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TWO PENETRATION MODELS FEATURING BOMBER DEFENSE MISSILES AGAINST AN AWACS AIR DEFENSE

THESIS

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THESIS

Presented to the Faculty of the School of Engineering of the Air Force Institute of Technology Air University in Partial Fulfillment of the Requirements for the Degree of Master of Science by

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Preface

I first became interested in the modeling of bomber defense missiles (BDMs) after reading a 1969 article by Clifford Fawcett and Chester Jones of ASD. In the article they presented a model for estimating BDM contributions to bomber effectiveness, asserting that no comparable models existed at that time.

Because BDMs seemed to be an attractive option for strategic bombers, I did a brief modeling survey, followed by conversations with strategic analysts. This led me to believe that most force-on-force analysis of BDMs is performed with models that are either too complex or too aggregated to be sensitive to subtle variations in BDM features. My thesis goal became to accurately model the strategic merits of BDMs.

I learned much through the thesis process in both professional and personal areas. For helping me through this process, I owe more than a published "thanks" to several people: Lt Col Jim Bexfield for his patient leadership, Capt Rick Wilkinson for his "real-world" experience, Capt Shawn O'Keefe for his all-around positive influence and good humor, and to all the instructors and students who generously showed me there is no such thing as a "dumb" question. Above all others I thank my wife, Susan, for her love and faith in me, at a time when she was single-handedly managing a full-time job, a newborn son, eight dogs and cats, and buying a house.
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Abstract

Two models were developed for evaluation of bomber defense missiles as penetration aids to bombers carrying cruise missiles. The defense consisted of a forward-based AWACS controlling airborne interceptors. Both models utilize a corridor concept with a single AWACS.

One of the models is a simulation using the Q-GERT computer language; penetrators and interceptors wait in "queues" to be paired by the AWACS "server" for interceptor attempts. The second model is a stochastic analytic approach recursively estimating a separate survival probability for each successive bomber to enter the corridor. This probability reflects delays between intercepts due to fighter attrition. Both models estimate the numbers of bombers surviving, cruise missiles launched and cruise missiles surviving.

The models yielded similar results for 24 different cases. The thesis models represent the effects of fighter attrition, BDM depletion and payload tradeoffs in greater detail than do other similar models.
I. **Introduction**

**Background**

A primary mission of the U.S. Air Force is to maintain an effective manned bomber force as part of the triad concept of strategic deterrence. One possible plan for a retaliatory bomber strike calls for many B-52s penetrating enemy forward air defenses (FAD) to deliver air-launched cruise missiles (ALCMs) toward inland targets. The ability of the B-52 fleet to perform this mission is an ongoing concern of defense planners; the structurally old B-52 is relatively slow and has a large radar cross-section (RCS), causing some to doubt its current ability to penetrate Soviet air defenses. Furthermore, Soviet defenses have improved greatly through recent technological advances in radar, surface-to-air missiles, and fighter interceptors, as well as the expected deployment of a Soviet Union Airborne Warning and Control System (SUAWACS) aircraft in the near future.

The gravity of this threat was underlined in a 1975 report by the Secretary of Defense:

1
Should the Soviet Union develop and deploy an AWACS/"Foxbat" "look-down, shoot-down" air defense system, we would have to counter it with new penetration devices and techniques such as the cruise missile, bomber defense missiles and improved ECM [Ref 17:196].

Today several options are being explored to improve the strategic bomber force. Among these are new aircraft to complement or replace the B-52s, longer-range cruise missiles to give the fleet greater standoff range, and penetration aids such as improved electronic countermeasures (ECM), believable decoys, and lethal bomber-defense missiles (BDMs).

The Strategic Planning branch of Aeronautical Systems Division's (ASD) Strategic Systems System Program Office (SPO) is currently studying proposed weapon system innovations to aid the effectiveness of cruise missile carrier aircraft (CMCA) against a Soviet forward air defense with an AWACS network. Part of the analysis consists of running many cases with the SPEED model (Ref 13). The relative merits of various electronic countermeasures and lethal defense missiles is a particular focus of the study, and the SPO is interested in comparing their results with those of alternative methods of modeling.

Statement of the Issue

Although many simulation models and analytical formulations exist for studying general bomber penetration issues, few have been designed to study a modern FAD; a
specialized model including BDMs, ALCMs and ECM would be of unique utility. There is also a need to compare the roles, advantages, and disadvantages of different types of bomber penetration models: large versus small, analytic versus simulation.

Research Objectives

The purpose of the thesis effort is to develop two separate but complementary models for assessing the relative effectiveness of BDMs and ECM against the FAD.

Simulation Model. The first model is a simulation using a Q-GERT network and FORTRAN "user functions." This is the first known application of the concise Q-GERT language to bomber penetration modeling.

Analytic Model. The second model is a sequence of analytic expressions leading to estimates of bomber effectiveness against the FAD.

Secondary Objective. A secondary objective of the study is to compare the two models in terms of:

1. Results obtained for certain research questions
2. Credibility and consistency of results
3. Methodology
4. Model versatility (number of addressable questions)
5. Ease of use and revision
6. Cost and time requirements
7. Sensitivity to key inputs

Scope and Limitations

The scenario modeled in both cases will be restricted to the time interval during which the bomber or CMCA, force is penetrating the enemy's Forward Air Defense (FAD), beginning with detection of the first CMCA or ALCM and ending for each when it is either killed or has successfully penetrated the FAD. The FAD to be considered has two main components: airborne fighter interceptors (AIs) to engage and kill penetrators; and a network of AWACS aircraft which detect CMCA's and ALCMs on radar and then assign and guide FI to intercept each penetrator detected. CMCA's will fire BDMs at interceptors but not at an AWACS.

The models focus on a single penetration "corridor," with only one AWACS aircraft and its associated MI combat air patrol (CAP). Penetrator flight paths are straight lines, and maneuvers to alter course while in radar coverage are not modeled.

Both models are based on the same major assumptions, but the analytic model requires occasional additional simplifications of the scenario. The inputs common to both models include:

1. Number of bombers (limited to 20 in simulation)
2. Number of ALCMs per bomber
3. Number of interceptors
4. AWACS detection ranges for bombers and ALCMs
5. Probabilities of engagement for AI against bombers and ALCMs
6. Kill probabilities by air-to-air missiles (AAMs) and BDMs
7. Bomber arrival rates and ALCM launch schedule

The models are essentially parametric; event probabilities and deterministic detection ranges are not estimated by the model, but must be provided by the user based on external analysis of specific weapon systems. For the purpose of model illustration and comparisons, inputs which are representative of actual capabilities are assumed.

The primary outputs of the models are the number of CMCAs surviving the FAD, number of ALCMs launched, and the number of surviving ALCMs.

**Methodology and Overview**

The overall approach of the study emphasizes the derivation of models and the comparison of methodologies. The study process consists of four main phases.

**Literature Survey**. The first phase consists of research about:

1. previous FAD issues studied;
2. previous bomber penetration modeling--simulation and analytic approaches;
3. derivation of probabilities of detection, conversion, kill, etc., based on pertinent features of ECM and BDMs; and
4. models considering BDMs or general air-to-air combat.

Chapter II summarizes the general findings about bomber penetration models, and Chapter III describes FAD issues and how they are treated in two particular models used at ASD.

Q-GERT Model Development. The simulation model was developed in three general steps: formulation, computerization, and verification.

1. Formulation tasks:
   a. decompose scenario into significant elements
   b. analyze and describe elements
   c. make assumptions necessary for modeling
   d. integrate elements into a network model

2. Computerization tasks:
   a. define constants, variables, inputs and outputs
   b. set initial values for parameters
   c. transform network model into code
   d. debug the program: obtain output
3. Verification: This step consists of inspecting sample runs to determine if the scenario is simulated as intended. Next, outputs for a base case are obtained, and excursions are tested to see if the output is altered in the expected direction when key parameters are changed.

Some iterations of the three steps were necessary. The final form of the model is reported in Chapter IV.

Analytic Model Development. The third phase of the study consists of six iterative steps:

1. Choose desired forms of results: expected value, upper bound, lower bound, and/or probability distribution.
2. Decompose problem into important events. Decide which are dependent and which are independent; analyze cause-and-effect relationships.
3. Make simplifying assumptions about offense, defense, and encounters.
4. Derive event probabilities, resulting in expressions for numbers of surviving CMCAs and ALCMs based on ECM and BDM parameters.
5. Computerize and debug the model. Confirm by hand that the desired calculations are performed correctly.
6. Verify that key factors identified in Step 2 are included. Check results for intuitive validity.
The analytic model is described in Chapter V.

**Model Comparisons.** The key steps in this phase are:

1. Using realistic inputs, generate Q-GERT and analytic results for the same variable cases. Each case is specified by:
   a. whether or not BDMs are loaded; the three levels are zero, four, and eight BDMs;
   b. which of four ECM capabilities (minimal ECM, high ECM vs. AIs, high ECM vs. AWACS radar, or high ECM vs. both) is used by bombers.

2. Compare how the two models rank the alternatives according to the three different measures of effectiveness.

3. Summarize the differences in approaches.

4. Make conclusions about relative advantages, disadvantages, and roles of methodologies; summarize additional insights.

The results of this final phase are reported in Chapter VI.
II. Review of Bomber Penetration Modeling

A large variety of models and techniques exist today for quantitative analysis of strategic bomber penetration issues. Therefore, analysis performed in this area must include selection of the means of analysis to be used in the study process itself. Before choosing a model, or choosing to develop one himself, the analyst should understand his purpose for using a model.

In a broad sense, all models serve the same purpose. They "shed light" on a problem (or problems). Hoeber lists three ways, in order of importance, that a model can do this (Ref 6:6). They are:

1. to increase the understanding, by both analyst and client, of the problem through the modeling process;
2. to aid in making choices between alternatives by providing useful relative numbers for measures of merit; and
3. to give valid absolute numbers as solutions to problems; however, applications where this is possible are rare.

Thus, one difference between models is the varying degrees to which the three purposes above motivate their development. The first purpose, "understanding," may
reflect the greatest value found in models, but the consideration that most often drives the choice of models is a set of specific study questions: how the alternatives will be compared, and the other issues to be addressed by data from a model. For bomber penetration studies, typical questions include:

1. Are bomber force improvements necessary to maintain or achieve a specified level of performance?
2. Which of several proposed changes in offensive tactics or capabilities would most improve bomber force effectiveness?
3. How will certain changes in the tactics, resources, or performance capabilities of the air defense threat affect the overall success of the bomber mission, and what are the best responses by the offense?

Selecting a Model or Models

After identifying the important study questions, the analyst must choose the appropriate modeling technique (or set of techniques) according to some set of criteria. He may wish to perform some level of cost-benefit analysis. The following is a possible checklist for such assessment.

Benefits. Answering several questions about alternative models can tell the analyst whether the techniques can generate the desired quality and quantity of information. The model(s) should:
1. **be applicable to the problem.** To what extent will the set of techniques address the study questions? Are the assumptions in each model appropriate for the problem?

2. **promote confidence in the results.** Are the techniques perceived as valid? Are the assumptions reasonable, and has the model been tested? Does the level of resolution enhance credibility?

3. **be acceptable to the client, or decision maker.** This is a prerequisite when there is a sponsor; if the client does not approve of the choice of models or find the assumptions credible, he will place little faith in the results.

4. **provide additional insights.** Regardless of results, will understanding of the problem increase?

**Costs.** Models can range widely in the resources required for their use. The costs are primarily of two types:

1. **Manpower Costs.** What level of effort is required to meet the deadline? What additional expertise is needed to obtain and input data, and analyze results?

2. **Computer Costs.** How much computer time is required? The expense of generating data should be assessed, including the possible interference with other computer operations.
**Constraints.** There is usually a minimum amount and quality of information desired from use of a model, and there are almost always clear manpower, computer, and budgetary limits for a study. The acceptability of a model to the client, mentioned under "benefits," is clearly also a real constraint. There are two additional constraints:

1. **Availability.** If the desired techniques are not already operational for use, can they be obtained and computerized in time? Also, can the necessary inputs be easily obtained?

2. **Time.** Can the entire study process be performed with these techniques by the project completion deadline?

A given model has five general features which determine the resources required for its use, and the utility of the information it can provide. These are:

1. the purpose it was originally developed for,
2. the output possible from its use,
3. its scope,
4. its level of detail or resolution, and
5. whether its methodology is simulation or analytic modeling.

An analytic model can be defined as

... a solution technique that allows us to write a functional relation between system parameters and a chosen performance criterion in terms of equations that are analytically solvable [Ref 10:17].
Examples of analytic formulations are queueing models, probability models and Lanchester equations. The category is broad, however, and includes simple "paper-and-pencil" calculations and "rules-of-thumb." For example, the function $P_s = e^{-pkI/B}$ has been widely used to express the relationship between a bomber's probability of survival $P_s$, the kill probability $P_k$, and the ratio of interceptors (I) to bombers (B) in a battle. There are cases where this highly aggregated equation gives misleading results, but it can easily yield a gross understanding of key factors in bomber performance.

In contrast, a simulation model means "a numerical technique for conducting an experiment (by a digital computer) or a system evolving in time [Ref 10:18]." Because its concept of time is explicit, a simulation can describe the dynamic behavior of a system. Simulations can represent reality in extensive detail, but can be time-consuming or expensive to use compared to an analytic model.

This chapter will summarize some representative simulation and analytic models which already exist for the study of bomber effectiveness. Key differences between the various types of models will be assessed, and the relative advantages of each will be discussed.

Simulation Models

"Simulation models are attempts to replicate reasonably well understood processes [Ref 1:71]." Such
techniques typically represent the behavior of systems by generating events and activities according to specified deterministic or probabilistic rules.

The critical processes represented in strategic bomber analysis are the interactions between and within offensive flight performance, tactics, and weapon systems and the defense environment. The simulation techniques used historically range from one-on-one models of a bomber-interceptor encounter to force-on-force models of an entire nuclear war. For illustrative purposes, three simulations of bomber penetration scenarios will be described in this section.

Advanced Penetration Model. The Advanced Penetration Model (APM) was created by Boeing Computer Services for Air Force Studies and Analysis in 1969 (Ref 7:2). The Air Force needed a "big picture" model of the bomber mission which could be combined with other war-gaming models for assessment of force-level issues, strategies and deployment decisions. In order to capture the effects of specific weapon systems and also utilize actual war plans, a high degree of resolution, or level of detail, was required.

Twenty analysts and forty programmers were initially committed to its development, and its first major operational application came three years later in the Joint
Strategic Bomber Study (JSBS). Since that time, the APM has been transferred to SAC Headquarters, and both versions continue to evolve and be improved.

The APM simulates the attempted escape of the bomber force from an SLBM attack on the home bases, then each surviving bomber's rendezvous and refueling with tankers, its penetration of the FAD and SAM defenses, weapons delivery and recovery. As a source of inputs, the model computes the assignment of bombers to targets, and many other initial conditions, in the Mission Planner phase.

The unique value of the APM is that it models many individual bomber and defense units simultaneously, capturing the complex effects of defense saturation, command-and-control limitations, and weapon assignment doctrines. Hence it can be used to compare a large variety of alternative forces, weapons and tactics.

The strength of the APM is also its greatest weakness. The wealth of data inputted and outputted gives the model great resolution, but it also makes it expensive and time-consuming to prepare, correct (if necessary) and interpret runs. The manpower and computer time required for each run also makes extensive sensitivity analysis extremely expensive and time-consuming compared to smaller models. For this reason, a major value of the APM is as a source of validation for other models, as well as a source of
input data for simpler models to use for their sensitivity analysis.

Therefore, the APM is at the top of a hierarchy (of bomber penetration models) of the type recommended by G. Clark in 1969 (Ref 3). The APM can be used to provide a detailed analysis of a base case and one or two excursions. Then smaller models borrowing inputs and estimated parameters, and hence some of its credibility from the APM can be used to analyze other excursions.

**SPEED Model.** The SPEED (Simulation of Penetrators Encountering Extensive Defenses) model was originally developed by Calspan Corporation for the Aeronautical Systems Division (ASD) of the Air Force Systems Command (AFSC) (Ref 13). Written in FORTRAN IV, the model was the result of the perceived need for a relatively fast-running penetration model with flexibility and validity comparable to that of APM. The several current versions of SPEED generally run in faster-than-real time. Depending on the type of computer and the complexity of the scenario, this is often not true of the APM.

SPEED simulates a shorter scenario than APM beginning with the entry of the first penetrator into defense air space, and ending with the last penetrator's exit. Although it cannot depict nearly as many elements as APM,
SPEED also models in-depth the performance capabilities of individual penetrators and the entire defense network.

The purpose for the development of SPEED was to provide a methodology for analysts to:

- obtain an understanding of the interactions among the penetrating forces and the various facets of an air defense system, and
- quantitatively assess the overall impact on bomber force effectiveness of penetration system variations including numbers of air vehicles, ECM used and decoy deployment [Ref 13:16].

One advantage of SPEED over APM is that its reduced scale of complexity makes it easier for the analyst to understand the simulated interactions and the assumptions behind them. In addition, a major application of SPEED is examining sensitivities to one-on-one effectiveness data and changes in threat capabilities. Thus it facilitates tradeoff analysis of competing or complementary weapon systems. In fact, because SPEED can be directly tied to APM's Mission Planner, Studies and Analysis may use SPEED to perform excursions on studies where the APM is the primary model.

Other simulation models of the penetration mission exist which are comparable to SPEED in purpose, scope and level of detail. For example, Rockwell International's Advanced Campaign Effectiveness (ACE) Model has been widely used since its development for B-1 bomber studies, and differs from SPEED only in relatively minor details (Ref 14).
Small-Scope Simulations. Many-on-many models such as APM or SPEED are designed for study of major bomber force issues, but it is often desirable to have separate one-on-one models simulating a single isolated aspect of the whole bomber mission. An early example was NORTAM (Northrop Terminal Attrition Model), which was developed for Headquarters USAF (AFDAP) by James L. Taylor of Northrop, Inc., in the late 1950s (Ref 19).

NORTAM is a model of the terminal engagement between a single bomber and a single fighter-interceptor. Its purpose was to

... evaluate realistically the effects of meteorological factors, airborne equipments (i.e., design parameter variation), bomber defensive measures, and fighter attack doctrine and countertactics upon fighter bomber engagement outcomes [Ref 19:790].

It used a large number of user-provided inputs for air-battle and equipment parameters such as radar power and frequencies and aircraft turning radius. After a statistically significant number of runs, separate estimates were summarized for probabilities of detection, conversion to firing, abort, engagement and kill for both bomber and fighter. Average ranges of detection and firing were also calculated.

NORTAM is not widely used today, but it is a prototype for many flexible, small-in-scope simulations which replaced it for analysis of terminal fighter bomber engagements. The applications of such models are obviously
different from those of APM and SPEED. They can provide more detailed insight into the terminal engagement process, and allow more subtle comparisons of different tactics and weapon system features. Thus they are often used to generate probability inputs for the large many-on-many models.

Analytic Models

Analytic formulations of bomber penetration do not represent reality in the detailed manner of simulations, but rather simplify and transform key elements of the scenario pictured into the abstract language of mathematics. Developing such abstractions is an art, because there are probably as many ways to mathematically model a given scenario as there are analysts. Thus it is no surprise that analytic models vary in form and complexity more than simulation models. Such models can also differ in scope, level of detail or aggregation, mathematical technique and especially the simplifying assumptions made. Several models are cited here as illustrations.

PENEX. PENEX is a mathematical model for estimating the number of bombers surviving a many-on-many air battle with manned interceptors. It reflects a level of aggregation somewhere between the detail of APM or SPEED and very aggregated models such as the survival function, $P_s = e^{-P_kI/B}$ (Ref 2). In fact, its development for AF/SA
in 1972 was a direct response to the need for a versatile, but not too simple, analytic model.

The relative complexity of PENEX permits its application to a variety of penetration problems. It computes the expected numbers of bombers and decoys surviving an air battle by analyzing the conflict as a sequence of discrete sub-battles. The model allows two types of interception policy by the defense in the same sub-battle: raid controlled and close controlled. Under the raid controlled mode, interceptors take off when their bases are warned of incoming penetrators, and they search individually over a large area until either finding and engaging penetrators or using up their fuel. Under close control, a radar sensor network detects bombers, charts tracks of each, and uniformly allocates interceptors to engage the perceived penetrators. By providing for false bomber tracks and radar failures, as well as fairly detailed decoy and interceptor capabilities, the model explicitly treats the effects of confusion on the defense's command, control and surveillance capabilities. The analysis includes a method for determining the number of fighter-bomber encounters and derives the number of iterations needed of a recursive formula for bomber survival.

The limitations of PENEX are characteristic of all aggregated models. By assuming the continuous processes of a battle progress in discrete stages, they tend to
ignore delays inherent in the systems. Treating time as
discrete also causes problems when input data is being
developed or when time-related output is desired.

PENEX assumes that penetrators are confined to
corridors and are identical in capabilities and tactics.
Although these constraints facilitate model development,
they can fail to reflect important variations in opera-
tional plans.

If the user is fully aware of the assumptions and
limitations, models like PENEX are attractive for several
reasons. It can yield information about many types of pos-
sible decisions such as choices between decoy types, the
best type of ECM or ECCM for low or high altitude penetra-
tion, and comparisons of standoff missile systems and dif-
ferent penetrating bomber systems, as well as assessing
the effects of different air defense strategies and capa-
bilities. It can also test a variety of sensitivities;
because it requires no replications for a given input case,
it can do so in much less computer time than a simulation
can.

COLLIDE. COLLIDE is a model first developed by the
Lambda Corporation in 1972 to estimate ECM effectiveness in
a bomber–interceptor engagement (Ref 11). The primary
output it provides is $P_{dc}$, the probability that an inter-
ceptor within maximum detection range of the bomber detects
it and maneuvers (converts) to a missile-firing position. The calculations are derived from a score of input parameters for aircraft and weapon performance, as well as the specific angles of approach of the interceptor to the bomber's path, and an assumed elliptical detection envelope around the bomber.

COLLIDE is essentially an analytic counterpart of one-on-one simulations like NORTAM, and has been used to provide off-line ECM inputs to the APM. However, it has been criticized for its highly simplified view of the complex ECM game (Ref 7:27).

There is little agreement among analysts about how to model ECM, and the difficulty in validating models such as COLLIDE can permit some subjectivity. Nevertheless, COLLIDE tends to be biased against the bomber, and equally so against all bombers; thus it is reasonably fair in comparisons of different penetrators. Because the total effectiveness of ECM for a number of bombers will be greater than the sum of one-on-one estimates, one-on-one models such as COLLIDE tend to be conservative in favor of the defense. Again, relative numbers for making comparisons are more important than absolute numbers, which one cannot be sure are accurate.

COLLIDE is not a high-resolution model. Its major utility is its flexibility for making comparisons. It permits consideration of command and control system
degradation, visual and radar detection, extensive aircraft maneuvering, and different types of AAM systems. Furthermore, as a small-scale analytic model, it is much quicker running than its simulation counterparts.

Other Analytic Models. PENEX and COLLIDE are two analytic approaches which address different-sized portions of the bomber penetration problem. This section briefly describes some other analytic approaches to modeling bomber effectiveness issues.

The Stanford Research Institute (SRI), under contract with the Defense Communications Agency, developed the Corridor Penetration Model (COPEM) in 1971 (Ref 12). Similar to PENEX, COPEM estimates penetration probabilities for a group of bombers attempting to penetrate through a corridor defended by a group of fighter-interceptors. Its unique feature is the assumption that the underlying stochastic process representing the number of intercepts possible in a time interval following a bomber detection is a time-dependent Poisson process.

ENROUTE is a computerized probability model developed at General Research Corporation (GRC) as an aid for computing CMCA effectiveness against forward-based interceptors assisted by AWACS and also against ship-based SAMs (Ref 5). Widely used by ASD, it estimates the probability of CMCA survival and the fraction of the cruise
missile payload that is successfully launched before the CMCA is killed. ENROUTE can be used to evaluate various levels of interceptor performance and numbers, the CMCA defense concepts, including BDMs. ENROUTE is of particular relevance to this thesis, and will be discussed more deeply in the next chapter.

In 1968, Clifford D. Fawcett and Chester G. Jones of ASD developed a many-on-many model of engagements between bombers with BDMs and/or decoys, and interceptors capable of several multi-pass sorties during a battle (Ref 4). The model uses a modified Lanchester difference equation to determine the attrition of the forces, and then calculates the expected number of targets reached by at least one bomber. Their analysis assumed that bombers never run out of defensive missiles, causing them to perform a Monte Carlo simulation to derive the number of BDMs the bombers must carry for the analytic model to yield valid results—an interesting example of complementary uses of simulation and analytic techniques.

Another interesting approach to bomber penetration is a Markov-type model developed by Lulejian (Ref 9). It estimates survival probabilities for a group of bombers penetrating an air defense which includes an AWACS, but without effective SAMs, AAA, or GCI radar coverage. Its key assumption is that the bomber penetration process can
be divided into short discrete time intervals and modeled as a Markov process.

Relative Advantages of Simulation and Analytic Models

The aforementioned bomber penetration models illustrate that there are many differences among and between simulation and analytic models. Some of the benefits and costs of both types will be discussed in this section.

The Value of Simulations. The benefits from simulation modeling are products of the great detail with which simulations can represent subject systems and forces. The unique depth of resolution that simulations may provide frequently inspires greater confidence in their results than in those of corresponding analytic models. However, this is not the primary value, as some have previously noted: "The purpose of simulation is insight, not numbers [Ref 8:238]."

The inclusion of many system components in simulations gives such models a capacity for extensive experimentation, which may reveal the nature of previous unclear interactions to the analyst. Furthermore, in contrast with expected-value models, simulation results can display some of the randomness and variation of key processes in a system.
The Value of Analytic Models. For a given problem, an analytic model should be sought whenever possible, since it can "evaluate the performance with minimal efforts and costs over a wide range of choices in parameters and configurations [Ref 10:18]." Thus, the cost and flexibility of use is the clearest advantage of analytic models over simulations.

If the analyst is interested in only the mean value of some measure, an analytic model is clearly preferable to a simulation; the latter is inefficient if only a small portion of its output is of interest. Most simulations require many replications to generate significant results, while analytic models require a single run. Thus, with more responsiveness and a generally faster running time, an analytic model is a much more efficient tool for evaluating sensitivities.

Analytic models also require less detailed input than simulations, saving both time and money. In addition, a simulation is very costly to create and debug, and cannot be developed as quickly as an analytic model. An extreme example of this is the three years required to make the APM operational.

In spite of the economy of analytic models, they cannot always be applied to a problem. If the subject system is extremely complex, the insight required to adequately model it analytically may not yet exist. Even if
the system is understood completely, and simplifying assumptions are made, the resulting analytic model may be mathematically untractable. A simulation may be the only alternative.

**Complementary Roles.** The reason for comparing simulations and analytic models is not to choose a "best type" of model. The credibility and insight from detailed simulations, combined with the practicality of aggregated analytic models, gives the two classes complementary features which can make them a synergistic set of methodologies.

In a hierarchical approach, an analytic model can be validated through comparisons with large-scale simulation results (Ref 8). In some cases, a curve can be fit to simulation outputs and then used as an analytic function of some difficult-to-model relationship. The analytic model can then be used more credibly. Furthermore, comparisons with simulations can clearly inform the analyst of the analytic model's limitations.

Conversely, if an analytic model provides results very close to a simulation's, it may be found that some elements unique to the simulation are irrelevant to the problem. In large studies, a relatively valid analytic model can also weed out obviously inferior alternatives.
in a problem, and then only the remaining ones would be compared via simulation.

Many limitations of models are not limitations of the corresponding types of models--invalid models can come in any type. Although simulation models tend to resemble reality more than analytic models do, the apparent validity can be deceiving. Even complex simulations must make assumptions, but these can be so numerous that many are hidden or undocumented; the analyst may be unaware of them all.

A complementary role of small-scope to large-scope models can be briefly summarized. The designer or user of an aggregated simulation or analytic model may desire additional insight, or realistic input parameter values, for some process such as radar reliability or missile kill probability. Such data is not always available from experimentation, so detailed modeling (simulation or analytic) of the small scenario is usually done externally.

Summary

It should be remembered that "models do not analyze anything. Analysts analyze, and models can help them in their task [Ref 6:7]." Modeling is only one of the aids to analysis. However, the choice of models used in a study can affect both the quality and quantity of information the decision makers receive. Therefore, care should be
used in weighting the value of insight gained through alternative models and the resources required by their use. In addition, the analyst should be aware that new modeling may be needed to appropriately address the defined problem.

The next chapter describes the key features required of a model for studying the effectiveness of ECM and BDMs as aids to a CMCA force penetrating an AWACS FAD. Subsequent chapters will present two new models of utility for addressing this issue. Their development will illustrate the separate and combined values of simulation and analytic modeling.
III.  **Modeling the Use of BDMs Against the FAD**

For studying the bomber effectiveness issues described in Chapter I, most bomber penetration models are relevant to some degree. They all feature an offense of one or more bombers encountering a defense of one or more penetrators, and together they contain the range of assumptions and techniques considered reasonable or useful in previous modeling efforts. The emphasis of this chapter is on describing how the FAD is modeled, with regard to the four major study elements for this thesis: BDMs, ECM, cruise missiles, and AWACS controlled interceptors.

Of the simulation models described thus far, the APM and the SPEED and ACE models encompass all four elements; of the analytic approaches, only ENROUTE does so. The major assumptions in SPEED and ENROUTE will be described to illustrate simulation and analytic modeling of the four key aspects of the FAD problem. Of the simulations, SPEED is chosen because it is widely used by the Air Force, particularly at ASD.

Before discussing specific models, the key factors in CMCA effectiveness against the FAD are identified.
Factors in CMCA Effectiveness

The air battle between bombers and the AWACS defense can be analyzed as a dynamic system. The scenario can be decomposed into the significant processes and elements affecting three measures of offensive success: bombers surviving, ALCMs launched, and ALCMs surviving. The three quantities are defined as follows:

**Number of Bombers Surviving.** "Surviving" bombers are the CMCAs which are no longer in AWACS radar range and will not be intercepted by an AI assigned to an AWACS CAP. They are assumed to have penetrated to the next phase of enemy defenses.

**Number of ALCMs Launched.** This result is the total number of cruise missiles dispensed from bombers to become separate penetrators. It is also a measure of the average distance CMCAs penetrate before being killed, and depends greatly on ALCM range.

**Number of ALCMs Surviving.** Because the defense may attempt to shoot down cruise missiles already launched, "surviving" means the same thing for ALCMs as for bombers.

There are many interactions between offensive capabilities, defensive capabilities, and events in an air battle, as shown by the causal loop diagram in Figure 1. This diagram was developed by isolating each pair of
Figure 1. Cause-Effect Interactions in the Air Battle
elements and drawing an arrow if an increase in one directly affected change in the other. A plus (+) at the end of the arrow denotes a positive effect; a minus (-) indicates a negative influence.

The complexity of the system makes it difficult to model. If a key element is not modeled, its effects are lost from the analysis; however, some important elements may be included in larger elements, in the manner that bomber hardening might be encompassed by the enemy's AAM kill probability. Therefore, the art of modeling involves simplifying the scenario and the number of model elements, while still retaining the factors which most influence the quantities of interest.

The two factors which influence bomber effectiveness the most against the FAD are the number of interceptor engagements and the probability of surviving engagements which do occur. Hence there are two basic approaches to improving bomber effectiveness.

Reducing the Number of Engagements. There are three general tactics the offense can employ to reduce the number of engagements it must survive.

1. Decrease each penetrator's time in coverage. Less intercepts may be attempted against a bomber if detection tactics such as ECM and low altitude penetration delay the first intercept attempt. The effect of these tactics
is to decrease the range at which radar detection occurs or to create uncertainty about the penetrator's position.

2. Reduce the probability of engagement per intercept attempt. Even if the defense knows the approximate position of the penetrator, some attacks may be avoided by using ECM specifically designed to jam interceptor radar.

3. Saturate the defense. If the defense perceives a greatly increased number of penetrators, the number of intercepts performed against each one will probably decrease; furthermore, if the defense cannot distinguish between different penetrator types, some of the most important ones may not be intercepted. This is the motivation behind the use of decoys, waves of bombers, and to some extent, ALCMs.

Increasing the Survival Probability per Encounter. If the enemy's fighters have difficulty shooting down the bombers they intercept, then reducing the number of engagements becomes less important. This is the value of bomber-defense missiles designed to either kill the fighter before it can attack or detonate the fighter's missiles after they are launched but before they arrive. Similarly, ECM might be used to jam the guidance systems of the AAMs.

A possible secondary effect of BDMs is to reduce the number of engagements in the long run by attriting the interceptor force.
A possible third approach to increasing bomber effectiveness is to decrease the required penetration distance by developing longer-range ALCMs to be launched sooner. However, the value of this tactic is unclear; it assumes that a bomber will not attempt to penetrate after launching all of its ALCMs. The chances of survival for each ALCM may also be reduced if they must travel further.

**Tradeoffs.** In addition to cost limitations, there are obvious constraints for the number and level of capabilities a bomber can have. The addition of ECM or BDMs usually displaces a portion of the lethal payload intended for strategic targets. Furthermore, the added weight may adversely affect aircraft performance.

Evaluating such tradeoffs involves cost-effectiveness analysis, and the "effectiveness" part of the study usually involves models such as SPEED and ENROUTE.

**Assumptions in SPEED and ENROUTE**

The appropriateness of SPEED and ENROUTE for studying the contribution of BDMs to bomber effectiveness against the FAD depends in part on the assumptions the models represent. The major assumptions of both models are discussed in five categories: bomber defense missiles, electronic countermeasures, ALCMs, the AI assignment policy, and the CAP replacement policy.
Bomber Defense Missiles. Two types of lethal defense missiles have been provided for in the APM, SPEED, ACE, COPEM, Markov, ENROUTE and the Fawcett-Jones model. The two types are short-range BDMs (SRBDMs) which are used to destroy fighter-launched AAMs before they can reach the bomber, and long-range BDMs (LRBDMs) which can kill an interceptor before it launches its AAMs. Strategically, the LRBDMs have two advantages over SRBDMs: less are required and the opposition is also attrited. A proposed third type is massive BDMs to be fired at SAM sites or AWACS from outside of their radar range. This type of BDM has not been featured in many penetration models.

SPEED does not explicitly model BDMs, while the APM and ACE do. However, one may attempt to represent them in SPEED by reducing the fighter $P_k$ to reflect BDM effectiveness against AAMs or AIs, and interceptor attrition due to LRBDMs may be modeled by artificially reducing the AI inventory.

ENROUTE models both types of BDMs. The number of attacks that a CMCA must survive to penetrate the FAD is calculated to be the expected number of engagements in which the AI is not killed by a LRBDM, given prior calculation of the number of engagements AIs will attempt with the CMCA. This number is identical for each bomber. When a surviving AI launches its AAMs, a salvo of SRBDMs is launched with an input probability of killing all of the AAMs.
One limitation of ENROUTE is that the number of BDMs carried by a bomber cannot be input by the user. The bomber is assumed to carry exactly enough BDMs to be armed for all its engagements and for all the AAMs launched against it. The expected numbers of BDMs required are calculated, and are assumed to be the same for each CMCA. The maximum number of cruise missiles carried can be user-specified, but the model automatically reduces this supply by the number of ALCMs equivalent in weight to the calculated BDM requirement.

Electronic Countermeasures. ECM modeling for penetration models falls into two categories: effects on AWACS capabilities and effects on AI performance. In SPEED, penetrators with ECM cause a jamming strobe on the AWACS radar scope as soon as they are within line of sight. Then a range for burnthrough or clear (no ECM) detection is provided, based on a user-specified detection criteria. It is a function of the penetrator's ECM modules and radar cross-section (RCS), the type of AWACS radar, and which of five aspect angle segments the penetrator is at with respect to the AWACS radar. Although AIs can be vectored along a strobe before burnthrough detection of a penetrator, the interception time is assumed to be 10 percent longer than it would be if the penetrator's precise location is known.
In analytic models such as ENROUTE, the effect of ECM is usually to reduce the radius of a detection range circle centered at the AWACS. This is a more simplistic view of ECM than SPEED's, but the results of the treatments are essentially the same; more powerful ECM reduces the number of possible intercepts made per bomber, by denying the defense information about the bomber's position.

The modeling of ECM effects on AIs is more straightforward, although not necessarily more accurate. In both SPEED and ENROUTE, ECM used against fighter radar reduces the input probability of encounter (also called detection and conversion). In addition, the AAM kill probabilities are degraded by certain types of ECM. In SPEED the input parameter for ECM performance may differ for each interceptor type and for each altitude segment.

**AI Assignment Policy.** The policy for assigning AIs to pursue penetrators is a critical part of any penetration model. In SPEED the closest AI to a given penetrator is vectored to it, and the intercept attempt begins whenever AWACS detects a clear, burnthrough, or strobe target and has an AI available. Reassignment of an AI already performing an intercept is possible if a higher priority target is detect. In ENROUTE, fighters are assigned to clear or burnthrough targets, but no "strobe-riding" is performed. Both models permit the user to
establish a limit to the number of simultaneous intercepts an AWACS can control.

Bomber penetration simulations, and a few analytic models, frequently assume that a single interceptor is assigned to a penetrator, apparently because multiple fighters per engagement is regarded as inefficient. This is true of ENROUTE, but SPEED allows AIs to be assigned in pairs or alone. In contrast, more aggregated models such as PENEX and the Markov Model assume that all available interceptors are uniformly or randomly distributed among the bombers in the corridor.

**CAP Maintenance Policy.** A dominant feature in any Forward Air Defense model is how the AWACS network is assumed to utilize the interceptor force. Both SPEED and ENROUTE can represent several AWACS aircraft positions (stations). In SPEED, two types of AWACS can be input with different radar and vectoring control capabilities. Each AWACS orbit is defined by the two endpoints of its patrol line, and overlapping radar coverage is possible. In ENROUTE, only one type of AWACS is represented. The AWACS orbit is a single point, and each station can be at a different standoff range from the coast. The spacing between stations is a user-provided constant. If the amount of spacing results in overlapping radar coverage, the user
can opt to have calculations based on a continuous strip of radar coverage instead of a chain of circles.

In both models an input number of fighter interceptors is assigned to each AWACS, with a portion initially located on CAP at the AWACS orbiting position. SPEED assumes each AWACS has its own inventory of several types of fighters, which are sent individually to replace AIs when they are assigned to penetrators from CAP. Each AWACS has a base for its own interceptor force, but it can also use AIs from other sites if its airbase is destroyed. In ENROUTE, up to five types of fighters can be input by the user for each AWACS. A single base holds the spare fighters for each CAP, and all bases are assumed to be the same distance inland.

In both models, loitering of AIs is possible only at the location of the AWACS CAP. In SPEED, AIs attempting engagements are returned to CAP only if they failed to encounter the target and also have sufficient fuel remaining for additional intercepts. ENROUTE returns the AIs to CAP differently, calculating a constant number of fighters on station and specifying a maximum number of total engagements that can be performed by each class of fighters.

In contrast to the APM and the ACE models, SPEED does not model the recycling of an AI to the airbase and then back to the battle. In some cases the initial AI inventory can be adjusted to reflect a portion of reusable
AIs, but this is a significant limitation of SPEED. In many scenarios the duration of the air battle may be much longer than the time required to recycle.

Summary and Possible Extensions

Both SPEED and ENROUTE are thought to be reasonably valid, and both have been used recently by ASD to study problems involving an AWACS forward air defense, BDMs, ECM, and ALCMs. SPEED's scope includes a large part of the traditional strategic penetration mission, but it also models most aspects of the FAD with greater detail than ENROUTE. On the other hand, ENROUTE is a faster-running analytic model designed specifically for FAD issues, permitting rapid generation of expected value outputs for many excursions.

SPEED, ACE and the APM can be used to study BDMs as an alternative to other penetration aids such as ECM. However, they are each complex simulations requiring a considerable amount of computer time; furthermore, the fact that they are campaign models with broad scenarios and many elements other than the FAD can make them relatively unresponsive to minor changes in BDM capabilities. A new simulation model can have unique utility for certain problems if it is designed to focus on the specific factors expected to affect CMCA performance against the FAD. Such a model is presented in the next chapter.
As far as the thesis research could discern, ENROUTE is the most appropriate analytic model existing for study of BDMs against the FAD. However, it has several limitations in this area:

1. ENROUTE assumes that all bombers have the same probability of survival, regardless of how many bombers have preceded each one in the battle. However, the effects of AI attrition due to BDMs will probably favor the later bombers to enter the battle.

2. The time each penetrator spends in coverage significantly affects its probability of survival. This time interval can vary in length for identical penetrators with different flight paths, yet ENROUTE uses a constant time in coverage for its estimation of this measure.

3. ENROUTE does not consider the possible depletion of a bomber's BDM supply, and because the number of ALCMs carried is determined by the model rather than the user, it lacks the flexibility to evaluate BDM-ALCM trade-offs in much depth.

An analytic model with more extensive treatments of these issues is presented in Chapter V.
IV. A Q-GERT Model

This chapter describes the development of a simulation model for studying counter-FAD options. First the conceptual model is explained and the major assumptions are identified. Then the computerized Q-GERT network is presented, and the results to be generated by the model are listed. Finally, the methods used to verify and validate the model are reported.

The AWACS Corridor

The model is based on a corridor concept, as illustrated in Figure 2. The scenario begins for each bomber when it enters the corridor at an imaginary line 1500 nautical miles from the enemy coastline. Proceeding in a straight line parallel to the corridor sides, the bomber begins to launch ALCMs at a user-input distance $R_{ALCM}$ from the border. After an initial lateral displacement from the bomber, the ALCMs are assumed to travel in straight parallel paths.

No ALCMs are launched prior to entering the corridor. The time between successive launches is a constant. A minor modification would allow the model to consider bombers with different target sets.
Fig. 2. AWACS Corridor Concept
A single AWACS aircraft is positioned somewhere within the corridor. Although an actual AWACS would patrol along a short line segment roughly parallel to the border, the model assumes it is located at a stationary point a distance SR (for standoff range) from the coast.

Associated with the AWACS is a single fighter base which refuels and reloads the interceptors utilized by the AWACS. The base is a distance \( R_b \) inland directly behind the AWACS center point. A portion of the interceptor inventory is assumed to be stationed at a single combat air patrol (CAP) point, located at the AWACS orbiting point. Operationally ready fighters are sent from the base to the CAP when requested by the AWACS. The distance from base to CAP is always \( R_b + SR \).

The width of the corridor and the position of the AWACS within it are user-specified. A coordinate system is established to facilitate the play of geometry in the model (Figure 3). For each of calculations, the model places the AWACS at the origin, and establishes coordinates (Y-values) for the bomber entry line, and ALCM launch line, and the position of the base according to the input value of SR. The right and left boundaries of the bomber corridor are specified by the input parameters \( X_L \) (a negative number) and \( X_R \), respectively.

For the cases studied in Chapter VI, the scenario consists of a single representative corridor with the AWACS
Fig. 3. Coordinate System for Calculations
centered in it, such that $X_L$ equals $-X_R$ as in Figure 3. The width of the corridor, $2X_R$, equals the maximum diameter of the AWACS detection zone, approximately 500 NM. This corresponds to the line-of-sight clear (no ECM) detection range for low-flying (500 feet) bombers, given that the AWACS is at an altitude of about 40,000 feet. This feature permits assessment of the value of ECM and RCS reduction by decreasing the effective detection range. Penetrators outside the effective range are undetected by the AWACS and hence are not engaged by interceptors.

**Major Simplifying Assumptions**

The corridor penetration scenario can be modeled as a set of six component processes:

1. The Bomber Entry Process
2. The ALCM Launch Process
3. The AWACS Detection Process
4. The AI Allocation Process
5. The Terminal Engagement Process
6. The CAP Maintenance Process

The general interactions between these components are depicted in Figure 4. The six processes serve as a guide for classifying the major assumptions of the model.

**Bomber Entry Process.** Bombers are assumed to cross the corridor threshold according to an exponentially distributed interarrival time truncated at sixty minutes.

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Fig. 4. Flowchart of Model Components
The user-specified mean time between arrivals can be calculated as the time between the first and last bombers divided by N-1, where N is the number of bombers. A bomber's entry point follows a uniform distribution from $X_L$ to $X_R$.

Bombers are homogeneous, fly at the same altitude, and travel at a constant speed $V_B$ through the corridor. They are assumed to gain little by altering a straight line course while penetrating the FAD.

Bombers carry identical payloads with a maximum of twenty ALCMs. Each pair of BDMs loaded (the number of BDMs per bomber is an input parameter) decreases the cruise missile load by one. Both the maximum ALCM load and the BDM-ALCM tradeoff rate can be adjusted by the user.

**ALCM Launch Process.** A bomber begins launching cruise missiles as soon as it is a distance $R_{ALCM}$ from the coastline. The missiles are dispatched at a constant rate of one every ten minutes, based on the assumption that:

1. the distance between the closest and farthest missile targets is about 1200 NM,
2. the target distances are roughly evenly spaced,
3. an ALCM is launched as soon as it is within range of its designated target, and
4. the bomber is flying at about 360 NM/hour, or six miles per minutes.
ALCMs are each assumed to fly at the same low altitude at a constant speed, $V_A$. At launch the ALCM is instantly displaced laterally from the bomber according to a uniform distribution from -10 to 10 NM. It then proceeds parallel to the bomber path.

**AWACS Detection Process.** A bomber is assumed to appear on AWACS radar when its course intersects an imaginary circle of radius $R_{\text{DET}}$ centered at the AWACS. Surviving bombers are lost from radar coverage when they cross the back side of the circle. $R_{\text{DET}}$ is the average of the detection ranges in all directions. It is provided by the user based on bomber altitude, RCS and ECM capabilities against the AWACS radar. Cruise missiles are detected by the AWACS when they intersect a smaller circle of radius $R_{\text{DCM}}$ centered at the AWACS. No penetrators are lost to coverage until they exit from their respective circles of coverage.

**AI Allocation Process.** The defense will attempt to intercept both bombers and ALCMs while they are in radar coverage, but bombers are higher priority targets (each bomber is probably still carrying several ALCMs). The AWACS mistakes ALCMs for bombers with probability $P_{\text{AMB}}$. Thus, available AIs on CAP are assigned first to bombers and ALCMs not distinguished from bombers, and then to known ALCMs. Penetrators appearing to be the same type
are treated on a first come, first serve basis. Interceptors are not assigned to a penetrator once that penetrator has left coverage.

The AI chosen for allocation to a penetrator is the one on CAP with the least amount of available fuel. No fighters attempting to intercept a penetrator are diverted to a different one. A single AI is assigned for each intercept.

**Terminal Engagement Process.** During an intercept attempt, an AI is vectored to the vicinity of the penetrator, where it locates and engages the target with probability $E_1$ if the target is a bomber, and $E_A$ if the target is an ALCM. If the intercept point is out of AWACS radar coverage, $E_1$ (or $E_A$) is multiplied by an exponential degrade factor $e^{-\lambda \Delta t}$, where $\Delta t$ is the time from the penetrator's departure from coverage to the moment it would have reached the projected intercept point.

If an ALCM is located by an AI, it is assumed to be successfully killed by AAMs. The duel between fighter and bomber is more complex, and is illustrated by the network in Figure 5. The model assumes the bomber's defensive missiles have a kill-before-launch capability against the interceptor. This means that the BDM range is enough greater than the AAM that when an AI is killed it is not yet close enough to the bomber to fire its own missiles.
Fig. 5. Network of Engagement Process
A BDM is fired only if: (1) the AI begins to make a pass (probability = $E_1$), (2) the bomber sees that an interceptor pass is forthcoming (probability $P_{bdi}$), and (3) the bomber has not depleted its BDM inventory. The BDM kills the fighter with probability $K_B$.

A surviving AI will attempt a head-on pass in which it will fire two AAMs with a combined kill probability of $K_1$. If the bomber is not killed, the AI will reengage with probability $E_2$, attacking from the rear. On a reengagement, the bomber gets the first shot if it still has missiles. If the AI survives this attack, it fires its last two AAMs with combined kill probability $K_2$.

**CAP Maintenance Process.** Fifteen fighters are assumed to be on CAP when the first penetrator is detected. This number can be easily adjusted by the user.

When an AI is assigned to a penetrator, an interceptor takes off from base to replace it, provided one is available in reserve. After the initial reserve of interceptors is depleted, AIs recycle to and from the base, spending only the service time $T_R$ on the ground. The number initially at the base is user specified.

AIs reaching a model-calculated maximum time on CAP are returned to the base. They recycle to CAP after a constant interval which includes the round trip cruise time. This maximum time on CAP is calculated so that any
AI still on CAP has sufficient fuel to perform the longest intercept and still recycle.

Fighters loiter only at the CAP, which is collocated with the AWACS. AIs attempting intercepts are returned to CAP only if: (1) they fail to engage a penetrator, and (2) they have sufficient fuel remaining for additional intercepts. Any AI engaging a penetrator and surviving is assumed to have used at least half of its AAMs, and no AI will return to CAP unless it is fully armed.

Computerization

After the necessary assumptions about the key processes were made, the conceptual model was refined with a Q-GERT network which could be translated directly into code.

Bombers, ALCMs and interceptors are modeled as transactions generated in the Bomber Entry, ALCM Launch, and CAP Maintenance processes, respectively. Penetrators enter a defense queue after detection, and await the arrival of interceptors at a separate queue. In the AI Allocation Process, an interceptor transaction is combined with the transaction at the front of the penetrator queue for an intercept attempt. Survivors emerge again from the Terminal Engagement Process as separate transactions.

The parameters required for calculations and for system branching decisions are carried by the transactions
as attributes. Documentation of the model, including a listing of the code, is provided in Appendix B.

Output Results Provided

For each input case, the simulation can generate estimates for the three numbers which measure offensive effectiveness: bombers surviving, ALCMs launched, and ALCMs surviving. The same standard deviation of each estimate is also calculated.

Other Results Possible. Additional insights can be obtained from model runs. For example, the Q-GERT variable NTC (NODE) records the number of transactions that have passed through NODE. This variable can be used to output the total number of engagements, the number of interceptors killed, and the frequencies of other events of interest.

In addition, the Q-GERT Analysis Program allows the user to obtain, from one or more simulations, extensive statistical summaries of designated events and activities (Ref 16). Examples are the minimum and maximum numbers of penetrators in coverage, and the average waiting time by detected penetrators until AI assignment.

Verification and Validation

In an attempt to verify that the model simulated what it was intended to, traces of several runs were
inspected for errors. Hand calculations were performed and found to conform to user function computations. Data and transactions were tracked manually through the network, confirming that the assumptions of the conceptual model were met in the simulations.

Measures were taken to evaluate the model's face validity. Common sense suggests that certain changes in input parameters should alter the outputs of the model in certain directions. Using the causal-loop diagram in Figure 1 as a guide, a list was made of parameters which, when increased, should improve bomber survivability. Another list was made of defense-favorable inputs. Some of these two types are listed in Table 1.

TABLE 1
EXPECTED EFFECTS OF KEY PARAMETERS

<table>
<thead>
<tr>
<th>Pro-Bomber</th>
<th>Pro-Defense</th>
</tr>
</thead>
<tbody>
<tr>
<td>V_B</td>
<td>V_I</td>
</tr>
<tr>
<td>K_B</td>
<td>K_I and K_2</td>
</tr>
<tr>
<td>P_BDI</td>
<td>E_1</td>
</tr>
<tr>
<td># of BDMs/bomber</td>
<td>E_2</td>
</tr>
<tr>
<td>T_RCYC = AI recycling time</td>
<td># of AI</td>
</tr>
<tr>
<td>R_B</td>
<td>R_DET</td>
</tr>
</tbody>
</table>

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A base case was established for the input parameters and 20 replications were performed to estimate the number of bombers surviving. Then a sequence of 12 excursion cases was run with 10 replications per case. In each excursion case only one of the above parameters differed from its base-case value. All comparisons of the average output to the base-case estimate showed the difference in the expected direction.

Validating the simulation model is difficult in the absence of applicable real-world data, but care was taken to incorporate only believable or acceptable assumptions. The major assumptions were based on precedents or analogues in recently published studies and models. This fact was confirmed during discussion of the model with an Air Force analyst specializing in strategic studies (Ref 20).

**Broader Applications**

The user may wish to study a scenario involving penetration of enemy defenses with multiple bomber corridors and many AWACS at once, as in Figure 6. Some of the AWACS may have overlapping radar coverage, and more than one fighter base may support a given CAP. The model can be used for such a study if the analyst divides each corridor into narrower subcorridors, so that the latter each contain a single AWACS. For example, corridors I, II and III (Figure 6) can be subdivided as in Figure 7. Then the
Fig. 6. Hypothetical Multiple-Corridor Scenario

Fig. 7. Example Corridor Subdivisions
number of bombers traveling through each subcorridor can be specified, and the results of the entire campaign can be estimated as the sum of independent runs of the model for each subcorridor. Fighters supporting a group of AWACS in a large corridor can be split equally among the CAPs, and a single base can be assumed for each CAP with distance SR plus $R_B$ equal to the weighted average of the distances from the supporting bases to the CAP.

Summary

The Q-GERT model can be used to rank alternative mixes of BDMs and ECM capabilities. This will be shown in Chapter VI, where this model will also be compared to the analytic model developed in the next chapter.
V. An Analytic Model of the FAD

This chapter describes the development of a mathematical model for evaluating CMCA effectiveness against the forward-based AWACS defense. First the approach taken to estimating three measures of the effectiveness is summarized, and then the sequence of computations comprising the model are explained. The computerization of the model is briefly described, and the validation efforts are reported.

Modeling Approach

The analytic model estimates the same three measures of effectiveness that the Q-GERT model provides:

1. Expected Number of Bombers Surviving
2. Expected Number of ALCMs Launched
3. Expected Number of ALCMs Surviving

The development utilizes the same corridor concept and coordinate system as the Q-GERT model (Figure 8), and the assumptions are the same except when otherwise stated.

The essential quantity estimated by the model is $P(i)$, $i = 1, 2, \ldots N_b$, where $N_b$ is the number of bombers entering the corridor and $P(i)$ is the probability that the $i$th bomber to enter penetrates the AWACS defense. It is calculated separately for each bomber, because the number
Fig. 8. Coordinate System and Corridor Concept
of fighters available, and the number of penetrators in coverage for the defense to attack, varies according to when a bomber enters the battle. Hence the average level of saturation may differ for each detected bomber's time in coverage, implying that the threat is greater for some bombers than for others.

The number of bombers surviving and the total number of ALCMs launched are estimated from the values of \( P(i) \); furthermore, estimation of the number of ALCMs surviving follows directly from measures taken to calculate \( P(i) \). \( P(i) \) is derived through five major steps.

**Step 1.** The first major quantity estimated is the probability \( P_s(j) \) that the bomber survives, given that the defense can make \( j \) attempts (engagements) to kill it. \( P_s(j) \) is based on the same engagement scenario assumed in the previous chapter, and also on the number, \( M \), of BDMs carried. Because a bomber may deplete its BDM supply, probabilities of surviving a single engagement are calculated for three cases; i.e., when the bomber has zero, one, or at least two BDMs available. \( P_s(j) \) is used to estimate the expected number of engagements the defense must perform to kill a single bomber; hence, it is used in computing the effects of defense saturation on other variables in the model. Later, \( P_s(j) \) is again used in the final calculation of survival probability, \( P(i) \).
Step 2. The next major variable estimated is $E_{\text{pic}}(i)$, the expected number of penetrators--bombers and ALCMs--in coverage during the $i$th bomber's passage through the FAD, assuming initially unlimited AI availability on CAP. It is determined from the average time between penetrator arrivals and also from the estimated time the penetrator is in coverage. Time in coverage is the minimum of the time needed to fly through the detection zone and the average time required for the defense to kill the penetrator; the latter depends on $P_s(j)$.

Step 3. Third, the average number $N_{ai}(i)$ of interceptors alive during the $i$th bomber's time in coverage is estimated, permitting approximation of the number available to perform intercepts. Expected interceptor attrition is computed based on the number of engagements performed per bomber, which in turn depends on the level of defense saturation by the offense. The ratio of $E_{\text{pic}}(i)$ to the number of fighters available on CAP leads to $E_{\text{dlay}}(i)$, the estimated delay between intercepts attempted on the $i$th bomber. $E_{\text{dlay}}(i)$ allows the model to capture the effects of saturation on the defense.

Step 4. Next a probability distribution, $P_{ni}(i,k)$, is derived for the number, $k$, of intercepts possible on the $i$th bomber. The bomber's entry point $x$ is assumed to follow a discrete uniform distribution across the width of the corridor. For each value of $x$, $E_{\text{dlay}}(i)$ is used to
calculate the expected maximum number of intercepts against the bomber. This expectation is then used to calculate \( P_{ni}(i,k) \).

**Step 5.** Finally, \( P_{ni}(i,k) \) is used to find the probability distribution, \( P_{ne}(i,j) \), for the number \( j \) of engagements possible against the \( ith \) bomber. This allows combining \( P_{ne}(i,j) \) and \( P_s(j) \) into an expression for \( P(i) \).

Because of the importance of times between intercepts in the analysis, the expected intercept time was calculated separately for each value of \( x \). The calculation flow of the model is illustrated in Figure 9. The estimates marked with asterisks are recalculated for each bomber.

**Extensions over ENROUTE**

The model has three significant features not found in ENROUTE. First, the number of BDMs carried is calculated by ENROUTE for the purpose of computing the number of cruise missiles offloaded, based on the expected number of encounters per CMCA. In doing so, ENROUTE assumes that the bomber is armed with BDMs for each encounter. It does not permit the user to input the number of BDMs (Ref 5). In contrast, the model reported here includes possible BDM depletion, permitting the user more flexibility to compare payload mixes.

A second major difference from ENROUTE is in the manner in which the number of engagements per bomber is
Fig. 9. Analytic Model Calculation Flow

* - function of bomber number, i.
calculated. ENROUTE utilizes a constant value for CMCA time in coverage and a single value for the average time between AI engagements (Ref 5). The model conditions time in coverage and intercept times on the lateral position $X$ of the penetrator in the corridor, and then calculates overall distributions based on a discrete uniform distribution of $X$.

The third major difference between this model and ENROUTE is that bombers are treated separately, and may have different probabilities of survival according to their arrival times. In ENROUTE, all bombers are independent and are treated identically (Ref 5). Hence, the extensions in this model may permit better assessment of AI saturation and attrition effects which may occur towards the end of the battle.

It is important to note that ENROUTE has advantages over this model in some cases. ENROUTE models multiple fighter types and AWACS kills by BDMs. In addition, its scenario includes multiple AWACS aircraft and the added threat of ship-based SAMs in the FAD.

**Model Development**

The derivation of the model's estimates is reported in the sequence that computations are performed.

**Finding $P_s(j)$.** The probability that a bomber survives given $j$ AI engagements depends upon how many BDMs
the bomber has for each engagement. It is derived from the engagement scenario in Figure 10. An engagement is said to occur whenever an AI that has been vectored to the vicinity of a bomber succeeds in locating the target and performing at least one pass in which he will fire AAMs if not killed first by a BDM.

Consider an engagement in which the bomber has no defense missiles. The AI will attempt to kill the bomber on two passes. Then the bomber's survival probability for the engagement, given at least one AAM pass is

\[
S_0 = \Pr\{\text{AAM miss on first shot}\} \times [\Pr\{\text{no second pass}\} + \Pr\{\text{second pass}\} \Pr\{\text{AAMs miss on second pass}\}] (1-K_1)(1-E_2+E_2(1-K_2))
\]

(1)

where

\[
K_1, K_2 = \Pr\{\text{AAM volley kills bomber}\} \text{ on the first and second passes, respectively, and}
E_2 = \Pr\{\text{AI converts to a second pass, given that both sides survived the first}\}
\]

If the bomber begins an engagement with a single BDM, he attempts to fire it on the first AI pass. If he fails to detect the fighter in time to fire the BDM then he is certain to fire on the second pass, providing he survived the first pass and there is a second pass. Hence his survival probability given at least one AAM pass is
Fig. 10. Network of Engagement Process
\[ S_1 = P_{\text{bdi}} [K_b + (1-K_b)S_0] + (1-P_{\text{bdi}}) \alpha \]  
\[ \alpha = (1-K_1)(1-E_2+E_2[K_b+(1-K_b)(1-K_2)]) \], 
\[ P_{\text{bdi}} = \text{Pr}\{\text{bomber detects interceptor in time to fire BDM on first pass}\}, \text{ and} \]
\[ K_b = \text{Pr}\{\text{launched BDM kills AI}\}. \]

Consider next the case in which the bomber has at least two BDMs. Because the AI will make at most two passes, the bomber will fire no more than two BDMs. If \( S_2 \) is the probability the bomber survives this engagement, then,

\[ S_2 = P_{\text{bdi}} [K_b + (1-K_b) \alpha] + (1-P_{\text{bdi}}) \alpha. \]

Thus the value of \( P_s(j) \) depends on how many of the \( j \) engagements are of each of the three types.

The expected number of engagements a bomber can survive and still have two BDMs is estimated by first calculating \( E_{\text{mfs}} \), the expected number of BDMs fired by a bomber per engagement, given that he survived the engagement and started it with at least two BDMs. Hence, using conditional probabilities,
\[ E_{\text{mfs2}} = 1 \cdot \Pr(1 \text{ BDM was fired} | \text{bomber survived}) + 2 \cdot \Pr(2 \text{ BDMs were fired} | \text{bomber survived}) \]

\[ = \frac{\Pr((\text{Survived}) \cap (\text{Fired 1 BDM})) + \Pr((\text{Survived}) \cap (\text{Fired 2 BDMs}))}{\Pr(\text{Survived})} \]

Thus,

\[ E_{\text{mfs2}} = \frac{\Pr(1 \text{ BDM fired}) + \Pr(2 \text{ BDMs fired})}{\Pr(\text{Survived})} \]

\[ = \frac{(1 - K_2)E_2 + 2(1 - K_2)\beta}{S_2}, \quad (S_2 < 0), \quad (5) \]

where

\[ \beta = (1 - K_b)E_2 [K_b + (1 - K_b)(1 - K_1)]. \quad (6) \]

Then the approximate number of engagements the bomber can survive before having less than two BDMs is

\[ E_{\text{nw2}} = \frac{M}{E_{\text{mfs2}}}, \quad (7) \]

where \( M \) is the initial number of BDMs.

The model assumes that the expected number of engagements the bomber can survive before having less than two BDMs is the integer part of \( E_{\text{nw2}} \), or

\[ N_2 = \text{Int}(E_{\text{nw2}}) \quad (8) \]

The fractional part is not discarded; it is used to help approximate \( N_1 \), the expected number of engagements survived in which the bomber carries a single BDM initially.
It is possible for a bomber to use his last two BDMs in the same engagement, and thus begin no engagements with exactly one defense missile. Note that the expected number of BDMs left after the bomber has survived \( N_2 \) engagements is

\[
E_{bml} = M - N_2(E_{mfs2}) = E_{mfs2}(E_{nw2} - N_2)
\] (9)

If \( E_{bml} \) is less than one-half, the model sets \( N_1 \) equal to zero. Otherwise, \( N_1 \) is calculated as follows. Assume that the bomber has a single BDM before an engagement. Then the expected number he fires given that he survives is

\[
E_{mfs1} = \text{Pr}(1 \text{ BDM fired|bomber survives})
= \frac{\text{Pr}(\text{bomber survives and fires 1 BDM})}{S_1}
= \frac{P_{bd1}[K_b+(1-K_b)(1-K_1)[1-E_2+E_2(1-K_2)]]}{S_1} + \frac{(1-P_{bd1})\alpha}{S_1}, \quad (S_1>0)
\] (10)

where \( \alpha \) and \( \beta \) are found in (3) and (6), respectively. Then

\[
N_1 = \text{Int}\{1/E_{mfs1}\}, \quad \text{if } E_{mfs1}>0. \quad (11)
\]

The results derived thus far enable estimation of the bomber's probability of surviving \( j \) engagements. The desired function is
Furthermore, the form of \( P_s(j) \) allows computation of \( E_{ntk} \), the expected number of AI engagements needed to kill the bomber.

\[
E_{ntk} = \sum_{j=0}^{\infty} P_s(j)
\]

\[
= \sum_{j=0}^{N_2} s_2^j + s_2 s_1 \sum_{j=N_2+1}^{N_1+N_2+1} s_2^{j-N_2} + s_2 s_1 \sum_{j=N_2+N_1+1}^{\infty} s_2^{j-(N_2+N_1)}
\]

\[
= \sum_{j=0}^{N_2} s_2^j + s_2 s_1 \sum_{j=1}^{N_1} s_2^j + s_2 s_1 \sum_{j=0}^{N_2} s_1^j + 1 - s_2 \frac{s_2^{N_2+1}}{1-s_2} + s_2^2 \frac{(s_1-s_1^{N_1+1})}{1-s_1}
\]

\[
= \frac{s_2^2 s_1^{N_1} s_0}{1-s_0}, \text{ provided } s_2, s_1, s_0 \neq 1. \quad (13)
\]
As a check, note that if \( S_2 = S_1 = S_0 \), then \( E_{ntk} = \frac{1}{1-S_0} \), which is the mean number of trials till failure (kill) for a geometric distribution.

Another key variable is \( S_I \), the probability an interceptor survives an engagement. Based on the same engagement scenario used for \( S_2, S_1 \) and \( S_0 \), it is calculated as follows:

\[
S_I = \left( P_{bd1}(1-K_b) + (1-P_{bd1}) \right) \left[ K_1 + (1-K_1)(1-E_2+K_2(1-K_b)) \right].
\] (14)

The values of \( E_{ntk} \) and \( S_I \) are used to compute \( E_{pic}(i) \) and \( N_{AI}(i) \).

Finding \( E_{pic}(i) \). The expected number \( E_{pic}(i) \) of penetrators (bombers and ALCMs) in AWACS coverage during the \( i \)th bomber's attempt to penetrate the FAD is estimated assuming an unlimited supply of interceptors on CAP. Later adjustments are made for delays due to availability of interceptors. One part of this estimate is \( E_{bic}(i) \), the average number of bombers in AWACS coverage during the \( i \)th bomber's passage. \( E_{bic}(i) \) is a factor in the saturation of the defense. It is based on the bomber's expected time in coverage and on the rate that bombers arrive in the corridor.

Suppose that a bomber enters the detection circle at lateral distance \( x \) from the center, and is at position
(x,y) when a fighter leaves the CAP to attempt interception. Recall that the CAP is located at (0,0) on the coordinate axis, and that the bomber flies parallel to the y-axis. If the speeds of the bomber and interceptor are $V_B$ and $V_I$, respectively, where $V_B < V_I$, then the minimum time till intercept can be shown to be

$$T_I(x,y) = \frac{-V_B y + \sqrt{(x^2 + y^2)V_I^2 - x^2 V_B^2}}{V_I^2 - V_B^2}$$

(15)

Let $R = R_{det}$, the radius of the AWACS detection circle for bombers. The bomber is detected when its path first intersects the circle and leaves radar coverage when it crosses the back side of the circle. Because $x$ is fixed, the bomber is detected if $|x| < R$, and the value of $y$ can range from $\sqrt{R^2 - x^2}$ to $-\sqrt{R^2 - x^2}$ while in the detection zone. Then the expected time between engagement attempts given the bomber entered a lateral distance $x$ from the AWACS is

$$E_{ti}(x) = \int_{-\sqrt{R^2 - x^2}}^{\sqrt{R^2 - x^2}} T_I(x,y)f_Y(y)dy.$$  

(16)

where $f_Y(y)$ is the probability density function of $y$. For simplicity, assume that a given intercept attempt is equally likely to have begun when the bomber was at any vertical point in coverage. Then $y$ is uniformly
distributed along the segment in coverage determined by its x-value. Thus,

\[ f_y(y) = 2\sqrt{R^2-x^2} \text{ for all } y \in (-\sqrt{R^2-x^2}, \sqrt{R^2-x^2}), \]

and equation (16) becomes

\[
E_{ti}(x) = \frac{1}{2\sqrt{R^2-x^2}} \int_{-\sqrt{R^2-x^2}}^{\sqrt{R^2-x^2}} \frac{V_B y + \sqrt{(x^2+y^2) V^2 - x^2 V_B^2}}{V_I - V_B} \, dy
\]

\[
= \frac{V_I}{2(V_I^2 - V_B^2)} \left[ 2\sqrt{R^2-x^2} \frac{V_B^2}{V_I^2} 
+ x^2(1 - \frac{V_B^2}{V_I^2}) \ln \left( \frac{\sqrt{R^2-x^2} + \sqrt{R^2-x^2} \frac{V_B^2}{V_I^2}}{-\sqrt{R^2-x^2} + \sqrt{R^2-x^2} \frac{V_B^2}{V_I^2}} \right) \right]
\]  

(17)

Note \( E_{ti}(x) \) is not a function of the bomber, i, since an unlimited number of interceptors on CAP has been assumed.

The penetrator's time in coverage is defined as the time the defense regards the penetrator as a target. It begins when the bomber enters AWACS radar coverage and ends when the bomber (1) is killed, or (2) exits the back side of the AWACS range without an AI assigned to it, or
(3) concludes an engagement initiated prior to existing but occurring after exiting the AWACS circle of detection.

\( E_{ti}(x) \) is used to calculate the bomber's expected time in coverage as a function of \( x \), given that no delays occur between the end of one intercept attempt and the beginning of the next one for the same bomber. Estimation of the time in coverage assumes that no delays occur, because this result will be used to quantify the demand a single bomber makes on the defense under ideal defensive conditions. Then the defense's ability to meet this demand will be determined, and delays may be estimated.

First let \( E_1 \) be the probability that an intercept attempt results in engagement with the bomber. Then an average of \( 1/E_1 \) intercepts must be attempted for one engagement to occur. Thus, the average number of intercepts needed to kill a bomber is \( E_{ntk}/E_1 \). Hence, if command and control delays between interceptor assignments on a bomber are negligible, the expected time in coverage given an entry point \( x \) within the radar range of AWACS is

\[
E_{tic}(x) = \begin{cases} 
\frac{E_{n+k} E_{ti}(x)}{E_1} & \text{if } \frac{(E_{n+k}-1)E_{ti}(x)}{E_1} < \frac{2\sqrt{R^2-x^2}}{V_B} \\
\frac{2\sqrt{R^2-x^2}}{V_B} + \frac{E_{ti}(x)}{2} & \text{o.w.}
\end{cases}
\]  

Note that \( E_{tic}(x) \) may include an intercept attempt which begins while the bomber is in coverage, but terminates
beyond AWACS radar range. For the case where less than \( E_{ntk} \) intercepts are initiated before the bomber leaves coverage, a final intercept attempt is assumed to be half over.

The bomber's entry point \( x \) is assumed to be a random variable uniformly distributed across the width of the corridor, whose boundaries are \( X_L \) and \( X_R \). The total width of the corridor is \( (X_R - X_L) \) and the portion of this that results in bombers entering AWACS coverage is

\[
D_x = \min\{X_R, R\} - \max\{X_L, R\}. \quad (19)
\]

The model gives an approximation for the conditional expected time a bomber is in AWACS coverage, given that it is detected. Ten evenly spaced entry points \( x_\tau \) are specified across the part of the corridor in coverage:

\[
x_\tau = \max\{X_L, -R\} - \frac{D_x}{20} + \frac{\tau D_x}{10}, \quad \tau = 1, 2, \ldots, 10. \quad (20)
\]

Each bomber entering AWACS coverage selects one of these ten points at random. The expected time in coverage given the bomber is detected by the AWACS is

\[
E_{tic|d} = \frac{1}{10} \sum_{\tau=1}^{10} E_{tic}(x_\tau). \quad (21)
\]
The probability that the bomber is detected is $D_x/(X_R - X_L)$; thus, the unconditional expected time in coverage per bomber is

$$E_{ticb} = \frac{D_x}{X_R - X_L} E_{ticd}.$$  \hspace{1cm} (22)

Assume that $T_{bb}$, the time between successive bomber entries into the corridor, is a constant. Then there are no other bombers in coverage when the first bomber attempts to penetrate, and approximately $E_{ticb}/T_{bb}$ at the moment it leaves coverage, assuming that this is less than the total number, $N_b$, of bombers. The average is

$$\frac{E_{ticb}}{T_{bb}} \hspace{1cm} (23)$$

For the $i$th bomber, the number of bombers still in coverage when it enters is $\text{Min}(i, E_{ticb}/T_{bb})$, and the number when it leaves is $\text{Min}(N_b - i, E_{ticb}/T_{bb})$. The average is

$$E_{bic}(i) = \frac{1}{2} \text{Min}(N_b - i, E_{ticb}/T_{bb}) + \frac{1}{2} \text{Min}(i, E_{ticb}/T_{bb}). \hspace{1cm} (24)$$

Note that as $N_b$ gets large, $E_{bic}(i)$ equals $E_{ticb}/T_{bb}$ for $i$ greater than or equal to $E_{ticb}/T_{bb}$.

The expected number of cruise missiles in coverage is calculated in a similar manner. The number of
intercepts required to kill an ALCM is $1/E_a$, where $E_a$ is the probability that a vectored fighter locates and fires AAMs at its target ALCM. Recall the assumption that AAMs kill the ALCM with probability one. Let $R_{dcm}$ be the detection range of ALCMs by the AWACS. It is assumed that:

1. $2R_{dcm}$ is less than $X_R - X_L$, and
2. the detected ALCM entry points, $x$, are uniformly distributed from $-R_{dcm}$ to $R_{dcm}$. Therefore, the probability an ALCM released by a bomber is detected by the AWACS is $2R_{dcm}/D_x$. Thus the expected time in coverage, $E_{tic|d}'$ for a detected ALCM is calculated the same way $E_{tic|d}$ is for the bomber in equations (18) to (22), with the following changes.

\[
R = R_{dcm} \text{ instead of } R_{det} \\
V = V_a \text{ instead of } V_b \\
E_{ntk} = 1/E_a.
\]

Thus, for a given ALCM,

\[
E_{tica} = E_{tic|d}' \left( \frac{2R_{dcm}}{D_x} \right)
\]

Each bomber carries $N_{cmpb}$ ALCMs, each of which progresses through the corridor after launch at about the same speed as the bomber. Therefore, the expected number of ALCMs in coverage during the $ith$ bomber's passage is

\[
E_{mic(i)} = E_{bic(i)} P_L E_{tica}/E_{ticb}
\]
where

\[ T_{ba} = \text{Time between ALCM launches for a single bomber,} \]

\[ P_L = \text{Portion of ALCM load a surviving bomber launches by the time it reaches } y = 0 \text{ (adjacent to AWACS)} \]

\[ \frac{R_{ALCM} - SR}{V_b} \left( \frac{1}{T_{ba}} \right) \]

\[ R_{ALCM} = \text{Distance from border at which bomber first launches ALCMs} \]

\[ SR = \text{AWACS distance (standoff range) from border} \]

Note that, although the model's coordinate system places the AWACS at the origin, the user-defined inputs \( R_{ALCM} \) and SR use a point on the defender's mainland as reference point.

The expected number of penetrators in coverage, \( E_{pic} \), while the \( i \)th bomber is detected, assuming no defense delays, is the sum of \( E_{bic}(i) \) and \( E_{mic}(i) \). However, for the purpose of capturing saturation effects on the defense's ability to assign fighters to its highest priority targets—which are assumed to be bombers--\( E_{pic}(i) \) includes only penetrators which the defense cannot distinguish from bombers. That is,

\[ E_{pic}(i) = E_{bic}(i) + P_{mb}E_{mic}(i), \] (28)
where

\[ P_{mb} = \text{Probability an ALCM is mistaken for a bomber.} \]

Clearly, the value of \( E_{pic}(i) \) can directly affect the availability of fighters on CAP to intercept the \( ith \) bomber. Whether or not delays occur depends also on the number of interceptors still alive when the \( ith \) bomber is in coverage.

Finding \( N_{ai}(i) \). Computing the expected number of fighters alive during the \( ith \) bomber's time in coverage depends on estimation of the AI attrition occurring in previous engagements with bombers, which may depend on whether or not delays occurred. First, the average number \( N_{epb} \) of engagements per bomber, given that no delays occurred, must be calculated.

Assume that enough AIs are on CAP to intercept all of the targets in coverage without delay, and that this number remains large enough throughout the time the \( ith \) bomber is in coverage. If this bomber enters with \( x \)-value \( x_t \), where \( x_t \) is defined in (20), then the expected number of engagements performed against it is

\[ N_{epb}(x_t) = \frac{E_{tic}(x_t)}{E_{ti}(x_t)}. \]  \hspace{1cm} (29)

When

\[ (E_{ntk-1})E_{ti}(x_t)/E_1 < 2 \sqrt{R^2 - x_t^2} \]

\[ / \sqrt{V_B} \]  \hspace{1cm} (30)
the number of engagements with the bomber is not time-constrained, and the last engagement can be assumed to result in a bomber kill. Thus, the number of engagements in which an AI might be killed is \( N_{\text{epb}}(x_T)-1 \). If, on the other hand,

\[
(E_{\text{ntk}}-1) E_{\text{i}}(x_T)/E_{1} \geq 2 \sqrt{R^2-x_T^2}/V_B ,
\]

then the bomber may not have been killed, and the number of engagements potentially lethal to an AI is \( N_{\text{epb}}(x_T) \).

Let \( N_{\text{EPB}} \) be the expected number of engagements the defense performs per bomber. This is the average for all values of \( \tau \), or

\[
N_{\text{EPB}} = \frac{\sum_{\tau=1}^{10} N_{\text{epb}}(x_T) / 10}{10} . \tag{31}
\]

Let \( \rho \) be the number of values of \( \tau \), \( \tau=1,2,...,10 \), for which inequality (30) is true, and let \( p=\rho/10 \). Then the expected number of potential lethal engagements against interceptors per detected bomber (recall the assumption of unlimited AI on CAP) is

\[
N_{\text{lep}}|d = N_{\text{EPB}}-p . \tag{32}
\]

Note that, if \( S_2=0 \), then (13), (18), (29), and (32) imply that \( N_{\text{epb}}(x_T)=1 \), \( p=1 \), and \( N_{\text{lep}}|d=0 \). Because a bomber kill and an AI kill are mutually exclusive events
for the same engagement, this makes intuitive sense—no attrition will occur to the AI force.

Because some bombers may go undetected if $D_x \leq X_R - X_L$, the number $N_{lep}\text{b}$ used in attrition calculations is

$$N_{lep\text{b}} = N_{lep\text{b}}|d D_x/(X_R - X_L).$$

(33)

Now consider the possibility of defense saturation. Saturation is said to occur when the number of penetrators in coverage exceeds the number of AIs available to intercept them. If delays are not to occur during a particular time interval, then the number of fighters performing intercepts must stay at least as large as the number of targets; if it is ever less, then at least one penetrator in coverage will not have a fighter assigned to it. Furthermore, because fighters perform intercepts only if vectored from the AWACS, where the CAP is, any AIs which have not arrived back at CAP after an unsuccessful intercept attempt are not available.

Thus, one factor determining whether or not saturation occurs is the total time interceptors spend flying from CAP to intercept points and back. From the interceptor's point of view, more engagements are performed against bombers with small $x$-distances than with larger ones, due to greater time in coverage and shorter
intercept times. Thus the expected time to fly from CAP to the intercept point is a weighted average, or

$$E_{itb} = \frac{\sum_{\tau=1}^{10} N_{epb}(x_\tau)E_{ti}(x_\tau)}{10N_{EPB}} = \frac{\sum_{\tau=1}^{10} E_{tic}(x_\tau)}{10N_{EPB}}$$

(34)

Each interceptor will perform at most one engagement per cycle on CAP, because it will either be killed or require replacement of AAMs. Thus an interceptor will perform two or more intercepts only if it fails to engage the target (bomber or ALCM) on the first intercept. Thus, the expected number of intercepts an AI will perform, given unlimited fuel is $1/P_e$, where $P_e$ is the probability of encountering the target on an intercept attempt. Note that

$$P_e = P_{blv} E_{lb} + P_{av} E_A,$$

(35)

where

$$P_{blv} = \Pr(\text{target is a bomber, given that AI is vectored})$$

$$P_{blv} = \frac{E_{bic}(i)}{E_{pic}(i)} = \frac{E_{bic}(i)}{E_{bic}(i) + E_{bic}(i)P_{mb}P_{L'tica} / E_{ticb}}$$

for all $i$

$$E_{ticb} = \frac{E_{ticb}}{E_{ticb} + P_{mb}P_{L'tica}},$$

(36)
\[ P_{a|v} = Pr(\text{target is an ALCM, given that AI is vectored}) = 1 - P_{b|v} \]  

(37)

The expected time to fly from CAP to the intercept point is

\[ E_{it} = P_{b|v} E_{itb} + P_{a|v} E_{ita'} \]  

(38)

where

\[ E_{ita} = \text{expected time to intercept an ALCM, calculated in a way analogous to } E_{itb}. \]

After an unsuccessful intercept attempt (fail to engage penetrator), an AI returns to CAP if he has enough fuel for additional intercepts. The fighter returns at the same speed as during the intercept attempt; therefore, his average time for the round trip is \(2E_{it}\). He is assumed to lose no additional time due to searching for the target.

In an intercept resulting in engagement, he will either be killed or require replacement of AAMs; hence his average time spent is \(E_{it}\). The engagement is assumed to take zero time, even if a second pass is made. Thus, with an average of \(1/P_e - 1 = (1-P_e)/P_e\) unsuccessful intercepts
and one engagement, an AI's expected time between arrival on CAP and when it is killed or must rearm is

\[ T_{ms} = \text{Mission time after saturation} \]

\[ = 2E_{it}(1-P_e)/P_e + E_{it} = E_{it}(2-P_e)/P_e. \]  

(39)

Note that, after saturation occurs, interceptors do not spend time waiting for assignments.

The user can input \( T_{ms\text{max}} \), the maximum time a fighter can remain in the battle if always flying at intercept speed. \( T_{ms\text{max}} \) is based on fuel capacity, distance from base to CAP, and fuel consumption at cruise speed and intercept speed. Thus, equation (39) is replaced by

\[ T_{ms} = \min\{T_{ms\text{max}}, E_{it}(2-P_e)/P_e\}. \]  

(40)

The portion of this time that the fighter is performing intercepts is

\[ T_{is} = \min\{T_{ms\text{max}}, E_{it}/P_e\}. \]  

(41)

The maximum steady-state fraction of the time a live AI will spend in the vectored state (performing intercepts) is

\[ P_{is} = \frac{T_{is}}{T_{ms} + T_r}, \]  

(42)

where \( T_r \) is the average time required for an AI to fly to and from its base and be serviced.
If the number of live fighters is $N_{ai}(i)$, the maximum steady-state number that can be simultaneously performing intercepts is estimated as $P_{is}N_{ai}(i)$. Therefore, saturation occurs when

$$P_{is}N_{ai}(i) < E_{pic}(i).$$

(43)

In this case, the fraction of the number of intercepts that can be performed is $\gamma(i) = P_{is}N_{ai}(i)/E_{pic}(i)$, and the number of engagements potentially lethal to AIs, which was $N_{lepb}$ in (33), is replaced by

$$N_{leas}(i) = N_{lepb} \gamma(i)$$

(44)

Therefore, fighter attrition is estimated from equations (14), (33), and (44). The number of fighters alive during the $i$th bomber's (expected) time in coverage is estimated recursively for $i=1, \ldots, N_b$.

$$N_{ai}(i) = \begin{cases} N_{ai}(i-1) - (1-S_t)N_{lepb} & \text{if } \gamma(i-1) > 1 \\ N_{ai}(i-1) - (1-S_t)N_{leas}(i-1) & \text{o.w.} \end{cases}$$

(45)

and $N_{ai}(0) =$ initial AI inventory.

The computation of $N_{ai}(i)$ is used directly to establish a function for expected delays between intercepts, which is used to estimate $P_{ni}(i,k)$. 

87
Finding $P_{ni}(i,k)$. The probability $P_{ni}(i,k)$ that $k$ is the maximum number of intercepts that can be attempted against the $i$th bomber is determined primarily by $E_{dlay}(i)$, the expected delay between intercept attempts.

If saturation has not occurred, then it can be assumed that no delay occurs. On the other hand, suppose that saturation has occurred then the number of penetrators in coverage is $1/\gamma(i)$ times the number of AI available to perform intercepts. Then $\gamma(i)E_{pic}(i)$ of the penetrators each have one AI assigned to them and the other $(1-\gamma(i))E_{pic}(i)$ are not being intercepted.

This implies that $1-\gamma(i)$ is the average fraction of the time that the typical live bomber does not have an interceptor assigned to it. Hence, for each intercept attempt performed against the bomber, the average delay is

$$E_{dlay}(i) = E_{it}(\frac{1-\gamma(i)}{\gamma(i)}) = E_{i} + (\frac{1}{\gamma(i)}) - 1$$

when $E_{pic}(i) > P_{ias}N_{ai}(i)$.

Thus the expected time between intercepts of a bomber entering the $x$-value $x_T$ is $E_{ti}(x_T) + E_{dlay}(i)$, and the number that can be performed before the bomber leaves coverage is approximately

$$N_{I}(i,t) = I_{nt} \left\{ .5 + \frac{2\sqrt{R^2-x_T^2}}{v_{b}(E_{ti}(x_T)+E_{dlay}(i))} \right\}.$$
The distribution $P_{ni}(i,k)$ is computed by calculating $N_i(i,\tau)$ for each $\tau=1, \ldots, 10$. Each time $N_i(i,\tau)=k$, the estimate of $P_{ni}(i,k)$ is increased by $0.1 \times D(x)(X_R-X_L)$, resulting in

$$P_{ni}(i,k) = \frac{(\text{Number of values of } \tau \text{ for which } N_i(i,\tau)=k)D(x)}{10(X_R-X_L)}, k=1,2,\ldots$$

$$1 - \frac{D(x)}{X_R-X_L}, k=0. \quad (48)$$

Finding $P_{ne}(i,j)$. Finding the probability that $j$ is the maximum number of times the defense can engage the $i$th bomber is straightforward from $P_{ni}(i,k)$. If the number of intercepts is $k$, then the probability that $j$ of these result in engagements is $\binom{k}{j} E_1^j (1-E_1)^{k-j}$, when $j \leq k$. Thus,

$$P_{ne}(i,j) = \sum_{k=j}^{\infty} P_{ni}(i,k) \binom{k}{j} E_1^j (1-E_1)^{k-j} \quad (49)$$

For computation purposes, the maximum number of intercepts is set at 15.

Finally, the probability that the $i$th bomber survives the battle is

$$P(i) = \sum_{j=0}^{15} P_S(j) P_{ne}(i,j)$$

$$= \sum_{j=0}^{15} P_S(j) \sum_{k=1}^{15} P_{ni}(i,k) \binom{k}{j} E_1^j (1-E_1)^{k-j} \quad (50)$$
The expected number of surviving bombers is

\[ E_{bs} = \sum_{i=1}^{N_b} P(i). \]  

(51)

Estimation of the number of ALCMs launched and the number of ALCMs surviving is straightforward after \( P(i) \) is found.

**Estimating \( E_{m1} \) and \( E_{ms} \).** If a bomber survives, it is assumed to penetrate far enough to launch all \( N_{cmpb} \) of its cruise missiles. Even the bombers killed succeed in launching a portion of their ALCMs before they are detected.

Assume that each surviving bomber spends exactly time \( T_{ic} = \frac{\pi R_{det}}{2V_b} \) in coverage; i.e., the detection zone is of constant width \( \frac{\pi R_{det}}{2} \), which is the average width of a circle of radius \( R_{det} \). Then the minimum number launched is

\[ E_{mlbd} = \frac{R_{ALCM} - SR - \frac{\pi R_{det}}{4}}{V_B T_b} \]  

(52)

The time until a bomber is killed can be modeled with an exponential distribution. While a bomber is in coverage, the probability that it survives for an interval of time of duration \( \Delta t \) is independent of what happened before. It is also independent of when the time interval
begins, if the mean time between intercepts is assumed to be constant.

P(i) can be used to estimate the exponential mean. Let $T_s(i)$ be the time the bomber survives after it is detected. Then

$$P(i) = P(T_s(i) > T_{ic}) = e^{-\lambda T_{ic}}. \quad (53)$$

and

$$\lambda = \frac{-\ln P(i)}{T_{ic}}, \quad 0 < P(i) < 1 \quad (54)$$

Suppose that a bomber is killed before it leaves coverage. Then the conditional expectation for its time in coverage is

$$E_{tk} = E(T_s(i) | T_s(i) < T_{ic})$$

$$= \int_0^{T_{ic}} \lambda x e^{-\lambda x} dx = \frac{1}{\lambda} - \frac{T_{ic} e^{-\lambda T_{ic}}}{1 - e^{-\lambda T_{ic}}} = \frac{1}{\lambda} - \frac{T_{ic} e^{-\lambda T_{ic}}}{1 - e^{-\lambda T_{ic}}}. \quad (55)$$

Therefore, the estimated total number of ALCMs launched is

$$N = \sum_{i=1}^{N_d} \left[ P(i) N_{cmpb} + (1-P(i))(E_{mlbd} + \frac{E_{mkl}}{T_{ba}}) \right] \quad (56)$$

The expected number of ALCMs detected per bomber by the defense is
\[ E_{mdpb} = P_L N_{cmpb} x \frac{2R_{dcm}}{X_{R}-X_{L}}, \]  

where  
\[ P_L = \frac{R_{ALCM} - SR}{V_b - T_{ba}}. \]

For each ALCM detected after launch from the \textit{i}th bomber, the approximate probability it is killed is  
\[ P_{mk|d(i)} = 1 - (1-P_{mb}E_a)^n_i \]  

where  
\[ n_i = \frac{E_{tic}}{E_{ita} + E_{dlay(i)}}, \]
\[ E_{tic} = \frac{\pi R_{dcm}}{2V_a}. \]

This result assumes that only ALCMs not distinguished from bombers will be intercepted. Therefore, the estimated number of ALCMs killed is  
\[ N_b \]
\[ E_{mk} = \sum_{i=1}^{N_b} E_{mdpb} P_{mk|d(i)}. \]  

Finally, the expected number of cruise missiles surviving is  
\[ E_{ms} = E_{ml} - E_{mk}. \]
Computerization and Verification. The model was encoded in FORTRAN V, using the same general notations for variables as just described. With one run required for each set of inputs, the program prints out the computed values of $E_{bs}$, $E_{ml}$, and $E_{ms}$, as well as the values of $P(i)$ for all $i=1,2,\ldots,N_b$. If desired, however, the user can insert additional PRINT statements to output other calculations used.

To confirm that the program performed the calculations as required by the model, a sample case was run. The calculations in the model were also performed by hand, and the results agreed with those obtained from the computer. Other than the research conducted before conceptualizing the scenario, no validation has been attempted for the analytic model. However, the next chapter describes efforts to compare the results generated by the Q-GERT model and the analytic model, providing an indication of their respective validities. The relative efficiency of the two models is also discussed.
VI. Results and Comparisons

The simulation model in Chapter IV and the analytic model in Chapter V were designed for study of the same scenario, and were built upon similar assumptions. Therefore, if both models are reasonably valid, the information they provide should not be conflicting. On the other hand, if major differences are found between the models' results, finding the source of the inconsistencies can be instructive.

This chapter reports the results generated to provide mutual verification of the models, and illustrates how the Q-GERT and analytic models can be useful for BDM issues.

Comparing Study Results

Data Generation. The Q-GERT model was used to generate estimates of the number of bombers surviving and the number of ALCMs launched for 24 different cases. Then results were obtained from the analytic model for the same 24 cases.

The five input parameters which varied among the cases were $M$, $E_1$, $E_2$, $R_{det}$ and $N_{cmpb}$, where
\[ M = \text{Number of BDMs carried per bomber} \]

\[ N_{\text{cmpb}} = \text{Number of cruise missiles per bomber} \]

\[ E_1 = \text{Probability engagement (at least one pass) occurs given an intercept attempt} \]

\[ E_2 = \text{Probability a second AI pass is made, given a first pass in which both bomber and fighter survived} \]

\[ R_{\text{det}} = \text{Distance at which the AWACS is assumed to detect bombers} \]

Three levels of \( M \) were tested: zero, four, and eight BDMs. In each case, the number of cruise missiles was

\[ N_{\text{cmpb}} = 20 - \frac{1}{2}M. \]

The maximum number of cruise missiles was assumed to be twenty per bomber. One was offloaded for every two BDMs.

As a measure of ECM effectiveness \( R_{\text{det}} \) was set at two levels: 200 NM and 125 NM. \( E_1 \) and \( E_2 \) are used to represent the effectiveness of bomber ECM against a fighter which relies on its radar when trying to find and engage the bomber. Thus, improving the ECM reduces both the probability a fighter makes a first pass (\( E_1 \)) and the probability of a second pass (\( E_2 \)) if a first one has occurred. \( E_1 \) was set at four levels; in each case, \( E_2 \) was input with the value


\[ E_2 = E_1 + \frac{(1-E_1)}{2} = \frac{1+E_1}{2}. \]  \hspace{1cm} (62)

\( E_2 \) was set higher than \( E_1 \) for a given ECM suite because retaining contact with a bomber is assumed to be easier than acquiring it initially. Hence, each input case assumes that a bomber has twice as high a chance of escaping detection as it has of eluding a fighter which has already made one pass; i.e., equation (62) can be rewritten as

\[ (1-E_1) = 2(1-E_2) \]  \hspace{1cm} (63)

The remaining inputs to the models were held constant for all cases. The values of these parameters and the levels of the variables are listed in Table 2.

Ten simulation replications were run for each case, and the estimates obtained were the average numbers of bombers surviving, ALCMs launched, and ALCMs surviving in the ten runs. The same three quantities were estimated by one run of the analytic model per case. Both models' results for bombers surviving and ALCMs launched are summarized in Table 3. The cases are numbered from 1 to 24 in the table.

**Similarity of Results (Verification).** Inspection of the data shows that, in most cases, the analytic model's estimates are fairly close in magnitude to the corresponding simulation results. More importantly, a high
<table>
<thead>
<tr>
<th>Label</th>
<th>Value(s)</th>
<th>Constants</th>
<th>Definition</th>
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</thead>
<tbody>
<tr>
<td>$N_b$</td>
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<td>Total Number of Bombers</td>
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<tr>
<td>$I_o$</td>
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<td>Initial Total Number of Interceptors</td>
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<td>$V_B$</td>
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<td>Bomber Speed</td>
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<td>$V_I$</td>
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<td>Fighter Intercept Speed</td>
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<td>$V_A$</td>
<td>GNM/min.</td>
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<td>Cruise Missile Speed</td>
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<td>$X_L$</td>
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<td>Left Boundary of Corridor: Distance from AWACS</td>
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<tr>
<td>$X_R$</td>
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<td>Right Boundary of Corridor: Distance from AWACS</td>
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<td>$R_{ALCM}$</td>
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<td>Distance from Border for First ALCM Launch</td>
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<tr>
<td>$S_R$</td>
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<td>AWACS Distance from Border</td>
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<td>$T_{RCYC}$</td>
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<td>Time to Recycle a Fighter</td>
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<td>$R_{dcn}$</td>
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<td>AWACS Radar Range for ALCM Detection</td>
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<tr>
<td>$T_{bb}$</td>
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<td>Mean Time Between Bomber Arrivals</td>
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<tr>
<td>$T_{ba}$</td>
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<td>Time Between ALCM Launches</td>
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TABLE 2—Continued

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**Variables**

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* - Only one value for each value of $M$

** - Only one value for each value of $E_1$
### Table 3

**Simulation and Analytic Results**

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<th>AWACS Radius of Detection</th>
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<td><strong>BDMs</strong></td>
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<tr>
<td></td>
<td>(1)</td>
<td>15.80</td>
</tr>
<tr>
<td></td>
<td></td>
<td>315.9</td>
</tr>
<tr>
<td></td>
<td>(2)</td>
<td>13.40</td>
</tr>
<tr>
<td></td>
<td></td>
<td>312.8</td>
</tr>
<tr>
<td></td>
<td>(3)</td>
<td>10.40</td>
</tr>
<tr>
<td></td>
<td></td>
<td>309.9</td>
</tr>
<tr>
<td></td>
<td>(4)</td>
<td>8.40</td>
</tr>
<tr>
<td></td>
<td></td>
<td>307.4</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>AWACS Radius of Detection</th>
<th>125 NM</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>8</td>
</tr>
<tr>
<td><strong>Fighting P x P k</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td><strong>BDMs</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td><strong>EBS</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>(1)</td>
<td>17.29</td>
</tr>
<tr>
<td></td>
<td></td>
<td>317.16</td>
</tr>
<tr>
<td></td>
<td>(2)</td>
<td>14.98</td>
</tr>
<tr>
<td></td>
<td></td>
<td>314.55</td>
</tr>
<tr>
<td></td>
<td>(3)</td>
<td>12.48</td>
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<td></td>
<td></td>
<td>311.48</td>
</tr>
<tr>
<td></td>
<td>(4)</td>
<td>10.62</td>
</tr>
<tr>
<td></td>
<td></td>
<td>308.98</td>
</tr>
</tbody>
</table>

**EBS = Expected Bombers Surviving; EML = Expected Missiles Launched**
correlation is found between how the two models rank alternative cases.

For example, consider how the cases in which $R_{\text{det}} = 200$ NM are ranked by the two models according to bomber survival as shown in Table 4.

**TABLE 4**
RANKING OF CASES WITH $R_{\text{det}} = 200$ NM

<table>
<thead>
<tr>
<th>Highest EBS</th>
<th>Lowest EBS</th>
</tr>
</thead>
<tbody>
<tr>
<td>Analytic</td>
<td>5 1 2 6 3 4 7 9 8 10 11 12</td>
</tr>
<tr>
<td>Simulation</td>
<td>5 1 6 2 3 7 4 8 9 10 11 12</td>
</tr>
</tbody>
</table>

The minor differences in ordering by the models may be due to variability of Q-GERT results; i.e., differences between adjacency-ranked simulation outputs may be statistically insignificant.

Similar high correlations are found when $R_{\text{det}} = 125$ or for other sets of cases in which one factor is held constant. Similar correlations are found for both EBS and EML. For example, the cases in which $M=8$ are ranked by EBS as shown in Table 5.

**TABLE 5**
RANKING OF CASES WITH EIGHT BDMs

<table>
<thead>
<tr>
<th>Highest EBS</th>
<th>Lowest EBS</th>
</tr>
</thead>
<tbody>
<tr>
<td>Analytic</td>
<td>13 1 14 2 15 3 16 4</td>
</tr>
<tr>
<td>Simulation</td>
<td>13 1 14 2 15 16 3 4</td>
</tr>
</tbody>
</table>
The sensitivities of estimates to a single parameter change are close for both models, with the notable exception of changes in EBS caused by reducing $R_{\text{det}}$ (from 200 NM to 125 NM) when $E_{1.1}$ is low (.2 or .4). When this occurs (Table 3), the analytic model predicts little or no effect on bomber survivability, while the simulation predicts a definite increase in EBS. Further analysis suggests that the cause for the difference is a subtle variation between the models' assumptions about interceptor allocation. 

In the analytic model, the defense is assumed to allocate AIs only to penetrators it believes are bombers. Thus, when $P_{mb}$ (the probability an ALCM is perceived to be a bomber) is input to be zero, as was the case for the runs in Table 3, no ALCMs are engaged.

In the Q-GERT model, all detected penetrators (including ALCMs) enter a queue to wait for available fighters—even if $P_{mb}=0$. Penetrators which are thought to be bombers are served (intercepted) first, but when no bombers are in the queue, the lower priority penetrators (ALCs) are assigned interceptors. Decreasing $R_{\text{det}}$ reduces the number of bombers in coverage, and frees interceptors to pursue ALCMs. Because the ratio of ALCMs to bombers (recall that $R_{\text{dcm}}$ is held constant regardless of $R_{\text{det}}$) increases, more of the fighters are intercepting ALCMs than $R_{\text{det}}=200$ NM; therefore, bombers arriving at the queue
(when detected or after surviving an engagement) will wait longer for interceptors to become available again. Even slight delays serve to reduce intercept attempts enough that, when $E_1K_1$ is small, the probability that detected bombers survive increases. Such delays do not result in the analytic model because no ALCMs are intercepted.

**Applicability to BDM Analysis**

The results in Table 3 reveal the sensitivity of both the simulation model and the analytic model to the number of BDMs carried, in terms of both bomber survival and the number of ALCMs launched. This sensitivity to BDM effects in the air battle is the primary advantage of the models developed in this thesis over SPEED and ENROUTE.

Four effects of BDM deployment impact the outcome of the bomber mission: survival of engagements, fighter attrition, BDM depletion, and payload tradeoffs. Realistic modeling of BDM issues should represent all four BDM consequences.

**First BDM Effect.** The first, and most obvious, effect of BDMs is to increase a bomber's probability of survival. Bomber defense missiles with sufficient range can kill an interceptor before it can fire its air-to-air missiles. This feature is modeled directly by the thesis models and by ENROUTE, but not by SPEED. In SPEED, reducing the fighter's kill probability is the only way to
represent BDMs. ENROUTE also provides for short-range BDMs used to destroy the fighter's AAMs after they have been fired.

**Second BDM Effect.** The next consequence of BDMs is that destroyed fighters reduce the number of AIs subsequently available to attack penetrators. If BDMs are carried, ENROUTE computes a factor by which the number of possible engagements will be reduced. However, it assumes that this reduced number of fighters available is constant for each bomber, regardless of when the bomber enters the battle. In SPEED, the effect of fighter attrition can be modeled only by artificially decreasing the fighter inventory at a base.

Both of the new models treat fighter attrition in greater depth. The Q-GERT model extensively models the allocation and recycling of individual interceptors. When one is killed, it is removed from the battle, so that less fighters are available as the battle progresses.

The analytic model estimates fighter attrition recursively. The average number of fighters available is estimated for a given bomber's time in radar coverage, based on the expected number of previous engagements performed. Hence both the number of engagements and the number of fighters available can vary among bombers.
**Third BDM Effect.** Another consequence of lethal defense is the possibility that a bomber is engaged after it has used all of its BDMs. When this occurs, the bomber presumably has less chance of surviving a fighter attack. However, ENROUTE assumes that a bomber is armed for every engagement, and SPEED would require significant modification to consider this effect. In contrast, the Q-GERT and analytic models in the thesis are sensitive to the input number of BDMs, because BDM depletion is possible. This is illustrated by cases in Table 3 where EBS is greater when four BDMs are carried instead of none.

**Fourth BDM Effect.** The fourth major consequence of carrying BDMs is that it usually means a portion of the strategic offensive weapons must be taken off the bomber to accommodate the added weight of the BDMs. ENROUTE computes the number of cruise missiles replaced by BDMs from its estimate of the expected number of engagements per bomber, and assumes that this ideal number of BDMs is carried by each bomber. This assumption limits the flexibility of the model for addressing tradeoffs. The two new models allow more extensive tradeoff analysis. The importance of such considerations is illustrated by cases in Table 3 where raising EBS by increasing the number of BDMs reduced the number of ALCMs launched. In addition, the results show that increasing the number of BDMs does not
necessarily improve a bomber's survivability, as valuable saturation effects may be lost because less ALCMs are carried.

**Summary**

The generally high correlations between the simulation results and estimates from the analytic model increase one's confidence that both the models correctly represent the major assumptions in the conceptual models.

Both models are capable of providing unique insights into problems involving bomber defense missiles. Furthermore, the simulation model is relatively fast-running, requiring an average of about 20 seconds of CDC 6600 computer time for ten replications. The analytic model is quicker, requiring about one second of CPU time per run.
VII. Conclusions and Recommendations

The purpose of this thesis was: (1) to develop two different specialized models for analysis of bomber penetration of the FAD, and (2) to compare the relative utilities of simulation and analytic modeling. This chapter summarizes the results of the study.

Value of the Models

The simulation model described in Chapter IV and the analytic model in Chapter V were designed to study an issue currently being addressed by ASD's Deputy for Strategic Systems. Specifically, what are the relative values of bomber defense missiles and electronic countermeasures as penetration aids for bombers penetrating an AWACS air defense? In addition, what are the relative gains or losses in terms of numbers of cruise missiles launched?

Analysis of this problem at ASD has been performed using the SPEED and ENROUTE models. The primary advantages of the models in this thesis over SPEED and ENROUTE are in the consideration of BDM effects. Four major consequences of BDM deployment were identified, and the new models were designed to model these effects more realistically than the models used by ASD. Therefore, the two
models in this thesis may be more useful for the study of certain cases involving BDMs.

The simulation model uses the Q-GERT simulation language, illustrating that an air battle can be described as a queueing system. The penetrators and interceptors are two types of "customers" waiting to be paired by the AWACS "server;" the resulting intercept attempts constitute "service." Use of Q-GERT networks is found to be useful when creating a conceptual model of bomber penetration and then translating it into computer code. The same is undoubtedly true of some other higher-order simulation languages based on network flowcharts.

The analytic model also has unique features. A bomber's estimated survival probability is dependent upon when it enters the battle. This is based on the interceptor population, which is estimated recursively for the sequence of bombers. In addition, probability distributions for the number of engagements per bomber are established, based on a distribution for the bomber's location in the corridor.

Model Comparisons

Chapter VI reports on a comparison of the two models using three measures of merit. The corresponding results are found to be relatively close in size for the two models. More importantly, the models tend to rank
alternative cases in about the same order. One source of minor inconsistencies is found and traced to different assumptions about the defense's engagement strategies.

Although the Q-GERT model runs much more quickly than larger simulations such as SPEED and APM, a single replication of the Q-GERT model takes about two times as long as one run of the analytic model. The greater detail in the simulation suggests greater confidence in its results, but the analytic model appears to be accurate enough to be useful when a rapid response is needed. Also, insights are often easier to see using analytic model results.

Recommendations

Preliminary inspection of the data generated by the two models reveals that they have at least face validity. However, more complete understanding of the value of the models can be gained from further study in two areas.

1. Output results should be obtained from the models for a wide range of input cases varying the value of each parameter from its minimum to its conceivable maximum. The results should be inspected, and compared between the models, to find the set of inputs for which each model gives meaningful results.

2. A formal comparison should be made between the results of the two new models and the corresponding results.
of SPEED, for the cases where SPEED is thought to be accurate. Additional comparisons among all four models--SPEED, ENROUTE, and the two models in this thesis--should be performed for a wider range of inputs.

Although the two thesis models are directly applicable to certain problems, they can be viewed as groundwork for more extensive tools of analysis. Several areas of extension are possible. Some possible model improvements are to:

1. Include multiple fighter types and multiple bomber types.

2. Model ECM and radar effects in greater detail, making detection distances dependent on altitude, aspect angles and relative velocities.

3. Allow defense allocation of multiple fighters per engagement under specified conditions.

4. Include multiple AWACS simultaneously, and multiple bomber corridors.

5. Model reassignment of vectored fighters to higher priority penetrators.

6. Include the possibility of bomber missiles designed to kill an AWACS.

7. Model fighter search tactics in the battle area after an AWACS is killed or when extra AIs are available.
8. Allow fighters to loiter in place after unsuccessful intercepts, or at multiple CAP positions for each AWACS.

9. Include subroutines to estimate key parameters from input features of actual weapon systems.

The desirability of such changes, of course, depends on the needs of the user in a specific study.

Summary

The analyst with a specific problem, such as finding the value of BDMs against a forward air defense, may find the specialized models developed in this thesis appropriate. Furthermore, both simulations and analytic models have unique values for providing insight. Therefore, if time permits, a parallel modeling approach using both methods can be beneficial.
Bibliography


Appendix A

Analytic Model FORTRAN Code
PROGRAM ANAMOD

DIMENSION PS(115), X(12), ETI(12), ETK(10), DY(11)
DIMENSION ETIB(10), ETIC(10), XNEPA(10), XNEPA(10)
DIMENSION ESEC(20), SECIC(20), ESEC(20), NA(10)
DIMENSION DFLAY(20), FNI(20), M(21), MZ(20), MZ(20), P20(21), P20(21)
DIMENSION P20(20), FCFRL(0:15)

REAL K1, K2, KB, NEPB, NLEPB, NEPA, NAI, LAMOA, NE
DATA XL/-25., XR/25., SCH/113., VGB/96.0/
DATA ROET/200.0/
DATA VA/5., VI/2., TBB/8., TBA/11., EA/3., FMB/121/
DATA RALK/1230., TR/130., NB/21., TTA/50.0/
DATA NCHPB/16/
DATA SR/30.0/

CALCULATE F(SURVIVE/ENCOUNTER, BOMBER AND AIR
S1=1*K1) * (1-E2+K2) (1-K2)
S2=(1-K1)^* (1-E2+K2)
S1=PBDI* (KB+1-K3)^*S0)+(1-PBDI)^*S2
S2=PBDI* (KB+1-K3)^*SP2)+(1-PBDI)^*S2
S3=(1-K1)^*E2* (KB+(1-K3)^* (1-K2))
IF (S2.GE.000..00..EQ.0) THEN
EN42=0.
ENFS2=FLOAT(NM)
ELSE
ENFS2=PBDI* (KB+1-KB)^* (1-K1)^* (1-E2)^* SP3* (1-PBDI +2.
* PBDI +1.
ENM2=FLOAT(NM)/ENFS2
ENDIF
IF (S1.GE.0.) THEN
ENFS1=(PBDI*(KB+1-KB)^* (1-K1)^* (1-E2+K2))
S1=(1-PBDI)*SP3/S1
EN1=1.0/ENFS1
ELSE
EN1=0.
ENDIF
SI=(PBDI*(1-KB)+1-PBDI) * (K1+1-K1)^* (1-E2+E2* (1-KB)))

CALCULATE P(BOMBER SURVIVES K ENGAGEMENTS)
NW=INT(ENW2)
ENML=ENFS2* (ENW2-FLOAT(NW2))
IF (ENML.LT.5.) NW1=0
IF (ENML.EQ.5.) NW1=INT(ENW1)
PS(0)=1.
DO 2 K=1,15
IF (K.LE.NW2) THEN
PS(K)=PS(K-1)*S2
ELSE
IF (K.LE.(NW2+NW1)) THEN
PS(K)=PS(K-1)*S1
ELSE
PS(K)=PS(K-1)*S0
ENDIF
ENDIF
PRINT *, 'PS OF ', K, '= ', PS(K)

114
CONTINUE

CALCULATE E(NUMBER OF ENGAGEMENTS TILL KILLED), BOMBER

IF (S1.EQ.1.) S2=.99
IF (S2.EQ.1.) S1=.99
IF (S3.EQ.1.) S0=.99

ENTK=(1.-S2**((NW2+1.)/(1.-S2)+(S2**NW2)*S1*)
      *(1.-S1**NW1)/(1.-S1)+(S2**NW2)*(S1**NW1)*S./(1.-S0))

PRINT *, 'ENTK = ',ENTK

PD=0.,
NEFB=0.,
EITB=0.,
NEFA=0.,
EITA=0.,
DO 65 N=1,2
IF (N.EQ.1.) THEN
   R=DET
   V=VR
   E=S1
   EKN=ENTK
   T=TEB
ELSE
   R=DOCH
   V=VA
   E=S2
   EKN=1.
   T=TEB
ENDIF

CALCULATE NUMBER OF BOMBEFS/ALCHS IN COVERAGE AT ONE TIME

FIRST CALCULATE AVERAGE INTERCEPT TIME

X1=AMIN1(XR,R)
X2=AMAX1(XL,-R)
DX=X1-X2
DO 4) L=1,10
   X(L)=X2-DX/26.*FLOAT(L)*D/16.
   A=SCRT(R*R-X(L)**2.0)
   IF (N.EQ.1.) OY(L)=A
   ELSE
      A=SCRT(R*R-(X(L)**2.0)**V*V/(VI*VI))
   ENDIF
   ETI(L)=VI*(2.*B+(X(L)*X(L))*((1.0-V*V/(V*V))/(4.*V*V)))
   IF (N.EQ.1.) ETI(L)=ETI(L)
   PRINT *, 'ETI = ',ETI(L)
   STK(L)=EKN*ETI(L)/E
   TICH=LX2.*A/V
ENDIF

NEXT CALCULATE EXPECTED TIME IN COVERAGE

IF (((EKN=1.)*ETI(L)/E).LT.TICMAX) THEN
   ETIC(L)=STK(L)
   IF (N.EQ.1.) PD=PD+.1
ELSE
   ETIC(L)=TICMAX+.5*ETI(L)
ENDIF

IF (N.EQ.1.) THEN
   XNEPB(L)=ETIC(L)/ETI(L)
   NEPB=XNEPB(XNEPB(L)/10.
   EITB=EITB+ETI(L)/10.
ELSE
   XNEPA(L)=ETIC(L)/ETI(L)
NEPA = NEPA + X * NEPA(L)/10.
EITA = EITA + ETI(L)/10.
ENDIF

40 CONTINUE
IF (N, EQ, 1) THEN
NEPA = (NEPA - PD) * CX/(XR - XL)
PRINT *, 'NLEPB=', NLEPB
PETIC = 0.
DO 5 L = 1, 10
PETIC = PETIC + ETI(L)/10.
END IF

50 CONTINUE
COMPUTE AVERAGE TIME IN COVERAGE, ALL X COMBINED
IF (N, EQ, 1) THEN
CX = DX
PETIC = PETIC * CX/(XR - XL)
BETIC = BETIC
ELSE
PETIC = PETIC * CX/(2.*ROET)
ENDIF

60 CONTINUE
COMPUTE NUMBER IN COVERAGE DURING EACH BOMBER PASSAGE
DO 6 I = 1, NB
3 = FLOAT(I)
IF (N, EQ, 1) THEN
EBIC(I) = (AMIN1(NB-B,PETIC/TBB) + AMIN1(B,PETIC/TBB))/2.
ELSE
PL = (RALCH - SR)/(V*T)
EBIC(I) = EBIC(I) * PL * PETIC/BETIC
PRINT *, 'EBIC(I)=' , EBIC(I)
END IF
PRINT *, 'EPIC = ', EPIC(I)
END IF

65 CONTINUE
P3 = EPIC(I)/EPIC(I)
P4 = 1 - P3
EIT = 0.5*(EIT + P3*EITA)
PE = 0.5*(EIT + P3*EA)
T0CAS = EIT*(2.*PE)/PE
EDLAY(I) = 0.
P1 = 0.5*(PE + (T0CAS + TR))
PRINT *, 'P1 = ', P1
EIT = EIT*PE/T0CAS
PRINT *, 'EIT = ', EIT
END IF
PRINT *, 'NAI(I) = NAI(I) - NLEPB*(1.-SI)
DO 71 I = 2, NB
SATPT = EPIC(I-1)/P1
PRINT *, 'SATPT = ', SATPT
IF (NAI(I-1) .LT. SATPT) THEN
NAI(I) = NAI(I-1) - NLEPB*(1.-SI)
EDLAY(I) = 0.
ELSE
NAI(I) = NAI(I-1) - NLEPB*(1.-SI)/SATPT
EDLAY(I) = EIT*(SATPT/NAI(I) - 1.)
ENDIF
PRINT *, 'NAI,EDELAY = ' , NAI(I), EDELAY(I)
ENDIF
CONTINUE
DO 75 I = 1, NB
P4(I) = 1. - DXB/(XR - XL)
DO 75 K = 1, 15

CONTINUE
DO 8 L=1,10
ETIX=ETIX(L)+ECLAY(I)
MIX=INT(.5+2.*CY(L)/(VB*ETIX))
3NI(I,NIX)=3NI(I,NIX)+1.DX8/(XR-XL)
CONTINUE
8 CONTINUE
FCTRL(0)=1.
DO 90 M=1,15
FCTRL(M)=FLOAT(M)*FCTRL(M-1)
CONTINUE
90 DO 1 J=0,15
DO 95 K=J,15
CHUZ(K,J)=FCTRL(K)/(FCTRL(J)*FCTRL(K-J))
CONTINUE
10 CONTINUE
EBS=1.
DO 12 I=1,NB
P(I)=0.
DO 110 J=0,15
PNE(I,J)=0.
DO 11 F K=J,15
PNE(I,J)=PNE(I,J)+PNI(I,K)*CHUZ(K,J)*(E1**J)*(1.-E1)**(K-J)
CONTINUE
P(I)=P(I)+PS(J)+PNE(I,J)
CONTINUE
EBS=EBS+P(I)
PRINT *, ' P(S) OF BOMBER ', I, ' = ', P(I)
CONTINUE
PRINT *, ' EBS = ', EBS.
TIC=3.1416*RCET/(2.*VB)
EMLBD=(RALCH-SP-3.1416*ROET/4.)/(V3*TBA)
DO 125 I=1,NB
LAMDA=ALOG(P(I))/TIC
IF (LAMDA*LE0.01) THEN
ETKB(I)=50.C.
ELSE
ETKB(I)=1./LAMDA-TIC*EXP(-LAMDA*TIC)/(1.-EXP(-LAMDA*TIC))
ENDIF
EMK=0.
EML=0.
125 CONTINUE
METIC=3.1416*RDCM/VA
DO 130 I=1,NB
EML=EML+HCM*PB*P(I)+(1.-P(I))*(EML83+ETKB(I)/TBA)
EMDPB=PL*2.*RDCM/(XR-XL)
NE=METIC/(EITA+ECLAY(I))
PHKD=(PHB+EA)**NE
EMK=EMK+EMDPB*PHKD
CONTINUE
EMS=EML-EMK
PRINT *, ' EML = ', EML, ' EMS = ', EMS
STOP
END
Appendix B

Q-GERT Model Code and Summary
FUNCTION UF(IFN)
COMMON/UCOM1,NCHM(2)/
COMMON/VQA/VDE,NFTBO(E1),NREL(511),NREL2(511),
1NRL1(11),NRL2(11),PARAM(11,L),T3EG,TNOW,
COMMON/USER/ATT(21),V1,V2,VA,RCMP,HRMP,RDET,FDCH,
SR,TRCYC,K9,E1,E2,EA,K1,K2,K3,P3DI,LAMDA,DS
REAL K1,K2,K3,LAMDA
DATA VI/2.1/,VB/6./,VA/3./,RALC4/12.1/,VCHMPb/2./,RDET/23G./,
1RCH/1.1/,SR/311/,TRCYC/E1/,89/51/,E1/91/,E2/9/,EA/3/,1
K1/5/,K2/6./,K3/3/G/,P3DI/95/,LAMDA/65./,DS/1/
CALL GETAT(ATT)
TH=T+ARK(ID)

C GO TO (1,2,3,4,5,6,7,8,9,1,11,12,13,14,15,16,17,18,19,21,22,23,24,25,26,27,28),IFN

C CALCULATE TIME TILL DETECTION; 150 = NOT DETECTED

1 X=ABS(ATT(2))
IF (X.LE.RDET) THEN
  Y=150.*-SR-SRT(RDET**2-X**2)
  UF=Y/VB
ELSE
  UF=150.*
ENDIF
RETURN

C CALCULATE TIME FOR INTERCEPT

2 X=ABS(ATT(2))
DELAY=TNOW-TH+LS-ATT(3)
IF (ATT(1).GE.34) THEN
  P=RDCH
  V=VA
ELSE
  R=RDET
  V=VB
ENDIF
IF (X.LE.R) THEN
  Y=P*SRT(1.-{(X/R)**2)-DELAY/V
  UF=(-V*Y+SRT((X*X+Y*Y)*V1*V-X*X*V*V))/(V1*V-V*V)+DS
ELSE
  UF=1.
ENDIF
RETURN

C CALCULATE TIME LEFT IN COVERAGE

3 SEEN=TNOW-(TH+ATT(3))
X=ABS(ATT(2))
IF (ATT(1).GE.34) THEN
  P=RDCH
  V=VA
  Y=:+(:-(5)-ATT(3).*V
119
ELSE
R=30ET
V=VB
   IF (X. LE. R) Y=SQT(R**2-X**2)
ENDIF
IF(X.LE.R) THEN
   UF=(Y+SQRT(R**2-R-X*X))/V-TSEEN
ELSE UF=1.
ENDIF
RETURN

C  SCHEDULE NEXT ALGM LAUNCH: 50G MEANS LAST ONE
C  4 IB=ATT(1)
    NCm(IB)=NCM(IB)+1
    IF (NCM(IB).EQ.1) UF=(1500.-FALCM1)/VB
    IF (NCM(IB).GE.2) UF=10.
    IF (NCM(IB).GT.NCM81) UF=5.0.
RETURN

C  FIND TIME TILL ALGM DETECTION: 50G=NOT SEEN
C  5 XA=ATT(2)+ATT(6)
    IF (APS(XA).LE.ROCM) THEN
       YA=SQT(ROCM**2-XA**2)
       UF=(ATT(5)-YA)/YA
       IF (UF.LT.6.) UF=9.
       IF (ATT(5).LT.-YA) UF=56.
    ELSE
       UF=99.
    ENDIF
RETURN

C  RETURN THE NUMBER OF AI USED FROM INVENTORY
C  6 UF=NTC(16)
RETURN

C  COMPUTE DEGRADE TO ENGAGEMENT PROBABILITY
C  7 TOC=ATT(5)-ATT(6)
    UF=EXP(-LAMDS*TOC)
RETURN

C  COMPUTE ONE MINUS DEGRADE FACTOR
C  8 UF=1.0*EXP(-LAMDS*(ATT(5)-ATT(6)))
RETURN

C  STORE AI TIME SPENT ON CAP
C  9 TSCC=ATT(7)
    UF=TSCC
RETURN
RETRIEVE AI TIME SPENT ON CAP

11 UF=TSOC 
RETURN

FIND TIME FOR RECYCLING ROUNDTRIP

11 UF=TRCYC+2.*(SR+R9)/(.5*VI) 
RETURN

FIND AI TIME FROM BASE TO CAP

12 UF=(RB+SR)/(.5*VI) 
RETURN

RECALCULATE AI TSOC

13 UF=ATT(7)+3.*ATT(5) 
RETURN

SCHEDULE AI RETURN TO CAP WITHOUT RECYCLING

14 UF=ATT(5)/.5 
RETURN

FIND ALCM Y-DISTANCE AT LAUNCH TIME

15 UF=ATT(7)-(THW-TH)*VB-SR 
RETURN

FIND X-VALUE OF ALCM POSITION

16 UF=ATT(2)+ATT(6) 
RETURN

COMPUTE MAXIMUM AI TIME ON CAP

17 UF=2.0-8.*(RB+SR)/VI 
RETURN

ASSIGN PROBABILITY (ENGAGEMENT, NO 3DM FIRST PASS)
OR P(ALCM IS KILLED, AI MUST RECYCLE FOR WEAPONS)

19 IF (ATT(1)<34) UF=E1*(1-PBDI) 
IF (ATT(1)<34) UF=0. 
RETURN

ASSIGN P(FIRST PASS, BDM SHOOTS FIRST), OR
P(ALCM IS KILLED, AI IS ARMED TO STAY ON CAP)

18 IF (ATT(1)<34) UF=E1*PBDI 
IF (ATT(1)<34) UF=EA 
RETURN
121
ASSIGN PROBABILITY TARGET IS NOT ENGAGED

2. IF (ATT(1), LT, 34) UF = 1. - E1
   IF (ATT(1), GE, 34) UF = 1. - EA
   RETURN

ASSIGN AAM KILL PROBABILITY, FIRST PASS

21 UF = K1
   RETURN

ASSIGN P (MISS ON FIRST PASS BUT KEEP CONTACT)

22 UF = (1. - K1) * E2
   RETURN

ASSIGN P (MISS ON FIRST PASS, LOSE CONTACT)

23 UF = (1. - K1) * (1. - E2)
   RETURN

ASSIGN BDM KILL PROBABILITY

24 UF = K9
   RETURN

ASSIGN BDM MISS PROBABILITY

25 UF = 1. - K8
   RETURN

ASSIGN AAM SECOND PASS P(KILL)

26 UF = K2
   RETURN

ASSIGN AAM SECOND PASS P(MISS)

27 UF = 1. - K2
   RETURN

SCHEDULE FIRST AI ON CAP

28 UF = (125C - SR) / VB
   RETURN

ENC
SUBROUTINE UI
COMMON/UCM1/NCM(20)
DO 1 I=1,20
NCM(I)=C
1 CONTINUE
RETURN
END

C
PRINT OUT IMPORTANT RESULTS FROM EACH:
C
SUBROUTINE UO
COMMON/ODR/NOD,NFTBU(500),NREL(500),NPELP(500),NREL2(500),
nrun,nrun,NTC(50),PARAM(11,4),T3ES,TCM
NB=NTC(62)
NBS=NTC(2)+NTC(9)
NCS=NTC(65)
NCMD=NTC(7)+NTC(5)
NCL=NTC(8)
IF (FUF.N2;1) THEN
PRINT*,"","RUN LIVE DEAD LIVE DEAD LAUNCHED","PRINT*,"","NO. BMRS BMRS ALCMS ALCMS ALCMS","PRINT*
ELSE
PRINT *
ENDIF
PRINT*,",NRUN,",NBS,",NBG,",NCS,",NCMD,"PRINT*,",NCML
RETURN
END
**NETWORK DESCRIPTION**

- Number of Statistics Nodes: 6
- Number of Sink Nodes: 0
- Number of Nodes to Realize the Network: 1
- The Simulated Time of One Run Not to Exceed: 6000
- Number of Simulations Requested: 2
- Summary Report Option: F
- Time from Which Statistics Will Be Kept: 0
- Number of Attributes Per Transaction: 20
- Largest User-Defined Node Number: 75
- Largest User-Defined Activity Number: 1

**SOURCE, SINK, REGULAR, AND STAT NODES**

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** PRIORITY SCHEME MAY BLOCK NODE FOR FOLLOWING NODES **

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** SELECTOR NODES **

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| NODE FOR Q-NODES ASSOCIATED BALKERS WITH SELECTOR |
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*** INPUT CARDS ***

GEN, RCRIECKS, BDY, (11) 6L, *2, (14) 2P, (21) 75, 1*
SOU, 1, 1, 0, 4*
BOMBERS ARRIVE

VAS, 1, 1, 1, 2, 1, 1, 3, UF, 1, 4, CO, 1, 3, CO, 1, 3, UF, 1*
REG, 2, 1, 1, A*
ACT, 1, 2*

ACT, 2, 4, 4, 2, 1, LT, 29*
PAP, 1, 9, -2, 4, 2, 30*
PAP, 2, 8, 2, 4*

ACT, 2, 3, 1, A1, GE, 20*
REG, 3, 1, 1*
ACT, 1, 4, 2, 1*
ACT, 1, 5*
REG, 4, 1, 1, A*

ACT, 9, 1, 4, 3, 1460*
ACT, 4, 1, 5, 4, 3, 16 LE, 14 'C, 3*
REG, 91, 1, 4*
REG, 5, 1, 1, A*
DEF, 1*
REG, 6, 1, 1, A*

ACT, 6, 6, 4, 9, 4, 4, 4, 13, LT, 4, 6, 3*
VAS, 6, 4, UF, 6, 4, 4, 3, 15, E, UN, 3*
ESN, 0*
DUP, 2*
DUP, 3*
DUP, 4*
DUP, 5*
DUP, 6*
DUP, 7*
DUP, 8*
DUP, 9*
DUP, 10*
DUP, 11*
DUP, 12*
DUP, 13*
DUP, 14*
DUP, 15*
DUP, 16*
DUP, 17*
DUP, 18*
DUP, 19*
DUP, 20*

LIN, 5, 51, UF, 4, 4, A1, ED, 1*
LIN, 3, 6, 2, UF, 4, 4, A1, ED, 2*
LIN, 3, 6, 3, UF, 4, 4, A1, ED, 3*
LIN, 3, 6, 4, UF, 4, 4, A1, ED, 4*
LIN, 5, 6, 5, UF, 4, 4, A1, ED, 5*
<p>| LIN | 5, 6/6, UF, | A1, 6* |
| LIN | 5, 6/7, UF, | A1, 7* |
| LIN | 5, 6/6, UF, | A1, 8* |
| LIN | 5, 6/9, UF, | A1, 9* |
| LIN | 5, 6/11, UF, | A1, 11* |
| LIN | 5, 6/12, UF, | A1, 12* |
| LIN | 5, 6/13, UF, | A1, 13* |
| LIN | 5, 6/14, UF, | A1, 14* |
| LIN | 5, 6/15, UF, | A1, 15* |
| LIN | 5, 6/16, UF, | A1, 16* |
| LIN | 5, 6/17, UF, | A1, 17* |
| LIN | 5, 6/18, UF, | A1, 18* |
| LIN | 5, 6/19, UF, | A1, 19* |
| LIN | 5, 6/20, UF, | A1, 20* |</p>
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<td>GE.</td>
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<td>1</td>
<td>4*</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

**BOMBER KILLED, ALCM NOT LAUNCHED**

**ALCM LAUNCHED**

**ALCM NOT DETECTED**

**ALCM SURVIVES**

**IS ALCM MISTAKEN FOR BOMBER?**

**YES, IT IS**

**NO, IT ISN'T**

**PENETRATORS WITHOUT AI AFTER THEM**

**NO TIME TO ASSIGN AI**

**INTERCEPT WILL BE ATTEMPTED**

**RECYCLE AI FOR FUEL**

**CHECK FOR PENETRATOR AGAIN IN 1 MIN**

**RECYCLE TIME**
SEND REPLACEMENT AI

DEGRADE INTERCEPT PROBABILITY

DEGRADE MAKES PENETRATOR SAFE

DOES PI SEE PENETRATOR SAFE

ATTEMPT TO REASSIGN AI TO BOMBER

DOES BOMBER HAVE BOMBS

YES BOMBS

NO BOMBS

BOMBER SHOOTS FIRST

ONE LESS BDMS

FIGHTER SHOOTS

BOMBER MISSES ON FIRST SHOT

142
ACT, 55, 51, CO, 1, ..., 1F
ACT, 55, 56, CO, 1, ..., 16
ACT, 55, 59, ..., 14
REG, 56, 1, 1, F
ACT, 55, 57, ..., A4, G T, 1
ACT, 56, 58, ..., A4, L, E, 1
REG, 57, 1, 1, P
VAS, 57, 4, CO, 1
ACT, 57, 6, ..., 17
ACT, 57, 58, ..., 13
ACT, 54, 6, ..., 17
REG, 53, 1, 1
REG, 59, 1, 1
ACT, 53, 52
ACT, 53, 52
ACT, 58, 61
REG, 61, 1, 1, P
ACT, 61, 6, ..., 2
ACT, 61, 62, ..., 19
VAS, 62, 6, UF, 3
ACT, 61, 1, CO, 1, ..., A6, GE, 1
ACT, 53, 75, ..., 45, LT, 1
REG, 75, 1, 1, A
ACT, 52, 42, ..., A1, LT, 35
ACT, 57, 66, ..., A1, GE, 35
ACT, 59, 62
REG, 62, 1, 1, A
ACT, 62, 21, ..., A1, EQ, 1
ACT, 62, 22, ..., A1, EQ, 2
ACT, 62, 23, ..., A1, EQ, 3
ACT, 62, 24, ..., A1, EQ, 4
ACT, 62, 25, ..., A1, EQ, 5
ACT, 62, 26, ..., A1, EQ, 6
ACT, 62, 27, ..., A1, EQ, 7
ACT, 62, 28, ..., A1, EQ, 8
ACT, 62, 29, ..., A1, EQ, 9
ACT, 62, 30, ..., A1, EQ, 10
ACT, 62, 31, ..., A1, EQ, 11
ACT, 62, 32, ..., A1, EQ, 12
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ACT, 62, 35, ..., A1, EQ, 15
ACT, 62, 36, ..., A1, EQ, 16
ACT, 62, 37, ..., A1, EQ, 17
ACT, 62, 38, ..., A1, EQ, 18
ACT, 62, 39, ..., A1, EQ, 19
ACT, 52, 4, ..., A1, EQ, 2
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REG, 24, 1, 1
REG, 25, 1, 1
REG, 26, 1, 1
REG, 27, 1, 1
REG, 28, 1, 1
REG, 29, 1, 1

AI MISSES BUT RETAINS CONTACT
AI MISSES AND LOSES CONTACT
FIGHTER KILLS BOMBER, FIRST SHOT
DOES BOMBER HAVE A BDM
YES HE HAS A BDM
BOMBER SHOOTS
BOMBER KILLS FIGHTER
BDM MISSES ON SECOND PASS
FIRST SHOT BOM KILLED FIGHTER
FIGHTER SURVIVES
FIGHTER WOA
FIGHTER WILL RECYCLE
RECYCLE TO BASE
FIGHTER GETS THE LAST SHOT
DOES HE GET THE BOMBER
IS BOMBER STILL IN AWACS COVERAGE
CALCULATE TO SEE
PENETRATOR IS STILL IN COVERAGE
PENETRATOR IS SAFE
WHICH BOMBER GOT KILLED
BOMBER NUMBER 1
BOMBER NUMBER 2
BOMBER NUMBER 3
BOMBER NUMBER 4
BOMBER NUMBER 5
BOMBER NUMBER 6
BOMBER NUMBER 7
BOMBER NUMBER 8
BOMBER NUMBER 9
BOMBER NUMBER 10
BOMBER NUMBER 11
BOMBER NUMBER 12
BOMBER NUMBER 13
BOMBER NUMBER 14
BOMBER NUMBER 15
BOMBER NUMBER 16
BOMBER NUMBER 17
BOMBER NUMBER 18
BOMBER NUMBER 19
BOMBER NUMBER 20
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REG111, 1, 1
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REG391, 1, 1
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REG461, 1, 1
REG471, 1, 1
REG481, 1, 1
REG491, 1, 1
REG501, 1, 1
DEAD BOMBERS
LIVE BOMBERS
FIN

*** NO ERRORS DETECTED IN INPUT DATA ***

*** EXECUTION WILL BE ATTEMPTED ***

RUN LIVE DEAD LIVE DEAD LAUNCHED
NO. BMBRS BMBRS ALCMS ALCMS ALCMS
1 1 27 268 132 278
Vita

Richard C. Riecks II was born on 24 August 1957 near Laon AB, France. He graduated from high school in Englewood, Colorado in 1975 and attended Colorado State University. There he received the degree of Bachelor of Science Degree in Mathematics in May 1980. After receiving a commission in the USAF through the ROTC program, he was assigned to the School of Engineering, Air Force Institute of Technology, in June 1980.

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# Two Penetration Models Featuring Bomber Defense Missiles Against an AWACS Air Defense

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**Key Words:**
- Bomber Defense Missiles
- Penetration AIDS
- AWACS
- Air Defense
- Bomber Aircraft

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**Abstract:**

(Continued on reverse side if necessary and identify by block number)
Two models were developed for evaluation of bomber defense missiles as penetration aids to bombers carrying cruise missiles. The defense consisted of a forward-based AWACS controlling airborne interceptors. Both models utilize a corridor concept with a single AWACS.

One of the models is a simulation using the Q-GERT computer language; penetrators and interceptors wait in "queues" to be paired by the AWACS "server" for interceptor attempts. The second model is a stochastic analytic approach recursively estimating a separate survival probability for each successive bomber to enter the corridor. This probability reflects delays between intercepts due to fighter attrition. Both models estimate the numbers of bombers surviving, cruise missiles launched and cruise missiles surviving.

The models yielded similar results for 24 different cases. The thesis models represent the effects of fighter attrition, BDM depletion and payload tradeoffs in greater detail than do other similar models.
FILME
7–8