CRACK CLOSURE AND CRACK GROWTH RATE: AN EMPIRICAL MODEL TO ACCOUNT--ETC(U)
AUG 81  G S JOST, L R GRATZER

UNCLASSIFIED

ARL/STRUC-387
CRACK CLOSURE AND CRACK GROWTH RATE:
AN EMPIRICAL MODEL TO ACCOUNT FOR R

by

G. S. JOST and L. R. GRATZER

Approved for Public Release.

© COMMONWEALTH OF AUSTRALIA 1981

COPY No 82 03 20 095 August 1981
CRACK CLOSURE AND CRACK GROWTH RATE:
AN EMPIRICAL MODEL TO ACCOUNT FOR R

by

G. S. JOST and L. R. GRATZER

SUMMARY

An empirical equation is proposed to account for stress ratio effects in fatigue crack growth based upon the crack closure concept. It is characterised by two fitted parameters.

The new equation is used to establish effective stress intensity ranges for three sets of extensive crack growth rate data from the literature covering the stress ratio range $-1 \leq R < 1$. Least squares best fitted Paris equations are then used to predict crack growth lives for comparison with the individual originals. Actual lives are almost invariably within the range from $\frac{1}{3}$ to 2 times those predicted.

The study shows that even better fits to the data would result from using a sigmoidal growth rate/effective stress intensity range relationship. This approach is being followed up.
**ABSTRACT**

An empirical equation is proposed to account for stress ratio effects in fatigue crack growth based upon the crack closure concept. It is characterised by two fitted parameters.

The new equation is used to establish effective stress intensity ranges for three sets of extensive crack growth rate data from the literature covering the stress ratio range $0.5 \leq R < 1$. Least squares best fitted Paris equations are then used to predict crack growth lives for comparison with the individual originals. Actual lives are almost invariably within the range from $1/3$ to $2$ times those predicted.

The study shows that even better fits to the data would result from using a sigmoidal growth rate effective stress intensity range relationship. This approach is being followed up.
CONTENTS

1. INTRODUCTION 1

2. MODELS FROM THE LITERATURE 1
   2.1 Elber, 1971 1
   2.2 Bell and Creager, 1974 1
       Bell and Wolfman, 1976 2
       Eidinoff and Bell, 1977 2
   2.3 Newman, 1976 2
   2.4 Schijve, 1979 2
   2.5 Summary 2

3. PROPOSAL 3

4. APPLICATION TO CRACK GROWTH RATE DATA 3

5. CRACK GROWTH PREDICTION 5

6. CONCLUSIONS 6

REFERENCES

TABLES

FIGURES

DISTRIBUTION
1. INTRODUCTION

The significance of the observation by Elber that the fracture surfaces of cracks in specimens undergoing tensile fatigue loadings remain closed during the lower stress part of the cycle has now become generally appreciated. There can be no doubt that the phenomenon must modify substantially the manner in which the behaviour of cracked regions to subsequent fatigue loading is assessed; in particular, allowance for consideration of crack closure in predictive fatigue crack growth modelling would seem to be mandatory.

In this Report, several empirical models which have been proposed for relating crack closure to stress ratio are considered briefly, and a more flexible empirical model, free of most of the limitations of earlier models, is presented. The ability of the new model to condense crack growth rate data obtained at several stress ratios is then examined. Finally, predicted crack growth curves obtained from the condensed growth rate data are compared with experimental data, and reveal (surprisingly) little scatter.

2. MODELS FROM THE LITERATURE

In this Section several models from the literature relating crack closure under constant amplitude loading and stress ratio are examined and compared.

2.1 Elber, 1971

The first proposal relating crack opening stress*, $S_{op}$, and the other cycle stresses $S_{max}$ and $S_{min}$ to stress ratio $R$ was made by Elber in the form

$$U = (S_{max} - S_{op}) (S_{max} - S_{min}) / (R)$$

where $U$ was defined as the effective stress range ratio. For 2024-T3 aluminum alloy data he found that

$$U \approx 0.4R - 0.5$$

approximately, in the range 0.1 ≤ $R$ ≤ 0.7.

From (1) and (2)

$$S_{op} \approx S_{max} - 0.4R^2 - 0.1R - 0.5.$$  \hspace{1cm} (3)

This expression is shown as the upper curve in Fig. 1, along with the data on which it is based. Schijve has pointed out that since (3) has a minimum at $R = 0.125$ it cannot physically be correct for $R < 0.125$ where the crack opening stress must continue either to decrease or be asymptotic to some constant low level.

2.2 Bell and Creager, 1974

Bell and Wolfman, 1976

Eldinof and Bell, 1977

In modelling crack growth under variable amplitude sequences, the above authors have also used the concepts of crack closure. They have fitted empirical equations of the following form

* No distinction is made here between opening stress and closing stress. Although the phenomenon is known as crack closure, it is actually crack opening during the increasing part of the loading cycle which is of interest.
to aluminum and titanium alloy data in the range \( 1 - R - 1 \).

\[
S_{\text{op}}/S_{\text{max}} = (S_{\text{op}}/S_{\text{max}})^{1 + (\frac{N_{\text{op}}/S_{\text{max}}}{N_{\text{op}}/S_{\text{max}} + 1}) (1 - R)^P
\]

(4)

where the subscripts \( 1 \) and 0 refer to \( R \) values. This equation has flexibility in that the opening stress ratio \( S_{\text{op}}/S_{\text{max}} \) must be specified at \( R = 0 \) and \( R = 1 \), along with the exponent \( P \). On the other hand, there is a major deficiency: at \( R = 1 \) the value of \( S_{\text{op}}/S_{\text{max}} \) does not necessarily equal one as is required; as \( R \rightarrow 1 \), the minimum stress approaches the maximum as must the opening stress. Representative values of the constants from Reference 5 for four aluminum alloys and three titanium alloys are given in Table 1, their plots being shown in Figure 2. Figure 3 shows a redrawing of Figure 2B of Reference 5, highlighting the variability which is a feature of crack opening measurements.

2.3 Newman, 1976\(^*\)

Newman carried out a two-dimensional finite element analysis using an elastic-perfectly plastic material to predict crack closure and crack opening stresses during cyclic loading. Predictions were made at \( R \) values of \( -1, 0 \), \( -0.5 \), \( 0 \), and \( 0.5 \) using two values of the ratio of maximum stress to yield stress. When \( R = 0 \), and irrespective of the \( S_{\text{max}}/S_{\text{yield}} \) values used, predictions agreed very closely with those of Elber, Figure 1. However, for \( R > 0 \) differences in predicted \( S_{\text{op}}/S_{\text{max}} \) appear, according to the ratio \( S_{\text{max}}/S_{\text{yield}} \); the larger this latter, the lower is \( S_{\text{op}}/S_{\text{max}} \). More importantly for present purposes, however, the predictions confirm the intuitive notion of an (asymptotic?) decrease in \( S_{\text{op}}/S_{\text{max}} \) for increasingly negative \( R \).

2.4 Schijve, 1979\(^*\)

Schijve has adopted a cubic polynomial and used it to condense 2024-T3 Al clad crack growth rate data in the range \( 1 - R < 1 \):

\[
S_{\text{op}}/S_{\text{max}} = (1 - R)^2 - 0.25R^2 - 0.5R - 0.45
\]

(5)

This equation is very similar to that of Elber for \( R = 0 \), but corrects its unsatisfactory performance for \( R < 0 \). Figure 1. Schijve points out that (5) has a very weak minimum at \( R = -0.67 \); it also has a weak maximum at \( R = 1 \). Both slope \( S_{\text{op}}/S_{\text{max}} \) and magnitude \( S_{\text{op}}/S_{\text{max}} \) are unity at \( R = 1 \).

Although no explanation is offered as to how the coefficients in (5) were determined, it will be seen later that the collective choice is a very happy one for the particular data used. Different data may well require other coefficient values - their determination will be no simple task if a satisfactory solution is to be found by trial and error.*

2.5 Summary

1. Schijve's formulation for 2024-T3 in the range \( 1 - R < 1 \) represents a significant improvement over that of Elber, which was limited to the range \( 0.1 < R < 0.7 \).

2. Newman's finite element predictions confirm the general trend of a flattening out of the \( S_{\text{op}}/S_{\text{max}} \) versus \( R \) curve for increasingly negative \( R \). For \( R > 0 \), the predictions agree well with those of Elber and Schijve.

3. The model of Bell and co-authors, for the range \( 1 - R < 1 \), although possessing unusual limiting values for some materials, does have a flexibility which the other analytical models lack. Although values are required for three constants in each formulation, the curves of Figure 2, for two alloy groups, suggest that a family of curves formed from some basic curve might suffice. This idea is pursued in the following.

* Since this Report was written, further Dutch activity in this area has been published (Refs. 12 and 13).
3. PROPOSAL

The crack opening data and the forms of the empirical models reviewed in Section 2 and shown in Figures 1 and 2, suggest the following boundary conditions, which have been adopted here, for the relationship between \( S_{op} \), \( S_{max} \), and \( R \):

1. \( S_{op} \leq S_{max} \) at \( R = 1 \)
2. \((S_{op} S_{max})' = 1 \) at \( R = 1 \)
3. \((S_{op} S_{max})' = 0 \) at some specified low value of \( R \), \( X \) say, below which \( S_{op} S_{max} \) is constant.
4. \((S_{op} S_{max})' = 0 \) in the range \( X < R < 1 \).

i.e. there are no points of inflexion. By setting \((S_{op} S_{max})' = 0 \) at \( R = X \), the second derivative (as well as the first) becomes continuous into the (constant \( S_{op} S_{max} \)) region beyond \( R = X \).

A cubic polynomial in \( R \) conforming to the above boundary conditions has, for given \( X \), the following unique solution:

\[
(S_{op} S_{max})_{int} = \frac{R^3}{12} - \frac{3AR^2}{4} + \frac{3AX-R}{(3X-2)} - \frac{3X}{(3X-1)^2}
\]

where the subscript refers to an intermediate stage in the process of establishing a relationship between \( S_{op} S_{max} \) and \( R \).

For \( X = 1 \), the lower limit of \( R \) considered by most of the models examined in Section 2, (6) becomes

\[
(S_{op} S_{max})_{int} = \frac{R^3}{12} - \frac{R^2}{4} + \frac{R}{4} - \frac{1}{12}
\]

(7)

This is shown in Figure 4 along with the relationships, eq. 6, for neighbouring values of \( X \).

Although (6) has flexibility in that \( X \) may be arbitrarily chosen, the choice automatically specifies the entire curve, including the level of \((S_{op} S_{max})_{int} \) at \( R = X \). According to material, Figures 1 and 2 suggest, for example, a range from about 0.3 to 0.5 at \( R = 1 \). Clearly, for given \( X \), a shift in \( S_{op} S_{max} \) is required to accommodate this variation, and a suitable one takes the following form:

\[
S_{op} S_{max} = (S_{op} S_{max})_{int} + Z(1 - S_{op} S_{max})_{int}
\]

(8)

where \((S_{op} S_{max})_{int} \) is a function only of \( R \) and \( X \), given here by eq. (6), and \( Z \) is a shift parameter.

Equation (8) also satisfies the boundary conditions given above. Representative families of shifted \( S_{op} S_{max} \) versus \( R \) for \( X = 1 \) and \( X = 0.5 \) are shown in Figure 5.

Thus in the proposed formulation (6) and its shifted version (8) there is complete freedom to choose:

(a) the limiting lower value of \( R (R = 1) \) at which the \( S_{op} S_{max} \) versus \( R \) curve reaches zero slope and curvature, and

(b) the required magnitude of \( S_{op} S_{max} \) at this point, by choice of an appropriate value of shift parameter, \( Z \).

Specification of these two parameters defines completely the relationship between \( S_{op} S_{max} \) and \( R \).

4. APPLICATION TO CRACK GROWTH RATE DATA

Fiber proposed that crack closure concepts might be useful in explaining the known qualitative effect of stress ratio on crack growth. In particular he suggested that crack growth rate was not so much a function of the stress intensity range \( \Delta K \) as of the effective stress intensity range \( \Delta K_{eff} \), i.e. the difference between the maximum stress intensity and that corresponding to crack opening. Thus, in place of \( \Delta K \) in the Paris equation:

\[
da \frac{dN}{dA} = B \Delta K^n
\]
where

\[ \Delta K = K_{\text{max}} - K_{\text{op}}. \]

there should be substituted \( \Delta K_{\text{eff}} \):

\[ da/dN = C \Delta K_{\text{eff}}^n \]  

(9)

where

\[ \Delta K_{\text{eff}} = K_{\text{max}} - K_{\text{op}}. \]

Since the crack opening stress, \( S_{\text{op}} \), is a function of \( R \) (eqs 6 and 8) the crack opening stress intensity \( K_{\text{op}} \) is also a function of \( R \). Thus, a plot of \( \log da/dN \) versus \( \Delta K_{\text{eff}} \), which includes data at differing \( R \) values, should have associated with it only the variability in crack growth rate itself, as was first demonstrated by [1] using data from Reference 8.

The effectiveness of the formulations proposed in Section 3 has been evaluated using three published sets of extensive crack growth data: those of Shuie [2] on 2024-T3 Al clad sheet and those of Hudson [8] on 2024-T3 and 7075-T6 aluminium alloy sheet. These data sets are shown in Figures 6(a), (b) and (c) in terms of the conventional \( da/dN \) versus \( \Delta K \) plots. The observable groupings of data are associated with the differing \( R \) values used on test. Shuie’s data on 2024-T3 Al clad sheet are given in the Appendix to Reference 2 in terms of \( da/dN \) and \( \Delta K \), and have been so used here; Hudson’s data are given in Reference 8 in terms of cycles to given crack lengths. These latter have been fitted by cubic splines, from which the derivatives at the given crack lengths have provided the required \( da/dN \) data. The corresponding stress intensities were obtained as indicated below.

For the centrally through-cracked specimens used in all three investigations the stress intensity, \( K \), at the crack tip is given by

\[ K = S \sqrt{a} f \]  

(10)

where \( S \) is the remote applied stress, \( a \) is the semi-crack length and \( f \) is the finite width correction factor given by

\[ f = \chi \left( \frac{a}{W} \right), \]  

(11)

\( W \) being the total width of the specimen.

The effective stress intensity, \( \Delta K_{\text{eff}} \), for use in eq. (9) is therefore given by

\[ \Delta K_{\text{eff}} = K_{\text{max}} - K_{\text{op}} = \left( S_{\text{max}} - S_{\text{op}} \right) \chi \left( \frac{a}{W} \right). \]  

(12)

where \( S_{\text{max}}, S_{\text{op}} \) is the function of \( R, A \) and \( Z \) given by equations (6) and (8).

Since \( \Delta K_{\text{eff}} \), for each \( R \) was not measured on test, but is being modelled here by equations (6), (8) and (12), there is flexibility in choosing \( A \) and \( Z \), and some appropriate criterion is required in making that choice. The least-squares criterion has been adopted here whereby the sum of squares of residuals obtained by fitting Paris equations to the data for a range of \( A, Z \) combinations are compared. That particular combination yielding the smallest sum of squares then provides the best possible fit of the present model to the data. The (computerised) procedure is as follows:

1. Select \( A \) and \( Z \) from the ranges 1.5 \( \leq A \leq 0 \) and 2 \( \leq Z \leq 2 \).
2. Calculate \( \Delta K_{\text{eff}} \) for every data point using equations (6), (8) and (12).
3. Fit a Paris equation (9) to the data expressed as \( \log da/dN \) versus \( \log \Delta K_{\text{eff}} \), and note the sum of squares of the \( \log da/dN \) residuals.
4. Repeat steps 1, 2, and 3 for all \( A, Z \) pairs to determine that pair yielding the minimum residual sum of squares of \( \log da/dN \).

* Although the crack growth rate data of Shuie and Hudson have not been derived in an identical manner from the original test data, it is thought unlikely that this fact is of consequence here. Data from different sources are not being compounded.
Examples of the residual sums of squares of log $da/dN$ associated with the regressions of $X$ and $Z$ combination yielding the minimum sum of squares is noted in each case. It can be seen that for both sets of 2024-13 data the influence of the shift parameter $Z$ on the residual sum of squares is very much less than that of the stress ratio asymptote $X$, for the 7075-16 data the influence of $Z$ is rather more significant than the family of $Z$ curves do not coincide to the same extent in the minimum sum of squares region.

A plot showing the optimum $S_{N_{ap}}$, $S_{max}$ versus $R$ curves for the three data sets, is shown in Figure 8, and the corresponding transformed crack growth rate data, expressed in terms of $\Delta K_{th}$, are shown in Figures 9(a), (b) and (c). The least squares fitted straight lines are also shown, along with their equations. The effectiveness of taking $R$ into account in this way is very clear.

Cumulative probability plots of the residuals are shown in Figures 10(a), (b) and (c) using normal ordinates. The plotting position, $P_i$, used for the $i$th residual has been determined from

$$P_i = (i - 0.5)/N$$

for large $N$, the total number of data points, after Reference 9. Also shown on the figures are smoothed Kolmogorov-Smirnov 95% confidence bounds based on the given number of data points in each case assumed perfectly normally distributed. Although Figures 9 indicate that the common assumption of normality of log crack growth rate data is not seriously challenged here for the transformed data, the application of the more stringent $\chi$ test shows that only the transformed 2024-13 data of Hudson form an empirical distribution which does not differ significantly from the corresponding normal. Table 2, column 2.

Relevant data associated with the regressions are tabulated in Table 3. Also given there are the results of applying Schijve's equation (5) to the data. The minimum sums of squares are seen to be little higher than those of the present formulations $F$ tests based on the variance of the residuals for both models show no significant differences for the 2024-13 data sets; for 7075-16 however the present formulation is significantly better. The variance ratio becomes significant at the 1% level. Meaningful comparisons of $C$ and $n$, the Paris constants, are not possible since they operate on different equations (16), (8) and (12) for the present formulation and (5) and (12) in Schijve's case.

5. CRACK GROWTH PREDICTION

The best fit Paris equations have now been used to predict the crack growth cycles between the successively long crack lengths as established on test and given in the original references. The expectation is, of course, that the predictions will, on average, agree with the data from which they derive. These predictions are shown in Figures 11(a), (b) and (c) expressed in terms of (test cycles predicted cycles), each between the same two crack lengths, plotted against $\Delta K_{th}$ based upon final interval crack length.

Figures 11 show that, overall, the above expectation is realised. There can be seen however, in all three graphs, very clear indications of a systematic "smearing" trend of the ratio with $\Delta K_{th}$, and the reason is readily seen from Figure 9. The data points there do, of course, fall in the intermediate section of the overall $S$ shaped or sigmoidal $da/dN$ versus $\Delta K_{th}$ plot. They should, therefore, be even better fitted by an appropriate sigmoidal curve. Points falling below the regression line on Figure 9 become points above unity on Figures 11, and vice versa: the tendency for increased variability at low $\Delta K_{th}$ is no doubt due at least in part, to inaccuracies

* Preliminary analysis showed that the circled data points in Figures 6(a) and 6(c) were falling into the increasingly steep inflexion region of the sigmoidal crack growth rate curve beyond the central (and nominally linear) region considered here. Those data points have been omitted in further analysis.

† In establishing some of the original crack growth data on 2024-13 Al clad sheet from which the growth rate data summarised in Reference 7 were based, it was found that the total number of $a$ vs $N$ data points exceeded substantially the total number of derived $da/dN$ data points. This fact accounts for the differing tallies entered on Figures 10(a), and 11(a) and 12(a).
associated with measurements and derivations at shorter crack lengths. Figures 11 show that the test cycles predicted cycles ratio falls, for the most part, between 1.3 and 2.

This same fact may be seen more readily from the cumulative frequency plots of Figure 12, where the ratio very seldom exceeds 2 and is seldom below 1.3. These graphs reflect behaviours similar to their counterparts of Figure 10, although, in the case of Figure 12c), an increased deviation from the normal takes it outside even the weak Kolmogorov-Smirnov 95% limits. The results of $\chi^2$ tests to check goodness-of-fit with the corresponding normals are similar to those of the data of Figures 10: only the 2024-T3 data of Hudson do not differ significantly from normal, Table 2, column 3. The variance ratio tests listed in Table 4 show that the differences between variability in log crack growth rate and that of log life ratio are not significant for either of the 2024-T3 data sets; for the 7075-T6 data they are, however, significant at the 0.5% level.

6. CONCLUSIONS

1. The crack opening concept leading to effective, as opposed to nominal, stress intensity ranges in fatigue cycling has provided a sound basis for quantifying the known effect of stress ratio on fatigue crack growth rate. Several existing models have been examined in this Report.

2. A flexible formulation relating crack opening stress ratio with conventional stress ratio (based on a third order polynomial and requiring for its definition two fitted parameters) has been applied to three extensive constant amplitude crack growth rate data sets from the literature (2024-T3 Al clad, 2024-T3 and 7075-T6 sheet materials). Paris equations have been least-squares fitted to the log da/dN data expressed in terms of log $\Delta K_{eq}$ in each case.

3. Actual growth cycles over given crack length increments fall almost invariably within the range from 1.3 to 2 times those predicted using these equations.

4. The study shows that even better fits would be attainable using, instead of the linear relationship assumed here, the known overall sigmoidal relationship between crack growth rate and effective stress intensity range. This approach is being followed up and should permit realistic extrapolation into the low and high crack growth rate regions beyond those of the present data.
REFERENCES


TABLE 1
Eidinoff and Bell's Equation, Ref. 5

<table>
<thead>
<tr>
<th>Material</th>
<th>$(S_{op} - S_{max})_1$</th>
<th>$(S_{op} - S_{max})_2$</th>
<th>$(S_{op} - S_{max})_3$</th>
<th>$P$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Ti-6Al-4V (EBWeld)</td>
<td>0.332</td>
<td>0.400</td>
<td>0.633</td>
<td></td>
</tr>
<tr>
<td>Ti-6Al-4V (STA)</td>
<td>0.347</td>
<td>0.400</td>
<td>0.633</td>
<td></td>
</tr>
<tr>
<td>Ti-6Al-4V (Annealed)</td>
<td>0.450</td>
<td>0.500</td>
<td>0.622</td>
<td></td>
</tr>
<tr>
<td>2219 1851</td>
<td>0.450</td>
<td>0.500</td>
<td>0.622</td>
<td></td>
</tr>
<tr>
<td>2024 1851</td>
<td>0.450</td>
<td>0.500</td>
<td>0.622</td>
<td></td>
</tr>
<tr>
<td>7075 1651</td>
<td>0.450</td>
<td>0.500</td>
<td>0.622</td>
<td></td>
</tr>
</tbody>
</table>

These equations are plotted in Figure 2.

TABLE 2
Summary of $A^2$ Tests on log Crack Growth Rate
Residuals and log Life ratios

<table>
<thead>
<tr>
<th>Data Source</th>
<th>Residuals (Figs 10)</th>
<th>Ratios (Figs 12)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Schijve 2024-13 Alclad</td>
<td>36.4</td>
<td>21.6</td>
</tr>
<tr>
<td>Hudson 2024-13</td>
<td>3.2</td>
<td>5.2</td>
</tr>
<tr>
<td>Hudson 7075-16</td>
<td>22.7</td>
<td>70.1</td>
</tr>
</tbody>
</table>

$95\% \chi^2$ 2.17
$5\% \chi^2$ 14.07
### TABLE 3
Least Squares Fitted Paris Equations

<table>
<thead>
<tr>
<th>Data</th>
<th>Present Formulation (eq. 6, 8, 12)</th>
<th>Schijve (eq 5, 12)</th>
<th>Significance Tests†</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Min. SS*</td>
<td>St. Dev.</td>
<td>X</td>
</tr>
<tr>
<td>2024-T3 Alelad</td>
<td>7</td>
<td>423</td>
<td>9.41</td>
</tr>
<tr>
<td>2024-T3</td>
<td>8</td>
<td>210</td>
<td>4.77</td>
</tr>
<tr>
<td>7075-T6</td>
<td>8</td>
<td>409</td>
<td>7.50</td>
</tr>
</tbody>
</table>

* Minimum sum of squares of log $da/dN$ about regression line

† $S_1^2$: Schijve variance

$S_2^2$: Variance of present formulation.
<table>
<thead>
<tr>
<th>Data</th>
<th>2024-13 Alcad (Schiye)</th>
<th>2024-13 (Hudson)</th>
<th>7075-16 (Hudson)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>S</td>
<td>r</td>
<td>S</td>
</tr>
<tr>
<td>Log (test life predicted life)</td>
<td>0.1604</td>
<td>0.1534</td>
<td>0.1511</td>
</tr>
<tr>
<td>Log (test dN)</td>
<td>0.1494</td>
<td>0.1510</td>
<td>0.1511</td>
</tr>
<tr>
<td>(S₁ S₂)^2</td>
<td>1.153</td>
<td>1.032</td>
<td>1.242</td>
</tr>
<tr>
<td>5&quot;. Fr₁. r₂</td>
<td>1.154</td>
<td>1.257</td>
<td>1.177</td>
</tr>
<tr>
<td>Significance</td>
<td>N.S.</td>
<td>N.S.</td>
<td>Sig.</td>
</tr>
</tbody>
</table>

* See footnote, Section 5.
FIG. 1 ELBER'S AND SCHIJVE'S EQUATIONS, ELBER'S DATA, NEWMAN'S PREDICTIONS

Legend

Elber's data range: I

Newman's predictions:

- $S_{\text{max}}/S_Y = 0.3$
- $S_{\text{max}}/S_Y = 0.4$

FIG. 2 EIDINOFF AND BELL'S EQUATIONS FOR FOUR ALUMINIUM AND THREE TITANIUM ALLOYS
FIG. 3 EIDINOFF AND BELL'S EQUATION FOR 2219–T851 ALUMINIUM ALLOY, AND MEASURED DATA RANGES

FIG. 4 FAMILY OF CURVES, eq (6)
FIG. 5(a) FAMILY OF CURVES, eqs (6) AND (8), $X = -1$

FIG. 5(b) FAMILY OF CURVES, eqs (6) AND (8), $X = -0.5$
MATERIAL: 2024-T3 ALCLAD SHEET

DATA SOURCE: SCHIJVE (REF. 7)

\[-0.66 \leq R \leq 0.73\]

Note: Circled data point is omitted in all further analysis.

FIG. 6(a) CRACK GROWTH DATA: 2024-T3 ALCLAD SHEET
MATERIAL: 2024-T3 Sheet

DATA SOURCE: HUDSON (REF 8)

\(-1 \leq R \leq 0.7\)

FIG. 6(b) CRACK GROWTH DATA: 2024-T3 SHEET
MATERIAL: 7075-T6 SHEET

DATA SOURCE: HUDSON (REF. 8)

$-1 \leq R \leq 0.8$

Note: Circled data points are omitted in all further analysis.

FIG. 6(c) CRACK GROWTH DATA: 7075-T6 SHEET
Total number of data points = 423
Optimum (X, Z) = (−0.62, −0.71)
Minimum sum of squares = 9.41

Total number of data points = 210
Optimum (X, Z) = (−1.09, 0.16)
Minimum sum of squares = 4.77
Total number of data points = 409
Optimum (X, Z) = (-0.68, 1.14)
Minimum sum of squares = 7.50

FIG. 7(c) ERROR SUMS OF SQUARES FOR 7075-T6 SHEET
(HUDSON, REF 8)

FIG. 8 BEST FIT CRACK OPENING RELATIONSHIPS FOR THREE DATA SETS
MATERIAL: 2024-T3 Alclad sheet

DATA SOURCE: SCHIJVE (REF 7)

\[ \begin{align*}
X &= -0.62 \\
Z &= -0.71 \\
\text{SUM OF SQUARES} &= 9.41
\end{align*} \]

\[ -0.66 \leq R \leq 0.73 \]

FIG. 9(a) TRANSFORMED CRACK GROWTH DATA: 2024-T3 ALCLAD SHEET
MATERIAL: 2024-T3 Sheet

DATA SOURCE: HUDSON (REF 8)

\[ x = -1.09 \quad z = 0.16 \]

SUM OF SQUARES = 4.77

\(-1 \leq R \leq 0.7\)

FIG. 9(b) TRANSFORMED CRACK GROWTH DATA: 2024–T3 SHEET
MATERIAL: 7075-T6 Sheet

DATA SOURCE: HUDSON (REF 8)

\[ X = -0.68 \quad Z = 1.14 \]

SUM OF SQUARES = 7.50

-1 \leq R \leq 0.8

FIG. 9(c) TRANSFORMED CRACK GROWTH DATA: 7075-T6 SHEET
Hudson 2024-T3
210 Data Points
Mean = 0.0000
SD = 0.1510

Fig. 10(b) Cumulative Probability & 95% Confidence Bounds of Log Crack Growth Rate Residuals - 2024-T3 Sheet
Hudson 7075-T6
409 Data Points
Mean = 0.0000
SD = 0.1356

FIG. 10(c) CUMULATIVE PROBABILITY & 95% CONFIDENCE BOUNDS OF LOG CRACK GROWTH RATE RESIDUALS - 7075-T6 SHEET
FIG. 11(a)  RATIO OF TEST CRACK GROWTH CYCLES TO THOSE PREDICTED VERSUS EFFECTIVE STRESS INTENSITY RANGE – 2024-T3 ALCLAD SHEET
FIG. 12(b) CUMULATIVE PROBABILITY & 95% CONFIDENCE BOUNDS OF RATIO (TEST/PREDICTED) CYCLES – 2024–T3 SHEET

Hudson 2024–T3
210 Data Points
Mean = -0.0235
SD = 0.1534
FIG. 12(c) CUMULATIVE PROBABILITY & 95% CONFIDENCE BOUNDS OF RATIO (TEST/PREDICTED) CYCLES – 7075-T6 SHEET

HUDSON 7075-T6
408 DATA POINTS
MEAN = -0.0181
SD = 0.1511
DISTRIBUTION

AUSTRALIA

Department of Defence
   Central Office
   Chief Defence Scientist  1
   Deputy Chief Defence Scientist  2
   Superintendent, Science and Technology Programmes  3
   Counsellor, Defence Science (U.S.A.)  21
   Defence Central Library  4
   Document Exchange Centre, D.I.S.B.  22
   Joint Intelligence Organisation  23

Aeronautical Research Laboratories
   Chief Superintendent  23
   Library  24
   Superintendent Structures  25
   Divisional File-Structures  26
   Authors: G. S. Jost  27
            I. R. Gratzer  28

Materials Research Laboratories
   Library  29

Defence Research Centre
   Library  30

Central Office
   Director General Army Development (NSO) (4)  31

Central Studies Establishment
   Information Centre  35

Engineering Development Establishment
   Library  36

RAN Research Laboratory
   Library  37

Victorian Regional Office
   Library  38

Navy Office
   Naval Scientific Adviser  39

Army Office
   Army Scientific Adviser  40
   Royal Military College Library  41

Air Force Office
   Aircraft Research and Development Unit, Scientific Flight Group  42
   Air Force Scientific Adviser  43
   Technical Division Library  44

Department of Industry and Commerce
   Government Aircraft Factories
   Library  45
Department of Transport
Library
Flying Operations and Airworthine Division

Statutory and State Authorities and Industry
Commonwealth Aircraft Corporation, Library

CANADA
CAARC Coordinator Structures

FRANCE
ONERA, Library

INDIA
CAARC Coordinator Structures
Defence Ministry, Aero Development Establishment, Library
Hindustan Aeronautics Ltd., Library
National Aeronautical Laboratory, Information Centre

ISRAEL
Technion-Israel Institute of Technology, Professor J. Singer

JAPAN
National Research Institute for Metals, Fatigue Testing Division

NETHERLANDS
National Aerospace Laboratory (NLR), Library
DELT University of Technology, Professor Schijve

NEW ZEALAND
Defence Scientific Establishment, Library
Universities
Canterbury, Library

SWEDEN
Aeronautical Research Institute, Library

SWITZERLAND
F. W (Swiss Federal Aircraft Factory)

UNITED KINGDOM
Ministry of Defence, Research, Materials and Collaboration
CAARC, Secretary (NPL)
British Library, Lending Division
CAARC Co-Ordinator, Structures
British Aerospace:
Hatfield-Chester Division, Library

UNITED STATES OF AMERICA
N.A.S.A. Scientific and Technical Information Facility
Metals Abstracts, Editor
Spares