BASIC CONCEPTS OF RADAR POLARIMETRY
AND ITS APPLICATIONS TO TARGET DISCRIMINATION,
CLASSIFICATION, IMAGING AND IDENTIFICATION

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Polarization, radar polarimetry, target discrimination, classification, imaging, optimal polarization nulls, polarization transformation matrix, Mueller matrix, radar scattering matrix, polarization fork.

In meticulous detail, a succinct summary of basic electromagnetic wave polarization descriptors, of the various scatterer polarization transformation and of the scatterer descriptive operators is introduced. It is then shown how the five (5) independent matrix parameter for the relative phase monostatic scattering matrix describing an isolated, yet regionally distributed, target in a reciprocal propagation medium can be recovered from (i) amplitude-only, (ii) mixed amplitude plus partial phase, (iii) complete two-step (Cont'd)
20. ABSTRACT

Amplitude-phase measurements. Basic properties of the radar target scattering matrix for linear (H, V) and circular (R, L) polarization basis are described in terms of geometrical target features as functions of the specular point surface coordinate parameters, known as gaussian principal, main and related curvature functions. Based upon this succinct background introduction on radar polarimetry, the concepts are applied mainly for the coherent case to various classes of increasing order of sophistication, as defined in detail in the INTRODUCTION, to the problem of radar target handling for the non-cooperative, limited-data case.
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In meticulous detail, a succinct summary of basic electromagnetic wave
polarization descriptors, of the various scatterer polarization transformation
matrices, and its invariants of the associated optimal matrix polarizations,
and of the scatterer descriptive operators is introduced. It is then shown
how the five (5) independent matrix parameter for the relative phase
correlation matrix describing an isolated, yet regionally distributed, target
in a reciprocal propagation medium can be recovered from (i) amplitude-only,
(ii) mixed amplitude plus partial phase, (iii) complete two-step amplitude-
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concepts are applied mainly for the coherent case to various classes of increasing
order of sophistication, as defined in detail in the INTRODUCTION, to the
problem of radar target handling for the non-cooperative, limited-data case.

KEYWORDS: Polarization, radar polarimetry, target discrimination, classification,
Imaging, optimal polarization nulls, polarization transformation matrix,
Mueller matrix, radar scattering matrix, polarization fork.

PREAMBLE: In the pursuit of this research on radar target recognition/handling
within the m-to-mm wavelength region we are dealing with wide interdisciplinary
research areas for which not all studies carried out in the past are available
in the open literature. In seeking for a unified approach of treating this
complex problem, it can happen that one may overlook some important base studies;
and, therefore, the paper presented here is a revised, highly updated version
of earlier similar papers. Specifically, we owe our apologies to Dr. Glendon
McCormick, Mr. Archibald Handry and Mr. Laverne E. Allan, Electromagnetics Div-
ision, NRC, Ottawa, Canada, for not having paid due attention to their outstanding
contributions to polarimetric radar meteorology. The major relevant contributions
of their research are now included in this paper.

1. INTRODUCTION

The interaction of electromagnetic waves with a geometrically bounded,
material body may best be described as a

"Polarization-Sensitive Scatterer Feature Spatial and Temporal
Resonance Phenomenon".

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particularly when the spatial and temporal periods become of the order of a target characteristic features and motion dimensions. Specifically, for the limited data, non-cooperative target case, there exists an hierarchy in complexity, amount, quality and accuracy of radar data required to obtain an "immediate (instantaneous) decision operator" in tactical (seeker) radar for the distinct radar problems of:

**Target versus clutter discrimination:** Various methods may be applicable, yet we found that in a hostile clutter and/or chaff environment such as (i) the marine boundary layer, (ii) the atmospheric ground-based battle-field scene, or (iii) for low-flying tactical aircraft involved in support of ground/sea-based battle actions, we must incorporate complete CW polarimetric target/clutter scattering matrix information. Specifically, we require to utilize the dynamic polarimetric fork properties. Whereas, for distributed clutter/chaff, the vector scattering centers are distributed more densely and separated by a small fraction of a wavelength, resulting in a more stable motion of the associated co-polarization nulls (prongs of polarization fork) on the Poincare sphere; those of isolated larger, more complex (man-made) objects are separated by distances being comparable to the wavelength and larger, resulting in a rapid circular path loci motion on the polarization sphere. Therefore, the highly varied behavior of the dynamic polarization fork motions of target (rapid) versus clutter/chaff (slow) on the polarization sphere will provide an indispensable target versus clutter/chaff discrimination operator as was demonstrated without further doubt by Poelman (1977 to 1982). We note that we also will need to reassess the merit factor definitions of optimal target signal versus clutter-plus-noise separation which need to be based on Huynen's N-target theories (Huynen, 1978) and Poelman's (1981) maximum entropy approach for extracting the most useful stochastic merit factor parametric diagrams based on Kennaugh's optimal target polarization null theory.

**Target-versus-target and clutter-versus-clutter classification:** Because of the fact that the vector scattering centers of larger, more complex isolated targets are separated by longer electric lengths resulting in a rapid circular path motion of the polarization fork, in general, over the entire Poincare sphere in case of "not-symmetrical" reciprocal targets, we find that a monochromatic CW, limited aspect, complete polarimetric approach for the backscatter (monostatic) radar case will not suffice; and, in addition, we require polarimetric target downrange silhouette resolution. Although the mean optimal polarization null locations and their spread can be obtained for clutter and/or chaff rather accurately if the polarimetric clutter matrix information is recovered within time frames lying below the clutter vector scattering center reshuffling time; improved clutter classifi-
cation (surface versus inhomogeneous volumetric underburden scatter can only result from broadband complete polarimetric clutter information (Fung and Eom, 1982; Morgan and Weisbrod, 1982; McCormick and Hendry, 1982; Boerner and Huynen, 1982).

We re-emphasize that, given complete broadband polarimetric scatter matrix information, target classification for the non-cooperative target versus target, target versus clutter, and clutter versus clutter case is guaranteed (Root, 1980, Banks, 1981).

Target imaging in inhomogeneous media and/or clutter environments: In case the target does not possess rotational symmetry but is of general "not-symmetrical" reciprocal shape, in addition to complete polarimetric downrange linear chirp maps along the rotational axis of invariance, we will require such data over a wide cone of the unit sphere of directions in dependence of data completeness, quality, etc. or, additional "equivalent a priori" target shape information. In case the target is embedded in weakly diffracting clutter, the G.O. superlimited parallelbeam methods of projection tomography do not suffice; then we must, at least, incorporate back-propagation tomographic methods based on the Born/Rytov approximation to apply, which represent a dramatic improvement over Radon's single ray (straight or bent) projection reconstruction theory. Furthermore, as we are strictly dealing with an electromagnetic vector inverse problem, the scalar back-propagation tomographic method must be extended to vector back-diffraction tomography for the general case of a target embedded in the type of clutter described above. For the application of general vector back-diffraction tomography to target imaging in dense depolarizing clutter, we also must develop direct scattering theories incorporating a polarimetric vector radiation transfer approach utilizing a Stokes' vector formulation which implicitly must also contain multi-scatter phase information.

Target identification: Complete single target identification in shape and material decomposition will strictly require solutions to all of the above three (3) tasks, plus incorporation of complete doppler and scatterometric information within the various windows of the m-to-sub-mm wavelength region. Therefore, we need to develop complete polarimetric broadband (discrete linear chirp) doppler radar systems within the various windows of the 1-400 GHz e.m. spectral region so that optimal target information can be extracted from electromagnetic wave/target interaction which is a "polarization-sensitive target feature spatial and temporal resonance phenomenon", i.e., amplitude, phase polarization, frequency, doppler information, all are equivalently and equally important.
Criteria for the Assessment of Available Complete Polarimetric Measurement Methods:

The main obstacle towards realizing incorporation of complete polarimetric radar target theory into target versus clutter discrimination, target versus target classification, target in clutter imaging, single target identification until recently was the underdeveloped state of broadband polarimetric antenna theory and design. It was not possible to recover for the general not-symmetrical reciprocal target case (which must be the basic requirement here), i.e., both amplitude and relative polarization phase of the scattering matrix elements at time frames below the vector scattering center reshuffling time of clutter/chaff. Until very recently, complete ellipsometric amplitude-only measurement principles had to be used which require nine (9) rather time-consuming independent amplitude-only measurements for a selected set of linear, circular and elliptical base polarizations. For the complete symmetric (H, V, aligned) target case, Copeland (1960) and Huynen (1960) independently developed polarization rotation-sweep techniques, which were shown to be sufficient to recover the optimal polarization nulls of aligned, symmetric targets only on the polarization sphere from co-polarized amplitude-only measurements. In a next step, a method of recovering the co/cross-polarization phase $\phi_{AB}$ or $\phi_{BA}$ for $S_{AA}/S_{AB}$ or $S_{BB}/S_{BA}$ measurements was developed using fast magnetic waveguide switches and/or pin-diode switches. This method, when re-designed for the circular left/right polarization base vector pair does provide a two-step complete measurement approach, as e.g., was developed by McCormick/Allan/Hendry (1977-1982) for polarimetric radar meteorology, for which target reciprocity must apply as well as complete target symmetry with respect to the linear H, V polarization basis which certainly is a rather unrealistic assumption for the case of tactical target detection in meteorological clutter. More recently with the advanced pin-diode switching technology, it is now possible to recover complete polarimetric scattering matrix information for the general "not-symmetrical" reciprocal target case within the time frames which lie below the reshuffling time of vector clutter scattering centers, i.e., we are now witnessing the realistic phase of incorporating complete radar polarimetric concepts into the general radar target description problem.

In the following sections, a survey of the important concepts of radar polarimetry is presented and relevant examples are provided.
2. POLARIZATION DESCRIPTORS

In this brief survey of optimal polarization descriptors, we will schematically introduce basic definitions (Table 1), describing the polarization ellipse in time and frequency domain (Fig. 1) and its relationship with the Poincare sphere.

### TABLE I: POLARIZATION DESCRIPTORS

<table>
<thead>
<tr>
<th>PARAMETER</th>
<th>DEFINITION</th>
</tr>
</thead>
<tbody>
<tr>
<td>a. Radar Cross Section</td>
<td></td>
</tr>
<tr>
<td>1) no polarization corrections</td>
<td></td>
</tr>
<tr>
<td>11) with polarization corrections</td>
<td></td>
</tr>
<tr>
<td>b. The Polarization Vector</td>
<td></td>
</tr>
<tr>
<td>1) time domain</td>
<td></td>
</tr>
<tr>
<td>11) frequency domain</td>
<td></td>
</tr>
<tr>
<td>111) geometric parameter</td>
<td></td>
</tr>
<tr>
<td>iv) polarization ratio</td>
<td></td>
</tr>
<tr>
<td>c. The Polarization Ellipse</td>
<td></td>
</tr>
</tbody>
</table>

![Fig. 1](Image)

\[
\alpha = 1 \text{Im} \frac{E_1^2}{E_2^2} = 1 \text{Im} \frac{H_1^2}{H_2^2}
\]  
(1)

\[
\alpha_{rt} = 1 \text{Im} \frac{|h^r \cdot E^s|^2}{|h|^2}
\]  
(2)

\[
h(t) = a_H \cos \omega t + a_V \cos (\omega t + \delta) \hat{h}_V
\]
where \( \delta = \delta_V - \delta_H \)
(3)

\[
h(t) = \Re(h e^{j\omega t}), \text{ where}
\]
(4)

\[
h = a_H e^{j\delta} \hat{h}_H + a_V e^{j\delta} \hat{h}_V
\]
(5)

\[
h = a_H e^{j\delta} (\hat{h}_H + p \hat{h}_V), \text{ where } p = \frac{a_V}{a_H} e^{j\delta}
\]
(6)

\[
\left(\frac{h_H}{a_H}\right)^2 + \left(\frac{h_V}{a_V}\right)^2 = 2 \left(\frac{h_H}{a_H}\right) \left(\frac{h_V}{a_V}\right) \cos \delta = \sin^2 \delta
\]
(7)

Linear: \( \delta = 0 \), horizontal \( (a_H = 0) \), vertical \( (a_V = 0) \), linear 45° \( (a_H = a_V) \)

Left circular (LC): \( \delta = 90^\circ \), \( a_H = a_V \)

Right circular (RC): \( \delta = 90^\circ \), \( a_H = a_V \)

Left elliptic: \( \sin \delta > 0 \)

Right elliptic: \( \sin \delta < 0 \)
Table 1 (Cont'd): Polarization Descriptors

**d. The Stokes Vector**

\[
g = \begin{pmatrix} g_0 \\ g_1 \\ g_2 \\ g_3 \end{pmatrix} = \begin{pmatrix} h^2 + |v|^2 \\ h^2 - |v|^2 \\ 2 \text{Re}(h^* v) \\ -2 \text{Im}(h^* v) \end{pmatrix} = \begin{pmatrix} a_H^2 + a_V^2 \\ a_H^2 - a_V^2 \\ 2a_H a_V \cos \delta \\
2a_H a_V \sin \delta \end{pmatrix} = \begin{pmatrix} a^2 \\ a^2 \cos 2\tau \cos 2\phi \\ a^2 \cos 2\tau \sin 2\phi \\ a \sin 2\tau \end{pmatrix} \begin{pmatrix} Q \\ U \\ V \end{pmatrix}
\]  

(8)

where

\[ g_0^2 = g_1^2 + g_2^2 + g_3^2 = 1^2 = Q^2 + U^2 + V^2 \]

H: \( g = \begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix} \), V: \( g = \begin{pmatrix} -1 \\ 0 \\ 0 \end{pmatrix} \), LC: \( g = \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix} \), RC: \( g = \begin{pmatrix} 0 \\ 0 \\ -1 \end{pmatrix} \)

Modified Stokes Vector \( g_m = \{ \frac{1}{2}(1 + Q), \frac{1}{2}(1 - Q), U, V \} \)

**e. The Polarization Ratio**

\[
\rho = \frac{h_v}{h_H} = \left( \frac{a_v}{a_H} \right) e^{i\delta} = \text{tanye}^{i\delta}
\]

(9)

Linear: \( \text{Im}(\rho) = 0 \), H: \( \rho = 0 \), V: \( \rho = \infty \)

Circular: \( \text{Re}(\rho) = 0 \), LC: \( \rho = j \), RC: \( \rho = -j \)

Elliptic: Left elliptic: \( \text{Im}(\rho) > 0 \), right elliptic: \( \text{Im}(\rho) < 0 \)
### TABLE 1 (Cont'd): Polarization Descriptors

<table>
<thead>
<tr>
<th>f. The Poincare Sphere</th>
</tr>
</thead>
<tbody>
<tr>
<td>i) Cartesian Coordinates = $(g_1, g_2, g_3)$</td>
</tr>
<tr>
<td>ii) Spherical = $(g_0, \frac{\pi}{2} - 2\tau, 2\phi)$</td>
</tr>
<tr>
<td>iii) In terms of Polarization ratio</td>
</tr>
</tbody>
</table>

$$ u = \frac{1 - |p|}{1 + |p|}, \quad \theta = \cos^{-1}\left(\frac{|u|^2 - 1}{|u|^2 + 1}\right), \quad \phi = \text{phase} (u) \quad (10) $$

![Poincare Sphere Diagram](image)

**Fig. 2 POINCARE SPHERE**

### 3. SCATTERING MATRICES: $[S]$, $[M]$, $[P]$

There exist three matrices of specific value to the description of hydrometeor ensembles in the coherent and the incoherent cases which are defined here and the interactions are derived (Boerner, et al, 1981, Chan, 1981).

#### 3.1 The Scattering Matrix $[S]$

The $2 \times 2$ complex scattering matrix $[S]$ is relating the polarization vector of the scattered field $\vec{h}^s$ to the corresponding one of the incident field $\vec{h}^i$ through the relation

$$ \vec{h}^s = [S] \vec{h}^i \quad (11) $$

Different representations for $[S]$ with absolute and relative phase in the bistatic and monostatic cases are summarized as follows: in the bistatic case, the scattering matrix with absolute phase is defined by
where $\phi_{AB}$ is the absolute phase, $[S]_{SMR}$ is the target scattering matrix with relative phase and it can be written in the bistatic case as

$$
[S]_{SMR} = \begin{pmatrix}
|S_{AA}|e^{j(\phi_{AA} - \phi_{AB})} & |S_{AB}| \\
|S_{BA}|e^{j(\phi_{BA} - \phi_{AB})} & |S_{BB}|e^{j(\phi_{BB} - \phi_{AB})}
\end{pmatrix}
$$

(12)

Eqs. (12), (14) satisfy the reciprocity condition $S_{AB} = S_{BA}$ in the monostatic case. In this paper, we are considering the monostatic case only.

3.2 The Mueller Matrices

The Mueller (Stokes reflection) matrix $[M]$, the modified Mueller matrix $[M_m]$, and the symmetrized Mueller matrix $[M_s]$ are presented in this section. The reconstruction of these matrices from the scattering matrix elements is given in Table 2. (Boerner et al., 1981, Jan & Sept.)

The $4 \times 4$ real Mueller matrix $[M]$ relates the scattered Stokes vector $\mathbf{g}^S$ to the corresponding incident vector $\mathbf{g}$ with the following relationship

$$
\mathbf{g}^S = [M] \mathbf{g},
$$

(14)

where the Stokes vector is defined in Table 1. A similar relationship relating the modified scattered and incident Stokes vectors is given by

$$
\mathbf{g}^S_m = [M_m] \mathbf{g}_m.
$$

(15)

The relationship between $[M]$ and $[M_m]$ is given by (Boerner et al., 1981), also by Gerrard & Burch et al., (1975).

$$
[M_m^\dagger] = [R] [M] [R^{-1}]
$$

(16)

and

$$
[M] = [R^{-1}] [M_m] [R]
$$

(17)

*We note that this specific choice need not be the best one as e.g., in the case of a circular polarization basis (also see Huynen, 1970).*
where the constant transformation matrix \([R]\) becomes

\[
[R] = \begin{pmatrix}
\frac{1}{2} & \frac{1}{2} & 0 & 0 \\
\frac{1}{2} & -\frac{1}{2} & 0 & 0 \\
0 & 0 & 1 & 0 \\
0 & 0 & 0 & 1
\end{pmatrix}
\]  

(18)

The Mueller matrices are 4 x 4 real and asymmetric. The symmetric Mueller matrix \([M_s]\) can be deduced as follows: the received power (Huynen, 1970; Kennaugh, 1949-54 #9) is

\[
P_r = \frac{1}{2} [g_s^r g_s^r + g_s^r g_s^r + g_s^r g_s^r - g_s^r g_s^r] = [Q] g_s^r g_r^r \]

(19)

where \(g_s^r\) and \(g_r^r\) are the scattered wave and receiving antenna Stokes vectors respectively and \([Q]\) is a constant matrix and is given by:

\[
[Q] = \frac{1}{4} \begin{pmatrix}
1 & 0 & 0 & 0 \\
0 & 1 & 0 & 0 \\
0 & 0 & 1 & 0 \\
0 & 0 & 0 & -1
\end{pmatrix}
\]

substituting (16) into (19), then

\[
P_r = [Q][M] g_s^l \cdot g_r^l = [M_s] g_s^l \cdot g_r^l \]

(20)

where \([M_s] = [Q][M]\) is a symmetric Mueller matrix.

3.3 Graves Power Scattering Matrix \([P]\) and its Associated \([P_h]\) and \([P_v]\)

The total backscattered power from a target is given by (Graves, 1956)

\[
P_b = h^s^* \cdot h^s = (h^s^*)^T h^s \]

(21)

where \(h^s\) is the backscattered polarization vector. Substituting (ii) into (21)

\[
P_b = (h^l)^T [S] h^l = (h^l)^T [P] h^l \]

(22)

where the matrix \([P]\) is known as Graves Power scattering matrix and it is given by

\[
[P] = [S]^T [S] = \begin{pmatrix}
a & c \\
c^* & b
\end{pmatrix}
\]

(23)
where \( a, b \) are real and \( c \) is complex. The reconstruction of the elements of \([P]\) in terms of the elements of the scattering matrix \([S]\) is given in Table 2 (Chan, 1981).

The matrix \([P]\) can be decomposed into two measureable matrices \([P_H]\) and \([P_V]\), where

\[
[P] = [P_H] + [P_V] .
\] (24)

The elements of \([P_H]\) and \([P_V]\) in terms of the elements of \([S]\) are also shown in Table 2 (Chan, 1981).

**TABLE 2**: RECONSTRUCTION OF \([M], [M_m], [P], [P_H], [P_V]\) AND OPTIMAL POLARIZATION FROM \([S]\)

<table>
<thead>
<tr>
<th>([M])</th>
<th>([M_m])</th>
</tr>
</thead>
<tbody>
<tr>
<td>(m_{11} = \frac{1}{2}(</td>
<td>S_{AA}</td>
</tr>
<tr>
<td>(m_{12} = m_{21} = \frac{i}{4}(</td>
<td>S_{AA}</td>
</tr>
<tr>
<td>(m_{13} = m_{31} = \text{Re}(S_{AA}S_{AB}^* + S_{AB}^*S_{BB}))</td>
<td>(M_{13} = \text{Re}(S_{AA}S_{AB}^*) = \frac{1}{2}M_{31})</td>
</tr>
<tr>
<td>(m_{14} = -m_{41} = \text{Im}(S_{AA}S_{AB}^* + S_{AB}^*S_{BB}))</td>
<td>(M_{14} = \text{Im}(S_{AA}S_{AB}^*) = \frac{1}{2}M_{41})</td>
</tr>
<tr>
<td>(m_{22} = \frac{1}{2}(</td>
<td>S_{AA}</td>
</tr>
<tr>
<td>(m_{23} = m_{32} = \text{Re}(S_{AA}S_{AB}^* - S_{AB}^*S_{BB}))</td>
<td>(M_{23} = \text{Re}(S_{AB}S_{BB}^*) = \frac{1}{2}M_{32})</td>
</tr>
<tr>
<td>(m_{24} = m_{42} = \text{Im}(S_{AA}S_{AB}^* - S_{AB}^*S_{BB}))</td>
<td>(M_{24} = \text{Im}(S_{AB}S_{BB}^*) = -\frac{1}{2}M_{42})</td>
</tr>
<tr>
<td>(m_{33} = \text{Re}(S_{AA}S_{BB}^*) +</td>
<td>S_{AB}</td>
</tr>
<tr>
<td>(m_{34} = m_{43} = \text{Im}(S_{AA}S_{BB}^*))</td>
<td>(M_{34} = \text{Im}(S_{AA}S_{BB}^*) = -M_{43})</td>
</tr>
<tr>
<td>(m_{44} = m_{33} + m_{22} - m_{11})</td>
<td>(M_{44} = M_{33} - 2M_{12})</td>
</tr>
</tbody>
</table>
TABLE 2: (Cont'd)

<table>
<thead>
<tr>
<th>[P] = [P_H] + [P_V]</th>
<th>CO-POL &amp; X-POL NULLS</th>
</tr>
</thead>
<tbody>
<tr>
<td>( a =</td>
<td>S_{HH}</td>
</tr>
<tr>
<td>( b =</td>
<td>S_{HV}</td>
</tr>
<tr>
<td>( c = S_{HH}^* S_{HV} + S_{HV}^* S_{VV} )</td>
<td>where: ( u = \frac{1 - \rho}{1 + \rho} )</td>
</tr>
</tbody>
</table>

where: \( \rho = -B + \sqrt{B^2 - 4AC} \)

and \( A = S_{BB} \)

\( B = 2S_{AB} \), \( C = S_{AA} \)

CO-POL Nulls

\( A' = S_{BB}^*S_{BA} + S_{AA}^*S_{AB} = -C^* \)

\( B = |S_{AA}|^2 - |S_{BB}|^2 \)

4. THE CONCEPT OF THE OPTIMAL POLARIZATION PAIRS

It was first shown by Kennaugh (1952) that there exist two pairs of optimal polarizations which can be useful in describing target properties at one aspect and at one frequency (Kennaugh, 1949-1952 #9). The concept is based on invariance of polarization state transformation under consideration of reciprocity as we will introduce next.

4.1 Polarization State Transformation

In the following we shall limit ourselves exclusively to the monostatic case \( \theta = \theta_1, \phi = \phi_1 \) and we may define the "normalized monostatic scattering matrix S with relative phase" in terms of two arbitrary elliptically orthogonal polarization base vectors \( h_A \) and \( h_B \) so that with \( h = h_A h_A^* + h_B h_B^* \)

\[ h^* = [S] h^1, [S] = \begin{pmatrix} S_{AA} & S_{AB} \\ S_{BA} & S_{BB} \end{pmatrix}, S_{AB} = S_{BA} \]

\( \phi_{AB} = \phi_{BA} = 0 \)

(25)
Thus, assuming reciprocity of the propagation path \( S_{AB} = S_{BA} \) and conservation of energy, we require five real quantities to determine \( S \) completely (Kennaugh, 1949-1952 #1, #4). However, we note that in case \( S_{AB} \neq S_{BA} \), i.e., reciprocity of the propagation paths is violated, the definition of \( S \) cannot be used (Kanareykin et al., 1966) as may be encountered for a propagation path within a highly ionized cloud containing various dense liquid and solid ice states of hydrometeors (Jeske, 1976).

Assuming reciprocity holds, there exists an infinite number of general pairs of orthonormal elliptical polarization vectors \( \mathbf{h}_A, \mathbf{h}_B \) and an infinite number of possible invariant transformations (Kennaugh, 1949-1952 #12). Numerically, the transformation properties of \( S(A,B) \) assuming no polarization losses from any one orthonormal polarization pair \( \mathbf{h} = \mathbf{h}_A \mathbf{h}_B + \mathbf{h}_B \mathbf{h}_A \) to another orthonormal pair \( \mathbf{h} = \mathbf{h}_A \mathbf{h}_A + \mathbf{h}_B \mathbf{h}_B \) can be expressed in terms of the geometric parameters \( \tau \) and \( \phi \), or polarization ratio parameters \( \gamma \) and \( \delta \), mathematically expressed in matrix form

\[
[T] = \begin{pmatrix}
\cos\phi & -\sin\phi \\
\sin\phi & \cos\phi
\end{pmatrix}
\begin{pmatrix}
\cos \tau & j \sin \tau \\
-j \sin \tau & \cos \tau
\end{pmatrix}
\] (26)

which implies rotation of coordinate axes and deformation of ellipticity of the polarization ellipse. \([T] \) may also be defined according to Maffett (Crispin, and Siegel, 1968)

\[
[T] = \begin{pmatrix}
e^{j\psi_1 \cos \gamma} & e^{j\psi_2 \sin \gamma} \\
e^{-j\psi_3 \sin \gamma} & e^{j\psi_4 \cos \gamma}
\end{pmatrix}
\] (27)

and in order for \([T] \) to be unitary, the following condition on \( \psi \)'s has to be imposed \( \psi_2 - \psi_1 = \psi_4 - \psi_3 \), where in our work, we chose \( \psi_1 = \psi_4 = 0 \). Thus, having \( \psi_2 = -\psi_3 = \delta \pm \pi \), the \([T] \) matrix reduces to

\[
[T] = \begin{pmatrix}
\cos \gamma & -e^{j\delta} \sin \gamma \\
e^{-j\delta} \sin \gamma & \cos \gamma
\end{pmatrix}
\] (28)

The two transformations \([26] \) and \([28] \) are equivalent, and can be represented on the Poincare sphere as shown in Fig. 3.

Fig. 3
DESCAMPS' SPHERE
Equation (26) can also be expressed in terms of polarization ratio \( \rho = \tan \gamma \) and after normalizing \([T]\) it takes on the following form

\[
[T] = (1+\rho \rho^*)^{\frac{1}{2}} \begin{pmatrix} 1 & -\rho^* \\ \rho & 1 \end{pmatrix}.
\] (29)

The transformed elements of the scattering matrix \([S'(A',B')] = [T]^T [S(A,B)] [T]\) are given for the general bistatic case by

\[
\begin{align*}
S'_{A'A'} &= (1+\rho \rho^*)^{-1} [S_{AA} + \rho^2 S_{BB} + \rho (S_{AB} + S_{BA})] \\
S'_{A'B'} &= (1+\rho \rho^*)^{-1} [-\rho^* S_{AA} + S_{BB} + S_{AB} - \rho \rho^* S_{BA}] \\
S'_{B'A'} &= (1+\rho \rho^*)^{-1} [\rho \rho^2 S_{AA} + S_{BB} - \rho \rho^* (S_{AB} + S_{BA})] \\
S'_{B'B'} &= (1+\rho \rho^*)^{-1} [\rho \rho^2 S_{AA} + S_{BB} - \rho \rho^* (S_{AB} + S_{BA})],
\end{align*}
\] (30)

satisfying the transformation invariants \( \det([S'(A',B')]) = \text{constant} \) when \( \det([T]) = \pm 1 \), otherwise \( \det([S'(A',B')]) \) is different by a factor of \( \exp(2\text{ARG} \det([T])) \) and

\[
\begin{align*}
\text{Span}([S(A,B)]) &= |S_{AA}|^2 + |S_{AB}|^2 + |S_{BA}|^2 + |S_{BB}|^2 = \rho \\
\text{Span}([S'(A',B')]) &= |S'_{A'A'}|^2 + |S'_{A'B'}|^2 + |S'_{B'A'}|^2 + |S'_{B'B'}|^2
\end{align*}
\] (31)

We note that if \( S_{AB} = S_{BA} \), then \( S'_{A'B'} = S'_{B'A'} \) for all \( \rho \); i.e., if reciprocity is satisfied for any one pair of orthogonal polarizations, it is satisfied for all such pairs. Furthermore, we must emphasize the important property that for any one given aspect and for one frequency, the transformation is polarization invariant, i.e., the transformation occurs on one and the same polarization sphere of radius \( \rho = \text{span}([S(A,B)]) \). Thus, if \([S(A,B)]\) is known and reciprocity as well as conservation of energy is satisfied, \([S'(A',B')]\) for any other orthogonal pair \((A',B')\) can be obtained as is known for example for the transformation from linear to circular polarization base vectors in Long (1975). In case of polarization losses properties of the coherency matrix need to be used (Kraus, 1966), and the transformation will not occur on the same polarization sphere (Deschamps, 1953; Deschamps and Mast, 1973).

4.2 Transformation from Linear \((H,V)\) to Circular \((R,L)\) Polarization Bases

Based on equation (28) we can construct a transformation from a linear to a circular polarization base. The parameters in equation (28) are set to the following values

\[
\gamma = \pi/4 \text{ and } \delta = 3\pi/2
\]

*We note that Huynen (1970) chose target maximum power \( m \) to represent the radius.
The resulting transformation matrix \([T]\) is

\[
[T] = \frac{1}{\sqrt{2}} \begin{bmatrix} 1 & j \\ j & 1 \end{bmatrix}
\]  

(32)

The relationship between the unit vectors \((\hat{h}_H, \hat{h}_V)\) of the linear basis and \((\hat{h}_R, \hat{h}_L)\) in the circular basis can now be written as:

\[
\begin{bmatrix} \hat{h}_R \\ \hat{h}_L \end{bmatrix} = [T] \begin{bmatrix} \hat{h}_H \\ \hat{h}_V \end{bmatrix}
\]  

(33)

Equation (33) holds in the incident system, its counterpart in the scattered system is

\[
\begin{bmatrix} \hat{h}_R \\ \hat{h}_L \end{bmatrix} = [T]^* \begin{bmatrix} \hat{h}_H \\ \hat{h}_V \end{bmatrix}
\]

the two systems are illustrated in Fig. 4.

![Figure 4: Incident and Scattered Field Coordinates](image)

FIGURE 4: INCIDENT AND SCATTERED FIELD COORDINATES

It can be shown that the scattering matrix in the circular basis \([C(R,L)]\) is

\[
[C(R,L)] = [T(RL, HV)] [S(HV)] [T(RL, HV)]^T
\]  

(34)

Substituting equations (32) into equation (34) we obtain the scattering matrix in the circular polarization basis in terms of elements of its counterpart in the linear basis.
4.3 Calculation of the Optimal Polarizations

It was shown by Kennough (1949-1952) that there exist two pairs of optimal polarizations, the Co-Polarization Null Pair for which $S'_A$ and $S'_B$ in (27) vanish and the Cross-Polarization Null Pair for which $S'_A$ and $S'_B$ vanish. In Table 2, the optimal polarization (CO-POL and X-POL) nulls are given in terms of $[S]$ elements and are represented on the Poincare sphere.

It should be noted that the CO-POL and X-POL nulls lie on one major circle on the Poincare polarization sphere and their locations define the polarization fork (Fig. 5). The X-POL nulls are anti-podal on this sphere and the line joining them bisects the angle between the CO-POL nulls as shown in Fig. 5. We note here that this unique description of a scatterer under monostatic conditions given for one frequency and aspect is of paramount importance to target description at one aspect and one frequency and its properties have been overlooked in practice (Kennough, 1949-1952; Kanareykin, et al, 1966).

\[
[C(RL)] = \begin{bmatrix}
\frac{S_{HH} - S_{VV}}{2} + jS_{HV} & \frac{S_{HH} + S_{VV}}{2} \\
\frac{S_{HH} + S_{VV}}{2} & \frac{S_{VV} - S_{HH}}{2} + jS_{HV}
\end{bmatrix}
\]

4.4 Reconstruction of $[S]_{SMR}$

The reconstruction of $[S]_{SMR}$ from $[M]$, $[M']$, $[P]$, $[P']$ and $[P]$ or optimal polarizations is shown in Table 3. This means Tables 2 and 3 give a complete interrelationship between these scattering matrices as well as the optimal polarizations. From a measurement point of view, this is very important because it suffices to measure one of the matrices or the optimal polarization to calculate the other matrices. The reconstruction of $[S]_{SMR}$ from the optimal polarizations is of great importance to target polarization synthesis. In these Tables, A and B are any two orthogonal bases e.g. horizontal and vertical. We note here that in the incoherent or quasi-coherent case, clustering properties of the CO-POL nulls need to be taken into consideration.

<table>
<thead>
<tr>
<th>elements of $[S]_{SMR}$</th>
<th>from $[M]$</th>
<th>from $[M_m]$</th>
<th>from $[P_A]$ and $[P_V]$ $(A=H, B=V)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$</td>
<td>s_{AA}</td>
<td>$</td>
<td>$\sqrt{\frac{m_{11} + 2m_{12} + m_{22}}{2}}$</td>
</tr>
<tr>
<td>$</td>
<td>s_{AB}</td>
<td>=</td>
<td>s_{BA}</td>
</tr>
<tr>
<td>$</td>
<td>s_{BB}</td>
<td>$</td>
<td>$\sqrt{\frac{m_{11} - 2m_{12} - m_{22}}{2}}$</td>
</tr>
<tr>
<td>$\phi_{AA} - \phi_{AB}$</td>
<td>$\tan^{-1} \left( \frac{m_{41} + m_{42}}{m_{31} - m_{32}} \right)$</td>
<td>$\tan^{-1} \left( \frac{M_{14}}{M_{13}} \right)$</td>
<td>$\tan^{-1} \frac{\text{Im} C_A}{\text{Re} C_A}$</td>
</tr>
<tr>
<td>$\phi_{BB} - \phi_{AB}$</td>
<td>$\tan^{-1} \left( \frac{m_{41} - m_{42}}{m_{31} - m_{32}} \right)$</td>
<td>$\tan^{-1} \left( \frac{M_{41}}{M_{32}} \right)$</td>
<td>$\tan^{-1} \frac{\text{Im} C_B}{\text{Re} C_B}$</td>
</tr>
</tbody>
</table>

from optimal polarizations

$[S(A,B)] = K \begin{bmatrix} x & z \\ z & y \end{bmatrix}$

CO-POL Nulls are known:

$K = \sqrt{\frac{P}{2}} \left( |\rho_1^{CO}| + |\rho_2^{CO}| \right)^{-\frac{1}{2}}$

$x = -2\rho_1^{CO} \rho_2^{CO} \exp(-j\phi_E)$, \( \phi_E = \text{phase}(\rho_1^{CO} + \rho_2^{CO}) \)

$y = -2 \exp(-j\phi_E)$

$z = |\rho_1^{CO}| + |\rho_2^{CO}|$

One CO-POL and one X-POL are known:

$K = \sqrt{\frac{P}{D}} = 2 \left( |\rho^{CO}|^2 + |\rho^X|^2 + |\rho^{CO}|^2 \left| |\rho^{CO}|^2 - |\rho^X|^2 \left| - |\rho^X|^2 \right| \right)^{-\frac{1}{2}}$

$x = \rho^{CO} \left( |\rho^{CO}|^2 - |\rho^X|^2 \right) \exp(-j\phi_E)$, \( \phi_E = \text{phase}(\rho^X)^2 (\rho^{CO})^2 + \rho^X \)

$y = -2 \rho^{CO} (\rho^X)^* - |\rho^X|^2 + 1 \exp(-j\phi_E)$

$z = (\rho^X)^*(\rho^{CO})^2 + \rho^X$
4.5 Optimal Polarizations for Different Isolated Simple Target Shapes

The CO-POL and X-POL nulls are calculated here for different target shapes. Table 4 shows the calculated nulls for simple shapes, e.g. ideally conducting flat plate or sphere, metallic trough, right and left metallic helices.

**Table 4: CO-POL AND X-POL NULL FOR SIMPLE TARGET SHAPES**

<table>
<thead>
<tr>
<th>TARGET</th>
<th>SCATTERING ([S]) AND MODIFIED MUeller ([M_m]) MATRICES</th>
<th>CO-POL ((C)) AND X-POL ((X)) NULLS ON POINCARE SPHERE</th>
</tr>
</thead>
<tbody>
<tr>
<td>a. Ideally conducting flat plate or sphere</td>
<td>([S] = \begin{pmatrix} 1 &amp; 0 \ 0 &amp; 1 \end{pmatrix})</td>
<td><img src="image1" alt="CO-POL and X-POL nulls" /></td>
</tr>
<tr>
<td></td>
<td>([M_m] = \begin{pmatrix} 1 &amp; 0 &amp; 0 &amp; 0 \ 0 &amp; 1 &amp; 0 &amp; 0 \ 0 &amp; 0 &amp; 1 &amp; 0 \ 0 &amp; 0 &amp; 0 &amp; 1 \end{pmatrix})</td>
<td><img src="image2" alt="POINCARE nulls" /></td>
</tr>
<tr>
<td>b. Metallic trough</td>
<td>([S] = \pm \begin{pmatrix} 1 &amp; 0 \ 0 &amp; -1 \end{pmatrix})</td>
<td><img src="image3" alt="POINCARE nulls" /></td>
</tr>
<tr>
<td></td>
<td>([M_m] = \begin{pmatrix} 1 &amp; 0 &amp; 0 &amp; 0 \ 0 &amp; 1 &amp; 0 &amp; 0 \ 0 &amp; 0 &amp; -1 &amp; 0 \ 0 &amp; 0 &amp; 0 &amp; -1 \end{pmatrix})</td>
<td><img src="image4" alt="POINCARE nulls" /></td>
</tr>
<tr>
<td>c. Metallic helix (right screw)</td>
<td>([S] = \pm \begin{pmatrix} 1 &amp; -j \ -j &amp; -1 \end{pmatrix})</td>
<td><img src="image5" alt="POINCARE nulls" /></td>
</tr>
<tr>
<td></td>
<td>([M_m]_{\pm} = \begin{pmatrix} 1 &amp; 1 &amp; 0 &amp; 1 \ 1 &amp; 1 &amp; 0 &amp; 1 \ 0 &amp; 0 &amp; 0 &amp; 0 \ 2 &amp; 2 &amp; 0 &amp; 2 \end{pmatrix})</td>
<td><img src="image6" alt="POINCARE nulls" /></td>
</tr>
<tr>
<td>d. Metallic helix (left screw)</td>
<td>([S] = \pm \begin{pmatrix} 1 &amp; j \ j &amp; -1 \end{pmatrix})</td>
<td><img src="image7" alt="POINCARE nulls" /></td>
</tr>
<tr>
<td></td>
<td>([M_m]_{\pm} = \begin{pmatrix} 1 &amp; 1 &amp; 0 &amp; -1 \ 1 &amp; 1 &amp; 0 &amp; -1 \ 0 &amp; 0 &amp; 0 &amp; 0 \ 2 &amp; 2 &amp; 0 &amp; -2 \end{pmatrix})</td>
<td><img src="image8" alt="POINCARE nulls" /></td>
</tr>
</tbody>
</table>
5. FREQUENCY-DEPENDENT RELATIONSHIP OF POLARIMETRIC SCATTERING MATRIX ELEMENTS WITH SPECULAR POINT CURVATURE

The time-domain first order polarization-dependent correction to the physical optics impulse response has been given by Bennett et al. (1973, 1977 and 1981) as

\[
\hat{r}_{o's}(p0) (\hat{r}, t) = \frac{K_u - K_v}{4\pi} \left[ a_u H_{ui} - a_v H_{vi} \right] \frac{dA}{dt}
\]

where \( \hat{a}_u, \hat{a}_v \) are unit vectors along the directions of the principal curvatures at the specular point; \( H_{ui}, H_{vi} \) are the components of the incident field in the directions of \( \hat{a}_u, \hat{a}_v \) respectively; \( K_u, K_v \) are the principal curvatures at the specular point; and \( A \) is the silhouette area of the scatterer as delineated by the incident impulsively plane wavefront moving at half the free space velocity.

An expression for the far scattered impulse response field was found in [Bennett et al, (1973) and (1977)]

\[
\hat{r}_{o's}(p0) (\hat{r}, t) = \frac{1}{\sqrt{2\pi}} \frac{d^2 A}{dt^2} \hat{a}_H
\]

which is the Kennaugh-Cosgriff formula, (Kennaugh & Cosgriff, 1958; Kennaugh & Moffatt, 1965). The corrected total field is thus

\[
\hat{r}_{o's}(\hat{r}, t) = \hat{r}_{o's}(p0) (\hat{r}, t) + \hat{r}_{o's}(p01) (\hat{r}, t)
\]

which is transformed to the frequency domain, and is directly related to the scattering matrix which exhibits total polarization/depolarization effects (Chaudhuri et al, 1982). Ignoring scale factors, the matrix elements are given by

\[
S_{11} = \frac{1}{2\pi} (jk)^2 A_F(k) - (jk) A_F(k) \frac{K_u - K_v}{4\pi} \cos 2\alpha
\]

\[
S_{22} = \frac{1}{2\pi} (jk)^2 A_F(k) + (jk) A_F(k) \frac{K_u - K_v}{4\pi} \cos 2\alpha
\]

\[
S_{12} = (jk) A_F(k) \frac{K_u - K_v}{4\pi} \sin 2\alpha
\]

where \( k \) is the wave number, \( A_F(k) \) is the Fourier transform of \( A(t) \), the polarization angle \( \alpha \) is defined in Figure 6a. The validity of (35) requires high frequency interrogation with smooth, convex, conducting targets. (Fig. 6b)

A complex function \( \frac{1 - R}{2} \) (where \( R = \frac{S_{22}}{S_{11}} \)) is defined, and curvature information can be extracted from it at high frequencies. A relationship between the phase factors of the scattering matrix elements and the principal curvatures is then established (Chaudhuri et al, 1982)

\[
\frac{K_u - K_v}{2} = - \frac{k}{\cos 2\alpha} \tan \frac{\phi_d}{2}
\]

(36)

where \( \phi_d = \phi_{22} - \phi_{11} \).
FIGURE 7: BEHAVIOR OF D AT HIGH FREQUENCIES (Foo, 1982)
The cross-polar nulls are also found to be along the directions of principal curvatures. These directions can be recovered from the scattering matrix elements (Chaudhuri et al., 1982).

The curvature recovery model is based on the first order correction to the Physical Optics approximation. Higher order corrections are investigated by directly extending the space-time integral equation approach of Bennett et al., (1977). The second order correction current is found to be very insignificant when compared to the first order one (Chaudhuri et al., 1982) and (Foo, 1982).

The phase-curvature relationship (36) is tested by applying it to theoretical, as well as, experimental backscattering data obtained for a prolate spheroidal scatterer, as shown in Figures 6 to 8. Both sets of data support the relationship well. Figure 8a is a direct verification of (36) with theoretical data; Figure 8b depicts that the quantity \( k \Im \frac{1-R}{1+R} \) converges to \( K_u/K_v \) as \( kb \) increases; Figure 8c is a plot of the imaginary part versus the real part of \( k \frac{1-R}{1+R} \), and shows that, as predicted from theory, it converges to (or hovers around) a point on the imaginary axis as frequency increases. The distance of this point on the imaginary axis from the origin equals the required value \( \sqrt{K_u/K_v} = 0.375 \) for the specular point of interest.

Figures 8a-8c refer to the broadside incidence to a 2:1 prolate spheroid in Figure 6b with \( \phi = 90^\circ \). For useful presentation of results from experimental data, the complex plot, such as that in Figure 8c, is found to be most interpretative (Chaudhuri et al., 1982) and (Foo, 1982). Figures 8d-8f are the experimental versions of Figures 8a, b and c. Deviations from the theoretical predictions are mainly attributed to the \( k \) factor and the tangent function in (36), and the relative phase error between the TE and TM incidences, all of which become more significant at higher frequencies (Chaudhuri et al., 1982) and (Foo, 1982).

While the phase difference of like-polarized terms, however small, contains curvature information, the phase sum, regardless of the type of orthogonal polarization bases, tends to a value which is twice the argument of the Fourier transform of the silhouette area of the target (Foo, 1982), i.e.,

\[
\phi_{11} + \phi_{22} + 2 \text{Arg } A_F(k) \tag{37}
\]

The phase sum also tends to the argument of the scattering ratio defined as the ratio of the determinant to the span of the scattering matrix

\[
D \equiv \frac{S_{11}S_{22} - S_{12}^2}{|S_{11}|^2 + |S_{22}|^2 + 2|S_{12}|^2} \tag{38}
\]

\[
\phi_{11} + \phi_{22} + \text{Arg } D \tag{39}
\]

The magnitude of the scattering ratio, whose definition is immaterial of whether linear, circular or general elliptic polarization is used, approaches 0.5 rapidly as frequency is increased, i.e.

\[
|D| + 0.5 \tag{40}
\]
a) Direct Verification of Phase-Curvature Relationship (theoretical)

b) Convergence of $k \text{ Im } \frac{(1-R)}{(1-R)}$

c) The Scattering Chart

d) Experimental version of (a)

e) Experimental Version of (b)

FIGURE 8 Comparison of Phase-curvature Relationships for theoretical and experimental data (Foo, 1982)
The magnitude of the ratio is interpreted as the ratio of the maximum radar cross section to the trace of the power scattering matrix \([P]\) at high frequencies, i.e.,

\[
\frac{\sigma_{\text{max}}}{\text{Tr}[P]} \rightarrow 0.5
\]

where \(\sigma_{\text{max}}\) is the optimum radar cross section defined in Kennaugh (1949-1954), Sinclair (1948), as

\[
\sigma_{\text{rt}} = |h^r \cdot [S] h^t|^2
\]

In the above, it is assumed that identical transmitting and receiving antennas are used; \(h^r\) and \(h^t\) are the antenna heights, and are normalized to unity; \([P]\) is the Graves-Dower scattering matrix defined in Graves (1956), as

\[
[P] = [S]^T [S]
\]

The complex plots of the scattering ratio provide a simple check on the accuracy of high frequency polarimetric measurements (Foo, 1982). The complex plot and the amplitude plot of the ratio are depicted in Figures 7a and 7c, respectively, for theoretical data. In Figures 7b and 7d, the respective plots for measurement data are provided which demonstrates the usefulness of introducing Eq. (38).

Another curvature recovery equation has been derived (Foo, 1982) in circular polarization basis vector notation

\[
\left(\frac{K_u - K_v}{2}\right)^2 = k^2 \frac{C_{RR} C_{LL}}{C_{RR}}
\]

where the C's denote the elements of the circular polarization scattering matrix. It is to be noted that the quantity \(|C_{RR}||C_{LL}| - |C_{RL}|^2\) (Morgan and Weisbrod 1982) can be interpreted as,

\[
(|C_{RR}| |C_{LL}| - |C_{RL}|^2) = [- (1/2\pi)^2 k^4 |A_p(k)|^2],
\]

which reveals area information for a smooth, convex, conducting target at high frequencies (Foo 1982).

The curvature recovery model is proven to satisfy the image reconstruction identities of invariant transformation (Foo 1982). It is found that the determinant of the scattering matrix is strictly transformation-invariant if (Foo 1982)

\[
\text{Det} [T] = \pm 1
\]

where \([T]\) is the unitary transformation matrix, whereas, the invariance of the span of the scattering matrix necessitates no restriction (Foo 1982), i.e.,
$$\text{Det} \ [C] = e^{i2\text{Arg} \ (\text{Det} \ [T])} : \text{Det} \ [S] \quad (44)$$

and

$$\text{Span} \ [C] = \text{Span} \ [S] \quad (45)$$

where [C] can be extended to the scattering matrix resulting from transforming [S] to the general elliptic polarization.

Finally, the values of \(k_b\) (Chaudhuri et al., 1982) have been found to be most potentially suitable for curvature recovery of the 6\(^{"} \times 12\(^{"} \) prolate spheroid (and probably targets of similar size and shape), provided that polarimetric measurements can be improved to a better accuracy. Not only is this range of \(k_b\) valid for the first order correction to physical optics, but it is also a compromise range between the high frequency condition required by the curvature recovery model and the drawback to lower frequencies required to prevent critical magnification of measurement errors (Chaudhuri et al., 1982).

6. MEASUREMENTS OF [M], [P], and [S]

The measurements of the scattering matrices [S], [P], and [M] are intricate, and various methods exist which have been summarized recently by Chan, 1981. Of particular interest here is the measurement of [S] and, specifically, the retrieval of both amplitude and phase of all of the relevant elements of [S]\(_{\text{SMR}}\), i.e., \(|S_{AA}|, |S_{BB}|, |S_{AB}|, \phi_{AA}, \phi_{BB}\) (assuming that \(\phi_{AB} = \phi_{BA} = \ldots\)). Since this brief introduction does not allow a complete treatment, we refer to the above report and point out only that it is necessary to recover the relative phase between the two co-polarized components in addition to the relative phase between the co-/cross-polarized components, as well as the amplitudes of \(|S_{AA}|, |S_{BB}|, |S_{AB}|\), which requires isolation of at least 25 to 30 dB between co- and cross-polarized channels.

6.1 Amplitude-Only Measurements

When amplitude alone is measured, cross-section measurements do not lead to a direct determination of the scattering matrix. But, rather, the measured data provide the coefficients of equations from which the magnitudes and relative phases of the matrix elements are deduced. Hence, only [S]\(_{\text{SMR}}\) can be derived from the measurements.

a) Measurement of \([M_0]\): The Mueller matrix \([M]\) of a target at one aspect angle can be readily obtained from its associated \(S_{\text{SMR}}\) as shown in Table 2. As for the measurement of the average Mueller matrix \(<[M]\rangle\) of a target, a set of nine (9) independent measurements of average power for various combinations of antenna polarizations are required to obtain \(<[M]\rangle\). All elements of the symmetric \([M_0]\), as defined in Eq. (20), can be obtained with the set of transmitting and receiving antenna combinations as shown in Table 5.
TABLE 5: ANTENNA POLARIZATION FOR THE MEASUREMENT OF THE SYMMETRIC $[\text{MS}(\vec{m}_{ij})]$ Transmission | Reception | Average received power |
<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>$\phi=45^\circ$</td>
<td>$\phi=45^\circ$</td>
<td>$\frac{1}{2}(\vec{m}<em>{11} + \vec{m}</em>{33} + 2\vec{m}_{13})$</td>
</tr>
<tr>
<td>$\phi=135^\circ$</td>
<td>$\phi=135^\circ$</td>
<td>$\frac{1}{2}(\vec{m}<em>{11} + \vec{m}</em>{33} - 2\vec{m}_{13})$</td>
</tr>
<tr>
<td>Horizontal</td>
<td>Horizontal</td>
<td>$\frac{1}{2}(\vec{m}<em>{11} - \vec{m}</em>{22} + 2\vec{m}_{12})$</td>
</tr>
<tr>
<td>Vertical</td>
<td>Vertical</td>
<td>$\frac{1}{2}(\vec{m}<em>{11} + \vec{m}</em>{22} - 2\vec{m}_{12})$</td>
</tr>
<tr>
<td>Left circ.</td>
<td>Left circ.</td>
<td>$\frac{1}{2}(\vec{m}<em>{11} + \vec{m}</em>{44} + 2\vec{m}_{14})$</td>
</tr>
<tr>
<td>Right circ.</td>
<td>Right circ.</td>
<td>$\frac{1}{2}(\vec{m}<em>{11} + \vec{m}</em>{44} - 2\vec{m}_{14})$</td>
</tr>
<tr>
<td>$\phi=45^\circ$</td>
<td>Horizontal</td>
<td>$\frac{1}{2}(\vec{m}<em>{11} + \vec{m}</em>{12} + \vec{m}<em>{13} + \vec{m}</em>{23})$</td>
</tr>
<tr>
<td>$\phi=45^\circ$</td>
<td>Left circ.</td>
<td>$\frac{1}{2}(\vec{m}<em>{11} + \vec{m}</em>{13} + \vec{m}<em>{14} + \vec{m}</em>{34})$</td>
</tr>
<tr>
<td>Horizontal</td>
<td>left circ.</td>
<td>$\frac{1}{2}(\vec{m}<em>{11} + \vec{m}</em>{12} + \vec{m}<em>{14} + \vec{m}</em>{24})$</td>
</tr>
</tbody>
</table>

Note that the expressions in the average received power column of Table 6.8 have different signs from those derived by Kennaugh [2: No. 7]. In Kennaugh's report, the Stokes vector for horizontal polarization is defined as

$$\mathbf{S}_H = \frac{1}{\sqrt{2}} \begin{bmatrix} 1 & 1 & 0 & 0 \end{bmatrix}^T,$$

whereas, here it is defined as

$$\mathbf{S}_H = \frac{1}{\sqrt{2}} \begin{bmatrix} 1 & 0 & 0 & 0 \end{bmatrix}^T.$$

b) Measurement of $[S]_{SMR}$: The monostatic $[S]_{SMR}$ is specified by five (5) independent parameters, namely three (3) unsigned amplitudes and two (2) relative phases. In order to determine these five (5) parameters, five (5) independent magnitude-determining measurements have to be made. In general, therefore, seven (7) amplitude measurements are needed to completely determine the $[S]_{SMR}$.

Kennaugh (1949-1954) suggested measurements using the same transmitting and receiving antenna polarizations. Only one of these measurements need be other than linear polarization. The measured data are used to locate the CO-POL nulls of the $[S]_{SMR}$ on the Poincare sphere. Once the CO-POL nulls have been determined, the corresponding $S$ $[S]_{SMR}$ can be completely specified (Boerner et al., 1981). The combinations of transmitting and receiving antenna polarizations are summarized in Table 6.
TABLE 6: TRANSMITTING AND RECEIVING ANTENNA POLARIZATIONS
FOR AMPLITUDE-ONLY MEASUREMENT

<table>
<thead>
<tr>
<th>Transmission</th>
<th>Reception</th>
<th>Measured Parameters</th>
<th>Calculated Parameters</th>
</tr>
</thead>
<tbody>
<tr>
<td>(1) Vertical</td>
<td>Vertical</td>
<td>(</td>
<td>S_{VV}</td>
</tr>
<tr>
<td>Vertical</td>
<td>Horizontal</td>
<td>(</td>
<td>S_{HV}</td>
</tr>
</tbody>
</table>

If \(|S_{HV}| \neq 0\) in (1), proceed with the following measurement:

(2) Vertical   Right circular  \(\phi=45^\circ\)
Vertical       Horizontal       \(|S_{HH}|\)
Horizontal     Horizontal       \(\phi=45^\circ\)
Horizontal     Right circular   \(\phi=45^\circ\)

If \(|S_{HV}| = 0\), replace (2) by (3):

(3) Horizontal Right circular  \(\phi=45^\circ\)
Horizontal     \(\phi=45^\circ\)
Horizontal     \(\phi=45^\circ\)

ROSS AND FREENY (1964)

c) Measurement of \([P]\): The power scattering matrix \([P]\) specifies the total power backscattered from the target for any transmitting antenna polarization, hence, it can be found by measuring only the total power in the backscattered return, no phase measurement is necessary. The form of the power scattering matrix is \(PB(HV)\) is
\[
[P(HV)] = [S(HV)]^* [S(HV)] \\
\begin{bmatrix}
|S_{HH}|^2 + |S_{HV}|^2 & S_{HH} S_{HV}^* + S_{HV} S_{VV}^* \\
(S_{HH} S_{HV} + S_{HV} S_{VV})^* & |S_{HV}|^2 + |S_{VV}|^2 \\
\end{bmatrix} \\
= \begin{bmatrix}
k_1 & K \\
K^* & k_2 \\
\end{bmatrix}
\] (46)

where \( k_1 \) and \( k_2 \) are real, \( K \) is complex. \([P]\) can be completely specified by transmitting at horizontal, vertical, \( \phi = 45^\circ \) linear polarization and right-handed circular polarization and measuring the total power backscattered, no phase measurements are necessary (Graves, 1956).

For example, if horizontal polarization is transmitted, i.e.,

\[
h^t = \begin{bmatrix} 1 \\ 0 \end{bmatrix}
\] (47)

then, from \( P_b = (h^t)^T [P] h^t \), we have

\[
P_b^H = \begin{bmatrix} 1 & 0 \\
K^* & k_2 \\
\end{bmatrix} \begin{bmatrix} 1 \\
0 \\
\end{bmatrix} = k_1
\] (48)

where the superscript indicates the polarization of the transmitting antenna. The result is summarized in Table 7. It should be noted that \([P]\) cannot be obtained using linear polarizations only. Rather, linear, as well as, circular polarizations are required.

**TABLE 7: TRANSMITTING ANTENNA POLARIZATIONS FOR THE MEASUREMENT OF \([P]\)**

<table>
<thead>
<tr>
<th>Transmitting Polarization</th>
<th>Total backscattered power</th>
<th>Measured Parameters</th>
</tr>
</thead>
<tbody>
<tr>
<td>Horizontal</td>
<td>( P_b^H )</td>
<td>( k_1 )</td>
</tr>
<tr>
<td>Vertical</td>
<td>( P_b^V )</td>
<td>( k_2 )</td>
</tr>
<tr>
<td>( \phi = 45^\circ )</td>
<td>( P_b^{45^\circ} )</td>
<td>( \frac{1}{2}(k_1 + k_2) + \text{Re}(K) )</td>
</tr>
<tr>
<td>Right Circ.</td>
<td>( P_b^{RC} )</td>
<td>( \frac{1}{2}(k_1 + k_2) + \text{Im}(K) )</td>
</tr>
</tbody>
</table>
d) Measurement of $[P_H]$ and $[P_V]$: The elements of $[P_H]$ and $[P_V]$ can be determined by power measurements similar to those for obtaining $[P]$. For the determination of $[P_H]$, we transmit horizontal, vertical, $\phi = 45^\circ$ and right circular, and then receive with horizontal polarization only, i.e., the power backscattered in the horizontal channel. As for $[P_V]$, we receive with horizontal polarization only, i.e., the power backscattered in the vertical channel. The results are tabulated in Table 8 and Table 9.

**TABLE 8: TRANSMITTING ANTENNA POLARIZATIONS FOR THE MEASUREMENT OF $[P_H]$**

<table>
<thead>
<tr>
<th>Transmitting polarization</th>
<th>Backscattered power in horizontal channel</th>
<th>Measured parameters</th>
</tr>
</thead>
<tbody>
<tr>
<td>Horizontal</td>
<td>$P_{bH}^H$</td>
<td>$</td>
</tr>
<tr>
<td>Vertical</td>
<td>$P_{bH}^V$</td>
<td>$</td>
</tr>
<tr>
<td>$\phi = 45^\circ$</td>
<td>$P_{bH}^{45^\circ}$</td>
<td>$\frac{1}{2}(</td>
</tr>
<tr>
<td>Right circ.</td>
<td>$P_{bH}^{RC}$</td>
<td>$\frac{1}{2}(</td>
</tr>
</tbody>
</table>

**TABLE 9: TRANSMITTING ANTENNA POLARIZATIONS FOR THE MEASUREMENT OF $[P_V]$**

<table>
<thead>
<tr>
<th>Transmitting polarization</th>
<th>Backscattered power in vertical channel</th>
<th>Measured parameters</th>
</tr>
</thead>
<tbody>
<tr>
<td>Horizontal</td>
<td>$P_{bV}^H$</td>
<td>$</td>
</tr>
<tr>
<td>Vertical</td>
<td>$P_{bV}^V$</td>
<td>$</td>
</tr>
<tr>
<td>$\phi = 45^\circ$</td>
<td>$P_{bV}^{45^\circ}$</td>
<td>$\frac{1}{2}(</td>
</tr>
<tr>
<td>Right circ.</td>
<td>$P_{bV}^{RC}$</td>
<td>$\frac{1}{2}(</td>
</tr>
</tbody>
</table>

6.2 Amplitude and Relative Phase Measurement

The technique of measuring amplitude and relative phase has been used to obtain scattering matrix data by Kennaugh (1949-1954), Ross and Freeny (1964), Crispin et al (1961), and Huynen (1965).
In the following, several more useful methods are summarized.

6.2A Linear Polarization Basis

The 10 cm CHILL Meteorological Radar is one such system that has the capability of measuring both the amplitude and relative phase of the scattering matrix. The CHILL system is a coherent radar which has the ability to rapidly switch from vertical to horizontal polarization. The polarization switching is accomplished by use of a switchable ferrite circulation which is located behind the parabolic antenna (Mueller, 1981).

Measurement with the "fast switch" requires that the switch be transferred after each transmitter pulse. First, a horizontal polarization, and then a vertical polarization measurement is obtained. Two separate channels are used to keep the horizontal and vertical measurements separate. It should be noted that the same logarithmic receiver, sample and hold, and A/D converter are used for both channels. Thus, any non-linearities of these analog circuits are reflected in both channels, and will tend to cancel. The signal processing is achieved with an A/D converter and a floating point integration. The five (5) most significant bits are separated from the eight (8) bit digital word and used to represent the horizontal power and then the vertical return power are integrated separately in the floating point integration.

With additional modification, the dual polarization antenna switching capability can be enhanced to 834 m sec which will complete recovery of the polarization phase information so that the relative phase scattering matrix $S_{RM}$ can be measured within less than 4 m sec time frames, which would be just below the projected decorrelation time for hydrometeor investigations.

Similar experiments have been carried out by Ross and Freeny (1964), and their results are summarized in Table 10.

<table>
<thead>
<tr>
<th>Transmission</th>
<th>Reception</th>
<th>Measured Parameters</th>
</tr>
</thead>
<tbody>
<tr>
<td>(1) Vertical</td>
<td>Vertical</td>
<td>$</td>
</tr>
<tr>
<td></td>
<td>Horizontal</td>
<td></td>
</tr>
</tbody>
</table>

If $|S_{HV}| \neq 0$, proceed with the following measurement

| (2) Horizontal | Horizontal | $|S_{HH}|$, $|S_{HV}|$, $\phi_{HH} - \phi_{HV}$ |
|               | Vertical   |                     |

Continued
If $|S_{HV}| = 0$, replace (2) by (3)

| $\phi = 45^\circ$ | Horizontal simultaneous reception $|S_{HH}|/\sqrt{2}$ | $|S_{VV}|/\sqrt{2}$ $\phi_{HH} - \phi_{VV}$ |

6.2B Circular Polarization Basis

The information of the backscattered characteristics of a radar target can be greatly improved if successful suppression of clutter return can be achieved. Advances in the field of radar polarimetry have clearly shown the usefulness of knowledge of the complete vector characteristics of the scattered fields which led to the development of dual-channel polarization diversity radar. Such a system is capable of transmitting arbitrary polarization and receiving the backscattered wave in two channels, one of which is polarized parallel to the transmitting channel, while the other is orthogonal to it. The complete scattering matrix can be measured in an arbitrary polarization basis by alternately switching the transmitting and receiving channels.

Successful use of this type of system has been documented by such investigators as G.C. McCormick and A. Hendry, F.E. Nathanson and I. Poelman. Their published works are referenced in the attached bibliography.

A brief description of the systems used by the above-mentioned investigators is given here:

Nathanson (1975): Has proposed a technique to discriminate effectively against rain clutter which is implemented in a two (2) channel system with the same and opposite sense of polarization (circular) to that of a transmitter available at the inputs. The schematic diagram is shown in Figure 15.

![Circuitry for adaptive circular polarization (Homodyne Modulator = same circuitry as Homodyne Detector).]
McCormick and Hendry (1975): Realized a system determining certain parameters of rain clutter using "the ideal polarization diversity radar" (1975). See Figure 16.

Poelman (1981): Introduced another way of suppressing rain clutter in an X-Band radar facility for polarization signature studies of targets and clutter. This system has a dual-polarized antenna which receives all back-scattered power of the target in the parallel and orthogonal polarizations in separate channels.

It can utilize both linear and circular polarization, the latter used primarily for rain clutter suppression; whereas, the first for target in clutter discrimination.

6.3 Amplitude and Absolute Phase Measurements

The method of measuring amplitude and absolute phase information can be used to determine the scattering matrix completely by transmitting only two (2) linear polarizations and receiving three (3) linear polarizations, see Table 11.
The amplitudes and phases of the elements of the scattering matrix were measured at ElectroScience Laboratory of Ohio State University (ESL-OSU). The experiments were conducted (Walton 1982) on a frequency domain range yielding the backscattered returns $S_{VV}$ and $S_{HH}$ ($S_{HV}$ and $S_{VH}$ being zero in this case).

### TABLE II: TRANSMITTING AND RECEIVING ANTENNA POLARIZATIONS FOR AMPLITUDE AND ABSOLUTE PHASE MEASUREMENT

<table>
<thead>
<tr>
<th>TRANSMISSIONS</th>
<th>RECEPTIONS</th>
<th>MEASURED PARAMETERS</th>
</tr>
</thead>
<tbody>
<tr>
<td>Vertical</td>
<td>Vertical</td>
<td>$</td>
</tr>
<tr>
<td>Vertical</td>
<td>Horizontal</td>
<td>$</td>
</tr>
<tr>
<td>Horizontal</td>
<td>Horizontal</td>
<td>$</td>
</tr>
</tbody>
</table>

FRENEY (1965)

7. ASPECT-DEPENDENT PROPERTIES OF OPTIMAL POLARIZATION NULL LOCI MOTION ON THE POLARIZATION SPHERE

7.1 Vector Scattering Center Interaction

Of particular relevance to this electromagnetic target scattering problem is the interaction of polarization/depolarization sensitive scattering centers on a single closed target of irregular shape, which was first attacked rigorously by Huynen (1960) in his dissertation. In this masterpiece, he developed his little understood "N-target decomposition theorem", utilizing canonical properties of the distributed target's Stokes matrix, which specifically applies to clutter analysis and multiple vector scattering center interaction of single targets. This theory is of paramount importance to further advancement in radar polarimetry. Although it still requires extensive extension, it clearly paves the single unique method of complete polarimetric description of radar clutter and average object, an immensely complicated electromagnetic inverse problem. We are fortunate to have the senior expert in the field, Dr. J. Richard Huynen (1982), join the efforts of this research task, and in a separate report entitled, "A revisitation of the phenomenological approaches with application to radar target decomposition", he has further developed on his N-target decomposition theory.

Whereas, in collaboration with Morgan and Weisbrod (Teledyne Micronetics) to polarimetric CW radar target characteristics description, we are following the direction of extracting a complete set of most simple canonical target shapes (such as the sphere, the linear wire target, the n-bounce corner reflector, the left/right winding helices, the cone-tip/ogival and/or spherical capped truncated cylinders with and without fins, bumps, protrusions, etc. treated in Boerner et al., Jan 15/Sept, 1982), in consultation with Bennett and Mieras (1982), Sperry Research Center, use of a CW vector dumbbell scattering center (matrix) interaction was chosen. Both methods have proven to provide useful results and can be used for interpretation of the motion of the Huynen polarization fork as function of frequency, relative aspect angle (with respect to line joining the vector scattering centers) and the electric separation of both.
7.2 Dynamic Polarization Fork Motion

Specifically, we observe that for linear (H, V) polarization base pair anchoring, the cross-polarization null move only whenever the principal target symmetry axis is rotated about the line of sight orthogonal to (H, V); and that the co-polarization null locations move on a quasi-circular spiral non-closing paths as function of differential change in aspect angle where, for small electric separation of vector scattering centers the circles remain with isolated patches, whereas, for large electric separation on large circles encircling the total polarization sphere. Furthermore, the specific character of the vector scattering center (as e.g. smooth versus cone-tipped) dictates the relative differential speed with which these loci are transversed (slow versus rapid) as functions of differential aspect angle. We also note that for a large ensemble of closely packed vector scattering centers, the loci of the co-polarization nulls remain within rather small isolated patches on the polarization sphere which is indicative of clutter-type. Furthermore, the specific quasi-circular paths drawn are indicative of clutter motion. We note that the analytical result was verified experimentally by Poelman (1980-1982) as explained in detail in (Boerner, STs 1914, Sept. 30, 1981), and this specific phenomenon of the dynamic fork motion of time frames of below the vector scattering center reshuffling time requires further extensive analytical and experimental studies. In general, the cross-polarization null location for linear symmetric targets is of slow precession type, and the rapid quasi-circular path motion of the co-polarization null location is nutative gyroscopic in nature. We note that this specific dynamic polarization fork behavior is well described by Huynen's "single target" description into five target characteristic parameters ($p_m$, $\gamma$, $\nu$, $\tau_m$,$\Psi$) as detailed in (Huynen, 1982).

However, the electromagnetic inverse problem of decomposing a single radar target into its characteristic polarimetric target vector scattering centers is very complicated and still not resolved.

7.3 Optimal Polarization Null Characteristics of Buoy-Target Models

(Using measurement data of TELEDYNE-MICRONETICS, L.A. Morgan and S. Weisbrod)

In (Boerner et al., Sept., 1981) the optimal target polarization null concept, introduced briefly in this paper, is applied to experimental amplitude-plus-phase matrix data measured by Teledyne-Micronetics for two specific classes of water submerged buoy targets (six types of dihedral corner reflectors and twelve types of cylindrical open-ended pipe sections with specific termination). For the purpose of extracting useful target classification algorithms and also in order to analyze the aspect-angle dependent behavior, the three measured radar cross sections ($\sigma_{HH}$, $\sigma_{VV}$, $\sigma_{VV'}$), the span {S}, the det {S}, $\text{Re} \{(C_{RR}C_{LL})/(C_{RL})^2\}$, $\{|C_{RR}|C_{LL} - |C_{RL}|^2\}$, the spherical angle spanned by two co-pol nulls, and the co/cross-polarization null loci are plotted as functions of aspect angle. In Figures 18a and b, the polarimetric target behavior for a horizontal, truncated open pipe-section above a sea bed, and a vertical pipe section of the same dimensions in isolation,
Figure 18a Polarimetric Target Behavior (Horizontal, Truncated Open Pipe-section above Sea-Level)
Target S31
Diameter 2.5"
Length 3'
Wall Thickness 1 1/8"
Frequency 3.15 GHz

Figure 18b Polarimetric Target Behavior (Vertical Truncated Open Pipe-Section in Isolation)
respectively, is presented, whereas in Figures 19a and b that for a four- 
corner dihedral reflector above a sea-bed and in isolation, respectively. 
Comparing the data obtained for targets above the sea-bed with those in 
isolation, the expected interference behavior for Figures 18a/19a versus 
Figures 18b/19b is apparent and will not be further analyzed in detail, and 
we refer to the associated report by L.A. Morgan and S. Welsbrod(1982) for 
进一步 relevant interpretations (also see Figure 20).

By analyzing the various plots for the two principal target categories 
considered, we are able to verify the fundamental theorems which can be 
derived from Huynen's polarization fork concept (Huynen, 1960, 1970, 1978, 
1982); the target vector scattering interaction theory (Bennett and Mieras, 
1982); the relationship between relative scattering matrix co-polarization 
phase and specular point curvature perturbations of Kennaugh's target silhouette 
area function(Foo, 1982); and the dynamic behavior of the polarization fork 
motion(Poelman, 1980-1982). Although, we have considered here the monochromatic 
(CW) backscattering case for reciprocal symmetrical targets only, it is evident 
that very definite target polarization properties exist which may be utilized as 
target classifiers and identifiers. In the following, we will first summarize our observations and then extract most important polarimetric 
target classifiers.

Observations

(i) Polarization Fork Behavior: The X-POL nulls are in all cases 
antipodal, and the line joining the X-POL nulls bisects the spherical 
angle between the COPOL nulls on the polarization sphere. NOTE, this 
specific polarization fork behavior represents a very efficient method 
of checking on the accuracy of monostatic scattering matrix data for 
the reciprocal target case (i.e. an isotropic target embedded in an 
isotropic propagation medium).

(ii) Huynen's Target Characteristic Angle γ: The spherical angle 
between the two COPOL nulls exhibits very distinct target characteristic 
behavior in dependence of strength and separation of the target's most 
definite vector scattering centers(We will have to await the detailed 
analyses of Bennett and Mieras(1982) to obtain more comprehension of 
this aspect of the problem).

(iii) Polarization Transformation Invariants: It was clearly established 
that the two target polarization transformation invariants, i.e. Det {[S]} 
and Span {[S]} do provide constants for whatever measurement polarization 
basis is being used. In addition, these invariants, which exhibit 
identical aspect dependence, but not frequency dependence(see Section 5; 
and in more detail(Foo, 1982)), are contributing parameters towards 
establishing useful target characteristic classification algorithms.

(iv) Relative Scattering Matrix Copolarization Phase (φ_{HH} - φ_{VV}): In 
strict compliance with the results presented in(Foo,1982), the relative 
scattering matrix copolarization phase (φ_{HH} - φ_{VV}) certainly plays a 
dominant role in classifying undulating smooth versus rugged edged 
surfaces. We note here that the circular polarization base identities, 
derived in Foo's thesis(Foo, 1982) and investigated, in parts, in the 
associated report by Morgan and Welsbrod(1982), i.e.\(|c_{RR}|^2 - |c_{RL}|^2\)}
Target 4C1
Height above water 2'
0° Conical cut
Frequency 3.15 GHz

Figure 19a: Polarimetric Target Behavior (Four Corner Dihedral Reflector above Sea-Level)
Figure 19b Polarimetric Target Behavior (Four-Corner Dihedral Reflector in Isolation)
and \( \{\text{Re}[(C_{RR}C_{L})/(C_{RL})^2]\} \), contain rather similar information to \( \{\phi_{HH} - \phi_{VV}\} \). From an examination of the relevant plots in Figures 18a/b, 19a/b, it is evident that the theories on target curvature at the specular point developed in (Foo, 1982) and summarized in Section 5 do present rather important target characteristic classification as well as identification algorithms. For example, the relevant figures(\(\{\phi_{HH} - \phi_{VV}\}; \text{Re}[(C_{LL}C_{RR})/(C_{RL})^2]\)) in Figure 18b for the vertical pipe section amplify the correctness of the formulas resulting from the theories developed in (Chaudhuri et al., 1982; Foo, 1982). In (Morgan and Weisbrod, 1982), the relevance of the quantity \( \{C_{RR}\|C_{LL}\| - \|C_{RL}\|^2\} \), closely related to \( \text{Det}[\{C\}] \), is emphasized. This peculiar expression is shown to be a maximum for dihedral, minimum for a plate or sphere, and zero for a linear or helical target. Here we reproduce Figure 5 of (Morgan and Weisbrod, 1982) in Figure 20, which is being explained and interpreted in detail in their report.

7.4 Recommendations on Pursuing Aspect-Plus-Frequency Dependent Dynamic Polarization Fork Analyses in terms of Vector Scattering Center Interaction Models and Huynen's \( N \)-target Decomposition Theory

There exists one major problem in applying polarimetric techniques derived from the optimal target polarization theory to the polarimetric transient time-
dependent problem of ramp response target cross-sectional area projection shape reconstruction, because this optimal polarization target null theory is derived in the frequency only as a mono-frequency theory. Both Kennough (1949-1981) and Huynen (1960, 1970, 1978, 1982) were aware of this complication in their pioneering and continuing research efforts of advancing the state-of-the-art in radar target polarimetry. What we urgently need is a time-dependent polarization-sensitive target feature descriptive theory which is still not completely available.

8. CONCLUSIONS

We have demonstrated in our analyses on radar target polarimetry that broadband radar polarimetry deserves the full attention of Naval Research Centers involved in target/clutter handling in the boundary layer of an ocean environment.

Specifically, the propagation assessment of electromagnetic waves along the marine boundary layer in an ocean environment is of justifiable concern to the Navy and applies to improving techniques of surveillance, communications, navigation, electronic warfare, and optimum sensor design. Considerable efforts have been made, primarily at NOSC, NADC, MWC, and NRL, to characterize the marine boundary layer using electromagnetic (EM: specifically microwaves and millimeter waves) and electro-optical (EO: infrared and visible) remote probing methods. Utilizing, in parts, the electromagnetic wave interrogation properties at frequencies from 200 MHz to 100 THz led to the development of such lower atmospheric assessment systems as IREPS (Integrated Refractive Effects Prediction System), PREOS (Prediction of Performance and Range for EO System), and TESS (Tactical Environment Support Systems).

A thorough literature analysis on the available EM/EO systems shows, however, that hitherto, no, or extremely little, use of the broadband polarization vector properties of the electromagnetic wave have been made. This apparent shortcoming can result in considerable ambiguity of environmental assessment, particularly in the spectral band of 1 GHz to 400 GHz. Therefore, the main objective of this research is to promote the efficient use of novel polarimetric radar techniques, to improve upon the assessment performance of existing m-to-mm-wave radar systems, and to develop complete polarimetric chirp radar systems which allow sub-millisecond acquisition of the radar scattering matrix and real-time sea clutter and/or target classification and identification. We emphasize that electromagnetic vector wave interrogation with material bodies can best be identified as a polarization-sensitive target feature (spatial and temporal) spectral frequency resonance phenomenon. Every effort needs to be made to implement these important polarimetric/scatterometric methods into existing and newly-to-be-developed vector wave scattering techniques for a more reliable propagation assessment of electromagnetic waves along the marine boundary layer in an ocean environment.

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10. REFERENCES


L.J. Battan, J.B. Theiss, "Depolarization of Microwaves by Hydrometeors in a Thunderstorm", J. ATMOS. SCI., 27, 974-977.


E. M. Kennaugh, "Effects of Type of Polarization on Echo Characteristics: Monostatic Case", Rept. 389-1 (Sept. 16, 1949) and 389-4 (July 16, 1950). The Ohio State University, Antenna Laboratory, Columbus, Ohio. 43212.


J. D. Kraus, Guest Editor, IEEE Trans. AP-12(7), Dec. 1964, "Special Issue on RADAR ASTRONOMY".


J. D. Kraus, 1966: Radioastronomie, McGraw-Hill.


A. Schrott, "Rahmenbetrachtungen and Experimentvorschläge zum geplanten 20/30-GHz- Satellitenexperiment", DFVLR, Institut für Hochfrequenztechnik, 1B 551-80/14 (1980).


G. Tricoles and E. L. Rope, "Polarization Independent Radomes", GACIAC PR-81-02, pp.121.


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