Recoil Distributions in Some Proton Reactions

J. B. Langworthy

Radiation Survivability & Detection Branch
Condensed Matter & Radiation Sciences Division

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The distributions of recoil products from the two most important double decays of silicon excited by proton absorption are calculated by analytical and numerical methods. The nuclear reactions treated are Si(p,p\alpha) and Si(p,2p) and it is assumed the decays are by evaporation. Conversion of the results to distributions of dose is discussed. The latter distributions are of interest in the soft upset of satellite electronic memories.
RECOIL DISTRIBUTIONS IN SOME PROTON REACTIONS

Single event upset in electronic memory devices has caused a great deal of concern in the last two or three years. One particular area of concern is upset produced in memories on board spacecraft which operate in the earth's radiation belts. Recent NRL calculations have shown that upset rates can be very high in spacecraft memories. Though protons do not deposit energy sufficiently fast to produce upsets directly, some of their secondary products are sufficiently heavy and energetic to do so and occur with sufficient frequency to contribute to observed rates. This paper calculates the energy distribution of the recoil nuclei from the two most important proton induced reactions, Si(p,\alpha)Mg and Si(p,2p)Al. The dose density from the recoil is easily obtained from this and is briefly discussed. It is assumed for each case that the two light products are emitted essentially simultaneously so that the recoil velocities for both steps of the two step decay are calculated using the mass of the residual nucleus.

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An excited nucleus with velocity $\vec{v}_0$ decays first with a recoil velocity distribution, $D_1(v_1)$, spherical relative to $\vec{v}_0$, and again with $D_2(v_2)$, also spherical relative to $\vec{v}_1$. In order to calculate the distribution of the energy deposited by the recoil, it is first necessary to obtain the combined distribution of the two steps in the frame of $\vec{v}_1$.

Assuming the decay steps are by evaporation

$$D_i(E_i) = c_i (E_i - B_i) e^{-E_i/t_i}, i = 1, 2, E_i > B_i$$

(1)

where $B_i$ and $t_i$ are reaction constants and $c_i$ is determined by

$$1 = \int dE D_i(E)$$

(2)

The energy parameter is center of mass reaction energy so

$$E_i = \frac{1}{2} m_i v_i^2$$

$$m_i = \frac{M_R}{M_R + M_i}, \quad M_i = M_p, \quad M_2 = \left\{\begin{array}{c} M_\alpha, \quad M_\gamma = \left\{\begin{array}{c} M(24Mg) \quad \text{for } M_\gamma = M(24Mg) \end{array}\right. \end{array} \right.$$  

(3)

Carrying out the normalization, one obtains

$$c_i = \frac{B_i}{t_i} e^{-B_i/t_i}$$

(4)
Since it is our intention to apply the vector convolution theorem in velocity space it is first necessary to change the given distributions from energy densities to velocity space densities. From (2)

\[ l = m \int D(v)vdv \]

\[ = (4\pi)^{-1} m \int v^{-1} D(v)d^3v. \quad (5) \]

Thus to convert \( D \) to a velocity space density one writes

\[ D'(v) = (4\pi)^{-1} m D(v). \quad (6) \]

Assuming no correlation between \( D_1 \) and \( D_2 \), the combined distribution is given as a function of \( \vec{v} = \vec{v}_1 + \vec{v}_2 \) in the same frame as \( \vec{v}_1 \) by the convolution theorem.

\[ D'(|\vec{u}|) = (4\pi)^{-1} m_1 m_2 \int v_1^{-1} v_2^{-1} D_1(v_1)D_2(v_2)\delta(\vec{u} - \vec{v}_1 - \vec{v}_2)d^3v_1 d^3v_2 \]

\[ = (4\pi)^{-1} m_1 m_2 \int v_1^{-1} v_2^{-1} D_1(v_1)D_2(v_2)\delta(\vec{u} - \vec{v}_2)d^3v_1 d^3v_2, \quad v_2 = |\vec{u} - \vec{v}_1|, \]

\[ = (4\pi)^{-1} m_1 m_2 \int D_1 D_2 v_1 v_2^{-1} dv_1 d\mu_1 d\phi_1 \]

\[ = (8\pi)^{-1} m_1 m_2 \int D_1 D_2 v_1 v_2^{-1} dv_1 d\mu_1, \quad (7) \]

where \( \mu_1 = \cos \theta_1 \). Note that \( v_2 = 0 \) does not occur. Since
\[ v_2^2 = u^2 - 2uv_1 u_1 + v_1^2, \quad (8) \]

the entire \( \mu_1 \) dependence is in \( v_2^{-1}D_2 \). Also one has from (8), holding \( u \) and \( v_1 \) fixed,

\[ v_2 dv_2 = -uv_1 d\mu_1 \]

\[ v_2^{-1}d\mu_1 = - (uv_1)^{-1}dv_2. \quad (9) \]

As \( \mu_1 \) goes from \(-1\) to \(1\), \( v_2 \) goes from \( v^+ = u + v_1 \) to \( v^- = |u-v_1| \).

For the \( \mu_1 \) integral then one has

\[
\int_{v^-}^{v^+} v_2^{-1}D_2 dv_2 = -(uv_1)^{-1} \int_{v^+}^{v^-} D_2 dv_2
\]

\[
= -(uv_1 t_2^2)^{-1} \int_{v^+}^{v^-} (\frac{1}{2} m_2 v^2 - B_2) e^{-(\frac{1}{2} m_2 v^2 - B_2)/t} dv
\]

\[
= -(uv_1)^{-1} \int_{v^+}^{v^-} \left( \frac{1}{2} m_2 v^2 - B_2 \right) e^{B_2/t_2} \int_{w^+}^{w^-} e^{-w^2} dw
\]

\[
= (uv_1)^{-1} (2t_2 m_2)^{-\frac{1}{2}} e^{B_2/t_2} \left[ \int_{w^+}^{w^-} e^{-w^2} dw \right] - \int_{w^+}^{w^-} \left( \frac{w^2 - B_2}{t_2 - 1} \right) e^{-w^2} dw
\]

\[
= (uv_1)^{-1} (2t_2 m_2)^{-\frac{1}{2}} e^{B_2/t_2} \left( \int_{w^+}^{w^-} e^{-w^2} + \frac{1}{2} \left( \pi \right)^{\frac{1}{2}} (2B_2/t_2 - 1) \text{erf}(w) \right) \mid_{w^+}^{w^-} \quad (10)
\]
The point \( u = 0 \) will not be a problem after changing back to energy density below. Since there is no remaining angular dependence, \( D'(u) = D'(u) \) and,

\[
D'(u) = \frac{m_1}{8\pi u} \left( \frac{m_2}{2t} \right)^{\frac{3}{2}} e^{\frac{B_2}{t}} \int_{v_m}^{\infty} dv_1 D_1(v_1) F(v_1)
\]

where

\[
F(v_1) = \left( we^{-w^2 + \frac{1}{2} (\pi)^{\frac{1}{2}} (2B_2/t - 1)erf(w)} \right) \left| w^- \right|^{w^+}
\]

and

\[
w^+ = \left( \frac{m_2}{2t} \right)^{\frac{3}{2}} (u + v_1), \quad w^- = \text{Max} \left\{ \left( \frac{m_2}{2t} \right)^{\frac{3}{2}} |u - v_1|, \left( \frac{B_2}{t} \right)^{\frac{3}{2}} \right\}
\]

with

\[
v_m = \left( \frac{2B_1/m_1}{} \right)^{\frac{3}{2}}. \tag{11}
\]

For the reactions of interest \( B_2 \geq B_1 \) and \( B_2/m_2 \geq B_1/m_1 \) so the lower limit for \( v_1 \) includes that for \( v_2 \). Also in evaluating (11), whenever \( (B_2/t)^{\frac{3}{2}} > w^+ \), \( F \), and thus the integrand, vanish at this particular \( v_1 \).

Of course at this point one has only the spherical distribution relative to \( v_0 \). This is a convenient point at which to check normalization. One requires

\[
1 = \int D'(u)d^3u
\]

\[
= 4\pi M^{-1} \int D'(E)dE \tag{12}
\]

so a simple check is just a sum over
\[ D(u) = 4\pi u \frac{M_k}{R} D'(u) \] (13)

on a grid linear in energy.

In the numerical evaluation it was convenient to program \( D_1 \) and \( F \) as statement functions. The error function (erf) is a single precision library function and ought to be fairly fast. Still each call of \( F \) is roughly equivalent to the evaluation of two one-dimensional integrals. Use of Simpson rule is made so one loop for evaluation and one for integration was written to allow printing one complete integrand for error checking. Another convenient error check was to require the sign of \( F(v_1) \) to be positive. The constants are \( B_1 = 2.4, B_2 = 4.6, t_1 = t_2 = 2.45 \) in MeV. Units of the \( D_1 \) are MeV\(^{-1} \) so writing the masses in MeV gives the velocities in units of light velocity, \( c \). Since the integral is in velocity and the normalization check is in energy, the corresponding loops are on different grids designed to evaluate overlapping regions in \( D_1(v_1) \) and \( D'(u) \). Thus it was convenient to begin each loop with a unitless energy scale using \( B_1 \) as the unit. Then, for example, if a range in \( E(u) \) from 0 to 10 (10 \( B_1 \) MeV) gives an adequate evaluation of \( D_1 \) the appropriate range for \( v_1/v_m \) would be 1 to 10\(^1/2 \). While in principle there is a serious complication of upper limits required by numerical evaluation, this is ignorable in practice because of the property of these distributions in having compact support. Thus any accuracy desired within computer precision may be attained simply by extending the upper limits sufficiently.

The program RECOIL (listed in the Appendix) has been written, the first half performing the preceding calculations, and the results from this part
are given in Tables I and II. Upper limits for the unitless energy grids were 49 for (p,pα) and 36 for (p,2p), corresponding to evaporation energies of 117.6 and 86.4 MeV. For comparison the peak of the evaporation distribution is at \( B_i + t_i \), or 7.05 and 4.85 MeV, respectively. The overall sums obtained were 0.996 and 0.989, respectively. It may be seen below that extending the energy range decreases this error still.

Consider now the transformation of these distributions to the frame of \( v_0 \). Assume absorption of a 30 MeV proton in the initial excitation, forming a compound nucleus. One has

\[
M_{pp} = M_{co}
\]

and obtains for the excitation energy

\[
E_x = (1 - \frac{M_2}{M_c})T, \quad T = \frac{1}{2} \frac{M_{pp} v^2}{c^2}
\]

Taking \( M_c = M(^{28}\text{Si} + p) \) one obtains \( E_x = 28.96 \) MeV, \( E_c = \frac{1}{2} M_{co}^2 = 1.04 \), \( v_0 = 0.008945c \). Thus the transformation is in part accomplished by the substitution

\[
u^2 = v^2 - 2v_0 \cos \theta + v_0^2, \quad (16)
\]
Table I

PROGRAM RECOIL ON (P.PA)

SPHERICAL ENERGY DISTRIBUTION (PER MEV)

<table>
<thead>
<tr>
<th>ENERGY DISTRIBUTION</th>
<th>0.0967</th>
<th>0.092224</th>
<th>0.2922</th>
<th>0.246142</th>
<th>0.4836</th>
<th>0.173525</th>
<th>0.34079</th>
<th>0.5771</th>
<th>0.372533</th>
<th>0.7738</th>
<th>0.408342</th>
<th>0.9673</th>
<th>1.0640</th>
<th>0.426253</th>
<th>1.1607</th>
<th>0.431509</th>
<th>1.2575</th>
<th>0.435221</th>
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<th>0.437758</th>
<th>1.4509</th>
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<th>0.436139</th>
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<td>0.429517</td>
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<td>0.080470</td>
<td>3.3855</td>
<td>0.069456</td>
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<td>0.019682</td>
<td>4.2560</td>
<td>0.016673</td>
<td>4.3528</td>
</tr>
</tbody>
</table>

OVERALL SUM = 0.996384.
Table II

PROGRAM Recoil on (P,2P)

SPHERICAL ENERGY DISTRIBUTION (PER MEV)

<table>
<thead>
<tr>
<th>ENERGY DISTRIBUTION</th>
<th>0.0000 0.000000</th>
<th>1.0797 0.288429</th>
<th>2.1161 0.003285</th>
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<tr>
<td>0.0864 1.042541</td>
<td>1.1666 0.211004</td>
<td>2.2025 0.002168</td>
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<tr>
<td>0.1296 1.079965</td>
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</tr>
<tr>
<td>0.1727 1.119379</td>
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<td></td>
</tr>
<tr>
<td>0.2159 1.099337</td>
<td>1.2956 0.128065</td>
<td>2.3321 0.001156</td>
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</tr>
<tr>
<td>0.2591 1.101681</td>
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<td>2.3752 0.000935</td>
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</tr>
<tr>
<td>0.3455 1.103173</td>
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</tr>
<tr>
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</tr>
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<td>0.9069 0.505051</td>
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<td>0.9501 0.443215</td>
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<td>1.0365 0.334745</td>
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<td></td>
<td></td>
</tr>
</tbody>
</table>

OVERALL SUM 0.988986
redefining \( \theta \) as the angle between \( v_o^+ \) and \( \vec{v} \). Similarly defining \( \theta_1 \), as the angle between \( v_o^+ \) and \( u \), one notes that (13) has the form

\[
D(u) = \frac{dN}{dE_1 d\Omega_1}
\]  

(17)

where \( d\Omega_1 = d\mu_1 d\phi_1 \), even though it lacks \( \Omega_1 \) dependence. The transformation is therefore completed by the Jacobian.

\[
\frac{\partial(E_1, \Omega_1)}{\partial(E, \Omega)} = \frac{\partial(E_1, \Omega_1)}{\partial(E, \Omega)} \frac{dN}{dE_1 d\Omega_1}
\]  

(18)

Calculations of this transformation to the lab have been added to RECOIL, forming the second half. In addition lab solid angle \( d\Omega \) is averaged to obtain \( dN/dE \). Further the norm and average energy are obtained. The resulting distributions are in Tables III and IV and Figure 1. These tables show peaks near 1.35 and 0.97 MeV and average energies of 2.55 and 1.55 MeV for \((p,\alpha)\) and \((p,2p)\) respectively.

Now notice that from (18) one easily obtains an energy deposition distribution, given the recoil energy loss \( dE/dx \),

\[
\frac{dN}{dx d\Omega} = \frac{dE}{dx} \frac{dN}{dE d\Omega}
\]  

(20)

If one assumes constant energy loss (a crude approximation) and sets \( dE/dx = \varepsilon \), then one recognizes that the multiplication of (18) by a constant is removed by renormalization. One then views (20) as nothing but a unit
**Table III**

**PROGRAM RECOMMEND (P,PA)**

**SPHERICAL ENERGY DISTRIBUTION**:  
POINTS: 101 EMAX: 9.6748 DU(EMAX): 0.666660 NORM: 0.999838

**LAB ENERGY DISTRIBUTION (PER MV)**

<table>
<thead>
<tr>
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<th>LAB ENERGY DISTRIBUTION</th>
</tr>
</thead>
<tbody>
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<td></td>
</tr>
<tr>
<td></td>
<td></td>
</tr>
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**LAB NORM: 9.94159E-01 EAVE: 2.8350**

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### Table IV

**Program RECOIL on (P,2P)**

**Spherical Energy Distribution:**

- Points: 129
- $E_{\text{MAX}}$: 5.5278
- $\Delta(E_{\text{MAX}})$: 0.000000
- $\text{NORM}$: 0.988988

**LAB Energy Distribution (PER MEV)**

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<th>0.000000</th>
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**LAB NORM**: 9.85881E-01 **EAVE**: 1.5531
Fig. 1 - Recoil distributions
change and accepts the numerical distribution (18) as giving the
distribution per \( \varepsilon^{-1} \) \( \mu m \), the distance in which 1 MeV is lost. Instead of
assuming constant energy loss one may utilize the code E-DEP-1 to include
straggling effects. A minor change will be required however since the
energy-deposited report of E-DEP-1 does not include ionization loss, a
significant part of dose. In making this change it will be permissible to
ignore radiation loss, a few percent effect. Short of using E-DEP-1 but
better than assuming constant \( dE/dx \), one can find expressions giving the
variation.

In fact \( \varepsilon = 1 \) MeV/\( \mu m \) is only about twenty percent low when averaged over
the first few \( \mu m \). Thus the implication of Figure 1 is that significant
energy, deposited in a spherical region of radius 3 to 4 \( \mu m \), should be added
to that previously considered.

Acknowledgments

The author wishes to thank E. L. Petersen for suggesting this problem
and P. Shapiro for helpful conversations and a critical reading.

References

1. C. S. Guenzer, R. G. Allas, A. B. Campbell, J. M. Kidd, E. L. Petersen,

5. Useful manipulations of vectorial distributions are given in J. B. Langworthy, Naval Research Laboratory Report 6391 (1966).

APPENDIX

SOURCE LISTING

ASC FAST FORTRAN COMPILER
CP OPTIONS = (M,X)

PROGRAM RECOIL
C SET UP FOR SIG(P,P ALPHA). FOR (P,2P) SET B2=B1, MA=MP & CHANGE MR
C TRANSFORMATION OF SPHERICAL DISTRIBUTION TO LAB ADDED

IMPLICIT REAL(M)

DIMENSION SG(241),DC282),UC(82),DV(66),EV(66)
DIMENSION DVC(22),EV2(22),DY3(22),EV3(22)

EQUIVALENCE (DV2,DV(23)),(EV2,EV(23)),(DV3,DV(45)),(EV3,EV(45))

DATA MC,MP,MA,MR/931.5016,938.2796,4.0026,23.98504/931.5016,938.2796,4.0026,23.98504/

E(V)=(V*V*MC-B1)*RT

OL(V)=RT*E(1(V)*EXP(-F1(V))

FW(V)=V*EXP(-OL(V))*FG*ERF(V)

J2=J1-1

OL(V)=RT*OL(V)*FLOAT(J2)

MA=MA*MC

MR=MR*MC

MO=0.5*(MR+MP)/MP

M1=MO*MR

M2=MR*(MR+MA)*0.5/MA

RT=1.0/RT

VW=SQRT(M2*RT)

F=M0*VW*EXP(B2*RT)

P=ATAN(1.0)

FC=(2.0*B2*RT-1.0)*SQRT(P4)

VM=SQRT(V1/M1)

SI=2.*OL(V1)*VW

VT=SQRT(B2*RT)

PRINT 61

FORMAT(***PROGRAM RECOIL ON (P,PA)**)

DO I=1,11

U=SQRT(DC*(I-1))*VW

EUC(I)=0.5*MR*U

DO J=2,J1

V1=VU*(1.0+OL*(J-1))

SG(J)=0.

VP=OL(V1)*VW

IF(VP.LE.VI) GO TO 2

IF(VP.LT.VT) VM=VT

SG(J)=FW(VM)-FW(VP)

IF(SG(J),G,1E-6) GO TO 7

SG(J)=FG*OL(V1)*SG(J)

CONTINUE

S=S+2.*SG(J)*SG(J-1)

PRINT 69,S,0,J1

FORMAT("***NEGATIVE INTEGRAND, V1,U,J1=""1P3E11.2,2I4)

S=0.5*(SG(J)-SG(I))

CONTINUE

DO J=1,2

S=S+2.*SG(J)+SG(J-1)
0048  1  DU(I)=SS1
0049       A=A+DU(I)
0050       A=A*EU(I)
0051       PRINT 62,['SHEERICAL ENERGY DISTRIBUTION:'," POINTS=",I4,
0052       ' EMAX=',F7.4,' DUCE_MAX=',F9.6,' NORM=',F9.6)
0053       M=M+MC
0054       EP=30.*NP/ME*MR/ME
0055       EV(I)=0.
0056       DV(I)=0.
0057       DO 10 K=2,K1
0058       EV(K)=0.1*(K-1)
0059       E2=2.0*SQR(EVK)*EP
0060       S=0.
0061       DO 11 J=1,100
0062           I0=1
0063           M=1.0-(2*J-1)/100.
0064           E0=EV(K)+EP-E2*MU
0065           F1=SQR(EVK)/EO
0066           DO 14 I=101,I1
0067           IF(EO.LT.EU())CONTINUE
0068           GOTO 15
0069          14 CONTINUE
0070          GOTO 16
0071          15 I0=I-1
0072          DO 11((I0)+DUC(I)-DUC(I0))*(EO-EUC(I0))/(EU(I)-EU(I0))
0073          S=S+DUC(I)
0074          11 CONTINUE
0075          16 DV(K)=S*0.01
0076          10 CONTINUE
0077          PRINT 64
0078          FORMAT(/  " LAB ENERGY DISTRIBUTION (PER MEV) ",
0079          */ " ENERGY DISTRIBUTION")
0079          PRINT 65,(EVK),DV(K),EV2(K),DV2(K),EV3(K),DV3(K),K=1,22
0080          FORMAT(3(F10.1,F10.6))
0081          S=0.
0082          A=0.
0083          DO 20 K=1,K1
0084          S=S+DV(K)
0085          20 A=A+(EVK)*0.05)*DV(K)
0086          A=A/S
0087          S=S+EV(2)
0088          PRINT 66+S*A
0089          FORMAT(/  " LAB NORM=",1PE13.5," EAVE="PF7.4)
0090          END