LIMITING PERFORMANCE OF NONLINEAR SYSTEMS WITH APPLICATIONS TO -- ETC(U)
LIMITING PERFORMANCE OF NONLINEAR SYSTEMS WITH APPLICATIONS TO HELICOPTER VIBRATION CONTROL PROBLEMS

FINAL REPORT

WALTER D. PILKEY

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This report summarizes the accomplishments of a study exploring new methods for the vibration control of helicopters. Reanalysis methodology permits a variety of vibration control problems to be solved efficiently. Both analytical and experimental studies have been conducted.
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PROBLEM STATEMENT AND ACCOMPLISHMENTS

The goal of this program is to develop the technology for the vibration control of dynamic systems. Particular emphasis is given to helicopter vibrations. There were several accomplishments. A method for designing dynamic vibration absorbers to obtain antiresonances at specified locations on a harmonically forced vibratory system has been developed. An efficient numerical method for the computation of the frequency response of a vibrating structure as a function of its structural parameters has been used to analyze and design structures for minimum vibration. A derivative-based sensitivity analysis has been used to determine which structural element changes are optimal for vibration reduction. A technique for the optimal reduction of the steady state vibration response over a frequency range has been formulated. The validity and usefulness of the methods described above have been justified by results obtained for a 44 degree-of-freedom, elastic line helicopter model. An experimental helicopter model has been built to investigate active vibration control.

Technical Summary of Analytical Work

I. Dynamic Vibration Absorbers: Antiresonance

A general method has been developed for designing dynamic vibration absorbers which, when attached to a harmonically forced vibratory structure, cause antiresonances at specified points on the original structure. The method is formulated for a conservative multi-degree-of-freedom system described by linear differential equations. These appendant dynamic systems may be attached to the main system at any number of points. The formulation is in terms of the receptance of the absorber at the attachment point; consequently, the absorber may be any mechanical system whose receptance is known in terms of the design parameters.

To describe the method it is necessary to define three integer sets, I, A, and N. I denotes the set of degrees of freedom at which the designer wishes to have antiresonance; A denotes the set of degrees of freedom where absorbers are to be attached; and N denotes the set of points where external forces are applied to the system. Suppose the steady-state equations of motion for the system in the frequency domain are

\[ (K - \omega^2 M) u = F \]  

(1)

The receptance matrix H for a system governed by Eq. (1) is defined as

\[ H = (K - \omega^2 M)^{-1} \]  

(2)
When the original system is considered as a free body with externally applied forces $P_i$, $i \in \mathbb{N}$, and with the forces $f_s$, $s \in A$, exerted by the absorbers on the original structure, the generalized displacement at each degree-of-freedom $i \in I$ is given by

$$u = \sum_{i \in I} H F + \sum_{g \in \mathbb{N}} H f$$

$$i$$

The displacement $u_a$ at each degree of freedom $a \in A$ is

$$u = \sum_{a \in A} H F + \sum_{a \in A} H f$$

$$a$$

Let $\alpha_a$ denote the receptance of the absorber to be designed and attached at degree of freedom $a \in A$. Then

$$u_a = -\alpha_a f_a$$

$$a$$

where $f_a$ is the force exerted by absorber $a$ on the original system, and $\alpha_a$ is a function of the properties of the absorber and the forcing frequency $\omega$.

Since the design goal is to make each $u_i$, $i \in I$, antiresonant, i.e., to have $u_i = 0$ for all $i \in I$, it follows from Eq. (3) that

$$\sum_{s \in A} H f = -\sum_{g \in \mathbb{N}} H F$$

$$s \in A$$

$$g \in \mathbb{N}$$

Suppose the sets $I$ and $A$ both contain $n$ elements. Equation (6) represents $n$ linear equations in $n$ variables $f_s$. If Eq. (6) is solved for $f_s$, $s \in A$, and the solution is substituted into Eqs. (4) and (5), the equation for $\alpha_a$ is

$$\alpha_a = \frac{1}{f_a} \left( \sum_{g \in \mathbb{N}} H F + \sum_{s \in A} H f \right)$$

$$a$$

$$s \in A$$

Equations (6) and (7) are the basis for the design procedure. For a given excitation frequency $\omega$, the forces $f_s$ are found from Eq. (6)
and the receptance $\alpha_a$ is calculated using Eq. (7). The remaining problem is to determine the structural parameters on which $\alpha_a$ depends, such that $\alpha_a$ takes on the value required by Eq. (7). These parameters define the absorber and the design is complete.

II. The Effect of Structural Properties on Helicopter Response

An efficient numerical method has been developed for the computation of the helicopter frequency response as a function of structural properties. This is a local modification method in which the entire helicopter is analyzed once, and the response as a function of mass and stiffness parameters is found by solving linear algebraic systems of equations of reduced order. This analysis allows the designer to reduce vibration levels by choosing values of system property parameters through a design-by-analysis approach since the variation of the system response as these parameters are changed can be quickly calculated and examined.

An outline of this method for a helicopter model made up of beam elements with $4 \times 4$ stiffness, mass, and damping matrices is given as an example. The receptance matrix $R$ for a damped vibratory system is

$$R = (-\omega^2 M + i\omega C + K)^{-1}$$  \hspace{1cm} (8)

Initially, a representative set of system parameter values is chosen to define a base receptance matrix $R$. For the helicopter described by this base receptance matrix $R$

$$R^{-1} x_o = f_o$$  \hspace{1cm} (9)

where $x_o$ is the response vector and $f_o$ the force vector. Suppose now that the stiffness and mass matrices of one fuselage beam element are changed. The modified system is described by

$$R^{-1} y_o + D y_o = f_o$$  \hspace{1cm} (10)

where $y_o$ is the modified system response and $D$ the matrix that is to be added to $R^{-1}$ to account for the mass and stiffness changes in the beam element. Equation (10) may be rewritten as

$$y_o + RDy_o = f_o$$  \hspace{1cm} (11)

The rest of the procedure exploits the sparsity of the matrix $D$. Since only one beam element has been changed, the nonzero
entries of $D$ form a $4 \times 4$ submatrix at the locations where the beam element contributes to the global mass and stiffness matrices. Hence, the nonzero entries of $RD$ form an $N \times 4$ submatrix where $N$ is the number of degrees of freedom of the system. Consequently, the product $RDv$ involves only four coordinates of the full response vector $y$. Let the 4-vector $u$ denote these four coordinates. Then it is possible to extract from Eq. (11) a set of four linear equations to be solved for $u$

$$u + Gu = w \tag{12}$$

where $G$ is a $4 \times 4$ matrix and $w$ a 4-vector of known entries. Now, since the product $RDv$ involves only the entries of $y$ given by $u$, it is clear that, once $u$ is solved by using Eq. (12), the remaining elements of $y$ can be calculated using Eq. (11). This is the frequency response of the modified system.

### III. Antiresonance using Structural Modification

A structural design method was developed to create antiresonances in a sinusoidally loaded vibrating system. In this method, a structural parameter of a beam element in the helicopter model is left as a design variable to be determined such that at a specified degree of freedom on the fuselage, an antiresonance is produced. Related to this design procedure is the direct design of appendant structures.

Suppose again that one beam element is to be modified. Because of the form of element property matrices, the matrix $D$ in Eq. (11) may be written as

$$D = \alpha D_0 \tag{13}$$

where $D_0$ is evaluated at some arbitrarily chosen property values. and $\alpha$ can be regarded as a percent change in the structural properties represented by the matrix $D$. Now Eq. (11) becomes

$$y_0 + \alpha RDy_0 = Rf_0 \tag{14}$$

and Eq. (12) takes the form

$$u + \alpha Gu = w \tag{15}$$

so that $u$ is given by

$$u = (I + \alpha G)^{-1}w \tag{16}$$
where $I$ is the $4 \times 4$ identity matrix. The vector $u$ can be written as the ratio of two polynomials in $\alpha$ if Leverrier's Algorithm is applied to Eq. (16)

$$u_j = \frac{sh_j(s)}{\Delta(s)} \quad 1 \leq j \leq 4$$ (17)

where $h_j(s)$ is a third degree polynomial, $s = 1/\alpha$, and $\Delta(s)$ is a fourth degree polynomial. Since the product $\alpha R_{y_0}$ involves only the four coordinates of $y_0$ given by the $u_j$. Eq. (14) will yield the entire response vector $y_0$ as ratios of polynomials

$$(y_0)_j = \frac{g_j(s)}{\Delta(s)} \quad 1 \leq j \leq N$$ (18)

If $(y_0)_k$ is to be antiresonant, then the required value of $\alpha = 1/s$ is found by solving for the roots of the polynomial $g_k(s)$, i.e., for antiresonance

$$g_k(s) = 0$$ (19)

provided the root $s$ is real and physically realizable. Equation (18) may also be used to study the frequency response as a function of the design parameter $\alpha$.

**IV. Vibration Reduction Over a Range of Excitation Frequencies**

As rotor speed varies, it is sometimes necessary to consider a range of excitation frequencies. Since realistic finite element models of helicopters usually have a large number of degrees of freedom, it becomes important to be able to carry out dynamic analyses quickly and efficiently as design variables are modified. Thus, a slightly extended version of the previously described reanalysis procedure has been coupled to a nonlinear programming optimization method to arrive at optimal, passive, frequency-response control over a selected interval of excitation frequencies.

A possible choice for an objective function is

$$G(b) = \max_{\Omega_2 \leq \omega \leq \Omega_1} \max_{j \in J} |u_j|$$ (20)
where the frequency range of concern is \([\Omega_L, \Omega_U]\). \(J\) is the integer set of degrees of freedom for which a response reduction is sought, and \(b\) is a vector of design variables such as flexural rigidity or damping constant. The design variables are bounded by physical realizability criteria so that the minimization of the cost function \(G\) is subject to the constraints

\[
\begin{align*}
    b_i^L &< b_i < b_i^U, & 1 \leq i \leq n
\end{align*}
\]

along with the frequency domain equations of motion

\[
(-\omega^2 M + i\omega C + K) u = f
\]

at every frequency \(\omega\) in the interval considered.

For a given impedance change \(\Delta Z\)

\[
\Delta Z = -\omega^2 \Delta M + i\omega \Delta C + \Delta K
\]

Eq. (22) may be rewritten in the form

\[
[I_N + Z_0^{-1}(\omega) \Delta Z(\omega)] u = Z_0^{-1}(\omega) f
\]

where \(Z_0\) is the impedance matrix of the original system and \(I_N\) is the \(N\)th order identity matrix. By taking advantage of the sparsity of the modification matrix \(\Delta Z\), an exact reanalysis procedure suitable for automatic machine computation is derived using Eq. (24) such that at any given iterative step in the optimization algorithm is a reduced order system of linear equations which is solved instead of the entire set (24). The reduced system has the form

\[
(I_p + D_p) u_A = u_{oA}
\]

where \(I_p\) and \(D_p\) are matrices of order \(p < N\), the vector \(u_A\) is a subvector of \(u\), and the zero subscript on the right-hand side denotes the unmodified system solution. The elements of the response vector \(u\) not appearing in Eq. (25) are expressible in terms of \(u_A\) by a relationship of the form

\[
\begin{align*}
    u_B &= u_{oB} + H_u u_A
\end{align*}
\]

This reanalysis algorithm makes feasible the repeated evaluation of the objective function (20) for systems with large...
The numbers of degrees of freedom. In fact, the same reanalysis procedure is also applicable to the computation of derivatives with respect to design variables since the relationship

\[
[I_n + Z_o^{-1}(\omega) \Delta Z(\omega)] \frac{\partial u}{\partial b} = -Z_o^{-1}(\omega) \frac{\partial \Delta Z(\omega)}{\partial b} u
\]  

(27)

has the same form as Eq. (24) as far as the sparsity of the product \(Z_o^{-1}(\omega) \Delta Z(\omega)\) is concerned. The efficient computation of gradients of response with respect to design variables is often of great importance in the use of format optimization algorithms.

The overall optimization algorithm may be summarized by the following steps:

1) Guess an initial value for the design vector \(b\).
2) Compute the displacement vector \(u(x, \omega_k)\) for all frequencies \(\omega_k\) of interest using the reanalysis procedure given above in Eqs. (24-27).
3) Compute cost function \(G(b)\) using Eq. (20).
4) Compute cost function derivatives using a reanalysis procedure analogous to that used in Step 2.
5) Apply a constrained optimization algorithm such as the method of feasible directions, enforcing the constraints (21) on \(b\) to compute a solution. The derivatives of step 5 are available for use here.
6) Repeat steps 2-5 until a convergence criterion has been satisfied.

B. Technical Summary of Experimental Work

A working model has been built to investigate a concept for active vibration control in helicopters. Following this concept, the response of each blade is used in the control equations to generate feedback signals to control actuators on the blades. Since "cross signals" are used, the control equations can be written so that the system resembles a gust alleviation system in a conventional airplane, in which the gust sensor is mounted on a probe ahead of the wing. Otherwise, it can have a conventional control logic, or it can combine the conventional system with the "gust" system.

A block diagram giving the important components is shown in Figure 1, while the present status of each component is discussed below. This status is also summarized in the figure by the degree of shading; full shading of a block indicates completion, half shading indicates partial completion, and no shading indicates that no work has been done. At the time of writing this report, the actuator control system has been "breadboarded."
Fig. 1. Block Diagram of Components in System for Active Vibration Control of Helicopter
board has been designed and etched, and actuators are complete. The speed control has not been perfected because it does not appear necessary.

1. Main DC Power Supply
A DC power supply (Figure 2) is used to provide current to drive the main rotor. The current is controlled by a bank of variacs driven by an electric motor. AC relays have been added so that this motor can be driven from TTL outputs or from push buttons mounted on the console.

2. DC Motor
A vertically mounted 28 volt DC motor from an aircraft cabin-heating system is used to drive the helicopter rotor directly.

3. Tower (Figure 3)
A tower, mounted on a wooden platform, holds the DC motor so that the helicopter blades are at a safe height above the ground. The tower contains an electronic cabinet and a shelf for electronic components.

4. Flow Environment
The tower is presently in a small room with thick walls designed for rotor experiments. Airflow conditions in the room are unfavorable, but, according to safety standards used by our rotor dynamics and centrifugal separation laboratories, we would otherwise need at least 1/10 inch of mild steel for protection should blades fly off. Ultimately, the tests will be run in moving air, either behind a small wind tunnel, or in a larger tunnel.

5. Speed Control
A TTL circuit has been breadboarded to control the speed of the motor through the relays on the main DC power supply. It was found to be unstable when using a once-per-revolution signal. The signal, from a proximity probe, has since been changed to 16 per revolution by addition of a sprocket-like wheel. However, the speed control has not been re-activated, largely because the present system has proved to be quite steady. The speed control can be seen on the control console in Figure 2.

6. Model Helicopter Rotor (Figure 4)
A helicopter model helicopter rotor was obtained commercially. This consists of the rotor hub, blades, axle, bearings, bearing blocks, swashplate, and cyclic and collective control rods. The assembly was mounted for direct drive from the motor, and provisions were made for adjustments to cyclic and collective pitch. The so-called "rigid rotor" configuration option was used.

7. Strain Gages
Short aluminum blocks were made and inserted into the blade roots. These have complete strain gage bridges capable of reading root bending moments, without being affected by tension or torsion loads. They extend the blade radius 35 mm.
8. 36 V Instrumentation Power Supply
A two-amp, 36 volt, unregulated DC power supply was built to provide power to the electronics on the hub.

9. Slip Rings
Two bronze slip rings were built and mounted on the axle. These supply the 36 volt power to the hub. The axle was grooved to admit the two conductors.

10. Strain Gauge Amplifier
A round circuit board, mounted just above the blades, contains regulated 5V and 18V power supplies. The 5V supply is used only to polarize the strain gauge bridges. The signal from each bridge is amplified by two LM 741 operational amplifiers. Three 20-turn potentiometers are installed for adjustments on each circuit. The board shows clearly in Figures 7 and 8.

11. DC to FM Conversion
The amplified DC signals are converted to FM by the VCO portions of two LM 562B phase-lock loops. Center frequencies are 1.0 and 1.414 MHz.

12. Annular Antennae
The two FM signals are transmitted by annular antennae which have been made of circuit board material from the patterns shown in Figures 5 and 6. One of the antennae is attached to the hub below the blades, the other is fixed to the mounting on the motor. The rotating half has two 180° segments for the 1.0 and 1.414 MHz signals. These correspond to a 360° segment on the fixed half, which receives the combined signal. Corresponding 360° segments carry the signal ground. The antennae can be seen mounted in Figures 7 and 8.

13. Times-16 Signal
The rotating antennae half also contains sixteen sectors. It was intended that these would be used in conjunction with a pair of short segments in the fixed half to generate a revolution times 16 signal from which the blade azimuth could be determined and supplied to the TRS-80 computer. However, the present configuration includes a sixteen tooth wheel on the rotor shaft from which a fixed proximity probe picks up the "times-16" signal. Circuits on the demodulation board detect the positive and negative going edges of this signal and thus convert it to a "times-32" signal.

The electronic cabinet on the tower contains a card rack with a +5V, ±15 power supply.

15. FM to DC Monitor
A card in the rack contains permanent circuits which demodulate the 1.0 and 1.414 MHz strain gauge signals or the .861 and 1.189 MHz control feedback signals using LM562B phase-lock loops. The demodulated signals can be displayed on an oscilloscope, or they can be read directly on a voltmeter.
16. Interface Board
A wire-wrap board, hinged to the top of an electrical box, contains the interface circuits for the TRS-80 Model III. The box contains power supplies for +5V and ±15V.

17. FM to DC Demodulator and Trigonometer Functions
The FM to DC demodulation circuit on the interface board demodulates the 1.0 and 1.414 MHz strain gauge signals. The two outputs are read into sample and hold circuits as $x_1$ and $x_4$; these are also multiplied by sines and cosines of the azimuth angles discussed below, using four DAC 7523 multiplying D/A converters. The times-32 signal derived from the proximity probe generates a 5-bit cosine which is used to address a fusible-link PROM containing a sine table. By logical operations on the 5-bit count, the following signals are generated:

$$x_2 = \sin \theta, \quad x_3 = \cos \theta, \quad x_5 = \sin(\theta + 180^\circ), \quad x_6 = \cos(\theta + 180^\circ)$$

where $\theta$ is the blade azimuth angle. The "32-times" signal also sets a flip/flop so that on completing the cycle, the computer can be programmed to count for new data before starting another cycle.

18. A/D Conversion
A CDAS 9528R A/D analyzer reads both inputs $X_1$ and $X_4$ together with their products with the sines and cosines of the azimuth angles $X_2, X_3, X_5$, and $X_6$.

19. Radio Shack TRS-80 Model III Computer
This is a standard Model III computer, with two disc packs, and is sold as a home computer. The microprocessor is a Z80, and there are 48K of dynamic storage. For test purposes, it can be programmed in BASIC, but for the present use, it must be programmed in machine language so that reasonable speed can be achieved. The interface board is mapped into input/output, using instructions shown in Table 1.

20. Machine Language Program
The complete control logic is incorporated in the computer program. Two conflicting approaches to this have been considered:

1. In the traditional control theory approach, each control output $Y$ (blade position) would be generated from the corresponding strain gauge reading $X$.

2. In the "gust alleviator" approach, each blade would act as a sensor to control the other, following blade. This assumes that the disturbances are quasi-cyclic, while the first approach assumes that they are completely random.
In generating an initial control program, an intermediate approach has been taken, with each input generated from both strain gauges. The control equation is

\[
\begin{bmatrix}
Y_1 \\
Y_2 \\
Y_3
\end{bmatrix} = [C][X_1 X_2 X_3 X_4 X_5 X_6]^T + D
\]

with the inputs \(X_1\), \(X_2\), \(X_3\), and \(X_6\) used in such a way that "once-per-revolution" signals, such as pilots control inputs, do not drive the actuators to the stops. Values for the C and D matrices can be inserted through a BASIC master program. The constant terms are included because all input signals represent positive voltages in the range of 0 to 5V converted to 8-bit digital numbers. Thus, \(\sin 0\) is read as 80 hexadecimal. At present, a multiplication subroutine with an 8-bit output is used. However, once some initial testing has been completed, it will be easy to modify this program. One important consideration is whether to continue reading the four trigonometric functions, suffering the long conversion delays, or to generate these internally.

**TABLE 1** TRS-80 Model III Instructions to Interface Board

<table>
<thead>
<tr>
<th>FUNCTION</th>
<th>ASSEMBLY CODE</th>
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<tbody>
<tr>
<td>LATCH (\sin \theta)</td>
<td>OUT (8).A</td>
</tr>
<tr>
<td>LATCH (\cos \theta)</td>
<td>OUT (1).A</td>
</tr>
<tr>
<td>SAMPLE CH #1</td>
<td>OUT (2).A</td>
</tr>
<tr>
<td>HOLD CH #1</td>
<td>OUT (3).A</td>
</tr>
<tr>
<td>LATCH (\sin(\theta + 180^\circ))</td>
<td>OUT (4).A</td>
</tr>
<tr>
<td>LATCH (\cos(\theta + 180^\circ))</td>
<td>OUT (5).A</td>
</tr>
<tr>
<td>SAMPLE CH #2</td>
<td>OUT (6).A</td>
</tr>
<tr>
<td>HOLD CH #2 = (X_1)</td>
<td>OUT (7).A</td>
</tr>
<tr>
<td>OUTPUT CH #1 = (Y_1)</td>
<td>OUT (17).A</td>
</tr>
<tr>
<td>OUTPUT CH #2 = (Y_2)</td>
<td>OUT (18).A</td>
</tr>
<tr>
<td>MUX (X_1)</td>
<td>OUT (32).A</td>
</tr>
<tr>
<td>MUX (X_1 \ast \cos \theta = X_2)</td>
<td>OUT (33).A</td>
</tr>
<tr>
<td>MUX (X_1 \ast \cos \theta = X_3)</td>
<td>OUT (34).A</td>
</tr>
<tr>
<td>MUX (X_4)</td>
<td>OUT (36).A</td>
</tr>
<tr>
<td>MUX (X_4 \ast \cos(\theta + 180^\circ) = X_5)</td>
<td>OUT (37).A</td>
</tr>
<tr>
<td>MUX (X_4 \ast \sin(\theta + 180^\circ) = X_6)</td>
<td>OUT (38).Z</td>
</tr>
<tr>
<td>READ STATUS</td>
<td>IN A. (9)</td>
</tr>
<tr>
<td>READ AID</td>
<td>IN A. (16)</td>
</tr>
<tr>
<td>CLEAR FLIP/FLOP</td>
<td>IN A. (32)</td>
</tr>
<tr>
<td>GENERATE TIMING SIGNAL</td>
<td>IN A. (48)</td>
</tr>
</tbody>
</table>

A = Contents of computers 8-bit accumulator  
\(\theta\) = Azimuth angle
21. D/A Converters
Output data \( Y_1, Y_4 \) for the two blades are latched in digital form on the interface board and converted to analog form in two DAC 7523 8-bit D/A converters.

22. DC to FM Conversion
The analog signals to the two blades are converted to FM signals at .841 and 1.189 MHz using LM 562B phase lock loops. By restricting frequency variations to within the range 92% to 1.09% of nominal values, cross-talk between the four channels should be eliminated. It should be noted that the VCO's on phase-lock loops do not produce pure sinusoidal signals, therefore the receivers can lock onto harmonics of the nominal frequencies so that the system is restricted to a 2:1 frequency range.

23. FM to DC Monitor
The same card can be used on the card rack to monitor the 1.189 and 1.682 MHz signals from the interface board, as is used to monitor the strain gauge signal.

24. Annular Antennae
The two FM signals are to be transmitted back onto the hub via the same pair of annular antennae as are used to transmit the strain gauge signals. The signals share a common ground track.

25. FM to DC Demodulation
A second circuit board, to be mounted above the strain gage board, will contain a demodulation circuit, using LM 562B phase-lock loops.

26. Actuator Solenoids
Circuitry on the demodulator board will drive the actuator solenoids which have already been mounted in place of the collective pitch linkages. These solenoids act one-way only, and have been provided with return springs. Feedback position sensors will be incorporated into a feedback control system, which includes four operational amplifiers and two transitors to drive each solenoid. This system has already been evaluated, and the circuit board is being assembled.
PARTICIPATING PERSONNEL

W.D. Pilkey. Principal Investigator
B.P. Wang, Associate Investigator
J.K. Haviland. Associate Investigator
A. Palazzolo. Received PhD
E. Woomer. Received PhD
L. Kitis. PhD Candidate
PUBLICATIONS

Several computational design techniques for the vibration control of helicopters have been developed. The following manuscripts describe this work.


2. "Helicopter Vibration Reduction by Local Structural Modification." Accepted for publication by the J. of American Helicopter Society.


The contents of the first three manuscripts were summarized and our more recent work (paper 5) was accorded a more detailed explanation in this report.
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