Scattering from a vegetation layer with an irregular vegetation soil boundary

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The findings in this report are not to be construed as an official Department of the Army position unless so designated by other authorized documents.

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A theoretical model is computed for the backscattering of electromagnetic waves from a layer of vegetation by using a first-order renormalization technique to determine volume scattering. The vegetation soil interface is assumed rough according to the tangent plane approximation and the scattering from this boundary is added incoherently to the volume scattering result. The mean wave in the vegetation is obtained using a bilocal approximation of the Dyson's equation. A free space dyadic Green's function is used, along with a correlation function of the dielectric fluctuations that are exponential in form and that also possess different correlation lengths $\xi$ and $\eta$. 
20. Continued

\( \mathbf{\nabla} \) in the \( x, y, \) and \( z, \) directions. Effective propagation constants are obtained for both horizontal and vertical polarization. The scattered wave is solved for by using a two-dimensional Fourier transform technique, and the boundary conditions at either end of the vegetation layer are matched. The far field backscatter coefficients are computed for both horizontal and vertical polarizations. The mean and variance of the dielectric fluctuations are calculated with the aid of Peake's model for the dielectric constant of vegetation. The theory is matched to experimental data taken from a corn field. The resulting values for the correlation parameters are then used to monitor the growth pattern of the corn field over a period of time. Comparisons between the theoretical and experimental results over this time period are shown. The theory is also matched to experimental data from spring and fall deciduous trees.
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The theory described is the result of in-house work and represents an application of the renormalization technique for studying scattering from certain types of vegetation. A solution for the mean wave is obtained by using the bilocal approximation of the Dyson's equation. A Fourier transform of the dyadic Green's function is used to compute a solution utilizing an anisotropic correlation function for the random dielectric fluctuations. The scattered waves are computed from the mean wave and finally the radar backscatter coefficient is calculated. The influence of a rough surface under the vegetation is considered by using a noncoherent technique.

This task was performed under the supervision of Dr. Frederick Rohde, Team Leader, Center for Theoretical and Applied Physical Sciences; Mr. Melvin Crowell, Jr., Director, Research Institute.

COL Daniel L. Lycan, CE and COL Edward K. Wintz, CE were Commanders and Directors and Mr. Robert P. Macchia was Technical Director of the Engineer Topographic Laboratories during the study period.
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SCATTERING FROM A VEGETATION LAYER WITH AN
IRREGULAR VEGETATION SOIL BOUNDARY

INTRODUCTION

This research report presents a theory for analyzing the nature of radar wave scattering from certain types of vegetation. The vegetation is simulated by a continuous random medium, and use is made of a first-order renormalization technique to calculate the radar backscatter coefficient. The influence of an irregular vegetation soil interface has also been considered, using a noncoherent approach.

In a previous report, a derivation was presented of the radar backscatter coefficient from a half space of random medium using a first-order renormalization solution for the scattered wave and an isotropic correlation function for the random dielectric fluctuations. Recently, Fung solved the problem of scattering from a vegetation layer by using a scalar first-order renormalization approach. In his solution, however, he did not consider the existence of a rough vegetation soil boundary. He did consider an anisotropic correlation function in which the horizontal variation is different from the vertical.

Tsang and Kong solved the problem of volume scattering from a half space random medium that contains lateral and vertical fluctuations. A radiative transfer approach was used to calculate the backscattering cross sections up to second order in approximation. This enabled the cross polarized terms to be obtained.

There are two important practical applications for developing and analyzing various radar scattering theories. The first application is radar image simulation of terrain features. In this problem, the radar system parameters and terrain parameters are known and used to calculate a radar response in the form of a gray tone or density.

The scattering theories can be used to compute the radar backscatter coefficient, which in turn is used to calculate gray tone. Using scattering theories in this type of application is straightforward, even though a solution for any one particular scattering problem may be extremely complicated.

The second application is in the field of remote sensing of terrain in which the sensor responses must be used to determine various terrain parameters. Using scattering theories for this application is not straightforward.

However, there are two important uses of scattering theories that bear directly on remote sensing. The first use is a parameter sensitivity study. The theory can be used to analyze the influence of various vegetation, terrain, and radar parameters upon the sensor response. Such parameters as surface roughness, soil moisture, vegetation height, and density could be varied one at a time to determine the influence on the sensor response. This type of analysis should lead to determining what radar parameters are most sensitive to certain terrain parameter changes. This type of analysis assumes the existence of scattering theories that have been developed and compared with existing experimental data.

The second use is to analyze the radar response for two different types of terrain features to see if the two features could be distinguished from each other on an image. Once again, this would assume the existence of scattering theories that have been developed and tested against experimental data. These applications provide the incentive for developing, analyzing, and testing various scattering theories.

In this report, the geometry of the scattering problem to be solved and the basic technique used for the solution will be discussed. In the analysis section, the derivation of the necessary equations will be provided. In the results section, the resulting theory will be compared with existing experimental data, and a study on the sensitivity of the input parameters will be provided.

In this report, the rationalized MKS system of units is used. A line under a symbol will be used to represent a vector quantity. A double line under a symbol will be used to represent a dyadic. A list of the most important symbols is provided at the end of this report.
In figure 1, the scattering geometry of the vegetation problem is shown. A plane wave with a time harmonic of \( \exp(j\omega t) \) is incident from free space at an angle \( \theta_i \) onto a layer of vegetation. The mean thickness of the vegetation is \( L \). The vegetation soil boundary is considered to be randomly rough according to the tangent plane approximation. The vegetation is simulated by a continuous random medium in which \( \varepsilon(\mathbf{r}) \) and \( \sigma(\mathbf{r}) \) represent the three-dimensional random dielectric and conductivity fluctuations, respectively. These fluctuations consist of the sum of an average and a fluctuating component. The standard deviations of the fluctuations are represented by \( \eta_1 \) and \( \eta_2 \). The angle of refraction of the mean wave in the random medium is \( \theta_o \).

\[
\varepsilon(\mathbf{r}) = \varepsilon_0 [\varepsilon_0 + \eta_1 \varepsilon'(\mathbf{r})] \\
\sigma(\mathbf{r}) = \sigma_0 + \eta_2 \sigma'(\mathbf{r})
\]

\[
k_3 = \beta_3 - j\alpha_3
\]
The soil below the vegetation represented by medium 3 is assumed homogeneous with a complex propagation constant \( k_3 \). The magnetic permeability for all three media is assumed to be that of free space. The electric field \( \mathbf{E}_1 \) incident onto the vegetation layer can be written as follows:

\[
\mathbf{E}_1 = \{a_1 a_x + a_2 a_y + a_3 a_z\} e^{-j k_0 (x \sin \theta + z \cos \theta)}
\]

where \( a_x, a_y, \) and \( a_z \) are unit vectors in the \( x, y, \) and \( z \) directions, respectively. The constants \( a_1, a_2, \) and \( a_3 \) are arbitrary, allowing for the consideration of both horizontal and vertical polarizations. A first-order renormalization method will be used to calculate the mean and scattered waves in the random medium. A solution will be developed first for the case where the vegetation soil boundary is a plane interface. The irregular boundary will be considered in a noncoherent manner afterwards. The dielectric and conductivity fluctuation terms \( \varepsilon'(\mathbf{r}) \) and \( \sigma'(\mathbf{r}) \) are considered as being generated by statistically homogeneous random processes. The means and correlation functions of the random processes are defined as follows:

\[
\langle \varepsilon'(\mathbf{r}) \rangle = \langle \sigma'(\mathbf{r}) \rangle = 0
\]

\[
\langle \varepsilon'(\mathbf{r}) \varepsilon'(\mathbf{r}') \rangle = \langle \sigma'(\mathbf{r}) \sigma'(\mathbf{r}') \rangle = e^{-|x - x'|/\ell_x} e^{-|y - y'|/\ell_y} e^{-|z - z'|/\ell_z}
\]

where \( \ell_x, \ell_y, \) and \( \ell_z \) are the correlation distances in the \( x, y, \) and \( z \) directions, respectively. The correlation functions have been chosen to be anisotropic. This representation, with unequal correlation distances, is believed to be closer to reality than an isotropic correlation function. This is because the size of vegetation scatterers in a horizontal plane is not the same as the size of the scatterers in a vertical plane. The mean wave in the random medium is determined from the bilocal approximation of the Dyson's equation:

\[
[\nabla \times \nabla - k^2_0] \langle \mathbf{E}(\mathbf{r}) \rangle - \int_{V'} \langle \xi(\mathbf{r}) \xi(\mathbf{r}') \rangle \langle \mathbf{E}(\mathbf{r}') \rangle \cdot \mathbf{G}(\mathbf{r}, \mathbf{r}') \, d\mathbf{r}' = 0
\]

where \( \langle \mathbf{E}(\mathbf{r}) \rangle \) is the mean wave in the random medium.
\[ \xi(I) = -j \omega \mu_0 \eta_2 \sigma(I) + \omega^2 \mu_0 \varepsilon_0 \eta_1 e'(I). \]

\[ k_s^2 = -j \omega \mu_0 \sigma_s + \omega^2 \mu_0 \varepsilon_0 \varepsilon_s. \]

\( \Gamma(I, I') \) is the dyadic Green's function.

\( V' \) is the volume of the random medium.

In the next section, plane wave solutions will be sought to the Dyson equation using an infinite space dyadic Green's function. It should be noted that the mean wave is located in the integrand, making any solution very difficult. Once the mean wave has been calculated and the appropriate boundary conditions have been matched for the mean waves in all three media, then the scattered wave in the vegetation layer can be calculated from the following equation:

\[ [\nabla \times \nabla - k_s^2] E_s(I) = \xi(I) \langle E(I) \rangle \]

where \( E_s(r) \) is the scattered wave.

The mean wave acts as a source term for the scattered wave, which will be computed using a Fourier transform technique. This in turn will enable the scattered waves in air to be determined. The necessary boundary conditions will be matched, and the backscatter coefficient will be calculated for horizontal and vertical polarizations. The influence of the rough boundary between the vegetation and the soil will be considered apart from the volume scattering solution, using the tangent plane method. The backscatter coefficient for rough surface scattering will be modified by the attenuation through the vegetation. This result will then be added to the volume scattering solution to obtain a final answer for the backscatter coefficient. An elementary permittivity model will be developed that will relate certain parameters of the random media to the parameters of actual vegetation. A discussion of results section will follow in which the theoretical results are compared with actual experimental data. Also, a parameter sensitivity study will be conducted on the theory to determine the influence of various parameter changes upon the final result.
ANALYSIS

In this section, the necessary mathematical derivations will be provided to enable a scattering model for vegetation to be obtained. First, a solution for the mean wave will be presented and then the scattered wave will be calculated.

MEAN WAVE SOLUTION

The first step in obtaining a solution to the Dyson equation is to write the dyadic Green's function:

\[
\Gamma(\mathbf{r}, \mathbf{r}') = \frac{\nabla \nabla'}{k^2} - \mathbf{1} G_o(\mathbf{r}, \mathbf{r}')
\]  

(1)

where \( \mathbf{1} \) is the unit dyadic, and \( G_o(\mathbf{r}, \mathbf{r}') \) is the scalar Green's function that satisfies the following equation:

\[
(\nabla^2 + k^2) G_o(\mathbf{r}, \mathbf{r}') = \delta(\mathbf{r} - \mathbf{r}')
\]

(2)

The solution for equation (2) is usually expressed in terms of \( R = |\mathbf{r} - \mathbf{r}'| \). However, this particular form is not useful when working with an anisotropic correlation function that is expressed in rectangular coordinates. We shall therefore seek a solution to (2) that uses rectangular coordinates. Let \( G_o(\mathbf{r}, \mathbf{r}') \) take the following form:

\[
G_o(\mathbf{r}, \mathbf{r}') = \frac{1}{(2\pi)^3} \int \frac{dk}{k} \mathcal{G}_o(k) e^{i\mathbf{k} \cdot (\mathbf{r} - \mathbf{r}')} 
\]

(3)

where

\[
\mathbf{k} = k_x \mathbf{a}_x + k_y \mathbf{a}_y + k_z \mathbf{a}_z
\]

Substituting (3) into (2) results in a solution for \( \mathcal{G}_o(k) \).

\[
\mathcal{G}_o(k) = \frac{1}{k^2 - k_x^2 - k_y^2 - k_z^2}
\]

(4)
When (4) is placed in (3) and integration is performed in the complex $k_z$ plane, $G_o (\xi, \xi')$ becomes

$$G_o (\xi, \xi') = \frac{j}{8\pi^2} \int_{-\infty}^{\infty} dk_x \int_{-\infty}^{\infty} dk_y \exp[j \{ k_x (x - x') + k_y (y - y') - k_z' |z - z'|\}]$$

\[
\frac{k_z'}{k_z} = \frac{\sqrt{k_x^2 - k_y^2 - k_z^2}}{k_z}
\]

Transforming (5) into polar form results in

$$G_o (\xi, \xi') = \frac{j}{8\pi^2} \int_0^{2\pi} dk \int_0^{2\pi} d\theta \exp[j \{ k (x - x') \cos \theta + k (y - y') \sin \theta - \sqrt{k_x^2 - k_y^2 - k_z^2} \} |z - z'|]$$

\[
k = \sqrt{k_x^2 + k_y^2} \quad k_x = k \cos \theta \quad k_y = k \sin \theta
\]

When (6) is used in (1), the dyadic Green’s function becomes

$$\Gamma (\xi, \xi') = \frac{j}{8\pi^2} \int_0^{2\pi} dk \int_0^{2\pi} d\theta \frac{k}{\sqrt{k_x^2 - k_y^2}} \left\{ C(k, \theta) B(k, \theta) / k_z^2 - 1 \right\}$$

\[
\exp[j \{ k (x - x') \cos \theta + k (y - y') \sin \theta - \sqrt{k_x^2 - k_y^2 - k_z^2} \} |z - z'|]
\]

where

$$C (k, \theta) = a_x (jk \cos \theta) + a_y (jk \sin \theta) + a_z f_2 (k)$$

$$B (k, \theta) = a_x (-jk \cos \theta) + a_y (-jk \sin \theta) + a_z f_1 (k)$$
Plane wave solutions to the Dyson equation take the following form:

\[
\langle E(\mathbf{r}) \rangle = A e^{-j k_0 \cdot \mathbf{r}}
\]

The vector \( A \) can be obtained by matching boundary conditions. The mean wave is seen to propagate with an effective propagation constant \( k_0 \), which must be determined. When the above equation is placed in the Dyson equation along with the dyadic Green's function given by (7), the following equation is obtained when the cross correlation terms between the dielectric and conductivity are ignored:

\[
\begin{align*}
\hat{\nabla} \times \hat{\nabla} - k_o^2 \hat{\mathbf{e}} \cdot \nabla e^{-j k_0 \cdot \mathbf{r}} &= 0 \\
\hat{\mathbf{e}} &= \frac{k_a^2}{k_o^2} \left[ 1 - \frac{1}{(2\pi)^3} \mathbf{M} \right] \\
\mathbf{M} &= j \pi (\eta_2^2 \eta_1^2 - k_o^2 \eta_1^2) \int \mathbf{d} \mathbf{r}' \int_0^1 dk \int_0^{2\pi} d\theta \\
&\quad \cdot d(k)e^{-j(k \cdot \mathbf{r})} \Phi_{\mathbf{e}}(k, \theta) \Phi_{\mathbf{e}}(k, \theta) e^{-j k_0 \cdot \mathbf{r}} \\
&\quad \cdot e^{+j k_0 \cdot (\mathbf{r} - \mathbf{r}')} \left[ \mathcal{C}(k, \theta) \mathcal{P}(k, \theta) \right] \exp \left[ j \left( k(x - x') \cos \theta - k(y - y') \sin \theta \right) \right] \\
&\quad \cdot \sqrt{k_a^2 - k^2} \left[ z - z' \right] \\
&\quad + k(y - y') \sin \theta - \sqrt{k_a^2 - k^2} \left[ z - z' \right] 
\end{align*}
\]
\[ d(k) = \frac{k}{\sqrt{k_a^2 - k^2}} \]

\[ \eta = \frac{\sqrt{\mu_0}}{\varepsilon_o} \]

The integral in \( \mathcal{L} \) is allowed to be over all space. For the case where \( \ell_a \) is very small this should be a good approximation except for the points extremely close to either boundary. By carrying out the integration in \( \mathcal{L} \), one obtains a result for \( M_{ij} \) that represents the \( ij \)th element of the dyadic \( M \).

\[
M_{ij} = 4K a \delta_{x_y} \int_0^1 \frac{dk}{k} \int_0^{2\pi} \, d\theta \, d(k) \left\{ g(k) \right\}
\]

\[
\cdot \left[ F_1(k, \theta) F_{1j}(k, \theta) + k_a^2 \delta_{ij} \right] + h(k) \left[ F_2(k, \theta) F_{2j}(k, \theta) \right.
\]

\[
+ k_a^2 \delta_{ij} \right\} / \left\{ 1 + \sqrt{k_a^2 (k_x + kcos\theta)^2} \right\}
\]

\[
\cdot \left[ 1 + \sqrt{k_a^2 (k_y + ksin\theta)^2} \right\} \quad (11)
\]

where

\[
g(k) = \frac{-1}{1 + j\frac{\eta}{k_a} (k_\theta + \sqrt{k_a^2 - k^2})} \quad h(k) = \frac{1}{j\frac{\eta}{k_a} (k_\theta - \sqrt{k_a^2 - k^2}) - 1}
\]

\[
F_{11}(k, \theta) = jkcos\theta \quad F_{12}(k, \theta) = jksin\theta \quad F_{13}(k, \theta) = j\sqrt{k_a^2 - k^2}
\]

\[
F_{21}(k, \theta) = jkcos\theta \quad F_{22}(k, \theta) = jksin\theta \quad F_{23}(k, \theta) = -j\sqrt{k_a^2 - k^2}
\]

\[
\delta_{ij} = 1 \quad \text{when} \quad i = j
\]

\[
\delta_{ij} = 0 \quad \text{when} \quad i \neq j
\]

\[
K = j\pi (\eta^2 \frac{\ell_a^2}{k_a^2} - k_\theta^2 \ell_a^2) / k_a^2
\]
The form of the incident wave dictates that $k_y = 0$ and $k_{ex} = k_o \sin \theta_i$. This result makes the following elements of the dielectric tensor become equal to zero:

\[ \varepsilon_{12} = \varepsilon_{21} = \varepsilon_{23} = \varepsilon_{32} = 0 \]

The above result is easily shown by considering a transformation of (11) back to rectangular coordinates and recognizing that the integrand is even in $k_y$. Carrying out the indicated differentiation in equation (8) and writing the result in matrix form produces

\[
\begin{bmatrix}
    k_{ex}^2 - k_o^2 \varepsilon_{11} & 0 & -k_{ex} k_o \sin \theta_i - k_o^2 \varepsilon_{13} \\
    0 & k_{ex}^2 + k_o^2 \sin \theta_i - k_o^2 \varepsilon_{22} & 0 \\
    k_{ex} k_o \sin \theta_i - k_o^2 \varepsilon_{13} & 0 & k_o^2 \sin^2 \theta_i - k_o^2 \varepsilon_{33}
\end{bmatrix}
\begin{bmatrix}
    A_x e^{i k e \cdot \mathbf{r}} \\
    A_y e^{i k e \cdot \mathbf{r}} \\
    A_z e^{i k e \cdot \mathbf{r}}
\end{bmatrix} = 0
\]

(12)

In forming the above matrix, use has been made of the fact that $k_y = 0$ and $k_{ex} = k_o \sin \theta_i$. Now, only solutions for $k_{ex}$ are needed. Two solutions can be developed for $k_{ex}$, one for a horizontally polarized wave and one for a vertically polarized wave. For a horizontally polarized wave, one has $A_x = A_z = 0$ and $A_y \neq 0$. The following equation can be used to determine $k_{ex}$ for this case:

\[
\begin{aligned}
\{ k_{ex}^2 + k_o^2 \sin^2 \theta_i - k_o^2 \varepsilon_{22} \} A_y e^{i k e \cdot \mathbf{r}} &= 0 \\
\end{aligned}
\]

(13)

Since the term outside the brackets is not zero, this means that the quantity inside the brackets must be zero.

\[
k_{ex} = \pm k_o \sqrt{\sin^2 \theta_i - \varepsilon_{22}}
\]

(14)

The above result is not an explicit solution for $k_{ex}$ since this quantity also appears in the integral of $M_{22}$. A first approximation for $k_{ex}$ can be obtained by letting $\eta_1 = \eta_2 = 0$. This represents the case where there are no random fluctuations at all.

\[
k_{ex}^{(0)} = -\sqrt{k^2 - k_o^2 \sin^2 \theta_i}
\]
The above value can be used to compute $M_{22}$, which in turn can be used to calculate a new value of $k_{ez}$ that will be called $k_h$. The minus sign on the square root is chosen over the plus sign to consider waves propagating in the minus $z$ direction.

$$k_h = -k_0 \sqrt{\sin^2 \theta_i - \varepsilon_{22}}$$

(15)

When computing $k_h$ in (15), the expression for $k_{ez}^{(0)}$ is used to calculate $M_{22}$. It is interesting to see that even when $k_\alpha$ is real, $k_h$ still comes out complex so that the mean wave decays as it propagates into the medium. This decay has been explained as resulting from multiple scattering. It is not clear, however, how much multiple scattering is being considered. For a vertical polarized wave, $A_x \neq 0, A_y = 0, A_z = 0$. This leads to the following determinant:

$$\begin{vmatrix}
    k_{ez}^2 - k_0^2 & -k_{ez} k_o \sin \theta_i - k_0^2 \varepsilon_{13} \\
    -k_0 k_{ez} \sin \theta_i - k_0^2 \varepsilon_{13} & k_0^2 (\sin^2 \theta_i - \varepsilon_{33})
\end{vmatrix} = 0$$

The above determinant provides another solution for $k_{ez}$:

$$k_{ez} = k_0 \left\{ \hat{\varepsilon}_{13} \sin \theta_i \pm \sqrt{\hat{\varepsilon}_{13}^2 \sin^2 \theta_i - \varepsilon_{33} (\hat{\varepsilon}_{11} \sin^2 \theta_i - \varepsilon_{11} \varepsilon_{33} + \varepsilon_{13}^2)} \right\} \hat{\varepsilon}_{33}$$

(16)

Once again a first approximation for $k_{ez}$ can be obtained for the case where the random dielectric and conductivity fluctuations disappear ($\eta_1 = \eta_2 = 0$).

$$k_{ez}^{(0)} = -\sqrt{k_\alpha^2 - k_0^2 \sin^2 \theta_i}$$

This value for $k_{ez}$ can be used to calculate the elements of the dielectric tensor. These elements are used in (16) to compute a new value for $k_{ez}$, which will be called $k_v$.

$$k_v = -k_0 \left\{ \frac{\hat{\varepsilon}_{13} \sin \theta_i + \sqrt{\hat{\varepsilon}_{13}^2 \sin^2 \theta_i - \varepsilon_{33} (\hat{\varepsilon}_{11} \sin^2 \theta_i - \varepsilon_{11} \varepsilon_{33} + \varepsilon_{13}^2)}}{\hat{\varepsilon}_{33}} \right\}$$
The sign associated with the square root has been chosen as minus in order to consider waves propagating in the minus \( z \) direction. The effective propagation constant for the mean wave has been determined, and now the amplitude must be calculated by matching appropriate boundary conditions. The total mean electric field in air can be written as:

\[
E_1(r) = \begin{cases} 
[a_1 \hat{a}_x + a_2 \hat{a}_y + a_3 \hat{a}_z] e^{i k_0 z \cos \theta_i} \\
+ [R_1 \hat{a}_x + R_2 \hat{a}_y + R_3 \hat{a}_z] e^{-i k_0 z \cos \theta_i} \end{cases} e^{-j k_0 x \sin \theta_i}
\]

\( z \geq 0 \) (17)

The first bracketed term in (17) is the incident wave, and the second bracketed term is the reflected wave. The unknowns in (17) are represented by \( R_1, R_2, \) and \( R_3 \). However, it will not be necessary to obtain an explicit solution for them since they are not needed in determining the scattered waves. The total mean electric field in the vegetation \((E_2(r))\) can be written in the following form, using previous results:

\[
E_2(r) = \begin{cases} 
T_2 \hat{a}_y e^{p_1 z} e^{i q_1 z} + (T_{1x} \hat{a}_x + T_{3z} \hat{a}_z) e^{p_2 z} e^{i q_2 z} + V_2 \hat{a}_y e^{p_2 z} e^{i q_2 z} \\
+ (V_{1x} \hat{a}_x + V_{3z} \hat{a}_z) e^{p_2 z} e^{i q_2 z} \end{cases} e^{-j k_0 x \sin \theta_i}
\]

\(-L \leq z \leq 0 \) (18)

where

\[
p_1 + jq_1 = -jk_h \\
p_1 = \text{Im}(k_h) \quad q_1 = -\text{Re}(k_h)
\]

also

\[
p_2 + jq_2 = -jk_v \\
p_2 = \text{Im}(k_v) \quad q_2 = -\text{Re}(k_v)
\]
In (18), both upward and downward waves have been considered. The unknowns are represented by the amplitudes $T_1$, $T_2$, $T_3$, $V_1$, $V_2$, and $V_3$. An explicit solution for each of these is required in terms of propagation constants, medium characteristics, and layer thickness. The explicit solution is needed to compute the scattered waves. An expression for the mean wave in the homogeneous soil medium can be written as

$$E_3(z) = [W_1 e^{ik_1 x} + W_2 e^{ik_2 x} + W_3 e^{ik_3 x}] e^{jz}$$

$$z < -L \quad (19)$$

In the soil, the mean wave propagates in the minus $z$ direction and the constants $W_1$, $W_2$, and $W_3$ are the unknown amplitudes. Once again, no explicit solution will be required for these amplitudes since they are not needed to compute the scattered waves. To compute the six amplitudes of the mean wave in the vegetation, the following boundary conditions are used:

$$E_{1x} = E_{2x} \text{ at } z = 0$$

$$E_{1y} = E_{2y} \text{ at } z = 0$$

$$\frac{\partial E_{1z}}{\partial y} - \frac{\partial E_{1x}}{\partial z} = \frac{\partial E_{2x}}{\partial y} - \frac{\partial E_{2y}}{\partial z} \text{ at } z = 0$$

$$\frac{\partial E_{1x}}{\partial z} - \frac{\partial E_{1x}}{\partial x} = \frac{\partial E_{2x}}{\partial z} - \frac{\partial E_{2y}}{\partial x} \text{ at } z = 0$$

$$E_{2x} = E_{3x} \text{ at } z = -L$$

$$E_{2y} = E_{3y} \text{ at } z = -L$$

$$\frac{\partial E_{2x}}{\partial y} - \frac{\partial E_{2x}}{\partial z} = \frac{\partial E_{3x}}{\partial y} - \frac{\partial E_{3y}}{\partial z} \text{ at } z = -L$$

16
\[
\frac{\partial E_{2x}}{\partial z} - \frac{\partial E_{3z}}{\partial x} = \frac{\partial E_{3x}}{\partial z} - \frac{\partial E_{3z}}{\partial x} \quad \text{at } z = -L
\]

\[D_{1x} = D_{2x} \text{ at } z = 0\]

\[D_{2z} = D_{3z} \text{ at } z = -L\]

\[D_1 = \epsilon_0 E_1\]

\[D_2 = \epsilon_v E_{2x} a_x + \epsilon_h E_{2y} a_y + \epsilon_v E_{2z} a_z\]

where

\[\epsilon_h = \frac{q_1^2 - p_1^2}{\omega^2 \mu_o} \text{ evaluated at } \theta_1 = 0^\circ\]

\[\epsilon_v = \frac{q_2^2 - p_2^2}{\omega^2 \mu_o} \text{ evaluated at } \theta_1 = 0^\circ\]

\[D_3 = \epsilon_3 E_3\]

where \(\epsilon_3\) is the dielectric constant of the soil. Two divergence conditions will also be used, along with the above boundary conditions.

\[\nabla \cdot E_1 = 0 \quad \text{and} \quad \nabla \cdot E_3 = 0\]

When the equations for the mean fields given by (17) through (19) are placed in the boundary conditions, the result is 12 equations and 12 unknowns. Explicit solutions are only required for the six amplitudes associated with the mean wave in the random medium. Solving for these six values yields the following results:

\[T_2 = \frac{2 j k_o a_1 a_2 \cos \theta_1}{a_1 a_2^2 - a_{11} a_{22}}\]  

(20)
\[ V_2 = \frac{-2jk_o \cos \theta_i a_1 b_1}{a_2 b_1 - b_1 a_2} \]  
\hspace{1cm} (21)

\[ T_1 = \frac{jk_o b_1 \left\{ 2a_1 \cos \theta_i + \sin \theta_i (1 - \epsilon_o / \epsilon_r) [a_3 + a_1 \tan \theta_i] \right\}}{b_2 b_1 - b_1 b_2} \]  
\hspace{1cm} (22)

\[ V_1 = \frac{-jk_o b_1 \left\{ 2a_1 \cos \theta_i + \sin \theta_i (1 - \epsilon_o / \epsilon_r) [a_3 + a_1 \tan \theta_i] \right\}}{b_2 b_1 - b_1 b_2} \]  
\hspace{1cm} (23)

\[ T_3 = \frac{1}{\epsilon_r \left\{ \epsilon_o e^{p_2 L e^{jq_2 L}} - e^{p_2 L e^{jq_2 L}} - \epsilon_o e^{p_2 L e^{jq_2 L}} \right\}} \cdot \left\{ [a_3 - \tan \theta_i (T_1 + V_1 - a_1)] \right\} \]  
\hspace{1cm} (24)

\[ V_3 = \frac{\epsilon_o}{\epsilon_r} [a_3 - \tan \theta_i (T_1 + V_1 - a_1)] - T_3 \]  
\hspace{1cm} (25)

The parameters used in the above equations are defined below:

\[ a_{11} = (jk_o \cos \theta_3 - p_1 - jq_1) e^{p_2 L e^{jq_2 L}} \]

\[ a_{12} = (jk_o \cos \theta_3 + n_2 + jq_1) e^{p_2 L e^{jq_2 L}} \]

\[ a_{21} = jk_o \cos \theta_i + p_1 + jq_1 \]

\[ a_{22} = jk_o \cos \theta_i - p_1 - jq_1 \]

\[ b_{11} = e^{p_2 L e^{jq_2 L}} (p_2 + jq_2 - \alpha) \]

\[ b_{12} = e^{p_2 L e^{jq_2 L}} (p_2 + jq_2 + \alpha) \]
\[ b_{21} = p_2 + jq_2 + jk_0 [\cos \theta_1 + \sin \theta_1 \tan \theta_1 (1 - \epsilon_0 / \epsilon_r)] \]

\[ b_{22} = - \{ p_2 + jq_2 - jk_0 [\cos \theta_1 + \sin \theta_1 \tan \theta_1 (1 - \epsilon_0 / \epsilon_r)] \} \]

where

\[ \alpha = j \{ k_3 \cos \theta_3 + k_0^2 \sin^2 \theta_1 (1 - \epsilon_3 / \epsilon_r) / (k_3 \cos \theta_3) \} \]

Now that the mean waves have been fully determined, the scattered waves can be calculated. The scattered waves in the upper medium (air) will then be used to compute the backscatter coefficient.

**SCATTERED WAVE SOLUTION**

In the random medium, the scattered or incoherent field is calculated from the following equation:

\[ [\nabla \times \nabla - k_s^2] E_s^{(2)}(z) = \xi(z) < \mathcal{E}(z) > \]  

(26)

For our problem, the mean wave \( < \mathcal{E}(z) > \) is given by \( \mathcal{E}_0(z) \) as shown in (18). A superscript 2 is used with \( E_s^{(2)}(z) \) to indicate clearly the scattered field in the random medium. A solution for this scattered field can be obtained by using a two-dimensional Fourier transform.

\[ E_s^{(2)}(z) = \frac{1}{(2\pi)^2} \int d \mathbf{k} \mathcal{G}_s(k_x, z)e^{i \mathbf{k} \cdot \mathbf{r}} \]  

(27)

where

\[ \mathbf{k} = k_x \mathbf{a}_x + k_y \mathbf{a}_y \quad \mathbf{r} = x \mathbf{a}_x + y \mathbf{a}_y \]

\[ \mathcal{G}_s(k_x, z) = G_{sx}(k_x, z) \mathbf{a}_x + G_{sy}(k_x, z) \mathbf{a}_y + G_{sz}(k_x, z) \mathbf{a}_z \]

When the complete expression for the mean wave in the random medium as given by (18) is placed in (26), a term of the form \( \xi(z) \exp(-jk_0 z \sin \theta_1) \) on the right side results. This term will be written as a two-dimensional Fourier transform.
\[ S(k_t, z) = \int dk_t \xi(\xi) e^{jk \cdot x \cdot \sin \theta_t} e^{-i(k_t \cdot x)} \]

\[ \xi(\xi) e^{jk \cdot x \cdot \sin \theta_t} = \frac{1}{(2\pi)^2} \int dk_t S(k_t, z) e^{i(k_t \cdot x)} \]  \hspace{1cm} (28)

When (27) and (28) are placed in (26) and the result is put in matrix form,

\[
\begin{bmatrix}
(k_y^2 - k_a^2 - D_z^2) & -k_x k_y & jk_x D_z \\
-k_x k_y & (k_x^2 - k_a^2 - D_z^2) & jk_y D_z \\
jk_x D_z & jk_y D_z & (k_x^2 + k_y^2 - k_a^2)
\end{bmatrix}
\begin{bmatrix}
G_{xx} \\
G_{xy} \\
G_{zz}
\end{bmatrix}
= 
\begin{bmatrix}
f_x(z) \\
f_y(z) \\
f_z(z)
\end{bmatrix}
\]  \hspace{1cm} (29)

where \( D_z \) represents the differential operator \( \frac{d}{dz} \). The quantities on the right side of (29) are defined below:

\[
f_x(z) = S(k_t, z)[T_1 e^{p_2 x} e^{i q_2 z} + V_1 e^{p_2 x} e^{i q_2 z}] \]

\[
f_y(z) = S(k_t, z)[T_2 e^{p_1 x} e^{i q_1 z} + V_2 e^{p_1 x} e^{i q_1 z}] \]

\[
f_z(z) = S(k_t, z)[T_3 e^{p_2 x} e^{i q_2 z} + V_3 e^{p_2 x} e^{i q_2 z}] \]

Solutions for the quantities \( G_{xx}, \ G_{xy}, \) and \( G_{zz} \) can be obtained by solving the three differential equations in (29) using the method of variation of parameters.

\[
G_{xx}(k_t, z) = A_1 e^{j k_x' z} + A_2 e^{j k_y' z} + \frac{1}{2 j k_x' k_y'} \left[ \int_L^f f(z) e^{j k_z' z} dz \right] \\
- e^{j k_z' z} = \frac{1}{2 j k_x' k_y'} \left[ \int_L^f f(z) e^{j k_z' z} dz \right] e^{j k_z' z} \]  \hspace{1cm} (30)
The quantities $A_1$, $A_2$, $B_1$, and $B_2$ are not functions of $z$ and at present, are unknown. Although the solutions for $G_{xx}$, $G_{sy}$, and $G_{ss}$ are rather formidable in appearance, it will be found after some mathematical manipulations that a solution will emerge. The three components of the scattered electric field in the upper medium ($z > 0$) can be written in the following form:

$$G_{sy}(k_z, z) = B_1 e^{jk_zz} + B_2 e^{jk_zz} - \frac{1}{2jk_zk_s^2} \left[ \int_L^z h(z) e^{jk_z'z} dz \right] e^{jk_zz}$$

$$G_{ss}(k_z, z) = \frac{(k_x A_1 + k_y B_1)}{k'_z} e^{jk'_z z} - \frac{(k_x A_2 + k_y B_2)}{k'_z}$$

$$+ \frac{e^{jk'_z z}}{2jk_z^2 k_s^2} \left[ \int z \left[ k_x f(z) - k_y h(z) \right] e^{jk_z'z} dz \right]$$

$$+ \frac{e^{jk'_z z}}{2jk_z^2 k_s^2} \int z \left[ k_x f(z) - k_y h(z) \right] e^{jk_z'z} dz - \frac{f_x(z)}{k_z'^2}$$

$$k'_z = \sqrt{k_s^2 - k_x^2 - k_y^2}$$

$$f(z) = f_x(z) \left[ k_x^2 - k_z^2 \right] - k_y k_x f_y(z) + jk_x D_z f_z(z)$$

$$h(z) = \frac{k_y(k_x^2 - k_y^2)}{k_x} f_x(z) - (k_x^2 - k_y^2) f_y(z) - jk_y D_z f_z(z)$$

$$+ \frac{k_y}{k_x} (k_x^2 + k_y^2 - k_z^2) f_x(z)$$

The quantities $A_1$, $A_2$, $B_1$, and $B_2$ are not functions of $z$ and at present, are unknown. Although the solutions for $G_{xx}$, $G_{sy}$, and $G_{ss}$ are rather formidable in appearance, it will be found after some mathematical manipulations that a solution will emerge. The three components of the scattered electric field in the upper medium ($z > 0$) can be written in the following form:
\[ E_{sx}^{(1)}(r) = \frac{1}{(2\pi)^2} \int dk_x A_x(k_x)e^{i(k_x x + k_y y - k_{1z} z)} \]  \hspace{1cm} (33)

\[ E_{sy}^{(1)}(L) = \frac{1}{(2\pi)^2} \int dk_y A_y(k_y)e^{i(k_x x + k_y y - k_{1z} z)} \]  \hspace{1cm} (34)

\[ E_{sz}^{(1)}(L) = \frac{1}{(2\pi)^2} \int dk_z A_z(k_z)e^{i(k_x x + k_y y - k_{1z} z)} \]  \hspace{1cm} (35)

where
\[ k_{1z} = \sqrt{k_0^2 - k_x^2 - k_y^2} \]

The parameter \( k_{1z} \) is obtained by taking any one of the field components given above and putting it into the free space scalar wave equation. The superscript 1 is used to refer to the field in the upper medium, which is air. Therefore, \( E_{sx}^{(1)}(L) \) would indicate the x component of the scattered electric field in air. The components of the scattered electric field in the soil (\( z < -L \)) can be written as follows:

\[ E_{sx}^{(3)}(L) = \frac{1}{(2\pi)^2} \int dk_x C_x(k_x)e^{i(k_x x + k_y y + k_{3z} z)} \]  \hspace{1cm} (36)

\[ E_{sy}^{(3)}(L) = \frac{1}{(2\pi)^2} \int dk_y C_y(k_y)e^{i(k_x x + k_y y + k_{3z} z)} \]  \hspace{1cm} (37)

\[ E_{sz}^{(3)}(L) = \frac{1}{(2\pi)^2} \int dk_z C_z(k_z)e^{i(k_x x + k_y y + k_{3z} z)} \]  \hspace{1cm} (38)

where
\[ k_{3z} = \sqrt{k_0^2 - k_x^2 - k_y^2} \]

The superscript 3 refers to the computation of the scattered fields in the soil. The expression for \( k_{3z} \) is obtained by putting any one of the field components into the
scalar wave equation, which has a propagation constant $k_3$. The unknowns associated with the scattered waves are represented by $A_x$, $A_y$, $A_z$, $A_1$, $A_2$, $B_1$, $B_2$, $C_x$, $C_y$, and $C_z$. The only unknowns for which an explicit solution is needed are $A_x$, $A_y$, and $A_z$. Since the only interest is in computing the backscattered far field in air, complete solutions for the scattered fields in the other mediums are not needed. The boundary conditions that the scattered waves must satisfy are provided below:

\[
\begin{align*}
\frac{\partial E^{(1)}_{sx}}{\partial y} - \frac{\partial E^{(1)}_{sy}}{\partial z} &= \frac{\partial E^{(2)}_{sx}}{\partial y} - \frac{\partial E^{(2)}_{sy}}{\partial z} & \text{at } z = 0 \\
\frac{\partial E^{(1)}_{sx}}{\partial z} - \frac{\partial E^{(1)}_{sx}}{\partial y} &= \frac{\partial E^{(2)}_{sx}}{\partial z} - \frac{\partial E^{(2)}_{sx}}{\partial y} & \text{at } z = 0 \\
E^{(2)}_{sx} &= E^{(3)}_{sx} = -L \\
E^{(2)}_{sy} &= E^{(3)}_{sy} & \text{at } z = L \\
\frac{\partial E^{(2)}_{sx}}{\partial y} - \frac{\partial E^{(2)}_{sy}}{\partial z} &= \frac{\partial E^{(3)}_{sx}}{\partial y} - \frac{\partial E^{(3)}_{sy}}{\partial z} & \text{at } z = -L \\
\frac{\partial E^{(2)}_{sx}}{\partial z} - \frac{\partial E^{(2)}_{sx}}{\partial y} &= \frac{\partial E^{(3)}_{sx}}{\partial z} - \frac{\partial E^{(3)}_{sx}}{\partial y} & \text{at } z = -L
\end{align*}
\]

The boundary conditions given above, along with the divergence equations in the two homogeneous media allow ten independent equations to be formulated.

\[k_x A_x + k_y A_y = k_{1z} A_z \quad (39)\]

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\[ k_x C_x + k_y C_y = -k_3 z C_z \quad (40) \]

\[ A_x = A_1 + A_2 + \frac{1}{2jk_z^2} \int_{-L}^{0} f(z)[e^{jk_z^2 z} - e^{jk_z^2 z}] dz \quad (41) \]

\[ A_y = B_1 + B_2 + \frac{1}{2jk_z^2} \int_{-L}^{0} h(z)[e^{jk_z^2 z} - e^{jk_z^2 z}] dz \quad (42) \]

\[ C_x = e^{jk_3 z L} \{ A_1 e^{jk_z L} + A_2 e^{-jk_z L} \} \quad (43) \]

\[ C_y = e^{jk_3 z L} \{ B_1 e^{jk_z L} + B_2 e^{-jk_z L} \} \quad (44) \]

\[ jk_y A_z + jk_1 z A_y = jk_y G_{sz}(k, \theta) - \frac{\partial G_{sy}}{\partial z} \bigg|_{z = 0} \quad (45) \]

\[ k_1 z A_x + k_x A_z = j \frac{\partial G_{sx}}{\partial z} \bigg|_{z = 0} + k_x G_{sx}(k, \theta) \quad (46) \]

\[ jk_y G_{sz}(k, -L) - \frac{\partial G_{sy}}{\partial z} \bigg|_{z = -L} = jk_y C_x e^{jk_3 z L} - jk_3 z C_y e^{jk_3 z L} \quad (47) \]

\[ \frac{\partial G_x}{\partial z} \bigg|_{z = -L} = -jk_x G_{sx}(k, -L) = jk_3 z C_x e^{-jk_3 z L} - jk_x C_y e^{-jk_3 z L} \quad (48) \]

A solution for \( A_x, A_y, \) and \( A_z \) using the above equations would be quite difficult for arbitrary values of \( k_x \) and \( k_y \). However, since we are only interested in the backscattered far field in air, a basic expression for the far field can be obtained by using the Stratton-Chu integral as modified by Silver.\(^4\) When equations (33) through (35) are evaluated on the surface \((z = 0)\) and placed into the Stratton-Chu integral, the following equation for the scattered far field \((E_{sf})\) will result:

\[^4\text{S. Silver, Microwave Antenna Theory and Design, McGraw Hill, New York, 1947.}\]

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\[ E_{sf} = \frac{2 \cos \theta (j k_o e^{j k_o R})}{4 \pi R} \left[ A_x(k_o \sin \theta, 0) + A_y(k_o \sin \theta, 0) + A_z(k_o \sin \theta, 0) \right] \]

\[ + \frac{2}{k_z} A_z(k_o \sin \theta, 0) \]  

(49)

Where \( R \) is the distance from the origin of the coordinate system to the field point where \( E_{sf} \) is required. It can now be seen that solutions for \( A_x \), \( A_y \), and \( A_z \) are only needed for values of \( k_x = k_o \sin \theta \) and \( k_y = 0 \). When the appropriate expressions for the derivatives are substituted into equations (45) through (48) and we let \( k_y = 0 \), the 10 equations (39) through (48) become.

\[ k_x A_x = k_1 A_z \]  

(50)

\[ k_x C_x = k_3 C_x \]  

(51)

\[ A_x = A_1 + A_2 + \frac{I_1}{2 j k_z k_x^2} \]  

(52)

\[ A_y = B_1 + B_2 + \frac{I_2}{2 j k_z k_x^2} \]  

(53)

\[ C_x = e^{j k_x z_L} \left[ A_1 e^{j k_x^L} + A_2 e^{-j k_x^L} \right] \]  

(54)

\[ C_y = e^{j k_x z_L} \left[ B_1 e^{j k_x^L} + B_2 e^{-j k_x^L} \right] \]  

(55)

\[ j k_1 z A_y = j k_x B_1 - j k_x^z B_z - \frac{I_3}{2 k_x^2} \]  

(56)

\[ k_1 z A_x + k_x A_z = k_x A_1 - k_x A_2 + \frac{1}{2 j k} \left( 1 + k_z^2 / k_x^2 \right) I_4 \]

\[ + \frac{k_x^2}{k_z^2} (A_1 - A_2) - \frac{k_x f(0)}{k_x^2} \]  

(57)
\[-k_x' B_1 e^{jk'zL} + k_x' B_2 e^{jk'zL} = k_3 z e^{jk_3 zL}\]  
\[A_2 (k_z'^3 + k_x' k_z') e^{jk_z' L} - A_1 (k_z'^3 + k_x' k_z') e^{jk_z' L} = k_x f_3 (-L)\]

The quantities \(I_1, I_2, I_3,\) and \(I_4\) used in the above equations are integrals in \(z\) and are defined below:

\[I_1 = \int_{-L}^{0} f(z) \left[ e^{jk_1 z} - e^{-jk_1 z} \right] dz\]

\[I_2 = \int_{-L}^{0} h(z) \left[ e^{-jk_1 z} - e^{jk_1 z} \right] dz\]

\[I_3 = \int_{-L}^{0} h(z) \left[ e^{jk_3 z} + e^{-jk_3 z} \right] dz\]

\[I_4 = \int_{-L}^{0} f(z) \left[ e^{jk_3 z} + e^{-jk_3 z} \right] dz\]

The functions \(f(z), h(z)\) and \(k_z'\) can be evaluated at \(k_y = 0\), before integration takes place. When all the above equations are used to solve for \(A_x, A_y,\) and \(A_z\), the following results are derived:

\[A_x (k_x, 0) = b_1 f_z (-L) + b_2 f_z (0) + \int_{-L}^{0} f(z) \{ b_5 e^{jk_1 z} + b_6 e^{-jk_1 z} \} dz\]  

(60)
\[ A_3(k_3, o) = \frac{-1}{j k_3^2 k_1(1 + \tilde{a}_2^2) + k_3^2(1 - \tilde{a}_1^2)} \int_{-1}^{0} h(z) \]

\[ \text{d} \left[ \tilde{a}_1 e^{-j k_3 z} + e^{j k_3 z} \right] dz \quad (61) \]

\[ A_2(k_3, o) = k_3 \left\{ b_1 f_2(-L) + b_2 f_2(o) + \int_{-1}^{0} f(z) \right\} \]

\[ \left[ e^{-j k_3 z} + \frac{b_6 e^{-j k_3 z}}{k_{1z}} \right] dz \right\} / k_{1z} \quad (62) \]

In the above three equations, \( k_3 \) is to be evaluated at \( k_3 \sin \theta_1 \). The new quantities introduced into the above equations are defined below:

\[ \tilde{a}_1 = (k_3' - k_3 z) e^{-2j k_3^2 L} / (k_3^2 + k_3 z) \]

\[ \tilde{a}_2 = \frac{\left[ k_3^2(1 + k_3^2 k_3') - k_3^2(1 + k_3^2 k_3') \right]}{k_3^2(k_3^2 + k_3^2 k_3') + k_3^2(1 + k_3^2)} e^{-2j k_3^2 L} \]

\[ \tilde{a}_3 = \frac{k_3^2 e^{-2j k_3^2 L}}{k_3^2(k_3^2 + k_3^2 k_3') + k_3^2(1 + k_3^2)} \]

\[ \tilde{a}_4 = k_3^2(1 + k_3 z) \]

\[ \tilde{a}_5 = k_4(1 + k_3^2 k_3') \left( 1 - \tilde{a}_2 \right) \]

\[ \tilde{a}_6 = k_6(1 + k_3^2 k_3') \tilde{a}_3 \]

\[ \tilde{a}_7 = k_7(1 + k_3^2 k_3') / (2j k_3^2) \]

\[ \tilde{a}_8 = \tilde{a}_5 + \tilde{a}_4 (\tilde{a}_2 + 1) \]
If the receiver in the far field is sensitive to a unit polarization vector \( \mathbf{e}_r \), then the received field (\( E_R \)) will be

\[
E_R = \mathbf{e}_r \cdot E_{sf}
\]

where \( \mathbf{e}_r \) in general has three components (\( \mathbf{e}_r = \mathbf{e}_{rx} + \mathbf{e}_{ry} + \mathbf{e}_{rz} \)). The next step in calculating the backscatter coefficient is to determine the statistical average of \( E_R E_{R}^* \), using (49).

\[
\langle E_R E_{R}^* \rangle = \frac{4k_0^2 \cos^2 \theta_i}{16\pi^2 R^2} \left\{ \mathbf{e}_{rx}^* \mathbf{e}_{rx}^* \langle A_x A_x^* \rangle + 2\Re(\mathbf{e}_{rx}^* \mathbf{e}_{ry}^*) - \langle A_x A_y^* \rangle + 2\Re(\mathbf{e}_{rx}^* \mathbf{e}_{rz}^*) \langle A_x A_z^* \rangle \right\}
\]

\[
+ \langle A_x A_y^* \rangle + 2\Re(\mathbf{e}_{ry}^* \mathbf{e}_{rz}^*) \langle A_y A_z^* \rangle + \mathbf{e}_{ry}^* \mathbf{e}_{rz}^*
\]

\[
+ \langle A_y A_y^* \rangle + 2\Re(\mathbf{e}_{ry}^* \mathbf{e}_{rz}^*) \langle A_y A_z^* \rangle + \mathbf{e}_{ry}^* \mathbf{e}_{rz}^*
\]

\[
+ \langle A_z A_y^* \rangle + 2\Re(\mathbf{e}_{rz}^* \mathbf{e}_{ry}^*) \langle A_z A_y^* \rangle + \mathbf{e}_{rz}^* \mathbf{e}_{ry}^*
\]

\[
\langle A_z A_z^* \rangle
\]

\[
(63)
\]
The brackets \(-\ldots\ldots\ldots\ldots\) around \(E_R E_R^*\) are used to indicate the calculation of the statistical average. The possibility of \(e_{rx}\) and \(e_{rz}\) being complex is anticipated, with \(\xi_{y'}\) always being real. What is required now is the computation of each of the six terms inside the brackets of (63). The determination of \(<A_y A_y^*\) will be considered first:

\[
<A_y A_y^*> = k_a^2 k_e^2 M_0 M_0^* \int_{-L}^{0} dz \int_{-L}^{0} dz' \int_{-\infty}^{\infty} dx \cdot \int_{-\infty}^{\infty} dx' \int_{-\infty}^{\infty} dy \int_{-\infty}^{\infty} dy' \langle \xi (I) \xi^* (I') \rangle
\]

where

\[
\langle \xi (I) \xi^* (I') \rangle = (\omega^2 \mu_o^2 \eta_2^2 + k_o^4 \eta_2^2) e^{-|x - x'|/\xi x e^{-|y - y'|/\xi y e^{-|z - z'|/\xi z}}
\]

\[
\hat{f}(z, z') = \sum_{T_1 T_2 T_2^* e^{D_{1z}} e^{D_{1z}'}} + \tilde{a}_1 T_2^* e^{D_{2z}} e^{D_{2z}'}
\]

\[
+ \tilde{a}_1 V_2^* T_2^* e^{D_{2z}} e^{D_{2z}'} + \tilde{a}_1 V_2^* e^{D_{1z}} e^{D_{1z}'}
\]

\[
+ \tilde{a}_1 V_2^* e^{D_{1z}} e^{D_{1z}'} + \tilde{a}_1 V_2^* e^{D_{2z}} e^{D_{2z}'}
\]

\[
+ \tilde{a}_1 T_2^* e^{D_{2z}} e^{D_{2z}'} + \tilde{a}_1 T_2^* e^{D_{2z}} e^{D_{2z}'}
\]

\[
+ \tilde{a}_1 V_2^* e^{D_{1z}} e^{D_{1z}'} + \tilde{a}_1 V_2^* e^{D_{2z}} e^{D_{2z}'}
\]

\[
+ V_2^* T_2^* e^{D_{1z}} e^{D_{1z}'} + V_2^* T_2^* e^{D_{2z}} e^{D_{2z}'}
\]

\[
+ V_2^* T_2^* e^{D_{1z}} e^{D_{1z}'} + V_2^* T_2^* e^{D_{1z}} e^{D_{1z}'}
\]

\[
\]
where

\[ D_1 = p_1 + j(q_1 - k'_1) \quad \text{and} \quad D_2 = -(p_1 + jq_1 + jk'_1) \]

\[ M_0 = \frac{-1}{jk'_1 k_1 (1 + a_1) + k'_1 (1 - a_1)} \]

The form of \( f(z, z') \) given above appears to be very complicated. However, each of the 16 terms in \( f(z, z') \) consist of simple exponentials in \( z \) and \( z' \) and therefore can be integrated easily. It should be remembered when computing \( D_1 \) and \( D_2 \) that \( p_1 \) and \( q_1 \) are real, but \( k'_1 \) will be complex. Making the substitution that \( u = x - x' \) and \( v = y - y' \) and transforming the \( x \) and \( y \) integrals into integrals in \( u \) and \( v \) produces

\[
<A_y A_y^* > = (\omega^2 \mu^2_0 \eta^2_2 + k_0^2 \eta^2_1)k'_1 k''^2 M_0 M_0^* \int_{-L}^{0} dz \\
\int_{-L}^{0} dz' \int_{-\infty}^{\infty} du \int_{-\infty}^{\infty} dv \int_{-\infty}^{\infty} dx' \int_{-\infty}^{\infty} dy' \\
f(z, z') e^{-2jk_0 usin\theta_i} e^{-1/2k_0 u} e^{-1/2k_0 v} e^{-1/2k_0 l} e^{z} e^{z'} (65)
\]

The integrals in \( x' \) and \( y' \) appear somewhat meaningless. These integrals actually represent the illuminated area in the \( xy \) plane, since it is physically unrealistic to have backscattered energy from a portion of the surface that is not illuminated. Considering the integrals in \( x' \) and \( y' \) to form the illuminated area \( (A_1) \) and carrying out the integrations in \( u \) and \( v \) will yield the following result:

\[
<A_y A_y^* > = \frac{4\pi \mu \nu_1 (\omega^2 \mu^2_0 \eta^2_2 + k_0^2 \eta^2_1)k'_1 k''^2 M_0 M_0^*}{(1 + 4k_0^2 \pi^2 \sin^2 \theta_i)} \\
\int_{-L}^{0} dz \int_{-L}^{0} dz' \hat{f}(z, z') e^{-z} e^{-z'} (66)
\]

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Consider now a typical term of \( \hat{\eta}(z, z') \) which is of the form \( A e^{az} e^{bz'} \) where \( A, a, \) and \( b \) are not functions of \( z \) or \( z' \), and make a transformation of variables from \( z \) and \( z' \) to \( n = z - z' \) and \( z'' = z' \). Then, the results of carrying out the integration for this one term becomes

\[
\int_{-L}^{0} dz \int_{-L}^{0} dz' A e^{az} e^{bx'} e^{-|z - z'|/\xi z} =
\]

\[
\frac{A \xi_z}{(a + b)} \left\{ \frac{1 - \xi_z b + \xi_z (a + b) e^{-L(s + 1/\xi z)} - (1 + \xi_z) e^{-L(s + b)}}{(1 + \xi_z) (1 - \xi_z b)} \right. \\
+ \left. \frac{1 - \xi_z a + \xi_z (a + b) e^{-L(1/\xi z + b)} - e^{-L(s + b)} (1 + b \xi_z)}{(1 + \xi_z) (1 - \xi_z a)} \right\}
\]

When the answer for the integration in \( z \) and \( z' \) given above is used for each of the 16 terms in \( \hat{\eta}(z, z') \), then the final result for \( \langle A_y A_y^* \rangle \) can be written.

\[
\langle A_y A_y^* \rangle = \frac{4\xi_z \xi_z \xi_z \xi_z \xi_z (\omega_2^2 \eta_2^2 + k_0^2 \eta_1^2) k_0^2 k_0^2 M_0 M_0^*}{(1 + 4k_0^2 \eta_2^2 \sin^2 \theta_i)}
\]

\[
\sum_{n = 1}^{16} A_n \left\{ \frac{1 - \xi_z d_n + \xi_z (c_n + d_n) e^{-L(c_n + 1/\xi z)} - (1 + \xi_z c_n) e^{-L(c_n + d_n)}}{(1 + \xi_z c_n) (1 - \xi_z d_n)} \right. \\
+ \left. \frac{1 - \xi_z c_n + \xi_z (c_n + d_n) e^{-L(d_n + 1/\xi z)} - e^{-L(c_n + d_n)} (1 + d_n \xi_z)}{(1 + \xi_z d_n) (1 - \xi_z c_n)} \right\}
\] (67)

The values for the \( A_n \)'s, the \( c_n \)'s, and \( d_n \)'s are provided in appendix A and simply come from the expression for \( \hat{\eta}(z, z') \). Using the methodology for computing \( \langle A_y A_y^* \rangle \), one can calculate all the remaining terms in (63). All of these other terms are given in appendix A. An expression for the radar backscatter coefficient \( \langle q_r^* \rangle \) can be written in terms of \( \langle E_R E_R^* \rangle \).
\[ \sigma_v = \frac{4\pi R^2}{\mathcal{A}_1} \frac{\langle E_R E_R^* \rangle}{\mathbf{E}_1 \cdot \mathbf{E}_1^*} \] (68)

The subscript \( v \) on \( \sigma_v \) is used to indicate a volume scattering result from a plane layer of random media. No consideration for rough surface scattering is given in \( \sigma_v \).

Using (63) and the expression for the incident wave given previously, one can write a final result for \( \sigma_v \):

\[ \sigma_v = \frac{k_0^2 \cos^2 \theta_i}{\pi (a_1 a_1^* + a_2 a_2^* + a_3 a_3^*)} \left\{ e_{xx} e_{xx}^* \alpha_{xx} + 2 \text{Re} [e_{xx} e_{yy} \alpha_{xy}] \\
+ 2 \text{Re} [e_{xx} e_{zz}^* \alpha_{xz}] + e_{yy}^2 \alpha_{yy} + 2k_0 \sin \theta_i \right\} \\
\cdot \left\{ \text{Re} [e_{yy} e_{rz}^* \alpha_{xy} / k_{1z}] + e_{rr} e_{zz}^* \alpha_{zz} \right\} \] (69)

where

\[ \alpha_{xx} = \frac{<A_x A_x^*>}{A_1} \]
\[ \alpha_{xy} = \frac{<A_x A_y^*>}{A_1} \]
\[ \alpha_{xz} = \frac{<A_x A_z^*>}{A_1} \]
\[ \alpha_{yy} = \frac{<A_y A_y^*>}{A_1} \]
\[ \alpha_{zz} = \frac{<A_z A_z^*>}{A_1} \]

Consider now the form of \( \sigma_v \) for horizontal and vertical polarizations. The following parameters are used to describe a wave that is transmitted with horizontal polarization and horizontal polarization is received.
\[ a_1 = 0 \quad e_{rx} = 0 \]
\[ a_2 = 1 \quad e_{ry} = 1 \]
\[ a_3 = 0 \quad e_{rz} = 0 \]

For these parameters, the backscatter coefficient can be given the additional sub-
scripts of HH to indicate horizontal polarization transmit, and horizontal polarization
receive.

\[ \sigma_{HH}^0 = k_0^2 \alpha_{x,y} \cos^2 \theta_i \pi \]  

(70)

The case of vertical polarization transmit, vertical polarization receive can be char-
acterized as follows:

\[ a_1 = \cos \theta_i \quad e_{rx} = \cos \theta_i \]
\[ a_2 = 0 \quad e_{ry} = 0 \]
\[ a_3 = \sin \theta_i \quad e_{rz} = \sin \theta_i \]

The backscatter coefficient associated with these parameters can be given the addi-
tional subscripts VV.

\[ \sigma_{VV}^0 = \frac{k_0^2 \cos^2 \theta_i}{\pi} \left\{ \alpha_{x,x} \cos^2 \theta_i + 2 \text{Re} [\alpha_{x,x} \cos \theta_i \sin \theta_i] \right\} + \alpha_{z,z} \sin^2 \theta_i \]  

(71)

If a result is computed for the cross-polarized backscatter coefficient (HV or VH),
the term will disappear. The reason for this is that the particular elements of the dyadic
\( M \), which would yield cross-polarized terms in the mean wave, are all zero. Next,
let's consider the influence of an irregular vegetation-soil boundary.
MODIFYING THE VOLUME SCATTERING RESULTS TO INCORPORATE THE INFLUENCE OF AN IRREGULAR VEGETATION–SOIL BOUNDARY

In this section, we will consider what must be done to equations (70) and (71) to include the influence of a rough ground surface. It is expected that the influence of the rough surface would be greater when the angle of incidence is small. Also, as the vegetation height or density gets larger, less scattering is expected from the ground surface below. In what follows, the horizontal and vertical polarizations will be considered separately.

Consider a horizontally polarized wave incident from free space onto a layer of vegetation that has an average thickness L. The interface between the vegetation and soil will be considered as randomly rough in such a way that the tangent plane approximation is applicable. The radar backscatter coefficient ($\sigma_{HH}^o$) will be considered as the sum of a term resulting from surface scattering and a term resulting from volume scattering.

$$\sigma_{HH}^o = \sigma_{HHS}^o \exp \left[ -4\alpha_{e1} L \sec \psi_{e1} \right] + \sigma_{HHv}^o$$

(72)

In equation (72), $\sigma_{HHS}^o$ represents the backscatter coefficient for a randomly rough surface with a Gaussian distribution of surface heights. The subscript s indicates surface scattering. The quantity $\sigma_{HHS}^o$ is multiplied by a decaying exponential in which $\alpha_{e1}$ is the imaginary part of the effective propagation constant and $\psi_{e1}$ is the true angle of refraction for the mean wave in the vegetation. The second subscript I on $\alpha_{e1}$ and $\psi_{e1}$ is used to indicate horizontal polarization since these parameters will have different values for vertical polarization. The effective propagation constant ($k_{e1}$) can be obtained from $k_h$.

$$k_{e1}^2 = k_{ex}^2 + k_{ex}^2 = k_h^2 + k_o^2 \sin^2 \theta_i$$

If we now let $k_{e1} = \beta_{e1} - j\alpha_{e1}$ and in place of $k_h$ we put $jp_1 - q_1$, then we can solve for $\beta_{e1}$ and $\alpha_{e1}$.

$$\beta_{e1} = \rho_{e1}^R \cos (\phi_{e1}/2)$$

$$\alpha_{e1} = \rho_{e1}^R \sin (\phi_{e1}/2)$$

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Where \( \rho_{e1} \) and \( \phi_{e1} \) are defined below:

\[
\rho_{e1} = \left\{ 4p_1^2 q_1^2 + (q_1^2 + k^2 \sin^2 \theta_1 - p_1^2)^2 \right\}^{1/4}
\]

\[
\phi_{e1} = \frac{2p_1 q_1}{q_1^2 + k^2 \sin^2 \theta_1 - p_1^2}
\]

The planes of constant phase for the mean wave in the random medium are used to calculate an expression for \( \sec \psi_{e1} \):

\[
\sec \psi_{e1} = \frac{\sqrt{k^2 \sin^2 \theta_1 + \rho_1^2 (\beta_{e1} \cos \gamma_1 - \alpha_{e1} \sin \gamma_1)}}{\rho_1 (\beta_{e1} \cos \gamma_1 - \alpha_{e1} \sin \gamma_1)}
\]

Where \( \rho_1 \) and \( \gamma_1 \) are given below:

\[
\rho_1 = \left\{ 4a_1^2 b_1^2 \sin^4 \theta_1 + [1 - (a_1^2 - b_1^2) \sin^2 \theta_1]^2 \right\}^{1/4}
\]

\[
\gamma_1 = \frac{1}{2} \tan^{-1} \left[ \frac{2a_1 b_1 \sin^2 \theta_1}{1 - (a_1^2 - b_1^2) \sin^2 \theta_1} \right]
\]

\[
a_1 = \frac{k \beta_{e1}}{\beta_{e1}^2 + \alpha_{e1}^2} \quad b_1 = \frac{k \alpha_{e1}}{\beta_{e1}^2 + \alpha_{e1}^2}
\]

Many derived expressions are available for the backscatter coefficient from a randomly rough surface using the tangent plane method. The following equation will be used:

\[
\Phi_{HHS} = \frac{R_0^2}{4m_c \cos^4 \psi_{e1}} \left\{ 1 - \sin^2 \psi_{e1} [2(1 - \theta_0 \cos^2 \psi_{e1}) - \sin^2 \psi_{e1} (1 + \theta_0^2)] \right\} \exp \left[ -\tan^2 \psi_{e1}/(4m_r^2) \right] \tag{73}
\]

---

The quantity $R_o$ in (73) represents the Fresnel reflection coefficient and can be computed as follows:

$$R_o = \frac{\beta_1 \cos \psi_{e1} - \sqrt{\beta_3^2 - \beta_{e1}^2 \sin^2 \psi_{e1}}}{\beta_1 \cos \psi_{e1} + \sqrt{\beta_3^2 - \beta_{e1}^2 \sin^2 \psi_{e1}}}$$

In calculating $R_o$, we have neglected the effect of the imaginary parts of the propagation constants. The term $m_s$ represents the ratio of the standard deviation of the surface fluctuations to the correlation distance. The quantity $g_o$ is defined by the following expression:

$$g_o = 1 - \frac{2\beta_1 \cos \psi_{e1}}{\sqrt{\beta_3^2 - \beta_{e1}^2 \sin^2 \psi_{e1}}}$$

A final equation can now be written for $\alpha_{HH}^o$ that considers both the volume-scattering and surface-scattering effects.

$$\alpha_{HH}^o = \frac{R_o^2}{4m_s^2 \cos^4 \psi_{e1}} \left\{ 1 - \sin^2 \psi_{e1} [2(1 - g_o \cos^2 \psi_{e1})]ight.$$

$$\left. - \sin^2 \psi_{e1} (1 + g_o^2)] \right\} \exp \left[-\tan^2 \psi_{e1} / (4m_s^2)\right]$$

$$\times \exp([-4\alpha_{e1} L \sec \psi_{e1}] + k_o^2 \alpha_{yy} \cos^2 \theta_i / \pi)$$

(74)

In the same manner, a complete solution for vertical polarization can be obtained:

$$\alpha_{VV}^o = \frac{r_o^2}{4m_s^2 \cos^4 \psi_{e2}} \left\{ 1 + \frac{r_o^2 \sin^2 \psi_{e2} \cos \psi_{e2}}{\gamma_o^2} \right\} [2r_o \sin \psi_{e2}$$

$$+ r_o' \cos \psi_{e2}] + \frac{2r_o'}{r_o} \sin \psi_{e2} \cos^3 \psi_{e2} \right\}$$

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\[
\exp \left( -\tan^2 \psi_{e2} / (4m^2) \right) \exp \left( -4 \alpha_{e2} L \sec \psi_{e2} \right)
\]
\[
+ \frac{k_0^2 \cos^2 \theta_1}{\pi} \left\{ \alpha_{xx} \cos^2 \theta_1 + 2 \text{Re} \left[ \cos \theta_1 \sin \theta_1 \alpha_{xz} \right] - \alpha_{zz} \sin^2 \theta_1 \right\}
\]

(75)

The new quantities introduced into the above equation are defined below:

\[
\beta_{e2} = \rho_{e2} \cos \left( \phi_{e2}/2 \right)
\]

\[
\alpha_{e2} = \rho_{e2} \sin \left( \phi_{e2}/2 \right)
\]

The quantities \( \rho_{e2} \) and \( \phi_{e2} \) are given as follows:

\[
\rho_{e2} = \left\{ 4p_2^2 q_2^2 + (q_2^2 + k_0^2 \sin^2 \theta_1 - p_2^2)^2 \right\}^{1/2}
\]

\[
\phi_{e2} = \tan^{-1} \left\{ \frac{2p_2 q_2}{q_2^2 + k_0^2 \sin^2 \theta_1 - p_2^2} \right\}
\]

\[
\sec \psi_{e2} = \frac{\sqrt{k_0^2 \sin^2 \theta_1 + \rho_2^2 \left( \beta_{e2}^2 \cos \gamma_2 - \alpha_{e2} \sin \gamma_2 \right)^2}}{\rho_2 \left( \beta_{e2} \cos \gamma_2 - \alpha_{e2} \sin \gamma_2 \right)}
\]

\[
\rho_2 = \left\{ 4a_2^2 b_2^2 \sin^4 \theta_1 + \left[ 1 - (a_2^2 - b_2^2) \sin^2 \theta_1 \right]^2 \right\}^{1/4}
\]

\[
\gamma_2 = \frac{1}{2} \tan^{-1} \left[ \frac{2a_2 b_2 \sin^2 \theta_1}{1 - (a_2^2 - b_2^2) \sin^2 \theta_1} \right]
\]

where \( a_2 \) and \( b_2 \) are
\[ a_2 = \frac{k_0 \beta e_2}{\beta e_2^2 + \alpha e_2^2} \quad \text{and} \quad b_2 = \frac{k_0 \alpha e_2}{\beta e_2^2 + \alpha e_2^2} \]

\[ r_2 = \frac{\beta_3^2 \cos \psi_{e_2} - \beta_{e_2} \sqrt{\beta_3^2 - \beta_{e_2}^2 \sin^2 \psi_{e_2}}}{\beta_3^2 \cos \psi_{e_2} + \beta_{e_2} \sqrt{\beta_3^2 - \beta_{e_2}^2 \sin^2 \psi_{e_2}}} \]

\[ r_0' = \frac{2\beta_3^2 \beta_{e_2} \sin \psi_{e_2} (\beta_3^2 - \beta_{e_2}^2)}{\sqrt{\beta_3^2 - \beta_{e_2}^2 \sin^2 \psi_{e_2}} \beta_{e_2} \cos \psi_{e_2} + \beta_{e_2} \sqrt{\beta_3^2 - \beta_{e_2}^2 \sin^2 \psi_{e_2}}} \]

Equations (74) and (75) are the final results for the radar backscatter coefficient for horizontal and vertical polarizations. Before the results of computing equations (74) and (75) are shown, an elementary vegetation permittivity model must be developed that relates some of the model input parameters to the complex dielectric constants of vegetation and water.

**DEVELOPMENT OF A VEGETATION PERMITTIVITY MODEL**

To determine the influence of various vegetation parameters (such as moisture content) upon the calculation of the backscatter coefficient, one must relate some of the permittivity parameters in the scattering model to the physical parameters of the vegetation. Peake and Oliver's model\(^6\) will be used to calculate the relative complex dielectric constant of vegetation \(\varepsilon_v\):

\[ \varepsilon_v = \left( \frac{F}{2} \right) \text{Re}[\hat{\varepsilon}_w] + j \left( \frac{F}{3} \right) \text{Im}[\hat{\varepsilon}_w] \]

where $F$ is the fraction of water by weight in the vegetation; $\text{Re}[\varepsilon_w]$ and $\text{Im}[\varepsilon_w]$ are the real and imaginary parts of the relative complex dielectric constant of water ($\varepsilon_w$), which can be written as

$$\varepsilon_w = 5 + \frac{75}{1 + j(1.85/\lambda)}$$

where $\lambda$ is the wavelength in centimeters. For particular values of $\lambda$ and $F$, we can now compute $\varepsilon_v$. With a knowledge of $\varepsilon_v$, one can estimate the average relative complex dielectric constant ($\varepsilon_a$) by using the following

$$\varepsilon_a = \left(\varepsilon_v + \varepsilon_a\right)V_a/V_t$$

$$\varepsilon_a = \text{Re}[\varepsilon_a]$$

where $V_v$ is the volume occupied by the vegetation; $V_A$ is the volume occupied by air; $V_T$ is the total volume equal to $V_v + V_A$; $\varepsilon_A$ is the relative dielectric constant of air, assumed equal to 1. The variances $\eta_1^2$ and $\eta_2^2$ can be computed by using the following formulas:

$$\eta_1^2 = \frac{V_v (e'_v - \varepsilon_a)^2 + V_A (e_A - \varepsilon_a)^2}{V_T}$$

$$\eta_2^2 = \frac{V_v (\sigma_v - \varepsilon_a)^2 + V_A \sigma_a^2}{V_T}$$

where

$$\sigma_v = -\omega e_0 F \text{Im}[\varepsilon_w]/3 \quad \text{and} \quad e'_v = \text{Re}[\varepsilon_v]$$

The symbol $R_v$ shall be used to designate the volume ratio $V_v/V_T$.

The developed model for the radar backscatter coefficient is now complete and calculations can be made. In the next section, computed results will be shown, and the theory will be compared with some existing experimental data.
DISCUSSION OF RESULTS

In this section, some numerical calculations will be shown for the theory derived in the previous section, and a study will be presented of the influence of the various input parameters on the backscatter coefficient. Two computer programs were developed for solving equations (74) and (75). One program solves for equation (74) and the second solves for (75). The solutions to the half-space and plane layer problems are also generated for comparison. A listing of the computer program for solving equation (74) is given in appendix B. The 10 input parameters to the programs are

1. Fraction of water by weight in the vegetation (F)
2. Volume of vegetation divided by the total volume (Rv)
3. Correlation distance in the x direction (Rx)
4. Correlation distance in the y direction (Ry)
5. Correlation distance in the z direction (Rz)
6. Mean thickness of the vegetation layer (L)
7. Relative dielectric constant of the soil below the vegetation (εs)
8. Conductivity of the soil below the vegetation (σ3)
9. Frequency (f)
10. Ratio of the standard deviation of the rough surface fluctuations to the correlation distance of the fluctuations (m).

The output of the computer programs is the backscatter coefficient in decibels as a function of incidence angle. The backscatter coefficient in decibels is related to the backscatter coefficient as follows:

\[ a^0 \text{ (in decibels)} = 10 \log_{10} a^0 \]

The backscatter coefficient on the right side of the above equation is computed by (74) or (75). The following discussion centers on figures 2 through 30, which show the results of computing equations (74) and (75). Figures 2 and 3 come from Ulaby and Bush and provide pertinent ground truth data associated with the experimental measurements. Figure 4 comes from Cihlar and Ulaby and provides a relationship between soil moisture and relative complex dielectric constant. Figures 5 through 15 provide a comparison of the developed theory with experimental data taken from a cornfield by Ulaby and Bush.

---


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<th>Soil Moisture</th>
<th>(g/cm³)</th>
<th>% Plant Moisture</th>
<th>Normalized Plant Water Content (g/m)</th>
<th>Plant Height (m)</th>
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<td>M 0.19</td>
<td>F 0.24</td>
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<td>M 0.10</td>
<td>F 0.12</td>
<td>87.1</td>
<td>0.58</td>
</tr>
<tr>
<td>June 5</td>
<td>N 0.10</td>
<td>M 0.13</td>
<td>F 0.10</td>
<td>88.5</td>
<td>0.88</td>
</tr>
<tr>
<td>June 13</td>
<td>N 0.34</td>
<td>M 0.34</td>
<td>F 0.34</td>
<td>89.9</td>
<td>1.25</td>
</tr>
<tr>
<td>June 20</td>
<td>N 0.30</td>
<td>M 0.31</td>
<td>F 0.32</td>
<td>89.9</td>
<td>1.90</td>
</tr>
<tr>
<td>June 26</td>
<td>N 0.06</td>
<td>M 0.09</td>
<td>F 0.08</td>
<td>83.9</td>
<td>2.30</td>
</tr>
<tr>
<td>July 1</td>
<td>N 0.05</td>
<td>M 0.05</td>
<td>F 0.07</td>
<td>82.4</td>
<td>2.60</td>
</tr>
<tr>
<td>July 8</td>
<td>N 0.21</td>
<td>M 0.27</td>
<td>F 0.30</td>
<td>84.7</td>
<td>2.60</td>
</tr>
<tr>
<td>July 11</td>
<td>N 0.17</td>
<td>M 0.15</td>
<td>F 0.15</td>
<td>81.5</td>
<td>2.60</td>
</tr>
<tr>
<td>July 16</td>
<td>N 0.04</td>
<td>M 0.07</td>
<td>F 0.06</td>
<td>81.5</td>
<td>2.60</td>
</tr>
<tr>
<td>July 22</td>
<td>N 0.04</td>
<td>M 0.04</td>
<td>F 0.04</td>
<td>73.4</td>
<td>2.60</td>
</tr>
<tr>
<td>August 5</td>
<td>N 0.06</td>
<td>M 0.07</td>
<td>F 0.07</td>
<td>62.4</td>
<td>2.60</td>
</tr>
<tr>
<td>August 15</td>
<td>N 0.26</td>
<td>M 0.27</td>
<td>F 0.26</td>
<td>52.9</td>
<td>2.70</td>
</tr>
<tr>
<td>September 5</td>
<td>0.31</td>
<td>M 0.30</td>
<td>F 0.29</td>
<td>47.5</td>
<td>2.70</td>
</tr>
<tr>
<td>September 19</td>
<td>0.12</td>
<td>M 0.12</td>
<td>F 0.10</td>
<td>16.5</td>
<td>0.23</td>
</tr>
<tr>
<td>July 30*</td>
<td>N 0.26</td>
<td>M 0.26</td>
<td>F 0.26</td>
<td>74.8</td>
<td>2.60</td>
</tr>
</tbody>
</table>

* = irrigated corn field  N = near range sample  M = medium range sample  F = far range sample

Figure 3. Data Record of Soil Moisture, Plant Moisture, Plant Height, and Precipitation as Measured During the Observation Period.

Figure 4. Representative Dielectric Constant Values as a Function of Volumetric Water Content.

The soil moisture is obtained from figure 2 for a particular set of measurements performed on a given date. This soil moisture is used along with the curves of figure 4 to determine the relative dielectric constant and the conductivity of the soil. Throughout all comparisons of theory with experiment, it has been assumed that the soil type is a loam. In comparing theory with experiment, remember that certain input parameters to the theoretical model were not known and had to be estimated; whereas, other input parameters were known from the ground truth data collected during the experiment. The unknown input parameters are $R_V$, $\ell_x$, $\ell_y$, $\ell_z$ and $m_s$.

In figure 5, the theory is matched to the experimental data for corn that is at a height of 30 centimeters. The large rise that occurs in $\sigma^o$ as $\theta_i$ goes from 10° to 0° is indicative of a rough surface effect. In this case, the rough surface is quasi-specular since $m_s$ is given such a small value. It can be seen that to match the theory with the experimental data, it was necessary to let $\ell_x$ be different from $\ell_y$ and to let $\ell_z$ be much smaller than $\ell_x$ or $\ell_y$. The fact that $\ell_x$ is different from $\ell_y$ shows an anisotropic effect in the horizontal plane, which probably arises from the corn being planted in rows.

Figure 6 matches the theory to experimental data for corn that is 2.3 meters high. It can be seen that a good match is obtained for $\ell_x$ equal to $\ell_y$, indicating that the anisotropic effect in the horizontal plane has essentially disappeared for 8.6 GHz. The values of the parameters used for $R_V$, $\ell_x$, $\ell_y$, $\ell_z$, and $m_s$ in figure 6 are also used in figures 7 through 10 to determine whether the model could provide a correct prediction of $\sigma^o$ for different values of $F$, soil moisture, and vegetation height.

Figures 7 through 9 show an excellent agreement between theory and experiment. Figure 10 shows an excellent agreement between theory and experiment for angles of incidence equal to and greater than 30°. For angles of incidence less than 30°, the agreement is poor. A possible reason for this poor agreement may be due to the rainfall that came prior to the August 15th measurements. The rainfall could have disturbed the soil surface in both a physical and an electrical manner such that its scattering behavior is no longer predictable from prior values.

Figures 11 and 12 show an attempt to match the theory to the experimental data for frequencies of 8.6 GHz and 13 GHz. It can be seen that to obtain a good match, the values of $\ell_y$ and $\ell_y$ must be altered from the values used at 8.6 GHz. This seems to indicate that as the frequency goes higher, the vegetation medium becomes more complicated and the anisotropic behavior becomes more pronounced.
Polarization: Horizontal
\[ F = 0.895 \quad f = 8.6 \text{ GHz} \]
\[ R_v = 0.0008 \]
\[ r_x = 4 \text{ mm} \quad r_y = 2.5 \text{ mm} \quad r_z = 0.05 \text{ mm} \]
Layer Thickness = 0.3 meter
Relative Dielectric Constant of Soil = 9.0
Conductivity of Soil = 1.44
\[ m_s = 0.04 \]

Ulaby's Experimental Data for Corn – May 20, 1974

Angle of Incidence in Degrees

Figure 5. Comparison of Theory with Experimental Data.
Polarization: Horizontal
F = 0.839  f = 8.6 GHz
Rv = 0.0002
\( \ell_x = \ell_y = 5\text{mm} \quad \ell_z = 0.095\text{mm} \)
Layer Thickness = 2.3 meters
Relative Dielectric Constant of Soil = 4.0
Conductivity of soil = 0.278
m_d = 0.035

Ulaby's Experimental Data for Corn – June 26, 1974

Figure 6. Comparison of Theory with Experimental Data.
Polarization: Horizontal
F = 0.815  f = 8.6 GHz
R_v = 0.0002
\( \ell_a = \ell_y = 5\text{mm} \quad \ell_z = 0.095\text{mm} \)
Layer Thickness = 2.6 meters
Relative Dielectric Constant of Soil = 3.5
Conductivity of Soil = 0.389
m_s = 0.035

Ulaby's Experimental Data for Corn — July 16, 1974

Figure 7. Comparison of Theory with Experimental Data.
Polarization: Horizontal
F = 0.734  f = 8.6 GHz
R_v = 0.0002
\( \ell_x = \ell_y = 5\text{ mm} \), \( \ell_z = 0.095\text{ mm} \)
Layer Thickness = 2.6 meters
Dielectric Constant of Soil = 3.0
Conductivity of Soil = 0.278
m_x = 0.035

Ulaby's Experimental Data for Corn – July 22, 1974

Figure 8. Comparison of Theory with Experimental Data.
Polarization: Horizontal
F = 0.624  f = 8.6 GHz
R_y = 0.0002
\ell_x = \ell_y = 5\,\text{mm} \quad \ell_z = 0.095\,\text{mm}
Layer Thickness = 2.6\,\text{meters}
Dielectric Constant of Soil = 4.0
Conductivity of Soil = 0.278
m_s = 0.035

Ulaby's Experimental Data for Corn - August 5, 1974

Figure 9. Comparison of Theory with Experimental Data.
Polarization: Horizontal

F = 0.529

\( \varepsilon \) = 8.6 GHz

\( R_y = 0.0062 \)

\( R_x = 0.05 \) mm

Layer Thickness = 9.7 meters

Relative Dielectric Constant of Soil = 10.5

m = 0.035

Ulaby's Experimental Data for Corn - August 15, 1974

Figure 10. Comparison of Theory with Experimental Data.

\( \theta \) in degrees

\( \phi \) in degrees

0

10

20

30

40

50

60

70

80

90

100

110

120

130

140

150

160

170

180

0

10

20

30

40

50

60

70

80

90

100

110

120

130

140

150

160

170

180

0
Polarization: Horizontal
F = 0.839  f = 11 GHz
R_v = 0.0002
ε_x = 4mm  ε_y = 6mm  ε_z = 0.095mm
Relative Dielectric Constant of Soil = 4.0
Conductivity of Soil = 0.278
m_s = 0.035

Ulaby's Experimental Data for Corn - June 26, 1974

Figure 11. Comparison of Theory with Experimental Data.
Polarization: Horizontal
F = 0.839  f = 13 GHz
R_r = 0.0002
ε_x = 2mm  ε_y = 8mm  ε_z = 0.0001
Layer Thickness = 2.3 meters
Relative Dielectric Constant of Soil = 4.0
Conductivity of Soil = 0.278
m_s = 0.035

Ulaby's Experimental Data for Corn - June 26, 1974

Figure 12. Comparison of Theory with Experimental Data.
Polarization: Vertical
$F = 0.839 \quad f = 8.6 \, \text{GHz}$
$R_v = 0.0002$
$\ell_x = 4\, \text{mm} \quad \ell_y = 7\, \text{mm} \quad \ell_z = 0.095\, \text{mm}$
Layer Thickness = 2.3 meters
Relative Dielectric Constant of Soil = 4.0
Conductivity of Soil = 0.278
$m_y = 0.035$

Ulaby's Experimental Data for Corn – June 26, 1974

Figure 13. Comparison of Theory with Experimental Data.
In figures 13 through 15 a match is shown of the theory with experimental data for vertical polarization. For all three curves, \( \xi_x \) must be made unequal to \( \xi_y \) to obtain a good match. In figure 16, the variations are shown in the experimental measurements of \( \sigma^0 \), which can occur throughout the spring and summer for alfalfa. Large variations in the backscatter coefficient occur prior to and after harvesting.

Figure 17 presents a study of sensor-look direction with respect to vegetation planted in rows. The two parameters \( \sigma_\perp^0 \) and \( \sigma_\parallel^0 \) represent the backscatter coefficient when the look direction is perpendicular and parallel to the rows, respectively. We see that when \( \theta_\parallel \) equals zero degrees, \( \sigma_\perp^0 \) and \( \sigma_\parallel^0 \) are equal. However, for \( \theta_\parallel \) greater than zero degrees, \( \sigma_\perp^0 \) is greater than \( \sigma_\parallel^0 \). The theoretical results agree only partially with the experimental results given in figure 18, which comes from Batlivala and Ulaby.\(^9\)

Figure 19 provides a study of backscatter coefficient versus layer thickness for two angles of incidence. For the \( \theta_\parallel \) equal to zero curve, the solution for \( \sigma^0 \) with a rough layer differs from the half-space solution by approximately 16dB (decibels) for a layer thickness of 0.5 meters. As the layer thickness is increased, the solution for \( \sigma^0 \) at \( \theta_\parallel \) equal to zero, approaches the half-space solution. The \( \theta_\parallel \) equal to 20° curve also approaches the half-space solution when the layer thickness is increased. In this case, \( \sigma^0 \) differs from the half-space solution by only about 3.5dB when the layer thickness is 0.5 meters.

Figure 20 presents a study of the skin depth of the mean wave versus incidence angle for three different frequencies. For a horizontally polarized wave, the skin depth is taken to be the reciprocal of \( p_1 \), which was derived earlier as part of the solution to the Dyson’s equation. It should be remembered that the mean wave decays for two reasons, absorption and scattering. For a frequency of 8.6 GHz, the skin depth goes from approximately 5 meters at \( \theta_\parallel \) equal to 0° down to 1 meter at \( \theta_\parallel \) equal to 80°. When the frequency is increased, the overall level of the curve is lowered considerably, but it does not drop off as fast with increasing incidence angle.

In figure 21, the solutions are compared for the half space, the plane layer, and the layer with a rough surface. For angles of incidence between 0° and 30°, the rough interface at the vegetation soil boundary can have a dramatic effect on \( \sigma^0 \). In this case, it is clearly not sufficient to use a plane layer model.


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Polarization: Vertical
F = 0.815, f = 8.6 GHz
R_w = 0.0002
\( \ell_x = 4 \text{mm} \), \( \ell_y = 7 \text{mm} \), \( \ell_z = 0.095 \text{mm} \)
Layer Thickness = 2.6 meters
Relative Dielectric Constant of Soil = 3.5
Conductivity of Soil = 0.389
m_s = 0.035

Ulaby's Experimental Data for Corn - July 16, 1974

Figure 14. Comparison of Theory with Experimental Data.
Figure 15. Comparison of Theory with Experimental Data.

Polarization: Vertical
F = 0.734  f = 8.6 GHz
R = 0.0002

\( \ell_x = 4\text{mm} \quad \ell_y = 7\text{mm} \quad \ell_z = 0.095\text{mm} \)

Layer Thickness = 2.6 meters
Relative Dielectric Constant of Soil = 3.0
Conductivity of Soil = 0.278
\( m_i = 0.035 \)
Polarization: Horizontal
\[ \theta_1 = 0^\circ \]

Ulaby's Experimental Data for Alfalfa taken in 1974

Figure 16. Study of the Experimental Variations of the \( \sigma^0 \) for Alfalfa.
Polarization: Horizontal
F = 0.8 f = 7.25 GHz
R_s = 0.001
\( \ell_z = 0.2 \text{mm} \)
Layer Thickness = 2.5 meters
Dielectric Constant of Soil = 7.5
Conductivity of Soil = 1.39
m_s = 0.09

Figure 17. Study of the Sensor Look Direction.
Figure 18. Scattering Coefficient $\sigma^o$ as a Function of Incidence Angle at (a) 2.75GHz, (b) 5.25GHz, and (c) 7.25GHz. Data set ≠ 1, July 16, 1974.

Polarization: Horizontal
F = 0.839  f = 8.6 GHz
R_τ = 0.0002
ε_x = ε_y = 5 mm  ε_z = 0.095 mm
Dielectric Constant of Soil = 4.0
Conductivity of Soil = 0.278
m_s = 0.035

Figure 19. Study of the Variation of $\sigma^\circ$ with Layer Thickness.
Figure 20. Study of the Skin Depth of the Mean Wave Versus Incidence Angle.
Polarization: Horizontal
$F = 0.6$  $f = 8.6\, \text{GHz}$
$R_x = 0.0005$
$\xi_x = \xi_y = 1.11\, \text{mm} \quad \xi_z = 0.194\, \text{mm}$
Layer Thickness = 0.3 meters
Relative Dielectric Constant of Soil = 9.0
Conductivity of Soil = 0.05
$m_s = 0.1$

Figure 21. Comparison of Half Space, Plane Layer and Layer with Rough Surface Solutions.
In figures 22 through 30, a sensitivity-of-parameters study is presented. In this study, the input parameters are varied individually to determine the overall effect on the scattering coefficient. In figure 22, a study is provided of \( \sigma^0 \) variations with \( F \) (the vegetation moisture content). When \( \theta_i \) is greater than 20°, the larger values of \( F \) produce higher levels of \( \sigma^0 \). The shape of the \( \sigma^0 \) versus \( \theta_i \) curve does not change much, but the overall level is significantly different. At approximately \( \theta_i = 10^\circ \), a crossover of the curves exists such that the curve with the highest moisture content now yields the lowest value of \( \sigma^0 \). The reason for the crossover is that two entirely different mechanisms are responsible for scattering. For \( \theta_i \) greater than 20°, the volume-scattering mechanism dominates so that higher moisture in the vegetation results in larger backscatter values. However, for angles of incidence \( \theta_i < 10^\circ \) when the mechanism for scattering is dominated by the rough surface under the vegetation, the higher moisture values result in lower \( \sigma^0 \) values. This is because the higher moisture values provide more attenuation of the mean wave, which means that less is available for scattering from the rough surface.

Figure 23 presents a study of \( \sigma^0 \) variations with the parameter \( R_v \). The smaller value of \( R_v \) yields a larger \( \sigma^0 \) value for small angles of incidence. This is again because rough surface scattering dominates for small angles, and smaller values of \( R_v \) mean that more energy gets down to the surface. A crossover occurs at approximately \( \theta_i = 15^\circ \) where the volume scattering result begins to dominate. Another crossover occurs between \( \theta_i = 40^\circ \) and \( \theta_i = 50^\circ \) such that for angles larger than \( 50^\circ \), \( \sigma^0 \) falls off faster for the larger value of \( R_v \). The reason for this second crossover is possibly because that for the larger value of \( R_v \), the lower interface between the vegetation and soil no longer provides a contribution for backscattering.

Figure 24 presents a study of \( \sigma^0 \) variations with \( L \) (the mean thickness of the vegetation layer). For small angles of incidence, the smaller value of \( L \) yields larger values of \( \sigma^0 \). The rough surface below the vegetation is dominating the return and the smaller value of \( L \) provides a lower attenuation, thus making more energy available for scattering from the surface. A crossover point occurs around \( \theta_i = 12^\circ \), which indicates that volume scattering is now beginning to dominate and so the thicker layer will yield a larger value of \( \sigma^0 \). Another crossover point occurs at approximately \( \theta_i = 67^\circ \). This crossover point possibly indicates that for \( L = 2 \) meters, the lower interface is having no influence, but for \( L = 0.5 \) meters, the lower interface still provides a contribution.
Figure 22. Study of $\sigma^\circ$ Variations with $F$. 

Polarization: Horizontal
$R_v = 0.0002 \quad f = 8 \text{ GHz}$
$\xi_z = \xi_y = 2\text{ mm} \quad \xi_x = 0.02\text{ mm}$
Layer Thickness = 1 meter
Relative Dielectric Constant of Soil = 8.2
Conductivity of Soil = 1.67
$m_s = 0.04$
Polarization: Horizontal
F = 0.8  f = 8 GHz
\( \ell_x = \ell_y = 2\text{mm} \quad \ell_z = 0.02\text{mm} \)
Layer Thickness = 1 meter
Relative Dielectric Constant of Soil = 8.2
Conductivity of Soil = 1.67
\( m_s = 0.04 \)

Figure 23. Study of \( o^\circ \) Variations with \( R_v \).
Polarization: Horizontal
F = 0.8  f = 8 GHz
R_v = 0.0002
r_x = r_y = 2mm  r_z = 0.02mm
Relative Dielectric Constant of Soil = 8.2
Conductivity of Soil = 1.67
m_z = 0.04

Figure 24. Study of $\sigma^o$ Variations with L.
Figure 25 shows a study of $\sigma^o$ variations with soil moisture. The soil moisture influences both the dielectric constant and the conductivity. The effect is shown of doubling the soil moisture underneath a 1-meter layer of vegetation. Thus, doubling the soil moisture content increases the level of $\sigma^o$ slightly over all angles of incidence.

In figure 26, a study is presented of $\sigma^o$ variations with frequency for two angles of incidence. We see only a slight frequency dependence at $\theta_1 = 10^\circ$, because the rough surface scattering is independent of frequency except for the attenuation portion. At $\theta_1 = 30^\circ$ where volume scattering is playing a more important role, we see a very significant frequency dependence.

In figure 27, $\sigma^o$ variations with the parameter $m_1$ are shown. This parameter represents the ratio of the standard deviation of the rough surface undulations to the correlation distance. The most specular surface ($m_1 = 0.04$) yields the highest value of $\sigma^o$ at $\theta_1 = 0^\circ$. However, for this surface, $\sigma^o$ falls off very fast with increasing incidence angle so that at $\theta_1 = 20^\circ$, the rough surface effect has disappeared. When $m_1$ is allowed to increase, the value of $\sigma^o$ at $\theta_1 = 0^\circ$ decreases. Also, as $m_1$ increases, the rough surface influences $\sigma^o$ over a larger range of incidence angles.

In figure 28, $\sigma^o$ variations with $\ell_x$ are shown. Notice first that a change in $\ell_x$ yields virtually no influence upon $\sigma^o$ for angles of incidence less than $10^\circ$. For angles of incidence greater than $10^\circ$, the curve associated with the larger value of $\ell_x$ is higher until a crossover point is reached around $\theta_1 = 48^\circ$. For angles of incidence greater than $48^\circ$, the curve associated with the larger value of $\ell_x$ falls off much faster than the curve associated with the smaller value of $\ell_x$. Increasing the value of $\ell_x$ will then move the curve upward for angles of incidence less than about $50^\circ$, but will lower the curve for angles of incidence greater than about $50^\circ$.

In figure 29, $\sigma^o$ variations with $\ell_y$ are shown. Once again, notice that a change in $\ell_y$ has virtually no influence on $\sigma^o$ for angles less than $10^\circ$. For angles of incidence greater than or equal to $20^\circ$, an increase in $\ell_y$ results in an increase in $\sigma^o$. Therefore, increasing $\ell_y$ simply increases the level of the curve for angles equal to and greater than $20^\circ$.

In figure 30, $\sigma^o$ variations with $\ell_z$ are shown. For angles of incidence less than $10^\circ$, changes in $\ell_z$ have no influence on $\sigma^o$ owing to the dominance of the rough surface. For angles of incidence greater than $20^\circ$, increasing the value of $\ell_z$ simply increases the overall level of the curve without changing the shape. It can be seen that the $\sigma^o$ curve is sensitive to slight changes in $\ell_z$. A change in $\ell_z$ of only a fraction of a millimeter produces a significant change in $\sigma^o$. This sensitivity may make any attempt to determine $\ell_z$ in a rigorous experimental manner very difficult.
Polarization: Horizontal
F = 0.8  f = 8 GHz
R_x = 0.0002
\ell_x = \ell_y = 2\text{mm}  \ell_z = 0.02\text{m}
Layer Thickness = 1\text{ meter}
m_s = 0.04

Soil Moisture = 0.4\text{ grams/cm}^3
Relative Dielectric Constant of Soil = 16.2
Conductivity of Soil = 4.17

Soil Moisture = 0.2\text{ grams/cm}^3
Relative Dielectric Constant of Soil = 8.2
Conductivity of Soil = 1.67

Figure 25. Study of \(\sigma^\circ\) Variations with Soil Moisture.
Polarization: Horizontal
F = 0.08  m_s = 0.04
R_r = 0.0002
w_x = w_y = 2mm   w_z = 0.02mm
Layer Thickness = 1 meter
Relative Dielectric Constant of Soil = 8.2
Conductivity of Soil = 1.67

Figure 25. Study of \( \phi \) Variations with Frequency.
Polarization: Horizontal
F = 0.8 \quad f = 8 \text{ GHz}
R_v = 0.0002
\ell_x = \ell_y = 2\text{ mm} \quad \ell_z = 0.02\text{ mm}
Layer Thickness = 1 \text{ meter}
Relative Dielectric Constant of Soil = 8.2
Conductivity of Soil = 1.67

Figure 27. Study of \( \sigma \) Variations with \( m_x \).
Polarization: Horizontal
F = 0.8  f = 8 GHz
R0 = 0.0002
\( \ell_y = 2\text{mm} \quad \ell_z = 0.02\text{mm} \)
Layer Thickness = 1 meter
Relative Dielectric Constant of Soil = 8.2
Conductivity of Soil = 1.67
\( m_z = 0.04 \)

Figure 28. Study of \( \phi^o \) Variations with \( \ell_z \).
Polarization: Horizontal
\[ F = 0.8 \quad f = 8 \text{ GHz} \]
\[ R_y = 0.0002 \]
\[ \ell_x = 2\text{mm} \quad \ell_z = 0.02\text{mm} \]
Layer Thickness = 1 meter
Relative Dielectric Constant of Soil = 8.2
Conductivity of Soil = 1.67
\[ m_s = 0.04 \]

Figure 29. Study of $\phi^o$ Variations with $\ell_y$. 

\[ \ell_y = 2\text{mm} \]
\[ \ell_y = 8\text{mm} \]
Polarization: Horizontal
\( F = 0.8 \)
(\( R_y = 0.0002 \)
\( \theta_y = 2 \text{mm} \)
Layer Thickness = 1 meter
Relative Dielectric Constant of Soil = 8.2
Conductivity of Soil = 1.67
\( m_z = 0.04 \)

\( \theta_x = 0.06 \text{mm} \)
\( \theta_y = 0.04 \text{mm} \)
\( \theta_z = 0.02 \text{mm} \)

Figure 30. Study of \( \theta \) Variations with \( \theta_z \).

\( \theta \) in degrees

Angle of Incidence in Degrees
This section concludes with a brief discussion on the limitations and difficulties encountered in developing the theory presented in this report. It appears valid to simulate a region of vegetation with a continuous random medium, although it is not certain as to how well the first-order renormalization technique does in solving the problem. It is not clear how much multiple scattering is being considered, and it is not even clear as to how much multiple scattering must be considered. A free space dyadic Green's function was used in solving the Dyson's equation; however, what should have been used was a Green's function applicable to a layered problem. Such Green's functions are very complicated to develop and work with. Also, it is not clear how much the final result will change if a more complicated Green's function is used.

An equation for the radar backscatter coefficient $\sigma^0$ was developed by first obtaining plane wave solutions to the Dyson's equation. Expressions were found for the $z$ component of the effective propagation constant for both horizontal and vertical polarizations. The mean wave was then used to calculate the scattered wave, which in turn was used to compute $\sigma^0$. The final result for $\sigma^0$ indicated the necessity to develop a permittivity model that would relate some of the permittivity parameters in the scattering model to the physical parameters of the vegetation. This was accomplished with only limited success since a rather elementary permittivity model was used. The final result for $\sigma^0$ still contained five input parameters that were unknown. The five parameters are $\ell_x$, $\ell_y$, $\ell_z$, $R_V$, and $m_s$.

Further work in this area should attempt to determine correlation functions and distances, as well as $R_V$ and $m_s$ to relate clearly theory and experiment. An anisotropic correlation function was used; however, this did not result in a depolarization term. The existence of depolarization is clearly evident from the experimental data. The reason for this depolarization is, as yet, unknown. A depolarization term could be obtained by computing the scattered field to a second-order approximation. It could also be obtained by initially allowing for an anisotropic random medium. At this time, it is unclear which approach is correct.
CONCLUSIONS

1. For certain types of vegetation, such as corn, the irregular vegetation soil boundary dominates the backscattering results for angles of incidence between 0° and 20°. Any remote sensing of surface phenomena beneath vegetation should be done in this angular range.

2. The effect of the rough surface boundary between the vegetation and soil increases with decreases in frequency, vegetation moisture content, vegetation volume, and layer thickness.

3. Increasing the soil moisture content increases the level of the σo curve slightly.

4. Using different correlation distances in x and y does not completely explain the effect of look direction on scattering from a row crop.

5. The σo curve versus incidence angle curve is sensitive to very slight changes in the correlation distance in z.

6. The predictability of the σo curve is dependent on meteorological phenomena, such as rain.

7. The correlation distances and the vegetation volume ratio do not stay constant throughout the entire growth cycle of the corn crop. However, once the crop matured, these parameters remained fairly constant.

8. Because different correlation distances were required to match the experimental data for horizontal and vertical polarizations, a more correct model for the vegetation may be an anisotropic random media model.

9. The predictability of the σo versus incidence angle curve depends upon a very detailed knowledge of the dielectric fluctuations of the vegetation and the surface roughness properties of the soil below. However, such knowledge for particular vegetation features does not exist at the present time. This detailed understanding should be obtained if theoretical models are to have ultimate usefulness in predicting scattering from vegetation features.
APPENDIX A. Definition of Terms Involved in Computing

\[ <A_y A_y^*>, \ <A_x A_x^*>, \ <A_z A_z^*>, \ <A_A A_A^*>, \ <A_A A_A^*> \]

Repeating equation (67) produces

\[ <A_y A_y^*> = \frac{4I_x I_y I_z A_1 (\omega^2 \mu_0^2 \eta_2^2 + k_0^2 \eta_1^2) k_0^2 k_0^2 M_0 M_0^*}{(1 + 4k_0^2 \theta^2 \sin^2 \theta)} \]

\[ \sum_{n=1}^{16} \frac{A_n}{(c_n + d_n)} \left\{ \frac{1 - \ell_z d_n + \ell_z (c_n + d_n)e^{-L(c_n + \ell_z)} - (1 + \ell_z c_n)e^{-L(c_n + d_n)}}{(1 + \ell_z c_n)(1 - \ell_z d_n)} \right\} \]

The values for \( A_n \), \( c_n \), and \( d_n \) are defined below:

\[
\begin{align*}
A_1 &= \bar{a}_1 \bar{a}_1^* T_2 T_2^* & c_1 &= D_1 & d_1 &= D_1^* \\
A_2 &= \bar{a}_1 T_2 \bar{a}_2^* V_2^* & c_2 &= D_1 & d_2 &= D_2^* \\
A_3 &= \bar{a}_1 T_2 T_2^* & c_3 &= D_1 & d_3 &= -D_2^* \\
A_4 &= \bar{a}_1 T_2 V_2^* & c_4 &= D_1 & d_4 &= -D_1^* \\
A_5 &= \bar{a}_1 V_2 \bar{a}_1^* T_2^* & c_5 &= D_2 & d_5 &= D_1^* \\
A_6 &= \bar{a}_1 \bar{a}_1^* V_2 V_2^* & c_6 &= D_2 & d_6 &= D_2^* \\
A_7 &= \bar{a}_1 V_2 T_2^* & c_7 &= D_2 & d_7 &= -D_2^* \\
\end{align*}
\]
\[ A_8 = \tilde{a}_1 V_2 V^* \quad c_8 = D_2 \quad d_8 = -D_1 \]
\[ A_9 = \tilde{a}_1^* T_2 T^* \quad c_9 = -D_2 \quad d_9 = D_1^* \]
\[ A_{10} = \tilde{a}_1^* T_2 V^* \quad c_{10} = -D_2 \quad d_{10} = D_2^* \]
\[ A_{11} = T_2 T^* \quad c_{11} = -D_2 \quad d_{11} = -D_1^* \]
\[ A_{12} = T_2 V^* \quad c_{12} = -D_2 \quad d_{12} = -D_1^* \]
\[ A_{13} = V_2 \tilde{a}_1^* T^* \quad c_{13} = -D_1 \quad d_{13} = D_1^* \]
\[ A_{14} = \tilde{a}_1^* V_2 V^* \quad c_{14} = -D_1 \quad d_{14} = D_2^* \]
\[ A_{15} = V_2 T^* \quad c_{15} = -D_1 \quad d_{15} = -D_2^* \]
\[ A_{16} = V_2 V^* \quad c_{16} = -D_1 \quad d_{16} = -D_1^* \]

A basic expression for \(<A_x A_x^*>\) can be written as follows:

\[
<A_x A_x^*> = \frac{4k_1^2 \gamma_z A_i (\omega^2 \mu^2 \eta_2^2 + k_0^2 \eta_2^2)}{(1 + 4k_0^2 \rho^2 \sin^2 \theta)}
\]

\[
\sum_{n=1}^{16} \frac{A_n}{(\beta_n + \gamma_n)} \left\{ \frac{1 - \frac{k_1}{\beta_n} \gamma_n + \frac{k_1}{\beta_n} (\beta_n + \gamma_n) e^{-L(\beta_n + 1/\gamma_z)} - (1 + \frac{k_1}{\beta_n}) e^{-L(\beta_n + \gamma_n)}}{(1 + \frac{k_1}{\beta_n} \gamma_n) (1 - \frac{k_1}{\beta_n} \gamma_n)} \right\} + \frac{1 - \frac{k_1}{\beta_n} \gamma_n + \frac{k_1}{\beta_n} (\beta_n + \gamma_n) e^{-L(\beta_n + \gamma_n)} (1 + \gamma_n \beta_n)}{(1 + \frac{k_1}{\beta_n} \gamma_n) (1 - \frac{k_1}{\beta_n} \beta_n)} \right\}
\]
Before giving the values for $\Lambda_n$, $\beta_n$, and $\gamma_n$, the following parameters are defined:

\[
D_1' = p_2 + jq_2 - jk_z'
\]

\[
D_2' = -(p_2 + jq_2 + jk_z')
\]

\[
h_1 = b_6 \{ T_1 (k_z^2 - k_o^2 \sin^2 \theta_i) - T_3 k_o k_z' \sin \theta_i \}
\]

\[
h_2 = b_6 \{ V_1 (k_z^2 - k_o^2 \sin^2 \theta_i) - V_3 k_o k_z' \sin \theta_i \}
\]

\[
h_3 = b_6 \{ T_1 (k_z^2 - k_o^2 \sin^2 \theta_i) + T_3 k_o k_z' \sin \theta_i \}
\]

\[
h_4 = b_6 \{ V_1 (k_z^2 - k_o^2 \sin^2 \theta_i) + V_3 k_o k_z' \sin \theta_i \}
\]

The values of $\Lambda_n$, $\beta_n$, and $\gamma_n$ will now be written in terms of the above parameters:

\[
\Lambda_1 = h_1 h_1^* \quad \beta_1 = D_1' \quad \gamma_1 = D_1'^*
\]

\[
\Lambda_2 = h_1 h_2^* \quad \beta_2 = D_1' \quad \gamma_2 = D_2'^*
\]

\[
\Lambda_3 = h_1 h_3^* \quad \beta_3 = D_1' \quad \gamma_3 = D_2'^*
\]

\[
\Lambda_4 = h_1 h_4^* \quad \beta_4 = D_1' \quad \gamma_4 = D_1'^*
\]

\[
\Lambda_5 = h_2 h_1^* \quad \beta_5 = D_2' \quad \gamma_5 = D_1'^*
\]

\[
\Lambda_6 = h_2 h_2^* \quad \beta_6 = D_2' \quad \gamma_6 = D_2'^*
\]

\[
\Lambda_7 = h_2 h_3^* \quad \beta_7 = D_2' \quad \gamma_7 = -D_2'^*
\]
\[ \Lambda_8 = h_2 h_4^* \quad \beta_8 = D'_2 \quad \gamma_8 = - D'_1^* \]
\[ \Lambda_9 = h_3 h_1^* \quad \beta_9 = - D'_2 \quad \gamma_9 = D'_1^* \]
\[ \Lambda_{10} = h_3 h_2^* \quad \beta_{10} = - D'_2 \quad \gamma_{10} = D'_2^* \]
\[ \Lambda_{11} = h_3 h_3^* \quad \beta_{11} = - D'_2 \quad \gamma_{11} = - D'_2^* \]
\[ \Lambda_{12} = h_4 h_4^* \quad \beta_{12} = - D'_2 \quad \gamma_{12} = - D'_1^* \]
\[ \Lambda_{13} = h_4 h_1^* \quad \beta_{13} = - D'_1 \quad \gamma_{13} = D'_1^* \]
\[ \Lambda_{14} = h_4 h_2^* \quad \beta_{14} = - D'_1 \quad \gamma_{14} = D'_2^* \]
\[ \Lambda_{15} = h_4 h_3^* \quad \beta_{15} = - D'_1 \quad \gamma_{15} = - D'_2^* \]
\[ \Lambda_{16} = h_4 h_4^* \quad \beta_{16} = - D'_1 \quad \gamma_{16} = - D'_1^* \]

The basic equation for \( <A_x A_y^*> \) can be written as

\[
<A_x A_y^*> = \frac{-4k_x \eta_x k_x A_x(\omega^2 \mu_0^2 \eta_2^2 + k_x^4 \eta_1^2) k_x^2 M_x^*}{(1 + 4k_x^2 \eta_x^2 \sin^2 \theta_i)}
\]

\[
\sum_{n=1}^{16} \frac{P_n}{(\nu_n + \rho_n)} \left\{ \frac{1 - \xi_z \rho_n + \xi_z (\nu_n + \rho_n)e^{-L(\nu_n + 1/\xi_z)} - (1 + \xi_z \nu_n)e^{-L(\nu_n + \rho_n)}}{(1 + \xi_z \nu_n) (1 - \xi_z \rho_n)} \right. \\
+ \left. \frac{1 - \xi_z \nu_n \gamma + \rho_n e^{-L(\rho_n + 1/\xi_z)} - e^{-L(\nu_n + \rho_n)}}{(1 + \xi_z \rho_n) (1 - \xi_z \nu_n)} \right\}
\]
The values of $\rho_n$, $\rho_n$, and $\nu_n$ can be defined in terms of previous parameters.

\begin{align*}
P_1 &= h_{1} T_{1}^{*} & \nu_1 &= D'_1 & \rho_1 &= D'_1 \\
P_2 &= h_{1} T_{2}^{*} & \nu_2 &= D'_1 & \rho_2 &= D'_2 \\
P_3 &= h_{1} T_{2}^{*} & \nu_3 &= D'_1 & \rho_3 &= -D'_2 \\
P_4 &= h_{1} T_{2}^{*} & \nu_4 &= D'_2 & \rho_4 &= -D'_2 \\
P_5 &= h_{1} T_{2}^{*} & \nu_5 &= D'_2 & \rho_5 &= D'_1 \\
P_6 &= h_{2} T_{2}^{*} & \nu_6 &= D'_2 & \rho_6 &= D'_2 \\
P_7 &= h_{2} T_{2}^{*} & \nu_7 &= D'_2 & \rho_7 &= -D'_2 \\
P_8 &= h_{2} T_{2}^{*} & \nu_8 &= D'_2 & \rho_8 &= -D'_1 \\
P_9 &= h_{2} T_{2}^{*} & \nu_9 &= -D'_2 & \rho_9 &= D'_1 \\
P_{10} &= h_{2} T_{2}^{*} & \nu_{10} &= -D'_2 & \rho_{10} &= D'_2 \\
P_{11} &= h_{2} T_{2}^{*} & \nu_{11} &= -D'_2 & \rho_{11} &= -D'_2 \\
P_{12} &= h_{2} T_{2}^{*} & \nu_{12} &= -D'_2 & \rho_{12} &= -D'_1 \\
P_{13} &= h_{2} T_{2}^{*} & \nu_{13} &= -D'_2 & \rho_{13} &= D'_1 \\
P_{14} &= h_{2} T_{2}^{*} & \nu_{14} &= -D'_1 & \rho_{14} &= D'_2
\end{align*}
\[
\begin{align*}
P_{15} &= h_4 T_2^* & \nu_{15} &= -D_1' & \rho_{15} &= -D_2^* \\
P_{16} &= h_4 V_2^* & \nu_{16} &= -D_1' & \rho_{16} &= -D_1^*
\end{align*}
\]

Expressions for \( \langle A_z A_z^* \rangle \) and \( \langle A_x A_x^* \rangle \) can be obtained in terms of previous results.

\[
\begin{align*}
\langle A_z A_z^* \rangle &= \frac{k_o^2 \sin^2 \theta_1}{k_{1z} k_{1z}^*} & \langle A_x A_x^* \rangle \\
\langle A_x A_x^* \rangle &= \frac{k_o \sin \theta_1}{k_{1z}^*} & \langle A_x A_x \rangle
\end{align*}
\]
APPENDIX B.

Computer Programs for Calculating the Backscattering Coefficients
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**Statement Level**

- **Non-Active**: 20
- **Active**: 12

**Loops**

- **Active**: 12
- **Non-Active**: 10
### Table: Variables

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### Table: Constants

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### Comments

- Description of the simulation:...
- Values of parameters:...
- Fluctuations and correlation distances:...
- Additional notes:...

---

**Page 2**

---

**Page 3**

---

**Page 4**
<table>
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<th>Parameter</th>
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<tr>
<td>Layer thickness (m)</td>
<td>5.94272477</td>
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<tr>
<td>Depth of the surface fluctuation</td>
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<tr>
<td>Volume of water in the fluctuations (m³)</td>
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<tr>
<td>Frequency (Hz)</td>
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<td>Height of the surface fluctuation</td>
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<td>Layer thickness in water (m)</td>
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<tr>
<td>Stability of the soil (kPa)</td>
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</tr>
<tr>
<td>Conductivity of the soil (S/m)</td>
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<td>Theoretical value of the conductivity (S/m)</td>
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</table>
## LIST OF SYMBOLS

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>( r )</td>
<td>Position vector</td>
</tr>
<tr>
<td>( e(I) )</td>
<td>Permittivity of the random medium</td>
</tr>
<tr>
<td>( \sigma(I) )</td>
<td>Conductivity of the random medium</td>
</tr>
<tr>
<td>( e_a )</td>
<td>Average relative dielectric constant of the random medium</td>
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<tr>
<td>( \sigma_a )</td>
<td>Average conductivity of the random medium</td>
</tr>
<tr>
<td>( \eta_1 )</td>
<td>Standard deviation of the dielectric fluctuations</td>
</tr>
<tr>
<td>( \eta_2 )</td>
<td>Standard deviation of the conductivity fluctuations</td>
</tr>
<tr>
<td>( L )</td>
<td>Mean thickness of the vegetation layer</td>
</tr>
<tr>
<td>( k_3 )</td>
<td>Complex propagation constant in the soil</td>
</tr>
<tr>
<td>( \theta_i )</td>
<td>Angle of incidence</td>
</tr>
<tr>
<td>( k_0 )</td>
<td>Free space propagation constant</td>
</tr>
<tr>
<td>( \Gamma(I,I') )</td>
<td>Infinite space dyadic Green's function</td>
</tr>
<tr>
<td>( &lt;E(I)&gt; )</td>
<td>Mean wave in the random medium</td>
</tr>
<tr>
<td>( G_o(I,I') )</td>
<td>Infinite space scalar Green's function</td>
</tr>
<tr>
<td>( E_s(I) )</td>
<td>Scattered electric field in the random medium</td>
</tr>
<tr>
<td>( \delta(I-I') )</td>
<td>Three-dimensional Dirac delta function equal to ( \delta(x-x')\delta(y-y')\delta(z-z') )</td>
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<tr>
<td>( \mathbb{I} )</td>
<td>Unit dyadic</td>
</tr>
<tr>
<td>( a_x, a_y, a_z )</td>
<td>Unit vectors in ( x, y, ) and ( z )</td>
</tr>
</tbody>
</table>
\( k_x, k_y \)  
Fourier transform variables

\( \sqrt{-1} \)  
Kronecker Delta

\( \delta_{ij} \)  
Components of the effective propagation constant

\( k_{ex}, k_{ey}, k_{ez} \)  
Value of \( k_{ez} \) for horizontal polarization

\( k_h \)  
Value of \( k_{ez} \) for vertical polarization

\( G_a(k_x, z) \)  
Two-dimensional Fourier transform of the scattered electric field in the random medium

\( A_x(k_x, k_y), A_y(k_x, k_y), A_z(k_x, k_y) \)  
Fourier transform of the amplitudes of the scattered electric field in air

\( \alpha_i \)  
Illuminated surface area

\( \xi_x, \xi_y, \xi_z \)  
Correlation distances in \( x, y, \) and \( z \), respectively

\( \sigma_{HHV}^0 \)  
Backscatter coefficient for volume scattering and for the case of horizontal polarization transmit, horizontal polarization receive

\( \sigma_{VVV}^0 \)  
Backscatter coefficient for volume scattering and for the case of vertical polarization transmit, vertical polarization receive

\( \sigma_{HHS}^0 \)  
Backscatter coefficient for a randomly rough surface for the case of horizontal polarization transmit, horizontal polarization receive

\( \sigma_{HH}^0 \)  
Final backscatter coefficient result that includes both volume scattering and rough surface scattering for the case of horizontal polarization transmit, horizontal polarization receive
\( o_{VV}^{o} \)

Final backscatter coefficient result that includes both volume scattering and rough surface scattering for the case of vertical polarization transmit, vertical polarization receive

\( F \)

Fraction of water by weight in the vegetation

\( R_{V} \)

Volume of vegetation divided by the total volume

\( m_{s} \)

Standard deviation of the rough surface fluctuations divided by the correlation distance

\( f \)

Frequency