THE RESPONSE OF A TWO-WIRE TRANSMISSION LINE TO INCIDENT FIELD AND VOLTAGE EXCITATION, INCLUDING THE EFFECTS OF HIGHER ORDER MODES

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The main objective of this report has been to derive the induced current on a two-wire transmission line due to incident field excitation. Special attention has been paid to the case of voltage source excitation, where the current may be given, at least approximately, using the so-called leaky-wave concept. The influence of the electrical dimensions of the structure on the currents and the relative importance of the higher modes is examined by numerical examples.
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I. Introduction

In this report a study has been made of the induced currents on a two-wire (parallel-cylinder) transmission line due to incident field excitation. The basic assumptions are that the wires are perfectly conducting, parallel, infinitely long and that their radii are very small compared to the wavelength of the excitation. Thus, the induced currents along the wires are oriented in the direction of the cylinder axis of each wire, they are independent of the azimuthal angle around each wire, and hence we may replace each wire by an equivalent filament. Boundary conditions are applied at the wire surface so that wire thickness is taken into account. Previous work related to this problem included Schelkunoff's Laplace transform solution [1] and Marin's transient solution [2] of the two-wire problem.

The formulation of the problem is effected by using the spectral concept. The excitation field has been resolved into its Fourier (spatial) components and the induced current has been expressed in terms of a superposition integral. First, the case of plane wave excitation has been treated. This led to a straightforward solution of the integral representation and a simple solution for the induced currents. Using the results, the variations of the induced currents have been shown as a function of the distance between the wires (while the ratio of the distance between the wires to the radius of the wire has been kept fixed). Also, the variation of the currents has been shown as a function of the radius of the wire (while the distance between the wires has been kept fixed).

Special attention has been paid to the case of voltage source excitation. In this case, by deforming the contour of integration in the induced current formulation and applying Cauchy's theorem, one
obtains the solution in terms of the so-called leaky modes, the TEM mode, and a continuous spectrum. The leaky modes are not proper solutions of Maxwell's equation and, in contrast to the discrete eigenmodes, which exist in regions of finite extent bounded by impermeable walls, do not possess orthogonality and completeness properties, and therefore must be supplemented by a continuous spectrum of characteristic modes to permit the representation of an arbitrary function. Nevertheless, despite their physically unacceptable behavior in the entire domain, they can be employed to obtain a convergent representation of the field solution in certain regions.

The preceding modal solution may be obtained via a straightforward approach using the general network parameters of the method of moments [3]. Solving for the zeros of the total impedance of the system leads as expected to the same equation that determines the poles of the integrand in the integral solution for the induced currents.

The influence of the electrical dimensions of the structure on the induced currents and the relative importance of the higher modes are examined by concrete numerical examples. By looking at the results, it can be readily seen, as one can expect, that the attenuation constants of the higher modes decrease as the distance between the wires is increased. In contrast, the attenuation constants increase for fixed distance between the wires as the radius is decreased. The magnitudes of the TEM and the higher order modal currents have also been compared at a fixed point along the z axis. As one proceeds along the z axis, the higher order modes are attenuated exponentially, while the TEM remains constant.

These studies apply particularly to problems involving high frequency excitation, such that the wire separation is comparable to or larger than a wavelength, in which case the usual TEM mode analysis is inadequate.
II. Formulation of the Problem

Consider the problem of two perfectly conducting, parallel, thin, infinitely long wires with radii $a$ separated by a distance $d$. The $z$-directed induced currents on the wires are due to an incident field excitation which may be either symmetric or antisymmetric as shown in Figure 1. The symmetric excitation (which consists of two plane waves) yields a magnetic wall at $y = \frac{d}{2}$. Similarly, the antisymmetric excitation yields an electric wall at $y = \frac{d}{2}$. An arbitrary plane wave can be represented in terms of a sum of symmetric and antisymmetric excitations. We assume that $\frac{a}{d} \ll 1$ and $\frac{a}{\lambda} \ll 1$, where $\lambda$ is the wavelength. Thus, the induced current is along the cylinder axis of each wire, and is independent of the azimuthal angle around each wire.

The magnetic vector potential for this problem is given by (for $\exp(j\omega t)$ time convention)

$$ A(x, y, z) = \hat{z} \hat{\Psi}(x, y, z) $$

where

$$ \hat{\Psi}(x, y, z) = \int_{-\infty}^{\infty} I_\pm(z') \left[ \frac{e^{-jk\sqrt{x^2 + y^2 + (z - z')^2}}}{4\pi \sqrt{x^2 + y^2 + (z - z')^2}} + \frac{e^{-jk\sqrt{(y - d)^2 + (z - z')^2}}}{4\pi \sqrt{(y - d)^2 + (z - z')^2}} \right] dz' $$

$I(z)$ denotes the current along the wire, $x, y, z$, are the coordinates of a field point in a rectangular coordinate system, $z'$ is the coordinate of a source point, $k$ is the wavenumber, and the upper and lower signs are for symmetric and antisymmetric excitations, respectively.

The Fourier transform of $\Psi(x, y, z)$ with respect to the cylinder
TWO-WIRE TRANSMISSION, SYMMETRIC EXCITATION, ANTISYMMETRIC

Fig. 1 The two-wire geometry.
axis z is,
\[
\overline{\mathbf{E}}_z(x,y,k_z) = \int_{-\infty}^{\infty} \mathbf{E}_z(x,y,z) e^{jk_zz} \, dz
\]
(3)
and it may be given by,
\[
\overline{\mathbf{E}}_z(x,y,k_z) = \frac{1}{4j} \overline{I}_z(k_z) \left[ H_1^{(2)}(k_r \rho_1) \pm H_1^{(2)}(k_r \rho_2) \right]
\]
(4)
where \( \overline{I}_z(k_z) \) is the transform of \( I_z(z) \); \( k_r \) the radial wavenumber is defined as
\[
k_r = (k^2 - k_z^2)^{1/2}
\]
(5)
and \( \rho_1 = (x^2 + y^2)^{1/2} \), \( \rho_2 = [x^2 + (y - d)^2]^{1/2} \).
The \( H_1^{(2)} \) is the Hankel function of the second kind of zero order, and we have used the identity [4]
\[
\frac{e^{-jk\sqrt{\rho_1^2 + z^2}}}{\sqrt{\rho_1^2 + z^2}} = \frac{1}{2j} \int_{-\infty}^{\infty} H_1^{(2)}(k_r \rho) e^{jk_r^2} \, dk_r
\]
(6)
According to our assumption \( \frac{d}{d} \ll 1 \). Therefore we can make the following approximation
\[
\sqrt{x^2 + (y - d)^2} = a \left[ 1 - \frac{y}{d} + \frac{1}{2} \frac{a^2}{d^2} \right] \approx d
\]
(7)
Using (7), (4) may be simplified for \( x^2 + y^2 = a^2 \) and we obtain,
\[
\Psi_{\pm}(x,y,k_z) \Big|_{x^2 + y^2 = a^2} = \frac{1}{4j} \overline{I}_z(k_z) \left[ H_1^{(2)}(k_r \rho_1) \pm H_1^{(2)}(k_r \rho_2) \right]
\]
(8)
The z-directed scattered field due to the induced currents is obtained from \( \Psi \) according to
\[
E_z^* (x,y,z) = \frac{1}{j\omega \varepsilon_c} \frac{\partial}{\partial z} \Psi_{\pm}(x,y,z)
\]
(9)
The field can also be written as a function of \( \Psi \), the Fourier transform
of \( \Psi \). Using the inverse transform of (3), namely

\[ \Psi_i(x,y,z) = \frac{1}{2\pi j} \int_{-\infty}^{\infty} \overline{\Psi_i}(x,y,kz) e^{jkz} dz \]

(10)

and substituting (10) into (9) results in

\[ E_z^S(x,y,z) = \frac{1}{2\pi j} \int_{-\infty}^{\infty} \frac{k^2 - k_z^2}{j\omega\epsilon} \overline{\Psi_i}(x,y,kz) e^{jkz} dz \]

(11)

Hence, the transformed solution to the field is readily given by

\[ \overline{E_z^S}(x,y,kz) = \frac{k^2 - k_z^2}{j\omega\epsilon} \overline{\Psi_i}(x,y,kz) \]

(12)

where \( \overline{\Psi} \) is given, for the general case, by (4).

The total z-directed field, with the conducting cylinders present, is the sum of the incident and scattered fields, that is,

\[ E_z = E_z^i + E_z^S \]

For each cylinder the boundary condition \( E_z = 0 \) must be satisfied on the cylinder surface. Without loss of generality, we can deal with the cylinder centered at the origin. Thus, we obtain

\[ E_z^i(x,y,z) + E_z^S(x,y,z) \bigg|_{x^2 + y^2 = a^2, -\infty < z < \infty} = 0 \]

(13)

The transformed solution of the field should also satisfy (13), that is,

\[ \overline{E_z^i}(x,y,kz) + \overline{E_z^S}(x,y,kz) \bigg|_{x^2 + y^2 = a^2} = 0 \]

(14)

where

\[ \overline{E_z^i}(x,y,kz) = \int_{-\infty}^{\infty} E_z^i(x,y,z)e^{-jkz} dz \]

(15)

and

\[ \overline{E_z^S}(x,y,kz) \] is given by (12).

Using \( \overline{E_z^S} \) together with the approximate \( \overline{\Psi_i} \) of (8) in (14) results in the
following expression for the transformed current

\[
\overline{I}_\pm(z) = \frac{4\omega \mathcal{E}_0^4(x,y,k_z) |x^2 + y^2 = a^2}{k_0^2 [H_o^{(2)}(k_0 a) \pm H_o^{(2)}(k_0 d)]}
\]

(16)

Applying straight-forward inverse transform on (16), one readily obtains the current on the wire. We have,

\[
I_\pm(z) = \frac{2\omega \mathcal{E}_0^4}{\pi} \int_{-\infty}^{\infty} \frac{E_z^i(x,y,k_z)}{k_0^2 [H_o^{(2)}(k_0 a) \pm H_o^{(2)}(k_0 d)]} |x^2 + y^2 = a^2 e^{jkz} |x^2 + y^2 = a^2 e^{jkz} dz
\]

(17)

In case of an incident wave which is neither symmetric nor antisymmetric, it may be represented as a superposition of symmetric and antisymmetric incident waves as follows

\[
E_z = E_z^+ + E_z^-
\]

and the currents will be given then by

\[
I_1(z) = I_+(z) + I_-(z)
\]

(18)

for the wire centered at \(x = 0, y = 0\) and

\[
I_2(z) = I_+(z) - I_-(z)
\]

(19)

for the wire centered at \(x = 0, y = d\). From (17) we quote

\[
I_+(z) = \frac{2\omega \mathcal{E}_0^4}{\pi} \int_{-\infty}^{\infty} \frac{E_z^i(x,y,k_z)}{k_0^2 [H_o^{(2)}(k_0 a) + H_o^{(2)}(k_0 d)]} |x^2 + y^2 = a^2 e^{jkz} |x^2 + y^2 = a^2 e^{jkz} dz
\]

\[
I_-(z) = \frac{2\omega \mathcal{E}_0^4}{\pi} \int_{-\infty}^{\infty} \frac{E_z^i(x,y,k_z)}{k_0^2 [H_o^{(2)}(k_0 a) - H_o^{(2)}(k_0 d)]} |x^2 + y^2 = a^2 e^{jkz} |x^2 + y^2 = a^2 e^{jkz} dz
\]

(20)
III. Plane Wave Excitation

Figure 2 shows a plane wave incident upon the two-wire structure.

The electric incident field has been divided into the sum,

\[ E_{\text{inc}} = E_h^{\text{inc}} + E_e^{\text{inc}} \]  

(21)

The subscripts \( h \) and \( e \) denote H and E polarizations, respectively. H modes are those for which \( E \) is parallel to the plane spanned by the \( \hat{z} \) and \( \hat{h} \). \( \hat{R} \) represents a unit vector in the direction of propagation; the carat represents a unit vector. E modes are those for which \( E \) is normal to this plane. Hence, we have

\[ E_e^{\text{inc}} = [E^{\text{inc}} \cdot (k \times \hat{z})] \frac{\hat{R} \times \hat{z}}{|\hat{R} \times \hat{z}|^2} \]  

(22)

\[ E_h^{\text{inc}} = E^{\text{inc}} - E_e^{\text{inc}} \]  

(23)

Since we have already limited ourselves to a thin wire problem which can be replaced with a filamentary model, the E-polarized component does not contribute to the induced current. Therefore we will consider only the H component. The last is given for the incident wave depicted in Figure 2 by

\[ E_h^{\text{inc}} (x,y,z) = E_0 (-\hat{x} \cos \theta_0 \cos \phi_0 - \hat{y} \cos \theta_0 \sin \phi_0 + \hat{z} \sin \theta_0) e^{+j(k_{x0} + k_{y0} + k_{z0} z)} \]  

(24)

where \( k_{x0}, k_{y0}, k_{z0} \), the axial wave numbers are

\[ k_{x0} = k \sin \theta_0 \cos \phi_0, \quad k_{y0} = \sin \theta_0 \sin \phi_0, \quad \text{and} \quad k_{z0} = k \cos \theta_0, \]  

(25)

and \( \theta_0, \phi_0 \) are, respectively, the polar and azimuthal angles of the incident wave in spherical coordinates.

The \( z \) component of \( E^{\text{inc}} \) may be represented as a sum of symmetric and antisymmetric incident waves as follows

\[ E_z^{\text{inc}} = E_{z+}^{\text{inc}} + E_{z-}^{\text{inc}} \]  

(26)
Fig. 2 A plane wave incident upon the two-wire structure.
where $E_{inc}^{+}$, the symmetric excitation, is given by

$$E_{inc}^{+}(x,y,z) = \frac{1}{2} E_0 \sin \theta_0 [e^{j(k_x x + k_0 y + k_z z)} + e^{j[k_x x + k_0 y (d - y) + k_z z]}]$$

and $E_{inc}^{-}$, the antisymmetric excitation, is given by

$$E_{inc}^{-}(x,y,z) = \frac{1}{2} E_0 \sin \theta_0 [e^{j(k_x x + k_0 y + k_z z)} - e^{j[k_x x + k_0 y (d - y) + k_z z]}]$$

The transformed field is readily obtained by applying the Fourier transform. We have

$$E_{z}^{\text{inc}} = E_{z+}^{\text{inc}} + E_{z-}^{\text{inc}}$$

where

$$E_{z+}^{\text{inc}}(x,y,k_z) = \pi E_0 \sin \theta_0 e^{j k_0 y} [e^{j k_0 (d - y)} + e^{-j k_0 (d - y)}] \delta(k_z - k_{zo})$$

and

$$E_{z-}^{\text{inc}}(x,y,k_z) = \pi E_0 \sin \theta_0 e^{j k_0 y} [e^{j k_0 (d - y)} - e^{-j k_0 (d - y)}] \delta(k_z - k_{zo})$$

where the $\delta$ denotes the impulse function. For $x^2 + y^2 = a^2 < \lambda^2$, (30) and (31) reduce, respectively, to,

$$E_{z+}^{\text{inc}}(x,y,k_z) \bigg|_{x^2 + y^2 = a^2} = \pi E_0 \sin \theta_0 (1 + e^{j k_0 y}) \delta(k_z - k_{zo})$$

and

$$E_{z-}^{\text{inc}}(x,y,k_z) \bigg|_{x^2 + y^2 = a^2} = \pi E_0 \sin \theta_0 (1 - e^{j k_0 y}) \delta(k_z - k_{zo})$$

substituting (32) and (33) in (18), (19) and (20) yields

$$I_1(z) = \frac{2E_o}{\pi k_0} \left[ \frac{j k_0 d}{H_0^{(2)}(k_0 a) + H_0^{(2)}(k_0 d)} + \frac{j k_0 d}{H_0^{(2)}(k_0 a) - H_0^{(2)}(k_0 d)} \right] e^{j k_{zo} z}$$

(34)
For the wire centered at $x = 0$, $y = 0$ and

$$I_2(z) = \frac{2E_0}{\eta k_{\rho o}} \left( \frac{j k \gamma d}{1 + e^{-j k \gamma d}} - \frac{1 - e^{-j k \gamma d}}{H_0^{(2)}(k_{\rho o} a) + H_0^{(2)}(k_{\rho o} d)} \right) e^{-j k \gamma z}$$

(35)

for the wire centered at $x = 0$, $y = d$ where $k_{\rho o}$, the radial wave number, is $k_{\rho o} = (k^2 - k_{\gamma o}^2)^{1/2} = k \sin \theta_0$ and $\eta$ denotes the characteristic impedance of the medium given by $\eta = (\mu_0 \epsilon_0)^{1/2}$.

Equations (34) and (35) reduce properly in some limiting cases.

For $\theta_0 = \frac{\pi}{2}$, $\phi_0 = \pi$ we deal with a plane wave, $E_z = E_0 e^{-jkx}$. Taking
the limits $kd \to \infty$ and $ka \to 0$, and using large argument and small argument formulas for $H_0^{(2)}$, we obtain

$$I(z) = j \frac{2\pi E_0}{\omega \mu \ln(ka)}$$

(36)

which agrees with Harrington's result [5] for the total current induced on a single thin wire due to identical excitation.
IV. Solution of the Superposition Integral in Terms of the TEM Mode and the Leaky Modes

The integral representation for the current [Equation (17)] is rewritten for convenience as

\[ \int_{-\infty}^{\infty} f_z(k_z, x, y) \frac{e^{j k z}}{x^2 + y^2 - a^2} dk_z \tag{37} \]

where

\[ f_z(k_z, x, y) = \frac{E_z^0(x, y, k_z)}{\sqrt{2\pi}} \frac{z}{k_z[H_0^{(2)}(k_z a) \pm H_0^{(2)}(k_z d)]} \]

The singularities of the integrand are two branch points at \( k_z = \pm k \).

On introducing a small loss perturbation one finds that for positive values in the \( z \) direction (\( z > 0 \)), the contour runs on the top sheet of the \( k_z \) plane along the real axis, below the branch point \(-k\) and above \(+k\). If the integrand in (37) does not have poles in the upper half of the top sheet of the \( k_z \) plane, it becomes possible to deform the original path of integration, the real axis, from \(-\infty\) to \(+\infty\), to a new path ABCD as shown in Figure 3. The path ABCD may be modified by letting it cross the branch cut twice, entering into the bottom sheet in this process. The portion of the path in the lower sheet, namely \( C' \), may now be deformed into a new path \( C'' \), as shown in Figure 4, with the purpose of improving the convergence of the integral [2, 6, 7]. In the process of carrying out the last step, we cross the poles of the integrand in (37). These poles are solutions of the equation

\[ H_0^{(2)}(\sqrt{k^2 - k_z^2}a) \pm H_0^{(2)}(\sqrt{k^2 - k_z^2}d) = 0 \tag{38} \]

Equation (38) corresponds to Equation (30) of Marin [2]. Lying in the second quadrant of the \( k_z \) plane, these poles are associated with fields that decay as they propagate in the \( z \) direction. However, since
Fig. 3  Illustration of the contours ABDC and ABC'D.
Fig. 4  Deforming the contour to cross the leaky poles.
these poles belong to the bottom sheet of the $k_z$ plane, the fields associated with them show a growth behavior in the transverse direction. For this reason, these modes are referred to as the improper, or leaky, modes [7].

In this fashion, when deforming the contour no contribution will arise from any segment of the infinite circle $(A + D)$ on the proper sheet of the complex $k_z$ plane. On applying Cauchy's theorem, one has

$$I_x(z) = \sum \text{residues of } \left[ f_{\pm} \left( k_z, x, y \right) \left| \frac{x^2 + y^2 = a^2 \exp(jk_zz)}{x^2 + y^2 = a^2} \right. \right]$$

at the leaky poles when $C'$ is deformed to $C''$.)

$$+ \int_{B + C} f_{\pm}(k_z, x, y) \left| \frac{x^2 + y^2 = a^2 \exp(jk_zz)dk_z}{x^2 + y^2 = a^2} \right. \right. \right)$$

The first term in Equation (39) is the contribution of the leaky poles, and it may be rewritten as (Appendix A1)

$$I_x(z) = \sum_{\text{leaky}} \frac{4j}{\pi} \frac{k}{k_{p\pm}} \frac{\bar{F}_{k_z}(x, y, k_{p\pm})}{k_{p\pm}[\bar{H}_1^{(2)}(k_{p\pm} a) + \bar{H}_1^{(2)}(k_{p\pm} d)\bar{H}_1^{(2)}(k_{p\pm} a)]} \frac{\exp(jk_{p\pm}z)}{c}$$

where $k_z = k_{p\pm}$ are the consecutive roots of the equation

$$\bar{H}_0^{(2)}(\sqrt{k^2 - k_z^2} a) + \bar{H}_0^{(2)}(\sqrt{k^2 - k_z^2} d) = 0$$

and will be referred to as even poles; $k_z = k_{p\pm}$ are the consecutive roots of the equation

$$\bar{H}_0^{(2)}(\sqrt{k^2 - k_z^2} a) - \bar{H}_0^{(2)}(\sqrt{k^2 - k_z^2} d) = 0$$

and will be referred to as odd poles, and

$$k_{p\pm} = (k_z^2 - k_{p\pm}^2)^{1/2}$$

In the above equations we exclude, of course, those roots for which $\Re(k_z) < -k$ or $\Re(k_z) > 0$ and $\Im(k_z) < 0$. This series, in (40), is often the dominant contribution to the current of the wires [6].
For negative values in the $z$ direction ($z < 0$), the contour runs on the top sheet of the $k$ plane along the real axis, above the branch point $-k$ and below $+k$. If the integrand in (37) does not have poles in the lower half of the top sheet of the $k$ plane, then making similar contour deformation in the lower part of the $k$ plane, one crosses the leaky poles, which are the roots of Equation (38) that lie in the fourth quadrant and are the image of the previous poles. Hence for $z < 0$ we may apply (40) replacing $k_{sp}$ by $-k_{sp}$.

The second term is in fact the integral along a branch cut starting at $-k$ and parallel to the imaginary axis in the upper half of the $k$ plane [8]. It results in the contribution of the branch point which, for antisymmetric excitation, may be interpreted as the TEM mode, represented by (Appendix A2)

$$I(z) = \frac{\pi}{\eta \ln(\frac{d}{a})} E_{\text{inc}}(x, y, -k) \left| x^2 + y^2 = a^2 e^{-j \kappa z} \right. (41)$$

For symmetric excitation the branch point contribution is zero. The other contribution is the branch cut contribution, which is rather time-consuming to evaluate, but is negligible [8].

Hence, the current induced in the case of an arbitrary incident wave will be given by

$$I_1(z) = I_{\text{TEM}}(z) + I_{\text{leaky}}(z) + I_{\text{leaky}}(z) (42)$$

for the wire centered at $x = 0, y = 0$, and by

$$I_2(z) = -I_{\text{TEM}}(z) + I_{\text{leaky}}(z) - I_{\text{leaky}}(z) (43)$$

for the wire centered at $x = 0, y = d$, where $I_{\text{TEM}}(z)$ and $I_{\text{leaky}}(z)$ are given by Equation (40) and Equation (41), respectively.
V. Voltage Source Excitation

Consider the ideal voltage generator centered at the origin as depicted in Figure 5. The equivalent superposition of symmetric and antisymmetric excitations is shown in Figure 6. For this case, the incident wave is given by [3]

$$E_{x}^{inc} = \begin{bmatrix} \nu \delta(z) & x^2 + y^2 = a^2 \\ 0 & \text{elsewhere} \end{bmatrix}$$ (44)

where $\delta$ denotes the impulse function. $E_{x}^{inc}$ in (44) may be cast into the form

$$E_{x}^{inc} = E_{x+}^{inc} + E_{x-}^{inc}$$

$$E_{x+}^{inc} = \begin{Bmatrix} v/2 \delta(z) & x^2 + y^2 = a^2 \\ v/2 \delta(z) & x^2 + (y - d)^2 = a^2 \\ 0 & \text{elsewhere} \end{Bmatrix}$$ (45)

$$E_{x-}^{inc} = \begin{Bmatrix} v/2 \delta(z) & x^2 + y^2 = a^2 \\ -v/2 \delta(z) & x^2 + (y - d)^2 = a^2 \\ 0 & \text{elsewhere} \end{Bmatrix}$$ (46)

The transformed field may be readily obtained and at $x^2 + y^2 = a^2$, we have

$$E_{x}^{inc}(x,y,k_z) \bigg|_{x^2 + y^2 = a^2} = \begin{Bmatrix} -E_{x+}^{inc}(x,y,k_z) + E_{x-}^{inc}(x,y,k_z) \end{Bmatrix} \bigg|_{x^2 + y^2 = a^2}$$ (47)

where

$$E_{x+}^{inc}(x,y,k_z) \bigg|_{x^2 + y^2 = a^2} = v/2$$ (48)

$$E_{x-}^{inc}(x,y,k_z) \bigg|_{x^2 + y^2 = a^2} = v/2$$ (49)
Fig. 5  Voltage-source excitation.
Substituting Equations (48) and (49) in Equations (41), (42) and (43), we obtain

\[ I_1(z) = \frac{n}{\pi} \left( \frac{y}{2} \right) e^{-jkz} + \]

\[
\sum_{\text{even poles}} \frac{jk e^{z \xi_p^+}}{k x_p^+ z_p^+ [a h_1^{(2)}(k x_p^+ a) + d h_1^{(2)}(k x_p^+ d)]} +
\]

\[
\sum_{\text{odd poles}} \frac{jk e^{z \xi_p^-}}{k x_p^- z_p^- [a h_1^{(2)}(k x_p^- a) - d h_1^{(2)}(k x_p^- d)]} \]

(50)

for the wire centered at \( x = 0, y = 0, \) and

\[ I_2(z) = \frac{n}{\pi} \left( \frac{y}{2} \right) e^{-jkz} + \]

\[
\sum_{\text{even poles}} \frac{jk e^{z \xi_p^+}}{k x_p^+ z_p^+ [a h_1^{(2)}(k x_p^+ a) - d h_1^{(2)}(k x_p^+ d)]} +
\]

\[
\sum_{\text{odd poles}} \frac{jk e^{z \xi_p^-}}{k x_p^- z_p^- [a h_1^{(2)}(k x_p^- a) + d h_1^{(2)}(k x_p^- d)]} \]

(51)

for the wire centered at \( x = 0, y = d. \)
VI. The Modal Solution of the Problem using Generalized Network Method

Equation (38), whose roots are the z-directed wave numbers of the higher-order leaky modes, may be obtained via straightforward analysis, as follows. Consider the problem of the two wires, having induced currents $I_1(z)$ and $I_2(z)$, respectively, due to an arbitrary impressed field $E_z^i$ produced by external sources. The currents $I_1(z)$ and $I_2(z)$ produce scattered fields $E_{z1}^s(z)$ and $E_{z2}^s(z)$, respectively. These fields can be found in terms of the currents by the potential integral method in a manner similar to that presented under "Formulation of the Problem."

We have,

$$E_{1,2}^s(x,y,z) = \frac{1}{j\omega} \left( \frac{\partial^2}{\partial z^2} \right) \psi_{1,2}(x,y,z)$$  \hspace{1cm} (52)

where

$$\psi_1(x,y,z) = \int_0^\infty I_1(z') \frac{e^{-jk\sqrt{x^2 + y^2 + (z - z')^2}}}{4\sqrt{x^2 + y^2 + (z - z')^2}} dz'$$  \hspace{1cm} (53)

$$\psi_2(x,y,z) = \int_0^\infty I_2(z') \frac{e^{-jk\sqrt{(y - d)^2 + (z - z')^2}}}{\sqrt{(y - d)^2 + (z - z')^2}} dz'$$  \hspace{1cm} (54)

At each cylinder the boundary condition $E_z = 0$ must be satisfied on the surface. Hence, we obtain

$$E_z^i(x,y,z) + E_{z1}^s(x,y,z) + E_{z2}^s(x,y,z) \bigg|_{x^2 + y^2 = a^2} = 0, \quad -\infty < z < \infty$$

(55)

and

$$E_z^i(x,y,z) + E_{z1}^s(x,y,z) + E_{z2}^s(x,y,z) \bigg|_{x^2 + (y - d)^2 = a^2} = 0, \quad -\infty < z < \infty$$

(56)
The transformed solutions of the fields should also satisfy (55) and (56), that is

\[
-\bar{E}_z(x, y, k_z) + \bar{E}_z(x, y, k_z) + \bar{E}_z(x, y, k_z) = 0
\]

\[
\bar{E}_z(x, y, k_z) + \bar{E}_z(x, y, k_z) + \bar{E}_z(x, y, k_z) = 0
\]

Substituting the appropriate transformations in a manner analogous to that in "Formulation of the Problem," (57) and (58) may be cast in a matrix form similar to that which occurs in the familiar method of moments formulation [3]. We have

\[
\begin{bmatrix}
Z_{11} & Z_{12} \\
Z_{12} & Z_{22}
\end{bmatrix} \begin{bmatrix}
J_1 \\
J_2
\end{bmatrix} = \begin{bmatrix}
V_1 \\
V_2
\end{bmatrix}
\]

(59)

where the elements of the matrix \([Z]\) are called "generalized impedances" and are given by,

\[
Z_{11} = \frac{k^2 - k_z^2}{4 \mu_0} H_0^{(2)} \left(\sqrt{k^2 - k_z^2} a\right)
\]

\[
Z_{22} = Z_{11}
\]

\[
Z_{12} = \frac{k^2 - k_z^2}{4 \mu_0} H_0^{(2)} \left(\sqrt{k^2 - k_z^2} d\right)
\]

(60)

the elements of the vector \([J]\) are called "generalized currents" and are given by

\[
J_1 = \frac{I_1(k_z)}{4 \pi a}, J_2 = \frac{I_2(k_z)}{4 \pi a}
\]

(61)

and the elements of the vector \([V]\) are called "generalized voltages" and are given by

\[
V_1 = \frac{-k_z^{-1}}{Z_1}, V_2 = \frac{-k_z^{-1}}{Z_2}
\]

(62)
In order to determine the modal solution of the problem, which is connected with the zeros of the total impedance of the system, we seek for all possible source-free solutions of (59). Hence the appropriate matrix equation is (59) with the right side zero; that is

\[
\begin{bmatrix}
Z_{11} & Z_{12} \\
Z_{12} & Z_{11}
\end{bmatrix}
\begin{bmatrix}
J_1 \\
J_2
\end{bmatrix}
= 
\begin{bmatrix}
0 \\
0
\end{bmatrix}
\]  

Equation (63) will have a non-trivial solution provided that the determinant of the impedance matrix is zero. That is

\[
Z_{11}^2 - Z_{12}^2 = 0
\]

which may be rewritten as

\[
Z_{11}^2 + Z_{12}^2 = 0
\]

Substituting (60) into (65) results in

\[
k_z = \pm k
\]

which represents the TEM mode; and,

\[
H_0^{(2)} (\sqrt{k^2 - k_z^2}) a \pm H_0^{(2)} (\sqrt{k^2 - k_z^2}) d = 0
\]

which is identical to (38). Once the proper values of \(k_z\) for which (63) is satisfied are found, the eigenvectors \(J_1, J_2\) may be obtained by solving (59) for each \(k_z\).
VII. Results and Conclusions

The case of plane wave excitation is shown in Figures 7-9. The example treats an incident plane wave for which \( \theta_0 \), the polar angle is 45° and \( \phi_0 \), the azimuthal angle is 45°. Figures 7-8 describe the magnitude of the currents induced on the wires - Eqs. (35),(36) - vs. the normalized distance between the wires, for a fixed ratio \( d/a = 10 \). Note the oscillations, which reflect the changes that occur in the mutual coupling between the wires as we vary the distance. Figure 9 shows the magnitude of the currents induced on the wires vs. the ratio \( d/a \) for a given distance between the wires \( d/\lambda = 2.5 \). One may observe that the currents decrease monotonically as the ratio \( d/a \) is increased (i.e., the radius \( a/\lambda \) is decreased).

The case of voltage source excitation is shown in Figures 10 through 20. Figures 10-13, show the variation of the imaginary part of the normalized \( z \)-directed wave numbers, which are the normalized attenuation constants in the \( z \)-direction, as a function of the normalized distance between the wires, for a given ratio \( d/a \). We have depicted only the two lowest even modes and the two lowest odd modes, and we have chosen \( d/a = 10 \) and \( d/a = 100 \) for Figures 10-11, and Figures 12-13 respectively. By looking at the graphs one can readily see, as may be expected, that for fixed \( d/a \), the attenuation constants decrease as the distance between the wires is increased. In contrast the attenuation constants increase for fixed distance \( d/\lambda \) as the ratio \( d/a \) is increased (i.e., when the radius, \( a/\lambda \), is decreased). This fact is also demonstrated in Figure 14 which shows the variation of
Fig. 7  The currents induced on the wires due to plane wave excitation, vs. the normalized distance between the wires ($d/a = 10, d/\lambda < 1$).
Fig. 5  The currents induced on the wires due to plane wave excitation, vs. the normalized distance between the wires \((d/a = 10, d/\lambda > 1)\).
Fig. 9  The currents induced on the wires due to plane wave excitation vs. the ratio $d/a$. ($d/\lambda = 2.5$).
Fig. 10 The normalized attenuation constants of the two lowest even and odd modes on the wires vs. the normalized distance between the wires ($d/a = 10$, $d/\lambda < 1$).
The normalized attenuation constants of the two lowest even and odd modes on the wires, the normalized distance between the wires (d/a = 10, d/λ = 1).
The normalized attenuation constants of the two lowest even and odd modes on the wires vs. the normalized distance between the wires, \(d/a = 100\) and \(d/\lambda < 1\).

Fig. 12
The normalized attenuation constants of the two lowest even and odd modes on the wires vs. the normalized distance between the wires, \(d/a < 1\).

Fig. 13
Fig. 14 The normalized attenuation constants of the lowest even and odd modes on the wires vs. the ratio $d/a$, $(d/\lambda = 2.0)$. 
the $z$-directed normalized attenuation constants as a function of the ratio $d/a$ for the fixed distance between the wires, $d/\lambda = 2$. The figure shows only the lowest even and odd modes.

Figures 15 and 16 describe, respectively, the variation of the magnitude of the lowest even and odd modes, normalized to the magnitude of the TEM mode as a function of the ratio $d/a$ while the distance $d$ is kept constant. The values are given at $z = 0$. It is important to remember that as we precede along the $z$ axis the higher order modes are attenuated exponentially, while the TEM mode remains constant. For instance, for $d/\lambda = 2.0$ and $a/\lambda = 0.04$ ($d/a = 50$) we have for the first even mode, which would have the lowest attenuation, at $z = 0$, \[ \frac{|I_{1\text{ even}}|}{|I_{\text{TEM}}|}_z = 0.744. \] However, since for this case $\text{Im}(k_z) = \frac{0.179}{\lambda}$, we would have at $z = 5\lambda$ along the wires \[ \frac{|I_{1\text{ even}}|}{|I_{\text{TEM}}|}_z = 0.744. \ e^{-0.895} = 0.304 \] which is only 40% of its initial value. Figures 17-18 and Figures 19-20 show, respectively, variation of the magnitude of the lowest even and odd modes, normalized to the magnitude of the TEM mode as a function of the normalized distance, $d/\lambda$, between the wires, when $d/a$ is a parameter and we have chosen to show for $d/a = 10$ and $d/a = 100$. The values are given at $z = 0$ and they are attenuated exponentially as one precedes along the $a$ axis.
Fig. 15: The lowest even-mode current at $z^2 = 0$ normalized to the TM current vs. the ratio $d/a (d/A = 2.60)$. 

LOWEST EVEN MODE

$0.76 \quad 0.74 \quad 0.72 \quad 0.70 \quad 0.68 \quad 0.66 \quad 0.64 \quad 0.62 \quad 0.60$

$0 \quad 20 \quad 40 \quad 60 \quad 80 \quad 100$
Fig. 16 The lowest odd-mode current at $z = 0$ normalized to the TEM Current vs. the ratio $d/a$ ($d/\lambda = 2.0$).
Fig. 17 The lowest even-mode current at $z = 0$ normalized to the TEM current vs. the normalized distance the wires ($d/a = 10,100; d/\lambda < 1$).
Fig. 18 The lowest even-mode current at $z = 0$ normalized to the TEM current vs. the normalized distance between the wires ($d/a = 10, 100$; $d/\lambda > 1$).
Fig. 19  The lowest odd-mode current at \( z = 0 \) normalized to the TEM current vs. the normalized distance between the wires (\( d/a = 10,100; \ d/\lambda < 1 \)).
Fig. 20 The lowest odd-mode current at $z = 0$ normalized to the TEM current vs. the normalized distance between the wires ($d/a = 10, 100; d/\lambda > 1$).
Appendix A

The Contribution of a Leaky Pole

In this appendix the first term of Equation (39), namely Equation (40), is evaluated. Figure Al which applies to this calculation is a more detailed version of Figure 4.

The integral around the complex pole, as shown in Figure Al is given by,

\[ I_{\pm} = \left[ \frac{2\omega}{\pi} \frac{1}{k_{z}^2} \right] \frac{v^{2} - a^{2}}{k_{z}^2 - k_{z}^2} e^{ik_{z}^2} dk_{z} \]  \hspace{1cm} (A1)

where

\[ h_{z}(k_{z}) = H_{0}^{(2)}(\sqrt{k_{z}^2 - k_{z}^2}) \]

Expanding \( h_{z}(k_{z}) \) in a Taylor series around \( k_{z} = k_{zp} \) one has,

\[ h_{z}(k_{z}) = h_{z}(k_{zp}) + h^{(1)}(k_{zp}) (k_{z} - k_{zp}) + \ldots \]

\[ \ldots + \frac{h^{(n)}(k_{zp})}{n!} (k_{z} - k_{zp})^{n} + 0 [(k_{z} - k_{zp})^{n+1}] \]  \hspace{1cm} (A3)

where \([x]\) means terms of the order of \( x \). For \( |k_{z} - k_{zp}| \to 0 \) we may write \( h_{z}(k_{z}) \) explicitly as

\[ h_{z}(k_{z}) = \left[ \frac{H_{1}^{(2)}(\sqrt{k_{z}^2 - k_{zp}^2})}{k_{zp}^2} \right] \frac{k_{zp}^2}{v^{2} - k_{zp}^2} (k_{z} - k_{zp}) \]  \hspace{1cm} (A4)

In expression (A4) terms of the order of \( (k_{z} - k_{zp})^{2} \) have been omitted and we assume that \( h^{(1)}(k_{zp}) \neq 0 \).
Fig. A1. Illustration of the contours in the \( k_z \) plane.

BRANCH CUTS WHICH SEPARATE THE BOTTOM SHEET FROM THE TOP SHEET.

BRANCH CUTS WHICH ARE ACTUALLY USED IN OUR INTEGRATION.
Also, $H_1^{(2)}$ is the Hankel function of the second kind of first order, and we have used the relation

$$\frac{d}{dx} H_0^{(2)}(x) = -H_1^{(2)}(x)$$

Using the residue theorem the integral in Equation (A1) results in the residue at the simple pole $k = k_{zp\pm}$. Thus we have,

$$I_p = \frac{4j}{\pi} \frac{k}{\sqrt{k^2 - k_{zp\pm}^2}} \frac{\text{inc} (x, y, k_{zp\pm})}{\sqrt{x^2 + y^2 - a^2}} e^{jk_{zp\pm}z}$$

where

$$\eta = \left( \frac{\mu}{\epsilon} \right)^{1/2}$$

Substituting $k_{zp\pm} = \sqrt{k^2 - k_{zp\pm}^2}$ into (A5) one readily obtains Equation (40).
The Contribution of the Branch Point

In this appendix the contribution of the branch point, namely Equation (41), is evaluated.

The integral around the branch point, as shown in Figure A1, is given by,

\[
\int_{CDE} \frac{2\text{inc} \left( x, y, k_z \right)}{E_z} \left[ x^2 + y^2 = a^2 \right] dk_z - e^{j k_z z} \left[ H_o^{(2)}(\sqrt{k^2 - k_z^2} a) \pm H_o^{(2)}(\sqrt{k^2 - k_z^2} d) \right]
\]

By changing the variable of integration to

\[ k_z = -k + e^{j \theta} \]  

where \( \theta \) varies from \(-\frac{\pi}{2}\) to \(\frac{\pi}{2}\), we obtain

\[ k^2 - k_z^2 = (k + k_z) (k - k_z) = e^{j \theta} (2k - e^{j \theta}) \]  

In the limit where \( \varepsilon \to 0 \) we have,

\[
\text{inc} \left( x, y, k_z \right) \left[ x^2 + y^2 = a^2 \right] + \text{inc} \left( x, y, -k \right) \left[ x^2 + y^2 = a^2 \right] 
\]

\[
k^2 - k_z^2 + 2k \varepsilon e^{j \theta} \]  

\[
\sqrt{k^2 - k_z^2} + \sqrt{2k \varepsilon e^{j \theta}} \]  

\[
H_o^{(2)}(\sqrt{k^2 - k_z^2} a) + 1 - \frac{j \pi}{2} \ln \left[ \frac{\sqrt{2k \varepsilon e^{j \theta}}}{2} a \right] \]  

\[
H_o^{(2)}(\sqrt{k^2 - k_z^2} d) + 1 - \frac{j \pi}{2} \ln \left[ \frac{\sqrt{2k \varepsilon e^{j \theta}}}{2} a \right] \]

Substituting Equations (A9-A13) in Equation (6), we obtain \( I_{+\text{DEF}}^{\text{DEF}} = 0 \)  

i.e., zero contribution for symmetrical excitation, and,
\[ \mathbf{I}_{\text{DEF}} = \frac{\omega c}{\pi} \mathbf{E}_{z}^{\text{inc}} (x, y, -k) \left| \begin{array}{l} x^2 + y^2 = a^2 e^{-jka} \end{array} \right. \]

\[ \int \frac{1}{2\kappa c \epsilon \eta \left( \frac{1}{\pi} \ln \frac{d}{a} \right) + j\kappa} e^{j\theta} e^{j\kappa} d\theta \]

\[ = \frac{\pi}{n \ln \left( \frac{d}{a} \right)} \mathbf{E}_{z}^{\text{inc}} (x, y, -k) \left| \begin{array}{l} x^2 + y^2 = a^2 e^{-jka} \end{array} \right. \]

(A15)

where

\[ n = \left( \frac{d}{a} \right)^{1/2} \]

It is easily seen that Equation (A15) can be written in the explicit form,

\[ \mathbf{I}_{\text{DEF}} = (\mathbf{Z}_0)^{-1} \mathbf{E}_{z}^{\text{inc}} (x, y, -k) \left| \begin{array}{l} x^2 + y^2 = a^2 e^{-jka} \end{array} \right. \]

(A16)

Where \( Z_0 = \frac{\pi}{n \ln \left( \frac{d}{a} \right)} \) is the characteristic impedance of the two-wire line as given by [9].
A3. The Contribution of the Branch Cut

In this appendix the contribution of the branch cut is evaluated.

In the bottom sheet we have,

\[
I_{\text{EF}}^{\text{EF}} = \int_{\text{EF}} \frac{2\omega_0 \pi}{(k^2 - k_z^2) [H_0^{(2)}(r k^2 - k_z^2 a) + H_0^{(2)}(r k^2 - k_z^2 d)]} \left( x^2 + y^2 = \varepsilon^2 \right) e^{j k_z z} \, dk_z
\]

(A17)

By changing the variable of integration to

\[
k_z = -k + ju
\]

where \( u \) varies from \( \varepsilon \) to \( R \), one obtains,

\[
k^2 - k_z^2 = (k + k_z) \left(k - k_z\right) = ju(2k - j) = u^2 + juk = u^2(1 + J_u^2)
\]

(A19)

\[
\sqrt{k^2 - k_z^2} = u\sqrt{1 + \frac{k^2}{u^2}} = u(1 + \frac{k^2}{u^2})^{1/2} e^{i \phi/2}
\]

(A20)

where

\[
\phi = \tan^{-1} \frac{k}{u}
\]

Taking the limit where \( \varepsilon \to 0 \) and \( R \to \infty \), Equation (A17) results in

\[
I_{\text{EF}}^{\text{EF}} = \lim_{\varepsilon \to 0} \int_{R \to \infty} \frac{2\omega_0 \pi}{(k^2 - k_z^2) [H_0^{(2)}(r u^2(1 + jk^2/2) \left(u^2(1 + jk^2/2) \right)^{1/2} e^{i \phi/2} du]}
\]

(A21)

In the top sheet we have,

\[
I_{\text{BC}}^{\text{BC}} = \int_{\text{BC}} \frac{2\omega_0 \pi}{(k^2 - k_z^2) [H_0^{(2)}(r k^2 - k_z^2 a) + H_0^{(2)}(r k^2 - k_z^2 d)]} \left( x^2 + y^2 = \varepsilon^2 \right) e^{j k_z z} \, dk_z
\]

(A22)
By changing the variable of integration in the same way as Equation (A18) where \( u \) varies from \( R \) to \( \epsilon \), one obtains

\[
k^2 - \kappa^2 = u^2(1 + j\mu_u^2) \tag{A23}
\]

\[
\sqrt{k^2 - \kappa^2} = -u(1 + \frac{\kappa^2}{u^2})^{1/2} e^{j\phi/2} \tag{A24}
\]

where \( \phi = \tan^{-1} \frac{k}{u} \)

Taking the limit where \( \epsilon \to 0 \) and \( R \to \infty \), equation (A22) results in

\[
I_\text{BC}^{\text{inc}} = \lim_{\epsilon \to 0} \lim_{R \to \infty} \left\langle \frac{2\mu_c}{\pi} \frac{\text{inc}(x,y,-k + ju)}{\sqrt{x^2 + y^2 = a^2}} e^{-j\kappa z} e^{-uz} \right\rangle_{u} \tag{A25}
\]
Appendix B

The Method for Finding the Roots of Equation (38)

A well known and proven method for finding the roots of equation (38), is that of the steepest decent. Equation (38) may be modified using a real function \( h_{\text{abs}} \) in the following fashion,

\[
\frac{\partial h_{\text{abs}}(k_z)}{\partial \theta_z} + (k_z) = 0 \quad (B1)
\]

where \( h_{\pm}(k_z) \) is given by Equation (A2) and \( k_z \) the \( z \)-directed axial wave number may be cast into the vector form

\[
\vec{k}_z = \begin{bmatrix} \theta_z \\ \nu_z \end{bmatrix} \quad (B2)
\]

where \( \theta_z \) and \( \nu_z \) are the real and imaginary parts of \( k_z \), respectively.

The search by this method starts with initial value \( \vec{k}_{z0} \) for the wave number vector. The normalized gradient of the absolute value of \( h_{\pm}(k_z) \), obtained by differentiating (B1):

\[
\vec{v}_+ = \begin{bmatrix} \frac{\partial h_{\text{abs}}(k_z)}{\partial \theta_z} \\ \text{Sign (} \frac{\partial h_{\text{abs}}(k_z)}{\partial \nu_z} \text{)} \end{bmatrix} \quad (B3)
\]

is measured and the vector \( \vec{k}_z \) is altered, in accordance with the negative of the value obtained. This procedure is repeated causing the value of \( h_{\text{abs}} \) to be successively reduced and the \( \vec{k}_z \) vector to approach the root of Equation (B1).

For convenience, the + and - subscripts will be omitted in the following discussion. The method can be described by the relation

\[
\vec{k}_{z_k + 1} = \vec{k}_{z_k} - [\nu_k] \vec{v}_k \quad (B4)
\]
where

\[
\mathbf{\beta}_k = \begin{bmatrix}
\mathbf{v}_k^1 \\
\mathbf{v}_k^2
\end{bmatrix} = \begin{bmatrix}
\text{Sign} \left( \frac{\partial h_{\text{abs}}}{\partial z}(k) \right) \\
\text{Sign} \left( \frac{\partial h_{\text{abs}}}{\partial \alpha}(k) \right)
\end{bmatrix} \\
k_z = k_{z_k}
\]

\[
\text{Sign}(z) = \begin{bmatrix}
1 & x > 0 \\
-1 & x < 0
\end{bmatrix}
\]

is the normalized gradient at a point in the \( k_z \) plane corresponding to \( k_z = k_{z_k} \) and

\[
[H_k] = \begin{bmatrix}
3/4 + i/4 \mathbf{v}_{k-1}^1 \mathbf{v}_k^1 & 0 \\
0 & 3/4 + 1/4 \mathbf{v}_{k-1}^1 \mathbf{v}_k^2
\end{bmatrix} \cdot \mu_k - 1
\]

\[
(B6)
\]

is a matrix that together with an initial matrix \( [H_0] \) control stability and rate of convergence.

This method is suitable for our purpose, since we are interested only in leaky modes having the smallest attenuation in the \( z \) direction. Thus, we are allowed to search for the zeros within a rectangular region of the second quadrant in the complex \( k_z = \beta_z + i \alpha_z \) plane, which is defined by

\[
0 < \beta_z < k \quad 0 < \alpha_z << k \quad (B7)
\]

The absolute value of the complex function Equation (B1) is reduced to less than \( 10^{-6} \) at the location of a root.
Appendix C

Program Listings and Output Samples

49
The first two programs calculate the induced currents in the case of a plane wave excitation. The first program varies the distance between the wires $d$ (in wavelengths) while keeping the ratio $d/a$ constant. The second program varies the ratio $d/a$ while keeping the distance between the wires constant.

The following four programs find the real and imaginary parts of the wave numbers associated with higher order modes in the case of volt excitation, and calculate the contribution of each mode to the total current. The first program varies the distance between the wires, while keeping the ratio $d/a$ constant, for the even leaky modes and the second carries it out for the odd leaky modes. The third program varies the ratio $d/a$, while keeping the distance between the wires constant, for the even leaky modes and the fourth carries it out for the odd modes.
THE PURPOSE OF THIS PROGRAM IS TO CALCULATE THE
INDUCED CURRENTS IN THE CASE OF A PLANE WAVE
EXCITATION.
THE PROGRAM VARIES THE DISTANCE BETWEEN THE WIRES-
"I"-(IN WAVELENGTHS) WHILE KEEPING THE RATIO "D/A"
CONSTANT.

COMPLEX KA,KD,CO,CE,CU1
COMPLEX HOA,HOD,JOA,JOD,YOA,YOD
COMPLEX CU2
REAL K,1JOA,1YOA,1JOD,1YOD

THE RATIO "D/A"

M=100
WRITE(5,50) M
50 FORMAT(///,1X,'D/A IS '*,6,///)
PI=3.1415927
K=2.*PI

THE VERTICAL AND AZIMUTHAL ANGLES OF THE
INCIDENT WAVE

Q=PI/4
PY=PI/4

INITIAL VALUE FOR THE DISTANCE-D

D=0.1
A=D/M
IF(D.GT.5.00) GO TO 20
WRITE (5,40) D
40 FORMAT(/,1X,'D IS '*,14,///)
KA=K*A*SIN(Q)*(1.,0.)
KD=KA*M
CALL CBESO(KA,JOA,YOA)
CALL CBESO(KD,JOD,YOD)
RJOA=REAL(JOA)
IJOA=AIMAG(JOA)
RYOA=REAL(YOA)
IYOA=AIMAG(YOA)
RJOD=REAL(JOD)
IJOD=AIMAG(JOD)
RYOD=REAL(YOD)
IYOD=AIMAG(YOD)

THE RELATIVE CONTRIBUTION OF THE EVEN AND ODD PARTS
AND THE TOTAL CURRENT INDUCED ON THE WIRES:

HOA=CMPLX(RJOA+IYOA,IJOA-RYOA)
HOD=CMPLX(RJOD+IYOD,1JOD-RYOD)
CE=2./377./K/SIN(Q)*(1+CEXP((0.,1.)*KD*SIN(PY)))/(HOA+HOD)
CO=2./377./K/SIN(Q)*(1-CEXP((0.,1.)*KD*SIN(PY)))/(HCA-HOD)
CU1=CE+CO
ACU1=CABS(CU1)
RAT=CABS(CE/CO)
WRITE(5,30)ACU1,RAT
30  FORMAT(1X,'CURRENT 1 IS',E15.8,1X,'EVEN/ODD IS',E15.8)
CU2=CE-CO
ACU2=CABS(CU2)
WRITE(5,37)ACU2
37  FORMAT(1X,'CURRENT 2 IS',E15.8)
RAT1=ACU1/ACU2
WRITE(5,38)RAT1
38  FORMAT(1X,'CUR1/CUR2 IS',E15.8,1X)

CHANGING THE DISTANCE-D

D=D+0.1
GO TO 10
20  CONTINUE
STOP
END

THE SUBROUTINE CBESO CALCULATES THE ZERO ORDER BESSEL
FUNCTIONS FOR A GIVEN INPUT VARIABLE Z

SUBROUTINE CBESO(Z,BSJO,BSYO)
COMPLEX Z,BSJO,BSYO,Y,W,F0,F0,CEXP
COMPLEX CCOS,CSIN,CSQRT,CLOG,SQ0,SO0,CO0,SN,SP1,SP1,SQ1
PI=3.141593
A=CABS(Z)
BSJO=(1.,0.,)
IF(A.EQ.0.) GO TO 2
IF(A.GT.3.) GO TO 1
Y=Z/9.
BSJO=1.+Y*(-2.2499997+Y*(1.2656208+Y*(-.3163866
1.+Y*.0444479+Y*(-.0039444+Y*.00021)))
GO TO 2
1  W=3/Z
F0=.79788456+W*(-.00000077+W*(-.0055274+W*(-.00009512
1.+W*.0137237+W*(-.00072805+W*.00014476))))
P0=.78539816+W*.04166397+W*.0003954+W*(-.00262573
BSJO=F0*CCOS(Z-P0)/CSQRT(Z)
2  CONTINUE
BSYO=(1.E30;0)
IF(A.EQ.0.) GO TO 3
IF(A.GT.3.) GO TO 4
BSYO=.63661977*CLOG(.5*Z)*BSJO+.36746691+Y*(-.60559366
1+Y*(-.74350384+Y*(-.25300117+Y*(-.04261214+Y*(-.00427916
2-Y*.00024846))))
GO TO 3
4  CONTINUE
BSYO=F0*CSIN(Z-P0)/CSQRT(Z)
3  CONTINUE
RETURN
END
Output Samples for Program No. 1

D/A IS 10

D IS .1000
CURRENT 1 IS 0.62013944E-03
EVEN/ODD IS 0.29726247E+01
CURRENT 2 IS 0.89227682E-03
CUR1/CUR2 IS 0.69500791E+00

D IS .2000
CURRENT 1 IS 0.71089397E-03
EVEN/ODD IS 0.20521233E+01
CURRENT 2 IS 0.12857444E-02
CUR1/CUR2 IS 0.55912665E+00

D IS .3000
CURRENT 1 IS 0.81071406E-03
EVEN/ODD IS 0.17450446E+01
CURRENT 2 IS 0.16215614E-02
CUR1/CUR2 IS 0.50501576E+00

D IS .4000
CURRENT 1 IS 0.92081023E-03
EVEN/ODD IS 0.15963233E+01
CURRENT 2 IS 0.19070097E-02
CUR1/CUR2 IS 0.50241131E+00

D IS .5000
CURRENT 1 IS 0.11598827E-02
EVEN/ODD IS 0.14851194E+01
CURRENT 2 IS 0.21185441E-02
CUR1/CUR2 IS 0.54749046E+00

D IS .6000
CURRENT 1 IS 0.14269433E-02
EVEN/ODD IS 0.13361567E+01
CURRENT 2 IS 0.21764565E-02
CUR1/CUR2 IS 0.65562683E+00

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THE PURPOSE OF THIS PROGRAM IS TO CALCULATE THE INDUCED CURRENTS IN THE CASE OF A PLANE WAVE EXCITATION. THE PROGRAM VARIES THE RATIO "D/A" WHILE KEEPING THE DISTANCE BETWEEN THE WIRES "D" CONSTANT. 

INITIAL VALUE FOR THE RATIO "D/A"

M=10
PI=3.1415927
K=2.*PI

THE VERTICAL AND AZIMUTHAL ANGLES OF THE INCIDENT WAVE

Q=PI/4
PY=PI/4

THE DISTANCE BETWEEN THE WIRES

D=2.5
WRITE (5,' D')

A=D/M
IF(M.GT.1000) GO TO 20
WRITE(5) M

THE RELATIVE CONTRIBUTION OF THE EVEN AND ODD PARTS AND THE TOTAL CURRENT INDUCED ON THE WIRES

CE=2./377./K/SIN(Q)*(1+CEXP((0.,1.)*K*PI/Q))/HOA+HOD
CO=2./377./K/SIN(Q)*(1-CEXP((0.,1.)*K*PI/Q))/HOA-HOD
CU1=CE+CO
ACU1=CABS(CU1)
RAT=CABS(CE/CO)
WRITE(5,30)ACU1,RAT
30 FORMAT(1X,'CURRENT 1 IS',E15.8,'/1X,'EVEN/ODD IS',E15.8)
   CU2=CE-C0
   ACU2=CABS(CU2)
WRITE(5,37)ACU2
37 FORMAT(1X,'CURRENT 2 IS',E15.8)
   RAT1=ACU1/ACU2
WRITE(5,38)RAT1
38 FORMAT(1X,'CUR1/CUR2 IS',E15.8,'///')
   C
   THE RATIO 'D/A'
   M=M+1
   GO TO 10
CONTINUE
STOP
END
C
THE SUBROUTINE CBES0 CALCULATES THE ZERO ORDER BESSEL
FUNCTIONS FOR A GIVEN INPUT VARIABLE Z.
C
SUBROUTINE CBES0(Z,BSJO,BSYO)
COMPLEX Z,BSJO,BSYO,Y,W,P0,F0,CEXP
COMPLEX COS,COS,COS,R,COS,COS,R,COS,R
PI=3.141593
A=CABS(Z)
BSJO=(1.,0.)
   IF(A.EQ.0.) GO TO 2
   IF(A.GT.3.) GO TO 1
   Y=Z*Z/9.
   BSJO=1.+Y*(-2.2499997*Y*(1.2656208+Y*(-.3163866
1+Y*(-.0444479+Y*(-.0039444+Y*.00021)))
   GO TO 2
1   W=3./Z
   F0=2.797884564+W*(-.00000077+W*(-.00009512
1+W*(-.00137237+W*(-.00072805+W*(-.00014476))))
   F0=2.785398164+W*(-.04166397+W*(-.00003954+W*(-.00262573
1+W*(-.00054125+W*(-.00029333-W*.00013558))))
   BSJO=F0*COS(Z-P0)/CSRT(Z)
2 CONTINUE
   BSYO=(-1.E30,0.)
   IF(A.EQ.0.) GO TO 3
   IF(A.GT.3.) GO TO 4
   BSYO=6.3661977*COS(.5*Z)*BSJO+.36746691+Y*(-.60559366
1+Y*(-.743507384+Y*(-.25300117+Y*(-.04261214+Y*(-.00427916
2-Y*.00024846))))
   GO TO 3
4 CONTINUE
   BSYO=F0*COS(Z-P0)/CSRT(Z)
3 CONTINUE
RETURN
END
The purposes of this program are:
1) To find the real and imaginary parts of the \(R\)-directed and \(Z\)-directed wave numbers associated with the higher order even leaky modes in the case of vol excitation.
2) To calculate the contribution of each mode to the total current.

The program varies the distance between the wires-\(D\)-(in wavelengths) while keeping the ratio \(D/A\) constant.

```
COMMON KR,AA,M,BR,AR
COMPLEX KR,KZP,KRP,RES
REAL IJOA,IJOA,IJ1A,IJ1A,IJOA,IJOA,IY0A,IY0A,IY1A,IY1A
COMPLEX KRA,KRD,JOA,YOA,JOD,YOD,HOA,HOD,J1A
COMPLEX Y1A,Y1A,Y1D,Y1D,GR1,GR2,H1A,H1A,F,GRA1,GRA2
COMPLEX KRP,KRPD
REAL KZ(2),K,KH
REAL GRAD(2),CON(2)

DEPARTURE POINT FOR THE SEARCHING IN THE KZ PLANE
I=1
BR=1.0
AR=-2.

START SEARCHING FOR THE ROOTS IN THE KZ PLANE
9 IF (BR.GT.100) GO TO 8
WRITE (5,18) I
18 FORMAT (/IX, 'THE INDEX OF THE MODE IS ',I2, ')
GRAD(1)=0.
GRAD(2)=0.
CON(1)=0.50
CON(2)=0.50
N=2
PI=3.1415927
K=2*PI
KH=K**2+AR**2-BR**2
BZ=SQRT(0.5*(KH+SQRT((KH**2+(2*AR*BR)**2))))
AZ=-AR*BR/BZ
KR=CMPLX(BR,-AR)
KZ(1)=BZ
KZ(2)=AZ

THE RATIO "D/A"
M=100
AA=1./M
WRITE (5,19) M
19 FORMAT (IX, ' D/A = ',F16.16)
3 CALL FUNCT(N,KZ,VAL,GRAD)
IF (VAL.LT.0.1E-5) GO TO 1
IF (GRAD(1)) 22,27,27
22 IF (GRAD(1)) 23,24,24
```

57
CON(1)=CON(1)/2,
KZ(1)=KZ(1)+CON(1)
OGRAD(1)=GRAD(1)
GO TO 31
27
IF (OGRAD(1)) 28,29,29
28
CON(1)=CON(1)/2
29
KZ(1)=KZ(1)-CON(1)
OGRAD(1)=GRAD(1)
31
CONTINUE
IF (OGRAD(1)) 32,37,37
32
IF (OGRAD(2)) 33,34,35
33
CON(2)=CON(2)/2,
KZ(2)=KZ(2)+CON(2)
OGRAD(2)=GRAD(2)
GO TO 3
37
IF (OGRAD(2)) 38,39,39
38
CON(2)=CON(2)/2
39
KZ(2)=KZ(2)-CON(2)
OGRAD(2)=GRAD(2)
GO TO 3
1
CONTINUE
C
INITIAL VALUE FOR THE DISTANCE-D
C
D=0.1
A=D/M
IF(D.GT.5.00) GO TO 60
BP=+BR/D
AP=+AR/D
KRP=CMPLX(BP,-AP)
KRPF=KRP*A
KRPD=KRP*D
K=2*PI
KH=K**2+AP**2-BP**2
BZ=SQRT(0.5*(KH+SQRT((KH**2+(2*AP*BP)**2))))
AZ=-AP*BP/BZ
KZP=CMPLX(-BZ,AZ)
WRITE (5,80) D
WRITE(5,90) KZP
80
FORMAT('/,1X,'D IS ','F6.4')
WRITE(5,90) KZP
90
FORMAT('/,1X,'REAL KZ IS ','F9.4,' IMAG KZ IS ','F9.4')
CALL CBES1(KRPF,J1A,Y1A)
CALL CBES1(KRPD,J1D,Y1D)
RJ1A=REAL(J1A)
IJ1A=AIMAG(J1A)
RY1D=REAL(Y1D)
IY1D=AIMAG(Y1D)
RJ1D=REAL(J1D)
IJ1D=AIMAG(J1D)
RY1D=REAL(Y1D)
IY1D=AIMAG(Y1D)
H1A=CMPLX(RJ1A+IY1A, IJ1A-RY1A)
H1D=CMPLX(RJ1D+IY1D, IJ1D-RY1D)
C
THE CONTRIBUTION OF THE LEAKY MODE
C
TO THE TOTAL CURRENT NORMALIZED TO THE
CONTRIBUTION OF THE TEM MODE

RES = 4. / 377,
RES = RES * (0, 1) * K/KRP/KZP/(A*H1A + D*H1D)
ARS = CABS(RES)
T = M
RAT = ARS * 377, * ALOG(T) / PI
WRITE(5, 40) ARS

FORMAT(1X, 'THE CURRENT IS:', E15.8)
WRITE(5, 41) RAT

FORMAT(1X, 'CUR/TEM IS:', E15.8)

CHANGING THE DISTANCE-D

D = D + 0.1
GO TO 70

CONTINUE

CHANGING THE DEPARTURE POINT IN ORDER TO
START THE SEARCHING FOR THE CONSECUTIVE ROOT

I = I + 1
BR = BR + 5
GO TO 9

CONTINUE

STOP

END

THE SUBROUTINE CBESO CALCULATES THE ZERO ORDER BESSEL
FUNCTIONS FOR A GIVEN INPUT VARIABLE Z

SUBROUTINE CBESO(Z, BSJO, BSYO)
COMPLEX Z, BSJO, BSYO, Y, W, PO, F0, CEXP
COMPLEX C, COS, SIN, CSORT, CLG, SQ0, SQ1, CS, SN, SP1, SQ1
PI = 3.141593
A = CABS(Z)
BSJO = (1, 0, 0)
IF(A.EQ.0.) GO TO 2
IF(A.GT.3.) GO TO 1
Y = Z * Z / 9.
BSJO = 1. + Y * (-2.2499997 + Y * (1.2656208 + Y * (-0.316386
1 + Y * (0.0444479 + Y * (-0.0039444 + Y * 0.0021)))))
GO TO 2

1
W = 3./Z
F0 = 0.7976846 + W * (-0.0000077 + W * (-0.005274 + W * (-0.000952
1 + W * (0.0137237 + W * (-0.0072805 + W * 0.0014476))))))
F0 = 0.78539816 + W * (0.04166397 + W * (0.0003954 + W * (-0.0262573
1 + W * (0.00054125 + W * (0.00029333 + W * 0.00013558)))))
BSJO = F0 * CSOS(Z - PO) / CSQRT(Z)
CONTINUE

2
BSYO = (-1.E30 + 0)
IF(A.EQ.0.) GO TO 3
IF(A.GT.3.) GO TO 4
BSYO = 0.63461977*CLG(.5*Z)*BSJO + 0.36746691 + Y* (0.6059366
1 + Y* (-7.4350384 + Y* (2.5300117 + Y* (-0.0426124 + Y* (0.00427916
2 - Y* 0.00024846))))))
THE SUBROUTINE CBES1 CALCULATES THE FIRST ORDER BESSEL FUNCTIONS FOR A GIVEN INPUT VARIABLE Z

SUBROUTINE CBES1(Z,BSJ1,BSY1)
COMPLEX Z,BSJ1,BSY1,Y+W,P1,F1,CEXP
COMPLEX CCOS,CSQRT,CLOG,CSIN,SP1,SP2,Q1,F,CS,SN,SP2,SO2
PI=3.141593
A=CCABS(Z)
BSJ1=0.
IF(A.EQ.0.) GO TO 2
IF(A.GT.3.) GO TO 1
Y=Z**9.
BSJ1=ZW**(-.56249985+YW**(.21093573+YW*(-.0395289
1+YW*(-.00443319)))*(-.0031761+YW*.0001109)))
GO TO 2

1
W=3./Z
F1=79788456+YW**(.00000156+W**(.01659667+W**(.00017105
1+W**(-.00249511+W**(.00113653-W**.00020033)))
P1=2.35619447+YW*(-.12499612+YW*(-.0005650+W**(.00637879
1+W**(-.00074348+W**(-.00079824+W**.00029166))))
BSJ1=F1*CCOS(Z-P1)/CSQRT(Z)
GO TO 2

2
BSY1=(-1.E30,0)
IF(A.EQ.0.) GO TO 3
IF(A.GT.3.) GO TO 4
BSY1=(-.63661977+YW*(-.2122091+YW(2.1682709+YW*(-1.3164827
1+YW*(-.3123951+YW*(-.04000976+YW*.0027873)))))/Z
BSY1=63661977*CLOG(.5*Z)*BSJ1
GO TO 3

4
BSY1=F1*CSIN(Z-P1)/CSQRT(Z)
GO TO 3

3
CONTINUE
RETURN
END

THE SUBROUTINE FUNCT CALCULATES FOR EACH VECTOR KZ THE VALUE OF THE FUNCTIONAL WHICH WE TRY TO MINIMIZE AND ITS GRADIENT

SUBROUTINE FUNCT(N,KZ,VAL,GRAD)
COMMON KR,KAA,K,KR,KR,KD,JOA,YOA,YOD,IOY1A,Y1DA,Y1D
DIMENSION GRAD(2)
REAL KZ(2),K,KH
REAL IJOA,JOA,YOA,YOD,IOYOD,IY1DA,Y1D
COMPLEX KR,KRA,KRD,KR,KOA,YOA,YOD,HOA,HOD,J1A
COMPLEX Y1A,J1A,Y1D,GR1,GR2,H1A,H1D,F,GRA1,GRA2
BZ=KZ(1)
AZ=KZ(2)
PI=3.1415927
K=2.*PI
KH = K**2 + A**2 * B**2
BR = SQRT(0.5 * (KH + SQRT((KH**2 + (2*A*B)**2))))
AR = -A*BZ/BR
KR = CMPLX(BR, -AR)
KRA = KR + A
KRD = KRA + M
CALL CBES0(KRA, JOA, YOA)
CALL CBES0(KRD, JOD, YOD)
RJOA = REAL(JOA)
IJOA = AIMAG(JOA)
RYOA = REAL(YOA)
IYOA = AIMAG(YOA)
RJOD = REAL(JOD)
IJOD = AIMAG(JOD)
RYOD = REAL(YOD)
IYOD = AIMAG(YOD)
HOA = CMPLX(RJOA + IYOA, IJOA - RYOA)
HOD = CMPLX(RJOD + IYOD, IJOD - RYOD)
F = HOA + HOD
VAL = ABS(F)
VAL = VAL**2
CALL CBES1(KRA, J1A, Y1A)
CALL CBES1(KRD, J1D, Y1D)
RJ1A = REAL(J1A)
IJ1A = AIMAG(J1A)
RY1D = REAL(Y1D)
IY1D = AIMAG(Y1D)
RJ1D = REAL(J1D)
IJ1D = AIMAG(J1D)
RY1D = REAL(Y1D)
IY1D = AIMAG(Y1D)
HI1A = CMPLX(RJ1A + IY1A, IJ1A - RY1A)
H11D = CMPLX(RJ1D + IY1D, IJ1D - RY1D)
GR1 = HI1A*(-CMPLX(BZ, -AZ)*AA/KR)
GR1 = GR1 - H11D*(-CMPLX(BZ, -AZ)*AA*MM/KR)
GR2 = GR1*(0.0, -1.0)
GRA1 = GR1*CONJG(F) + CONJG(GR1)*F
GRA2 = GR2*CONJG(F) + CONJG(GR2)*F
GRAD(1) = REAL(GRA1)
GRAD(2) = REAL(GRA2)
RETURN
END
Output Sample for Program No. 3

D/A = 10

D IS : .1000
REAL KZ IS : -11.9813  IMAG KZ IS : 32.2049
THE CURRENT IS : 0.40988265E-03
CUR/TEM IS: 0.11325743E+00

D IS : .2000
REAL KZ IS : -6.3111  IMAG KZ IS : 15.2847
THE CURRENT IS : 0.85170465E-03
CUR/TEM IS: 0.23534023E+00

D IS : .3000
REAL KZ IS : -4.6324  IMAG KZ IS : 9.2551
THE CURRENT IS : 0.13608340E-02
CUR/TEM IS: 0.37602118E+00

D IS : .4000
REAL KZ IS : -4.0584  IMAG KZ IS : 5.9422
THE CURRENT IS : 0.19572516E-02
CUR/TEM IS: 0.54082134E+00

D IS : .5000
REAL KZ IS : -4.0681  IMAG KZ IS : 3.7940
THE CURRENT IS : 0.25318940E-02
CUR/TEM IS: 0.69960463E+00

D IS : .6000
REAL KZ IS : -4.4116  IMAG KZ IS : 2.4296
THE CURRENT IS : 0.27964980E-02
CUR/TEM IS: 0.77271914E+00
THE PURPOSES OF THIS PROGRAM ARE:

1) TO FIND THE REAL AND IMAGINARY PARTS OF THE R-DIRECTED AND Z-DIRECTED WAVE NUMBERS ASSOCIATED WITH THE HIGHER ORDER ODD LEAKY MODES IN THE CASE OF VOLT EXCITATION CURRENT.

2) TO CALCULATE THE CONTRIBUTION OF EACH MODE TO THE TOTAL CURRENT.

THE PROGRAM VARIES THE DISTANCE BETWEEN THE WIRES - "D" (IN WAVELENGTHS) WHILE KEEPING THE RATIO "D/A" CONSTANT.

---

```plaintext
COMMON KR,AA,AR,AA
REAL KZ(2),KH
REAL GRAD(2)
REAL OGRAD(2),CON(2)

DEPARTURE POINT FOR THE SEARCHING IN THE KZ PLANE

I=1
BR=1.
AR=-2.

START SEARCHING FOR THE ROOTS IN THE KZ PLANE

10 IF (BR.GT.100) GO TO 8
WRITE (5,18) I
18 FORMAT (1X,'THE INDEX OF THE MODE IS ',I2,/) 
OGRAD(1)=0.
OGRAD(2)=0.
CON(1)=0.50
CON(2)=0.50
N=2
PI=3.1415927
K=2*PI
KH=K**2+AR**2-BR**2
BZ=SQRT(0.5*(KH+SQRT((KH**2+(2*AR*BR)**2)))
AZ=-AR*BR/BZ
KR=CMPLX(BR,-AR)
KZ(1)=BZ
KZ(2)=AZ

THE RATIO "D/A"

M=100
AA=1./M
WRITE (5,19) M
19 FORMAT (1X,' D/A =',I6 ) 
3 CALL FUNCT(N,KZ,VAL,GRAD)
IF (VAL.LT.0.1E-5) GO TO 1
IF (GRAD(1)) 22,27
22 IF (OGRAD(1)) 23,24
```

---

63
CON(1)=CON(1)/2,
KZ(1)=KZ(1)+CON(1)
OGRAD(1)=GRAD(1)
GO TO 31
IF (OGRAD(1)) 28,29,29
CON(1)=CON(1)/2
KZ(1)=KZ(1)-CON(1)
OGRAD(1)=GRAD(1)
CONTINUE
IF (GRAD(2)) 32,37,37
IF (OGRAD(2)) 33,34,34
CON(2)=CON(2)/2,
KZ(2)=KZ(2)+CON(2)
OGRAD(2)=GRAD(2)
GO TO 3
IF (OGRAD(2)) 38,39,39
CON(2)=CON(2)/2
KZ(2)=KZ(2)-CON(2)
OGRAD(2)=GRAD(2)
GO TO 3
CONTINUE
C
INITIAL VALUE FOR THE DISTANCE-D
C
D=0.1
A=D/M
IF(D.GT.5.00) GO TO 60
BP=BR/D
AP=AR/D
KRP=CMPLX(BP,-AP)
KRPA=KRP*A
KRPD=KRP*D
K=2*PI
KH=K**2+AP**2-BP**2
BZ=SQR(0.5*(KH+SQR((KH**2+(2*AP*BP)**2))))
AZ=-AP*BP/BZ
KZP=CMPLX(-BZ,AZ)
WRITE (S,80) D
WRITE(5,90) KZP
FORMAT(//,1X,’D IS ’,F6.4)
FORMAT(’/dX,’REAL KZ IS ’,F9.4,’ IMAG KZ IS ’,F9.4)
CALL CBES1(KRPA,J1A,Y1A)
CALL CBES1(KRPD,J1D,Y1D)
RJ1A=REAL(J1A)
IJ1A=AIMAG(J1A)
RY1D=REAL(Y1D)
IY1D=AIMAG(Y1D)
RJ1D=REAL(J1D)
IJ1D=AIMAG(J1D)
RY1D=REAL(Y1D)
IY1D=AIMAG(Y1D)
H1A=CMPLX(RJ1A+IY1D;IJ1A-RY1D)
H1D=CMPLX(RJ1D+IY1D;IJ1D-RY1D)
C
THE CONTRIBUTION OF THE LEAKY MODE
C TO THE TOTAL CURRENT NORMALIZED TO THE
CONTRIBUTION OF THE TEM MODE

RES=4./377.
RES= RES*(O,1)*K/KZP/(A*H1A+D*H1D)
ARS=CABS(RES)
T=M
RAT=ARS*377.*ALOG(T)/PI
WRITE(5,40) ARS
FORMAT(1X,'THE CURRENT IS :',F15.8)
WRITE(5,41) RAT
FORMAT(1X,'CUR/TEM IS :',F15.8)

CHANGING THE DISTANCE-D

D=0.1
GO TO 70
CONTINUE

CHANGING THE DEPARTURE POINT IN ORDER TO
START THE SEARCHING FOR THE CONSECUTIVE ROOT

I=I+1
BR=BR+5
GO TO 9
STOP
END

THE SUBROUTINE CBESO CALCULATES THE ZERO ORDER BESSEL
FUNCTIONS FOR A GIVEN INPUT VARIABLE Z

SUBROUTINE CBESO(Z,BSJ0,BSYO)
COMPLEX Z,BSJ0,BSYO,Y,W,PO,CZEXP
COMPLEX CCOS,CSIN,CSQRT,CLGB,SP0,SQ0,CO,CS,SN,SP1,SPQ
PI=3.141593
A=CABS(Z)
BSJ0=(1.,0.,)
IF (A.EQ.0.,) GO TO 2
IF (A.GT.3.,) GO TO 1
Y=Z/Z
BSJ0=1.+Y*(-2.2499997+Y*(1.2656208+Y*(-.31638661+Y*(-.0044479+Y*(-.0039444+Y*0.0021))))))
GO TO 2
W=3./Z
F0=.79788456+*W*(-.00000077+W*(-.0055274+W*(-.00095121+W(*.00137273+W*(-.00014476)))))
1+W(*.00137273+W*(-.00014476))
PO=.78539816+W*(-.04166397+W*(-.0003954+W*(-.00262573
1+W*(-.00054125+W*(-.00029333-W*(-.00013558))))))
BSJ0=F0*CCOS(Z-P0)/CSQRT(Z)

CONTINUE
BSYO=(-1.E30,0)
IF (A.EQ.0.,) GO TO 3
IF (A.GT.3.,) GO TO 4
BSYO=.63661977*CLGB(5.*Z)*BSJ0+.36746691*Y*(-.31638661+Y*(-.0044479+Y*(-.0039444+Y*(-.0021))))))
1+Y*(-.74350384+Y*(-.25300117+Y*(-.04261214+Y*(-.00427916
2-Y*0.00324846))))

65
THE SUBROUTINE CBES1 CALCULATES THE FIRST ORDER BESSEL FUNCTIONS FOR A GIVEN INPUT VARIABLE Z

SUBROUTINE CBES1(Z,BSJ1,BSY1)
COMPLEX Z,BSJ1,BSY1,Y,W,P1,F1,CEXP
COMPLEX CCOS,CSQRT,CLOG,CSIN,SP1,SQ1,Q1,F,CS8N,SP2,SQ2
PI=3.141593
A=CABS(Z)
BSJ1=0.
IF(A.EQ.0.) GO TO 2
IF(A.GT.3.) GO TO 1
Y=Z*Z/9.
BSJ1=Z*(.5+Y*(-.56249985+Y*(.21093573+Y*(-.03954289+
1+Y*(-.0443319+Y*(-.00031761+Y*0.0001109)))))
GO TO 2
W=3./Z
F1=.79788456+W*(-.00000156+W*(-.01659667+W*(-.0017105
1+W*(-.00249511+W*(-.0013653-W*0.0020033))))))
P1=2.35619449+W*(-.12499612+W*(-.00005650+W*(-.00637879
1+W*(-.00074348+W*(-.00079824+W*.00029166))))))
BSJ1=F1*CCOS(Z-P)/CSQRT(Z)
2 CONTINUE
BSY1=(-1.E30,0)
IF(A.EQ.0.) GO TO 3
IF(A.GT.3.) GO TO 4
BSY1=(-.63661977+Y*(2.2212091+Y*(2.1682709+Y*(-.1.3164827
1+Y*(-.3123951+Y*(-.04009764*Y*.0027873)))))1/Z
2+.63661977*CLOG(.5*Z)*BSJ1
GO TO 3
4 BSY1=F1*CSIN(Z-P)/CSQRT(Z)
3 CONTINUE
RETURN
END

THE SUBROUTINE FUNCT CALCULATES FOR EACH VECTOR KZ
THE VALUE OF THE FUNCTIONAL WHICH WE TRY
TO MINIMIZE AND ITS GRADIENT

SUBROUTINE FUNCT(N,KZ,VAL,GRAD)
COMMON KR,AA,H,BR,AR
DIMENSION GRAD(2)
REAL KZ(2),K,KH
REAL IJOA,JJOA,IP1A,IP1D,IP1A,IP1D
COMPLEX KR,KRD,JOA,YOA,J0D,YOD,H0A,H0D,J1A
COMPLEX Y1A,J1D,Y1D,GR1,GR2,H1A,H1D,F,GRA1,GRA2
BZ=KZ(1)
AZ=KZ(2)
PI=3.1415927
K=2*PI
\[ KH = K^2 + AZ^2 - BZ^2 \]
\[ BR = \sqrt{0.5 \left( (KH^2 + (2AZBZ)^2) \right)} \]
\[ AR = -AZBZ/BR \]
\[ KR = \text{COMPLEX}(BR - AR) \]
\[ KRA = KRAA \]
\[ KRD = KR*M \]
\[ \text{CALL CBES0(KRA, J0A, Y0A)} \]
\[ \text{CALL CBES0(KRD, J0D, Y0D)} \]
\[ RJ0A = \text{REAL}(J0A) \]
\[ RJ0A = \text{AIMAG}(J0A) \]
\[ RY0A = \text{REAL}(Y0A) \]
\[ IY0A = \text{AIMAG}(Y0A) \]
\[ RJ0D = \text{REAL}(J0D) \]
\[ RJ0D = \text{AIMAG}(J0D) \]
\[ RY0D = \text{REAL}(Y0D) \]
\[ IY0D = \text{AIMAG}(Y0D) \]
\[ HOA = \text{COMPLEX}(RJOA + IY0A, IJOA - RY0A) \]
\[ HOA = \text{COMPLEX}(RJOA + IY0A, IJOA - RY0A) \]
\[ F = HOA - HOD \]
\[ \text{CALL CBESI(KRA, J1A, Y1A)} \]
\[ \text{CALL CBESI(KRD, J1D, Y1D)} \]
\[ RJ1A = \text{REAL}(J1A) \]
\[ RJ1A = \text{AIMAG}(J1A) \]
\[ RY1D = \text{REAL}(Y1D) \]
\[ IY1D = \text{AIMAG}(Y1D) \]
\[ RJ1D = \text{REAL}(J1D) \]
\[ RJ1D = \text{AIMAG}(J1D) \]
\[ RY1D = \text{REAL}(Y1D) \]
\[ IY1D = \text{AIMAG}(Y1D) \]
\[ H1A = \text{COMPLEX}(RJ1A + IY1A, IJ1A - RY1A) \]
\[ H1D = \text{COMPLEX}(RJ1D + IY1D, +IJ1D - RY1D) \]
\[ GR1 = -H1A*\left( -\text{COMPLEX}(BZ, -AZ)*AA/KR \right) \]
\[ GR1 = -H1A*\left( -\text{COMPLEX}(BZ, -AZ)*AA/M/KR \right) \]
\[ GR2 = GR1*\left( 0.0, -1.0 \right) \]
\[ GRA1 = GR1*\text{CONJG}(F) + \text{CONJG}(GR1)*F \]
\[ GRA2 = GR2*\text{CONJG}(F) + \text{CONJG}(GR2)*F \]
\[ \text{GRAD(1)} = \text{REAL}(GRA1) \]
\[ \text{GRAD(2)} = \text{REAL}(GRA2) \]
\[ \text{RETURN} \]
\[ \text{END} \]
Output Sample for Program No. 4

\[ D/A = 10 \]

\[ D IS : .1000 \]
REAL KZ IS : \(-12.3694\) IMAG KZ IS : 68.1768
THE CURRENT IS : \(0.13376462E-03\)
CUR/TEM IS : \(0.36961400E-01\)

\[ D IS : .2000 \]
REAL KZ IS : \(-6.2624\) IMAG KZ IS : 33.8657
THE CURRENT IS : \(0.27066794E-03\)
CUR/TEM IS : \(0.74790076E-01\)

\[ D IS : .3000 \]
REAL KZ IS : \(-4.2655\) IMAG KZ IS : 21.9671
THE CURRENT IS : \(0.41419069E-03\)
CUR/TEM IS : \(0.11444781E+00\)

\[ D IS : .4000 \]
REAL KZ IS : \(-3.3017\) IMAG KZ IS : 15.9635
THE CURRENT IS : \(0.56857456E-03\)
CUR/TEM IS : \(0.15710665E+00\)

\[ D IS : .5000 \]
REAL KZ IS : \(-2.7587\) IMAG KZ IS : 12.2276
THE CURRENT IS : \(0.73941571E-03\)
CUR/TEM IS : \(0.20431292E+00\)

\[ D IS : .6000 \]
REAL KZ IS : \(-2.4367\) IMAG KZ IS : 9.6135
THE CURRENT IS : \(0.93456462E-03\)
CUR/TEM IS : \(0.25823583E+00\)
Program No. 5

THE PURPOSES OF THIS PROGRAM ARE:
1) TO FIND THE REAL AND IMAGINARY PARTS OF THE R-DIRECTED AND Z-DIRECTED WAVE NUMBERS ASSOCIATED WITH THE HIGHER ORDER EVEN LEAKY MODES IN THE CASE OF VOLT EXCITATION.
2) TO CALCULATE THE CONTRIBUTION OF EACH MODE TO THE TOTAL CURRENT.

THE PROGRAM VARIES THE RATIO "D/A", WHILE KEEPING THE DISTANCE BETWEEN THE WIRES "D" CONSTANT.

COMMON KR,AA,M,DR,AR
COMPLEX KR,KZP,KRP,RES
REAL IJOA,IJOD,IJ1A,IJ1D,IYOA,IYOD,IY1A,IY1D
COMPLEX KRA,KRD,JOA,YOA,JOD,YOD,HOA,HOD,J1A
COMPLEX Y1A,J1D,Y1D,GR1,GR2,H1A,H1D,F,GRA1,GRA2
COMPLEX KRP,KRPD
REAL KZ(2),KH
REAL OGRAD(2)
REAL OG50(2),CON(2)

DEPARTURE POINT FOR THE SEARCHING IN THE KZ PLANE

I=1
BR=1.
AR=-2.

START SEARCHING FOR THE ROOTS IN THE KZ PLANE

IF (BR.GT.100) GO TO 8
WRITE (5,18) I
18 FORMAT(///,1X,'THE INDEX OF THE MODE IS ',I2,//)

THE DISTANCE BETWEEN THE WIRES

D=2.0
WRITE (5,80) D
80 FORMAT(///,1X,'D IS ',F6.4,//)

INITIAL VALUE FOR THE RATIO "D/A"

M=10
IF(M.GT.1000) GO TO 60
OGRAD(1)=0.
OGRAD(2)=0.
CON(1)=0.50
CON(2)=0.50
N=2
PI=3.1415927
K=2*PI
KH=K**2+AR**2-DR**2
BZ=SQRT(0.5*(KH+SQRT((KH**2+(2*AR*BR)**2)))))
AZ=-AR*BR/BZ
KR=COMPLX(BR,-AR)
KZ(1)=BZ
KZ(2)=AZ
AA=1./M
WRITE (5,19) M
FORMAT(/,1X,'D/A = '',14)
19 CALL FUNCT(N,KZ,VAL,GRAD)
IF(VAL.LT.0.1E-3) GO TO 1
IF (GRAD(1)) 22,27,27
22 IF (OGRAD(1)) 23,24,24
23 KZ(1)=KZ(1)+CON(1)
OGRAD(1)=GRAD(1)
GO TO 31
27 IF (OGRAD(1)) 28,29,29
28 CON(1)=CON(1)/2.
29 KZ(1)=KZ(1)-CON(1)
OGRAD(1)=GRAD(1)
30 CONTINUE
IF (GRAD(2)) 32,37,37
32 IF (OGRAD(2)) 33,34,34
33 KZ(2)=KZ(2)+CON(2)
OGRAD(2)=GRAD(2)
GO TO 3
37 IF (OGRAD(2)) 38,39,39
38 CON(2)=CON(2)/2.
39 KZ(2)=KZ(2)-CON(2)
OGRAD(2)=GRAD(2)
GO TO 3
1 CONTINUE
A=D/M
BP=+BR/D
AP=+AR/D
KRP=CMPLX(BP,-AP)
KRPA=KRP&A
KRPD=KRP&D
K=2*PI
KH=K**2+AP**2-BP**2
BZ=SQRT(0.5*KH+SQRT((KH**2+(2*AP*BP)**2)))
AZ=-AP*BP/BZ
KZP=CMPLX(-BZ,AZ)
WRITE(5,90) KZP
90 FORMAT(/,1X,'REAL KZ IS ''F9.4,4'' , IMAG KZ IS ''F9.4,4'')
CALL CDES1(KRPA,J1A,Y1A)
CALL CDES1(KRPD,J1D,Y1D)
RJ1A-REAL(J1A)
IJ1A=AIMAG(J1A)
RY1D-REAL(Y1D)
IY1D=AIMAG(Y1D)
RJ1D-REAL(J1D)
IJ1D=AIMAG(J1D)
RY1D-REAL(Y1D)
IY1D=AIMAG(Y1D)
H1A=CMPLX(RJ1A+IY1A,IJ1A-RY1A)
H1D=CMPLX(RJ1D+IY1D,IJ1D-RY1D)
C THE CONTRIBUTION OF THE LEAKY MODE
C TO THE TOTAL CURRENT NORMALIZED TO THE
C CONTRIBUTION OF THE TEM MODE
C

70
C
RES=4./377.
RES= RES*(0,1)K/KRP/KZP/(A%H1A+D%H1D)
ARS=CABS(RES)
WRITE(5,40) ARS
40 FORMAT(/1X,'THE CURRENT IS :',E15.8)
M=M
RAT=ARS*377.*ALOG(T)/PI
WRITE(5,47) RAT
47 FORMAT(/1X,'THE CURRENT IS :',E15.8)
C
CHANGING THE RATIO 'D/A'
M=M+1
GO TO 41
60 CONTINUE
C
CHANGING THE DEPARTURE POINT IN ORDER TO
START THE SEARCHING FOR THE CONSECUTIVE ROOT
I=I+1
BR=BR+5
GO TO 9
8 CONTINUE
STOP
END
C
THE SUBROUTINE CBESO CALCULATES THE ZERO ORDER BESSEL
FUNCTIONS FOR A GIVEN INPUT VARIABLE Z
C
SUBROUTINE CBESO(Z,BSJO,BSYO)
COMPLEX Z,BSJO,BSYO,Y,W,P0,F0,CXP
COMPLEX CCOS,CSIN,CSQRT,CLG,S00,Q0,CS,SN,SP1,SG1
PI=3.141593
A=CABS(Z)
BSJO=1.*0.
IF(A.EQ.0.) GO TO 2
IF(A.GT.3.) GO TO 1
Y=Z/Z
BSJO=1.+Y*(-.2.2499997+Y*(.12656208+Y*(-.3163866
1+Y*(.0444479+Y*(-.0039444+Y*.00021)))))
GO TO 2
1 W=3./Z
F0=79788456+W*(-.00000077+W*(-.0055274+W*(-.00009512
1+W*(-.00137237+W*(-.00072805+W*(-.00014476))))))
P0=78539816+W*(-.04166397+W*(-.0003954+W*(-.00262573
1+W*(-.0054125+W*(-.00029333-W*(-.00013558))))))
BSJO=F0*CCOS(Z-P0)/CSQRT(Z)
2 CONTINUE
BSYO=1.-E30,0)
IF(A.EQ.0.) GO TO 3
IF(A.GT.3.) GO TO 4
BSYO=.36661977*CLG(.5*Z)*BSJO+.36746691+Y*(-.6059366
1+Y*(-.74350384+Y*(-.25300117+Y*(-.04261214+Y*(-.00427916
2-Z*-.00024846))))
GO TO 3
CONTINUE
BSY1=FOCSIN(Z-P0)/CSQRT(Z)
CONTINUE
RETURN
END

THE SUBROUTINE CBES1 calculates the first order Bessel functions for a given input variable Z

SUBROUTINE CBES1(Z, BSJ1, BSY1)
COMPLEX Z, BSJ1, BSY1, Y, W, P1, F1, CEXP
COMPLEX CCOS, CSQRT, CLOG, CSIN, SP1, SQ1, Q1, F1, CS, SN, SP2, SQ2
PI=3.141593
A=CABS(Z)
BSJ1=0.
IF(A.EQ.0.) GO TO 2
IF(A.GT.3.) GO TO 1
Y=Z**2/9.
BSJ1=Z**1.5+Y*(-.56249985+Y*(-.21093573+Y*(-.03954289
+Y*(-.00443319+Y*(-.00031761+Y*.0001109))))
GO TO 2
1
Y=Z**3/2
F1=+.75788456+W*(-.00000156+W*(-.01659667+W*(-.00017105
+W*(-.00249511+W*(-.00113653+W*(-.00020033))))))
P1=2.35619449+W*(-.12499612+W*(-.0005650+W*(-.00437879
+W*(-.00074348+W*(-.0079824+W*(-.00029166))))))
BSJ1=F1*CCOS(Z-P1)/CSQRT(Z)
2
CONTINUE
BSY1=-(1.*E30,0)
IF(A.EQ.0.) GO TO 3
IF(A.GT.3.) GO TO 4
BSY1=-(.63661977+Y*(-.2212091+Y*(2.1682709+Y*(-.164827
+Y*(-.3123951+Y*(-.040976+Y*.0027873))))))/Z
BSY1=F1*CLOG(.5*Z)*BSJ1
GO TO 3
3
CONTINUE
RETURN
END

THE SUBROUTINE FUNCT calculates for each vector KZ the value of the functional which we try

TO MINIMIZE AND ITS GRADIENT

SUBROUTINE FUNCT(N, KZ, VAL, GRAD)
COMMON KR, AA, M, BR, AR
DIMENSION GRAD(2)
REAL KZ(2), K, KH
REAL IJOA, IJOD, IJ1A, IJ1D, IYOA, IYOD, IY1A, IY1D
COMPLEX KR, KRA, KRD, J0A, YOA, JOD, YOD, HOA, HOD, J1A
COMPLEX Y1A, J1D, Y1D, GR1, GR2, H1A, H1D, F, GRA1, GRA2
BZ=KZ(1)
AZ=KZ(2)
PI=3.1415927
K=2*PI
KH=K**2*AZ**2-BZ**2

72
\begin{verbatim}
DR=SQRRT(0.5*(KHI+SQRT((KHI^2+(2*AIZBZ)^2))))
AR=-AIZBZ/DR
KR=CMPLX(DR,-AR)
KRA=KRAAA
KRD=KRA**M
CALL CBES0(KRA,JOA,YOA)
CALL CBES0(KRD,JOD,YOD)
RJOA=REAL(JOA)
IJOA=AIMAG(JOA)
RYOA=REAL(YOA)
IYOA=AIMAG(YOA)
RJOD=REAL(JOD)
IJOID=AIMAG(JOD)
RYOD=REAL(YOD)
IYOD=AIMAG(YOD)
HOA=CMPLX(RJOA+IJOA,RYOA-IYOA)
HOD=CMPLX(RJOD+IJOID,RYOD-IYOD)
F=HOA+HOD
VAL=CABS(F)
VAL=VAL**2
CALL CBES1(KRA,1A,Y1A)
CALL CBES1(KRD,1D,Y1D)
R1A=REAL(1A)
I1A=AIMAG(1A)
RY1D=REAL(Y1D)
IY1D=AIMAG(Y1D)
R1D=REAL(J1D)
I1D=AIMAG(J1D)
RYID=REAL(Y1D)
IY1D=AIMAG(Y1D)
H1A=CMPLX(R1A+I1A,RY1D-IY1D)
H1D=CMPLX(R1D+I1D,RY1D-IY1D)
GR1=-H1A*(-CMPLX(BZ,-A)*AA/KR)
GR1=GR1-H1D*(-CMPLX(BZ,-A)*AA*M/KR)
GR2=GR1*(0.0,-1.0)
GRA1=GR1*CONJG(F)+CONJG(GR1)*F
GRA2=GR2*CONJG(F)+CONJG(GR2)*F
GRAD(1)=REAL(GRA1)
GRAD(2)=REAL(GRA2)
RETURN
END
\end{verbatim}
Output Sample for Program No. 5

D IS 2.0000

D/A = 10
REAL KZ IS : -6.0968  IMAG KZ IS : 0.1582
THE CURRENT IS : 0.23093203E-02
THE CUR/TEM IS : 0.63810379E+00

D/A = 11
REAL KZ IS : -6.1045  IMAG KZ IS : 0.1595
THE CURRENT IS : 0.22558292E-02
THE CUR/TEM IS : 0.64912435E+00

D/A = 12
REAL KZ IS : -6.1111  IMAG KZ IS : 0.1605
THE CURRENT IS : 0.2211599E-02
THE CUR/TEM IS : 0.65935864E+00

D/A = 13
REAL KZ IS : -6.1169  IMAG KZ IS : 0.1616
THE CURRENT IS : 0.21676725E-02
THE CUR/TEM IS : 0.66721214E+00

D/A = 14
REAL KZ IS : -6.1220  IMAG KZ IS : 0.1626
THE CURRENT IS : 0.21300598E-02
THE CUR/TEM IS : 0.67457789E+00

D/A = 15
REAL KZ IS : -6.1265  IMAG KZ IS : 0.1636
THE CURRENT IS : 0.2093244E-02
THE CUR/TEM IS : 0.68048655E+00
THE PURPOSES OF THIS PROGRAM ARE:

1) TO FIND THE REAL AND IMAGINARY PARTS OF THE R-DIRECTED AND Z-DIRECTED WAVE NUMBERS, ASSOCIATED WITH THE HIGHER ORDER ODD LEAKY MODES IN THE CASE OF VOLT EXCITATION.

2) TO CALCULATE THE CONTRIBUTION OF EACH MODE TO THE TOTAL CURRENT.

THE PROGRAM VARIES THE RATIO "R/A", WHILE KEEPING THE DISTANCE BETWEEN THE WIRES "D" CONSTANT.

COMMON KR, AA, M, BR, AR
COMPLEX KRA, KRP, KR
REAL IJCA, IJDB, IJ1A, IJID, IYOA, IYOD, IY1A, IY1D
COMPLEX KRA, KRD, JOA, YOA, JOD, YOD, HOA, HOD, JA
COMPLEX Y1A, J1D, Y1D, BR1, BR2, H1A, H1D, F, GRA1, GRA2
COMPLEX KRP, KRD
REAL KZ(2), K, KH
REAL GRAD(2)
REAL OGRAD(2), CON(2)

DEPARTURE POINT FOR THE SEARCHING IN THE KZ PLANE

I=1
BR=1.
AR=-2.

START SEARCHING FOR THE ROOTS IN THE KZ PLANE

IF (BR.GT.100) GO TO 8
WRITE (5,10) I
FORMAT(/*//,1X, 'THE INDEX OF THE MODE IS ', I2, '/**)

THE DISTANCE BETWEEN THE WIRES

D=2.0
WRITE (5,80) D
FORMAT(/*//,1X, 'D IS ', F6.4, '/**)

INITIAL VALUE FOR THE RATIO "R/A"

M=10
IF (M.GT.1000) GO TO 60
OGRAD(1)=0.
OGRAD(2)=0.
CON(1)=0.50
CON(2)=0.50
N=2
PI=3.1415927
K=2*PI
KH=K**2-(AR**2-BR**2)
BZ=SQR(1.05*(KH+SQRT(KH**2+(2*AR*BR)**2))))
AZ=-AR*KH/BZ
KR=CMPLX(BR,-AR)
KZ(1)=BZ
KZ(2)=AZ
AA=1./M
WRITE (5,19) M
FORMAT(//1X,'/D/A = ',I4)
CALL FUNCT(N,KZ,VAL,GRAD)
IF(VAL.LT.0.1E-5) GO TO 1
IF (GRAD(1)) 22,27,27
IF (OGRAD(1)) 23,24,24
CON(1)=CON(1)/2.
KZ(1)=KZ(1)+CON(1)
OGRAD(1)=GRAD(1)
GO TO 31
IF (OGRAD(1)) 28,29,29
CON(1)=CON(1)/2
KZ(1)=KZ(1)-CON(1)
OGRAD(1)=GRAD(1)
CONTINUE
IF (GRAD(2)) 32,37,37
IF (OGRAD(2)) 33,34,34
CON(2)=CON(2)/2.
KZ(2)=KZ(2)+CON(2)
OGRAD(2)=GRAD(2)
GO TO 3
IF (OGRAD(2)) 38,39,39
CON(2)=CON(2)/2
KZ(2)=KZ(2)-CON(2)
OGRAD(2)=GRAD(2)
GO TO 3
1 CONTINUE
A=D/M
BP=+BR/D
AP=+AR/D
KRP=CMPLX(BP,-AP)
KRPA=KRP*A
KRPD=KRP*D
K=2*PI
KH=K**2+AP**2-BP**2
BZ=SQRT(0.5*(KH+SQRT((K**2+(2*AP*BP)**2))))
AZ=-AP*BP/BZ
KZP=CMPLX(-BZ,AZ)
WRITE(5,90) KZP
90 FORMAT(/1X,'REAL KZ IS : ',F9.4,' IMAG KZ IS : ',F9.4)
CALL CBES1(KRPA,J1A,Y1A)
CALL CBES1(KRPD,J1D,Y1D)
RJ1A=REAL(J1A)
IY1A=AIMAG(J1A)
RY1D=REAL(Y1D)
IY1D=AIMAG(Y1D)
RJ1D=REAL(J1D)
I1D=AIMAG(J1D)
RY1D=REAL(Y1D)
IY1D=AIMAG(Y1D)
H1A=CMPLX(RJ1A+IY1A,IJ1A-RY1A)
H1D=CMPLX(RJ1D+IY1D,1J1D-RY1D)

C THE CONTRIBUTION OF THE LEAKY MODE
C TO THE TOTAL CURRENT NORMALIZED TO THE
C CONTRIBUTION OF THE TEM MODE
RES=4./377.
RES= RES*(0,1)*K/KRP/KZP/(A*H1A+B*H1D)
ARS=CABS(RES)
WRITE(5,'40) ARS
40 FORMAT(/1X,'THE CURRENT IS:',E15.8)
T=M
RAT=ARS*377.*ALOG(T)/PI
WRITE(5,'47) RAT
47 FORMAT(/1X,'THE CUR/TEM IS:',E15.8)

C CHANGING THE RATIO "D/A"
C
M=M+1
GO TO 41
CONTINUE
C

C CHANGING THE DEPARTURE POINT IN ORDER TO
C START THE SEARCHING FOR THE CONSECUTIVE ROOT
C
I=I+1
BR=BR+5
GO TO 9
8 CONTINUE
STOP
END
C

C THE SUBROUTINE CBESO CALCULATES THE ZERO ORDER BESSEL
C FUNCTIONS FOR A GIVEN INPUT VARIABLE Z
C
SUBROUTINE CBESO(Z,BSJO,BSYO)
COMPLEX Z,BSJO,BSYO,Y,W,F0,F1,EXP
COMPLEX CCOS,CSIN,CSQRT,CLG,CP0,CPQ,CXP,CO,CS,SN,SP1,SPQ
PI=3.141593
A=CABS(Z)
BSJO=(1.+0.)
IF(A.EQ.0.) GO TO 2
IF(A.GT.3.) GO TO 1
Y=Z*Z/9,
BSJO1=+Y*(-2.2499997+Y*C1 .2656208+Y*(-.31638666
+Y*(.0444479+Y*(-.0039444+Y*.00021)))
1 CONTINUE
BSJO0=(-j.1.E30.v0)
IF(A.EQ.0.) GO TO 3
IF(A.GT.3.) GO TO 4
BSYO=.63661977*CLG(.5*z)*BSJO+.36746691+Y*(-.60559366
+Y*(-.74350384+Y*(-.25300117+Y*(-.04261214+Y*(-.00427916
2-Y*.00024846))))
GO TO 3

CONTINUE
BSY0=F0*CSIN(Z-P0)/CSQRT(Z)
CONTINUE
RETURN
END

THE SUBROUTINE CBES1 CALCULATES THE FIRST ORDER BESSEL FUNCTIONS FOR A GIVEN INPUT VARIABLE Z

SUBROUTINE CBES1(Z,BSJ1,BSY1)
COMPLEX Z,BSJ1,BSY1,Y,W,P1,F1,CEXP
COMPLEX CCOS,CSQRT,CLOG,CSIN,SP1,SQ1,Q1,F,CS,SN,SP2,SQ2
PI=3.141593
A=CABS(Z)
BSJ1=0.
IF(A.EQ.0.) GO TO 2
IF(A.GT.3.) GO TO 1
Y=Z/Z/9.
BSJ1=Z*(5.5+Y*(-.56249985+Y*(.21093573+Y*(-.03954289
1+Y*(-.00443319+Y*(-.00031761+Y*.0001109))))))
GO TO 2
1
W=3./Z
F1=.79788456+W*(-.00000156+W*(-.01659647+W*(-.00017105
1+W*(-.00249511+W*(-.00113653-W*(-.00020033))))))
P1=2.35619449+W*(-.12499612+W*(-.0005650+W*(-.00637879
1+W*(-.00074348+W*(-.00079824+W*(-.00029166))))))
BSJ1=F1*CCOS(Z-P1)/CSQRT(Z)
CONTINUE
2
BSY1=1.3*E3001
IF(A.EQ.0.) GO TO 3
IF(A.GT.3.) GO TO 4
BSY1=(-.63661977+Y*(-.2212091+Y*(2.16827094+Y*(-1.316487
1+Y*(-.3123951+Y*(-.0400976+Y*.0027873))))))/Z
2+.63661977*CLOG(.5*Z)*BSJ1
GO TO 3
4
BSY1=F1*COS(Z-P1)/CSQRT(Z)
CONTINUE
RETURN
END

THE SUBROUTINE FUNCT CALCULATES FOR EACH VECTOR KZ THE VALUE OF THE FUNCTIONAL WHICH WE TRY TO MINIMIZE AND ITS GRADIENT

SUBROUTINE FUNCT(N,KZ,VAL,GRAD)
COMMON KR,AA,M,BR,AR
DIMENSION GRAD(2)
REAL KZ(2),K,KH
REAL IJOA, IJOD, IJ1A, IJ1D, IYOA, IYOD, IY1A, IY1D
COMPLEX KR,KRA,KRD,JOA,YOA,JOD,YOD,HOA,HOD,J1A
COMPLEX Y1A,J1D,Y1D,GR1,GR2,H1A,H1D,F,GRA1,GRA2
BZ=KZ(1)
AZ=KZ(2)
PI=3.1415927
K=2*PI
KH=K**2+AZ**2-BZ**2
BR = SQRT(0.5*(KH + SQRT((K * H**2 + (2 * AZ * BZ)**2))))
AR = -AZ * BZ / BR
KR = COMPLEX(BR, -AR)
KRA = K * RA
KRD = KRA * M
CALL CBES0(KRA, JOA, YOA)
CALL CBES0(KRD, JOD, YOD)
RJOA = REAL(JOA)
IJOA = IMAG(JOA)
RYOA = REAL(YOA)
IYOA = IMAG(YOA)
RJOD = REAL(JOD)
IJOD = IMAG(JOD)
RYOD = REAL(YOD)
IYOD = IMAG(YOD)
HOA = COMPLEX(RJOA + IYOA, IJOA - RYOA)
HOD = COMPLEX(RJOD + IYOD, IJOD - RYOD)
F = HOA - HOD
VAL = CABS(F)
VAL = VAL**2
CALL CBES1(KRA, J1A, Y1A)
CALL CBES1(KRD, J1D, Y1D)
RJ1A = REAL(J1A)
IJ1A = IMAG(J1A)
RY1A = REAL(Y1A)
IY1A = IMAG(Y1A)
RJ1D = REAL(J1D)
IJ1D = IMAG(J1D)
RY1D = REAL(Y1D)
IY1D = IMAG(Y1D)
H1A = COMPLEX(RJ1A + IY1A, IJ1A - RY1A)
H1D = COMPLEX(RJ1D + IY1D, IJ1D - RY1D)
GR1 = H1A*(-COMPLEX(BZ, -AZ) * AA / KR)
GR1 = GR1 - H1D*(-COMPLEX(BZ, -AZ) * AA*M / KR)
GR2 = GR1*(0.0, -1.0)
GRA1 = GR1*CONJG(F) + CONJG(GR1)*F
GRA2 = GR2*CONJG(F) + CONJG(GR2)*F
GRAD(1) = REAL(GRA1)
GRAD(2) = REAL(GRA2)
RETURN
END
Output Sample for Program No. 6

D IS 2.0000

D/A = 10
REAL KZ IS : -5.3197  IMAG KZ IS :  0.3963
THE CURRENT IS : 0.17374900E-02
THE CUR/TEM IS : 0.48009754E+00

D/A = 11
REAL KZ IS : -5.3473  IMAG KZ IS :  0.3998
THE CURRENT IS : 0.16872877E-02
THE CUR/TEM IS : 0.48552414E+00

D/A = 12
REAL KZ IS : -5.3704  IMAG KZ IS :  0.4036
THE CURRENT IS : 0.16398675E-02
THE CUR/TEM IS : 0.48900164E+00

D/A = 13
REAL KZ IS : -5.3899  IMAG KZ IS :  0.4071
THE CURRENT IS : 0.15975309E-02
THE CUR/TEM IS : 0.49172189E+00

D/A = 14
REAL KZ IS : -5.4066  IMAG KZ IS :  0.4106
THE CURRENT IS : 0.15589255E-02
THE CUR/TEM IS : 0.49370289E+00

D/A = 15
REAL KZ IS : -5.4212  IMAG KZ IS :  0.4135
THE CURRENT IS : 0.15255773E-02
THE CUR/TEM IS : 0.49577245E+00
References


