BACKSCATTER AND EXTINCTION IN WATER CLOUDS

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# Backscatter and Extinction in Water Clouds

Atmospheric clouds can adversely affect the operation of military electro-optical systems, particularly under slant path scenarios. The probing of clouds by the lidar technique, in which a short pulse of laser radiation scattered backwards by the cloud droplets is detected, is attractive for two reasons. First, it is a remote sensing technique, and measurements at ranges of several kilometers are possible. Second, it can give a two-dimensional picture of the cloud and trace its time-development. Unfortunately, in some...
cases meaningful information from a lidar backscatter signal is hard to obtain. If the backscatter signal could somehow be related to a more interesting quantity, for example the extinction coefficient, the value of the lidar measurement would increase substantially. In this report a linear relation between the volume extinction coefficient \( \sigma_e \) (km\(^{-1}\)) and backscatter coefficient \( \sigma_b \) (km\(^{-1}\)sr\(^{-1}\)) of atmospheric cloud at visible and near-infrared wavelengths is derived. The relation is independent of the droplet-size distribution and has the form \( \sigma_e = \frac{8\pi}{g(\lambda)} \sigma_b \), where \( g(\lambda) \) is the averaged value of the backscatter gain \( G(\lambda) \) over the range of droplet-size parameters realistic for clouds. At a wavelength \( \lambda = 1.06 \mu m \) the relation is \( \sigma_e = 15.8 \sigma_b \). The relation is in good agreement (within 50 percent) with Mie calculations of extinction and backscatter coefficients based on 156 measurements of cloud droplet spectra in cumulus and stratus type clouds. The relation suggests that visible or near-infrared extinction coefficients in a cloud of unknown type could be inferred from lidar backscatter measurements alone, at least near the cloud boundary where the contribution of multiply scattered photons to the lidar return signal can be neglected. No similar size-distribution-independent relation between backscatter coefficient and liquid water content of cloud is found, suggesting that cloud liquid water content cannot be inferred solely from lidar backscatter measurements.
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1. INTRODUCTION

Despite considerable effort toward the application of the lidar technique to remote measurement of clouds, no one seems to have looked for (or at least to have found) a unique relation between cloud backscatter and extinction coefficients. If such a relation exists, it would increase the value of lidar measurements, since the cloud extinction coefficient (and, interestingly, also the total droplet surface area) could be inferred from the backscatter signal.

We will show that for ruby (\(\lambda = 0.694\mu m\)) and Nd-YAG (\(\lambda = 1.064\mu m\)) laser wavelengths a relation exists between the cloud extinction coefficient \(\sigma_e (km^{-1})\) and the backscatter coefficient \(\sigma_b (km^{-1}sr^{-1})\), independent of the form of the cloud drop-size distribution. We will show, however, that cloud liquid water content for clouds of unknown drop-size distribution cannot be inferred from visible, infrared, or near-millimeter (elastic) backscatter measurements alone.

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Finally, we will show that previously derived relations between infrared ($\lambda = 11\mu m$) extinction and liquid water content in fog and between infrared ($\lambda = 3.8\mu m$) absorption and liquid water content in fog\(^7\) can also be applied to most clouds.

2. BACKSCATTER AND EXTINCTION IN CLOUD

The volume extinction and backscatter coefficients $\sigma_e$ and $\sigma_b$ of a polydispersion of spherical cloud droplets characterized by a size distribution $n(r)$ and refractive index $m$ are given by

$$\sigma_e = \int \pi r^2 Q_e(m, x) n(r) dr$$

(1)

$$\sigma_b = \frac{1}{4\pi} \int \pi r^2 G(m, x) n(r) dr$$

(2)

where $Q_e(m, x)$ is the Mie efficiency factor for extinction for a particle with refractive index $m$ and size parameter $x = 2\pi r/\lambda$, and $G(m, x)$ is the backscatter gain defined as $4\pi$ times the ratio of the differential backscatter cross section to the geometric area.

To find a relation between the extinction and backscatter coefficients $\sigma_e$ and $\sigma_b$ that holds for all cloud types independent of drop size, we have to resort to approximate expressions for the Mie efficiencies $Q_e$ and $G$ in equations (1) and (2). Since extinction in cloud is dominated by droplets with radii $2\mu m < r < 85\mu m$ (corresponding to size parameters $12 < x < 500$ at a wavelength $\lambda = 1.06\mu m$), the extinction efficiency in equation (1) can justifiably be approximated by $Q_e \approx 2$. However, because of the complicated functional behavior of the backscatter gain $G(x)$ for large $x$, we might think that no simple approximation could be used for it in equation (2).

---


Nevertheless, let us examine the form of the Mie backscatter gain $G(x)$ for water droplets over a range of size parameters representative of atmospheric cloud droplets (we must of course restrict our attention to warm clouds containing no irregular ice particles). Calculations of $G(x)$ for $0 < x < 500$ in figure 1 show the familiar oscillating behavior with several distinct resonance periods. To show more detail in the backscatter gain, we made additional calculations with increased size resolution $\Delta x = 0.01$ from $x = 300$ to $x = 325$ (figure 2). The resonances with periods $\Delta x = 0.41, 0.83, 1.4$ predicted by Nussenzveig$^9$ (using complex angular momentum theory) and found by Shipley and Weinman$^1$ are now readily apparent.

Since our interest here is in lidar backscattering from a polydispersion of many cloud droplets of different sizes (rather than backscattering from single droplets), we are motivated to investigate some averaging scheme for the backscatter gain. A realistic size distribution of cloud droplets has a fairly uniform distribution of droplets throughout small ranges of drop size, so (following Shipley and Weinman$^1$) let us calculate a running mean of the backscatter gain over intervals of a resonance period evident in figure 2, $\Delta x = 0.83$. This running mean (figure 3) shows that to first order the averaged backscatter gain oscillates about some nearly constant value with a period of oscillation $\Delta x \approx 14$. If cloud droplet distributions are assumed to be somewhat uniform over intervals $\Delta x = 14$ (corresponding to $\Delta r \approx 2.4 \mu m$ at $\lambda = 1.06 \mu m$), the exact Mie values $G(x)$ can be averaged over these intervals. These averaged values $\overline{G(x)}$ over $\Delta x = 14$ for the entire range of size parameters realistic for cloud are shown in figure 4. The averaged values $\overline{G(x)}$ are slowly varying (except for small drops with size parameters less than 14, which are unimportant), and thus we can replace $G[m(\lambda),x]$ in equation (2) by a constant value $g(\lambda)$ that is independent of size parameter and depends only on the radiation wavelength $\lambda$. Note that in doing so we constrain the cloud to have a fairly uniform distribution of droplets over intervals $\Delta r \approx 2.4 \mu m$. (Actually this constraint can be loosened to require uniformity over intervals $\Delta r \approx 1.2 \mu m$ if we can choose the intervals to be over half-cycles of the $\Delta x \approx 14$ oscillation in the running mean of the backscatter gain.)

The use of these approximations for the Mie extinction efficiency ($Q_e = 2$) and backscatter gain [$G = g(\lambda)$] in equations (1) and (2), leads to the cloud extinction coefficient being linearly related to the backscatter coefficient.

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Figure 1. Backscatter gain $G(x)$ for water droplets at a wavelength
$\lambda = 1.06 \mu m$ (refractive index $m = 1.326 - 5x10^{-4}i$) calculated
for size parameters (lower size parameter, upper size parameter, step $\delta x$): (0.001, 0.1, 0.001), (0.1, 1, 0.005),
(1, 10, 0.01), (10, 20, 0.05), (20, 500, 0.1).
Figure 2. More detailed calculations of the backscatter gain $G(x)$ around $x = 300$ with resolution in size parameters $\Delta x = 0.01$. The quasiperiodic structure with periods $\Delta x \approx 0.41, 0.83, 14$ are apparent.
Figure 3. Running mean of the backscatter gain near size parameter 300, suggesting that if the backscatter gain is averaged over size parameter intervals $\Delta x = 14$ the result will be constant (independent of size parameter).

Figure 4. Averaged values of the backscatter gain $G(x)$ over intervals $\Delta x = 14$ for size parameters realistic in atmospheric cloud (size parameters 0-500 correspond to radii 0-85\,\mu m at $\lambda = 1.06\,\mu m$). The averaged values are nearly constant (except for $x<14$) and can be approximated by a single value $g = 1.59$ shown by the dashed line. Consequently, the extinction and backscatter coefficients in cloud can be unambiguously related according to relation (3).
\[ \sigma_e = \frac{8\pi}{\lambda} \sigma_b \]

(3)

where \( g(\lambda) \) is determined by numerically averaging the values of \( G(x) \) as in figure 4. The resulting averages are \( g(\lambda = 0.694\,\mu m) = 1.64 \) and \( g(\lambda = 1.06\,\mu m) = 1.59 \).

Shipley and Weinman\(^1\) have also averaged the backscatter gain for water drops over intervals \( \Delta x = 14 \) for size parameters realistic for rain at visible wavelengths. (Actually they averaged the normalized backscatter phase function, which is the ratio of the backscatter gain to the total scattering cross section, for size parameter intervals between \( x \geq 500 \) and \( x \leq 5000 \).) They found, as we found for size parameters \( x < 500 \), that the averaged gain is nearly constant. In terms of our parameter \( g \), this average value (from their figure 9) is \( g = 1.6 \pm 0.3 \). Thus, the size-distribution-independent relation (3) between extinction and backscatter coefficients can also be applied to rain; namely, \( \sigma_e = 15.7 \sigma_b \). (Previously, Shipley\(^1\) pointed out a unique relation between extinction and backscatter in rain.) Unfortunately, in the case of raindrops with diameters greater than 0.3 mm, the question regarding the effect of their nonsphericity\(^2\) (which is not taken into account in the derivation of (3)) still remains.

3. VERIFICATION OF THE EXTINCTION BACKSCATTER RELATION

To test the validity of the extinction-backscatter relation (3) for clouds, we applied the Mie theory and used the indexes of refraction of water given by Hale and Querry\(^3\) to calculate the extinction coefficient according to equation (1) and the backscatter coefficient according to equation (2) for 156 cloud droplet size distributions measured in the major cloud types. The sources of these measurements together with the range of droplet sizes measured and other pertinent information are listed in table 1. The main sampling technique employed to obtain the cloud droplet-size distributions was that of impaction of droplets onto coated slides or replicators whose collection efficiencies were known. The practical lower limit for detection of cloud droplets by the impaction technique is around 1.5\(\mu\)m radius. The sole


\(^{4}\)G. M. Hale and M. R. Querry, 1973, "Optical Constants of Water in the 200\(\mu\)m to 20\(\mu\)m Wavelength Region," Appl Opt, 12:555-563
### TABLE 1. CLOUD SIZE DISTRIBUTION MEASUREMENTS

<table>
<thead>
<tr>
<th>Source*</th>
<th>Cloud Type</th>
<th>Range of Droplet Sizes Measured (Radii in μm)</th>
<th>No. of Drop-Size Measurements</th>
</tr>
</thead>
<tbody>
<tr>
<td>aufm Kamne and Weickmann (1952)</td>
<td>Cumulus congestus</td>
<td>1.5 to 92</td>
<td>1</td>
</tr>
<tr>
<td>Battan and Reitan (1957)</td>
<td>Cumulus and Cumulus congestus</td>
<td>1.75 to 58</td>
<td>5</td>
</tr>
<tr>
<td>Diem (1948)</td>
<td>Cumulus and Cumulus congestus</td>
<td>1 to 42</td>
<td>6</td>
</tr>
<tr>
<td>Durbin (1959)</td>
<td>Cumulus</td>
<td>0.75 to 30</td>
<td>22</td>
</tr>
<tr>
<td>Fitzgerald (1972)</td>
<td>Continental cumulus</td>
<td>3.5 to 11.5</td>
<td>7</td>
</tr>
<tr>
<td>Fitzgerald et al (1973)</td>
<td>Cumulus and Stratocumulus</td>
<td>1.75 to 10.5</td>
<td>4</td>
</tr>
<tr>
<td>Jiusto (1967)</td>
<td>Maritime cumulus</td>
<td>1.5 to 24</td>
<td>4</td>
</tr>
<tr>
<td>Ryan et al (1972)</td>
<td>Continental cumulus</td>
<td>2 to 42</td>
<td>33</td>
</tr>
<tr>
<td>Singleton and Smith (1960)</td>
<td>Stratus</td>
<td>1.5 to 62</td>
<td>17</td>
</tr>
<tr>
<td>Spyers-Duran (1972)</td>
<td>Altostratus and Altocumulus</td>
<td>2.5 to 24</td>
<td>8</td>
</tr>
<tr>
<td>Squires (1958)</td>
<td>Orographic tradewind cumulus</td>
<td>2.5 to 82</td>
<td>10</td>
</tr>
<tr>
<td>Warner (1973b)</td>
<td>Maritime cumulus</td>
<td>1.5 to 13</td>
<td>4</td>
</tr>
<tr>
<td>Weickmann and aufm Kampe (1953)</td>
<td>Cumulus congestus and Cumulonimbus</td>
<td>2.5 to 100</td>
<td>3</td>
</tr>
</tbody>
</table>

*Source document information is listed separately.
SOURCE DOCUMENTS FOR TABLE 1


cloud size determination by a light scattering counter\(^1\) was calibrated by means of uniformly sized water droplets. Only nonprecipitating clouds were used in the analysis, and measurements which showed evidence of glaciation were excluded.

The numerical integrations for these Mie calculations were performed only over the range of droplet sizes measured, with no extrapolation to smaller or larger sizes. The results are shown in figure 5. Plotted for each cloud size distribution are values of the extinction coefficient as a function of the backscatter coefficient at the Nd:YAG laser wavelength \(\lambda = 1.06\mu\text{m}\). The linear relation between extinction and backscatter coefficients predicted using the size-distribution-independent relation (3) is shown by the straight line. For all considered cloud size distributions, the relation (3) is within 50 percent of the numerical results. Thus if errors of this order are acceptable, cloud extinction coefficients can be inferred from measurement of the backscatter coefficients directly from equation (3), without need to know details of the cloud droplet-size distribution.

Of course cloud backscatter coefficients can be determined from lidar return signals (in a straightforward way) only in the absence or neglect of multiple scattering contributions to the lidar signal. It follows that application of (3) to obtain cloud extinction coefficients from lidar returns might be restricted to the edges of clouds where the contribution of multiply scattered photons is small.

It would be desirable to compare the extinction-backscatter relation (3) to direct measurements of these quantities. The only known simultaneous measurements of backscatter and extinction in cloud are by Curcio and Knestrick\(^1\). They found empirically a proportionality between extinction and backscatter coefficients of the form \(\sigma_e \propto \sigma_b^{0.66}\) for weather conditions including fog, fog and drizzle, and clear weather. However, there is considerable leeway in determining the exponent in this proportionality from their measured data (their figure 4). In addition, the effects of fog inhomogeneities and multiple scatter contributions to both the backscatter and transmission signals are potential uncertainties in comparing our relation (3) with their data.

We should not necessarily expect the extinction-backscatter relation (3) to be applicable at all wavelengths, since the requirement that both the backscatter gain and the extinction efficiency be well approximated by constant values (independent of size parameter) is generally not satisfied. To prove this conjecture, we calculated the extinction and backscatter coefficients for the previously mentioned 156 cloud size distributions at several laser wavelengths. The results at the DF and \(\text{CO}_2\) laser wavelengths (figures 6 and

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Figure 5. Volume extinction coefficient versus volume backscatter coefficient at a wavelength $\lambda = 1.06 \mu m$ for 156 droplet size distributions measured in the major cloud types: Cu denotes cumulus, cumulus congestus, continental cumulus, maritime cumulus, tropical cumulus, altocumulus, and tradewind cumulus; Sc, stratocumulus; Cb, cumulonimbus; Or, orographic; St, stratus; As, altostratus; Ns, nimbostratus, and Mst, maritime stratus. The results are in good agreement with the size-distribution-independent prediction (3) (shown by the straight line) relating extinction uniquely to backscatter.

Figure 6. Same as figure 5 except for the DF laser wavelength $\lambda = 3.8 \mu m$. At this longer wavelength the extinction is no longer unambiguously related to backscatter, as the extinction coefficient for a particular backscatter coefficient varies by about an order of magnitude with the droplet size distribution.
7) show that for a particular backscatter coefficient the extinction varies by an order of magnitude for different size distributions of droplets. (Our neglect of gaseous absorption, which is at most 0.05 and 0.4 km\(^{-1}\) at these wavelengths, does not significantly affect the results in figures 6 and 7.) Therefore, a DF or CO\(_2\) single-ended lidar measurement could not be used (by itself without constraints on cloud homogeneity or similarity of drop-size distributions along the path) to deduce infrared extinction in cloud.

We also cannot anticipate an unambiguous extinction-backscatter relation at near-millimeter wavelengths. For these long wavelengths the Rayleigh approximation holds and we have \(G(x) \propto x^6\) and \(Q_e(x) \propto x\). Upon substitution of these Rayleigh formulas into (1) and (2), we see that the ratio of extinction to backscatter contains the ratio of the 3rd to 6th moments of the drop-size distribution, destroying any chance for a unique extinction-backscatter relation. Our numerical results based on the 156 drop distributions (figures 8, 9, and 10) support this conclusion. The contribution of the molecular absorption to the extinction, which is on the order of 0.30 km\(^{-1}\), 0.12 km\(^{-1}\), and 0.058 km\(^{-1}\) at these frequencies (220, 140, and 94 GHz), has not been accounted for in these figures.

4. BACKSCATTER AND LIQUID WATER CONTENT IN CLOUD

Having been encouraged by the success of the extinction-backscatter relation (3) at visible and near-infrared wavelengths, we extended our investigation to see if a similar relation might exist between cloud liquid water content and backscatter coefficient; the motivation of course being the prospect of using lidar for remote measurement of cloud liquid water content.

The liquid water content \(W\) of clouds with droplet-size distribution \(n(r)\) is given by

\[
W = \int \frac{4}{3} \pi r^3 n(r) dr
\]

where \(\rho\) is the density of water.

We already know that the backscatter gain (averaged over about 2\(\mu\)m radius intervals) at a wavelength \(\lambda = 1.06\mu m\) is nearly constant \([G(x) = g]\). Hence there can be no size-distribution-independent relation between liquid water content and backscatter coefficient at this wavelength as the ratio of these quantities

\[
\frac{W}{\sigma_b} = \frac{16\pi \rho}{3g} \cdot \frac{\int r^3 n(r) dr}{\int r^2 n(r) dr}
\]
Figure 7. Same as figure 6 except for the CO₂ laser wavelength \( \lambda = 10.6 \mu \text{m} \).

Figure 8. Same as figure 6 except for the near-millimeter wavelength \( \lambda = 1364 \mu \text{m} \) (frequency 220 GHz).
Figure 9. Same as figure 6 except for the near-millimeter wavelength \( \lambda = 2143\mu m \) (frequency 140 GHz).

Figure 10. Same as figure 6 except for the near-millimeter wavelength \( \lambda = 3192\mu m \) (frequency 94 GHz).
contains the ratio of the third-to-second moments of the droplet-size distribution. In other words, at $\lambda = 1.06\mu m$ the liquid water content of cloud is related to the backscatter coefficient only through a parameter that depends on droplet-size distribution. To obtain a quantitative measure of this size distribution dependence, we again performed Mie calculations of the backscatter coefficient by using equation (2) and the liquid water content by using equation (4) for the previously mentioned 156 cloud size distributions. The results are presented in figure 11 and show that for a particular backscatter coefficient the cloud liquid water content can vary by more than a factor of ten with the droplet-size distribution.

Similar investigations of a possible relation between cloud liquid water content and backscatter coefficient at other infrared, visible, and near-millimeter laser wavelengths ($\lambda = 0.55\mu m$, 0.694$\mu m$, 3.8$\mu m$, 10.6$\mu m$, 1364$\mu m$ [220 GHz], 2143$\mu m$ [140 GHz], and 3192$\mu m$ [94 GHz]) show again that no unambiguous relations exist and that for a fixed backscatter coefficient at these other wavelengths the cloud liquid water content shows an even larger variation than at $\lambda = 1.06\mu m$. Examples of these results at $\lambda = 3.8\mu m$ and 10.6$\mu m$ are shown in figures 12 and 13.

We can therefore conclude that for clouds of unknown size distribution a determination of liquid water content cannot be made from a single-wavelength (elastic) backscatter lidar measurement alone.

If the total surface area of cloud droplets $S = \int 4\pi r^2 n(r) dr$ is of interest (rather than liquid water content), it follows from equation (2) that the surface area can be unambiguously related to the backscatter coefficient by

$$S = \frac{16\pi}{g(\lambda) a_b}$$

for ruby and Nd-YAG laser wavelengths. At the ruby wavelength $\lambda = 0.694\mu m$, the relation is $S = 30.7a_b$ where $S$ is the total droplet surface area per unit volume of cloud (in $m^2/m^3$) and the units of $a_b$ are $m^{-1}$ steradian$^{-1}$.

5. EXTINCTION AND LIQUID WATER CONTENT IN CLOUD

Chýlek\textsuperscript{7} and Pinnick et al\textsuperscript{8} have previously shown that a linear relation exists between the infrared extinction around $\lambda = 11\mu m$ and the liquid water content of fogs. The relation is

\textsuperscript{7}Petr Chýlek, 1978, "Extinction and Liquid Water Content of Fogs and Clouds," J Atmos Sci, 35:296-300

Figure 11. Volume backscatter coefficient at a wavelength \( \lambda = 1.06\mu m \) vs liquid water content for 156 measured droplet size distributions of cumulus and stratus type clouds. The results show cloud liquid water content is not uniquely related to the backscatter coefficient irrespective of cloud type.

Figure 12. Same as figure 11 except for \( \lambda = 3.8\mu m \).
Figure 13. Same as figure 11 except for $\lambda = 10.6\mu m$.

$$a_e = \frac{3\pi c}{2\lambda} W,$$  \hspace{1cm} (7)

where $a_e$ is the extinction coefficient at the wavelength $\lambda$, $W$ is the fog liquid water content, and the coefficient $c$ is equal to the slope of a straight line that approximates the Mie extinction efficiency curve by $Q_e(\kappa, \lambda) \approx c(\lambda)\kappa$. (The reader is referred to Pinnick et al. for values of $c[\lambda]$). The success of relation (7) depends on the fact that fog droplets have radii predominately less than $r \approx 14\mu m$.

---


Since cloud droplets can be much larger than fog droplets, we might not expect relation (7) to be applicable to all clouds, particularly if droplets with radius $r > 14\mu m$ dominate either extinction or liquid water content. To investigate quantitatively the magnitude of the error involved in the application of (7) to clouds, we again made Mie calculations of the extinction coefficient (at $\lambda = 10.6\mu m$) and the liquid water content for the previously considered 156 cloud droplet-size distributions summarized in table 1. The results of these calculations are compared to the size-distribution-independent prediction (7) in figure 14. As noted previously, the effect of gaseous absorption on the extinction coefficient is small and has been neglected. Except for cumulonimbus, nimbostratus, cumulus congestus, and some stratus type clouds (which contain significant numbers of large [$r > 14\mu m$] droplets), the relation (7) is within a factor two of the numerical results.

This agreement reaffirms the conclusion of Chylek that at $\lambda \approx 11\mu m$ a nearly unique relation exists between the infrared extinction coefficient and liquid water content of the form of equation (7) for nonprecipitating clouds.

![Figure 14. Volume extinction coefficient at a wavelength $\lambda = 10.6\mu m$ vs liquid water content for 156 cloud droplet size distribution measurements of cumulus and stratus clouds. Except for cumulonimbus, nimbostratus, cumulus congestus, orographic, and some stratus cloud types the results are close to the equation (7) prediction (shown by the straight line) relating infrared extinction coefficient uniquely to liquid water content.](image)

6. ABSORPTION AND LIQUID WATER CONTENT IN CLOUD

The cloud size distribution measurements summarized in table 1 can also be used to check the applicability of the relation derived by Pinnick et al. connecting the volume absorption coefficient \( \sigma_a \) at wavelengths around \( \lambda = 3.8 \mu m \) to liquid water content in fog. The relation has the form

\[
\sigma_a = \frac{3\pi c'}{2\lambda \rho} W, \tag{8}
\]

where the parameter \( c'(\lambda) \) is determined by approximating the Mie absorption efficiency \( Q_\alpha(x, \lambda) \) by a linear function of size parameter for particles less than a certain size (see Pinnick et al. for details). As for the extinction-liquid water content relation (7), the absorption-liquid water content relation (8) is independent of the form of the drop-size distribution and depends only on drops having radii less than a certain value. At the 3.8 \( \mu m \) wavelength, this value is \( r = 13 \mu m \).

Numerical calculations of cloud absorption (at the 3.8 \( \mu m \) wavelength) and liquid water content based on the 156 size distributions are compared to the relation (8) prediction in figure 15. The gaseous absorption at this wavelength is small (at most 0.05 km\(^{-1}\)) and has again been neglected here. The comparison shows that for most cloud types the relation (8) can be used to connect cloud absorption (or cloud emissivity) to cloud liquid water content with not more than a 50 percent error. Thus equation (8) can be applied to fogs and most clouds without regard to their type or the character of their drop-size distributions.

7. CONCLUSION

A relation between extinction and backscatter coefficients at visible and near-infrared wavelengths has been derived for all types of atmospheric clouds consisting of spherical water droplets. The relation is independent of cloud droplet-size distribution. The relation should enable the determination of cloud extinction coefficient (or total droplet surface area) solely from lidar return signals, providing the contribution of multiply scattered photons to the lidar return can be neglected. However, no size-distribution-independent relation exists between cloud liquid water content and backscatter coefficient at visible, infrared, or near-millimeter wavelengths. This suggests that single-wavelength lidar by itself cannot be used to remotely measure cloud liquid water content for clouds of unknown type, at least without some constraint on cloud inhomogeneity or backscatter-extinction ratio along the lidar path.

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Figure 15. Volume absorption coefficient at a wavelength $\lambda = 3.8\mu m$ vs liquid water content for 156 cloud droplet size distributions of cumulus and stratus clouds. For most cloud types the results are close to the equation (8) prediction (shown by the straight line) relating cloud infrared absorption unambiguously to cloud liquid water content.
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