a rule's conditions. The elements in the problem are identified by parsing procedures. An example of this pattern-matching method was discussed above in connection with the computer simulation model. To use verbal rules to produce correct answers for a domain of problems, one must have a set of rules that provides adequate solution methods for possible problems in the domain, and a set of parsing procedures to supply data to select these rules. In general there is
not a single correct parsing for a problem; thus, producing correct answers depends on an interaction of available parsing procedures and available rules. If the same objective problem can be parsed in different ways, then alternative rules may be needed for a problem to be solved correctly by a stored verbal rule.

In practice, a person has a limited set of stored rules whose selection can be affected by difficulty in executing relevant parsing procedures, or by parsing a problem contrary to the expectations of the stored rules. To get the gist of this result, consider S2’s stated signed-number rules, which included a rule to add two numbers if their signs are positive and a rule to put a plus sign on the answer if both numbers are positive. Presumably, he would apply these two rules to a problem such as +46 - 12. In fact, he parsed this problem as “positive 46, negative 12”, and applied another rule for problems in which the signs of the two numbers are different. There are plausible interpretations that make “negative 12” a correct parse of the problem. But with respect to S2’s rules, this is an incorrect parse which led him to apply incorrectly one of his stated verbal rules.

Although in most cases a person would probably select rules by pattern-matching, it appears that the selection of verbal rules is not always determined solely by pattern-matching application conditions. Apparently a more general selection process may sometimes be involved as well. This process does not select a specific rule, rather it identifies a particular set of rules that may be adequate for solving
a given problem. For signed-number arithmetic, students try to decide, on some occasions, if the problem is a signed-number problem. This decision affects whether the student will try to apply signed-number rules to that problem. There are protocols to show that even if a student applied successfully a correct set of rules, she can still hesitate from applying these rules to a problem like "6 - 10" because she was not sure that they were signed numbers. Conversely, one student solved a simple addition "47 + 7" in a different way than she probably would have if she did not think of them as signed numbers. The following protocol shows that her method for producing her answer, -54, was guided by her determination that this was a signed number problem. She said, "47 plus 7. These are signed numbers, right? (E: Yes.) OK. The way I was taught to do this, you have here a positive 7 and it's an invisible plus sign next to the 7. So I would say that would be a--. I don't know if you would subtract or add. I would say that when you add two positive numbers it comes out negative."

In the case of textbook physics problems, expert problem solvers can usually identify a problem as solvable by energy equations or force equations, without always knowing simultaneously which specific equation(s) should be used (Chi, Feltovich, & Glaser, 1981). The two examples just presented illustrate a similar situation in which students seemed to evaluate whether signed-number rules were an appropriate general method for a particular problem. These students were not unable to carry out pattern-matching operations needed for rule-selection, rather they hesitated or were reassured by another
process that determined whether signed numbers were involved. The two examples also show an aspect of human performance not captured by the computer model discussed above. A small extension along the lines of a context-checking production should correct this shortcoming.

2. **Parsing difficulties can interfere with successful execution of a selected rule.**

As shown in the last section, parsing knowledge can have a central role in the proper selection of a rule. Parsing knowledge is also important for the proper execution of procedures: it supplies the specific data required by the procedures. If the needed data cannot be determined properly, then a student may have trouble executing a selected rule. Consider the beginning of S1's protocol for "39 - -25". "39 minus negative 25. You change the sign of the number being subtracted. So you change 25 to a positive and you change----- I just hope that I'm changing the right sign." S1, who used the instructed rules correctly, had selected a rule to apply, but was unsure that action specifications were applied to the proper symbols. I think the cause of this difficulty was lack of adequate parsing knowledge. This was the first problem of the form X - -Y that S1 had seen in the interview, thus she was not sure that she was changing the sign specified in the action prescription. It seems unlikely that she had forgotten the rule, because on the previous problem (10 - +49) she recalled the entire subtraction rule and explicitly applied it step-by-step.
The results in this and the previous section reinforce the idea that successful application of verbal rules depends on more than being able to recite the rules from memory. A person must be able to accurately parse problems if (s)he is to apply stored verbal rules consistently. Parsing knowledge enables a person to determine if a problem has the features or conditions that are needed to apply a rule (or procedure), and to provide the data, e.g., signs of numbers, needed to apply the selected procedures.

3. Mental procedures derived from instructed verbal rules can have a different structure than the procedures that implement these rules and can include prior knowledge not presented with the verbal rule.

A verbal rule must be elaborated into procedures in order to make its plan psychologically operable. Unsuccessful attempts at elaborating verbal rules can result in incorrect procedures which if applied consistently can be called ill-composed. Ill-composed mental procedures can produce systematically flawed performance, similar to the manner described by Brown and Burton's (1978) BUGGY system for elementary arithmetic. Comparison of the structure of the instructed verbal rules with the structure of the students's mental procedures reveals that students can acquire problem-solving procedures structured differently from the procedures that implement the verbal rules they were taught. Thus, students need not acquire the structure of instructed verbal rules in order to create effective procedures. An example of this result can be demonstrated with the instructed
verbal rule mentioned above. This rule was stated as: When adding unlike-signed numbers, take the difference and keep the sign of the larger. The structure of this rule can be described as "condition 1, condition 2 —> action 1, action 2", where the conditions are "adding" and "unlike-signed numbers", and the actions are "take the difference" and "keep the sign of the larger". The entire rule is self-contained—both the conditions must be satisfied simultaneously, and then both actions are performed without any intervening processing. On the other hand, one student apparently used two independent rules for adding two numbers with unlike signs. The structure of his two rules are "condition 1 —> action 1", and "condition 2 —> action 2". Evidence from his performance and his protocols is used to argue for this alternative organization for his procedures.

Other declarative or procedural knowledge not included in the instruction of verbal rules may be used to interpret or create procedures from verbal rules. This knowledge can contribute to a student’s construction of structurally different problem-solving procedures. For example, one student used a set of incorrect rules, identical to the sign assignment rules for signed multiplication and division (SMD), to give the sign of addition and subtraction answers. The student learned the four signed operations at the same time. It appears that he overgeneralized the application of the SMD sign assignment operations and included it as part of his addition and subtraction procedures.
The process of converting verbal rules into problem-solving procedures is a special case of the more general problem of converting declarative knowledge into procedural knowledge. One process by which a person might learn a procedure from a verbal rule is to translate and apply each instruction step by step. Neves & Anderson's (1981) proceduralization mechanism is a plausible account of such a process. This automatic, practice-driven mechanism transforms interpretative application of a declarative description of a procedure into smooth, compiled performance.

The obtained results show, unsurprisingly, that explicit instruction in the use of verbal rules does not prevent persons from constructing ill-composed procedures. In particular, these procedures may incorporate additional knowledge, and may not be structured like the rule that was taught. These results suggest that, additional learning mechanisms, that may or may not use verbal rules in a central way, will be needed to describe processes by which verbal rules might be used to develop operable procedures.

The three results just discussed considered requirements needed for successful application of verbal rules, and the consequences of trying to convert verbal rules into operable procedures. The next two results consider the conditions under which verbal rules would be recalled and used explicitly in problem solving performance.
4. Explicit recall and use of verbal rules in problem solving performance is optional and tends to disappear with practice.

While performance sometimes includes the use of rules in a significant way, it often does not. With sufficient practice, cognitive skills can apparently be organized in such a way that a person can omit explicit recall and interpretation of verbal rules in their problem-solving performance (e.g., Fitts, 1964; Neves & Anderson, 1981).

For example, on the first unlike-signs addition in the problem set, a student explicitly recalled and used a verbal rule. The problem was $30 - 29$. The student's protocol was:

OK. Thirty plus a negative 29. OK. Now what I do, the first thing I do is take the largest absolute value of these numbers which would be 30 and subtract the negative 29 and keep the sign of the larger absolute value. So the answer would be 1. It would be positive 1. Because the larger absolute value is positive.

On the sixth instance of an unlike-signs addition problem in the problem set, there was no evidence that verbal rules were used in a central way as in the first instance. One can also see that a number of explicit steps have dropped out of her performance. The problem was $-35 + 39$, and the protocol was: "Negative 35 plus positive 39. That's subtracting, 4. That would be positive 4."

The protocols that were collected confirm the well-established empirical and theoretical result that performance becomes abbreviated with practice (Fitts, 1964; Gal’perin, 1969; Lewis, 1978; Neves &
Anderson, 1981; Rumelhart & Norman, 1978), and show that this general point holds as well for performance of procedural skills derived from verbal rules. This result is related to the next result which describes another condition in which verbal rules tend to be used.

5. **Explicit recall and use of verbal rules tends to occur when directly applicable mental procedures are inadequate.**

Students are sometimes faced with problems to solve that they have not encountered recently. If a directly applicable mental procedure is not readily available for solution, then there is a tendency to attempt to retrieve and apply interpretatively an appropriate verbal rule. This result is an instance of Ryle's (1946) claim that "people who appeal much to principles show they don't know how to act."

The protocol for $30 + -29$ in the previous section is an instance of this result. This was the first occurrence of a problem of this form; thus the student recalled a verbal rule as a plan for action. Explicit recalls were not limited to the early part of the problem set: some explicit recalls occurred even on the 25th problem of this set if it was a new form or if such a form had not been solved recently. This result also supports the idea that the preceding result is not a consequence of the student becoming familiar with the interview procedure, thereby saying less as the interview proceeded.
These last two results suggest that verbal rules tend to be used by persons who are not practiced or competent at a skill. However, these results also show that verbal rules can play an important role in the development of efficient mental procedures. If a procedure is not immediately available for solving a problem, then a person might attempt some general problem solving such as means–ends analysis, or a person could use a verbal rule to aid in (re)constructing an effective procedure. This method of plan generation may involve nothing more than memory retrieval. In using memorized plans to solve a problem, a person must decide which recalled plan to use and possibly how to implement it, but surely this is easier than constructing and implementing a plan from scratch. In fact, the students who were interviewed tended to attempt to recall verbal rules rather than use general methods (e.g., semantic analysis) to solve problems. This use of verbal rules as a problem solving method is particularly feasible because verbal rules can be memorized and retrieved verbally. This adds a new and useful point to Sacerdoti’s (1977) planning formalism—a verbal rule provides a guide to action that is stored in a relatively accessible verbal form.

III.

A plan is a combination of procedures that attempt, if applied, to achieve a goal state from an initial state. A verbal rule can be viewed as a concise verbal description of a plan that results from a planning process. Verbal rules are different from plans that a person usually constructs in two ways. First, verbal rules eliminate the
need for much of the initial stages of planning because the verbal rule is usually acquired by instruction rather than construction. Second, when a novice is first learning to solve problems, an instructed verbal rule may be a more effective plan for correctly solving a problem than the novice's self-generated plans. Verbal rules are similar to self-generated plans in that they must be elaborated into specific procedures that implement the plan, and there must be adequate selection procedures so that these stored plans can be retrieved and applied in appropriate situations. As the empirical results reported above show, successful use of the procedures derived from verbal rules requires some mechanism to select and apply these rules.

It seems likely that verbal rules have been developed as a general problem solving method because they provide an efficient method for coding a plan into a concise and easily retrievable form. The analysis in this paper has identified limiting conditions on the effectiveness of verbal rules in problem solving and problems that can arise in making them operable. Five main points were presented. Students must add a non-trivial amount of knowledge to make the rules effective. Adequate parsing knowledge is needed if the rules are to be effective. Providing verbal rules as a technique for solving problems does not guarantee that those rules will be used in a central way, nor that they will be learned the way they were taught. Difficulties can arise from attempting unsuccessfully to elaborate the verbal rules into operable procedures. They may, however, be useful when a person's normal procedures are not available, or as a "training
wheel" in the process of developing efficient mental procedures.

Although the analysis in this paper was developed in the context of signed-number arithmetic, these conclusions are potentially applicable to verbal rules in other domains as well. The verbal rule as a theoretical concept provides a way to analyze the role of descriptions of procedures in learning and performance, and provides an analysis of a significant performance mechanism.
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Note 1. The model was written in the ACTP production system language (Anderson, 1976; Greeno, 1978).
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