THE MELTING OF SOLID AND POROUS ALUMINUM UNDER SHOCK COMPRESSION

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The melting point of shock compressed aluminum is calculated according to the prediction of the Lindemann law and the Gruneisen equation of state. The position of this point on the Lindemann melting curve varies with two parameters - the initial state and the porosity of aluminum. Because the Hugoniot of aluminum depends upon these parameters, variation in either the initial state or the porosity implies a family of Hugoniots. Normally, a characteristic (complete)
20. (Cont.)

equation of state is needed to determine these Hugoniots. But, in this work a family of Hugoniots is derived for each parameter by using the Gruneisen equation of state and a single experimental Hugoniot. Several melting points are calculated corresponding to various values of the parameter of a Hugoniot. These melting points, Hugoniots, and other thermal properties calculated from the theory are in excellent agreement with the data cited.
NSWC TR 79-279

FOREWORD

This report is a part of a study to make a reliable assessment of how certain materials behave when placed in a shock loading system. An important aspect of this study is the equation of state which describe the thermodynamics of each material during the loading process. Since aluminum is a significant feature in the design of many systems, a case is made for using the Gruneisen equation of state to describe the thermodynamic properties of aluminum. The Lindemann law of melting is used in conjunction with the Gruneisen equation of state to determine when the shock strength is great enough to melt the aluminum.

The author acknowledges the valuable assistance of Drs. D. J. Pastine, D. J. O'Keeffe and H. D. Jones. This work was prepared under the task number ZR01305.

J. F. PROCTOR
By direction
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Zel'Dovich\(^1\) and Kormer\(^2\) have pointed out the difficulties in attempting to measure temperatures in opaque solids. As a result, only limited data on temperatures at the melting state of a solid at ultra high pressures are available. However, a number of attempts have been made to calculate melting state temperatures by constructing the solid's equations-of-state. This approach was used by Urlin\(^3,4\) and Ivanov\(^3\) in attempts to calculate the melting points of the elements aluminum, copper, nickel, and lead at high, Hugoniot pressures. Except for some agreement on the pressures required for melting these elements, there is considerable difference between the melting temperatures. In the first treatment, Urlin uses the Lindemann law in conjunction with an equation of state to calculate the melting states of each metal. This particular procedure is supported by O'Keeffe\(^5\) who has shown, in the case of copper, that the Lindemann law gives an accurate description of the experimentally determined melting states of a solid.


O'Keeffe's technique minimizes the number of approximations needed in the equation of state. Thus, more reliability is provided to his predictions. On the other hand, the equation-of-state used by Urlin contain additional approximations, such as, constant specific heat capacity and the Dugdale-MacDonald formula for the Gruneisen parameter. Pastine, White, and Anderson have demonstrated the inadequacy of the Dugdale-MacDonald formula.

In this report, O'Keeffe's procedure will be used to recalculate the melting point of aluminum on its Hugoniot by using a Gruneisen equation of state which does not contain any of the approximations already mentioned. Pastine, Forbes, and O'Keeffe have shown that this equation-of-state gives an accurate description of the thermal properties of a metal. A more detailed description of this equation and how it is used to calculate the temperature on the Hugoniot as a function of pressure are discussed in Chapters 2 and 4.

A Hugoniot describes the behavior of a specific property of a material as it is compressed through various shock states. A Hugoniot for any flow or thermal property is derivable from the shock relations and a characteristic equation of state. It is clear from the shock relations that the initial state and the initial porosity are parameters of the Hugoniot. A family of Hugoniots can be generated by simply varying either parameter.

A characteristic equation of state is not used for porous aluminum. Instead, a Gruneisen equation of state is used to generate a family of Hugoniots from a single experimental Hugoniot. This procedure is described in Chapter 3.

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The Lindemann law is used to describe the melting states in both solid and porous aluminum. Therefore, the melting point of aluminum is in a shock state satisfying both the Lindemann law and the Hugoniot of aluminum. Since the Hugoniot is parametric, there is a "family" of melting points corresponding to a family of Hugoniots due to a specific parameter. These melting points and their Hugoniots are compared with data cited from other reports. Thermal properties—e.g., the zero-degree isotherm, the enthalpy at zero pressure, the temperatures and pressures on the various Hugoniots—are calculated and compared to the results of other investigators.
CHAPTER 2

EQUATION OF STATE FOR ALUMINUM

Temperatures on a Hugoniot cannot be determined without an equation of state unless they are measured experimentally. In this section, an equation of state for aluminum will be described.

In simple solids such as aluminum, copper, lead, etc., the Gruneisen equation of state is accurate enough for calculating Hugoniot temperatures. The components of this equation are the zero-degree isotherm, \( P_0(x) \), the Gruneisen function, \( \Gamma(x) \), and the specific thermal energy, \( E_T(x,T) \). The variables are \( x \), the compression ratio, and \( T \), the temperature. In this case, the compression ratio is defined as

\[
x = \frac{V}{V_0}
\]

where \( V \) is the specific volume. The density of the solid is \( \rho = 1/V \). A zero subscript on variables denotes their values at zero absolute temperature and zero pressure.

The Gruneisen equation of state for the pressure-volume-temperature relationship is

\[
P(x,T) = P_0(x) + \rho_0 \Gamma(x) \frac{E_T(x,T)}{x}
\]

in which \( P(x,T) \) is the pressure. The second term in this equation is referred to as the thermal pressure.

The specific thermal energy for the metals of interest can be described reasonably well by the Debye relation

\[
E_T(x,T) = 3 N_0 kTD(\theta/T)/M.
\]

The constants in this equation are \( N_0 \), \( k \), and \( M \) which are respectively Avogadro's number, the Boltzmann constant, and the atomic or molecular weight of the solid. In Equation (2), \( D(\theta/T) \) is the Debye function, and \( \theta \) is the Debye temperature (see Appendix A). In the limit, \( T = 0 \), \( E_T(x,0) = 0 \).

The specific internal energy of a material may be written as

\[
E(x,T) = \phi(x) + E_T(x,T)
\]
where \( \phi(x) \) is associated with the zero-degree isotherm and is defined as

\[
\phi(x) = E(x,0).
\]  

(4)

The zero-degree isotherm is defined by

\[
P_0(x) = -\rho_0 \frac{d\phi(x)}{dx}
\]

(5)

and the Gruneisen function is defined by the relation

\[
\rho_0 \Gamma(x) = x \left( \frac{3P}{\partial E} \right)_x.
\]

(6)

However, a function defined by

\[
\psi(x, T_1) = \rho_0 \left[ \phi(x) - \phi(x_i) + E_T(x, T_1) \right]
\]

(7)

is found more convenient to work with in describing the zero-degree isotherm along a Hugoniot which takes the form

\[
S = a + bu.
\]

(8)

Here, \( S \) is the shock velocity and \( u \) is the particle velocity. The subscript \( i \) on a variable is used to indicate the initial value of that variable. It is clear from Equation (5) that

\[
P_0(x) = (\frac{\partial \psi}{\partial x})_{T_1}
\]

(9)

in terms of \( \psi(x, T_1) \) as expressed by Equation (7). The function \( \psi(x, T_1) \) and \( (\frac{\partial \psi}{\partial x})_{T_1} \) are calculated by solving the differential equation,

\[
(\frac{\partial \psi}{\partial x})_{T_1} + \Gamma(x) \psi/x = \rho_0 a^2(x-x_i) \left[ 2x - \Gamma(x) (x-x_i) / 2x \right] x_i - b(x_i-x) \]

(10)

which is valid on the shock Hugoniot. See Appendix B for a derivation of this equation. In this equation, \( \rho_0 = 2.7334 \) g/cc, the initial state parameters\( 12,13 \) are \( a = 5.4 \) km/sec, \( b = 1.351 \), and \( x_i = 0.0127 \) at


\[ 13 \] Mitchell, A. C. and Shaner, J. W., Lawrence Livermore Laboratory, University of California, Livermore, California 94550 (Private Communication).
\[ T_1 = 298^{\circ}K \text{ for aluminum. The computed boundary conditions are} \]
\[
\psi(x_i, T_i) = \rho_0 \varepsilon_T(x_i, T_i) = 4.4918 \text{ kbars cc/g},
\]
\[
P_0(x_i) = \frac{3\psi}{x_i} T_i = -\Gamma(x_i) \psi(x_i, T_i)/x_i = -9.96911 \text{ kbars}. (12)
\]

\[ E_T(x_i, T_i) \text{ is calculated from Equation (2) with an initial Debye temperature taken from Reference 14 and adjusted to the value } \]
\[ \psi_i = 386.059 \text{ so that the condition } P(1,0) = P_0(1) = 0 \]

is satisfied. This condition describes the uncompressed state of aluminum at \( V = V_0 \) and \( T = 0^\circ K \). An explicit expression for the Gruneisen function, defined by Equation (6), has been derived by Pastine and Forbes. The value of \( \Gamma(x_i) \) in Equation (12) is calculated by using this expression. See Appendix C.

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CHAPTER 3
GENERATING A FAMILY OF HUGONIOTS

A strong shock wave can raise the temperature of a solid to the point of melting. This melting point is one of the shock states specified by a Hugoniot of the solid. Thus, a Hugoniot must be known before the melting point on it can be determined. In this section, a procedure will be described for calculating a family of Hugoniots from a single experimental Hugoniot, the Gruneisen equation of state, and the shock relation for the conservation of energy.

From an experimental Hugoniot of the type given by the shock velocity, particle velocity relation the pressure in Equation (8), can be calculated by eliminating $E(x,T)$, $\phi(x)$, and $E_T(x,T)$ from equations (1), (3), (7), and the Hugoniot relation

$$E - E_i = (x_i-x)(P+P_i)/2\rho_o$$

(13)

for the conservation of energy across a shock front (see Appendix B). The result at $P_i = 0$ is

$$P(x, T_i) = \frac{P_o(x) + (x) \psi(x, T_i)/x}{1 - \Gamma(x)(x_i-x)/2x}$$

(14)

This form of the Hugoniot clearly indicates that the pressure is dependent upon the initial state of the material. All of the initial state parameters associated with Equation (14), i.e., $\bar{a}$, $b$, $x_i$, $\phi(x_i)$, $E_T(x_i, T_i)$, etc. can be viewed as functions of the initial temperature, $T_i$. Therefore, Equation (14) represents a one-parameter family of Hugoniots, and each member of the family is distinguished by its initial temperature. Consequently, a different Hugoniot is obtained each time the initial temperature, $T_i$, changes to another initial temperature, $T_i'$. The resulting Hugoniot pressure, $P(x, T_i')$, can readily be obtained from Equation (14) by evaluating $\psi(x, T_i')$ and $x_i'$. 

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The compression ratio, $x_i^'$, at a temperature of $T_i^'$ can be calculated from experimental information on the density of the material. The function, $\psi(x, T_i^')$, may be evaluated by observing that for an initial temperature of $T_i$, Equation (7) becomes

$$\psi(x, T_i^') = \rho_o \left[ \phi(x_i^') - \phi(x) + E_T(x_i^', T_i^') \right]. \quad (15)$$

If $x = x_i^'$ in Equation (7), then its value is

$$\psi(x_i^', T_i^') = \rho_o \left[ \phi(x_i^') - \phi(x_i^') + E_T(x_i^', T_i^') \right]. \quad (16)$$

Elimination of $\phi(x)$, $\phi(x_i^')$, $\phi(x_i^')$, and $E_T(x_i^', T_i^')$ from Equations (7), (15), and (16) leads immediately to

$$\psi(x, T_i^') = \psi(x, T_i^') - \psi(x_i^', T_i^') + \rho_o E_T(x_i^', T_i^'). \quad (17)$$

Making use of the boundary conditions from Equations (11) and (12) for an initial temperature of $T_i$, Equation (17) reduces to

$$\psi(x, T_i^') = \psi(x, T_i^') - \psi(x_i^', T_i^') - x_i^' \rho_o (x_i^') / \Gamma(x_i^'). \quad (18)$$

Each term on the right of Equation (18) can be evaluated from the solution of Equation (10) which can be obtained from a single known Hugoniot. (The third term on the right side of Equation (17) could be calculated using Equation (2), but this is not needed.)

Since the new initial temperature, $T_i^'$, is arbitrary, a family of Hugoniots, $P(x, T_i^')$, in the form of Equation (14), can be calculated by using Equation (18), which is evaluated from data along a single Hugoniot.
A procedure is described in this section for calculating the melting temperature on any member of the family of Hugoniot's for aluminum. The first step in this procedure is to eliminate $x$ between Equations (1) and (14) to obtain the temperature which can be expressed in the form

$$T = f(P, T_i)$$

Without the single experimental Hugoniot of Equation (8), this result could not be achieved without a characteristic equation of state. For example, an equation of state in the form $E = E(x, P)$ is sufficient to produce a family of Hugoniot's of the form $P = P(x, T_i)$ from Equation (13). However, without additional information, the temperatures on these Hugoniot's cannot be calculated.

Next, apply the Lindemann law for melting in the form

$$T = T_s \left( \frac{\theta}{\theta_s} \right)^2 \left( \frac{x}{x_s} \right)^{2/3}.$$  

Here the subscript $s$ refer to the static values of $T$, $\theta$, and $x$ for melting at a pressure of $P = 0$. These values are $T_s = 9320 K$, $\theta_s = 324.085 K$, and $x_s = 1.07106$ for aluminum. Elimination of $x$ between Equations (1) and (20) leads to a melting law which can be expressed in the form

$$T = g(P).$$

The simultaneous solution to Equations (19) and (21) is the melting point of aluminum on a family-member Hugoniot. The solution will be denoted by

\[ T_m = T_m(T_i), \quad P_m = P_m(T_i) \]  

(22)

which shows that the melting state \((T_m, P_m)\) is dependent upon the initial temperature, \(T_i\).
CHAPTER 5
THE HUGONIOT FOR POROUS ALUMINUM

Let $V'$ represent the specific volume of porous aluminum, $V_h$, the specific volume of the holes, and $V$, the specific volume of the solid portion of aluminum, then

$$V' = V + V_h.$$  

Dividing all specific volumes by $V_o$ leads to

$$x' = x + x_h.$$ 

(23)

Porosity, $M$, is defined by the ratio

$$M = \frac{V'}{V} = \frac{x'}{x}.$$ 

The initial porosity is

$$M_i = \frac{x_i'}{x_i}.$$ 

(24)

Assuming that the Gruneisen equation of state and the conservation law apply to porous aluminum, it follows immediately that Equation (14) is the Hugoniot of porous aluminum with the quantities $x$, $x_i$, and $T_i$ replaced by $x'$, $x_i'$, and $T_i'$. Except for the initial values, thermal quantities do not depend significantly upon the details of compaction behavior at high pressures. This means that the shock pressure is too high to experience any substantial resistance to collapse in the porous part of the solid. After collapse, $x_h = 0$, and Equation (14) reduces to

$$P(x, T_i') = \frac{P_o(x) + \Gamma(x) \psi(x, T_i')}{1 - \Gamma(x)} \left( M_i x_i - x \right) / x.$$ 

(25)


The fact that $\psi(x, T'_i) = \psi(x, T_1)$ is a direct consequence of the theory that the initial energy of a solid does not change with its porosity. Thus, the Hugoniot of a porous material is related to that of a solid by the relation,

$$P(x, T'_i) = \frac{2x - \Gamma(x) (x_i - x)}{2x - \Gamma(x) (M_i x_i - x)} P(x, T_1). \quad (26)$$

In this relation, $P(x, T_1)$ is the Hugoniot of the solid defined by Equation (14). This relation shows that the pressure in a porous solid is greater than the pressure in a non-porous solid. The temperature corresponding to the porous pressure, $P(x, T'_i)$, is calculated from Equation (1). The shock velocity ($S'$) and particle velocity ($u'$) for a porous solid may be calculated from the shock relations in the form

$$S' = x'_i \sqrt{\frac{P(x, T'_i)}{\rho_o(x'_i - x)}} \quad (27)$$

$$u' = \sqrt{(x'_i - x) \frac{P(x, T'_i)}{\rho_o}} \quad (28)$$
The results from certain components of the Gruneisen equation of state are discussed and compared in this section to the work of other investigators. Also the numerical results for the pressures and temperatures on several Hugoniot-family members are presented and the numerical values of the melting points on these Hugoniots are given.

The Gruneisen equation of state is a sum of two important terms: the zero-degree isotherm and the thermal pressure. (See Chapter 2)

The zero-degree isotherm, $P_0(x)$, defined by Equation (9), was evaluated from the numerical solution to Equation (10). The values $P_0(x)$ is tabulated in Table 1. A corresponding graph of $P_0(x)$ is compared to Pastine's\textsuperscript{22} calculation in Figure 1. Although Pastine used a different approach to compute $P_0(x)$ for aluminum, his results are quite close to the current results, especially at lower pressures.

Since $P_0(x)$ is connected to the function $\psi(x, T_i)$ through the differential Equation (10), there is an indirect way in which $P_0(x)$ can be compared to experimental data. Consider the thermal quantity, enthalpy, which is given

$$H = E + PV.$$  

At a pressure of $P = 0$, the enthalpy,

$$H = E.$$  

From Equations (3) and (7), it is obvious that

$$\Delta H = E(x_i', T_i') - E(x_i, T_i) = V_0 \psi(x_i', T_i') - V_0 \psi(x_i, T_i')$$

### TABLE 1 NUMERICAL EVALUATION OF THE ZERO ISOTHERM AND RELATED FUNCTIONS

<table>
<thead>
<tr>
<th>X</th>
<th>$\Psi(X,T_1)$ (Kbars · cc/g)</th>
<th>$\Gamma(X)$</th>
<th>$P(X,T_1)$ (Kbars)</th>
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<td>1658.89</td>
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</table>

X = $V/V_o$, THE COMPRESSION RATIO

$\Psi(X,T_1)$ = ENERGY DENSITY OF EQ (7)

$\Gamma(X)$ = GRÜNEISEN FUNCTION

$P(X,T_1)$ = HUGONIOT PRESSURE

$P_o(X)$ = ZERO ISOTHERM
FIGURE 1  ZERO DEGREE ISOTHERM FOR ALUMINUM
Using Equation (18), the change in enthalpy reduces to

$$H = - V_0 \psi (x'_1, T_1) - V'_1 \frac{P_0(x'_1)}{\Gamma(x'_1)}$$  \hspace{1cm} (29)

which was readily evaluated numerically and listed in Table 2. The graph of these calculations in Figure 2 is in good agreement* with the experimental data\textsuperscript{18} plotted in Figure 2.

A list of values from the Gruneisen function, $\Gamma(x)$ is also included in Table 1. The Gruneisen function is a factor used to calculate the thermal pressure. A listing of thermal pressure on three Hugoniots of the same family for initial temperatures of $T_i = 278^0K$, $298^0K$, and $318^0K$, is in each of the Tables 3, 4, and 5.

The pressures and temperatures on these same three Hugoniots were calculated using the procedure of Section 4 as represented by Equation (19). These results are also listed in Tables 3, 4, and 5. The experimental Hugoniot at $T_i = 298^0K$ is in Figure 3 and is compared with data from the other two Hugoniots at $T_i = 278^0K$ and $318^0K$.

A list of temperatures and pressures calculated from the Lindemann law in the form of Equation (21) is in Table 6. This list is used to graph the melting states of aluminum in Figure 3. The intersections of this curve from Table 6 with the three Hugoniots are the melting points of aluminum for three of its initial states. These three points are listed in Table 7 along with other parameters dependent upon the initial temperature. Since these three points and all other similar melting points must fall on the Lindemann melting curve, this curve gives the melting point of aluminum as a function of initial temperature. Equation (22) is the parametric representation of this melting curve.

The results in Table 7 for the melting point of aluminum may compared with the calculations of Uurlin.\textsuperscript{3,4} He calculated the melting point to be (1.15 Mbars, 40000 K) or (1 Mbar, 30000 K), the results of two different procedures. Evidence of melting in aluminum at about 1 Mbar of shock pressure has been indicated also by experiments of Isbell and Sakharon.\textsuperscript{23,24}


*This agreement is quite good even though $\Delta H$ is calculated beyond the range defined by Equation (29). Equation (29) defines $\Delta H$ only within the range of $x_i^1 \leq x_i$, i.e., within the limiting range of the Hugoniot ($x \leq x_i$).
### TABLE 2 ENTHALPY AS A FUNCTION OF TEMPERATURE AT P = 0

<table>
<thead>
<tr>
<th>T (°K)</th>
<th>X</th>
<th>(\Delta H) (CAL/MOL)</th>
<th>(\Delta H_{\text{exp}}) (CAL/MOL)</th>
</tr>
</thead>
<tbody>
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</tr>
<tr>
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<td>610</td>
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<tr>
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<td>1.02976</td>
<td>1352.56</td>
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<td>2005.82</td>
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<td>1.04739</td>
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<td>1.07496</td>
<td>4423.84</td>
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</table>

\(T = \text{ABSOLUTE TEMPERATURE}\)

\(X = V/V_0, \text{ THE COMPRESSION RATIO}\)

\(\Delta H = \text{THE CALCULATED CHANGE IN ENTHALPY}\)

\(\Delta H_{\text{exp}} = \text{THE EXPERIMENTAL CHANGE IN ENTHALPY (REFERENCE 18)}\)
FIGURE 2 CHANGE OF ENTHALPY WITH ABSOLUTE TEMPERATURE AT P = 0
TABLE 3 HUGONIOT PRESSURES AND TEMPERATURES FOR T, = 278°C

<table>
<thead>
<tr>
<th>X</th>
<th>T(X,Ti) (°K)</th>
<th>P(X,Ti) (Kbars)</th>
<th>U(X,Ti) (KM/SEC)</th>
<th>Pt(X,Ti) (Kbars)</th>
</tr>
</thead>
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X = V/Vo, THE COMPRESSION RATIO
T(X,Ti) = HUGONIOT TEMPERATURE
P(X,Ti) = HUGONIOT PRESSURE
U(X,Ti) = HUGONIOT PARTICLE SPEED
Pt(X,Ti) = THERMAL PRESSURE ON HUGONIOT
### TABLE 4: HUGONIOT PRESSURES AND TEMPERATURES FOR \( T_i = 298^\circ K \)

<table>
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<th>( X )</th>
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<th>( P_T(X,T_i) ) (Kbars)</th>
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\( X = V/V_o \), THE COMPRESSION RATIO

\( T(X,T_i) \) = HUGONIOT TEMPERATURE

\( P(X,T_i) \) = HUGONIOT PRESSURE

\( U(X,T_i) \) = HUGONIOT PARTICLE SPEED

\( P_T(X,T_i) \) = THERMAL PRESSURE ON HUGONIOT
### Table 5: Hugoniot Pressures and Temperatures for $T_i = 318^\circ$ K

<table>
<thead>
<tr>
<th>$X$</th>
<th>$T(X,T_i)$ ($^\circ$K)</th>
<th>$P(X,T_i)$ (Kbars)</th>
<th>$U(X,T_I)$ (KM/SEC)</th>
<th>$P_T(X,T_i)$ (Kbars)</th>
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</table>

$X = V/V_o$, the compression ratio

$T(X,T_i) =$ Hugoniot Temperature

$P(X,T_i) =$ Hugoniot Pressure

$U(X,T_I) =$ Hugoniot Particle Speed

$P_T(X,T_i) =$ Thermal Pressure on Hugoniot
### TABLE 6 MELTING TEMPERATURE AND PRESSURE AS A FUNCTION OF COMPRESSION RATIO

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FIGURE 3  THE MELTING POINT OF ALUMINUM AS THE INTERSECTION OF LINDEMANN'S MELTING CURVE AND THE HUGONIOT (SHIFTS IN HUGONIOT DUE TO INITIAL TEMPERATURE DIFFERENCES IS ALSO SHOWN)
TABLE 7
MELTING POINT DEPENDENT ON INITIAL STATE OF Al

<table>
<thead>
<tr>
<th>( T_i ) ((^{\circ})K)</th>
<th>( P_m ) (Mbars)</th>
<th>( T_m ) ((^{\circ})K)</th>
<th>( a(T_i) )</th>
<th>( b(T_i) )</th>
<th>( x_i )</th>
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</table>
Once the Hugoniot is established, a similar procedure is used to calculate the melting point of porous aluminum at porosities of $M_i = 1.4$ and 1.7. To accomplish this, the Gruneisen relation is used to express the Hugoniot, Equation (25), of a porous solid into the form of Equation (19).

The intersection of these Hugoniots with the melting curve is in Figure 4. These points show that the melting point of aluminum decreases as its porosity increases. At $M_i = 1.7$, Figure 4 shows that the melting point is about 66 kbars and 1260 K. This result is in excellent agreement with the experimental estimate of 66 kbars 1220 K reported by Asay and Hayes. They calculated a value of 75 kbars and 1340 K as their best theoretical result for this point. This indicates that the theoretical result from Figure 4 is more accurate than their result.

The straight-line appearance of the Hugoniot graphs in Figure 4 indicate that the slope of these graphs may be constant. Plots of the slope, $\frac{dT}{dP}$, as a function of pressure are in Figure 5. These plots show that the slope of the Hugoniot, $\frac{dT}{dP}$, does vary with the pressure, but is not much different from the initial value,

$$
\frac{dT}{dP} \bigg|_{p=0} = \frac{x'_i - x_i}{2 \rho \ C_v} + \Gamma_i \frac{T_i x_i - \Gamma_i (x'_i - x_i)}{2 / \rho a^2}.
$$

(Note that $C_v$ is the initial specific heat at constant volume, $\Gamma_i = \Gamma(x_i)$, and $x'_i = M_i x_i$.)

The theoretical Hugoniots of porous aluminum are calculated from Equations (27) and (28). Figure 6 shows that these Hugoniots are in good agreement with reported data. Pastine has also calculated the results shown in Figure 6, but he used first principles to derive the Gruneisen equation of state.

The specific values of $M_i = 1.4$ and 1.7 above no porosity of $M_i = 1$ were chosen because these values clearly show the dramatic change that porosity can produce in the melting point of a simple solid. Figure 4 clearly shows the large changes in the melting point of aluminum as the porosity of aluminum takes on the values of

---


FIGURE 4  EFFECT OF POROSITY ON THE MELTING POINT OF ALUMINUM
FIGURE 5 CHANGE IN TEMPERATURE – PRESSURE SLOPE WITH PRESSURE ON THE HUGONIOT OF POROUS ALUMINUM
DATA:
- ○ X $M_i = 1.4, 1.7$ (Anderson, et al)
- □ $M_i = 1.7$ (J.R. Asay & D.B. Hayes)

FIGURE 6  SHOCK SPEED VERSUS PARTICLE SPEED FOR POROUS ALUMINUM
M_i = 1, 1.4, and 1.7. In addition, these specific values for M_i were chosen because data and other calculations have been reported on these same values of \( \mu_i \) so that the current results can be compared to these previous works.

There was concern with whether small changes in the initial state of aluminum would change significantly the melting point of aluminum. As can be seen from Table 7 and Figure 3 that small changes in the initial state of aluminum from \( T_i = 298^\circ K \) to \( T_i = 318^\circ K \) or \( T_i = 278^\circ K \) do not produce any great change in the melting point.
CHAPTER 7

SUMMARY

The dependence of a Hugoniot on the initial state of aluminum is represented by a one-parameter family of pressure-volume Hugoniots. The initial temperature is used as the parameter of the family. Instead of a characteristic equation of state for aluminum, the Gruneisen equation of state is used to generate a family of Hugoniots from a single experimental Hugoniot. In the absence of available experimental temperatures, the P-V-T form of the Gruneisen equation of state is used to calculate temperatures on these Hugoniots.

The Lindemann law of melting is used to calculate the melting point of aluminum on three Hugoniots. The pressures producing melting were calculated and are in good agreement with the results of other investigators. However, the corresponding melting temperatures differed appreciable from values reported by others. This difference is attributed to differences in the more exact form of the Gruneisen equation of state chosen for these calculations as compared to the equations used by other investigators. For this reason, the current results should be an improvement over previous calculations.

Using the theory of compaction, a one-parameter family of Hugoniots has been derived for porous aluminum with porosity as the family parameter. The melting points and the Hugoniots of porous aluminum are in good agreement with reported data.
REFERENCES


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APPENDIX A

DEBYE FUNCTION

In terms of the Gruneisen function, the Debye temperature is

\[ \Theta(x) = \Theta(x_1) \exp \left( \int_{x}^{x_1} \frac{\Gamma(z)}{z} \, dz \right). \]

Correspondingly, the Debye function is defined by the integral relation

\[ D(y) = \frac{3}{y^3} \int_{0}^{y} \frac{z^3 \, dz}{e^z - 1} \]

for \( y = \frac{\delta}{T} \).

---

\( A^\text{-1} \)


\( A^\text{-2} \)

APPENDIX B

SHOCK RELATIONS AND THE HUGONIOT

The shock relations\textsuperscript{B-1} for the conservation of mass, momentum, and energy across the shock front are

\begin{align*}
x_i (S - u) &= xS \\ x_i (P - P_i) &= \rho_0 Su \\ \frac{(S - u)^2}{2} + \frac{P}{\rho} + E &= \frac{S^2}{2} + P_i/\rho_i + E_i.
\end{align*}

(B1) (B2) (B3)

Elimination of $S$ and $u$ among the Equations (8), (B1), and (B2) leads to a Hugoniot of the form

\[P(x, T_i) = \frac{\rho_0 a^2 (x_i - x)}{\left[x_i - b(x_i - x)\right]^2}, \quad (x \leq x_i).\]

(B4)

Elimination of $S$ and $u$ among Equations (B1), (B2), and (B3) leads to the conservation of energy Equation (13). Comparing Equations (14) to (B4) and making use of Equation (9) lead to the differential Equation (10).

APPENDIX C
GRUNEISEN FUNCTION

The Pastine-Forbes\textsuperscript{C-1} expression for the Gruneisen function is

$$\Gamma(x) = \frac{2x\left[2bx - \delta(x_i - bn)\right] + 2x^2(x_i - bn) + 4x^2v}{2n^3v + n^2\left[2bx - s(x_i - bn)\right] + v(x + x_i)(x_i - bn) + 2(x_i - bn)^3C_p/aa^2}$$

In this equation, $\eta = x_i - x$. The unidentified parameters in this equation are $C_p$, the specific heat capacity at constant pressure; $\alpha$, the volume coefficient of thermal expansion; and the others are defined by the relations\textsuperscript{C-2}

$$\delta = 1 - \frac{1}{\alpha} \left[ \frac{\partial}{\partial T_i} \log \alpha^2 \right]_{P_i=0}$$

$$\nu = \frac{1}{\alpha} \left[ \frac{\partial b}{\partial T_i} \right]_{P_i=0}.$$
The values of the new parameters in the Gruneisen function are $\alpha = 69.6 \times 10^{-6}/\text{O}K$, $C_p = 5.82 \text{ cal/mole O}K = 0.90298 \text{ J/gO}K$, $\delta = 2.95135$, and $\nu = 1.9407$ for aluminum at $T_i = 298^\circ\text{OK}$.


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