ASYMPTOTIC METHODS ESPECIALLY IN COMBUSTION

FINAL REPORT

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Asymptotic Methods Especially in Combustion

Modern asymptotic methods have been applied to a wide range of problems in combustion science as well as a few in magnetohydrodynamic and fluid mechanical questions. Details are contained in the 39 Technical Reports, 6 Ph.D. Theses, and 1 Master's Thesis listed. A list of participating scientists is given.
Objectives and Results

Asymptotic methods were applied to a variety of combustion problems including

(i) quenching, in all its aspects,
(ii) the deflagration to detonation transition (DDT),
(iii) near-stoichiometric behavior of two-reactant mixtures,
(iv) propagation limits of mixtures,
(v) chambered diffusion flames,
(vi) thermal runaway in two-reactant systems.
(vii) burning of droplets.

The most important results may be summarized as follows.

(a) Mathematically rigorous treatment of thermal runaway.
(b) Description of chambered diffusion flames for different supply temperatures.
(c) A theory of quenching in shear flows.
(d) Discovery that differential diffusion can account for shift in flame-velocity maximum of mixtures; and demonstration that dissociation plays a minor role, as does the dilution of the mixture.
(e) Commonly accepted ignition and extinction criteria justified by a stability analysis.
(f) Rational analysis of flame tips.
(g) Limitations of accepted droplet theory; and its invalidity for variations via the ambient pressure.
(h) Simple and comprehensive description of detonation structure.
(i) Existence and non-existence criteria for the structure of detonation waves.
(j) Analysis of the concept of flame stretch.
(k) Potential uses of activation-energy asymptotics in turbulent combustion theory.
(l) Description of fast deflagration waves, whose theory is outside the combustion approximation; and the discovery of very slow deflagrations.
(m) Incorporation of fluid mechanics into the problems of flame tips and stretch.
(n) Theory of quasi-steady flame acceleration and deceleration, as an aspect of DDT.
(o) Stability analysis of near extinction for the burning fuel drop.
(p) Demonstration of stable steady states for flames held in the stagnation-point flow behind a wire or bluff body.
(q) Theory of the excess-enthalpy flame.
(r) Theory of flammability limits.

Most of these results have been incorporated into a book entitled "Theory of Laminar Flames", written by J. Buckmaster and G.S.S. Ludford for the Cambridge Monograph Series on Mechanics and Applied Mathematics. In addition a few problems in magnetohydrodynamics and fluid mechanics have been addressed.

Details of all the results are contained in Progress Reports Nos. 25-32.

Publications

39 Technical Reports were written, as follows.

90. J.D. Buckmaster: Combustion at a stagnation point.


120. G.S.S. Ludford & D.S. Stewart: Mathematical questions from combustion theory. Transactions of the 26th Conference of Army Mathematicians, Lebanon (NH) 1980, p. 53. (Published by ARO, Report 81-1.)

122. D.S. Stewart & G.S.S. Ludford: Deflagration and detonation for small heat release. Submitted for publication.


6 Ph.D. theses were completed, as follows.


Samuel Paolucci: Langmuir circulations as a convective instability mechanism and its effect on the ocean mixed layer, x + 218 pp., May 1979.


Karen Ann Ames: Comparison results for related properly and improperly posed Cauchy problems, with application to mechanics, vi + 156 pp., August 1980.

Donald Scott Stewart: The transition from deflagration to detonation, xii + 193 pp., May 1981.
In addition there was 1 Master's thesis, namely

**Participating Scientists.**

K.A. Ames (Ph.D.), C. Holmes (M.S.), H.V. McConnaughey, R.D.Janssen, L.C. Li (M.S.), G.C. Lu, A. Oyediran, A.K. Sen (Ph.D.), J.A. Simmen (M.S.) and D.S. Stewart (Ph.D.) were supported for various periods as Research Assistants. Advanced degrees that they earned are shown in parentheses. In addition R.O. Ayeni and S. Paolucci, who were supported at earlier times, earned Ph.D.'s during the report period.

M. Matalon and D.S. Stewart had postdoctoral appointments. Prof. J.D. Buckmaster of the University of Illinois has been a consultant every summer.