COMPUTER PROGRAM FOR ELECTROMAGNETIC COUPLING TO A CONDUCTING BODY OF REVOLUTION WITH A HOMOGENEOUS MATERIAL REGION

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**ABSTRACT (Continue on reverse side if necessary and identify by block number)**

A computer program is given to calculate electromagnetic coupling to a perfectly conducting body of revolution with a loss-free homogeneous material region. The material region is also a body of revolution. It is bounded by an aperture and part of the surface of the conducting body. The maximum dimensions of the conducting body and its associated material region are of the order of a few wavelengths. The electromagnetic excitation is an obliquely incident, polarized plane wave.
Application of the equivalence principle and subsequent enforcement of the boundary conditions for tangential fields give a set of integral equations. These equations are then solved by means of the method of moments. The computer program calculates the electric current induced on the surface of the conducting body and the equivalent electric and magnetic currents in the aperture. These currents radiate the field scattered by the conducting body and its associated material region. They also radiate the field transmitted through the aperture into the material region. The computer program is described and listed along with sample input and output data.
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I. INTRODUCTION

The computer program presented here calculates the Fourier coefficients [1, Eqs. (87)-(89)] of the electric and magnetic currents on the object shown in [1, Fig. 1] excited by an obliquely incident $\theta$-polarized plane wave. The $I$'s and $V$'s in [1, Eqs. (87)-(89)] are the elements [1, Eq. (68a)] of $X_n^\theta$ obtained by solving [1, Eq. (65a)] for non-negative values of $n$ with $B_n^\theta$ given by [1, Eq. (101)].

The computer program consists of a main program and the subprograms YZA, BLOC, PLANE, DECOMP, SOLVE, and PRNT. The subroutine YZA calculates the matrix elements [1, Eqs. (35) and (36)] which appear in expression [1, Eq. (33)] for the moment matrix $T_n$. $T_n$ is needed in [1, Eq. (65a)]. The subroutine YZA calls the function BLOC. The subroutine PLANE calculates $V_{n1}^0$, $-V_{n1}^\phi$, $-V_{n1}'$, and $V_{n1}'$, which, according to [1, Eq. (103)], determine $B_n^\theta$ of [1, Eq. (101)]. The subroutine DECOMP decomposes $T_n$ into the product of a lower triangular matrix with an upper triangular matrix. The subroutine SOLVE uses these triangular matrices to calculate the solution $X_n^\theta$ to [1, Eq. (65a)]. The subroutine PRNT uses the elements of $X_n^\theta$ of [1, Eq. (68a)] to calculate the Fourier coefficients [1, Eqs. (87)-(89)]. The subroutine PRNT also prints out [1, Eqs. (87)-(89)].

The main program calls the subroutines YZA, PLANE, DECOMP, SOLVE, and PRNT in order to calculate and print out [1, Eqs. (87)-(89)]. The main program is rather long because it has to rearrange the Z's and Y's calculated by the subroutine YZA and the V's calculated by the subroutine PLANE. Unfortunately, the Z's and Y's come out of the subroutine YZA.
arranged not as in [1, Eqs. (35) and (36)] but as on the right-hand sides of [1, Eqs. (47) and (50)] for consecutive values of $i'$ and $j'$. Likewise, the $V's$ exit the subroutine PLANE arranged not as on the left-hand sides of [1, Eq. (103)] but as on the right-hand sides of [1, Eq. (103)] for consecutive values of $i'$. The main program has to rearrange these $Z's$, $Y's$, and $V's$ so as to realize the transition from $W_{n\downarrow}^{r\downarrow}$, and $J_{-n\downarrow}^{s\downarrow}$, to $W_{n\downarrow}^{p\downarrow}$ and $J_{-n\downarrow}^{q\downarrow}$.

II. THE SUBROUTINE YZA

The subroutine YZA($M_1$, $M_2$, $NP$, $NPHI$, $NT$, $IN$, $RH$, $ZH$, $X$, $A$, $XT$, $AT$, $Y$, $Z$) calculates the elements of matrices $Y'$ and $Z_{n}$ defined by

$$Y'_{n} = \begin{bmatrix} Y_{n}^{t\downarrow} & Y_{n}^{t\phi} \\ Y_{n}^{\phi t} & Y_{n}^{\phi\phi} \end{bmatrix} , \quad n=M_1, M_1+1, \ldots M_2$$

(1)

$$Z_{n} = \begin{bmatrix} Z_{n}^{t\downarrow} & Z_{n}^{t\phi} \\ Z_{n}^{\phi t} & Z_{n}^{\phi\phi} \end{bmatrix} , \quad n=M_1, M_1+1, \ldots M_2$$

(2)

where the $ij$th elements of the submatrices on the right-hand sides of (1) and (2) are given by

$$Y_{nij}^{rs} = - \langle w_{n\downarrow}^{r} , H(J_{-n\downarrow}^{s} , 0) \rangle$$

(3)

$$Z_{nij}^{rs} = - \langle w_{n\downarrow}^{r} \frac{1}{n} E(J_{-n\downarrow}^{s} , 0) \rangle$$

(4)
The quantities on the right-hand sides of (3) and (4) are those on the right-hand sides of [1, Eqs. (50) and (47)] with the \( \pm \) notation omitted and with \( i' \) and \( j' \) replaced by \( i \) and \( j \). The \( < > \) notation in (3) and (4) denotes the same symmetric product as in [1, Eqs. (50) and (47)]. The \( \pm \) notation in [1] serves to distinguish the surface \((S^+ + A)\) from the surface \((S^- + A)\). The \( \pm \) notation is not needed in (3) and (4) because the subroutine YZA deals with only one surface of revolution at a time. Later on in Section V, the main program calls YZA twice, once for \((S^+ + A)\) and once for \((S^- + A)\). For a particular value of \( n \), \( Y'_n \) of (1) is stored by columns in \( Y \) and \( Z_n \) of (2) is stored by columns in \( Z \). \( Y'_n \) is placed immediately after \( Y'_n \) in \( Y \). Similarly, \( Z_{n+1} \) follows \( Z_n \) in \( Z \). \( Y \) and \( Z \) are the only output arguments of YZA. The rest of the arguments of YZA are input arguments.

The input arguments of YZA have the same meaning as those of the subroutine YZ presented in [2, pages 65-79]. In comparison with YZA, the subroutine YZ of [2] puts \( Y_n \) in \( Y \) and \( Z_n \) of (2) in \( Z \) where

\[
Y_n = \begin{bmatrix} \gamma_{tt}^n & \gamma_{t}^n \\ \gamma_{nt}^n & \gamma_{t}^n \\ \gamma_{nt}^n & \gamma_{t}^n \\ \gamma_{nt}^n & \gamma_{t}^n \end{bmatrix}
\]  

The \( ij \)th elements of the submatrices on the right-hand side of (5) are given by

\[
y_{nij}^{rs} = - \langle w_{n1}, n \times H(j_{nj}^s, 0) \rangle
\]

\[
\begin{cases} r = t, \phi \\ s = t, \phi \end{cases}
\]

where [2, Eq. (4)]

\[
n = u_\phi \times u_t
\]

As defined by (7), \( n \) is a unit vector normal to the surface of revolution.
Although we have been omitting the vector designation from vectors inside the symmetric product, we decided to designate $\mathbf{n}$ and $\mathbf{H}$ as vectors in (6) to clearly indicate that the vector product $\mathbf{n} \times \mathbf{H}$ is intended there.

YZ of [2] allows for the Ampere's law contribution to the magnetic field $\mathbf{H}$ in (6). The Ampere's law contribution to the magnetic field is the contribution due to the value of the electric current at the field point. The Ampere's law contribution in the subroutine YZ of [2] is controlled by means of the input argument IN of YZ of [2]. The action of IN is described in [2, page 39] under the assumptions that

1) $\mathbf{n}$ is given by (7)
2) $\mathbf{n}$ points outward from the surface of revolution.

However, the action of IN can be described without recourse to assumption 2) in the following manner.

$$
\begin{align*}
\text{IN} &= 1, \text{Magnetic field evaluation on the side of the electric current sheet where the tail of $\mathbf{n}$ is.} \\
\text{IN} &= 0, \text{No Ampere's law contribution.} \\
\text{IN} &= -1, \text{Magnetic field evaluation on the side of the electric current sheet where the head of $\mathbf{n}$ is.}
\end{align*}
$$

(8)

In (8), $\mathbf{n}$ is visualized as piercing the sheet of electric current which produces the magnetic field. Apparently, assumption 2) for $\mathbf{n}$ can be eliminated from [2] by replacing the description [2, page 39] of the action of the input argument IN of YZ of [2] by (8).

Presumably, the magnetic field $\mathbf{H}$ in (3) contains no Ampere's law contribution because the magnetic field $\mathbf{H}^+$ in [1, Eq. (50)] had no Ampere's law contribution. However, we decided to allow for the Ampere's
law contribution associated with \( H \) in (3) in order to enhance the usefulness of YZA. This Ampere's law contribution is controlled by means of the input argument IN of YZA. The action of IN is described by (8) where it is assumed that \( n \) is given by (7). We do not require \( n \) to point outward in (8).

Most of the statements in YZA are exactly the same as those in YZ of [2]. In the remaining part of this section, the differences between YZA and YZ of [2] are pointed out and explained and then YZA is listed. Unless stated otherwise, all line numbers and statement numbers cited henceforth in this section refer to the listing of YZA.

In line 4, more space is allotted to \( Y \) and \( Z \) than in YZ of [2]. Nevertheless, the minimum allocations given in [2, pages 65-66] apply to YZA. Line 9 in YZ of [2] has no counterpart in YZA because UG and UH are not used in YZA. On the other hand, line 17 has no counterpart in YZ of [2]. Line 17 enables YZA to calculate the moment matrix for the E-field solution [3] for the two conducting bodies in [1, Fig. 13]. This E-field solution was obtained by connecting the generating curves ABC and DEF in [1, Fig. 13] to obtain the single curve ABCDEF and then deleting all matrix elements associated with the connecting line CD. Line 17 sets \( k \rho = 1 \) at the midpoint of the connecting line CD. Here, \( k \) is the propagation constant and \( \rho \) is the distance from the z axis. If line 17 were absent, this \( k \rho \) would be zero and divisions by zero would occur.

The value 1 assigned to \( k \rho \) in line 17 is not critical because, as stated earlier, all matrix elements associated with the connecting line CD are deleted. Line 17 also enables YZA to calculate the moment matrix for
an H-field solution for the two conducting bodies in [1, Fig. 13]. However, this H-field solution, being a solution to [2, Eq. (1)] without the $n \times$, would fail according to the discussion in the third from the last paragraph in [1, Section III]. The H-field solution shown in [1, Fig. 15] was obtained by using the subroutine YZ of [2] modified by setting $k_0 = 1$ at the midpoint of the connecting line CD in [1, Fig. 13].

The major difference between YZ of [2] and YZA lies in the fact that there is an $n \times$ in (6) for $\gamma_{r,s}^{\text{ij}}$ calculated by YZ of [2] but not in (3) for $\gamma_{r,s}^{\text{ij}}$ calculated by YZA. Because $u_{r,s}^{\text{ij}}$ in (3) is a tangential vector, (3) can be written as

$$\gamma_{r,s}^{\text{ij}} = -\langle n \times W_{\text{ni}}, n \times H(J_{\text{nj}}^S, 0) \rangle \begin{cases} r = t, \phi \\ s = t, \phi \end{cases}$$ (9)

In view of (7) and the assumptions that $W_{\text{ni}}^t$ has only a $t$ component and $W_{\text{ni}}^\phi$ has only a $\phi$ component, (9) becomes

$$\gamma_{t,s}^{\text{ij}} = -\phi_s^{\text{ij}}$$ (10a)

$$\gamma_{\phi,s}^{\text{ij}} = \gamma_{t,s}^{\text{ij}}$$ (10b)

where

$$\gamma_{r,s}^{\text{ij}} = -\langle W_{\text{ni}}^r, n \times H(J_{\text{nj}}^S, 0) \rangle \begin{cases} r = t, \phi \\ s = t, \phi \end{cases}$$ (11)

where

$$W_{\text{ni}}^t = u_t (W_{\text{ni}}^\phi \cdot u_\phi)$$ (12a)

$$W_{\text{ni}}^\phi = u_\phi (W_{\text{ni}}^t \cdot u_t)$$ (12b)
According to (10)-(12), $Y_{nij}^{\text{ts}}$ is the negative of $Y_{nij}^{\phi}$ modified from (6) and $Y_{nij}^{\phi}$ is $Y_{nij}^{\text{ts}}$ modified from (6). The expression "modified from (6)" means (6) with the $t$ and $\phi$ components of $W_{nij}^r$ interchanged.

Now, the matrix elements (6) are given by [2, Eq. (12)] and it is evident from [2, Eq. (9)] that

$$W_{ni}^t \cdot u_\tau = \frac{T_i(t)}{\rho} e^{-jn\phi}$$  \hspace{1cm} (13a)

$$W_{ni}^\phi \cdot u_\tau = \frac{P_i(t)}{\rho_1} e^{-jn\phi}$$  \hspace{1cm} (13b)

Hence, (10) becomes

$$Y_{nij}^{tt} = -\phi_{nij}^{tt}$$  \hspace{1cm} (14a)

$$Y_{nij}^{\phi t} = \phi_{nij}^{tt}$$  \hspace{1cm} (14b)

$$Y_{nij}^{t\phi} = -\phi_{nij}^{\phi \phi}$$  \hspace{1cm} (14c)

$$Y_{nij}^{t\phi} = \phi_{nij}^{t\phi}$$  \hspace{1cm} (14d)

where

$$\phi_{nij}^{\phi t} = y_{nij}^{\phi t} \text{ of [2, Eq. (12b)] with } \frac{P_i(t)}{\rho_1} \text{ replaced by } \frac{T_i(t)}{\rho}$$  \hspace{1cm} (15a)

$$\phi_{nij}^{tt} = y_{nij}^{tt} \text{ of [2, Eq. (12a)] with } \frac{T_i(t)}{\rho} \text{ replaced by } \frac{P_i(t)}{\rho_1}$$  \hspace{1cm} (15b)

$$\phi_{nij}^{\phi \phi} = y_{nij}^{\phi \phi} \text{ of [2, Eq. (12d)] with } \frac{P_i(t)}{\rho_1} \text{ replaced by } \frac{T_i(t)}{\rho}$$  \hspace{1cm} (15c)

$$\phi_{nij}^{t\phi} = y_{nij}^{t\phi} \text{ of [2, Eq. (12c)] with } \frac{T_i(t)}{\rho} \text{ replaced by } \frac{P_i(t)}{\rho_1}$$  \hspace{1cm} (15d)

Obviously, the interchange of functions in (15) must be accompanied by
appropriate changes in the limits of integration with respect to $t$ in [2, Eq. (12)]. Also, because the magnetic fields in [2, Eq. (12)] as it stands are evaluated on the side of the electric current sheet where the tail of $\mathbf{n}$ is, the Ampere's law contributions to [2, Eq. (12)] should be multiplied by the input argument $IN$ of YZA. These contributions are the single integrals with respect to $t$ in [2, Eq. (12)].

Consequently, the Ampere's law contributions to (14) are given by

$$
\tilde{\nabla}_{\mathbf{n}ij} \cdot \mathbf{t} = \frac{\pi(IN)}{\rho_i} \int_{t_i}^{t_{i+1}} P_i(t) T_j(t) dt
$$

and

$$
\tilde{\nabla}_{\mathbf{n}ij} \cdot \mathbf{t} = - \frac{\pi(IN)}{\rho_j} \int_{t_j}^{t_{j+1}} T_i(t) P_j(t) dt
$$

where the horizontal bar on the left-hand sides of (16) denotes Ampere's law contribution. If the $q$th interval is defined to be $(t_q, t_{q+1})$, then the contributions to (16) due to the integrations over the $q$th interval are given by

$$
\tilde{\nabla}_{\mathbf{n}qj} \cdot \mathbf{t} = \frac{\pi(IN)}{\rho_q} \int_{t_q}^{t_{q+1}} P_i(t) T_j(t) dt \quad j = q-1, q
$$

and

$$
\tilde{\nabla}_{\mathbf{n}iq} \cdot \mathbf{t} = - \frac{\pi(IN)}{\rho_q} \int_{t_q}^{t_{q+1}} T_i(t) P_j(t) dt \quad i = q-1, q
$$

where $NP$ is one of the input arguments of YZA. The dot on the left-hand sides of (17) denotes Ampere's law contribution due to integration over the $q$th interval. Thanks to the definitions [2, Eqs. (5) and (6)], (17) reduces to
\[ y_{nq} = \frac{\Delta \pi (IN)}{2\rho_q} \]

\[ y_{ni} = -\frac{\Delta \pi (IN)}{2\rho_q} \]

where

\[ \Delta_q = t_{q+1} - t_q \]

Expressions (18) are the Ampere's law contributions to the matrix elements \( y_{nij} \) of (14) due to the integrations over the \( q \)th interval.

The contributions to (14) referred to in this paragraph and the next paragraph are exclusive of the Ampere's law contributions. If the \( pq \)th region is the region for which

\[ t_p^- \leq t \leq t_{p+1}^- \]

\[ t_q^- \leq t' \leq t_{q+1}^- \]

then the contributions to (14) due to the integrations over the \( pq \)th region are given by expressions similar to [2, Eq. (18)]. If the generating curve of the surface of revolution is assumed to be a series of straight line segments connecting the points \( t_1^- \), \( t_2^- \), ..., \( t_{NP}^- \) and if the integrations with respect to \( t \) are approximated by sampling the integrands at \( t = t_p \) and multiplying by \( \Delta_p \), then the contributions to (14) due to the integrations over the \( pq \)th region reduce to
\[ \dot{\gamma}_{nij}^{tt} = - \frac{1}{2} \text{ (right-hand side of [2, Eq. (22b)])} \] (20a)

\[ \dot{\gamma}_{npj}^{\phi t} = 2 \text{ (right-hand side of [2, Eq. (22a)])} \] (20b)

\[ \dot{\gamma}_{niq}^{t\phi} = - \frac{1}{2} \text{ (right-hand side of [2, Eq. (22d)] without the } \delta_{pq} \text{ term)} \] (20c)

\[ \dot{\gamma}_{npq}^{\phi \phi} = 2 \text{ (right-hand side of [2, Eq. (22c)])} \] (20d)

The asterisk on the left-hand sides of (20) denotes contribution due to integration over the pqth region. The ranges of values of i and j in (20) are given by

\[
\begin{align*}
i &= p-1, p \\
i &\neq 0 \\
i &= NP-1 \\
j &= q-1, q \\
j &\neq 0 \\
j &\neq NP-1
\end{align*}
\] (21a)

(21b)

The factors \(\frac{1}{2}\) and 2 in (20) are due to sampling the integrands at \(t = t_p\).

When \(t = t_p\), the functions \(\frac{P_i(t)}{\rho_i}\) and \(\frac{T_i(t)}{\rho}\) which are being interchanged in (15) reduce to

\[
\begin{align*}
P_i(t_p)_{\rho_i} &= \begin{cases} 
\frac{1}{\rho_p}, & i = p \\
0, & i \neq p 
\end{cases} \\
\rho_i
\end{align*}
\] (22)

\[
\begin{align*}
T_i(t_p)_{\rho} &= \begin{cases} 
\frac{1}{2\rho_p}, & i = p-1, p \\
0, & \text{otherwise}
\end{cases} \\
\rho
\end{align*}
\] (23)
Since [2, Eq. (22)] is equal to [2, Eq. (24)], (20) can be re-written as

\[
\begin{align*}
\hat{y}_{n,p-1,q-1}^{tt} &= -\frac{1}{2} (UC - UD), \ p \neq 0, q \neq 0 \\
\hat{y}_{np,q-1}^{tt} &= -\frac{1}{2} (UC - UD), \ p \neq NP-1, q \neq 0 \\
\hat{y}_{n,p-1,q}^{tt} &= -\frac{1}{2} (UC + UD), \ p \neq 0, q \neq NP-1 \\
\hat{y}_{npq}^{tt} &= -\frac{1}{2} (UC + UD), \ p \neq NP-1, q \neq NP-1 \\
\hat{y}_{np,q-1}^{tt} &= 2 (UA), \ q \neq 0 \\
\hat{y}_{npq}^{tt} &= 2 (UB), \ q \neq NP-1 \\
\hat{y}_{n,p-1,q}^{tt} &= -\frac{1}{2} (UF), \ p \neq 0 \\
\hat{y}_{npq}^{tt} &= -\frac{1}{2} (UF), \ p \neq NP-1 \\
\hat{y}_{npq}^{tt} &= 2 (UE) \\
\end{align*}
\]

where

\[
\begin{align*}
UA &= \text{right-hand side of [2, Eq. (24a)] for } j = q-1 \\
UB &= \text{right-hand side of [2, Eq. (24a)] for } j = q \\
UC-UD &= \text{right-hand side of [2, Eq. (24b)] for } j = q-1 \\
UC+UD &= \text{right-hand side of [2, Eq. (24b)] for } j = q \\
UE &= \text{right-hand side of [2, Eq. (24c)]} \\
UF &= \text{right-hand side of [2, Eq. (24d)] without the } \delta_{pq} \text{ term}
\end{align*}
\]
The notation UA, UB,...UF on the left-hand sides of (25) is the same as the notation used in YZ of [2]. Expressions (24) are the contributions to the matrix elements $Y_{nij}^{rs}$ of (14) due to the integrations over the pqth region. No Ampere's law contributions are included in (24). The Ampere's law contributions are given by (18).

The Ampere's law contributions in (18) are different from those in [2] which consist of [2, Eq. (23)] and the $\delta_{pq}$ term in [2, Eq. (24d)]. Hence, the statements which realize the Ampere's law contributions in YZA are different from those in YZ of [2]. Lines 50-51, 74-75, 86-88, 93-99, and 424-429 of YZ of [2] obtain the Ampere's law contributions. Line 50 of YZA puts $\pi(IN)$ of (18) in P1. Line 73 puts the right-hand side of (18a) in P1.

The index JQ of DO loop 15 obtains the subscript q which appears in (18a) and (24). The index IP of DO loop 16 obtains the subscript p which appears in (24). The purpose of the statement IF(IP.NE.JQ) in lines 121, 270, and 424 of YZ of [2] is to check if the IPth interval coincides with the JQth interval. If the IPth and JQth intervals coincide, then the numerical evaluation of the integrals $G_{m0}$ in [2, Section III] is affected and Ampere's law contributions are taken into account. However, YZ of [2] was not designed to accommodate a generating curve which closes upon itself.

The subroutine YZA allows for a generating curve which closes upon itself. Such a curve is treated in the following manner. A triangle function whose peak is at the first data point is needed and is obtained by overlapping the last interval of the generating curve with the first
interval as in [1, Fig. 5]. This triangle function is called \( T_{NP-2}(t) \).

In addition to \( T_{NP-2}(t) \), the process of overlapping obtains the pulse function \( P_{NP-1}(t) \). \( P_{NP-1}(t) \) is not wanted because it is identical to \( P_1(t) \). The effect of \( P_{NP-1}(t) \) can be eliminated by discarding all matrix elements (3) and (4) for which either \( W_{ni}^T \) or \( J_{nj}^S \) contain \( P_{NP-1}(t) \). These elements must be discarded after exit from YZA, because YZA does not contain any logic for discarding matrix elements.

Unfortunately, (18) was derived for a generating curve which does not overlap on itself. If the last interval of the generating curve overlaps the first interval, the quantities on the right-hand side of (18) are still correct but more values of \( i \) and \( j \) are needed in (18) when \( q = 1 \) and \( q = NP-1 \) to allow for the overlapping so as to account for all Ampere's law contributions to the matrix elements (3). It will be shown that all Ampere's law contributions can be accounted for by defining ZIP to be the electrical distance from the center of the \( JQ \)th interval to the center of the \( IP \)th interval and using the statement \( IF(ZIP.NE.0.) \) instead of the statement \( IF(IP.NE.JQ) \). The statement \( IF(ZIP.NE.0.) \) appears in lines 110, 259, and 408. If the generating curve does not overlap on itself, the action of the statement \( IF(ZIP.NE.0.) \) is the same as the action of the statement \( IF(IP.NE.JQ) \) and all Ampere's law contributions are accounted for. However, if the last interval of the generating curve overlaps the first interval, then it remains to be shown that \( ZIP = 0. \) obtains all Ampere's law contributions.

If the last interval of the generating curve overlaps the first interval, then ZIP is zero not only for
$$IP = JQ = m, \quad m=1,2,\ldots NP-1$$  \hspace{1cm} \text{Case 1}

but also for

$$\begin{cases} IP = 1 \\ JQ = NP-1 \end{cases}$$  \hspace{1cm} \text{Case 2}

and for

$$\begin{cases} IP = NP-1 \\ JQ = 1 \end{cases}$$  \hspace{1cm} \text{Case 3}

In what is to follow, the matrix elements $Y_{ni}^{rs}$ of (3) are viewed as interactions between testing functions and expansion functions. The combination of Cases 1, 2, and 3 will give all Ampere's law contributions if it covers all possible interactions between parts of testing functions on the $m$th interval and parts of expansion functions on the $m$th interval for $m = 1,2,\ldots NP-2$.

With regard to the Ampere's law contributions (18), the last statement is more general than necessary because not all interactions between testing functions and expansion functions are involved in (18). For example, the interactions $Y_{ni}^{tt}$ between $t$ directed testing functions and $t$ directed expansion functions are not involved in (18) and neither are the interactions $Y_{ni}^{\phi \phi}$ between $\phi$ directed testing functions and $\phi$ directed expansion functions. However, the matrix elements $Y_{ni}^{tt}$ and $Y_{ni}^{\phi \phi}$ of (6) calculated by YZ of [2] do have Ampere's law contributions. It would be useful to know that YZ of [2] can be modified to allow for a closed generating curve by using the same technique as in YZA. This technique consists of overlapping the last interval of the generating curve with the first, replacing the statement IF(IP.NE.JQ) by the statement IF(ZIP.NE.0.),
and deleting the matrix elements associated with $P_{NP-1}(t)$ after exit from the subroutine.

It is evident that the portion of Case 1 for which

$$IP = JQ = m, m = 2, \ldots, NP-2$$

covers all possible interactions between parts of testing functions on the $m$th interval and expansion functions on the $m$th interval for $m = 2, 3, \ldots, NP-2$. However, clarification is needed with regard to the first interval because, due to the overlapping, the first interval is sometimes disguised as the $(NP-1)$th interval.

All possible interactions between parts of testing functions on the first interval and parts of expansion functions on the first interval are listed as

1) $T^{a}_{NP-2}(t)$ with $T^{b}_{NP-2}(t)$
2) $T^{a}_{1}(t)$ with $T^{b}_{NP-2}(t)$
3) $P_{1}(t)$ with $T^{b}_{NP-2}(t)$
4) $T^{b}_{NP-2}(t)$ with $T^{a}_{1}(t)$
5) $T^{a}_{1}(t)$ with $T^{a}_{1}(t)$
6) $P_{1}(t)$ with $T^{a}_{1}(t)$
7) $T^{b}_{NP-2}(t)$ with $P_{1}(t)$
8) $T^{a}_{1}(t)$ with $P_{1}(t)$
9) $P_{1}(t)$ with $P_{1}(t)$

In each of the foregoing 9 interactions, the first function is the part of the testing function and the second function is the part of the expansion function. $T^{b}_{NP-2}(t)$ is the second part of the $(NP-2)$th triangle.
function, the downward sloping part. $T(t)$ is the first part of the first triangle function, the upward sloping part.

The interactions covered by Cases 2 and 3 and the parts of Case 1 for which $IP = JQ = 1$ and $IP = JQ = NP-1$ are listed as

<table>
<thead>
<tr>
<th>Interaction</th>
<th>Details</th>
</tr>
</thead>
<tbody>
<tr>
<td>2) $T^a(t)$ with $T^b_{NP-2}(t)$</td>
<td>$IP = 1$</td>
</tr>
<tr>
<td>3) $P_1(t)$ with $T^b_{NP-2}(t)$</td>
<td>$JQ = NP-1$ Case 2</td>
</tr>
<tr>
<td>10) $T^a(t)$ with $P_{NP-1}(t)$</td>
<td></td>
</tr>
<tr>
<td>11) $P_1(t)$ with $P_{NP-1}(t)$</td>
<td></td>
</tr>
<tr>
<td>4) $T^b_{NP-2}(t)$ with $T^a(t)$</td>
<td></td>
</tr>
<tr>
<td>12) $P_{NP-1}(t)$ with $T^a(t)$</td>
<td>$I'' = NP-1$ Case 3</td>
</tr>
<tr>
<td>7) $T^b_{NP-2}(t)$ with $P_1(t)$</td>
<td>$JQ = 1$</td>
</tr>
<tr>
<td>13) $P_{NP-1}(t)$ with $P_1(t)$</td>
<td></td>
</tr>
<tr>
<td>5) $T^a(t)$ with $T^a(t)$</td>
<td></td>
</tr>
<tr>
<td>6) $P_1(t)$ with $T^a(t)$</td>
<td>$IP = JQ = 1$ Part of Case 1</td>
</tr>
<tr>
<td>8) $T^a(t)$ with $P_1(t)$</td>
<td></td>
</tr>
<tr>
<td>9) $P_1(t)$ with $P_1(t)$</td>
<td></td>
</tr>
<tr>
<td>1) $T^b_{NP-2}(t)$ with $T^b_{NP-2}(t)$</td>
<td></td>
</tr>
<tr>
<td>14) $P_{NP-1}(t)$ with $T^b_{NP-2}(t)$</td>
<td>$IP = JQ = NP-1$ Part of Case 1</td>
</tr>
<tr>
<td>15) $T^b_{NP-2}(t)$ with $P_{NP-1}(t)$</td>
<td></td>
</tr>
<tr>
<td>16) $P_{NP-1}(t)$ with $P_{NP-1}(t)$</td>
<td></td>
</tr>
</tbody>
</table>
The preceding 16 interactions were numbered so as to facilitate comparison with the 9 interactions in the last paragraph.

Of the 16 interactions in the last paragraph, numbers 1 to 9 are the interactions in the second from the last paragraph and numbers 10 to 16 are to be discarded because they contain $P_{NP-1}(t)$. Drawing an equivalence between the interactions in the last paragraph and the interactions in the second from the last paragraph, we conclude that the combination of Cases 1, 2, and 3 covers all possible interactions between parts of testing functions on the $m$th interval and parts of expansion functions on the $m$th interval for $m = 1, 2, \ldots NP-2$. Hence, $(ZIP = 0.)$ obtains all Ampere's law contributions when the last interval on the generating curve overlaps the first interval. If such overlapping is intended, then the input arguments RH and ZH of YZA must satisfy

\[
\begin{align*}
RH(NP-1) &= RH(1) \\
RH(NP) &= RH(2) \\
ZH(NP-1) &= ZH(1) \\
ZH(NP) &= ZH(2)
\end{align*}
\]

exactly. Otherwise, the computed values of ZIP will not be zero in Cases 2 and 3.

With the intention of showing that YZA implements (24), we list in Table 1 some variables in YZA whose values are different from those in YZ of [2].
Table 1. Comparison of some variables in YZA with those in YZ of [2].

<table>
<thead>
<tr>
<th>Line in YZA</th>
<th>Variable in YZA</th>
<th>Expression in terms of variables in YZ of [2]</th>
</tr>
</thead>
<tbody>
<tr>
<td>379</td>
<td>W1</td>
<td>2.*W1</td>
</tr>
<tr>
<td>380</td>
<td>W2</td>
<td>2.*W2</td>
</tr>
<tr>
<td>385</td>
<td>H1C</td>
<td>-.5*H1C</td>
</tr>
<tr>
<td>387</td>
<td>H3C</td>
<td>2.*H3C</td>
</tr>
<tr>
<td>388</td>
<td>H2C</td>
<td>2.*H2C</td>
</tr>
<tr>
<td>389</td>
<td>H3C</td>
<td>2.*H3C</td>
</tr>
<tr>
<td>390</td>
<td>W3</td>
<td>-.5*W3</td>
</tr>
<tr>
<td>391</td>
<td>W4</td>
<td>-.5*W4</td>
</tr>
<tr>
<td>392</td>
<td>W5</td>
<td>-.5*W5</td>
</tr>
<tr>
<td>405</td>
<td>UC</td>
<td>2.*UC</td>
</tr>
<tr>
<td>406</td>
<td>UB</td>
<td>2.*UB</td>
</tr>
<tr>
<td>407</td>
<td>UF</td>
<td>-.5*UF</td>
</tr>
<tr>
<td>411</td>
<td>UA</td>
<td>2.*UA</td>
</tr>
<tr>
<td>412</td>
<td>UB</td>
<td>2.*UB</td>
</tr>
<tr>
<td>415</td>
<td>UC</td>
<td>-.5*(UC-UD)</td>
</tr>
<tr>
<td>416</td>
<td>UD</td>
<td>-.5*(UC+UD)</td>
</tr>
<tr>
<td>417</td>
<td>UE</td>
<td>2.*UE</td>
</tr>
</tbody>
</table>

In Table 1, the variable in the second column is defined by the statement in YZA whose line number is given in the first column. The expression in the third column is the value of the variable in the second column. The values of the variables appearing in the third column are the values which exist in YZ just after the counterpart to the variable in the second column has been defined in YZ. For example, the value of W1 in the first line of the third column is defined in line 391 of YZ of [2]. As an exception, the
value of UD intended in the third from the last line of the third column is that defined by line 433 of YZ of [2]. Table 1 was constructed under the assumption that no Ampere's law contributions come into play. This amounts to assuming that lines 425-429 of YZ of [2] are not executed and that lines 409-410 of YZA are not executed. The Ampere's law contributions (18) will be considered later.

The last 6 variables in the second column of Table 1 are used to construct the matrix elements \( \gamma'_{ni,ij} \) of (3). The statements in YZA which perform this task are listed in Table 2.

### Table 2. Construction of matrix elements \( \gamma'_{ni,ij} \)

<table>
<thead>
<tr>
<th>Line in YZA</th>
<th>Statement in YZA</th>
<th>Matrix element</th>
</tr>
</thead>
<tbody>
<tr>
<td>461,476</td>
<td>( Y(K1) = Y(K1) + UC )</td>
<td>( \gamma'_{n,p-1,q-1}^{tt} )</td>
</tr>
<tr>
<td>466,483</td>
<td>( Y(K2) = Y(K2) + UC )</td>
<td>( \gamma'_{n,p,q-1}^{tt} )</td>
</tr>
<tr>
<td>448,477</td>
<td>( Y(K3) = Y(K3) + UD )</td>
<td>( \gamma'_{n,p-1,q}^{tt} )</td>
</tr>
<tr>
<td>453,484</td>
<td>( Y(K4) = UD )</td>
<td>( \gamma'_{npq}^{tt} )</td>
</tr>
<tr>
<td>458,471</td>
<td>( Y(K5) = Y(K5) + UA )</td>
<td>( \gamma'_{np,q-1}^{t\phi} )</td>
</tr>
<tr>
<td>445,472</td>
<td>( Y(K6) = UB )</td>
<td>( \gamma'_{npq}^{t\phi} )</td>
</tr>
<tr>
<td>449,462,478</td>
<td>( Y(K7) = Y(K7) + UF )</td>
<td>( \gamma'_{n,p-1,q}^{t\phi} )</td>
</tr>
<tr>
<td>454,467,485</td>
<td>( Y(K8) = UF )</td>
<td>( \gamma'_{npq}^{t\phi} )</td>
</tr>
<tr>
<td>489</td>
<td>( Y(K9) = UE )</td>
<td>( \gamma'_{npq}^{t\phi} )</td>
</tr>
</tbody>
</table>

In Table 2, the variable being modified by the statement in the second column represents the matrix element in the third column. For example,
Y(K1) which appears in the first line of the second column of Table 2 is
the storage location of the matrix element $Y_{n,p-1,q-1}^{tt}$. The subscripts $n,p,$
and $q$ appearing in the third column of Table 2 are given in terms of vari-
ables in YZA by

\[ n = M + M_1 - 1 \]
\[ p = IP \]
\[ q = JQ \]

Here, $M$ is the index of DO loop 31, $M_1$ is one of the input arguments of
YZA, $IP$ is the index of DO loop 16, and $JQ$ is the index of DO loop 15.
In view of the fact that the variables $UA$, $UB$, $UC$, $UD$, $UE$, and $UF$ appear-
ing in the second column of Table 2 are given by the last 6 entries of
the third column of Table 2, it is now evident that YZA implements (24).

From (18), the correct values of the Ampere's law contributions are

\[ t(\Delta \pi(IN))/(2\rho_q) \]. In the paragraph which follows (25) we established that
line 73 stores $(\Delta \pi(IN))/(2\rho_q)$ in $PI$. In the paragraph prior to the intro-
duction of Table 1, we concluded that the Ampere's law contributions come
into play whenever $ZIP = 0$. Accordingly, lines 409-410 are executed when
$ZIP = 0$. The effect of line 409 is to add $PI$ to the values of $UA$ and $UB$
calculated by lines 411 and 412. Line 410 subtracts $PI$ from the value of $UF$
calculated by line 407. The previously mentioned variables $UA$, $UB$, and $UF$
appear in the second column of Table 1 and were transferred to the second
column of Table 2. It is now evident from Table 2 that execution of
lines 409-410 adds $PI$ to $Y_{np,q-1}^{t\phi}$ and $Y_{np}^{t\phi}$ and subtracts $PI$ from $Y_{n,p-1,q}^{t\phi}$
and $Y_{npq}^{t\phi}$. Hence, YZ takes proper account of the Ampere's law contributions.
LISTING OF THE SUBROUTINE YZA

THE SUBROUTINE YZA CALLS THE FUNCTION SLOG

DIMENSION RH(43),ZH(43),X(48),A(48),XT(10),AT(10),RS(42),ZS(42)
DIMENSION D(43),DR(42),DZ(42),DN(42),C1(48),C2(48),C3(48),C4(200)
DIMENSION C5(200),C6(200),RZ(10),ZT(10),Z7(10)

NP = 1
10 CONTINUE
M3=M2-M1+1
M4=M1-1
P12=1+570796
PP=9.669604
DO 11 K=1,NPHI
PHM=P12*(X(K)+1.)
C1(K)=PHM
C2(K)=PHM
SN=5IN(.5*PHM)
C3(K)=4.*SN*SN
A1=P12*A(K)
D4=.5*A1*C3(K)
D5=A1*COS(PHM)
D6=A1*SN(PHM)
N5=K
DO 29 M=1,N3
PHM=(M4+N)*PHM
A2=COS(PHM)
C4(N5)=D4*A2
C5(N5)=D5*A2
C6(N5)=D6*5IN(PHM)
N5=N5+NPHI
29 CONTINUE
DO 29 M=1,N3
P12=1+570796
PP=9.669604
DO 11 K=1,NPHI
PHM=P12*(X(K)+1.)
C1(K)=PHM
C2(K)=PHM
SN=5IN(.5*PHM)
C3(K)=4.*SN*SN
A1=P12*A(K)
D4=.5*A1*C3(K)
D5=A1*COS(PHM)
D6=A1*SN(PHM)
N5=K
DO 29 M=1,N3
PHM=(M4+N)*PHM
A2=COS(PHM)
C4(N5)=D4*A2
C5(N5)=D5*A2
C6(N5)=D6*5IN(PHM)
N5=N5+NPHI
29 CONTINUE
061  KQ=2
062  IF(JQ.EQ.1) KQ=1
063  IF(JQ.EQ.MP) KQ=3
064  R1=RS(JQ)
065  Z1=ZS(JQ)
066  D1=D0(JQ)
067  D2=OR(JQ)
068  D3=DZ(JQ)
069  D4=D2/R1
070  D5=D1/R1
071  S4=D2/D1
072  C4=D3/D1
073  P1=PN1#05
074  P3=Z+01
075  P4=Z+04
076  P5=04#04
077  P6=D1#01
078  P7=P6+01
079  T6=CT#01
080  T62=T6+D01
081  T62=Z6#T62
082  R6=CP#R1
083  R62=RS#R6
084  DO 12 L=1+NT
085  D6=XT(L)
086  R2(L)=R1+D2#06
087  Z2(L)=Z1+D3#06
088  12 CONTINUE
089  DO 16 IP=1+MP
090  R3=RS(IP)
091  Z3=ZS(IP)
092  R4=RI-R3
093  Z4=Z1-Z3
094  U3=D2#U1
095  U4=O3#U1
096  DO 40 L=1+NT
097  D7=R2(L)-R3
098  D8=Z2(L)-Z3
099  R7(L)=R3*R2(L)
100  Z7(L)=D7+D8*08
101  40 CONTINUE
102  PH=R4#SV+Z4#CV
103  A1=ABS(PH)
104  A2=ABS(R4#CV-Z4#SV)
105  D6=A2
106  IF(A1.LE.D1) GO TO 26
107  D6=A1-D1
108  D6=SQR(T(D6+D6+A2*A2)
109  26 ZIP=R4#R4+Z4#Z4
110  IF(ZIP-NE.0.AND.(R6#GT.D6.OR.T6.LE.06)) GO TO 41
111  Z5=ZIP
112  R5=R3#R1
113  PHM=-S3#R3#SV
114  DO 33 K=1+NP1
115  A1=C3(K)
116  RR=Z5+R5#A1
117  M1=0.
118  M2=0.
119  M3=0.
120  M4=0.
HSA=0.
IF(RR.LT.62) GO TO 34
DO 35 L=1,NT
W=27(L)+R7(L)*A1
R=SQRT(W)
SN=-SIN(R)
CS=COS(R)
D6=AT(L)/R
H1B=D6/W#CMPLX(CS-R*SN,SN+R*CS)
H1A=H1B#H1A
H2B=XT(L)*H1B
H2A=H2B#H2A
H3A=XT(L)*H2B+H3A
H4B=D6#CMPLX(CS,SN)
H4A=H4B+H4A
H5A=XT(L)*H4B+H5A
CONTINUE
GO TO 36
DO 37 L=1,NT
W=27(L)+R7(L)*A1
R=SQRT(W)
IF(RR.GT.-5) GO TO 14
CS=R/W+(-694444E-2-W*11736111E-3-1.125)
SN=W/.3333333E-1-W*.1190476E-2-.3333333
H1B=AT(L)#CMPLX(CS,SN)
C5=RW+(-41666667E-1-1.3888889E-2#W)+.5)
SN=W/.1984126E-3-W*.8333333E-2+.16666667-1.
H4B=AT(L)#CMPLX(CS,SN)
GO TO 43
CONTINUE
A1=PH+PHN*A1
A2=ABS(A1)
R=RR-A2#A2
D6=A2.-D1
DT=A2+01
D62=D6*D6
D72=D7#D7
D69=SQRT(D62+R)
D9=SQRT(D72+R)
IF(R-(RR*1.E-5)) 52,52,53
IF(D6.LT.0.) STOP
W4=.5/D62-.5/D72
GO TO 54
W6=(D7/09-D6/D81)/R
IF(D6#GE.0.) GO TO 39
W=ALO6((D7+09)*(-D6+081)/R)
GO TO 39
W1=(W4+.5#W)/D1
181 \( W = A_2/D_1 \)
182 \( W_2 = (1.5 \times (99 - 08) - 1.0) / D_1 + 1.0 / D_1 \) \( P_6 - W_5 \times W_1 \)
183 \( W_3 = (0.25 \times (97 + 09 - 06) - 0.92) + W - R \times (W_4 + 0.25 \times W_1) / P_7 - W_5 + (2.0 \times W_2 + W_5 \times W_1) \)
184 \( N_4 = W / D_1 \)
185 \( W_5 = (99 - 08 - A_2 \times W_1) / P_6 \)
186 IF \( (A_1 \times GE > 0.1) \) GO TO 27
187 \( W_2 = W_2 \)
188 \( W_5 = W_5 \)
189 27 \( H_1 = W_1 + H_1 A \)
190 \( H_2 = W_2 + H_2 A \)
191 \( H_3 = W_3 + H_3 A \)
192 \( H_4 = W_4 + H_4 A \)
193 \( H_5 = W_5 + H_5 A \)
194 36 \( G(A(K)) = H_1 A \)
195 \( G_6(K) = H_2 A \)
196 \( G_6(K) = H_3 A \)
197 \( G_6(K) = H_4 A \)
198 \( G_6(K) = H_5 A \)
199 33 CONTINUE
200 \( K_1 = 0 \)
201 \( D_0 = 45 \times N - 1 \times N_3 \)
202 \( H_1 = 0 \)
203 \( H_2 = 0 \)
204 \( H_3 = 0 \)
205 \( H_4 = 0 \)
206 \( H_5 = 0 \)
207 \( H_6 = 0 \)
208 \( H_7 = 0 \)
209 \( H_8 = 0 \)
210 \( H_9 = 0 \)
211 \( H_10 = 0 \)
212 \( H_11 = 0 \)
213 \( H_12 = 0 \)
214 \( H_13 = 0 \)
215 \( H_14 = 0 \)
216 \( H_15 = 0 \)
217 \( D_0 = 46 \times K = 1 \times NPH \)
218 \( K_1 = K_1 + 1 \)
219 \( D_6 = C_4(K_1) \)
220 \( D_7 = C_5(K_1) \)
221 \( D_8 = C_6(K_1) \)
222 \( U_A = G(A(K)) \)
223 \( U_B = G_6(K) \)
224 \( U_C = G(C(K)) \)
225 \( U_D = G_6(K) \)
226 \( U_E = G(E(K)) \)
227 \( H_1 = D_6 \times U_A + H_1 A \)
228 \( H_2 = D_7 \times U_B + H_2 A \)
229 \( N_3 = D_8 \times U_A + H_3 A \)
230 \( H_1 A = D_6 + H_1 B \)
231 \( H_2 A = D_7 + H_2 B \)
232 \( H_3 A = D_8 + H_3 B \)
233 \( H_1 C = D_6 + U_6 + H_1 C \)
234 \( H_2 C = D_7 + U_6 + H_2 C \)
235 \( H_3 C = D_8 + U_6 + H_3 C \)
236 \( H_4 A = D_6 + U_6 + H_4 A \)
237 \( H_5 A = D_7 + U_6 + H_5 A \)
238 \( H_6 A = D_8 + U_6 + H_6 A \)
239 \( H_6 B = D_6 + U_6 + H_6 B \)
240 \( H_5 B = D_7 + U_6 + H_5 B \)
H6B=88#UE+H6B
CONTINUE
G1A(M)=H1A
G2A(M)=H2A
G3A(M)=H3A
G1B(M)=H1B
G2B(M)=H2B
G3B(M)=H3B
G1C(N)=H1C
G2C(M)=H2C
G3C(M)=H3C
G4A(M)=H4A
G5A(M)=H5A
G6A(M)=H6A
G4B(M)=H4B
G5B(M)=H5B
G6B(M)=H6B
CONTINUE
IF(ZIP.NE.0.) GO TO 47
A1=D5/05
A8=0.
D9=0.
DO 63 K=1,NPHI
D8=D8+A(K)/SQRT(C2(K)+A1)
D9=D9+A(K)*BLOG(D8/C1(K))
CONTINUE
A2=3.*141593/05
A8=(BLOG(A2)-PI*08)/(R5*R1)
D9=2./R1*BLOG(A2)+A2*BLOG(1./A2))-3.141593/01+D9
DO 67 M=1,M3
G1A(M)=D8+G1A(M)
G2A(M)=0.
G2B(M)=0.
G2C(M)=0.
G3A(M)=0.
G3A(M)=0.
G4A(M)=0.
G5A(M)=0.
G6A(M)=0.
G4B(M)=0.
G5B(M)=0.
G6B(M)=0.
CONTINUE
GO TO 47
DO 25 N=1,M3
G1A(M)=0.
G2A(M)=0.
G3A(M)=0.
G1E(M)=0.
G2B(M)=0.
G3B(M)=0.
G1C(M)=0.
G2C(M)=0.
G3C(M)=0.
G4A(M)=0.
G5A(M)=0.
G6A(M)=0.
G4B(M)=0.
G5B(M)=0.
G6B(M)=0.
CONTINUE
DO 13 L=1,MT
H5=R7(L)
Z5=Z7(L)
DO 17 K=1,NPHI
W=W+Z5*R5*C3(K)
R = SQR(W)
SN = SIN(R)
CS = COS(R)
GA(K) = CMPLX(CS*R + SN, SN*R + CS)/(W*R)
GD(K) = CMPLX(CS, SN)/R
17 CONTINUE
IF(R62 <= ZS) GO TO 51
DO = 0.
D7 = 0.
D9 = 0.
DO 62 K = 1, NPNI
W2 = C2(K)
W1 = W/(Z5*R5*W2)
W1 = A(K)*SQR(T(W)
D6 = D6 + W1*W2
D7 = D7 + W1*(1. + 1.125*W5*R5*W2*W2)
D9 = D9 + W1
62 CONTINUE
W1 = R5/Z5
W2 = PP*W1
W = SQR(W2)
W = 1.0/Z5
R = SQR(W3)
W3 = SQR(R5)
5 = ALOG(W + R)
D8 = PI2*6 - (W/R - W5)/(R5*W4)
D6 = 5*08
D7 = ((W/R + W125 + 1.6666667*W2) + 1.125*W5*R5*W2*W2)/W4*PI2*07
D9 = W5/W4*PI2*09
51 A1 = AT(L)
A2 = XT(L)*A1
A3 = XT(L)*A2
K1 = 0
DO 30 K = 1, K3
W = H + M4
H1A = 0.
H2A = 0.
H3A = 0.
H4A = 0.
H5A = 0.
H6A = 0.
30 DO 32 K = 1, NPNI
K1 = K1 + 1
H1B = GA(K)
W4 = C4(K1)
W5 = C5(K1)
W6 = C6(K1)
H1A = W4*H1B + H1A
H2A = W5*H1B + H2A
H3A = W6*H1B + H3A
H1B = GD(K)
H4A = W4*H1B + H4A
H5A = W5*H1B + H5A
H6A = W6*H1B + H6A
32 CONTINUE
IF(R62 <= ZS) GO TO 44
H1A = D6 + H1A
H2A = D7 - (W*W + 1.)*D6 + H2A
H3A = D8 + H3A
H5A = 0 + H5A
G1A(M) = A1 * H1A + G1A(N)
G2A(M) = A1 * H2A + G2A(N)
G3A(N) = A1 * H3A + G3A(N)
G1B(N) = A2 * H1A + G1B(N)
G3B(M) = A2 * H3A + G3B(N)
G1C(M) = A3 * H1A + G1C(N)
G2C(M) = A3 * H2A + G2C(N)
G3C(N) = A3 * H3A + G3C(N)
G5A(M) = A1 * H5A + G5A(N)
G6A(N) = A1 * H6A + G6A(N)
G5B(N) = A2 * H5A + G5B(N)
G6B(N) = A2 * H6A + G6B(N)

30 CONTINUE
13 CONTINUE
47 A2 = D(IP)

W1 = A2 * (R4 * D3 - Z4 * D2)
W2 = A2 * R3 * D3
A3 = D2(IP)
D6 = DR(IP)
D7 = Z4 * D6
D9 = D3 * D6
M1C = (D2 * (R3 * A3 + D7) - R1 * D9) * U1
D8 = A2 * D1
H3C = D8 * U2
H2C = Z4 * H3C
H3C = D3 * H3C
W3 = D1 * (D7 - R4 * A3)
W4 = D1 * (D9 - D2 * A3)
W5 = D1 * R1 * A3
A1 = DR(IP)
U5 = A1 * U3
U6 = A3 * U4
D6 = D2 * A2
D7 = D1 * A1
A3 = D3(IP)
JN = JN
DO 31 N = 1, M3
H2A = G2A(N)
H1A = G1A(N)
H2B = G2B(N)
H1B = G1B(N)
UC = UC + H2A + W2 * H1A
UB = UC + H2B + W2 * H1B
UF = UC + H2C + W2 * H2B + W5 * (H2B + D4 * G2C(N)) + W5 * (H1A + P4 * H1B + P5 * G1C(N))
IF (ZIP = NE0) GO TO 48
UC = UC + P1
UF = UF - P1
40 UC = UC - UB
41 UB = UC + UB
H3A = G3A(N)
H3B = G3B(N)
UC = UC + (H3A - H3B)
UD = UC + (H3A + H3B)
UE = UC + (H3A + H3B)
HF = UC + (H3A - H3B)
H1A = G1A(N)
H5B = G5B(N)
H4A = G4A(N) + H5A

H4B = G4B(N) + H5B
422 H6A = G6A(N)
423 H6B + G6B(N)
424 H3A = U5H5A + U6H4A
425 H1B = U5H5B + U6H4B
426 H1A = H3A - H1B
427 H2A = H3A + H1B
428 H3A = U1H4A
429 H1B = D6H6A
430 = = = =
431 A1 = W1A3
432 H2B = D6H6B - A1H4A
433 H3B = D7H6A + D4H6B
434 H4A = W05H5A + H4A
435 K1 = IP + JM
436 K2 = K1 + 1
437 K3 = K1 + N
438 K4 = K2 + N
439 K5 = K2 + MT
440 K6 = K4 + MT
441 K7 = K3 + N2N
442 K8 = K4 + N2N
443 K9 = K8 + NT
444 GO TO (18, 20, 19), K0
445 18 Y(K6) = UB
446 Z(K6) = H1B + H2B
447 IF(IP = EQ = 1) GO TO 21
448 Y(K3) = Y(K3) + UD
449 Y(K7) = Y(K7) + UF
450 Z(K3) = Z(K3) + H2A - H3A
451 Z(K7) = Z(K7) + H3B - H4A
452 IF(IP = EQ = MP) GO TO 22
453 21 Y(K4) = UD
454 Y(K8) = UF
455 Z(K4) = H2A + H3A
456 Z(K8) = H3B + H4A
457 GO TO 22
458 19 Y(K5) = Y(K5) + UA
459 Z(K5) = Z(K5) + H1B + H2B
460 IF(IP = EQ = 1) GO TO 23
461 Y(K1) = Y(K1) + UC
462 Y(K7) = Y(K7) + UF
463 Z(K1) = Z(K1) + H1A + H3A
464 Z(K7) = Z(K7) + H3B - H4A
465 IF(IP = EQ = MP) GO TO 22
466 23 Y(K2) = Y(K2) + UC
467 Y(K8) = UF
468 Z(K2) = Z(K2) + H1A - H3A
469 Z(K8) = H3B + H4A
470 GO TO 22
471 20 Y(K5) = Y(K5) + UA
472 Y(K6) = UB
473 Z(K5) = Z(K5) + H1B - H2B
474 Z(K6) = H1B + H2B
475 IF(IP = EQ = 1) GO TO 24
476 Y(K1) = Y(K1) + UC
477 Y(K3) = Y(K3) + UD
478 Y(K7) = Y(K7) + UF
479 Z(K1) = Z(K1) + H1A + H3A
480 Z(K3) = Z(K3) + H2A - H3A
481 Z(K7) = Z(K7) + H3B - H4A
482 IF(IP = EQ = MP) GO TO 22
483 Y(K2) = Y(K2) + UC
484 Y(K4) = UD
485 Y(K8) = UF
486 Z(K2) = Z(K2) + H1A - H3A
487 Z(K4) = H2A + H3A
488 Z(K5) = H3B + H4A
489 22 Y(K9) = UE
490 Z(K9) = U2D6H5A + D4H5B - A1H4A
491 JM = JM + H2
492 31 CONTINUE
493 16 CONTINUE
494 JMP = JM + N
495 15 CONTINUE
496 RETURN
497 END
III. THE FUNCTION BLOG AND THE SUBROUTINES PLANE, DECOMP, AND SOLVE

The function BLOG is exactly the same as in [3, page 56]. The only difference between the subroutine PLANE (of the present report) and the subroutine PLANE of [3, pages 57-62] is that the statement

\[ \text{IF}(R1.\text{EQ}.0.) \quad R1 = .5 \]

in line 39 of PLANE has no counterpart in the subroutine PLANE of [3, pages 57-62]. The action of this statement is similar to that of the statement in line 17 of the subroutine YZA. Except for a difference in the actual space allocated to the variables UL, SCL, and IPS, the subroutine DECOMP is the same as the subroutine DECOMP of [3, pages 63-64]. Minimum allocations in DECOMP are the same as those in the subroutine DECOMP of [3, pages 63-64]. The subroutine SOLVE differs from the subroutine SOLVE of [3, pages 63-64] only in the actual space allocated to the variables UL, B, X, and IPS. Minimum allocations in SOLVE are the same as those in the subroutine SOLVE of [3, pages 63-64].
LISTING OF THE FUNCTION BLOG

FUNCTION BLOG(X)
IF(X.GT.1) GO TO 1
X=X**2
BLOG=(1.075*X**2-.166667)*X**2+1.1*X
RETURN
1 BLOG=ALOG(X+SQRT(1.+X**2))
RETURN
END

LISTING OF THE SUBROUTINE PLANE
SUBROUTINE PLANE(M1,M2,NF,NP,NT,RH,ZH,XT,AT,THR,R)
COMPLEX R(240),U,U1,UA,UB,FA(I0),FB(I0),F2A,F2B,F1A,F1B,U2,U3,U4
DIMENSION RH(433),ZH(43),XT(10),AT(10),THR(3),CS(3),SN(3),R2(10)
DO 11 K=1,NF
X=THR(K)
CS(K)=COS(X)
SN(K)=SIN(X)
11 CONTINUE
U=(0.,1.)
UI=3.141593+U0*N1
M=N1+1
M2=M+3
IF(M.EQ.0) M3=2
M5=M1+2
M6=M2+2
DO 12 IP=1,NP
K2=K+1
I=IP+1
DR=.5*(RH(I)-RH(IP))
DZ=.5*(ZH(I)-ZH(IP))
DI=SQRT(DR*DR+DZ*DZ)
R1=.25*(RH(I)+RH(IP))
IF(R1.EQ.0) R1=.5
Z1=.5*(ZH(I)+ZH(IP))
DR=.5*DR
D2=DR/R1
DO 13 L=1,NT
R2(L)=R1+DR*XT(L)
Z2(L)=Z1+DZ*XT(L)
13 CONTINUE
DO 14 K=1,NF
CC=CS(K)
SS=SN(K)
D3=DR*CC
D4=CC*SS
D5=DI*CC
DO 23 M=M3+M4
FA(M)=0.
FB(M)=0.
23 CONTINUE
DO 15 L=1,NT
X=SS*R2(L)
IF(X.GT.1.E-7) GO TO 19
DO 20 M=M3+M4
061  BJ(1)=0
062  20 CONTINUE
063  BJ(1)=1
064  S=1
065  GO TO 18
066  18 M=2.8*X+4.-/X
067  IF(X.LT.5)  N=11.8+ALOG10(X)
068  IF(M.LT.M4; N=M4
069  BJ(1)=0
070  JM=N-1
071  BJ(JM)=1
072  DO 16 J=4,N
073  JM=J
074  JM=JM-1
075  JM=JM-1
076  BJ(JM)=J1/X*BJ(J2)-BJ(JM+2)
077  16 CONTINUE
078  S=0.
079  IF(M.LE.4) GO TO 24
080  DO 17 J=4,M+2
081  S=S+BJ(J)
082  17 CONTINUE
083  24 S=BJ(2)+2.*S
084  18 ARG=22(L)*/CC
085  UA=AT(L)/5*CNPLX(COS(ARG),SIN(ARG))
086  UB=XT(L)*UA
087  DO 25 N=3,N4
088  FA(N)=BJ(N)*UA+FA(N)
089  FB(N)=BJ(N)*UB+FB(N)
090  25 CONTINUE
091  15 CONTINUE
092  IF(M1.NE.0) GO TO 26
093  FA(1)=-FA(3)
094  FA(1)=-FB(3)
095  UB=UA
096  DO 27 N=5,N6
097  MT=M-1
098  N8=N+1
099  F2A=UA*(FA(N8)+FA(N7))
100  F2B=UA*(FB(N8)+FB(N7))
101  UB=UBA
102  FA=UB*(FA(N8)-FA(N7))
103  FB=UB*(FB(N8)-FB(N7))
104  U4=U4A
105  U2=DF1A+U4*FA(N)
106  U3=DF1B+U4*FB(N)
107  U4=U4A+F2A
108  U5=DF#2B
109  K1=K2-1
110  K4=K1*N
111  K5=K2*N
112  R(K2+NF)=-DF*(F2A+D2#F2B)
113  R(K5+MT)=0l*(F1A+D2#18)
114  IF(IP.EQ.1) GO TO 21
115  R(K1)=R(K4)+U2-U3
116  R(K4)=R(K4)+U4-U5
117  IF(IP.EQ.NP) GO TO 22
118  21 R(K2)=U2+U3
119  R(K5)=U4+U5
120  22 K2=K2+N2
SUBROUTINE OECOMP(N, IPS, UL)
DIMENSION IPS(79), S(79)
DO N = 1, N
IF(RN-ULM) 1, 2
1 RN=ULM
2 CONTINUE
SCL(1)=1/RN
5 CONTINUE
DO K = 1, N
BIG=0
DO J = 1, N
BIG=MAX(BIG, ABS(UL(J)))
10 BIG=SIZE
11 CONTINUE
IF(IPV-K) 14, 15, 16
14 J=IPS(K)
15 IPS(K)=IPS(IPV)
16 IPS(IPV)=J
17 KPP=IPS(K)+K2
18 IPS(KPP)=IPS(IPV)
19 IPV=UL(KPP)
20 IPV=IPS(IPV)
21 K2=K-N
22 IPV=UL(I)+IPS(IPV)*Pivot
23 IPV=UL(I)
24 KPP=K+1
25 IPS(KPP)=IPS(IPV)
26 IPS(IPV)=-IP
27 K2=K-N
28 X(N)=X(N)*UL(IP)
29 K=K+1
30 KPP=K+1
31 IPS(KPP)=IPS(IPV)
32 IPS(IPV)=K2
33 XK2=X(N)/UL(IP)
34 X(N)=X(N)*UL(IP)
35 K2=K-N
36 X(N)=X(N)*UL(IP)
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SUBROUTINE SOLVE(N, IPS, LI, X)
DIMENSION IPS(79), IPS(79)
DO 5 I = 1, N
5 IPS(I) = 1
RETURN
END

LISTING OF THE SUBROUTINE OECOMP
SUBROUTINE OECOMP(N, IPS, UL)
DIMENSION S(79), IPS(79), IPS(79)
DO 5 I = 1, N
5 IPS(I) = 1
RETURN
END

LISTING OF THE SUBROUTINE SOLVE
SUBROUTINE SOLVE(N, IPS, LI, X)
DIMENSION IPS(79), IPS(79)
DO 5 I = 1, N
5 IPS(I) = 1
RETURN
END

SUBROUTINE OECOMP(N, IPS, UL)
DIMENSION S(79), IPS(79), IPS(79)
DO 5 I = 1, N
5 IPS(I) = 1
RETURN
END

SUBROUTINE SOLVE(N, IPS, LI, X)
DIMENSION IPS(79), IPS(79)
DO 5 I = 1, N
5 IPS(I) = 1
RETURN
END
IV. THE SUBROUTINE PRNT

The subroutine PRNT uses knowledge of the superscripted $I_{nj}^+$'s or $V_{nj}^+$'s and the $k^+ \rho^+$'s in a portion of [1, Eqs. (87a), (88a), and (89a)] and in the corresponding portion of [1, Eqs. (87b), (88b), and (89b)] in order to calculate and print out these portions of [1, Eqs. (87)-(89)]. By calling the subroutine PRNT repeatedly, it is possible to print out all the quantities on the right-hand sides of [1, Eqs. (87)-(89)] except those for the cases in which $j=1$ and $j=N^+$ in [1, Eq. (87a)] and $j=M^++1$ and $j=M^++M$ in [1, Eq. (89a)]. PRNT is not designed to treat these cases. They are included in [1, Eqs. (87a) and (89a)] merely for convenience.

Except for the cases in which $j=1$ and $j=N^+$ in [1, Eq. (87a)] and $j=M^++1$ and $j=M^++M$ in [1, Eq. (89a)], any portion of [1, Eqs. (87a), (88a), and (89a)] can be written as

$$C_1 \ast XX(J + JX)/RA(J + J3), \quad J = 1,2,...J1$$  \hspace{1cm} (27)

and the corresponding portion of [1, Eqs. (87b), (88b), and (89b)] can be written as

$$C_2 \ast XX(J+J1+JX)/(RA(J+J4-1) + RA(J+J4)), \quad J = 1,2,...J2$$  \hspace{1cm} (28)

In (27) and (28), XX represents the $I_{nj}^+$'s or $V_{nj}^+$'s of [1], and RA represents the $k^+ \rho^+$'s of [1]. In (27), $C_1$ is either $\varepsilon_n$ or $2j$. In (28), $C_2$ is either $4j$ or $2\varepsilon_n$. The sum of the two RA's in the denominator of (28) is due to [1, Eq. (12)]. The complex numbers (27) are directed currents, either electric or magnetic. The complex numbers (28) are directed currents. The set of complex numbers (28) is linked to the set
by the fact that the XX's used in (28) occur immediately after those used in (27).

First, the subroutine PRNT (J1, J2, J3, J4, RA) prints out the real part, the imaginary part, and the magnitude of each of the complex numbers (27) under the heading which appears in statement 10. Then, PRNT prints out the real part, the imaginary part, and the magnitude of each of the complex numbers (28) under the heading which appears in statement 13. The variables in (27) and (28) enter PRNT by means of the arguments of PRNT and the statement

\[
\text{COMMON C1, C2, XX, JX}
\]

in line 6. If \( J1 = 0 \), nothing from (27) is printed out. If \( J2 = 0 \), nothing from (28) is printed out. All the arguments of PRNT are input arguments. The common variables C1, C2, and XX are input variables. However, the common variable JX functions as both an input variable and an output variable. PRNT adds \( J1 + J2 \) to the original value of JX.

Minimum allocations in PRNT are given by

\[
\text{COMPLEX XX(J2} + J1 + JX) \\
\text{DIMENSION RA(Max(J1} + J3, J2 + J4))}
\]

where Max denotes the larger of the two values in the parentheses. Of course, the space allocated to XX in the calling program must be exactly the same as the space allocated to XX in line 4 of PRNT.

DO loop 11 prints out the complex numbers (27). Line 14 obtains (27). DO loop 14 prints out the complex numbers (28). Line 27 obtains (28).
LISTING OF THE SUBROUTINE PRNT

SUBROUTINE PRNT(J1,J2,J3,J4,RA)

COMPLEX C1,C2,U

COMPLEX XX(7)

DIMENSION RA(4)

COMMON C1,C2,XX,JX

IF(J1.EQ.0) GO TO 15

WRITE(3,10)

10 FORMAT(*0 REAL JT IMAG JT MAG JT*)

K3=J3

DO 11 J=1,J1

K3=K3+1

JX=JX+1

U=C1/RA(K3)*XX(JX)

W=CABS(U)

WRITE(3,12) U,W

12 FORMAT(*0 REAL JP IMAG JP MAG JP*)

K4=J4

DO 14 J=1,J2

JX=JX+1

K3=K4

K4=K4+1

U=C2/(RA(K3)+RA(K4))*XX(JX)

W=CABS(U)

WRITE(3,12) U,W

14 CONTINUE

RETURN

END
V. THE MAIN PROGRAM

The main program accepts input data and calls the subroutines YZA, PLANE, DECOMP, SOLVE, and PRNT in order to calculate and print out [1, Eqs. (87)-(89)]. The input data are read from punched cards according to

```
READ(1,15) NT, NPHI
15 FORMAT(2I3)
READ(1,10)(XT(K), K=1, NT)
READ(1,10)(AT(K), K=1, NT)
10 FORMAT(5E14.7)
READ(1,10)(X(K), K=1, NPHI)
READ(1,10)(A(K), K=1, NPHI)
READ(1,16) NA, NB, MA, MB, MC, LA, LB, LC,
           MI, M2, BK, UR, ER, THR(1)
16 FORMAT(10I3/4E14.7)
READ(1,18)(RA(I), I=1, NA)
READ(1,18)(ZA(I), I=1, NA)
18 FORMAT(10F8.4)
READ(1,18)(RB(I), I=1, NB)
READ(1,18)(ZB(I), I=1, NB)
```

Most of the input variables in the main program represent variables in [1] and [2]. Table 3 relates input variables in the main program to variables in [1] and [2]. The input variable NT in Table 3 represents both \( n_t \) and \( n_T \). It is assumed that \( n_t = n_T \) in the main program. The Gaussian quadrature data \( x_{\chi}', A_{\chi}', x_{\phi}', A_{\phi}' \) are given in [4, Appendix A]. It is
Table 3. Input data for the main program.

<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>NT</td>
<td>$n_t = n_T$</td>
<td>[2, Eqs. (35) and (79)]</td>
</tr>
<tr>
<td>NPHI</td>
<td>$n_\phi$</td>
<td>[2, Eq. (36)]</td>
</tr>
<tr>
<td>XT</td>
<td>$x_{\ell'}(n_t)$, $\ell' = 1,2, \ldots n_t$</td>
<td>[2, Eq. (35)]</td>
</tr>
<tr>
<td>AT</td>
<td>$A_{\ell'}(n_t)$, $\ell' = 1,2, \ldots n_t$</td>
<td>[2, Eq. (35)]</td>
</tr>
<tr>
<td>X</td>
<td>$x_{\ell}(n_\phi)$, $\ell = 1,2, \ldots n_\phi$</td>
<td>[2, Eq. (37)]</td>
</tr>
<tr>
<td>A</td>
<td>$A_{\ell}(n_\phi)$, $\ell = 1,2, \ldots n_\phi$</td>
<td>[2, Eq. (36)]</td>
</tr>
<tr>
<td>NA</td>
<td>$N^+$</td>
<td>[1, Fig. 4]</td>
</tr>
<tr>
<td>NB</td>
<td>$N^-$</td>
<td>[1, Fig. 5]</td>
</tr>
<tr>
<td>MA</td>
<td>$M^+$</td>
<td>[1, Fig. 4]</td>
</tr>
<tr>
<td>MB</td>
<td>$M^-$</td>
<td>[1, Fig. 5]</td>
</tr>
<tr>
<td>MC</td>
<td>$M$</td>
<td>[1, Figs. 4 and 5]</td>
</tr>
<tr>
<td>BK</td>
<td>$k^+$</td>
<td>[1, Eqs. (73)-(84)]</td>
</tr>
<tr>
<td>UR</td>
<td>$\mu^-/\mu^+$</td>
<td>[1, Fig. 1]</td>
</tr>
<tr>
<td>ER</td>
<td>$\epsilon^-/\epsilon^+$</td>
<td>[1, Fig. 1]</td>
</tr>
<tr>
<td>THR(1)</td>
<td>$\theta_t$</td>
<td>[1, Eq. (92)]</td>
</tr>
<tr>
<td>RA</td>
<td>$\rho_j^+$, $j = 1,2, \ldots N^+$</td>
<td>[1, Eq. (11)]</td>
</tr>
<tr>
<td>ZA</td>
<td>$\pi_j^+$, $j = 1,2, \ldots N^+$</td>
<td>[1, Eq. (11)]</td>
</tr>
<tr>
<td>RB</td>
<td>$\rho_j^-$, $j = 1,2, \ldots N^-$</td>
<td>[1, Eq. (11)]</td>
</tr>
<tr>
<td>ZB</td>
<td>$\pi_j^-$, $j = 1,2, \ldots N^-$</td>
<td>[1, Eq. (11)]</td>
</tr>
</tbody>
</table>
assumed that \( N^+ > 3, N^- > 3 \), and \( M \geq 1 \). It is also assumed that either

\[
\begin{align*}
M^+ &= 0 \\
M^- &= 0
\end{align*}
\]

or

\[
\begin{align*}
M^+ &= 0 \\
M^- &= 0
\end{align*}
\]

and that either

\[
\begin{align*}
N^+ - M^+ - M &= 0 \\
N^- - M^- - M &= 0
\end{align*}
\]

or

\[
\begin{align*}
N^+ - M^+ - M > 0 \\
N^- - M^- - M > 0
\end{align*}
\]

If \( \{ M^+ = 0 \} \) or \( \{ M^- = 0 \} \), then there is no conductor below the aperture as in [1, Fig. 18]. If \( \{ N^+ - M^+ - M = 0 \} \), then there is no conductor above the aperture as in [1, Fig. 10]. In Table 3, THR(1) is in radians.

The input variables \( LA, LB, LC, M_1, \) and \( M_2 \) are not listed in Table 3.

The input variables \( LA, LB, \) and \( LC \) allow for generating curves different from those in [1, Figs. 4 and 5]. \( LA, LB, \) and \( LC \) are either 0 or 1.

- \( LA = 0 \) if the generating curve of the surface \( (S^+ + A) \) does not close upon itself.
- \( LA = 1 \) if the generating curve of the surface \( (S^+ + A) \) closes upon itself.
- \( LB = 0 \) if the generating curve of the surface \( (S^- + A) \) does not close upon itself.
- \( LB = 1 \) if the generating curve of the surface \( (S^- + A) \) closes upon itself.
- \( LC = 0 \) if the generating curve of the aperture does not close upon itself.
- \( LC = 1 \) if the generating curve of the aperture closes upon itself.

For example, \( LA = 0, LB = 1, \) and \( LC = 0 \) for the generating curves in [1, Figs. 4 and 5]. If \( LA = 1 \), it is assumed that \( M^+ > 1 \) and \( N^+ - M^+ - M \geq 2 \).
If LB = 1, it is assumed that M ≥ 1 and that N ≥ M ≥ 2. LC = 1 implies that the object under consideration reduces to a homogeneous material obstacle with parameters (μ, ε) in contrast with the parameters of the external environment. As for M1 and M2, [1, Eqs. (87)-(89)] are printed out for n = M1, M1 + 1,...,M2.

Minimum allocations in the main program are given by

\[
\begin{align*}
\text{COMPLEX XX(N), YP(M*JA*JA), ZP(M*JA*JA),} \\
\text{YM(M*JB*JB), ZM(M*JB*JB), R(2*M*JA),} \\
\text{T(N*N), Y(N)}
\end{align*}
\]

\[
\text{DIMENSION XT(NT), AT(NT), X(NPHI), A(NPHI), RA(NA),} \\
\text{ZA(NA), RB(NB), ZB(NB), IPS(N)}
\]

where

\[
\begin{align*}
N &= \sum_{I=1}^{4} L1A(I) + \sum_{I=1}^{4} L1B(I) + L2(1) + L2(2) + L3(1) + L3(2) \\
M &= M2 - M1 + 1 \\
JA &= 2 * NA - 3 \\
JB &= 2 * NB - 3
\end{align*}
\]

(29) (30) (31) (32)

The L's on the right-hand side of (29) are calculated in the main program. At any rate, N is the order of the moment matrix \( T_n \) given by [1, Eq. (33)].

At the end of this section, the main program is listed along with sample input and output data. The sample data are for the object of [1, Fig. 13] with \( k^+ = 2.5 \) and \( \varepsilon_r = 2 \) as in [1, Fig. 17]. The output data printed under the heading "Electric current on first part of outside conductor" are, with reference to [1, Eq. (87)],

\[
J_n^{t+\theta}(c_j^+), \quad j = 2,3,...,M^+
\]

(33)

and

\[
J_n^{\phi+\theta}(c_j^+), \quad j = 1,2,3,...,M^+
\]

(34)
The real and imaginary parts and magnitudes of (33) are printed under the heading "Real JT Imag JT Mag JT." The real and imaginary parts and magnitudes of (34) are printed under the heading "Real JP Imag JP Mag JP." The output data printed under the heading "Electric current on second part of outside conductor" are

\[ J_{n}^{t+\theta}(t_{j}^{+}), \quad j = M^{+} + M + 1, M^{+} + M + 2, \ldots N^{+} - 1 \] (35)

and

\[ J_{n}^{\phi+\theta}(t_{j}^{+}), \quad j = M^{+} + M, M^{+} + M + 1, \ldots N^{+} - 1 - LA \] (36)

The output data printed under the heading "Electric current on first part of inside conductor" are, with reference to [1, Eq. (88)],

\[ J_{n}^{t-\theta}(t_{j}^{-}), \quad j = 2, 3, \ldots M^{-} \] (37)

and

\[ J_{n}^{\phi-\theta}(t_{j}^{-}), \quad j = 1, 2, 3, \ldots M^{-} \] (38)

The output data printed under the heading "Electric current on second part of inside conductor" are

\[ J_{n}^{t-\theta}(t_{j}^{-}), \quad j = M^{-} + M + 1, M^{-} + M + 2, \ldots N^{-} - 1 \] (39)

and

\[ J_{n}^{\phi-\theta}(t_{j}^{-}), \quad j = M^{-} + M, M^{-} + M + 1, \ldots N^{-} - 1 - LB \] (40)

The output data printed under the heading "Electric current in aperture" are, with reference to [1, Eq. (87)],

\[ J_{n}^{t+\theta}(t_{j}^{+}), \quad j = M^{+} + 1, M^{+} + 2, \ldots M^{+} + M \] (41)

and

\[ J_{n}^{\phi+\theta}(t_{j}^{+}), \quad j = M^{+} + 1, M^{+} + 2, \ldots M^{+} + M - 1 - LC \] (42)
If, as in [1, Fig. 18], there is no conducting surface below the aperture, then \( j = M' + 1 \) is deleted in (41). If, as in [1, Fig. 10], there is no conducting surface above the aperture, then \( j = M' + M \) is deleted in (41). The output data printed under the heading "Magnetic current in aperture" are, with reference to [1, Eq. (89)],

\[
M_n^{t\theta}(t^+), \quad j = M' + 2, M' + 3, \ldots M' + M - 1
\] (43)

and

\[
M_n^{\phi\theta}(t^+), \quad j = M' + 1, M' + 2, \ldots M' + M - 1 - LC
\] (44)

If, in any of (33)-(44), the upper limit on \( j \) is less than the starting value of \( j \), then no values of \( j \) are to be taken. For example, if \( M' = 1 \) in (33), then no values of \( j \) are to be taken in (33).

Since [1, Eq. (12)] is used for \( \rho^+_j \), the factors 2, 4, 2, and 4 are needed in [1, Eqs. (87)-(89)] when \( n \geq 1 \). In the main program, lines 48-51 store these factors in C3, C4, C5, and C6. Line 53 stores in BB the propagation constant of the medium characterized by \((\mu^- , c^-)\) in [1, Fig. 1]. This propagation constant is called \( k^- \). With regard to [1, Eq. (33)], line 54 stores \( \eta^- \) in ET and line 55 stores \( 1/\eta^- \) in ET1. DO loop 23 puts \( k^+ q^+_I \) in RA(I) and \( k^+ z^+_I \) in ZA(I). DO loop 24 puts \( k^- q^-_I \) in RB(I) and \( k^- z^-_I \) in ZB(I).

The elements \( y_{nji}^{pq} \) and \( z_{nij}^{pq} \) of the submatrices on the right-hand side of [1, Eq. (33)] are given by [1, Eqs. (36) and (35)]. The testing functions \( w_{nij}^p \) and the expansion functions \( j_{nij}^q \) appear in [1, Eqs. (36) and (35)]. However, the submatrices supplied by the subroutine YZA are \( y_{nij}^{rs} \) and \( z_{nij}^{rs} \) whose elements are defined by
Equations (45) and (46) are (3) and (4) with the superscript \( \pm \) appended, with \( i \) replaced by \( i' \), and with \( j \) replaced by \( j' \). In (45) and (46), the testing functions are \( Y_{ni'i'}^{r^\pm} \) and the expansion functions are \( J_{nji'}^{s^\pm} \). To obtain \( Y_{niij}^{pq^\pm} \) from \( Y_{ni'i'}^{r^\pm} \), and \( Z_{niji'}^{pq^\pm} \) from \( Z_{ni'i'}^{r^\pm} \), we must express \( Z_{nji'}^{p^\pm} \) in terms of \( Z_{ni'i'}^{r^\pm} \) and \( J_{nji'}^{q^\pm} \) in terms of \( J_{nji'}^{s^\pm} \). From [1, Eqs. (23), (24), (27), (28), (30), and (37)], we obtain

\[
W_{ni}^{p^\pm} = (J_{nji'}^{s^\pm})^* \tag{47}
\]

where * denotes complex conjugate. [1, Eq. (48)] and (47) state that the testing functions are the complex conjugates of the expansion functions.

Hence, the expressions for \( W_{ni}^{p^\pm} \) in terms of \( W_{ni'}^{r^\pm} \) will be similar to the expressions for \( J_{nji'}^{q^\pm} \) in terms of \( J_{nji'}^{s^\pm} \). Consequently, it suffices to express \( J_{nji'}^{q^\pm} \) in terms of \( J_{nji'}^{s^\pm} \).

[1, Eq. (16)] with the choice of superscript \( \pm \) is rewritten as

\[
J_{nji'}^{q^\pm} = \begin{cases} 
J_{nji'}^{+}, \ j' = 1 + M_1A(1), 2 + M_1A(1), \ldots \ L_1A(1) + M_1A(1) \\
J_{nji'}^{+}, \ j' = 1 + M_1A(2), 2 + M_1A(2), \ldots \ L_1A(2) + M_1A(2) \\
J_{nji'}^{+}, \ j' = 1 + M_1A(3), 2 + M_1A(3), \ldots \ L_1A(3) + M_1A(3) \\
J_{nji'}^{+}, \ j' = 1 + M_1A(4), 2 + M_1A(4), \ldots \ L_1A(4) + M_1A(4)
\end{cases} \tag{48}
\]

where
\[ j_{n_j}^+ = \begin{cases} \frac{j^n}{n_j}, & j' = 1, 2, \ldots, N^+ - 2 \\ \frac{j^n}{n_j}, j' = N^+ - 1, N^+, \ldots, 2N^+ - 3 \end{cases} \] (49a)

The vector functions (49a) are the \( t \) directed expansion functions on the surface \( (S^+ + A) \) in [1, Fig. 4]. The vector functions (49b) are the \( n \) directed expansion functions on the surface \( (S^+ + A) \) in [1, Fig. 4]. Instead of expressing each \( j' \) in (48) in terms of \( j \), we state that \( j_{n_j}^{j+} \) is the \( j \)th vector function listed on the right-hand side of (48). For example, if \( j = \text{L1A}(1)+3 \) and if \( \text{L1A}(2) \geq 3 \), then \( j_{n_j}^{j+} = \frac{j^n}{n_j}, 3+\text{M1A}(2) \).

In (48),

\[ \text{L1A}(1) = \text{Max}(0, M^+ - 1) \] (50a)
\[ \text{L1A}(2) = M^+ \] (50b)
\[ \text{L1A}(3) = \text{Max}(0, N^+ - M^+ - M - 1) \] (50c)
\[ \text{L1A}(4) = N^+ - M^+ - M - \text{LA} \] (50d)
\[ \text{M1A}(1) = 0 \] (51a)
\[ \text{M1A}(2) = N^+-2 \] (51b)
\[ \text{M1A}(3) = M^+ + M - 1 \] (51c)
\[ \text{M1A}(4) = N^+ + M^+ + M - 3 \] (51d)

where \( \text{Max} \) denotes maximum value. The parameters \( (\text{L1A}(I), I=1,2,3,4) \) and \( (\text{M1A}(I), I=1,2,3,4) \) are calculated by lines 64-71 of the main program.

The vector functions (48a) are the \( t \) directed expansion functions on the first part of the outside conductor. The first part of the
outside conductor is the part of S⁺ below the aperture in [1, Fig. 4]. The vector functions (48b) are the φ directed expansion functions on the first part of the outside conductor. The vector functions (48c) are the t directed expansion functions on the second part of the outside conductor. The second part of the outside conductor is the part of S⁺ above the aperture in [1, Fig. 4]. The vector functions (48d) are the φ directed expansion functions on the second part of the outside conductor.

\[
\begin{align*}
\bar{j}^-_{nj} &= \begin{cases} 
  j^-_{nj}, & j' = 1 + MIB(1), 2 + MIB(1), \ldots, L1B(1) + MIB(1) \\
  j^-_{nj}, & j' = 1 + MIB(2), 2 + MIB(2), \ldots, L1B(2) + MIB(2) \\
  j^-_{nj}, & j' = 1 + MIB(3), 2 + MIB(3), \ldots, L1B(3) + MIB(3) \\
  j^-_{nj}, & j' = 1 + MIB(4), 2 + MIB(4), \ldots, L1B(4) + MIB(4)
\end{cases} 
\end{align*}
\]

where

\[
\begin{align*}
\bar{j}^-_{nj}' &= \begin{cases} 
  j^t^-_{nj}, & j' = 1, 2, \ldots, N^- - 2 \\
  j^\phi^-_{nj, j'-N^-+2}, & j' = N^- - 1, N^-, \ldots, 2N^- - 3
\end{cases} 
\end{align*}
\]

The vector functions (53a) are the t directed expansion functions on the surface (S⁻ + A) in [1, Fig. 5]. The vector functions (53b) are the φ directed expansion functions on the surface (S⁻ + A) in [1, Fig. 5]. Equation (52) means that \( \bar{j}^-_{nj} \) is the jth vector function listed on the right-hand side of (52). In (52),

\[
\begin{align*}
L1B(1) &= \text{Max}(0, M^- - 1) \\
L1B(2) &= M^- \\
L1B(3) &= \text{Max}(0, N^- - M^- - M - 1) \\
L1B(4) &= N^- - M^- - M - L1B
\end{align*}
\]
\[ M_{1B}(1) = 0 \quad (55a) \]
\[ M_{1B}(2) = N^{-} - 2 \quad (55b) \]
\[ M_{1B}(3) = M^{-} + M - 1 \quad (55c) \]
\[ M_{1B}(4) = N^{-} + M^{-} + M - 3 \quad (55d) \]

The parameters \((L_{1B}(I), I = 1, 2, 3, 4)\) and \((M_{1B}(I), I = 1, 2, 3, 4)\) are calculated by lines 72-79 of the main program.

The vector functions \((52a)\) are the \(t\) directed expansion functions on the first part of the inside conductor and \((52b)\) are the \(\phi\) directed expansion functions there. The first part of the inside conductor is the part of \((S^{-} + A)\) below the aperture in [1, Fig. 5]. The vector functions \((52c)\) are the \(t\) directed expansion functions on the second part of the inside conductor and \((52d)\) are the \(\phi\) directed expansion functions there. The second part of the inside conductor is the part of \((S^{-} + A)\) above the aperture in [1, Fig. 5].

[1, Eq. (17)] with the choice of superscript + is rewritten as

\[
\frac{j_{n_j}^{2+}}{j_{n_j}} = \begin{cases} 
  \frac{j_{n_j}^{+}}{j_{n_j}} , & j' = 1 + M_2A(1), 2 + M_2A(1), \ldots, L_2(1) + M_2A(1) \\
  \frac{j_{n_j}^{+}}{j_{n_j}} , & j' = 1 + M_2A(2), 2 + M_2A(2), \ldots, L_2(2) + M_2A(2) 
\end{cases} \quad (56a) \]

\[
\frac{j_{n_j}^{2+}}{j_{n_j}} = \begin{cases} 
  M^+ \neq 0, \ L_{1A}(4) \neq 0 \\
  M = 0, \ L_{1A}(4) \neq 0 \\
  M = 0, \ L_{1A}(4) = 0 \\
  M = 0, \ L_{1A}(4) = 0 
\end{cases} \quad (57a) \]

where \(j_{n_j}^{+}\) is given by (49). Equation (56) means that \(j_{n_j}^{2+}\) is the \(j\)th vector function listed on the right-hand side of (56). In (56),
\[ L_2(2) = M - 1 - LC \]  \hspace{1cm} (57b)
\[ M_{2A}(1) = \text{Max}(0, M^+ - 1) \]  \hspace{1cm} (58a)
\[ M_{2A}(2) = N^+ - 2 + M^+ \]  \hspace{1cm} (58b)

where L1A(4) is given by (50d). The parameters L2(1), L2(2), M2A(1), and M2A(2) are calculated by lines 80-85 of the main program.

The vector functions (56a) consist of the \( t \) directed expansion function which straddles the first part of the outside conductor and the aperture, the \( t \) directed expansion functions in the aperture, and the \( t \) directed expansion function which straddles the aperture and the second part of the outside conductor. The vector functions (56b) are the \( \phi \) directed expansion functions in the aperture. Four different cases are necessary in (57a) because there is no \( t \) directed expansion function which straddles the first part of the outside conductor and the aperture if the first part of the outside conductor is absent and there is no \( t \) directed expansion function which straddles the aperture and the second part of the outside conductor if the second part of the outside conductor is absent.

\[ \text{[1, Eq. (17)] with the choice of superscript - is rewritten as} \]

\[
\begin{align*}
\frac{J^-}{n_j}, \ j' &= 1+M_{2B}(1), 2+M_{2B}(1), \ldots \ L_2(1) + M_{2B}(1) \quad (59a) \\
\frac{J^{2-}}{n_j} &= \begin{cases} \\
\frac{J^-}{n_j}, \ j' &= 1+M_{2B}(2), 2+M_{2B}(2), \ldots \ L_2(2) + M_{2B}(2) \quad (59b)
\end{cases}
\end{align*}
\]

where \( \frac{J^-}{n_j} \) is given by (53). Equation (59) means that \( \frac{J^{2-}}{n_j} \) is the jth vector function listed on the right-hand side of (59). In (59), L2(1)
and $L_2(2)$ are given by (57). $M_{2B}(1)$ and $M_{2B}(2)$ are given by

$$M_{2B}(1) = \text{Max}(0, M^- - 1)$$  
(60a)

$$M_{2B}(2) = N^- - 2 + M^-$$  
(60b)

$M_{2B}(1)$ and $M_{2B}(2)$ are calculated by lines 86-87 of the main program. The vector functions (59a) consist of the $t$ directed expansion function which straddles the first part of the inside conductor and the aperture, the $t$ directed expansion functions in the aperture, and the $t$ directed expansion function which straddles the aperture and the second part of the inside conductor. The vector functions (59b) are the $\phi$ directed expansion functions in the aperture. Because of [1, Eqs. (15b) and (15d)], the vector functions in the aperture in (59) are the same as the vector functions in the aperture in (56).

[1, Eq. (22)] is rewritten as

$$[1, +, J, l^+, J = 1 + M_3A(1), 2 + M_3A(1), \ldots L_3(1) + M_3A(1)$$  
(61a)

$$\begin{cases} 
J^+_{nj}, & j = 1 + M_3A(1), 2 + M_3A(1), \ldots L_3(1) + M_3A(1) \\
J^+_{nj}, & j = 1 + M_3A(2), 2 + M_3A(2), \ldots L_3(2) + M_3A(2) 
\end{cases}$$  
(61b)

where $J^+_{nj}$ is given by (49). Equation (61) means that $J^+_{nj}$ is the $j$th vector function listed on the right-hand side of (61). In (61),

$$L_3(1) = \text{Max}(0, M - 2)$$  
(62a)

$$L_3(2) = L_2(2)$$  
(62b)

$$M_3A(1) = M^+$$  
(63a)

$$M_3A(2) = N^- - 2 + M^+$$  
(63b)

where $L_2(2)$ is given by (57b). $L_3(1)$, $L_3(2)$, $M_3A(1)$, and $M_3A(2)$ are
calculated by lines 88-91 of the main program. The vector functions
(61a) are the t directed expansion functions in the aperture. The vector
functions (61b) are the \( \phi \) directed expansion functions in the aperture.

Replacement of the superscript \(+\) in [1, Eq. (22)] by the super-
script \(-\) gives

\[
\begin{align*}
\bar{J}_{nj}^{3-} &= \begin{cases} 
\bar{J}_{-n,j+M}^t, & j = 1, 2, \ldots, M-2 \\
\bar{J}_{-n,j+M-M+2}^\phi, & j = M-1, M, \ldots, 2M-3
\end{cases} 
\end{align*}
\]

Equation (64) is consistent with [1, Eqs. (22), (37), (15b), and (15d)].

Equation (64) can be rewritten as

\[
\begin{align*}
\bar{J}_{nj}^{3-} &= \begin{cases} 
\bar{J}_{-n,j+M}^{-}, & j' = 1+M3B(1), 2+M3B(1), \ldots, L3(1) + M3B(1) \\
\bar{J}_{-n,j+M-M+2}^{-}, & j' = 1+M3B(2), 2+M3B(2), \ldots, L3(2) + M3B(2)
\end{cases} 
\end{align*}
\]

where \( \bar{J}_{-n,j}^{-} \) is given by (53). Equation (65) means that \( \bar{J}_{nj}^{3-} \) is the jth
vector function listed on the right-hand side of (65). In (65), L3(1)
and L3(2) are given by (62a) and (62b). M3B(1) and M3B(2) are given by

\[
\begin{align*}
M3B(1) &= M^- \\
M3B(2) &= N^- - 2 + M^-
\end{align*}
\]

M3B(1) and M3B(2) are calculated by lines 92-93 of the main program. The
vector functions on the right-hand side of (65) are the same as those on
the right-hand side of (61).

To accompany the expansion functions \( \bar{J}_{nj}^{1+} \), of (49) and (53), testing
functions \( \bar{W}_{n1}^{1+} \), are defined by
Equations (49), (53), and (67) allow (45) and (46) to be recast as

\[ Y_{ni'}^{+j'} = -<W_{ni'}^{+} \ , H_{nj'}^{+} , 0> \]  \hspace{1cm} (68)

\[ Z_{ni'}^{+j'} = -<W_{ni'}^{+} \ , \frac{1}{n^z} E^{-}(J_{nj'}^{+}) , 0> \]  \hspace{1cm} (69)

Expressions (68) and (69) are the matrix elements supplied by the subroutine YZA.

Line 94 stores in JA the order of the matrices \( Y_{ni}^{+} \) and \( Z_{ni}^{+} \) whose \( i'j' \)th elements are given by (68) and (69). Line 95 stores in JB the order of the matrices \( Y_{ni}^{-} \) and \( Z_{ni}^{-} \) whose \( i'j' \)th elements are given by (68) and (69). Line 96 stores in IA the total number of vector functions listed on the right-hand side of (48). IA is the order of the submatrix \( Z_{ni}^{11+} \) in [1, Eq. (33)]. Line 97 stores in IB the total number of vector functions listed on the right-hand side of (52). IB is the order of the submatrix \( Z_{ni}^{11-} \) in [1, Eq. (33)]. Line 98 stores in N the order of the matrix \( T_{ni} \) of [1, Eq. (33)].

Line 99 puts \( Y_{ni}^{+} \) of (68) in \( Y_P \) \( ((n-M1)\times JA+1) \) to \( Y_P ((n-M1+1)\times JA+JA) \) and \( Z_{ni}^{+} \) of (69) in the corresponding region of \( Z_P \) for \( n = M1, M1+1, \ldots, M2 \). Storage of \( Y_{ni}^{+} \) and \( Z_{ni}^{+} \) in \( Y_P \) and \( Z_P \) is by columns. Line 100 puts \( Y_{ni}^{-} \) of (68) in \( Y_M ((n-M1)\times JB+1) \) to \( Y_M ((n-M1+1)\times JB+JB) \) and \( Z_{ni}^{-} \) of (69) in the corresponding region of \( Z_M \) for \( n = M1, M1+1, \ldots, M2 \). Storage of \( Y_{ni}^{-} \) and \( Z_{ni}^{-} \) in \( Y_M \) and \( Z_M \) is by columns. With reference to [1, Eq. (103)], line 101 puts
\[
\begin{align*}
V_{n_i}^\phi, & \text{ in } R(i' + J_1), \quad i' = 1, 2, \ldots NA-2 \\
-V_{n_i}^\phi, & \text{ in } R(i' + J_1 + NA - 2), \quad i' = 1, 2, \ldots NA-1 \\
-V_{n_i}, & \text{ in } R(i' + J_1 + JA), \quad i' = 1, 2, \ldots NA-2 \\
V_{n_i}^\psi, & \text{ in } R(i' + J_1 + JA + NA-2), \quad i' = 1, 2, \ldots NA-1 \\
\end{align*}
\]

where
\[
J_1 = (n - M_1) \times 2 \times J_1
\]

and
\[
n = M_1, M_1 + 1, \ldots M_2
\]

DO loop 82 puts \( k_{i,j}^+ \) in RB(J). \( k_{i,j}^- \) is needed in order to calculate [1, Eq. (88a)].

DO loop 86 obtains \( n \) according to \( n = M-1 \). If \( n \neq M_1 \), DO loop 89 moves \( Y_{n_i}^+ \) down into YP(1) to YP(JA*JA) and \( Z_{n_i}^+ \) down into ZP(1) to ZP(JA*JA), DO loop 90 moves \( Y_{n_i}^- \) down into YM(1) to YM(JB*JB) and \( Z_{n_i}^- \) down into ZM(1) to ZM(JB*JB), and DO loop 91 moves the \( V \)'s of (70) down into R(1) to R(2*JA).

The only difference between \( V_{nij}^{pq,\psi} \) of [1, Eq. (36)] and \( Y_{nij}^{\psi} \) of (68) lies in the testing functions and expansion functions. Likewise, the only difference between \( Z_{nij}^{pq,\psi} \) of [1, Eq. (35)] and \( Z_{nij}^{\psi} \) of (69) lies in the testing functions and expansion functions. The expansion functions in [1, Eqs. (36) and (35)] are related to the expansion functions in (68) and (69) by (48), (52), (56), (59), (61), and (65). According to (67) and (47), the testing functions are the complex conjugates of the expansion functions. Hence, the complex conjugates of (48), (52), (56), (59), (61), and (65) relate the testing functions in [1, Eqs. (36) and (35)] to the testing functions in (68) and (69). As a result,
\[
\begin{align*}
\gamma_{ni}^{pq+} &= \gamma_{ni'}^{+} (73) \\
Z_{ni}^{pq+} &= Z_{ni'}^{+} (74)
\end{align*}
\]

where \( j' \) is the subscript of the \( j \)th vector function listed on the right-hand side of either (48), (52), (56), (59), (61), or (65), whichever is appropriate. Similarly, \( i' \) is the subscript of the \( i \)th vector function listed on the right-hand side of either (48), (52), (56), (59), (61), or (65), whichever is appropriate. For example, if \( p = 2 \) and \( q = 1 \) and if the superscript \( + \) is chosen in (73), then (73) becomes

\[
\gamma_{ni}^{21+} = \gamma_{ni'}^{+} (75)
\]

where \( j' \) is the subscript of the \( j \)th vector function listed on the right-hand side of (48) and \( i' \) is the subscript of the \( i \)th vector function listed on the right-hand side of (56).

With regard to [1, Eq. (33)], DO loop 25 uses (73) and (74) to store

\[
\begin{bmatrix}
Z_{ni}^{11+} \\
0 \\
Z_{ni}^{21+} \\
\gamma_{ni}^{31+}
\end{bmatrix}
\]

by columns in \( T \). The elements of the submatrices in (76) depend on the expansion functions (48). Hence, \( j' \) in (73) and (74) is the subscript of the \( j \)th vector function listed on the right-hand side of (48). The index of DO loop 25 is \( JJ \). If \( JJ = 1 \), inner DO loop 26 obtains the values of \( j \) for which \( j' \) is given by (48a). If \( JJ = 2 \), DO loop 26 obtains the values
of $j$ for which $j'$ is given by (48b). Similarly, if $JJ = 3$, DO loop 26 obtains the values of $j$ covered by (48c). Finally, if $JJ = 4$, DO loop 26 obtains the values of $j$ covered by (48d).

The elements of $Z_{11n}^{1+}$ in (76) depend on the testing functions $W_{n1i}^{1+}$. These testing functions are related to $W_{n1i}^{+}$ of (67) by the testing function version of (48). The testing function version of (48) is (48) with $J_{nj}^{1+}$ replaced by $W_{n1i}^{1+}$, $J_{nj}^{1+}$ replaced by $W_{n1i}'$, and $j'$ replaced by $i'$. It is now apparent that $i'$ in (74) is the subscript of the $i$th vector function listed on the right-hand side of the testing function version of (48). Inside nested DO loops 27 and 28, line 141 stores the appropriate element of $Z_{11n}^{1+}$ of (76) in $T$. The index of DO loop 27 is $II$. If $II = 1$, DO loop 28 obtains the values of $i$ for which $i'$ is given by the testing function version of (48a). If $II = 2$, DO loop 28 obtains the values of $i$ for which $i'$ is given by the testing function version of (48b). Similarly, if $II = 3$, DO loop 28 obtains the values of $i$ covered by the testing function version of (48c). Finally, if $II = 4$, DO loop 28 obtains the values of $i$ covered by the testing function version of (48d).

DO loop 29 takes care of the 0 in (76).

The elements of $Z_{12n}^{1+}$ in (76) depend on the testing functions $W_{n1i}^{2+}$. These testing functions are related to $W_{n1i}^{+}$ of (67) by the testing function version of (56). Hence, $i'$ in (74) is the subscript of the $i$th vector function listed on the right-hand side of the testing function version of (56). Inside nested DO loops 30 and 31, line 157 stores the appropriate element of $Z_{12n}^{1+}$ of (76) in $T$. The index of DO loop 30 is $II$. If $II = 1$, DO loop 31 obtains the values of $i$ for which $i'$ is
given by the testing function version of (56a). If II = 2, DO loop 31 obtains the values of i for which i' is given by the testing function version of (56b).

The elements of $Y_{n}^{31+}$ in (76) depend on the testing functions $W_{n1}^{3+}$. These testing functions are related to $W_{n1}^{+}$, of (67) by the testing function version of (61). Hence i' in (73) is the subscript of the ith vector function listed on the right-hand side of the testing function version of (61). Inside nested DO loops 32 and 33, line 168 stores the appropriate element of $Y_{n}^{31+}$ of (76) in T. The index of DO loop 32 is II. If II = 1, DO loop 33 obtains the values of i for which i' is given by the testing function version of (61a). If II = 2, DO loop 33 obtains the values of i for which i' is given by the testing function version of (61b).

With regard to [1, Eq. (33)], DO loop 34 uses (73) and (74) to store

\[
\begin{bmatrix}
0 \\
\eta_{r} Z_{n}^{11-} \\
\eta_{r} Z_{n}^{21-} \\
Y_{n}^{31-}
\end{bmatrix}
\]

by columns in T. Nested DO loops 34 and 35 obtain the values of j in (73) and (74). In (73) and (74), j' is the subscript of the jth vector function listed on the right-hand side of (52). DO loop 36 obtains the 0 in (77). Nested DO loops 37 and 38 obtain the values of i for $Z_{nij}^{11-}$ of (74) in which i' is the subscript of the ith vector function listed on the right-hand side of the testing function version of (52). Nested DO loops 39 and 40 obtain the values of i for $Z_{nij}^{21-}$ of (74) in which i' is determined by the testing function version of (59). Nested DO loops 41 and 42 obtain the values of i for
\[ y_{ij}^{31-} \text{ of (73) in which } i' \text{ is determined by the testing function version of (65).} \]

With regard to [1, Eq. (33)], DO loop 43 uses (73) and (74) to store

\[
\begin{pmatrix}
Z_{n}^{12+} \\
\eta_{r} Z_{n}^{12-} \\
Z_{n}^{22+} + \eta_{r} Z_{n}^{22-} \\
\gamma_{n}^{32+} + \gamma_{n}^{32-}
\end{pmatrix}
\]

by columns in \( T \). Nested DO loops 43 and 44 obtain the values of \( j \) in (73) and (74) in which \( j' \) is determined by (56) for the submatrices with superscript + in (78) and by (59) for the submatrices with superscript - in (78). Nested DO loops 45 and 46 obtain the values of \( i \) for \( Z_{nij}^{12+} \) of (74) with \( i' \) determined by the testing function version of (48). Nested DO loops 47 and 48 obtain the values of \( i \) for \( Z_{nij}^{12-} \) of (74) with \( i' \) determined by the testing function version of (52). Nested DO loops 49 and 50 obtain the values of \( i \) for \( Z_{nij}^{22+} + \eta_{r} Z_{nij}^{22-} \) of (78). \( Z_{nij}^{22+} \) is given by (74) with \( i' \) determined by the testing function version of (56). \( Z_{nij}^{22-} \) is given by (74) with \( i' \) determined by the testing function version of (59). Nested DO loops 51 and 52 obtain the values of \( i \) for \( \gamma_{nij}^{32+} + \gamma_{nij}^{32-} \) of (78). \( \gamma_{nij}^{32+} \) is given by (73) with \( i' \) determined by the testing function version of (61). \( \gamma_{nij}^{32-} \) is given by (73) with \( i' \) determined by the testing function version of (65).

With regard to [1, Eq. (33)], DO loop 53 uses (73) and (74) to store
by columns in T. Nested DO loops 53 and 54 obtain the values of j in (73) and (74) with j' determined by (61) for the submatrices with superscript + in (79) and by (65) for the submatrices with superscript - in (79). Nested DO loops 55 and 56 obtain the values of i for $Y_{nij}^{13+}$ of (73) with i' determined by the testing function version of (48). Nested DO loops 57 and 58 obtain the values of i for $Y_{nij}^{13-}$ of (73) with i' determined by the testing function version of (52). Nested DO loops 59 and 60 obtain the values of i for $-Y_{nij}^{23+} - Y_{nij}^{23-}$ of (79). $Y_{nij}^{23+}$ is given by (73) with i' determined by the testing function version of (56). $Y_{nij}^{23-}$ is given by (73) with i' determined by the testing function version of (59). Nested DO loops 61 and 62 obtain the values of i for $Z_{nij}^{33+} + \left( \frac{1}{n_r} \right) Z_{nij}^{33-}$ of (79). $Z_{nij}^{33+}$ is given by (74) with i' determined by the testing function version of (61). $Z_{nij}^{33-}$ is given by (74) with i' determined by the testing function version of (65).

Now that the moment matrix $T_n$ has been stored in T, we turn to the excitation vector $B_n^0$ of [1, Eq. (101)]. The V's on the right-hand side of [1, Eq. (103)] reside in R. Because DO loop 91 has been executed, storage in R is not according to (70) as it stands but according to (70) without the offset J1. The auxiliary equation involving the W's in
[1, Eq. (103a)] is the testing function version of (48). Likewise, the auxiliary equation in [1, Eq. (103b)] is the testing function version of (56), and the auxiliary equation in [1, Eq. (103c)] is the testing function version of (61).

Based on the considerations in the preceding paragraph, lines 336–378 store $\hat{\theta}_n^i$ of [1, Eq. (101)] in Y. Nested DO loops 63 and 64 store $\hat{\nu}_n^i\theta$ of [1, Eq. (101)] in Y. The sign factor S in line 346 compensate for the minus sign in front of $\nu_n^i\theta$ in (70). The number of elements in the row vector $0$ in [1, Eq. (101)] is IB because this row vector can be traced back to [1, Eq. (39b)]. DO loop 65 stores this row vector in Y. Nested DO loops 76 and 77 store $\hat{\nu}_n^i\theta$ of [1, Eq. (101)] in Y. Line 364 compensates for the minus sign in front of $\nu_n^i\theta$, in (70). Nested DO loops 66 and 67 store $\hat{\nu}_n^i\theta$ of [1, Eq. (101)] in Y. The offset JA in line 370 is mandated by the superscript $\phi$ in $\nu_n^i\phi$, in [1, Eq. (103c)]. Line 376 accounts for the net effect of the minus sign in front of $\nu_n^i\phi$, in [1, Eq. (103c)] and the minus sign in front of $\nu_n^i\phi$, in (70).

Line 379 decomposes $T_n$ into the product of a lower triangular matrix with an upper triangular matrix. Line 380 stores the solution $\hat{X}_n^\phi$ to [1, Eq. (65a)] in XX. The elements [1, Eq. (68a)] of $\hat{X}_n^\phi$ are the I's and V's in [1, Eqs. (87)–(89)]. Lines 384–385 store $\varepsilon_n^i$ in C1. Line 386 stores 4j in C2. If LIA(2) = 0, then it is evident (50a) and (50b) that LIA(1) = 0 so that no values of j are to be taken in (33) and (34). If LIA(2) > 0, line 392 prints (33) and (34). If LIA(4) = 0, then it is evident from (50c) and (50d) that LIA(3) = 0 so that no values of j are to be taken in (35) and (36). If LIA(4) > 0, line 398 prints (35) and (36). Line 403 prints (37) and (38). Line 409 prints (39) and (40).
If $L_2(1) = L_2(2) = 0$, then it is evident from (57a), (57b), (62a), and (62b) that $L_3(1) = L_3(2) = 0$ so that no values of $j$ are to be taken in (41) – (44). If either $L_2(1) > 0$ or $L_2(2) > 0$, then line 417 prints (41)-(42). If $L_2(2) = 0$, then it is evident from (57b), (62a) and (62b) that $L_3(1) = L_3(2) = 0$ so that no values of $j$ are to be taken in (43) and (44). If $L_2(2) > 0$, line 421 puts 2j in $C_l$, lines 422 and 423 put $2e_n$ in $C_2$, and line 424 prints (43) and (44).
LISTING OF THE MAIN PROGRAM

THE SUBPROGRAMS YZA, BLOG, PLANE, DECOMP, SOLVE, AND PRNT ARE NEEDED

003 //PGM JOB (XXXX,XXXX,2.2), *MAUTZ,JOE*, REGION=275K

004 // EXEC WATFIV

005 // GO SYSIN DD *

006 $JOB MAUTZ,TIME=5,PAGES=60

007 COMPLEX C1, C2, C3, C4, C5, C6, XX, YY(2209), ZZ(2209), YM(2209)

008 COMPLEX ZM(2209), R(2209), T(16241), Y(79)

009 DIMENSION XT(10), AT(10), X(48), A(48), THR(3), RA(43), ZA(43), RB(43)

010 DIMENSION ZB(43), L1(4), M1A(4), L1B(4), M1B(4), L2(2), M2A(2), M2B(2)

011 DIMENSION L3(2), N3A(2), M3B(2), IPS(79)

012 COMMON C1, C2, XX, JX

013 READ(1,15) NT, NPHI

014 FORMAT(2I3)

015 WRITE(3,9) NT, NPHI

016 FORMAT* NT NPHI/I(1X,13,1S)

017 READ(1,10) (XT(K), K=1, NT)

018 READ(1,10) (AT(K), K=1, NT)

019 FORMAT(5E14.7)

020 WRITE(3,11) (XT(K), K=1, NT)

021 WRITE(3,12) (AT(K), K=1, NT)

022 FORMAT* XT*/(1X,5E14.7))

023 FORMAT* AT*/(1X,5E14.7))

024 READ(1,10) (X(K), K=1, NPHI)

025 READ(1,10) (A(K), K=1, NPHI)

026 WRITE(3,13) (X(K), K=1, NPHI)

027 WRITE(3,14) (A(K), K=1, NPHI)

028 FORMAT* X*/(1X,5E14.7))

029 FORMAT* A*/(1X,5E14.7))

030 READ(1,16) NA, NB, MA, MB, MC, LA, LB, LC, M1, M2, BK, UR, ER, THR(1)

031 FORMAT(I013/4E14.7)

032 WRITE(3,17) NA, NB, MA, MB, MC, LA, LB, LC, M1, M2, BK, UR, ER, THR(1)

033 FORMAT* NA NB MA MB MC LA LB LC M1 M2*/(1X,1013/7X.*BK*/12X.*UR*,

034 112X.*ER*/(1X,4E14.7))

035 READ(1,18) (RA(I), I=1, NA)

036 READ(1,18) (ZA(I), I=1, NA)

037 FORMAT* NA NB MA MB MC LA LB LC M1 M2*/(1X,1013/7X.*BK*/12X.*UR*,

038 112X.*ER*/(1X,4E14.7))

039 WRITE(3,19) (RA(I), I=1, NA)

040 WRITE(3,20) (ZA(I), I=1, NA)

041 FORMAT* RA*/(1X,10F8.4))

042 FORMAT* ZA*/(1X,10F8.4))

043 READ(1,18) (RB(I), I=1, NB)

044 WRITE(3,21) (RB(I), I=1, NB)

045 FORMAT* RB*/(1X,10F8.4))

046 WRITE(3,22) (ZB(I), I=1, NB)

047 FORMAT* ZB*/(1X,10F8.4))

048 C3=2e

049 C4=4u*(0+1)

050 C5=2u*(0+1)

051 C6=4u

052 UE=SQRT(UR*ER)

053 BB=8K*UE

054 ET=SQRT(UR/ER)

055 ET1=1./ET

056 DO 23 I=1, NA

057 RA(I)=8K*RA(I)

058 ZA(I)=8K*ZA(I)

059 CONTINUE

060 DO 24 I=1, NB

061 CC=2e

062 C4=4u*(0+1)

063 C5=2u*(0+1)

064 C6=4u

065 UE=SQRT(UR*ER)

066 BB=8K*UE

067 ET=SQRT(UR/ER)

068 ET1=1./ET

069 CONTINUE

070 CONTINUE
RB(1)=BB*RB(1)
ZB(1)=BB*ZB(1)
24 CONTINUE
L1A(1)=MAX(0,MA-1)
L1A(2)=NA
L1A(3)=MAX(0,NA-MA-NC-1)
L1A(4)=NA-MA-NC-LA
M1A(1)=0
M1A(2)=NA-2
M1A(3)=NA+NC-1
M1A(4)=NA+MA+NC-3
L1B(1)=MAX(0,MB-1)
L1B(2)=MB
L1B(3)=MAX(0,MB-NC-1)
L1B(4)=MB-NC-LB
M1B(1)=0
M1B(2)=MB-2
M1B(3)=MB+NC-1
M1B(4)=MB+NB+NC-3
L2(1)=NC
IF(MA*EQ.0) L2(1)=NC-1
IF(L1A(4)*EQ.0) L2(1)=L2(1)-1
L2(2)=NC-1-LC
M2A(1)=MAX(0,NA-1)
M2A(2)=NA-2+MA
M2B(1)=MAX(0,NA-1)
M2B(2)=NB-2+MB
L3(1)=MAX(0,NC-2)
L3(2)=L2(2)
M3A(1)=MA
M3A(2)=NA-2+MA
M3B(1)=NB
M3B(2)=NB-2+MB
J=2*NC-3
I=LI(1)+LI(2)+L1A(3)+L1A(4)
IB=L1B(1)+L1B(2)+L1B(3)+L1B(4)
N=I+IB+L2(1)+L2(2)+L3(1)+L3(2)
CALL YZA(M1+M2,NA+NB+MAT,0,RA,ZA,XT,AT,YP,ZP)
CALL YZA(M1+M2,NA+NB+MAT,0,RA,ZA,XT,AT,YN,ZM)
CALL PLANE(M1+N2,1,NA+MT,RA,ZA,XT,AT,THR,R)
DO 82 J=1,NB
RB(J)=RB(J)/UE
82 CONTINUE
JA2=JA*JA
JB2=JB*JB
M=M1+1
N=M2+1
DO 86 N=N3,N4
IF(N*EQ.N3) GO TO 88
J=J+1
J=J+1
J=J+1
ZP(J)=ZP(J+1)
89 CONTINUE
J=J+J
J=J+J
DO 90 J=1,JB
YN(J)=YN(J+J)
ZM(J)=ZM(J+J)
90 CONTINUE
J3=2*JA
J1=J2+J3
DO 91 J=1,J3
R(J)=R(J+J1)
91 CONTINUE
88 JT=0
DO 25 J=1,4
J2=1A(JJ)
IF(J2.EQ.0) GO TO 25
J1=1A(JJ)+JA
DO 26 J=1,J2
II=1,I+4
I3=1A(I)
IF(I3.EQ.0) GO TO 27
J3=J1+1A(I)
I1=I3+I
I2=I3+I3
DO 28 I=11,12
JT=JT+1
T(JT)=ZP(I)
28 CONTINUE
27 CONTINUE
IF(I8.EQ.0) GO TO 79
DO 29 I=I+18
JT=JT+1
T(JT)=0.
29 CONTINUE
79 DO 30 I=1,2
I3=2(I)
IF(I3.EQ.0) GO TO 30
J3=J1+2A(I)
I1=I3+1
I2=I3+I3
DO 31 I=11,12
JT=JT+1
T(JT)=ZP(I)
31 CONTINUE
30 CONTINUE
DO 32 I=1,2
I3=3(I)
IF(I3.EQ.0) GO TO 32
J3=J1+3A(I)
I1=I3+1
I2=I3+I3
DO 33 I=11,12
JT=JT+1
T(JT)=YP(I)
33 CONTINUE
32 CONTINUE
J1=J1+JA
32 CONTINUE
36 CONTINUE
DO 34 JJ=1,4
J2=1B(JJ)
IF(J2.EQ.0) GO TO 34
J1=1B(JJ)+JB
DO 35 J=1,J2
IF(I9.EQ.0) GO TO 80
DO 36 I=1,1A
181 JT=JT+1
182 T(JT)=0.
183 36 CONTINUE
184 80 DO 37 II=1,4
185 13=L1B(II)
186 IF(I3.EQ.0) GO TO 37
187 J3=J1+M1B(II)
188 11=J3+1
189 12=J3+1
190 DO 38 I=II+1,II+12
191 JT=JT+1
192 T(JT)=ET*ZM(I)
193 38 CONTINUE
194 37 CONTINUE
195 DO 39 II=1,2
196 13=L2(II)
197 IF(I3.EQ.0) GO TO 39
198 J3=J1+M2B(II)
199 11=J3+1
200 12=J3+1
201 DO 40 I=II+1,II+2
202 JT=JT+1
203 T(JT)=ET*ZM(I)
204 40 CONTINUE
205 39 CONTINUE
206 DO 41 II=1,2
207 13=L3(II)
208 IF(I3.EQ.0) GO TO 41
209 J3=J1+M3B(II)
210 11=J3+1
211 12=J3+1
212 DO 42 I=II+1,II+2
213 JT=JT+1
214 T(JT)=YM(II)
215 42 CONTINUE
216 41 CONTINUE
217 J1=J1+J8
218 35 CONTINUE
219 34 CONTINUE
220 DO 43 JJ=1,2
221 J2=L2(JJ)
222 IF(J2.EQ.0) GO TO 43
223 J1=M2A(JJ)+J1
224 J1=M2B(JJ)+J8
225 DO 44 J=J2,II
226 DO 45 I=1,II+4
227 13=L1A(II)
228 IF(I3.EQ.0) GO TO 45
229 J3=J1+M1A(II)
230 11=J3+1
231 12=J3+1
232 DO 46 I=II+1,II+12
233 JT=JT+1
234 T(JT)=ZP(I)
235 46 CONTINUE
236 45 CONTINUE
237 DO 47 II=1,4
238 13=L1B(II)
239 IF(I3.EQ.0) GO TO 47
240 J3=J1B+M1B(II)
II=J3+1
J2=J3+13
DO 48 I=I1+12
JT=JT+1
T(JT)=ET*ZN(I)
48 CONTINUE
47 CONTINUE
DO 49 I=I1+2
I3=L2(I1)
IF(I3.EQ.0) GO TO 49
J3=J1A+M2A(I1)
I1=J3+1
I2=J3+13
J4=J1B+M2B(I1)
DO 50 I=I1+12
J4=J4+1
JT=JT+1
T(JT)=ZP(I)+ET*ZN(J4)
50 CONTINUE
49 CONTINUE
DO 51 I=I1+2
I3=L3(I1)
IF(I3.EQ.0) GO TO 51
J3=J1A+M3A(I1)
I1=J3+1
I2=J3+13
J4=J1B+M3B(I1)
DO 52 I=I1+12
J4=J4+1
JT=JT+1
52 CONTINUE
51 CONTINUE
J1A=J1A+JA
J1B=J1B+JB
44 CONTINUE
43 CONTINUE
DO 53 J=1+2
J2=L3(JJ)
IF(J2.EQ.0) GO TO 53
J1A=M3A(JJ)+JA
J1B=M3B(JJ)+JB
DO 54 J=1+J2
DO 55 I=I1+4
I3=L4(I1)
IF(I3.EQ.0) GO TO 55
J3=J1A+M4A(I1)
I1=J3+1
I2=J3+13
DO 56 I=I1+12
JT=JT+1
56 CONTINUE
55 CONTINUE
DO 57 I=I1+4
I3=L1B(I1)
IF(I3.EQ.0) GO TO 57
J3=J1B+M1B(I1)
I1=J3+1
300 I2=J3+13
301  DO 58 I=11,12
302      JTE=JT+1
303      T(JT)=YM(I)
304  58 CONTINUE
305  57 CONTINUE
306  DO 59 I=1,2
307      I3=L2(I)
308  IF(I3.EQ.0) GO TO 59
309      J3=J1A+M2A(I1)
310      I1=J3+1
311      I2=J3+13
312      J4=J1B+M2B(I1)
313      DO 60 I=11,12
314      J4=J4+1
315      JT=JT+1
316      T(JT)=YP(I)+YM(J4)
317  60 CONTINUE
318  59 CONTINUE
319  DO 61 I=1,2
320      I3=L3(I)
321  IF(I3.EQ.0) GO TO 61
322      J3=J1A+M3A(I1)
323      I1=J3+1
324      I2=J3+13
325      J4=J1B+M3B(I1)
326      DO 62 I=11,12
327      J4=J4+1
328      JT=JT+1
329      T(JT)=ZP(I1)+ET1+ZM(J4)
330  62 CONTINUE
331  61 CONTINUE
332      J1A=J1A+JA
333      J1B=J1B+JB
334  54 CONTINUE
335  53 CONTINUE
336      JY=0
337      S=1
338  DO 63 I=1,4
339      I3=L1A(I1)
340  IF(I3.EQ.0) GO TO 63
341      J3=M1A(I1)
342      I1=J3+1
343      I2=J3+13
344  DO 64 I=11,12
345      JY=JY+1
346      Y(JY)=S*RI(I1)
347  64 CONTINUE
348  63 CONTINUE
349      S=S+S
350  IF(I8.EQ.0) GO TO 61
351  DO 65 I=1,18
352      JY=JY+1
353      Y(JY)=0
354  65 CONTINUE
355  DO 76 I=1,2
356      I3=L2(I)
357  IF(I3.EQ.0) GO TO 76
358      J3=M2A(I1)
359      I1=J3+1
360      I2=J3+13
DO 77 I=11,12
  JY=JY+1
  Y(JY)=R(I)
  IF(I1.EQ.2) Y(JY)=-Y(JY)
  77 CONTINUE
  76 CONTINUE
DO 66 II=1,2
I3=L3(II)
  IF(I3.EQ.0) GO TO 66
  J3=M3A(II)+JA
  I1=J3+1
  12=J3+13
DO 67 II=11,12
  JY=JY+1
  Y(JY)=R(I)
  IF(I1.EQ.2) Y(JY)=-Y(JY)
  67 CONTINUE
  66 CONTINUE
CALL DECOMP(NeIPS,T)
CALL SOLVE(NeIPS,T,XX)
  MS=MI-1
  WRITE(3,96) MS
  96 FORMAT('0TH MODE ELECTRIC AND MAGNETIC CURRENTS')
  C1=C3
  IF(MS.EQ.0) C1=1.
  C2=C4
  JK=0
J2=L1A(2)
  IF(J2.EQ.0) GO TO 84
  WRITE(3,68)
  68 FORMAT('ELECTRIC CURRENT ON FIRST PART OF OUTSIDE CONDUCTOR')
  CALL PRNT(LIA(1),J2,J1,1,RA)
  84 J2=L1A(4)
  IF(J2.EQ.0) GO TO 74
  WRITE(3,69)
  69 FORMAT('ELECTRIC CURRENT ON SECOND PART OF OUTSIDE CONDUCTOR')
  J3=MA+MC
  CALL PRNT(LIA(3),J2,J3,J3,RA)
  74 J2=L1B(2)
  IF(J2.EQ.0) GO TO 85
  WRITE(3,70)
  70 FORMAT('ELECTRIC CURRENT ON FIRST PART OF INSIDE CONDUCTOR')
  CALL PRNT(LIB(1),J2,J3,RA)
  85 J2=L1B(4)
  IF(J2.EQ.0) GO TO 75
  WRITE(3,71)
  71 FORMAT('ELECTRIC CURRENT ON SECOND PART OF INSIDE CONDUCTOR')
  J3=M8+MC
  CALL PRNT(LIB(3),J2,J3,J3,RA)
  75 J1=L2(1)
  J2=L2(2)
  IF((J1+J2).EQ.0) GO TO 86
  WRITE(3,72)
  72 FORMAT('ELECTRIC CURRENT IN APERTURE')
  J3=M2A(1)+I
  J4=MA+1
  CALL PRNT(J1,J2,J3,J4,RA)
  IF(J2.EQ.0) GO TO 86
  WRITE(3,73)
  73 FORMAT('MAGNETIC CURRENT IN APERTURE')
421      CI=C5
422      C2=C6
423      IF(MS.EQ.0) C2=2,
424      CALL PNR(L3(I),J2,J4,J4,RA)
425  86 CONTINUE
426      STOP
427      END

$DATA
 2 20
-.5773503E+00 0  .5773503E+00
 0.1000000E+01 0.1000000E+01
-0.9931286E+00-0.9639719E+00-0.9122344E+00-0.391170E+00-0.763319E+00
-0.6360537E+00-0.5108670E+00-0.3737061E+00-0.2277859E+00-0.7652652E-01
 0.7652652E-01 0.2277859E+00 0.3737061E+00 0.5108670E+00 0.6360537E+00
 0.7463317E+00 0.8391170E+00 0.9122344E+00 0.9639719E+00 0.9931286E+00
 0.1764104E-01 0.4060143E-01 0.6267205E-01 0.8327674E-01 0.1019301E+00
 0.1181945E+00 0.1316886E+00 0.1420961E+00 0.1491730E+00 0.1527534E+00
 0.1527534E+00 0.1491730E+00 0.1420961E+00 0.1316886E+00 0.1181945E+00
 0.1019301E+00 0.8327674E-01 0.6267205E-01 0.4060143E-01 0.1764104E-01

15 13 5 4 5 0 0 0 1 1
0.2500000E+01 0.1000000E+01 0.2000000E+01 0.0000000E+00
0.0000 0.2588 0.5000 0.7071 0.8660 0.9659 0.9914 1.0000
0.8660 0.7071 0.5000 0.2588 0.0000
-1.0000-0.9659 0.9914 0.9659 0.9914 1.0000
-1.0000 0.9659 0.9914 0.9659 0.9914 1.0000
0.0000 0.2415 0.4830 0.7244 0.9659 0.9914 1.0000
0.2415 0.4830 0.7244 0.9659 0.9914 1.0000
-0.2588 -0.2588 -0.2588 -0.2588 -0.1305 0.0000
0.2588 0.2588 0.2588

$STOP
/*
//
PRINTED OUTPUT
NT NPHI
2 20
XT
-0.5773503E+00 0.5773503E+00
AT
0.1000000E+01 0.1000000E+01
X
-0.9931286E+00 0.9639719E+00 0.9122344E+00 0.391170E+00 0.763319E+00
-0.6360537E+00 0.5108670E+00 0.3737061E+00 0.2277859E+00 0.7652652E-01
 0.7652652E-01 0.2277859E+00 0.3737061E+00 0.5108670E+00 0.6360537E+00
 0.7463317E-00 0.8391170E+00 0.9122344E+00 0.9639719E+00 0.9931286E+00
 0.1764104E-01 0.4060143E-01 0.6267205E-01 0.8327674E-01 0.1019301E+00
 0.1181945E+00 0.1316886E+00 0.1420961E+00 0.1491730E+00 0.1527534E+00
 0.1527534E+00 0.1491730E+00 0.1420961E+00 0.1316886E+00 0.1181945E+00
 0.1019301E+00 0.8327674E-01 0.6267205E-01 0.4060143E-01 0.1764104E-01

NA NB MA MB MC LA LB LC MI M2
15 13 5 4 5 0 0 0 1 1
BK UR ER TRH(1)
0.2500000E+01 0.1000000E+01 0.2000000E+01 0.0000000E+00
RA
0.0000 0.2588 0.5000 0.7071 0.8660 0.9659 0.9914 1.0000
0.8660 0.7071 0.5000 0.2588 0.0000
ZA
-1.0000-0.9659 -0.8660 -0.7071 -0.5000 -0.2588 -0.1305 0.0000
0.5000 0.7071 0.8660 0.9659 0.9914 1.0000
0.2588 0.2588 0.2588

RB
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<th>REAL</th>
<th>IMAG</th>
<th>MAG</th>
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<tr>
<td>0.403</td>
<td>0.2115</td>
<td>0.0000</td>
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<tr>
<td>0.4830</td>
<td>0.7244</td>
<td>0.9659</td>
</tr>
<tr>
<td>0.9914</td>
<td>0.7244</td>
<td>0.9659</td>
</tr>
<tr>
<td>1.0000</td>
<td>0.9914</td>
<td>0.9914</td>
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Electric Current on First Part of Outside Conductor

Real | Imag | Mag |
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<tr>
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<tbody>
<tr>
<td>0.4126E+00</td>
<td>-0.4741E+00</td>
<td>0.1077E+01</td>
</tr>
<tr>
<td>0.4098E+00</td>
<td>0.3222E-01</td>
<td>0.5940E+00</td>
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<td>0.2590E+00</td>
<td>0.7627E+00</td>
<td>0.8055E+00</td>
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<td>-0.4285E+00</td>
<td>0.1183E+01</td>
<td>0.1261E+01</td>
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Electric Current on Second Part of Outside Conductor

Real | Imag | Mag |
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<tr>
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<tbody>
<tr>
<td>-0.4426E+00</td>
<td>-0.1791E+01</td>
<td>0.1844E+01</td>
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<tr>
<td>0.4008E+00</td>
<td>0.1904E+01</td>
<td>0.1957E+01</td>
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<tr>
<td>0.1105E+01</td>
<td>-0.1607E+01</td>
<td>0.1950E+01</td>
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<tr>
<td>0.1478E+01</td>
<td>0.1761E+01</td>
<td>0.1943E+01</td>
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Electric Current on First Part of Inside Conductor

Real | Imag | Mag |
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<tr>
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<tr>
<td>0.1767E+01</td>
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<td>0.2635E+00</td>
<td>0.5877E-01</td>
<td>0.2673E+00</td>
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<td>0.1498E+00</td>
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Electric Current on Second Part of Inside Conductor

Real | Imag | Mag |
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<tr>
<td>0.1595E+00</td>
<td>0.3328E-01</td>
<td>0.1558E+01</td>
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<tr>
<td>0.1118E+01</td>
<td>-0.6796E+01</td>
<td>0.1118E+01</td>
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<tr>
<td>0.7626E+00</td>
<td>0.7621E+00</td>
<td>0.7103E+00</td>
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<tr>
<td>0.1767E+00</td>
<td>0.3173E+00</td>
<td>0.3603E+00</td>
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Electric Mode Electric and Magnetic Currents
<table>
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<tr>
<th>ELECTRIC CURRENT IN APERTURE</th>
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<tr>
<td>REAL JT</td>
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<tr>
<td>-0.1129E+01</td>
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<tr>
<td>-0.1584E+01</td>
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</table>

| REAL JT  | IMAG JT | MAG JT       |
| -0.1102E+01 | 0.6770E+00 | 0.1294E+01 |
| -0.1261E+01 | 0.3286E+00 | 0.1303E+01 |
| -0.1339E+01 | -0.4550E-01 | 0.1339E+01 |
| -0.1305E+01 | -0.4478E+00 | 0.1380E+01 |
| -0.1196E+01 | -0.9023E+00 | 0.1498E+01 |

| REAL JP  | IMAG JP | MAG JP       |
| -0.1580E+00 | 0.2558E+00 | 0.3092E+00 |
| -0.2728E+00 | 0.1534E+00 | 0.3130E+00 |
| -0.3750E+00 | 0.5005E-01 | 0.3783E+00 |
| -0.5571E+00 | -0.5482E-01 | 0.5598E+00 |

<table>
<thead>
<tr>
<th>MAGNETIC CURRENT IN APERTURE</th>
</tr>
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<tbody>
<tr>
<td>REAL JT</td>
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<tr>
<td>0.1429E+00</td>
</tr>
<tr>
<td>0.1589E+00</td>
</tr>
<tr>
<td>0.1049E+00</td>
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</tbody>
</table>

| REAL JP  | IMAG JP | MAG JP       |
| 0.9528E+00 | 0.1541E+00 | 0.6708E+00 |
| 0.1559E+00 | 0.3855E+00 | 0.4158E+00 |
| -0.1000E+00 | 0.4523E+00 | 0.4633E+00 |
| -0.8395E+00 | 0.4811E+00 | 0.9675E+00 |
REFERENCES


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