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GEODETTIC DATUM TRANSFORMATION BY MULTIPLE REGRESSION
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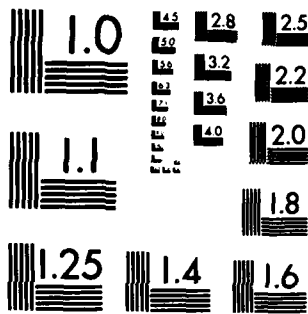
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REPORT DOCUMENTATION PAGE		READ INSTRUCTIONS BEFORE COMPLETING FORM
1. REPORT NUMBER N/A	2. GOVT ACCESSION NO.	3. RECIPIENT'S CATALOG NUMBER
4. TITLE (and Subtitle) GEODETIC DATUM TRANSFORMATION BY MULTIPLE REGRESSION EQUATIONS		5. TYPE OF REPORT & PERIOD COVERED Final Report
		6. PERFORMING ORG. REPORT NUMBER
7. AUTHOR(s) L. T. Appelbaum GDGB		8. CONTRACT OR GRANT NUMBER(s)
9. PERFORMING ORGANIZATION NAME AND ADDRESS Defense Mapping Agency Aerospace Center ATTN: GD St. Louis Air Force Station, Missouri 63118		10. PROGRAM ELEMENT, PROJECT, TASK AREA & WORK UNIT NUMBERS N/A
11. CONTROLLING OFFICE NAME AND ADDRESS Defense Mapping Agency Aerospace Center ATTN: GD St. Louis Air Force Station, Missouri 63118		12. REPORT DATE February 1982
		13. NUMBER OF PAGES 17
14. MONITORING AGENCY NAME & ADDRESS (if different from Controlling Office) N/A		15. SECURITY CLASS. (of this report) UNCLASSIFIED
		15a. DECLASSIFICATION/DOWNGRADING SCHEDULE N/A
16. DISTRIBUTION STATEMENT (of this Report) A - Approved for public release; distribution unlimited		
17. DISTRIBUTION STATEMENT (of the abstract entered in Block 20, if different from Report) N/A		
18. SUPPLEMENTARY NOTES Presented at Third International Geodetic Symposium on Satellite Doppler Positioning, sponsored by Defense Mapping Agency and National Ocean Survey, and hosted by Physical Science Laboratory of the New Mexico State University, Las Cruces, New Mexico, February 8-12, 1982		
19. KEY WORDS (Continue on reverse side if necessary and identify by block number) Geodetic Coordinate Transformation Datum Shift Multiple Regression Regression World Geodetic System		
20. ABSTRACT (Continue on reverse side if necessary and identify by block number) A computer program incorporating the least squares stepwise multiple regression procedure has been used to derive polynomial equations for con- verting coordinates from one geodetic datum to another (datum shifts) as a function of latitude and longitude. Reference coordinate differences through- out the geographic area or datum are used for the regression. Therefore, the regression exhibits sensitivity to regional variations in the coordinate dif- ferences. (These variations are closely related to changes in the geodetic		

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L. T. Appelbaum

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Presented

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DEFENSE MAPPING AGENCY
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GEODETIC DATUM TRANSFORMATION
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MULTIPLE REGRESSION EQUATIONS

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ABSTRACT

A computer program incorporating the least squares stepwise multiple regression procedure has been used to derive polynomial equations for converting coordinates from one geodetic datum to another (datum shifts) as a function of latitude and longitude. Reference coordinate differences throughout the geographic area or datum are used for the regression. Therefore, the regression exhibits sensitivity to regional variations in the coordinate differences. (These variations are closely related to changes in the geodetic control in localized areas.) The resulting equations reflect this in precision. Contrariwise, the traditionally used Molodensky Coordinate Transformation Formulas, as conventionally applied over a geographic area, utilize area average rectangular coordinate differences (ΔX , ΔY , ΔZ).

The stepwise regression program evaluates as many variables as desired (typically 100); e.g., $U^7 V^5$ is a single variable, where U and V are normalized latitude and longitude, respectively. It sequentially incorporates into the equation the variable providing the most improvement in fitting the reference coordinate differences. After such incorporation of a variable, all variables previously incorporated into the equation are examined, and any no longer significant are removed. This continues until an equation of specified precision is obtained. Thus, an equation of given precision contains a minimum number of terms and is therefore relatively computer efficient. This has particular relevance to near real time applications involving computer storage and computation time constraints.

INTRODUCTION

During the last two decades, considerable progress has been made within the Department of Defense (DoD) in reducing the number of geodetic datums used for mapping, charting, and geodetic (MC&G) purposes. This has been accomplished through the development and use of various world geodetic systems, the latest being DoD World Geodetic System 1972 (WGS 72), (Reference 1). Although the number of geodetic datums actively used by the DoD has decreased considerably, multiple datums are still in use for many geographic areas and the transformation of the geodetic coordinates of sites from one datum to another is often required. These coordinate transformations are accomplished using the Molodensky Coordinate Transformation Formulas. Another technique for accomplishing coordinate transformations, multiple regression formulas (equations), is examined in this paper for its applicability to provide precise coordinate transformations in near real time.

REGRESSION PROCEDURE

The terms $\Delta\phi$, $\Delta\lambda$, ΔH , ΔX , ΔY , and ΔZ represent reference coordinate differences (reference datum shifts) as well as regression equation computed datum shifts. This is applicable to geodetic coordinates latitude (ϕ), longitude (λ), height (H), and rectangular coordinates X , Y , and Z . Derived datum shift regression equations provide datum shifts to the equation user at any point within the applicable regression area. These equations are derived by regressing, for known orientation (reference) stations throughout the area, the reference coordinate difference dependent variables $\Delta\phi$, $\Delta\lambda$, ΔH , ΔX , ΔY , and ΔZ individually on selective variables, sometimes called independent variables. For the application subsequently presented herein, these latter variables consist of products of powers of normalized latitude (U) and normalized longitude (V); e.g., $U^3 V^4$ is a single variable. Since regression is on these polynomial two-dimensional variables, the resulting regression equations contain these variables.

The basic idea of the stepwise multiple regression procedure used for this paper is to perform the regression as a series of straight line regressions (steps). As many variables as desired are evaluated in the regression, but only a relatively small number are normally incorporated in the derived equation. The stepwise multiple regression procedure sequentially adds one variable at a time (step) to the equation; namely, the variable that provides the greatest improvement in fitting the reference coordinate differences. After a variable is added, all variables previously incorporated into the equation are examined for significance, and if any is no longer significant it is removed (another step). Correlations and F-tests provide the basis for entering and removing variables. The correlation coefficients are adjusted at each step. This stepwise regression continues until statistical parameters (F-values), used in the F-tests, are satisfied; in actual practice, until the desired equation precision is obtained. The greater the number of variables in the regression equation, the more precise (better fitting) is the equation. Coefficients for each equation variable, and a constant term, are determined.

This stepwise addition and removal of variables assures that only significant variables are retained in the final equation. Thus, a derived equation of given precision contains a minimum number of terms and therefore is relatively computer efficient. This has particular relevance to near real time applications involving computer storage and computation time constraints. For maximum computer efficiency in the computation of datum shifts, the coefficients and exponents of the variables in the set(s) of regression equations are utilized as data in an efficient algorithm.

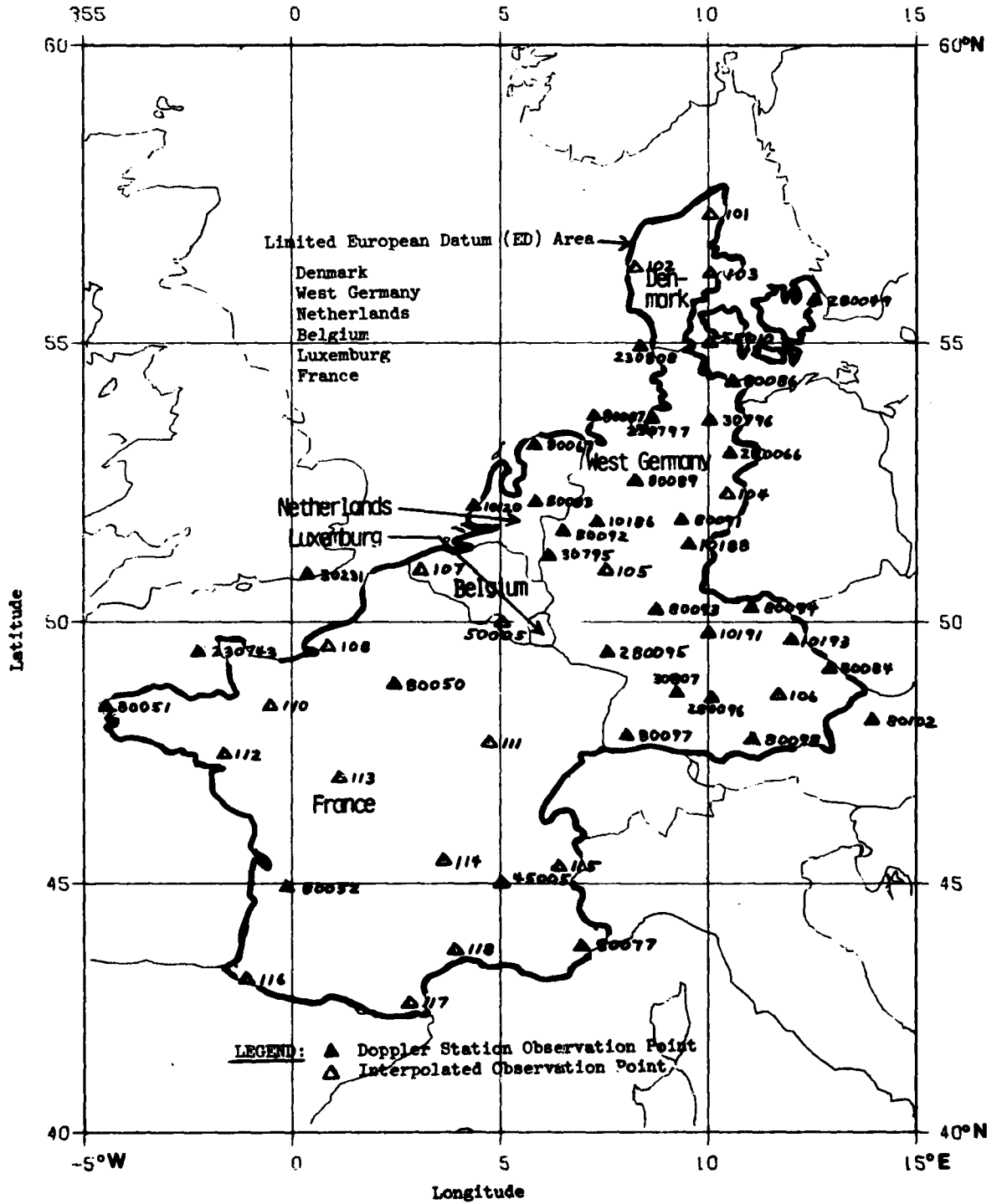
The stepwise multiple regression procedure utilized herein enjoys relatively high favor in the literature. Alternative multiple regression procedures include backward elimination, forward selection, stagewise, all possible, and variations thereof. Procedures are described in Reference 2.

REGRESSION APPLICATION

Datum shift regression equations herein provide shifts between a local geodetic datum and a world geodetic system; namely, between European Datum 1950 (ED 50) and WGS 72. The equations cover a "limited ED 50 area" consisting of Denmark, West Germany, Netherlands, Belgium, Luxemburg, and France, as outlined in Figure 1.

FIGURE 1

LIMITED EUROPEAN DATUM AREA WITH REGRESSION OBSERVATION POINTS



Herein, latitude implies geodetic latitude, and longitude implies geodetic (and geocentric) longitude. Table 1 contains limited ED 50 area datum shift equations for geodetic coordinates latitude (ϕ), longitude (λ), and height (H), and rectangular coordinates X, Y, and Z. The equations were derived to provide a maximum deviation from the reference coordinate differences of 1.5 meters.

These equations are used as shown in Table 1 when transforming from ED 50 to WGS 72 (WGS 72 minus ED 50), and are used with a change of sign when transforming from WGS 72 to ED 50 (ED 50 minus WGS 72). The latter can be readily accomplished by merely changing the sign of the equation datum shift result. Subsequently discussed precision analysis provides verification for the dual direction utility, wherein the latitude and longitude on either datum may be used regardless of the direction of transformation. In practice, latitude and longitude values are generally available on the datum that is being transformed, and are used.

Although variables composed of powers of latitude and longitude may be used in the regression, it is generally more efficient and convenient to use a normalized latitude (U) and a normalized longitude (V) as shown in Table-1. For example, in Table 1 the normalized latitude $U = 3 \phi - 2.61$ is obtained from $U = 3 (\phi - 0.87)$, where 0.87 is the approximate average latitude in radians over the regression area, and 3 is a convenient factor which inhibits large values of equation coefficients. Variables evaluated in the regressions total 99 and consist of all product combinations of U and V containing single digit exponents, selections from which were made for the Table 1 equations.

Figure 1, which outlines the limited ED 50 area, also shows the 53 observation points at which reference coordinate differences were used in the regressions to derive the equations. Table 2 contains this reference data. The first 33 observation points are Doppler stations at which reference differences consist of the differences between ED 50 coordinates obtained from ground survey and WGS 72 coordinates obtained from satellite Doppler observations. The final 20 observation points were located to provide more complete area coverage, and reference coordinate differences thereat were obtained by interpolation. This interpolation was from the reference coordinate differences at the 33 Doppler stations contained in Figure 1 and Table 2 as well as additional (not shown) Doppler stations external to the limited ED 50 area.

The Defense Mapping Agency Aerospace Center (DMAAC) Datum Transformation Stepwise Regression Program was used to derive the datum shift regression equations. The reference Doppler station data was provided by Mr. J. F. Vines, DMAAC, who obtains most of such data from the Satellite Records Desk of the DMA Hydrographic Topographic Center, which accumulates it from various agencies and governments. Mr. D. N. Huber, DMAAC, performed quality evaluation of this data. Aforementioned interpolated reference coordinate differences at 20 observation points were interpolated using datum shift contour charts prepared by Messrs. Huber and Vines using a Point Plotting and Contouring Program developed by the Ohio State University. Mr. D. Holland, DMAAC, performed the Molodensky Formula computations discussed at the end of this paper.

Table 3 is a summary of subsequently discussed tables and contains deviations of regression equation datum shifts relative to reference coordinate differences for each

TABLE 1

Datum Shift Regression Equations Between Limited ED 50 Area and WGS 72

Limited ED 50 Area: Denmark, West Germany, Netherlands, Belgium, Luxemburg, and France.

ED50: European Datum 1950. Identical to the terminology European Datum (ED).

WGS 72: World Geodetic System 1972.

Geodetic Latitude Datum Shift ($\Delta\phi$) in seconds (")

$$\Delta\phi(") = - 3.17250 + 1.96761 U + 0.747893 V - 0.252615 V^2 + 4.68674 U^2 V^2$$

Geodetic (and Geocentric) Longitude Datum Shift ($\Delta\lambda$) in seconds (")

$$\Delta\lambda(") = - 5.03830 - 1.40710 U + 1.60471 V - 0.521318 U^2 + 0.263364 V^2$$

Geodetic Height Datum Shift (ΔH) in meters (m)

$$\Delta H(m) = 47.1915 - 35.1158 U - 18.2122 V + 15.8592 U^2 + 264.165 U^5$$

X Rectangular Coordinate Datum Shift (ΔX) in meters (m)

$$\Delta X(m) = - 83.7539 - 1.98841 U + 4.26886 V$$

Y Rectangular Coordinate Datum Shift (ΔY) in meters (m)

$$\Delta Y(m) = - 107.458 + 10.2703 U - 12.8061 U V + 46.6228 U^2 V$$

Z Rectangular Coordinate Datum Shift (ΔZ) in meters (m)

$$\Delta Z(m) = - 121.619 + 1.72649 V - 4.42630 V^2 - 625.330 U^6 - 1292.75 U^3 V^4 + 10532.7 U^9$$

where $U = \text{Normalized latitude} = 3\phi - 2.61$
 where $\phi = \text{Geodetic latitude in radians.}$

$V = \text{Normalized longitude} = 3\lambda - 0.24$
 where $\lambda = \text{Geodetic (and geocentric) longitude in radians.}$

Use positive λ from 0° to 180° east of Greenwich.
 Use negative λ from 0° to 180° west of Greenwich.

ϕ and λ on either datum (ED 50 or WGS 72) may be used regardless of datum shift direction. There is no significant difference in the computed datum shift.

NOTES:

1. For datum shifts from ED 50 to WGS 72 (WGS 72 minus ED 50), use equations as shown above.
2. For datum shifts from WGS 72 to ED 50 (ED 50 minus WGS 72), use above equations with a change of sign. This can be readily accomplished by changing only the sign of the equation datum shift result.

TABLE 2

LIMITED EUROPEAN DATUM AREA
DATA FOR OBSERVATION POINTS

OBSER. SEQUENCE NO.	OBSER. POINT NO.	GEODETTIC COORDINATES			RECTANGULAR COORDINATES		
		LATITUDE	LONGITUDE	HEIGHT	X	Y	Z
		SECONDS	SECONDS	METERS	METERS	METERS	METERS
1	280049	-2.224	-4.773	30.62	-83.20	-103.75	-122.85
2	30231	-3.252	-5.355	48.07	-85.62	-106.02	-123.08
3	10120	-2.959	-5.261	43.96	-83.52	-106.73	-121.63
4	10186	-2.906	-4.958	41.57	-82.94	-106.28	-122.24
5	10188	-2.834	-4.719	39.79	-83.06	-106.38	-121.88
6	10191	-3.011	-4.588	42.10	-82.25	-107.65	-122.07
7	10193	-2.955	-4.401	39.40	-82.63	-107.71	-122.90
8	30795	-2.938	-5.079	44.39	-83.82	-106.18	-120.10
9	30796	-2.594	-4.907	36.65	-82.84	-106.18	-121.37
10	30807	-3.114	-4.483	45.10	-83.24	-106.39	-120.93
11	80067	-2.747	-5.218	40.57	-84.28	-105.90	-121.74
12	80083	-2.901	-5.104	42.99	-82.79	-105.91	-121.55
13	80084	-2.945	-4.249	41.93	-81.80	-107.05	-120.29
14	80086	-2.460	-4.848	34.96	-82.97	-104.57	-121.80
15	80087	-2.669	-5.082	38.42	-83.32	-104.94	-122.13
16	80089	-2.744	-4.921	39.37	-83.49	-105.74	-121.61
17	80091	-2.793	-4.740	39.37	-83.12	-105.59	-121.85
18	80092	-2.911	-5.000	41.65	-84.17	-106.30	-122.12
19	80093	-3.000	-4.684	42.43	-82.95	-106.59	-121.97
20	80094	-2.930	-4.468	40.49	-82.19	-106.12	-122.12
21	80097	-3.269	-4.551	47.71	-82.94	-107.13	-121.56
22	80098	-3.164	-4.303	44.36	-82.59	-107.31	-121.80
23	80102	-3.020	-4.115	41.82	-81.75	-107.86	-121.03
24	230797	-2.614	-4.995	36.67	-83.51	-105.39	-122.50
25	230808	-2.480	-5.187	36.63	-82.32	-105.44	-121.45
26	280066	-2.594	-4.814	36.94	-83.05	-106.55	-121.26
27	280095	-3.118	-4.706	46.07	-82.53	-106.60	-120.88
28	280096	-3.128	-4.461	43.05	-83.04	-107.56	-122.68
29	80050	-3.337	-5.161	50.53	-85.94	-108.94	-121.51
30	80051	-3.692	-5.621	38.19	-86.94	-109.11	-122.77
31	80052	-3.873	-5.088	62.77	-82.94	-111.38	-121.96
32	80077	-3.698	-4.428	55.70	-82.61	-109.85	-122.39
33	230743	-3.556	-5.585	33.69	-85.05	-109.24	-123.90
34	107	-3.100	-5.240	46.00	-84.29	-107.00	-122.00
35	101	-2.150	-5.240	32.10	-82.93	-103.43	-122.00
36	102	-2.330	-5.210	34.60	-82.71	-104.04	-122.00
37	103	-2.320	-5.040	33.10	-83.00	-103.65	-122.00
38	104	-2.690	-4.740	37.90	-82.59	-106.06	-122.00
39	105	-2.930	-4.870	42.90	-82.50	-106.62	-122.00
40	106	-3.030	-4.330	42.30	-82.15	-107.19	-121.29
41	50005	-3.130	-5.040	46.90	-83.74	-107.56	-122.00
42	55010	-2.450	-4.930	34.20	-83.00	-104.00	-122.00
43	108	-3.360	-5.300	50.80	-85.33	-107.69	-122.33
44	110	-3.520	-5.330	34.20	-85.35	-108.23	-122.55
45	111	-3.400	-4.870	51.40	-83.33	-108.72	-122.00
46	112	-3.640	-5.350	57.20	-85.26	-108.92	-122.76
47	113	-3.590	-5.150	55.90	-84.44	-109.48	-122.08
48	114	-3.660	-4.810	57.00	-83.19	-110.22	-122.24
49	115	-3.580	-4.560	54.20	-82.73	-109.38	-122.00
50	116	-4.050	-5.100	65.20	-84.53	-114.00	-123.86
51	117	-4.010	-4.750	61.90	-83.87	-112.33	-125.27
52	118	-3.810	-4.700	58.80	-83.27	-111.00	-123.30
53	43005	-3.830	-4.670	38.20	-82.88	-109.91	-122.19

TABLE 3

REGRESSION EQUATION DATUM SHIFT PRECISION

Summary at 53 Observation Points Over Limited European Datum Area

Coordinate	Deviation between Regression Equation Datum Shifts and Reference Coordinate Differences			
	* ED 50 to WGS 72		** WGS 72 to ED 50	
	RMS Meters	Maximum Meters	RMS Meters	Maximum Meters
Geodetic Latitude	0.63	1.21	0.63	1.21
Geodetic Longitude	0.63	1.38	0.63	1.38
Geodetic Height	0.67	1.34	0.67	1.34
X	0.53	1.36	0.53	1.36
Y	0.60	1.46	0.60	1.46
Z	<u>0.57</u>	<u>1.41</u>	<u>0.57</u>	<u>1.41</u>
Overall	0.61	1.46	0.61	1.46

* ED 50 geodetic latitude and longitude used for regression equations.

** WGS 72 geodetic latitude and longitude used for regression equations.

coordinate. Transforming from ED 50 to WGS 72 using ED 50 geodetic latitudes and longitudes for the equations results in an overall root mean square (RMS) deviation of 0.61 meter and an overall maximum deviation of 1.46 meters. These same overall deviations are obtained when transforming from WGS 72 to ED 50 using WGS 72 geodetic latitudes and longitudes for the equations.

For each coordinate, Tables 4 through 9 contain results at each of the 53 observation points when transforming from ED 50 to WGS 72 (WGS 72 minus ED 50) using ED 50 geodetic latitudes and longitudes for the regression equations. The tables contain the reference coordinate differences, the regression equation datum shifts, and the deviations of the latter from the former. At the bottom of each table are listed the RMS deviation and maximum deviation for the 53 points. The tables also contain normalized weights used in the equation derivations.

Similar tables, not here presented, were obtained using the same regression equations, with sign change, for transformation in the opposite direction; namely, from WGS 72 to ED 50 (ED 50 minus WGS 72) using WGS 72 latitudes and longitudes for the equations. The maximum difference of results between the two directions at the 53 points for the six coordinates is 0.02 meter. Thus, the same equations, with the appropriate sign, may be used for transformation in both directions.

Table 10 contains the Standard and Abridged Molodensky Coordinate Transformation Formulas. The Standard Molodensky Formulas are longer and provide slightly better precision than the Abridged Molodensky Formulas. For geodetic coordinates, Table 11 contains a summary of deviations of Standard Molodensky Formula datum shifts from the previously discussed 53 reference coordinate differences over the limited ED 50 area. A single set of area average rectangular coordinate differences was used as data ($\Delta X = -83.39$ meters, $\Delta Y = -107.27$ meters, and $\Delta Z = -122.07$ meters). Transforming from ED 50 to WGS 72 results in an overall RMS deviation of 1.58 meters and an overall maximum deviation of 6.45 meters. In comparison, overall regression equation deviations in Table 3 are 0.61 meter RMS and 1.46 meters maximum. Thus, in this illustration, regression equation precision is about three times better than that of the Standard Molodensky Formulas.

Since reference coordinate differences throughout the area are used for the regression, the regression exhibits sensitivity to regional variations in the coordinate differences. (These variations, transmitted from the local ED 50 coordinates, are closely related to changes in the geodetic control in localized areas.) The resulting datum shift regression equations reflect this in precision. Contrariwise, the traditionally used Molodensky Coordinate Transformation Formulas, as conventionally applied over a geographic area, utilize area average rectangular coordinate differences (ΔX , ΔY , ΔZ).

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2. Draper, N.R. and H. Smith; Applied Regression Analysis; John Wiley and Sons, Inc.; New York, New York; 1966.

TABLE 5

LIMITED EUROPEAN DATUM AREA

RESULTS AT OBSERVATION POINTS

LONGITUDE REFERENCE VS REGRESSION DATUM SHIFTS FROM LOCAL DATUM TO MGS 72 (UGS 72 MINUS LOCAL DATUM)

OBSERVATION SEQUENCE NO.	OBSERVATION POINT NO.	NORMALIZED WEIGHT	REFERENCE SECONDS	REGRESSION EQUATION SECONDS	D E V I A T I O N SECONDS	EQUATOR EQUIVALENT SECONDS	METERS
1	1	0.04	73287.00	73287.00	0.00	2071.17	529349
2	2	0.04	73287.00	73287.00	0.00	2071.17	529349
3	3	0.04	73287.00	73287.00	0.00	2071.17	529349
4	4	0.04	73287.00	73287.00	0.00	2071.17	529349
5	5	0.04	73287.00	73287.00	0.00	2071.17	529349
6	6	0.04	73287.00	73287.00	0.00	2071.17	529349
7	7	0.04	73287.00	73287.00	0.00	2071.17	529349
8	8	0.04	73287.00	73287.00	0.00	2071.17	529349
9	9	0.04	73287.00	73287.00	0.00	2071.17	529349
10	10	0.04	73287.00	73287.00	0.00	2071.17	529349
11	11	0.04	73287.00	73287.00	0.00	2071.17	529349
12	12	0.04	73287.00	73287.00	0.00	2071.17	529349
13	13	0.04	73287.00	73287.00	0.00	2071.17	529349
14	14	0.04	73287.00	73287.00	0.00	2071.17	529349
15	15	0.04	73287.00	73287.00	0.00	2071.17	529349
16	16	0.04	73287.00	73287.00	0.00	2071.17	529349
17	17	0.04	73287.00	73287.00	0.00	2071.17	529349
18	18	0.04	73287.00	73287.00	0.00	2071.17	529349
19	19	0.04	73287.00	73287.00	0.00	2071.17	529349
20	20	0.04	73287.00	73287.00	0.00	2071.17	529349
21	21	0.04	73287.00	73287.00	0.00	2071.17	529349
22	22	0.04	73287.00	73287.00	0.00	2071.17	529349
23	23	0.04	73287.00	73287.00	0.00	2071.17	529349
24	24	0.04	73287.00	73287.00	0.00	2071.17	529349
25	25	0.04	73287.00	73287.00	0.00	2071.17	529349
26	26	0.04	73287.00	73287.00	0.00	2071.17	529349
27	27	0.04	73287.00	73287.00	0.00	2071.17	529349
28	28	0.04	73287.00	73287.00	0.00	2071.17	529349
29	29	0.04	73287.00	73287.00	0.00	2071.17	529349
30	30	0.04	73287.00	73287.00	0.00	2071.17	529349
31	31	0.04	73287.00	73287.00	0.00	2071.17	529349
32	32	0.04	73287.00	73287.00	0.00	2071.17	529349
33	33	0.04	73287.00	73287.00	0.00	2071.17	529349
34	34	0.04	73287.00	73287.00	0.00	2071.17	529349
35	35	0.04	73287.00	73287.00	0.00	2071.17	529349
36	36	0.04	73287.00	73287.00	0.00	2071.17	529349
37	37	0.04	73287.00	73287.00	0.00	2071.17	529349
38	38	0.04	73287.00	73287.00	0.00	2071.17	529349
39	39	0.04	73287.00	73287.00	0.00	2071.17	529349
40	40	0.04	73287.00	73287.00	0.00	2071.17	529349
41	41	0.04	73287.00	73287.00	0.00	2071.17	529349
42	42	0.04	73287.00	73287.00	0.00	2071.17	529349
43	43	0.04	73287.00	73287.00	0.00	2071.17	529349
44	44	0.04	73287.00	73287.00	0.00	2071.17	529349
45	45	0.04	73287.00	73287.00	0.00	2071.17	529349
46	46	0.04	73287.00	73287.00	0.00	2071.17	529349
47	47	0.04	73287.00	73287.00	0.00	2071.17	529349
48	48	0.04	73287.00	73287.00	0.00	2071.17	529349
49	49	0.04	73287.00	73287.00	0.00	2071.17	529349
50	50	0.04	73287.00	73287.00	0.00	2071.17	529349
51	51	0.04	73287.00	73287.00	0.00	2071.17	529349
52	52	0.04	73287.00	73287.00	0.00	2071.17	529349
53	53	0.04	73287.00	73287.00	0.00	2071.17	529349
54	54	0.04	73287.00	73287.00	0.00	2071.17	529349
55	55	0.04	73287.00	73287.00	0.00	2071.17	529349
56	56	0.04	73287.00	73287.00	0.00	2071.17	529349
57	57	0.04	73287.00	73287.00	0.00	2071.17	529349
58	58	0.04	73287.00	73287.00	0.00	2071.17	529349
59	59	0.04	73287.00	73287.00	0.00	2071.17	529349
60	60	0.04	73287.00	73287.00	0.00	2071.17	529349
61	61	0.04	73287.00	73287.00	0.00	2071.17	529349
62	62	0.04	73287.00	73287.00	0.00	2071.17	529349
63	63	0.04	73287.00	73287.00	0.00	2071.17	529349
64	64	0.04	73287.00	73287.00	0.00	2071.17	529349
65	65	0.04	73287.00	73287.00	0.00	2071.17	529349
66	66	0.04	73287.00	73287.00	0.00	2071.17	529349
67	67	0.04	73287.00	73287.00	0.00	2071.17	529349
68	68	0.04	73287.00	73287.00	0.00	2071.17	529349
69	69	0.04	73287.00	73287.00	0.00	2071.17	529349
70	70	0.04	73287.00	73287.00	0.00	2071.17	529349
71	71	0.04	73287.00	73287.00	0.00	2071.17	529349
72	72	0.04	73287.00	73287.00	0.00	2071.17	529349
73	73	0.04	73287.00	73287.00	0.00	2071.17	529349
74	74	0.04	73287.00	73287.00	0.00	2071.17	529349
75	75	0.04	73287.00	73287.00	0.00	2071.17	529349
76	76	0.04	73287.00	73287.00	0.00	2071.17	529349
77	77	0.04	73287.00	73287.00	0.00	2071.17	529349
78	78	0.04	73287.00	73287.00	0.00	2071.17	529349
79	79	0.04	73287.00	73287.00	0.00	2071.17	529349
80	80	0.04	73287.00	73287.00	0.00	2071.17	529349
81	81	0.04	73287.00	73287.00	0.00	2071.17	529349
82	82	0.04	73287.00	73287.00	0.00	2071.17	529349
83	83	0.04	73287.00	73287.00	0.00	2071.17	529349
84	84	0.04	73287.00	73287.00	0.00	2071.17	529349
85	85	0.04	73287.00	73287.00	0.00	2071.17	529349
86	86	0.04	73287.00	73287.00	0.00	2071.17	529349
87	87	0.04	73287.00	73287.00	0.00	2071.17	529349
88	88	0.04	73287.00	73287.00	0.00	2071.17	529349
89	89	0.04	73287.00	73287.00	0.00	2071.17	529349
90	90	0.04	73287.00	73287.00	0.00	2071.17	529349
91	91	0.04	73287.00	73287.00	0.00	2071.17	529349
92	92	0.04	73287.00	73287.00	0.00	2071.17	529349
93	93	0.04	73287.00	73287.00	0.00	2071.17	529349
94	94	0.04	73287.00	73287.00	0.00	2071.17	529349
95	95	0.04	73287.00	73287.00	0.00	2071.17	529349
96	96	0.04	73287.00	73287.00	0.00	2071.17	529349
97	97	0.04	73287.00	73287.00	0.00	2071.17	529349
98	98	0.04	73287.00	73287.00	0.00	2071.17	529349
99	99	0.04	73287.00	73287.00	0.00	2071.17	529349
100	100	0.04	73287.00	73287.00	0.00	2071.17	529349

LONGITUDE DATUM SHIFT ROOT MEAN SQUARE DEVIATION 0.32787 SECONDS 0.69051 SECONDS
 LONGITUDE DATUM SHIFT MAXIMUM DEVIATION 0.73382 EQUATOR EQUIVALENT SECONDS 1.575960 METERS
 0.629349 METERS

TABLE 6

LIMITED EUROPEAN DATUM AREA

RESULTS AT OBSERVATION POINTS
 GEODETIC HEIGHT REFERENCE VS REGRESSION DATUM SHIFTS FROM LOCAL DATUM TO MGS 72 (UGS 72 MINUS LOCAL DATUM)

OBSERVATION SEQUENCE NO.	OBSERVATION POINT NO.	NORMALIZED HEIGHT	REGRESSION HEIGHT METERS	REGRESSION HEIGHT METERS	REGRESSION HEIGHT METERS	REGRESSION HEIGHT METERS
1	1	1	1	1	1	1
2	2	2	2	2	2	2
3	3	3	3	3	3	3
4	4	4	4	4	4	4
5	5	5	5	5	5	5
6	6	6	6	6	6	6
7	7	7	7	7	7	7
8	8	8	8	8	8	8
9	9	9	9	9	9	9
10	10	10	10	10	10	10
11	11	11	11	11	11	11
12	12	12	12	12	12	12
13	13	13	13	13	13	13
14	14	14	14	14	14	14
15	15	15	15	15	15	15
16	16	16	16	16	16	16
17	17	17	17	17	17	17
18	18	18	18	18	18	18
19	19	19	19	19	19	19
20	20	20	20	20	20	20
21	21	21	21	21	21	21
22	22	22	22	22	22	22
23	23	23	23	23	23	23
24	24	24	24	24	24	24
25	25	25	25	25	25	25
26	26	26	26	26	26	26
27	27	27	27	27	27	27
28	28	28	28	28	28	28
29	29	29	29	29	29	29
30	30	30	30	30	30	30
31	31	31	31	31	31	31
32	32	32	32	32	32	32
33	33	33	33	33	33	33
34	34	34	34	34	34	34
35	35	35	35	35	35	35
36	36	36	36	36	36	36
37	37	37	37	37	37	37
38	38	38	38	38	38	38
39	39	39	39	39	39	39
40	40	40	40	40	40	40
41	41	41	41	41	41	41
42	42	42	42	42	42	42
43	43	43	43	43	43	43
44	44	44	44	44	44	44
45	45	45	45	45	45	45
46	46	46	46	46	46	46
47	47	47	47	47	47	47
48	48	48	48	48	48	48
49	49	49	49	49	49	49
50	50	50	50	50	50	50

GEODETIC HEIGHT DATUM SHIFT ROOT MEAN SQUARE DEVIATION -0.66167 METERS
 GEODETIC HEIGHT DATUM SHIFT MAXIMUM DEVIATION 1.337848 METERS

TABLE 9

LIMITED EUROPEAN DATUM AREA

RESULTS AT OBSERVATION POINTS

2 RECTANGULAR COORDINATE REFERENCE VS REGRESSION DATUM SHIFT FROM LOCAL DATUM TO UGS 72 (UGS 72 MINUS LOCAL DATUM)

OBSERVATION SEQUENCE	OBSERVATION POINT NO.	NORMALIZED WEIGHT	REFERENCE METERS	REGRESSION EQUATION METERS	DEVIATION METERS
1	1	1.000000	1000000.000000	1000000.000000	0.000000
2	2	1.000000	1000000.000000	1000000.000000	0.000000
3	3	1.000000	1000000.000000	1000000.000000	0.000000
4	4	1.000000	1000000.000000	1000000.000000	0.000000
5	5	1.000000	1000000.000000	1000000.000000	0.000000
6	6	1.000000	1000000.000000	1000000.000000	0.000000
7	7	1.000000	1000000.000000	1000000.000000	0.000000
8	8	1.000000	1000000.000000	1000000.000000	0.000000
9	9	1.000000	1000000.000000	1000000.000000	0.000000
10	10	1.000000	1000000.000000	1000000.000000	0.000000
11	11	1.000000	1000000.000000	1000000.000000	0.000000
12	12	1.000000	1000000.000000	1000000.000000	0.000000
13	13	1.000000	1000000.000000	1000000.000000	0.000000
14	14	1.000000	1000000.000000	1000000.000000	0.000000
15	15	1.000000	1000000.000000	1000000.000000	0.000000
16	16	1.000000	1000000.000000	1000000.000000	0.000000
17	17	1.000000	1000000.000000	1000000.000000	0.000000
18	18	1.000000	1000000.000000	1000000.000000	0.000000
19	19	1.000000	1000000.000000	1000000.000000	0.000000
20	20	1.000000	1000000.000000	1000000.000000	0.000000
21	21	1.000000	1000000.000000	1000000.000000	0.000000
22	22	1.000000	1000000.000000	1000000.000000	0.000000
23	23	1.000000	1000000.000000	1000000.000000	0.000000
24	24	1.000000	1000000.000000	1000000.000000	0.000000
25	25	1.000000	1000000.000000	1000000.000000	0.000000
26	26	1.000000	1000000.000000	1000000.000000	0.000000
27	27	1.000000	1000000.000000	1000000.000000	0.000000
28	28	1.000000	1000000.000000	1000000.000000	0.000000
29	29	1.000000	1000000.000000	1000000.000000	0.000000
30	30	1.000000	1000000.000000	1000000.000000	0.000000
31	31	1.000000	1000000.000000	1000000.000000	0.000000
32	32	1.000000	1000000.000000	1000000.000000	0.000000
33	33	1.000000	1000000.000000	1000000.000000	0.000000
34	34	1.000000	1000000.000000	1000000.000000	0.000000
35	35	1.000000	1000000.000000	1000000.000000	0.000000
36	36	1.000000	1000000.000000	1000000.000000	0.000000
37	37	1.000000	1000000.000000	1000000.000000	0.000000
38	38	1.000000	1000000.000000	1000000.000000	0.000000
39	39	1.000000	1000000.000000	1000000.000000	0.000000
40	40	1.000000	1000000.000000	1000000.000000	0.000000
41	41	1.000000	1000000.000000	1000000.000000	0.000000
42	42	1.000000	1000000.000000	1000000.000000	0.000000
43	43	1.000000	1000000.000000	1000000.000000	0.000000
44	44	1.000000	1000000.000000	1000000.000000	0.000000
45	45	1.000000	1000000.000000	1000000.000000	0.000000

2 RECTANGULAR COORDINATE DATUM SHIFT ROOT MEAN SQUARE DEVIATION .570817 METERS
 2 RECTANGULAR COORDINATE DATUM SHIFT MAXIMUM DEVIATION 1.408366 METERS

Table 10
COORDINATE TRANSFORMATION FORMULAS
GEODETC DATUM TO WGS 72

A. The Standard Molodensky Formulas

$$\Delta\phi'' = \left\{ -\Delta X \sin \phi \cos \lambda - \Delta Y \sin \phi \sin \lambda + \Delta Z \cos \phi \right. \\
\left. + \Delta a (R_N e^2 \sin \phi \cos \phi) / a \right. \\
\left. + \Delta f [R_M (a/b) + R_N (b/a)] \sin \phi \cos \phi \right\} \cdot [(R_M + H) \sin 1'']^{-1}$$

$$\Delta\lambda'' = [-\Delta X \sin \lambda + \Delta Y \cos \lambda] \cdot [(R_N + H) \cos \phi \sin 1'']^{-1}$$

$$\Delta H = \Delta X \cos \phi \cos \lambda + \Delta Y \cos \phi \sin \lambda + \Delta Z \sin \phi \\
- \Delta a (a/R_N) + \Delta f (b/a) R_N \sin^2 \phi$$

B. The Abridged Molodensky Formulas

$$\Delta\phi'' = [-\Delta X \sin \phi \cos \lambda - \Delta Y \sin \phi \sin \lambda + \Delta Z \cos \phi + (a\Delta f + f\Delta a) \sin 2\phi] \\
\cdot [R_M \sin 1'']^{-1}$$

$$\Delta\lambda'' = [-\Delta X \sin \lambda + \Delta Y \cos \lambda] \cdot [R_N \cos \phi \sin 1'']^{-1}$$

$$\Delta H = \Delta X \cos \phi \cos \lambda + \Delta Y \cos \phi \sin \lambda + \Delta Z \sin \phi + (a\Delta f + f\Delta a) \sin^2 \phi - \Delta a$$

C. Definition of Terms in the Molodensky Formulas

ϕ, λ, H = geodetic coordinates (old ellipsoid)

ϕ = geodetic latitude. The angle between the earth's equatorial plane and the ellipsoidal normal at a point (measured positive north from the equator, negative south).

λ = geodetic longitude. The angle between the plane of the Greenwich meridian and the plane of the geodetic meridian of the point (measured in the plane of the equator, positive east from Greenwich).

H = the distance of a point from the ellipsoid measured along the ellipsoidal normal through the point.

$$H = N + {}^*h$$

*Indicates parameters which do not appear in the Abridged Molodensky Formulas.

Table 10 (Cont'd)

N = geoid-ellipsoid separation. The distance of the geoid above (+N) or below (-N) the ellipsoid.

* h = distance of a point from the geoid (elevation above or below mean sea level).

$\Delta\phi, \Delta\lambda, \Delta H$ = corrections to transform the geodetic coordinates from the old datum to WGS.

$\Delta X, \Delta Y, \Delta Z$ = shifts between ϵ ellipsoid centers of the old datum and WGS.

a = semimajor axis of the old ellipsoid.

* b = semiminor axis of the old ellipsoid.

* $b/a = 1 - f$

f = flattening of the old ellipsoid.

$\Delta a, \Delta f$ = differences between the parameters of the old ellipsoid and the WGS ellipsoid (WGS minus old).

e = eccentricity.

$e^2 = 2f - f^2$

R_N = radius of curvature in the prime vertical.

** $R_N = a/(1 - e^2 \sin^2 \phi)^{1/2}$

R_M = radius of curvature in the meridian.

** $R_M = a(1 - e^2)/(1 - e^2 \sin^2 \phi)^{3/2}$

NOTE: All Δ -quantities are formed by subtracting old ellipsoid values from WGS ellipsoid values.

*Indicates parameters which do not appear in the Abridged Molodensky Formulas.

**For desk calculator computations involving commonly used ellipsoids, these values are given in Latitude Function Tables, i.e., Latitude Functions Clarke 1866 Spheroid, AMS TM No. 68, 1957.

TABLE 11

STANDARD MOLODENSKY COORDINATE TRANSFORMATION FORMULA DATUM SHIFT PRECISION

Summary at 53 Points over Limited European Datum Area

Coordinate	Deviation between Standard Molodensky Formula Datum Shifts and Reference Coordinate Differences	
	* ED 50 to WGS 72	
	RMS Meters	Maximum Meters
Geodetic Latitude	1.07	2.81
Geodetic Longitude	2.26	6.45
Geodetic Height	<u>1.12</u>	<u>2.77</u>
Overall	1.58	6.45

* ED 50 geodetic latitude and longitude, and a single set of area average ΔX , ΔY , ΔZ rectangular coordinate differences used for Standard Molodensky Formulas.