SIGNAL PROCESSING OF SHOCK-WAVE OVERPRESSURE RECORDS (U)

by

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ABSTRACT

Successful methods for numerically processing overpressure-time measurements of shock and blast waves and sonic booms are presented and illustrated with sample results. These methods include: a) the elimination of high-frequency random and other noise from the measurement by employing a fast-Fourier-transform technique and judiciously altering the high-frequency part of the Fourier spectrum; b) the removal of low-frequency periodic fluctuations including baseline shift and drift from the record by using specially developed techniques described in the report; c) the extraction of information from the processed shock-wave record including shock-wave arrival time, positive-phase duration and impulse, peak overpressure, rise time, and overpressure decay rate.
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1.0 INTRODUCTION

Good quality field measurements of, for example, the air blast wave from a gun muzzle or a chemical or nuclear explosion, the shock wave from a bursting balloon or high-pressure vessel, and the sonic boom from a supersonic projectile or aircraft are normally difficult to obtain, because equipment recording conditions in the field are generally nonideal. A field overpressure record may have a baseline shift and also drift. Furthermore, the record may contain random and other noise from instrumentation, low-frequency periodic noise from power lines, extraneous spikes from known or unknown origins, and additional noise from other sources such as wind turbulence and mechanical vibration of the gage mount. Even when the most modern instrumentation is used in a field trial, the past problem of some records
being of poor quality (i.e., noisy) is still with us, although this problem is less severe today. Past measurements of shock and blast waves that exhibit various types of noise including baseline shift and drift can be found, for example, in Refs. 1 to 6.

A poor quality field measurement cannot always be discarded in favour of obtaining a new and better one, because a field trial cannot always be repeated easily or economically. Consequently, each less-than-ideal measurement has to be processed carefully to eliminate the interference noise or its effects before accurate information can be extracted from the record. In the case of an overpressure-versus-time record of a shock or blast wave, this information could include the shock-wave arrival time, positive-phase duration and impulse, peak overpressure, rise time, and overpressure decay rate.

Ethridge's method (Ref. 2) of reading and smoothing an overpressure-time record of a blast wave is a combination of first plotting the measured signature on semilogarithmic graph paper ($\ln(\Delta p)$ versus $t$ and $\Delta p$ versus $\ln(t)$) and then fitting specially selected exponential and exponential-like (Friedlander) functions to different but overlapping segments of the profile. Similarly, Harvey and Fisher's method (Ref. 3) of smoothing a blast-wave measurement consists of fitting a polynomial equation or other specially selected function (modified Friedlander expression) to the entire signature. For the case of a blast-wave record containing superposed random noise and extraneous spikes, both of these approaches can yield fairly good estimates of the peak overpressure, decay rate, and positive-phase duration. Furthermore, these methods give a smooth analytical expression that can be used easily for other purposes, such as in a calculation of the positive-phase impulse.

The success of Ethridge's and Harvey and Fisher's methods depends to a large extent on choosing an appropriate function to represent the measured overpressure signature. In the case of a blast wave from a conventional chemical or nuclear explosion, an exponential-like function is known from theory and experiment to be a good choice. For the more general case of shock and blast waves and sonic booms having nonexponential wave forms, the choice of a suitable function to yield good results is not necessarily
straightforward. Furthermore, such curve-fitting methods are normally not very successful when the overpressure record contains baseline drift or low-frequency periodic noise.

A method of processing, evaluating, and extracting air-blast information from an overpressure record, which does not utilize curve-fitting techniques, has been developed by Vortman and Long (Ref. 5). In their work, baseline shift is successfully eliminated by subtracting it from the record, after the shift or offset value has been determined from a consideration of both the overpressure record and the time integral of this record. Similarly, linear baseline drift is eliminated by subtraction, after the rate of change of the drift has been determined. High-frequency noise in a record having an unfavourable signal-to-noise ratio is removed by using a specially developed low-pass digital filter. A special peak-averaging technique is employed which effectively removes high-frequency fluctuations without significantly increasing the rise time of the steep-fronted shock. Low-frequency periodic noise of constant amplitude is eliminated from the record by subtracting a suitable periodic signal having a predetermined amplitude and frequency. Blast-wave information was then retrieved from the modified or corrected overpressure signature. Note that the record processing and information retrieval are done automatically with a digital computer.

Wells (Ref. 7) recently investigated the usefulness of the Fourier transform in processing digital data to remove unwanted frequency content or noise from a measurement. In his work the fast Fourier transform is used first to obtain the Fourier amplitude versus frequency spectrum, from which unwanted frequency content can be removed easily. The inverse fast Fourier transform is then employed to obtain the modified record. It is shown that a signal can be readily isolated from both high-frequency noise and low-frequency fluctuations including baseline shift and drift, provided that no significant signal frequencies are eliminated with the noise. In other words, the success of this method depends to a large extent on how well the dominant signal frequencies do not overlap the noise frequencies.

The present work is concerned primarily with the application of signal-processing methods to remove baseline shift and drift, low-frequency periodic noise, and high-frequency random and other noise from an overpressure-
time record of shock and blast waves and sonic booms. Ideas from previous work are incorporated freely and extended when necessary. In Chapter 2, a method which successfully removes high-frequency noise from a measurement is presented and illustrated with results. This method, which makes use of the fast Fourier transform and its inverse, goes beyond the past technique of removing only a band of or all high-frequency content from the Fourier spectrum. In the present work the Fourier amplitude or frequency content is judiciously altered so that the high-frequency content associated with only the noise is removed, and high-frequency content associated with the overpressure signature is, therefore, retained. Then, in Chapter 3, two methods for removing baseline shift and drift and low-frequency periodic noise from a record are presented and illustrated with results. The first and more general method involves a specialized procedure of determining the baseline shift and the amplitude, frequency, and shape of the periodic noise from the baseline signal preceding the shock-wave signature. The second and more limited method involves fitting a special function consisting of a constant (baseline shift), a linear time term (baseline drift), and a harmonic term of unknown amplitude and phase but specified frequency (periodic noise) to the baseline signal preceding the shock-wave signature. Then, in both methods, the low-frequency noise can be subtracted out of the overpressure record.

The present work is concerned also with algorithms for retrieving information from the modified overpressure measurement. In Chapter 4, methods are described briefly for obtaining the shock-wave arrival time, positive-phase duration and impulse, peak overpressure, rise time, and overpressure decay rate.

The signal processing work reported herein stems mainly from having to overcome problems encountered in the computer processing of numerous sonic-boom measurements of supersonic projectiles fired from military guns (106-mm M344A1, 3-inch/50 MK33, 3-inch/70 MK 34, 5-inch/54 MK41). These experimental results will be published separately as DRES reports. Because of nonideal recording conditions encountered during the field trials in which overpressure signatures were measured at different distances from the supersonic projectile, some of the overpressure records contained various types
of noise. Almost all of the records had some degree of baseline shift, but only a few had noticeable baseline drift. Although very few of the records contained significant high-frequency random electronic noise with an unfavourable signal-to-noise ratio, a few of the records contained significant high-frequency mechanical noise from gage-mount vibration or ringing. A few other records had marked low-frequency periodic noise of about 60 Hz, owing to electrical interference from 110-volt power lines. The methods presented in Chapters 2 to 4, to remove such undesirable noise and extract accurate information from the processed records, are described and illustrated with results that pertain directly to sonic-boom signatures from supersonic projectiles. It should be remembered, however, that these methods are much more general and can be used to remove similar noise and extract similar information from most shock- and blast-wave measurements.

2.0 HIGH-FREQUENCY FILTERING

The present procedure of removing high-frequency noise from an overpressure-time measurement or other type of analog record consists, essentially, of the following four sequential steps: the conversion of the measurement or analog record into a digital record; the use of the Fourier transform to obtain the Fourier amplitude versus frequency spectrum; the judicious alteration of the Fourier spectrum to remove undesirable noise; the use of the inverse Fourier transform to obtain the modified or corrected digital record (free of undesirable noise). These four basic steps are now described in more detail.

The equipment and computer programming techniques for the analog-to-digital conversion process are not described in this report, because they are specific to the type of tape recorder and computer available to the user. It is worth mentioning, however, that the sampling rate is normally chosen to be sufficiently high to ensure that significant high-frequency content of interest is retained in the digital record. Other considerations such as the maximum computer storage capability or computer execution cost may sometimes limit the maximum sampling rate.

The discrete Fourier transform can be obtained efficiently from the digital record by using a relatively new algorithm known as the fast
Fourier transform (Refs. 8 to 10), which stems from the original work of Cooley and Tukey (Ref. 11). A brief description of this algorithm and the computer program subroutine of Ref. 12, including the FORTRAN subroutine listing, are given for interest and reference in Appendix A.

Undesirable noise in the discrete Fourier amplitude versus frequency spectrum can now be identified and removed. For the simplest case when the noise contribution to the Fourier spectrum is distinct from the signal contribution, the identification and removal of this noise content are achieved readily. For example, consider an idealized situation for which the signal contribution to the Fourier spectrum occurs at low frequencies and the noise contribution occurs distinctly at higher frequencies. The high-frequency noise content can then be removed easily from the Fourier spectrum by simply setting to zero all of the noise amplitudes at the higher frequencies. In the reverse situation, low-frequency noise content can be deleted similarly from the Fourier spectrum when the signal contribution occurs predominantly at higher frequencies. Furthermore, noise content occupying a distinct frequency band or bands can also be deleted easily.

For the more complex case when the noise and signal contributions to the Fourier spectrum overlap significantly, identification and removal of the noise may then be quite difficult or even impossible. If all of the Fourier amplitudes in a frequency band that contains noise content were simply set equal to zero, then not only the noise contribution but also the signal contribution in this band would be deleted from the Fourier spectrum. In order to circumvent this difficulty, the Fourier amplitudes should be judiciously altered to remove only the noise content and, therefore, retain most of the signal content. Such a Fourier spectrum alteration can normally be achieved only if the noise contribution can be identified accurately. Hence, the basic shape of the Fourier spectrum of a signal without significant noise must generally be known beforehand.

Noise quite often causes a distinct peak or peaks in an otherwise fairly smooth Fourier spectrum of a signal. If the basic spectrum shape of the noise-free signal is known in advance, these noise peaks and their bandwidths are then recognized easily. Each noise peak can then be eliminated
from the frequency domain and the resulting gap bridged by a smooth polynomial curve which is joined to the smooth part of the spectrum on each side of the peak. In this manner the dominant noise contribution is effectively removed, while the signal contribution is left intact.

The last of the four steps in the procedure is to take the inverse Fourier transform of the altered Fourier spectrum to obtain the modified or corrected overpressure record. This can be done easily by using the same efficient algorithm for the fast Fourier transform, as is described in Appendix A.

The following results given in this chapter illustrate the usefulness of the foregoing procedure for removing high-frequency noise from an overpressure measurement. Firstly, consider an ideal sonic-boom signature having two equal amplitude front and rear shocks with instantaneous rise times and separated by a linear decay in overpressure. The Fourier spectrum as calculated by the fast Fourier transform is shown in Fig. 1, with the N-shaped overpressure signature appearing as an insert. For this and all other results presented in this chapter, the analog-to-digital conversion rate is 96,000 points per second and the Fourier amplitudes are calculated at 512 equally spaced frequencies between zero and about 24 kHz.

The results shown in Fig. 1 for an ideal N-wave of similar amplitude and duration to N-waves measured in a field trial for a supersonic projectile illustrate some of the basic features of the Fourier spectrum. At certain frequencies the spectrum is peaked, whereas at others the amplitude drops to nearly zero. The frequency at which the first peak occurs is approximately \( T^{-1} \) Hz or 500 Hz, as might be expected (Ref. 13), where \( T \) is the N-wave duration. Most of the energy spectral density of the N-wave is contained in the lower frequencies, less than about 10 \( T^{-1} \) Hz or 5000 Hz. It should be noted that analytical results are available in Refs. 13, 14, and 15 for the Fourier spectrum or energy spectral density of such a simple shaped N-wave.

The Fourier spectrum for a measured N-wave from a supersonic projectile (3-inch/50 MK33) is presented in Fig. 2. The original overpressure signature (insert) differs only slightly from the ideal N-shape and has negligible superposed high-frequency noise. Consequently, the
Fourier spectrum for this measured signature differs only to a small extent from that for an ideal N-wave of similar amplitude and duration.

In Figure 3 the Fourier spectrum of a measured N-shaped overpressure signature (insert) with significant high-frequency noise resulting from gage-mount vibration is displayed. By comparing this spectrum to the previous two cases for the ideal and measured N-waves (Figs. 1 and 2), the noise contribution to the spectrum can be identified fairly easily. Additional energy now appears in the frequency bands of 9 to 11 kHz and 17 to 19 kHz. These two bands correspond directly to the first two vibration modes of the gage mount.

Now that the distortion in the Fourier spectrum from the high-frequency noise has been identified, it can be removed in a number of different ways. The simplest method would be to set the Fourier amplitude to zero throughout each of the affected frequency bands. However, in this case some energy not associated with the noise would also be deleted. A more sophisticated approach would consist of using a polynomial equation to bridge each frequency band, being matched by a linear least squares method to the spectrum on each side of the band. In the present work it was found that a straight line was quite adequate for this purpose.

The Fourier spectrum appearing in Fig. 3 has been reproduced in Fig. 4, with the gage-mount ringing removed from only the 9 to 11 kHz band. The noise in the 17 to 19 kHz band is not very significant. After applying the inverse fast Fourier transform to this altered spectrum, one obtains the modified overpressure signature shown as an insert. Much of the high-frequency ringing no longer appears in this modified signature. This is more clearly illustrated in Fig. 5, where a direct comparison is made between the original measurement containing gage-mount ringing and the modified record with the first mode of gage-mount ringing removed.

Results such as those given in Fig. 5 aptly demonstrate that certain high-frequency noise can be successfully reduced in overpressure time measurements by the present method. The success of this method depends to a large extent, however, on how accurately the noise can be identified and removed from the Fourier spectrum. This can be more art than science. Noise that is concentrated in a narrow frequency band or
bands can normally be deleted most easily.

3.0 LOW-FREQUENCY FILTERING

Overpressure measurements from field trials frequently contain low-frequency noise including baseline shift and drift. Baseline shift and drift generally stem from a fixed or changing voltage bias in the recording equipment and gage response to changes in, for example, ambient temperature. Periodic noise can be the result of interference from 110-volt power lines or other electronic wiring and equipment. In the present work the periodic noise had a fixed frequency of about 60 Hz and was sinusoidal in shape. However, periodic noise having a nonsinusoidal shape is also very often encountered in practice.

Two methods were devised to remove low-frequency noise from an overpressure measurement. The first method is quite general and has the capability of numerically filtering out of a digitized record both baseline shift and low-frequency periodic noise having almost any unknown shape and frequency. The second method is simpler, requires significantly less computer storage and execution time, but is more limited in capability. Both baseline shift and drift and also low-frequency sinusoidal shaped noise of specified frequency (e.g., 60 Hz) can be numerically filtered out of an overpressure record. These two methods, labelled A and B, are described and illustrated with graphical results in the following two sections.

3.1 Method A

Consider an overpressure record from a field trial, which has an initial length of baseline signal preceding the shock-wave signature (e.g., see Fig. 8a). Suppose that this baseline signal ahead of the shock-wave signature contains both a baseline shift and low-frequency noise which is nonsinusoidal but periodic. This shift and low-frequency noise is assumed, of course, to continue through the shock-wave signature to the end of the record. Method A consists, essentially, of using the first part of the overpressure measurement preceding the shock-wave signature to establish one cycle of a reference baseline signal that accurately represents
the baseline shift and low-frequency periodic noise. The resulting reference signal can then be subtracted from the overpressure measurement, in order to remove the baseline shift and periodic noise, leaving a modified overpressure record containing a much more definitive shock-wave signature with only superposed random noise (e.g., see Fig. 8b).

The first step in Method A is to determine the number of points in the discrete overpressure record or baseline signal preceding the shock-wave signature. This number which is denoted by the symbol \( N_b \) can be obtained fairly easily by conducting a computer search for the arrival of the shock wave in the overpressure record.

The second step is to establish an initial reference signal to represent the shift and periodic noise in the baseline signal. The periodic noise frequency is first guessed or estimated, making certain that the chosen value is overestimated slightly (e.g., 65 Hz if the noise frequency is about 60 Hz). For an overpressure record stored in discrete form (e.g., 96,000 points per second), the total number of points \( N_0 \) in one cycle of this initial reference signal can then be obtained (e.g., 1476 points for a frequency guess of 65 Hz). The successive points from the start of the overpressure record \( (N = 1) \) to the \( N_0 \)-th point \( (N = N_0) \), therefore, establishes the first cycle of the initial reference signal. Successive duplication of these values then establishes subsequent cycles of the initial reference signal, having a total number of points equal to \( N_b \) or greater.

The third step is to compare the first \( N_b \) values of the reference signal to the first \( N_b \) values of the overpressure record. A convenient indicator of how well these two signals agree is the well-known standard deviation:

\[
\sigma_0 = \sum_{N=1}^{N_b} \frac{D_N^2}{N_b},
\]

(1)

If each \( N \)-th difference \( D_N \) between the reference signal and overpressure record was zero, then the standard deviation would also be equal to zero, and the correlation between the two signals would be perfect. Since the initial frequency guess was slightly high, however, the standard deviation
in this first case would be nonzero.

The fourth step is to pick a slightly lower frequency than the first estimate, and then repeat the comparison to obtain a new standard deviation. This is achieved easily by using $N_0 + 1$ or $N_1$ successive values of the overpressure record as one cycle of a new reference signal. For this case with $N_1$ points per cycle, let the new value of the standard deviation be denoted by $o_1$.

The fifth step is to continue the procedure of slowly increasing the number of points in one cycle of the reference signal (or decreasing the frequency) and monitoring the new values of the standard deviation. As the frequency of the reference signal decreases and approaches the noise frequency, the standard deviation diminishes slowly at first, then quite rapidly, and finally levels out at small nonzero values (see Fig. 6). When the frequency of the reference signal is decreased further, below the noise frequency, the standard deviation begins eventually to increase rapidly from the small nonzero values, and then increases more slowly. The fairly flat minimum in the resulting standard-deviation distribution is nonzero (Fig. 6), because perfect matching of the reference signal and overpressure record is unachievable, owing to random noise in the overpressure record. Furthermore, such random noise also causes small erratic variations in the discrete standard-deviation values in the vicinity of the minimum. Although the exact minimum is difficult to locate precisely, a good estimation can be made. For example, a parabolic curve can be fitted by a linear least-squares method to the standard-deviation values in the vicinity of the minimum (Fig. 6), and the location of the minimum can then be obtained from this parabola. This location yields directly, of course, a good estimate of the frequency of the periodic noise.

The sixth step is to refine the previous frequency estimate of the periodic noise. A straight line is first fitted by a linear least-squares method to a few values at the beginning of the reference signal. This straight line is then used to yield an average amplitude at the start of the reference signal. The effects of random noise on this average amplitude are, therefore, minimized. Similarly, a straight line is fitted to a few values at the end of the first cycle of the reference signal, and an
average amplitude is also obtained at this location. If these two amplitudes at the start and end of the first cycle are found to be different, the following adjustment to the frequency is then made. The end of the first cycle of the reference signal is lengthened or shortened such that the average amplitude at the end of the first cycle changes to match exactly the average amplitude at the beginning. Each cycle of the reference signal, therefore, starts and ends with the same average amplitude. The preceding adjustment to the periodic noise frequency is normally quite small, but it can be an important step in determining a precise value of the frequency.

The seventh step is to obtain a final reference signal for the periodic noise, having the previously determined frequency. A number of sequential cycles of periodic noise in the overpressure record, starting with the first cycle, are now averaged to obtain a final reference signal. The number of cycles that is employed is made as large as possible, utilizing almost all of the available baseline signal preceding the shock-wave signature. Such an averaging procedure helps to minimize the effects of anomalous noise (e.g., an infrequent spike) that might have occurred in the first cycle of the overpressure record.

The eighth and final step is to now use the final reference signal to remove the periodic noise from the overpressure record. This is done quite simply by subtracting the reference signal from the overpressure record, leaving a modified overpressure record containing an insignificant baseline shift and a relatively unfluctuating baseline signal with a well-defined shock-wave signature, along with some minor superposed random noise.

The capability of Method A is well demonstrated by the two sets of results given in Figs. 7 and 8. In each set the original overpressure measurement appears at the top of the figure and the modified record is shown below. From these two sets of results it can be clearly seen that Method A is very successful in removing the baseline shift and undesirable low-frequency periodic noise.

For the sake of interest the overpressure measurement shown in Fig. 7 is a shock-wave record from a supersonic projectile (3-inch/50 MK33). Its speed was 460 m/s, corresponding to a Mach number of 1.4, and the
measurement was taken at a distance of 15 m to one side of the flight path. The shock-wave overpressure measurement shown in Fig. 8 shows only the first part of the muzzle blast wave from the artillery gun that fired the supersonic projectile. This measurement was taken 185 m out front of the gun muzzle.

3.2 Method

Method B can be used to eliminate baseline shift, drift and low-frequency periodic noise from an overpressure measurement. The periodic noise is assumed to be sinusoidal in shape with a known frequency. The following equation is used to represent the low-frequency noise including baseline shift and drift.

\[ N(t) = a \sin(2\pi ft + \phi) + \beta + \gamma t \]  
(2)

The constant \( a \) is an initially unknown amplitude of the periodic noise, \( f \) denotes the known frequency of the periodic noise, \( t \) denotes time, and \( \phi \) is an initially unknown phase angle. The constant \( \beta \) denotes an initially unknown value of the baseline shift, and the constant \( \gamma \) is an initially unknown rate of change of baseline drift. Baseline drift, therefore, is assumed to be linear.

Equation 2 can be expressed in the following alternate form, as a combination of linear functions, which is required for the present work.

\[ N(t) = a \cos(\phi) \sin(2\pi ft) + a \sin(\phi) \cos(2\pi ft) + \beta + \gamma t \]  
(3)

A linear least-squares method can then be employed to fit this equation to the discrete baseline data preceding the shock-wave signature, resulting in a direct determination of the following four coefficients: \( a \cos(\phi) \), \( a \sin(\phi) \), \( \beta \), and \( \gamma \). From the values of the first two coefficients, individual values of \( a \) and \( \phi \) can now be obtained.

Now that the constants \( a \), \( \beta \), \( \gamma \), and \( \phi \) have been determined, Eq. 2 or 3 can be used to eliminate the sinusoidal noise and baseline shift and drift from the overpressure record. This is accomplished easily by subtracting the noise signal (Eq. 2 or 3) from the overpressure record. The resulting modified record should have insignificant baseline shift and
drift and consist of a relatively unfluctuating baseline signal with a well-defined shock-wave signature.

The capability of Method B is aptly demonstrated by the two sets of results given in Figs. 9 and 10. The original set of overpressure measurements appears first in Fig. 9. In these measurements the periodic noise is most severe in the second record, whereas the worst baseline shift occurs in the third record. The set of modified overpressure records is shown for comparison in Fig. 10. It can be readily seen that the periodic noise (assumed to be 60 Hz) and baseline shift have been almost entirely removed. Some minor low-frequency noise remains superposed on only the second record. In conclusion, Method B is quite effective in removing such noise from an overpressure measurement.

In the process of fitting Eq. 2 to the discrete overpressure data, in order to determine the values of $\alpha$, $\beta$, $\gamma$, and $\phi$, data for a short time interval of only 5 ms was used. This corresponds to only three-tenths of a cycle of the periodic noise. Such a small time interval was found to be quite satisfactory for our overpressure measurements for supersonic projectiles. A larger time interval, when available, produced only marginal improvements in removing the periodic noise and baseline shift. Furthermore, it was discovered that the baseline drift ($\gamma t$) relative to the baseline shift ($\beta$) was negligible over the short time interval of interest (10 to 40 ms). As a consequence, only the baseline shift ($\beta$) needed to be taken into account, and the value of the rate of change of baseline drift ($\gamma$) could be set equal to zero from the outset.

Methods A and B should be considered to be complementary. When only one cycle or a fraction of a cycle of periodic noise preceding the shock-wave signature is available for defining the reference signal, then Method B has to be employed. When two or preferably more cycles of noise are available, Method A should be used. Since Method A is more general in that nonsinusoidal noise with an unknown frequency can be handled, it is normally superior in eliminating low-frequency periodic noise.

For the sake of interest, the overpressure measurements presented in Fig. 9 are of the shock wave from a supersonic projectile (3-inch/70 MK34). This projectile was moving 3 m above and parallel to the ground.
at a speed of 610 m/s, corresponding to a Mach number of 1.8. The four measurements labelled a, b, c, and d were recorded at different distances of 7, 15, 36, and 42 m, respectively, to one side of the flight path. In the top three records of Figs. 9 and 10 the N-shaped shock wave is followed by another N-wave. This latter N-wave is a direct consequence of the ground acting as a reflecting plane. In the bottom record, the initial and reflected N-waves just begin to overlap.

4.0 INFORMATION RETRIEVAL FROM AN OVERPRESSURE RECORD

Numerous overpressure records and other signals very often need to be processed quickly and efficiently to obtain various types of information. Automated computer processing and extraction of information is most often the only feasible approach. Although this approach is usually quick and convenient for a computer-program user, precise definitions and algorithms must be developed initially to ensure that the user obtains accurate numerical information from the records. In the following six sections of this chapter our methods of obtaining information from overpressure records for shock-wave arrival time, positive-phase duration and impulse, peak overpressure, rise time, and initial overpressure decay rate are described briefly. The methods are not necessarily new; however, they are presented herein for both completeness and future reference. Researchers in the past have not documented the details of their methods.

4.1 Time of Arrival

The time of arrival of the shock wave relative to a reference time is determined, essentially, in the following manner. Near the beginning of each overpressure record an initial timing mark is normally available as a convenient reference point (e.g., see Fig. 11). Starting from this reference point (time $t_0$), or the beginning of the record in the absence of such a mark, the discrete overpressure data can be examined numerically, in a sequential point-by-point manner. Eventually, a few consecutive overpressure values will consistently increase (at time $t_3$), thereby marking the first few points of the front shock of the N-wave. The difference
between the times corresponding to the first value of the ascending sequence and the reference point \((t_3 - t_0)\) is, therefore, the time of arrival of the shock wave.

Although this procedure is simple and correct conceptually, additional computational steps or checks are required in practice to avoid erroneous results. Firstly, low-level random noise in the overpressure record may contain a sufficiently long run of ascending overpressure values (Fig. 11), which could stop the point-by-point search prematurely (at \(t_1\)). In order to circumvent this problem a threshold overpressure level can be specified. The search then stops only when the run of ascending values all exceed this threshold. Secondly, an infrequent spike in an overpressure record can also cause a premature stopping (at \(t_2\)), yielding an erroneous time of arrival. To avoid this problem the shape of the following signature can be explored. The point-by-point search is stopped only if the signature is N-shaped. Other additional safeguards to prevent erroneous results sometimes have to be devised to overcome problems peculiar to a particular record.

4.2 Positive-phase Duration

The duration of the initial positive phase of the overpressure signature (Fig. 11) is defined as the time interval from the arrival of the N-wave \((t_3)\) to the time \((t_6)\) when the overpressure signature crosses the horizontal axis and becomes negative (i.e., \(t_6 - t_3\)). The latter time \((t_6)\) must be chosen carefully from the discrete overpressure data so that extraneous crossings (e.g., at \(t_5\)) resulting from random or other noise are excluded. One common procedure of obtaining the crossing time accurately is to first fit a straight line to the data in the vicinity of the crossing. Then the crossing time can be obtained from this straight line with confidence. Another common procedure is to first obtain the time integral of the overpressure record, starting the integration from the arrival time \((t_3)\). The crossing time then corresponds to the time at which the integral is a maximum. These two procedures normally yield very similar results for the positive-phase duration. In the authors' work with shock waves from supersonic projectiles the latter integral procedure is
4.3 Positive-phase Impulse

The positive-phase impulse \( I \) is defined as the value of the time integral of the positive-phase overpressure \( \Delta p \) from the N-wave arrival time \( t_3 \) to the end of the positive-phase duration \( t_6 \), as shown below.

\[
I = \int_{t_3}^{t_6} \Delta p(t) \, dt
\]

(4)

The integration of the discrete overpressure data by using a trapezoidal or Simpson's rule is quite straightforward. Note that the impulse value is normally obtained simultaneously with the determination of the end of the positive-phase duration \( t_6 \) when the integral method is used.

4.4 Peak Overpressure

The peak overpressure of the front shock of an N-wave is easily picked from the discrete overpressure data. The time at which this peak occurs is also obtained easily \( t_4 \) in Fig. 11. To minimize the effects of random noise on this peak value, it can be modified slightly by averaging it with a few preceding and following overpressures. This averaging procedure is meaningful only when the digitization rate is sufficiently high so that the averaged overpressures are all an essential part of the peak. The peak overpressure obtained in this manner is denoted by the symbol \( \Delta p_0 \) for future reference.

An alternate estimate of the peak overpressure can be obtained in the following way. An exponential curve is first fitted to the discrete overpressure data in the time interval starting at \( t_4 \) and ending at \( (t_4 + t_6)/2 \) (Fig. 11). The peak overpressure is then obtained directly from this exponential curve at the time \( t_4 \), and denoted by the symbol \( \Delta p_1 \). In the author's work with shock waves from supersonic projectiles, both of the peak overpressures (\( \Delta p_0 \) and \( \Delta p_1 \)) were determined and found to be in good agreement.
4.5 Rise Time

The rise time of the front shock of the N-wave is defined as the time interval starting from the arrival of the N-wave \( t_3 \) and ending at the time \( t_4 \) when the overpressure is a maximum (i.e., \( t_4 - t_3 \)). From the previously determined data for \( t_4 \) and \( t_3 \) the rise time can be obtained easily.

Other definitions of shock-wave rise time are encountered in the open literature. For the fully dispersed shock structure of weak waves and impulse sound effects on hearing the rise time is often defined as 1.25 times the time for the overpressure to rise from 10 to 90% of its peak value (Ref. 16). The rise time obtained with this definition is generally 10 to 30% shorter than in the present case.

4.6 Overpressure Decay Rate

The decay rate of the overpressure just behind the front shock of the N-wave is determined directly from the exponential curve which was fitted previously to the discrete overpressure data in Section 4.4. This decay rate is equal to the derivative of the overpressure at the time \( t_4 \) where the peak overpressure \( \Delta p_1 \) was taken.

4.7 Sample Results

The previous methods of obtaining the shock-wave time of arrival, positive-phase duration and impulse, peak overpressure, rise time, and overpressure decay rate have been applied to the four shock-wave overpressure signatures shown in Fig. 10. The results are summarized for interest in Table 1. Note that the information has been extracted from the modified overpressure records from which the baseline shift and low-frequency and high-frequency noise have been removed. This ensures that the information is of the highest possible degree of accuracy.

5.0 CONCLUDING REMARKS

Numerical methods of successfully processing digitized overpressure-versus-time measurements of shock waves from supersonic projectiles
have been presented, illustrated with results, and discussed in detail. It has been shown that, once high-frequency noise is identified in the Fourier spectrum of the measurement, this noise can then be removed successfully by judiciously altering the spectrum before applying the inverse Fourier transform. As a prerequisite to the spectrum alteration, a fair knowledge of the spectrum shape without noise effects is normally required.

It has also been shown that baseline shift and drift and low-frequency periodic noise can be removed successfully from a measurement by two complementary methods. In Methods A and B baseline shift is eliminated equally well, but only Method B has the capability of removing baseline drift. Method A is generally superior to Method B for removing low-frequency periodic noise, since it was devised to remove periodic noise of initially unknown shape and frequency. Method B was devised to remove periodic noise with a known sinusoidal shape and specified frequency. It should be remembered that, in order to use Method A, two or preferably more cycles of periodic noise in the baseline signal preceding the shock-wave signature are required for characterizing the noise shape and frequency. When only one cycle or a fraction of a cycle of periodic noise is available, then Method B has to be employed.

The present methods of removing low- and high-frequency noise and retrieving accurate information from a modified overpressure measurement have been applied in the data reduction of numerous overpressure measurements of the shock wave from supersonic projectiles fixed from military guns (106-mm M344A1, 3-inch/50 MK33, 3-inch/70 MK34, 5-inch/54 MK41). These experimental results will be published separately as DRES reports. It is important to note that these methods are not restricted in use to only shock-wave records from supersonic projectiles, but can also be applied to other similar records. For example, the methods are almost directly applicable to shock-wave measurements from both shock tubes and blast-wave simulators and also to blast-wave records from chemical explosions.
6.0 REFERENCES


2. N.H. Ethridge A Procedure for Reading and Smoothing Pressure-Time Data from H.E. and Nuclear Explosions. BRL Memorandum Report No. 1691, September 1965. UNCLASSIFIED.


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TABLE I

DATA FROM A SHOCK WAVE OF A SUPERSONIC PROJECTILE
(3-inch/70 MK34, projectile speed and Mach number are 610 m/s and 1.6)

<table>
<thead>
<tr>
<th>Shock-Wave Properties</th>
<th>Shock-Wave Data for Signatures shown in:</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Fig. 10a</td>
</tr>
<tr>
<td>Relative time of arrival (ms)</td>
<td></td>
</tr>
<tr>
<td>Positive-phase duration (ms)</td>
<td>0.698</td>
</tr>
<tr>
<td>Positive-phase impulse (kPa ms)</td>
<td>0.554</td>
</tr>
<tr>
<td>Peak overpressure ( \Delta p_0 ) (kPa)</td>
<td>1.77</td>
</tr>
<tr>
<td>Initial decay rate ( \Delta p_1 ) (kPa)</td>
<td>1.71</td>
</tr>
<tr>
<td>Shock rise time (( \mu s ))</td>
<td>31</td>
</tr>
<tr>
<td>Initial decay rate (kPa/ms)</td>
<td>-4.10</td>
</tr>
</tbody>
</table>
FIG. 1. FOURIER SPECTRUM OF AN IDEAL N-WAVE.
FIG. 2. FOURIER SPECTRUM OF AN N-WAVE FROM A SUPersonic Projectile.
GAGE-MOUNT RINGING:

1st MODE

2nd MODE

FIG. 3. FOURIER SPECTRUM OF AN N-WAVE WITH SUPERPOSED GAGE-MOUNT RINGING.
FIG. 4. FOURIER SPECTRUM OF AN N-WAVE WITH THE FIRST MODE OF THE SUPERPOSED GAGE-MOUNT RINGING REMOVED.
FIG. 5. OVERPRESSURE SIGNATURES OF AN N-WAVE FROM A SUPERSONIC PROJECTILE: — WITH GAGE-MOUNT RINGING, ——— WITHOUT THE FIRST MODE OF THE GAGE-MOUNT RINGING.
FIG. 6. STANDARD DEVIATION VERSUS NUMBER OF POINTS PER CYCLE OF PERIODIC NOISE.
FIG. 7. ORIGINAL (a) AND MODIFIED (b) OVERPRESSURE SIGNATURES OF A SHOCK WAVE FROM A SUPersonic Projectile.
FIG. 8. ORIGINAL (a) AND MODIFIED (b) OVERPRESSURE SIGNATURES OF PART OF THE SHOCK WAVE FROM THE MUZZLE BLAST OF AN ARTILLERY GUN.
FIG. 9. ORIGINAL OVERPRESSURE RECORDS OF THE SHOCK WAVE FROM A SUPERSONIC PROJECTILE.
FIG. 10. MODIFIED OVERPRESSURE RECORDS OF THE SHOCK WAVE FROM A SUPERSONIC PROJECTILE.
$t_3 - t_0$: RELATIVE TIME OF ARRIVAL
$t_6 - t_3$: INITIAL POSITIVE-PHASE DURATION
$t_7 - t_6$: INITIAL NEGATIVE-PHASE DURATION
$t_8 - t_7$: SUBSEQUENT POSITIVE-PHASE DURATION

FIG. II. SAMPLE SHOCK-WAVE OVERPRESSURE RECORD.
APPENDIX A

FAST FOURIER TRANSFORM SUBROUTINE

A fast-Fourier-transform subroutine, similar to that in Ref. A1, is used to compute the discrete Fourier transform by the nlog n method. This subroutine is called FFT, and a sample FORTRAN listing is given at the end of this appendix. The extremely efficient nlog n method requires that the data consists of q values, where q equals 2^n and n is some positive integer. Subroutine FFT does not make use of any other subroutines. Its call statement is simply

CALL FFT(N, X, SIGN).

The subroutine inputs are

N = n,

X = X_0, X_1, X_2, ........ X_{q-1} (complex numbers)

SIGN = -1.0 or +1.0.

The subroutine output is

X = Y_0, Y_1, Y_2, ........ Y_{q-1} (complex numbers),

where

\[ Y_j = \sum_{k=0}^{q-1} X_k \exp(2\pi ikj/q), \text{ if } SIGN = -1.0, \]

or

\[ Y_j = \frac{1}{q} \sum_{k=0}^{q-1} X_k \exp(-2\pi ikj/q), \text{ if } SIGN = +1.0. \]

If X_k (k = 0, 1, ... q-1) represents a time function, then the subroutine output Y_k (k = 0, 1, ... q-1) in the case of SIGN = -1.0 represents the frequency spectrum at frequencies f = j/q cycles per time unit, where j = 0, 1, ... q-1. In particular, if X_k is real valued, then

\[ \text{Re} \{Y_j\} = \sum_{k=0}^{q-1} X_k \cos(2\pi kj/q) \]

is the cosine spectrum, and
is the sine spectrum. To recover the time function $X_k$ we may call Subroutine FFT again, but this time with $\text{SIGN} = +1.0$. That is, given the array $X$, the call statements

\begin{align*}
&\text{CALL FFT}(N, X, -1.0), \\
&\text{CALL FFT}(N, X, +1.0),
\end{align*}

one after another, yield the same $X$ as final output, because

$$\frac{1}{q} \sum_{j=0}^{q-1} \left[ \sum_{k=0}^{q-1} X_k \exp(-2\pi i k j/q) \right] \exp(2\pi i m/q) = X_m.$$ 

Likewise, given the array $X$, the call statements

\begin{align*}
&\text{CALL FFT}(N, X, +1.0), \\
&\text{CALL FFT}(N, X, -1.0),
\end{align*}

one after another, yield the same $X$ as final output.

As a numerical example, suppose we have the function

$$X = (1, 2, 3, 4).$$

Then the call statement

$$\text{CALL FFT}(2, X, +1.0)$$

would yield the Fourier transform

$$X = (2.5, -0.5-0.5i, -0.5, 0.5+0.5i).$$

If we execute the call statement

$$\text{CALL FFT}(2, X, -1.0),$$

we return to the given function

$$X = (1, 2, 3, 4).$$

For a number of different reasons the user of Subroutine FFT may want to alter part of the Fourier frequency spectrum $Y_k$ before obtaining the time function $X_k$. Before one alters the frequency spectrum, however, it is important to know that the second half of the frequency spectrum is a mirror image of the first half. In other words, the value of $Y_k$ in the first half
equals the value of $Y_{q-k}$ in the second half, where $q$ equals $2^n$. Therefore, if some of the values of $Y_k$ are changed in the first half of the frequency spectrum, the corresponding values of $Y_k$ in the symmetric half must similarly be changed. This procedure is not difficult, but it is necessary.

**REFERENCE**


```
C-----------------------------
 SUBROUTINE FFT (N, X, SIGN)
C-----------------------------
C NMAX = LARGEST VALUE OF N
C NONDUMMY DIMENSION M(NMAX)
C FOR EXAMPLE, IF NMAX = 8 THEN
 DIMENSION M(8)
C DIMENSION X(2**N)
 DIMENSION X(2)
 COMPLEX X, WK, HOLD, Q
 LX = 2**N
 DO 1 I=1,N
 1 M(I) = 2**N-I
 DO 4 L=1,N
 NBLOCK = 2**(L-1)
 LBLOCK = LX/NBLOCK
 LBHALF = LBLOCK/2
 K = 0
 DO 4 IBLOCK=1, NBLOCK
 FK = K
 FLX = LX
 V = SIGN*6.2831853*FK/FLZ
 WK = CMPLX(COS(V),SIG(V))
 ISTART = LBLOCK*(IBLOCK - 1)
 DO 2 !=1, LBHALF
 J = ISTART + I
 JH = J + LBHALF
```
Q = X(JH)*WK
X(JH) = X(J) - Q

2 CONTINUE
   DO 3 I=2,N
      II = I
      IF (K.LT.M(I)) GO TO 4
   3 K = K - M(I)
   4 K = K + M(II)
      K = 0
      DO 7 J=1,LX
         IF (K.LT.J) GO TO 5
         HOLD = X(J)
         X(J) = X(K+I)
         X(K+L) = HOLD
   5 DO 6 I=1,N
      II = I
      IF (K.LT.M(I)) GO TO 7
   6 K = K - M(I)
   7 K = K + M(II)
      IF (SIGN.LT.0.0) RETURN
      DO 8 I=1,LX
  8 X(I) = X(I)/FLX
      RETURN
   END
Successful methods for numerically processing overpressure-time measurements of shock and blast waves and sonic booms are presented and illustrated with sample results. These methods include: a) the elimination of high-frequency random and other noise from the measurement by employing a fast-Fourier-transform technique and judiciously altering the high-frequency part of the Fourier spectrum; b) the removal of low-frequency periodic fluctuations including baseline shift and drift from the record by using specially developed techniques described in the report; c) the extraction of information from the processed shock-wave record including shock wave arrival time, positive-phase duration and impulse, peak overpressure, rise time, and overpressure decay rate.
KEY WORDS

Shock Waves
Signal Processing
Pressure
Overpressure
Measurement
FFT (Fast Fourier Transform)
Signal Noise
Filtering
Least Squares

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