SOME MATHEMATICAL METHODS FOR MODELING THE PERFORMANCE OF A DISTRIBUTED DATA BASE SYSTEM

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Foreword

This paper was presented to the International Working Conference on Model Realism, which was held at Bad Honnef, West Germany, April 20-23, 1982.

The conference dealt with the modeling aspects of different systems theories and approaches. To get a common basis for discussion, three problem studies were introduced. This paper was a contribution to problem study 2: Reorganization in a Socio-Technical System, which postulated a distributed data base system with communication facilities for data transfer.
SOME MATHEMATICAL METHODS FOR MODELING THE PERFORMANCE OF A DISTRIBUTED DATA BASE SYSTEM

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This paper presents some mathematical methods for evaluating the performance of a distributed data base system (DDBS). Performance is measured by the speed and accuracy with which data are transmitted from one location to another. Five techniques are described: the Data Flow Model, a semi-Markov model for determining the spatial and temporal distribution of data that are to be transferred from one location to another; Optimal Sample Size Estimation, a method for determining the amount of data to be collected for input to the Data Flow Model; Confidence Interval Estimation, a method for estimating confidence intervals for the outputs of the Data Flow Model; Sensitivity Estimation, a method for estimating the sensitivity of DDBS performance to changes in the parameters of the Data Flow Model; and the Aggregation of Stratified Semi-Markov Processes, a method for combining semi-Markov Data Flow Models developed for subsystems (e.g., geographic regions) into a single model that is representative of the entire DDBS.

INTRODUCTION

An important consideration in establishing a distributed (decentralized) data base system (such as described in problem study 2: Reorganization in a Socio-Technical System) is a method for evaluating the performance of the system after it has been implemented. Distributed Data Base Systems (DDBS) often do not perform as planned: data transfers take too long, mistakes are made in updates, or data bases contain errors.

In this paper we describe some mathematical methods for modeling the performance of a distributed data base system. These methods were developed and used in an evaluation of the U.S. Marine Corps' Joint Uniform Military Pay System/Manpower Management System (JUMPS/MMS) [1].

The particular aspects of performance we wish to evaluate are the speed and accuracy with which data are transmitted from one location to another. To do this we must measure the propagation of errors in the DDBS and the delays and losses of data when transmitted from one location to another. Specific questions that we want to answer in the evaluation are these:

1. What percentage of data is transmitted from one location (source) to another (destination) without loss or error?
2. How long do these transmissions take?
3. What happens to data that are not successfully transmitted from source to destination?
4. How much will the percentage of successful transmissions increase if a decrease is made in a particular loss/error rate?

5. How much will the total transmission time decrease if a decrease is made in the delay at a particular location?

6. What degree of confidence do we have in our results?

For the purposes of our evaluation we assume that the DDBS can be modeled as a stochastic network in which the data flow from one location to another. More specifically, we assume that, by an appropriate definition of states, the flow of data can be modeled as a semi-Markov process with two types of states: (1) transient states, which are occupied temporarily by data before they move to another state, and (2) absorbing states, which are terminal points for data, since once reached they are never left (at least for the purposes of our evaluation). The absorbing states of the model are the intended destination of the data and the "data error or data loss" states associated with the transient states of the process.

To illustrate these ideas before proceeding, consider the simple state/flow diagram in figure 1. The dark lines in this illustrative example indicate what we call the primary flow, which is the path that data are supposed to take from source (state $S_1$) to destination (state $S_7$) through intermediate state $S_2$. The light lines indicate the secondary flow, which are paths taken by data that are either misdirected, erroneous, or lost.

![State Transition Diagram](image-url)
As indicated in the diagram, we assume that errors, losses, and delays can occur at each transient state of the process. Data that are erroneous or lost are modeled as transitioning to an absorbing state, such as $S_4$, $S_5$, or $S_6$. Only error-free data move from one transient state to another, or to the destination.

In the remainder of this paper we describe the following analytical methods for evaluating DDBS performance:

- **Data Flow Model**, a semi-Markov model consisting of $q$ transient states and $r$ absorbing states. Estimates of the transition probabilities and times are assumed to be based on sample data. The outputs of the model are estimates of (1) the distribution of source data among the absorbing states and (2) the time between source and destination.

- **Confidence Interval Estimation**, a method for estimating confidence intervals for the outputs of the Data Flow Model, whose inputs are estimates, based on sample data, of the transition probabilities and times.

- **Optimal Sample Size Estimation**, a method for estimating the amount of data to be collected at each transient state so as to obtain a given level of confidence in the output of the Data Flow Model at minimum cost of data collection.


- **Aggregation of Stratified Semi-Markov Processes**, a method for combining semi-Markov Data Flow Models developed for subsystems (e.g., geographic regions) into a single model that is representative of the entire system.

**DATA FLOW MODEL**

The flow of data in the DDBS is modeled as a finite, stationary, absorbing, semi-Markov process. The inputs to the model are the transition probabilities $p_{ij}$ from state $S_i$ to state $S_j$ and the delay time $t_i$ at each transient state $S_i$. The outputs of the model are (1) the probability that an item of data reaches its intended destination; (2) the probability that an item is erroneously changed or lost at each transient state; and (3) the mean time and the distribution of time for an item of data to reach its intended destination.
Let \( P = (p_{ij}) \) be the matrix of true (but unknown) transition probabilities. \( P \) can be partitioned into submatrices

\[
P = \begin{bmatrix}
Q & R \\
0 & I
\end{bmatrix}
\]

where the submatrix \( Q \) contains the transition probabilities between transient states, the submatrix \( R \) contains the transition probabilities from transient states to absorbing states, the zero submatrix \( 0 \) contains the transition probabilities from absorbing to transient states, and the identity submatrix \( I \) contains the transition probabilities between absorbing states.

The expected number \( h_{ij} \) of times the process is in transient state \( S_j \), given that it originated in transient state \( S_i \), is [2]

\[
(h_{ij}) = B = (I - Q)^{-1}
\]

The probability \( b_{ij} \) that the process terminates in absorbing state \( S_j \), given that it was initially in state \( S_i \), is [2]

\[
(b_{ij}) = B = (I - Q)^{-1}R = HR
\]

The matrix \( B \) provides the answers to questions 1 and 3 of our evaluation objectives. From it we can determine the percentage of data that reaches the destination without loss or error and the percentage that is lost or erroneously changed at each transient state of the process.

If the process terminates at absorbing state \( S_k \), the expected number of times that the process was in transient state \( S_j \), given that it originated in transient state \( S_i \), is [2]

\[
(m_{ij}) = M = D^{-1}HD
\]
where $D$ is a diagonal matrix whose diagonal elements are the probability $b_{jk}$ that a process reaches absorbing state $S_k$ given that it started in transient state $S_j$.

If the process terminates at absorbing state $S_k$, the expected time between arrival at transient state $S_j$ and absorption in state $S_k$ is

$$(v_j) = V = MT$$  \hspace{1cm} (1)$$

where $T = (t_{ij})$ is the column vector whose components are the expected times the process is in each transient state $S_j$.

The vector $V$ provides part of the answer to question 2 of our evaluation objectives. From it, we can determine $v_1$, the expected time from source (state $S_1$) to destination by designating absorbing state $S_k$ as the destination. The distribution of time from source to destination is obtained by simulation rather than by analytic methods, unless the individual delay times are exponentially distributed.

**CONFIDENCE INTERVALS FOR $\hat{B}$**

We assume that, in practice, the inputs to the Data Flow Model (the transition probabilities and delay times) must be estimated from sample data and, therefore, are known only with a certain degree of accuracy. Consequently, the outputs of the Data Flow Model ($B$ and $V$) are likewise known only with a certain degree of accuracy.

In this section, we present a method for estimating confidence intervals for the matrix $\hat{B}$ (an estimator for $B$) when the transition probabilities are based on sample data. Similar methods, described in [1], can be used for estimating confidence intervals for $\hat{V}$, an estimator for $V$.

**The Variance of $\hat{B}$**

Let $N_1$ be the number of observations at transient state $S_1$ and let the random variable $n_{1j}$ be the number of observations at $S_1$ having
outcome $S_j$. Then an unbiased, minimum variance, maximum likelihood estimator for $p_{ij}$ is $\hat{p}_{ij} = n_{ij}/N_i$.

We then estimate the matrix $B$ by $\hat{B} = (I - \hat{Q})^{-1}\hat{R}$, where $\hat{Q}$ and $\hat{R}$ are submatrices of $\hat{P}$:

$$\hat{B} = (\hat{B}_{ij}) = \begin{bmatrix} \hat{Q} & \hat{R} \\ \hat{0} & \hat{I} \end{bmatrix}.$$ 

Since $\hat{B}$ is a function of the $\hat{p}_{ij}$, we can expand $\hat{B}$ in a Taylor series about $(p_{ij})$. The $p_{ij}$ are not all independent variables, however, since $\sum_i p_{ij} = 1$ for all $i$. If, for each $i$, we arbitrarily let $\hat{p}_{id} \in \hat{R}$ be the dependent variable such that $\hat{p}_{id} = 1 - \sum_{j \neq d} \hat{p}_{ij}$, then the $\hat{p}_{ij}$ for $j \neq d$ are independent variables. Hence $\hat{B}$ can be written

$$\hat{B} = B + \sum_i \sum_{j \neq d} \frac{\partial \hat{B}}{\partial \hat{p}_{ij}} (\hat{p}_{ij} - p_{ij}) + \text{higher order terms}$$

where $\frac{\partial \hat{B}}{\partial \hat{p}_{ij}}$ is understood to be evaluated at $\hat{p}_{ij} = p_{ij}$, $i$ is an element of the index set of transient states and $j$ is an element of the index set of transient and absorbing states.\(^*\)

If we ignore the higher order terms we have

$$\hat{B} = B + \sum_i \sum_{j \neq d} \frac{\partial \hat{B}}{\partial \hat{p}_{ij}} (\hat{p}_{ij} - p_{ij})$$

and

$$\text{var}(\hat{B}) = \text{E}[(\hat{B} - B)^2] = \text{E} \left( \sum_i \sum_{j \neq d} \frac{\partial \hat{B}}{\partial \hat{p}_{ij}} (\hat{p}_{ij} - p_{ij}) \right)^2$$

As we shall see later, $\frac{\partial \hat{B}}{\partial \hat{p}_{id}} \equiv 0$, hence we can actually sum over all $j$ without difficulty.

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\(^*\) As we shall see later, $\frac{\partial \hat{B}}{\partial \hat{p}_{id}} \equiv 0$, hence we can actually sum over all $j$ without difficulty.
where $(\hat{B} - B)^{(2)} = (\hat{B} - B) \ast (\hat{B} - B)$ denotes the congruent product* of $(\hat{B} - B)$ with itself.

Expanding (2) yields

$$\text{var}(\hat{B}) = \sum_{i} \sum_{j} \sum_{m} \sum_{k} \frac{\partial \hat{B}}{\partial \hat{P}_{ij}} \cdot \frac{\partial \hat{B}}{\partial \hat{P}_{mk}} E \left[ (\hat{P}_{ij} - P_{ij})(\hat{P}_{mk} - P_{mk}) \right]$$

$$= \sum_{i} \sum_{j} \sum_{m} \sum_{k} \frac{\partial \hat{B}}{\partial \hat{P}_{ij}} \cdot \frac{\partial \hat{B}}{\partial \hat{P}_{mk}} \text{cov}(\hat{P}_{ij}, \hat{P}_{mk})$$

$$= \sum_{i} \sum_{j} \sum_{k} \frac{\partial \hat{B}}{\partial \hat{P}_{ij}} \cdot \frac{\partial \hat{B}}{\partial \hat{P}_{ik}} \frac{P_{ij}(\delta_{jk} - P_{ik})}{N_i}$$

where $\delta_{jk}$ is the Kronecker delta and $N_i$ is the sample size at state $S_i$.

For convenience, let $a_{ij} = \frac{\partial \hat{B}}{\partial \hat{P}_{ij}}$ and let $A_i$ be the vector of matrices $A_i = (a_{ij}) = (a_{i1}, \ldots, a_{in})$. Also, let $P_i$ denote the $i$th row of $P$.

Then var$(\hat{B})$ can be written in matrix notation as

$$\text{var}(\hat{B}) = \sum_{i} \frac{1}{N_i} \left[ A_i^{(2)} P_i^T - (A_i P_i^T)^{(2)} \right]$$

where $P_i^T$ is the transpose of $P_i$, $A_i P_i^T = \sum_{j} a_{ij} P_{ij}$, and $A_i^{(2)} = (a_{ij}^{(2)})$.

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* The congruent product is defined in general for a matrix $A = (a_{ij})$ as $A \ast A = A^{(2)} = (a_{ij}^2)$. Thus congruent matrix multiplication is the multiplication of corresponding elements. More generally, the congruent product of two commensurate matrices $A = (a_{ij})$ and $b = (b_{ij})$ is defined by $A \ast b = (a_{ij} b_{ij})$ for all $i$ and $j$. 

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The Derivative of $\hat{B}$

The expression for var($\hat{B}$) contains the derivative of $\hat{B}$, which is

$$\frac{\partial \hat{B}}{\partial \hat{B}_{ij}} = \hat{B} \left[ \frac{\partial ^2 \hat{B}}{\partial \hat{B}_{ij}^2} + \frac{\partial \hat{B}}{\partial \hat{B}_{ij}} \right].$$

By definition, $\hat{Q} = \hat{P}_{rc}$ when $r$ and $c$ are both elements of the index set of transient states and $\hat{R} = \hat{P}_{rc}$ when $r$ is an element of the index set of transient states but $c$ is an element of the index set of absorbing states.

Therefore,

$$\frac{\partial \hat{B}}{\partial \hat{B}_{ij}} = \begin{cases} 0, & \hat{P}_{ij} \in \hat{R} \\ (\delta_{ri} \delta_{cj}), & \hat{P}_{ij} \in \hat{Q} \end{cases}$$

$$\frac{\partial \hat{B}}{\partial \hat{B}_{ij}} = \begin{cases} (\delta_{ri} (\delta_{cj} - \delta_{cd})), & \hat{P}_{ij} \in \hat{R} \\ 0, & \hat{P}_{ij} \in \hat{Q} \end{cases}.$$  \hspace{1cm} (4)

Hence

$$\frac{\partial \hat{B}}{\partial \hat{B}_{ij}} = \begin{cases} (\hat{P}_{ri} \delta_{cj}) - (\hat{P}_{ri} \delta_{cd}), & \hat{P}_{ij} \in \hat{R} \\ (\hat{P}_{rj} \delta_{jc}), & \hat{P}_{ij} \in \hat{Q} \end{cases} .$$

As noted previously, when $\hat{P}_{ij} \in \hat{R}$ and $j = d$, $\partial \hat{B} / \partial \hat{B}_{ij} \equiv 0$.

Confidence Intervals

To estimate confidence intervals for $\hat{B}$ we assume that $\hat{B}$ is normally distributed with mean $\hat{B}$ and variance var($\hat{B}$). Then an approximate $1 - \alpha$ confidence interval for $\hat{B}$ is

$$(\hat{B}^* - Z_{\alpha/2}(\text{var}(\hat{B}))^{1/2}, \hat{B}^* + Z_{\alpha/2}(\text{var}(\hat{B}))^{1/2}) ,$$

where $\hat{B}^*$ and var*(\hat{B}) are the sample values of $\hat{B}$ and var($\hat{B}$),
\((\text{var}(\hat{B}))^{(1/2)})\) is the matrix such that \(((\text{var}(\hat{B}))^{(1/2)})^{(2)} = \text{var}(\hat{B})\),

and \(\int_{-\infty}^{\infty} \frac{1}{\sqrt{2\pi}} e^{-x^2/2} \, dx = \alpha/2\).

OPTIMAL SAMPLE SIZES

Now that we have an estimate of a confidence interval for \(\hat{B}\), we can estimate the sample sizes at each transient state \(S_i\) that will produce a confidence interval for a particular \(b_{ij}\) at minimum cost.

Let \(W_{ij}\) be the desired width of the confidence interval for \(b_{ij}\) and let

\[
G_{ij} = e_i^T \left[ A_1^{(2)} P_1^T - (A_1 P_1^T)^{(2)} \right] e_j
\]

where \(e_i\) and \(e_j\) are the unit column vectors \(e_i = (\delta_{ij})\) and \(e_j = (\delta_{jj})\).

Then we can write \(W_{ij}\) as

\[
W_{ij} = 2\sqrt{\alpha/2} \left[ \sum_i \frac{G_{ij}}{N_i} \right]^{1/2}
\]

Let \(C_i(N_i)\) be the cost of collecting a sample of size \(N_i\) at transient state \(S_i\). We assume \(C_i\) has the form \(C_i(N_i) = \beta_i + \gamma_i N_i\).

We wish to determine the values of the \(N_i\) such that the total cost of data collection will be minimized and the desired confidence interval width will be obtained. Thus, our problem is to

\[
\text{minimize} \sum_i C_i(N_i) = \sum_i (\beta_i + \gamma_i N_i) \\
\text{subject to} \quad W_{ij} = 2\sqrt{\alpha/2} \left[ \sum_i \frac{G_{ij}}{N_i} \right]^{1/2}
\]

\[\text{minimize} \sum_i C_i(N_i) = \sum_i (\beta_i + \gamma_i N_i) \quad (5)\]
The solution to this constrained optimization problem (using Lagrange multipliers) yields \( N^*_1 \), the optimal value of \( N_1 \):

\[
N^*_1 = \left( \frac{2\pi c/2}{w_{11}} \right)^2 \left( \frac{G_{11}}{\gamma_1} \sum_j G_{1j} \gamma_j \right)^{1/2}
\]

\textbf{SENSITIVITY ANALYSIS}

Two of our evaluation objectives are to determine the sensitivity of the percentage of successful transmissions to changes in individual error/loss rates (question 4) and the sensitivity of the total transmission time to changes in the individual sojourn times (question 5).

If \( y = f(x_1, \ldots, x_n) \), then the sensitivity of \( y \) to \( x_1 \) is \( \frac{\partial y}{\partial x_1} \). The sensitivity of \( y \) to \( x_1 \) is the percentage change in \( y \) that results from a 1 percent change in \( x_1 \).

\textbf{Sensitivity to Errors and Losses}

In our evaluation we want to determine the sensitivity of \( b_{1k} \) (the probability of successful transmission from source (state \( S_1 \)) to destination (state \( S_k \)) to some error/loss transition probability \( p_{1j} \in \mathbb{R} \).

Using the expression for the derivative of \( B \) (equation (4)), we have

\[
\Gamma_{b_{1k}p_{1j}} = -h_{11} \left( \frac{p_{1j}}{b_{1k}} \right)
\]

Thus the sensitivity is directly proportional not only to \( p_{1j} \), but also to \( h_{11} \), the expected number of times the process is in state \( S_1 \), given that it began in state \( S_1 \). The negative sign indicates that an increase in either \( h_{11} \) or \( p_{1j} \) results in a decrease in \( b_{1k} \).
Sensitivity to Delays

We also want to determine the sensitivity of \( v_1 \) (the expected time from source (state \( S_1 \)) to destination (state \( S_k \)) to the expected delay time \( t_j \) at transient state \( S_j \).

From equation (1), \( V = Mt \), so \( \partial v_1 / \partial t_j = m_{1j} \).

Therefore \( \Gamma_{v_1} t_j = m_{1j} \left( \frac{t_j}{v_1} \right) \).

AGGREGATION OF STRATIFIED SEMI-MARKOV PROCESSES

When modeling the performance of a DDBS, it might be necessary or desirable to stratify the system into subsystems and model the performance of each of these individual subsystems. For example, in our DDBS evaluation, the flow of data might be modeled for separate geographic regions (subsystems) to determine their performance. The Data Flow Model for each region would have the same state space, but the input data to the model might be different for each region because of differences in performance. When the above procedure is used, the resulting semi-Markov process of each subsystem becomes a conditional semi-Markov process in that its transition probabilities and times are relative to its associated subsystem.

In this section we derive the method for aggregating these separate conditional processes (each of which has the same state space) into a single (unconditional) process that is representative of the total system and has the same state space as the conditional processes.

Illustrative Example

Figure 2 illustrates a simple four-state process \( Y \) which has been stratified into two subprocess \( Y_1 \) and \( Y_2 \).

Suppose data has been collected for each subprocess and transition probabilities have been estimated as shown in the matrices of transition probabilities \( P_1 \) and \( P_2 \). Further, suppose we know the fraction of time
FIG. 2: FOUR-STATE PROCESS WITH TWO SUBPROCESSES
(i.e., the probability) that the process originates in each subprocess, say $f_1$ and $f_2$, and also know that the process always begins in state $S_1$. Given this information, how do we determine $P$? This example illustrates the general problem addressed in this section. As we shall see later, what might be considered as two "obvious" methods of determining $P$ do not, in general, work: (1) aggregating the data from the subprocesses and (2) defining $P = f_1P_1 + f_2P_2$.

**Derivation of $P$**

Suppose the process $Y$ under study can be stratified into $m$ subprocesses $Y_k$, $k \in \{1, 2, \ldots, m\}$, each with the same state space. We assume that $Y$ and all the $Y_k$ are being modeled as an irreducible absorbing semi-Markov process with $q$ transient states and $r$ absorbing states. We also assume that the matrix $P_k$ of transition probabilities for subprocess $Y_k$ can be partitioned into submatrices $Q_k$, $R_k$, $0$, and $I$ in the same manner that $P$ has been partitioned earlier in the paper.

As shown earlier, the probability that $Y_k$ is absorbed in $S_j$, given that the process began in transient state $S_i$, is

$$B_k = (b_{1jk}) = (I - Q_k)^{-1}R_k = H_kR_k.$$  

We assume, without loss of generality, that $Y_k$ always begins in a particular transient state, say $S_1$. Then the probability that $Y_k$ terminates in $S_j$ is $b_{1k} = e_{1k}^T = e_1^T H_k R_k$, where $e_1$ is the unit column vector $e_1 = (\delta_{11})$.

If we let $f_k$ be the probability that the process originates in $Y_k$ ($\sum f_k = 1$), then the probability that the process terminates in $S_j$ is

$$b_1 = \sum_k f_k b_{1k} = e_1^T \sum_k f_k (I - Q_k)^{-1}R_k.$$

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Our problem is to determine the stochastic matrix \( P \) of order \( q + r \) such that

\[
P = \begin{bmatrix} Q & R \\ 0 & I \end{bmatrix}
\]

and 
\[
e_1^T (I - Q)^{-1} R = e_1^T \sum_k f_k (I - Q_k)^{-1} R_k.
\]

To obtain an expression for \( P \), let

\[
H_{lk}^d = \text{a diagonal matrix of order } q \text{ whose diagonal elements are from the first row of } H_k,
\]

\[
A_k = \begin{bmatrix} f_k A_k^d & 0 \\ 0 & I \end{bmatrix} = \text{a diagonal matrix of order } q + r.
\]

Then it is easily shown [1] that

\[
P = \left( \sum_k f_k A_k \right)^{-1} \sum_k f_k A_k P_k,
\]

\[
Q = \left( \sum_k f_k H_{lk}^d \right)^{-1} \sum_k f_k H_{lk}^d Q_k,
\]

and

\[
R = \left( \sum_k f_k H_{lk}^d \right)^{-1} \sum_k f_k H_{lk}^d R_k.
\]

PRACTICAL EXPERIENCE WITH THE MODELS

The Data Flow Model described in this paper has been used in three separate evaluations of the performance of distributed data base systems ([1], [3], and [4]). The methods for estimating confidence intervals, for estimating sensitivity, and for aggregating stratified semi-Markov processes were also used in [1]. The system under study in [1] was stratified into five subsystems by geographic location. The Data Flow Model for each subsystem comprised 57 transient states and 32 absorbing states. Data for estimating transition probabilities and times were collected by a combination of automated and manual methods. Sample sizes were determined heuristically rather than by the method described...
in this paper, which was not developed until after the completion of [1].

CONCLUSION

This paper has presented a framework and a set of mathematical methods for modeling the performance of a distributed data base system, where performance is measured by the speed and accuracy with which data are transmitted from one location to another. A semi-Markov Data Flow Model was described that provides estimates of the temporal and spatial distribution of data that are to be transmitted from a source to a destination. Methods were presented for estimating confidence intervals for model outputs and for estimating optimal sample sizes for model inputs. Techniques were derived for determining the sensitivity of model outputs to model parameters. A method was presented for aggregating a set of stratified semi-Markov Data Flow Models into a single model.

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