NONLINEAR EVOLUTION OF PLASMA ENHANCEMENTS IN THE AURORAL IONOSPHERE

OCT 81 M J KESKINEN, S L OSSAKOW

UNCLASSIFIED NRL-MR-4611
NRL Memorandum Report 4611

Nonlinear Evolution of Plasma Enhancements in the Auroral Ionosphere: Long Wavelength Irregularities

M. J. Keskinen and S. L. Ossakow

Naval Research Laboratory
Washington, DC 20375

Defense Nuclear Agency, Washington, DC 20305 and
Office of Naval Research, Arlington, VA 22217

Naval Research Laboratory
Washington, DC 20375

Approved for public release; distribution unlimited.

This research was sponsored partially by the Defense Nuclear Agency under Subtask S99QAHC, work unit 00010, work unit title "High Latitude Effects" and partially by the Office of Naval Research.

The linear stability and nonlinear evolution of plasma enhancements in arbitrary ambient electric fields in the diffuse auroral F region ionosphere have been studied using analytical and numerical simulation techniques. Our results indicate that equatorward convecting plasma slabs initially limited in latitudinal extent are primarily destabilized on their poleward sides by a combination of the effects of convection and field aligned currents. Furthermore we find that the plasma enhancements break up into primary striation-like structures (elongated in the north-south direction for equatorward convection) which can...
form and cascade from large ($\approx 100$ km) to smaller ($\approx 3$ km) scale sizes on the order of an hour. The primary and associated smaller scale structures can be oriented either in the north-south or east-west (L-shell alignment) direction depending on the ambient electric field magnitude and direction. For wavenumbers $(k_x, k_y)$ in Fourier space corresponding to the east-west and north-south directions, respectively, the one-dimensional spatial power spectra on the irregularities in the east-west direction $P(k_x) \propto k_x^{-n_x}$ with $n_x \approx 2.5$ for $2\pi/k_x$ between 100 km and 3 km while in the north-south direction $P(k_y) \propto k_y^{-n_y}$ with $n_y \approx 2$ for $2\pi/k_y$ between 256 km and 3 km.
CONTENTS

1. INTRODUCTION .................................................. 1

2. EQUATIONS OF MOTION AND LINEAR THEORY ................ 3

3. NUMERICAL SIMULATIONS ........................................ 7

4. RESULTS .......................................................... 9

5. SUMMARY AND DISCUSSION ..................................... 12

ACKNOWLEDGEMENTS ............................................... 19

REFERENCES ..................................................... 20
1. INTRODUCTION

Recently, large scale equatorward convecting plasma enhancements in the diffuse auroral F-region ionosphere have been identified and studied [Vickrey et al., 1980] using both radar and satellite measurements. Observed in regions of diffuse auroral particle precipitation and associated field aligned currents, these enhancements have overall latitudinal dimensions of a few hundred kilometers, contain relatively steep poleward and equatorward edges, and have been shown to be approximately field-aligned resembling vertical slabs of ionization. Their occurrence, which is maximized in the evening-midnight sector, is apparently not strongly related to magnetic activity nor to E-region processes. The presence of plasma density irregularities associated with these enhancements has been verified using satellite scintillation studies [Fremouw et al., 1977; Rino et al., 1978; Vickrey et al., 1980]. The scintillation data have indicated that the electron density irregularities are structured like L-shell aligned sheets [Fremouw et al., 1977; Rino et al., 1978]. In addition, Rino and Matthews [1979] have shown that the scintillation enhancements resulting from these irregularities cannot be explained in terms of a geometrical enhancement alone. A purely geometrical enhancement occurs when the signal propagation path intercepts an axis transverse to the magnetic field along which axis the irregularities have a high degree of spatial coherence. Moreover, the source region of these scintillation causing irregularities has been demonstrated to be latitude limited [Rino and Owen, 1980] and contained in a vertical slab of F region plasma.

Since these ionization enhancements are observed to convect equatorward, their poleward edges could be unstable to the E x B gradient drift instability [Simon, 1963; Linson and Workman, 1970] as observed in artificial ionospheric plasma clouds. Indeed, for observed [Vickrey et al., 1980] poleward
density gradient scale lengths of $L = 10$–50 km and convection velocities of approximately 200 m/sec ($E_o = 10$ mV/m) reasonable growth rates for the $E \times B$ gradient drift instability can be expected since $\gamma^{-1} = (BL/cE_o) \approx 50$–250 sec where $\gamma$ is the $E \times B$ growth rate, $B$ is the ambient magnetic field and $c$ is the speed of light. Moreover, it has been shown [Ossakow and Chaturvedi, 1979] that by applying the current convective instability [Lehnert, 1958; Kadomtsev and Nedospasov, 1960] the $E \times B$ stable equatorward side of the plasma enhancements can be driven unstable by the ambient field aligned particle precipitation currents in conjunction with the equatorward density gradients. Other mechanisms that might account for these irregularities are structured low energy particle precipitation and irregular field aligned currents. Keskinen et al. [1980] showed that the nonlinear state of the irregularities in the equatorward edges of these plasma enhancements could be characterized by poleward convecting plasma depletions and equatorward-moving enhancements. In addition, it was demonstrated that these irregularities could be characterized by inverse power laws in the nonlinear regime. However, these studies addressed only the linear and nonlinear evolution of the equatorward side of the plasma enhancements and did not include the $E \times B$ unstable poleward edge.

In this report we study the stability and nonlinear evolution of "two sided" models of plasma enhancements initially latitudinally confined in order to provide a more realistic picture of the evolution of ionization enhancements in the auroral F region ionosphere. In Section 2 we present a linear stability analysis of the plasma fluid equations which describe the evolution of plasma enhancements in the auroral F region ionosphere. The effects of ambient auroral electric fields of arbitrary magnitude and direction are included. In Section 3 we describe the numerical methods used to solve these equations. The results of these simulations are presented in Section 4 and a discussion of these results is given in Section 5.
2. EQUATIONS OF MOTION AND LINEAR THEORY

For wavelengths greater than the ion mean free path we use fluid equations to describe the ion and electron plasma. The following geometry is used: the y-axis is in the north-south direction, the x-axis points west, and the z-axis is downward along the magnetic field. In this report we ignore the vertical density gradient which is weaker than the horizontal plasma density gradients [Vickrey et al., 1980] in the typical diffuse auroral plasma enhancements. The ion and electron fluids then obey the following equations [Chaturvedi and Ossakow, 1979]:

\[ \frac{\partial n}{\partial t} + \nabla \cdot (n \mathbf{v}) = 0 \]  
(1)

\[ \frac{\partial n}{\partial t} + \nabla \cdot (n \mathbf{v}_i) = 0 \]  
(2)

\[ \mathbf{v}_e = \frac{c_T e}{B} \frac{V_{in}^2 \mathbf{z}}{n} + \frac{c_E x^2}{B} \frac{V_{in}^2 \mathbf{z}}{n} - \frac{v_{ei} c_s^2}{\eta e_i} \frac{V_{in} n}{n} - \frac{e E_z}{m_v e_i} \]

\[ - \left( \frac{T_e}{m_v e_i} + \frac{c_s^2}{\nu_{in}} \right) \frac{1}{n} \frac{\partial n}{\partial z} \mathbf{z} + V_o \mathbf{z} \]  
(3)

\[ \mathbf{v}_i = \frac{c_E x^2}{B} + \frac{V_{in} c_E^2}{B} - \frac{c_T i^4}{e B} \frac{V_{in}^2 \mathbf{z}}{n} - \frac{v_{in} c_T i^4 \mathbf{z}}{n} \]

\[ - \frac{v_{ei} c_s^2}{\eta e_i} \frac{V_{in} n}{n} - \frac{c_s^2}{\nu_{in}} \frac{1}{n} \frac{\partial n}{\partial z} \mathbf{z} + V_o \mathbf{z} \]  
(4)

\[ V \cdot \mathbf{J} = 0 \]  
(5)

Here \( n_\alpha (\alpha = i \text{ or } e) \) is the species density and \( \mathbf{E} \) is the total electric field. Since we will be interested in low frequencies and long wavelengths, we have ignored inertial terms in the electron and ion momentum equations.
(3) and (4). Equation (5) results from the assumption of quasineutral fluctuations \( n_e \approx n_i \equiv n \). In addition, \( v_o \) and \( V_o \) refer to the electron and ion velocities along the magnetic field giving rise to the diffuse auroral current. The symbol \( v_{in} \) denotes the ion-neutral collision frequency, \( v_{ei} \) the electron-collision frequency, \( c \) the speed of light, \( T_e \approx T_i \equiv T \) the species temperature, \( c_s \) the ion acoustic speed and \( \Omega_i(\Omega_e) \) the ion (electron) gyro-frequency. We have neglected \( v_{en} \) compared with \( v_{ei} \) and taken \( v_\alpha/\Omega_\alpha \ll 1 \) for \( \alpha = i,e \) (F region approximation).

Any two of equations (1), (2), and (5) provide a complete description of the problem. We will use the ion continuity equation (1) and (5). After separating the total electric field into an ambient and fluctuating part \( E_\perp = E_o - V_\perp \delta \psi \) and transforming to a frame drifting with velocity \( V_o = -(c/B) [\hat{\bar{\omega}} \times E_o - (v_{in}/\Omega_1)E_o] \) we can write

\[
\frac{\partial n}{\partial t} + \frac{c}{B} \left[ \hat{\bar{\omega}} \times \left( V_\perp \delta \psi \right) n - \left( v_{in}/\Omega_1 \right) V_\perp \delta \psi \cdot V_\perp n \right] =
\]

\[
\left( \frac{v_{in}}{\Omega_1} \frac{cT_i}{eB} + \frac{v_{ei} c_s^2}{\Omega_e \Omega_1} \right) \frac{c_s^2}{\frac{\partial n}{\partial z^2}} + \frac{c_s^2}{v_{in} \frac{\partial n}{\partial z^2}}
\]  

(6)

\[
V_\perp \cdot (nV_\perp \delta \psi) + \frac{\Omega_i (\Omega_e \delta \psi)}{v_{in} \frac{\partial n}{\partial z}} \left( n \frac{\partial \delta \psi}{\partial z} \right) = \left( E_o - \frac{\Omega_i B}{v_i c} V_\perp \right) \cdot Vn
\]

\[- \frac{T_e}{\bar{\omega}} \left( n V_\perp \frac{\partial n}{\partial z} - \frac{\Omega_i \Omega_e}{v_{in} v_{ei}} \frac{\partial^2 n}{\partial z^2} \right) \]  

(7)

where \( V_d = \hat{\bar{\omega}} (v_o - V_o) \). Linearizing (6) and (7) by separating \( n = n_o(y) + \delta n \) with \( \delta n, \delta \psi \propto \exp[i(k_x x + k_y y + k_z z - \omega t)] \), \( \omega = \omega_r + i\gamma \), \( kL \gg 1, L^{-1} = (1/n_o)(\partial n_o/\partial y) \) we find a growth rate \( k || \equiv k_z \).
where we have assumed $k_{\perp} = k_x + k_y = \hat{\kappa} k \cos \alpha + \hat{\gamma} k \sin \alpha$ and $E_0 = E_0 \cos \beta \hat{x} + E_0 \sin \beta \hat{y}$ with $k_{\perp}^2 = k_x^2 + k_y^2$, $k_z > k_{\perp}^2$, $D_\perp = (v_e / \Omega_i \Omega_e) c_s^2$ and $D_\parallel = (c_s^2/v_{in}) \left\{1 + \left[\left(v_{in}/\Omega_i\right)^2/\left(v_e v_{in}/\Omega_i \Omega_e\right) + (k_z^2/k_{\perp}^2)\right]\right\}$. The growth rate $\gamma$ in eq. (8) is maximized for $k$-vectors propagating at angles $\alpha$ satisfying

$$\sin(2\alpha - \beta) = \zeta \sin \alpha$$

(9)

where $\zeta = (k_z/k_{\perp})(\Omega_i/v_{in})(BV_d/cE_o)$ and we have taken $k_z/k_{\perp}$ to be fixed. For typical parameters, $k_z/k_{\perp} \approx 10^{-4}$, $v_{in}/\Omega_i \approx 10^{-4}$, $v_d \approx 60$ m/sec (\parallel = 1 \mu A/m^2 at a density of $n_0 = 10^5$ cm$^{-3}$) and $E_0 = 10$ mV/m ($cE_o/B \approx 200$ m/sec) we find $\zeta \approx 0.3$. To lowest order for fixed $\beta$ we find from (9) the result that $\alpha = \beta/2 + \frac{\zeta}{2} \sin(\beta/2)$. In other words, for $\beta \neq 0$, the linear growth rate maximizes away from the direction perpendicular to the initial density gradient by the angle $\alpha = \beta/2 + \frac{\zeta}{2} \sin(\beta/2)$. As a result the maximum growth rate from (8) can be written to lowest order

$$\gamma = -\cos(\beta/2) \frac{v_e}{\Omega_e} \left[\frac{v_{in}}{\Omega_i} \frac{cE_o}{B} \cos(\beta/2) - \frac{k_z}{k_{\perp} \Omega_e} \frac{v_{in}}{\Omega_i} \frac{cE_o}{B} \cos(\beta/2) - \frac{k_z}{k_{\perp} \Omega_e} \frac{v_{in}}{\Omega_i} \frac{cE_o}{B} \cos(\beta/2) - \frac{k_z}{k_{\perp} \Omega_e} \frac{v_{in}}{\Omega_i} \right] - D_\perp k_{\perp}^2 - D_\parallel k_z^2$$

(10)

For ambient electric fields perpendicular to the initial density gradient ($\beta = 0$), eq. (8) implies that the maximum growth rate occurs for $\alpha = 0$, i.e., $k$ along $E_0 \hat{x}$. This can be written
\[
\gamma = -\frac{\nu_{ei}}{\Omega_e} \left( \frac{cE_{ox}}{B} - \Theta \nu_d \right) \theta^2 + \frac{\nu_{in} \nu_{ei}}{\Omega_i} - \frac{D_k}{k^2} = \frac{1}{\Omega_i} \left( \frac{cE_{ox}}{B} \right) - \frac{D_k}{k^2} \] (11)

where \( \Theta = k_z/k_x \). Note that by comparing eq. (10) and (11) the linear growth rate, for arbitrary \( k \), is reduced when \( E_0 \) is not exactly perpendicular to \( V_{n_0} \), i.e., \( \beta \neq 0 \) because of the \( \cos^2 (\beta/2) \) factor in (10). In regions of the plasma enhancements where \( \partial n_0/\partial y < 0 \) and for wavelengths whose perpendicular and parallel diffusive damping are negligible we find the condition for unstable growth \( [(\nu_{in}/\Omega_i)(cE_{ox}/B) + \Theta |\nu_d|] > 0 \) where we have taken, for example, the currents to be downward, i.e., \( \nu_d < 0 \). For westward electric fields \( E_{ox} > (B/c)(\Omega_i/\nu_{in})|\Theta| |\nu_d| \), the effects of the field-aligned currents will be to reduce \( (\Theta < 0) \) or enhance \( (\Theta > 0) \) the \( E \times B \) instability growth rate. However, when \( \partial n_0/\partial y > 0 \) the condition for unstable growth becomes \( [(\nu_{in}/\Omega_i)(cE_{ox}/B) + \Theta |\nu_d|] < 0 \) and could be satisfied for large enough currents with \( |\nu_d| > (\nu_{in}/\Omega_i)(cE_{ox}/B|\Theta|) \) if \( \Theta < 0 \). The expression for the growth rate \( \gamma \) in equation (11) can be maximized as a function of \( \Theta = k_z/k_x \), a measure of field alignment, using \( \partial \gamma / \partial \Theta |_{\Theta = 0} = 0 \) giving

\[
\Theta_m = \frac{\nu_{in} cE_{ox}}{\Omega_i B V_d} \pm \left[ \left( \frac{cE_{ox}}{B V_d} \right)^2 \left( \frac{\nu_{in}}{\Omega_i} \right)^2 + \left( \frac{\nu_{ei} \nu_{in}}{\Omega_i} \right) \right]^{1/2}
\] (12)

Using typical diffuse auroral F region parameters \( \nu_{in}/\Omega_i = 10^{-6} \), \( \nu_{ei}/\Omega_e = 10^{-4} \)

\( E_{ox} = 10 \text{ mV/m}, j_{||} = n_0 e V_d \approx 1 \mu \text{A/m}^2, B = 0.5 \text{G}, n_o \approx 10^5 \text{cm}^{-3} \) this gives \( |\Theta_m| \approx 10^{-4} \), i.e., approximate field alignment. Inserting these parameters into eq. (11) with \( L \approx 20 \text{ km}, D_\perp \approx 0.2 \text{ m}^2/\text{sec} \) and \( D_\parallel \approx 10^8 \text{m}^2/\text{sec} \) we find that the fastest growing linear modes have growth times \( \gamma^{-1}_{\text{max}} \approx 10^2 \text{ sec.} \)
3. NUMERICAL SIMULATIONS

Equations (6) and (7) can be written in dimensionless form by introducing the following scaled quantities \( \tilde{n} = n_0/N_0, \tilde{\varphi} = \delta \varphi / \delta L, \tilde{x} = x/L, \tilde{y} = y/L, \tilde{z} = z/L, \tilde{t} = ct/L \) as follows (where we have dropped the tilde for clarity)

\[
\begin{align*}
\frac{\partial \tilde{n}}{\partial \tilde{t}} + \frac{\partial \tilde{\varphi}}{\partial \tilde{x}} \frac{\partial \tilde{n}}{\partial \tilde{y}} - \frac{\partial \tilde{n}}{\partial \tilde{y}} \frac{\partial \tilde{n}}{\partial \tilde{x}} &- c_1 \left( \frac{\partial ^2 \tilde{n}}{\partial \tilde{x}^2} + \frac{\partial ^2 \tilde{n}}{\partial \tilde{y}^2} \right) = c_2 \left( \frac{\partial ^2 \tilde{n}}{\partial \tilde{x}^2} + \frac{\partial ^2 \tilde{n}}{\partial \tilde{y}^2} \right) + c_3 \frac{\partial ^2 \tilde{n}}{\partial \tilde{z}^2} \\
\frac{\partial ^2 \tilde{\varphi}}{\partial \tilde{x}^2} + \frac{\partial ^2 \tilde{\varphi}}{\partial \tilde{y}^2} + \frac{1}{n} \left( \frac{\partial \tilde{n}}{\partial \tilde{y}} \frac{\partial \tilde{\varphi}}{\partial \tilde{x}} + \frac{\partial \tilde{n}}{\partial \tilde{x}} \frac{\partial \tilde{\varphi}}{\partial \tilde{y}} \right) + c_4 \left( \frac{\partial ^2 \tilde{\varphi}}{\partial \tilde{z}^2} + \frac{1}{n} \frac{\partial \tilde{n}}{\partial \tilde{z}} \frac{\partial \tilde{\varphi}}{\partial \tilde{z}} \right) \\
&= c_5 \frac{\partial \tilde{n}}{\partial \tilde{x}} + c_6 \frac{\partial \tilde{n}}{\partial \tilde{y}} - c_7 \frac{\partial \tilde{n}}{\partial \tilde{z}} - c_8 \frac{1}{n} \left( \frac{\partial ^2 \tilde{n}}{\partial \tilde{x}^2} + \frac{\partial ^2 \tilde{n}}{\partial \tilde{y}^2} \right) + c_9 \frac{1}{n} \frac{\partial ^2 \tilde{n}}{\partial \tilde{z}^2}
\end{align*}
\]

with \( c_i, i = 1, \ldots, 9 \) dimensionless constants given by

\[
c_1 = \nu_{in}/\Omega_1, \\
c_2 = (\nu_{in}/\Omega_1)(T_1/eBL) + (\nu_{ei}/\Omega_e)(c_s^2/\Omega_1 c_L), \\
c_3 = c_s^2/\nu_{in} c_L, \\
c_4 = \Omega_e/\nu_{ei}, \\
c_5 = E_{ox}/B, \\
c_6 = E_{oy}/B, \\
c_7 = (\Omega_1/\nu_{i1})(V_d/c), \\
c_8 = T/eBL, \\
c_9 = (\Omega_e/\nu_{ei} c_L) c_8.
\]

In the following numerical simulations we take advantage of the fact that the fastest growing, most dangerous modes from linear theory are almost field-aligned, i.e., \( k_{||}/k_{\perp} \ll 1 \) where \( k_{||}(k_{\perp}) \) is the component of \( k \) parallel (perpendicular) to the magnetic field. These waves are of most interest to us and, as a result, we solve equations (13) and (14) in a plane containing these modes which is nearly perpendicular to the magnetic field while fixing the value of \( k_{||}/k_{\perp} \ll 1 \). A similar approach has been adopted in numerical studies of drift-wave [Lee and Okuda, 1976] and trapped-particle [Matsuda and Okuda, 1976] instabilities in laboratory plasmas. The system of equations (13) and (14) was first transformed to the \( x'y'z' \) coordinate system (as shown in Fig. 1) by a simple rotation about the \( y \)-axis by the angle \( \theta = k_{||}/k_{\perp} \ll 1 \) using
\[ \frac{\partial}{\partial x} = \cos \theta \frac{\partial}{\partial x'} - \sin \theta \frac{\partial}{\partial z'} \]

\[ \frac{\partial}{\partial z} = \sin \theta \frac{\partial}{\partial x'} + \cos \theta \frac{\partial}{\partial z'} \]

\[ \frac{\partial}{\partial y} = \frac{\partial}{\partial y'} \]

where \( \theta \) is the angle for maximum linear growth rate defined by eq. (12) for a definite set of parameters \( v_{in}/U_i, cE_{ox}/BV_d, v_{ei}/U_e \). Since \( \theta \ll 1 \) this transformation can be written \( \partial/\partial x = \partial/\partial x', \partial/\partial z \approx \theta \partial/\partial x', \partial/\partial y = \partial/\partial y' \) with \( \partial/\partial z \approx 0 \). As a consequence the three dimensional problem is reduced to two-dimensions. By solving equations (13) and (14) in the \( x'y'z' \) coordinate system a small but finite \( k_x \) is effectively introduced into the model.

Equations (13) and (14) were then solved numerically on a mesh consisting of 258 grid points in the north-south direction (y-direction) and 102 grid points in the east-west direction (x-direction) with constant grid spacing of 1 km. As a result, the simulation plane, which is taken to be essentially horizontal at an altitude of 350 km in the diffuse auroral F region, has a north-south and east-west extent of 256 and 100 km, respectively. The field aligned currents are taken to be constant in space and time over the grid. The plasma density \( n \) in equation (13) was advanced in time using a multi-dimensional flux-corrected variable timestep leapfrog-trapezoid scheme [Zalesak, 1979] which is second order in time and fourth order in space. At each timestep the self-consistent electrostatic potential \( \delta \phi \) of the plasma enhancement in eq. (14) was determined using a Chebychev iterative method [McDonald, 1980] with a convergence criterion of \( 10^{-4} \). Periodic boundary conditions were imposed in the east-west direction with Neumann boundary conditions (\( \partial/\partial y = 0 \)) in the north-south direction. A slab
approximation is used to model the zero order convecting plasma enhancements in the diffuse auroral ionosphere with the north-south profile given by

\[ n(y') = N_0 \left\{ 1 + 4.5 \left[ \tanh(y'-y_1)/L + \tanh(y_2-y')/L \right] \right\} \left( 1 + \epsilon(x',y') \right) \]

with \( N_0 = 1 \times 10^5 \text{cm}^{-3}, y_1 = 50 \text{ km}, \) and \( y_2 = 125 \text{ km}. \) This gives a maximum plasma enhancement density to background ratio of approximately 10. The initial perturbation \( \epsilon(x',y') \) has a root mean square value of 0.01 and has a radially Gaussian dependence on \( x' \) and \( y'. \) We now drop the prime notation for clarity.

4. RESULTS

In the following we consider the linear and nonlinear evolution of plasma enhancements in the diffuse auroral F region ionosphere in an approximately horizontal plane at 350 km altitude almost perpendicular to the magnetic field. We take the following typical parameters [Vickrey et al., 1980; Schunk and Walker, 1973; Banks and Kockarts, 1973] \( L = 20 \text{ km}, \) \( \nu_{in}/\Gamma_i = 2 \times 10^{-4}, \nu_{ei}/\nu_e = 2 \times 10^{-5}, \) \( E_{ox} = 10 \text{ mV/m} \) \( T_e = T_i = 1000 \text{K} \) and \( J_\parallel = 1 \mu\text{A/m}^2 \)

(which gives a current velocity of \( \nu_d = 60 \text{ m/sec} \) with \( N_0 \approx 1 \times 10^5 \text{cm}^{-3}. \) In addition, we assume that the diffuse auroral particle precipitation current \( J_\parallel \) is downward \( (\nu_d < 0) \) and spatially and temporally uniform over the entire plasma enhancement. In order to find the location and magnitude of the maximum linear growth rates to be expected with this set of parameters we first compute \( \theta_m = k_\parallel/k_x \) as given in eq. (12) with \( \nu_d = -|\nu_d| = -60 \text{ m/sec}. \) This gives two values for \( \theta_m \) which are \( \theta^+ = 1.4 \times 10^{-5} \) and \( \theta^- = -6.5 \times 10^{-4}. \) The first value \( \theta^+ \) gives a maximum linear growth rate \( \gamma_{\text{max}} = 1.1 \times 10^{-2} \text{sec}^{-1} \) on the poleward side \( (\partial n/\partial y < 0) \) with linearly damped perturbations \( \gamma_{\text{max}} = 3 \times 10^{-3} \text{sec}^{-1} \) on the equatorward side \( (\partial n/\partial y > 0) \). The second value \( \theta^- \) gives only a marginally unstable growth rate of \( \gamma_{\text{max}} = 2.1 \times 10^{-4} \text{sec}^{-1} \) on the
equatorward side with damped fluctuations $\gamma_{\text{max}} = 2.3 \times 10^{-4} \text{sec}^{-1}$ on the poleward side. These results agree with the experimental observations [Vickrey et al., 1980] that the largest linear growth rates occur on the poleward side of the convecting plasma enhancements. In this case the effect of the field-aligned currents is to enhance the $E \times B$ gradient-drift instability growth rate on the poleward side. The currents are too weak for the cases studied observationally to give appreciable growth on the equatorward side of the plasma enhancements. We will then consider the evolution of modes satisfying $0^+ = k_x/k^x = 1.4 \times 10^{-5}$.

We consider two models with different initial electric field configurations. Model 1 has $E_{ox} = 10 \text{ mV/m}$, $E_{oy} = 0$ while Model 2 takes $E_{ox} = 8 \text{ mV/m}$, $E_{oy} = 2 \text{ mV/m}$. Figure 2a-2d gives the evolution of the plasma enhancement using Model 1 (no northward electric field). Figure 2a shows the initial configuration which includes the small perturbation. Figure 2b illustrates the linear regime of the simulations and shows unstable growth on the poleward side of the plasma enhancement as predicted by the linear result given by eq. (11). One can note the depletion jetting to the equatorward side of the enhancement in analogy to the initial evolution of the $E \times B$ gradient drift instability in artificial ionospheric plasma clouds [Zabusky et al., 1973; Scannapieco et al., 1976]. Figure 2c gives the structure of the plasma enhancement at $t = 1000 \text{ sec}$ and shows steepened fingers which are beginning to elongate. Finally Figure 2d displays the plasma enhancement at $t = 1600 \text{ sec}$ in the well-developed nonlinear regime. The poleward edges of the principal fingers (striations) have steepened, become quasi-one dimensional and bifurcated. The length scales on Figure 2a-d are distorted with the depletions longer and narrower than is depicted.
Figure 3a-b give sample one-dimensional spatial power spectra at \( t = 1600 \) sec both in the east-west \((P(k_x))\) and north-south \((P(k_y))\) directions respectively for Model 1. These power spectra are defined as follows

\[
P(k_x) = \int dk_y \overline{P}(k_x, k_y)
\]

and

\[
P(k_y) = \int dk_x \overline{P}(k_x, k_y)
\]

where \( \overline{P}(k_x, k_y) = \left( L_x L_y \right)^{-1} \left( \delta n(k_x, k_y) / N_0 \right)^2 \) is the spectral density, \( \delta n = N - N_0 \) with \( N_0 \) the peak plasma enhancement density, and \( L_x L_y \) is the area of the numerical simulation plane. For both cases these power spectra are well-fitted with an inverse power law with spectral index \( n_x \approx 2 - 2.5 \) for \( 2\pi/k_x \) between approximately 100 and 3 km and \( n_y \approx 2 \) for \( 2\pi/k_y \) between 256 and 3 km.

Figure 4a-4d illustrate the evolution of the plasma enhancement using Model 2 with a westward electric field \( E_{ox} = 8 \) mV/m together with a northward component \( E_{oy} = 2 \) mV/m using the same initial conditions as in the previous case \( (E_{oy} = 0) \) at \( t = 0, 550, 1000, \) and \( 1600 \) sec. As in model 1, these electric field components give two values for \( \theta_m \) which are \( \theta^+ = 1.7 \times 10^{-5} \) and \( \theta^- = -5.03 \times 10^{-5} \). Since \( \theta^+ \) gives the largest linear growth rate of \( \gamma_{\text{max}} = 8.0 \times 10^{-3} \) sec\(^{-1} \) on the poleward side, we consider in Model 2 those modes satisfying \( \theta^+ = k_x \left| k_y \right| = 1.7 \times 10^{-5} \). Figure 4a gives the initial configuration which is identical to Fig. 2a. Figure 4b shows the isodensity contours of the plasma enhancement at \( t = 550 \) sec where one can note a westward tilt to the fingers on the unstable poleward side and decreased depletion jetting to the equatorward side in comparison to Fig. 2b. This tilt is reminiscent of ionospheric plasma cloud structuring [Perkins and Doles, 1975] in ambient electric fields which are not initially perpendicular to the initial plasma cloud density gradient. The tilt can be explained, in part, by referring to the discussion following eq. (9) which
states that the linear growth rate maximizes away from the direction perpendicular to the initial plasma enhancement density gradient when \( \mathbf{E} \cdot \nabla n_0 \neq 0 \). The decreased depletion jetting to the equatorward side of the plasma enhancement in Fig. 4b as opposed to Fig. 2b can be resolved by noting that the linear growth rate in Model 2 (\( \beta = 14^\circ \)) is reduced from Model 1 (\( \beta = 0 \)) due to the \( \cos^2 (\beta/2) \) factor in eq. (10). Figure 4c gives the structure of the plasma enhancement at \( t = 1000 \) sec for Model 2 which also appears slightly less developed than in Fig. 2c which contains no northward electric field component. Finally, Fig. 4d details the plasma enhancement at \( t = 1600 \) sec for Model 2. One notes that the eastward edge of the large finger on the westward side of the grid is slightly stepper than the westward edge \( (\mathbf{E}_0 \cdot \nabla n_0 \neq 0) \). This may lead to L-shell aligned east-west kilometer size structures due to secondary \( \mathbf{E} \times \mathbf{B} \) instabilities although the present simulations do not have adequate spatial resolution to develop such an hypothesis. In addition, there is more pronounced bending of the fingers in Fig. 4d in comparison to Fig. 2d and less bifurcation on the poleward tips of the striations. As mentioned previously, these features can be explained, in part, by the small northward electric field components.

Figure 5a-b give the one-dimensional east-west \( P(k_x) \) and north-south \( P(k_y) \) spatial power spectra at \( t = 1600 \) sec for Model 2. The power laws and spectral indices are similar to Model 1.

5. SUMMARY AND DISCUSSION

We have studied, through numerical simulations, the nonlinear evolution of plasma enhancements in the diffuse auroral F region ionosphere. We have
shown that equatorward convecting plasma slabs initially limited in latitudinal extent can be destabilized on the poleward sides by a combination of the effects of convection and field aligned currents. These simulations indicate that this destabilization leads to striation-like structures (elongated in the north-south direction) which can form and cascade from long wavelengths ($\sim 100$ km) to shorter scale sizes ($\sim 1$ km) on the order of an hour. The one-dimensional irregularity spatial power spectra in the east-west direction $P(k_x) \propto k_x^{-n_x}$, $n_x = 2-2.5$, for $2\pi/k_x$ between 100 km and 3 km while in the north-south direction $P(k_y) \propto k_y^{-n_y}$, $n_y = 2$, for $2\pi/k_y$ between 256 km and 3 km.

In this paper we have studied the quasi-two dimensional linear and nonlinear evolution of models of plasma enhancements in the diffuse auroral $F$ region ionosphere. This has been accomplished by solving the plasma fluid equations in a horizontal plane approximately perpendicular to the magnetic field. The observed plasma enhancements are three dimensional [Vickrey et al., 1980]. However the horizontal gradients are much steeper than the vertical density gradients allowing one to approximately model the plasma enhancements by vertical slabs. In addition, we have not included a full spectrum of finite $k_\parallel$ modes in these simulations. However, since the modes with maximum linear growth rate have $k_\parallel/k_\perp \ll 1$, the important structuring processes will occur in the plane nearly perpendicular to the magnetic field.

Finally, we note that we have not addressed the source mechanism of the plasma enhancements, their coupling to other levels, e.g., $E$-region, the nature of the intermediate wavelength irregularities ($\lambda \sim 1$ km) in the plasma enhancements, and the role of neutral winds. These topics will be discussed in future studies.
Fig. 1 — Coordinate system used in simulations. The $x'y'$ is the simulation plane. The $x', x, x', z$ axes are coplanar.
Fig. 2 — Real space isodensity contour plots of $n(x',y')/N_0$ for model 1 at (a) $t = 0$ sec, (b) $t = 550$ sec, (c) $t = 1000$ sec, (d) $t = 1600$ sec. The y-axis is compressed by a factor of 2.58. The distance between tic marks in the x-direction (y-direction) is 5 km (12.8 km). Eight contours are plotted in equal increments of 1.25 beginning at 1.25. The observer is looking upward along the magnetic field lines.
Fig. 3 – One dimensional (a) x power spectra $P(k_x)$ and (b) y power spectra $P(k_y)$ at $t = 1600$ sec for model 1. In (a) $k_{F_x} = \frac{2\pi}{100}$ km$^{-1}$ while in (b) $k_{F_y} = \frac{2\pi}{256}$ km$^{-1}$. The dots represent the numerical simulation results; the solid curve is a least squares fit to modes 2-30 in the x-direction and to modes 2-80 in the y-direction. The units of $P(k_x)$, $P(k_y)$ are km.
Fig. 4 — Real space isodensity contour plots of $n(x',y')/N_o$ for model 2 at (a) $t = 0$ sec, (b) $t = 550$ sec, (c) $t = 1000$ sec, (d) $t = 1600$ sec using the same format as Fig. 2.
Fig. 5 — One-dimensional (a) x-power spectra $P(k_x)$ and (b) y-power spectra $P(k_y)$ at $t = 1600$ sec for model 2 in same format as Fig. 3.
ACKNOWLEDGEMENTS

We wish to thank J.F. Vickrey, and C.L. Rino for useful discussions.

This work was supported by the Defense Nuclear Agency and the Office of Naval Research.
REFERENCES


### DISTRIBUTION LIST

**DEPARTMENT OF DEFENSE**

**ASSISTANT SECRETARY OF DEFENSE**
Comm. AID, Cont & Intellig.
Washington, D.C. 20301
- OTCY ATTN J. B. BIRCOCK
- OTCY ATTN M. E. NEITHE

**DIRECTOR**
Command Control Technical Center
Pentagon 4h 2200
Washington, D.C. 20301
- OTCY ATTN C-325
- OTCY ATTN C-232

**DIRECTOR**
Defense Advanced R&D Proj. Agency
1400 Wilson Blvd.
Arlington, Va. 22209
- OTCY ATTN CODE R202
- OTCY ATTN CODE R101
- OTCY ATTN CODE R202
- OTCY ATTN STRATEGIC TECH OFFICE

**DEFENSE COMMUNICATION ENGINEER CENTER**
1600 Wheeler Avenue
Weston, Va. 22920
- OTCY ATTN CODE R202
- OTCY ATTN CODE R101

**DIRECTOR**
Defense Communications Agency
Washington, D.C. 20305
- OTCY ATTN CODE 101B

**DEFENSE TECHNICAL INFORMATION CENTER**
Cameron Station
Alexandria, Va. 22314

12 COPIES (OPEN PUBLICATION, OTHERWISE 2 COPIES)
- OTCY ATTN TC

**DIRECTOR**
Defense Intelligence Agency
Washington, D.C. 20301
- OTCY ATTN 01-08
- OTCY ATTN 02-18
- OTCY ATTN 01-18
- OTCY ATTN 01-19
- OTCY ATTN 01-01

**DIRECTOR**
Defense Nuclear Agency
Washington, D.C. 20305
- OTCY ATTN 01-01
- OTCY ATTN 01-01
- OTCY ATTN 01-01

**COMMANDER**
Field Command
Defense Nuclear Agency
Kirtland AFB, N.M. 87115
- OTCY ATTN FPCR

**DIRECTOR**
InterService Nuclear Weapons School
Kirtland AFB, N.M. 87115
- OTCY ATTN DOCUMENT CONTROL

**JOINT CHIEFS OF STAFF**
Washington, D.C. 20335
- OTCY ATTN C35 EVALUATION OFFICE

**DIRECTOR**
Joint Strategic Planning Staff
DeFuniak AFB
DeFuniak, Fl. 32433
- OTCY ATTN JUP-2
- OTCY ATTN JPD-1

**CHIEF**
LIVERMORE DIVISION FLD COMMAND DNA
Department of Defense
Lawrence Livermore Laboratory
P. 0. Box 808
Livermore, Ca. 94550
- OTCY ATTN FOPML

**DIRECTOR**
National Security Agency
Department of Defense
P. O. Box 20755
Washington, D.C. 20356
- OTCY ATTN C20755

**COMMANDANT**
NATO School (SHAPE)
APO New York 20312
- OTCY ATTN U.S. DOCUMENTS OFFICER

**UNDER SECY OF DEF FOR R&D & ENG**
Department of Defense
Washington, D.C. 20301
- OTCY ATTN STRATEGIC & SPACE SYSTEMS (CS)

**AMC&C**
System Engineering Org
Washington, D.C. 20305
- OTCY ATTN K. CRAWFORD

**DIRECTOR/COMMANDER**
Atmospheric Sciences Laboratory
U.S. Army Electronics Command
White Sands Missile Range, N.M. 88007
- OTCY ATTN JEAJ-240 T. NILES

**DIRECTOR**
JND Advanced Tech CTR
Huntsville, Al. 35807
- OTCY ATTN ATE-19
- OTCY ATTN ATE-19
- OTCY ATTN ATE-19

**PROGRAM MANAGER**
JND Advanced Tech CTR
Huntsville, Al. 35807
- OTCY ATTN DCR-240 T. SHEA

**CHIEF C-5 SERVICES DIVISION**
U.S. Army Communications CMD
Pentagon, N.M. 87115
- OTCY ATTN C-5 SERVICES DIVISION

---

23
PENNSYLVANIA STATE UNIVERSITY
IONOSPHERE RESEARCH LAB
318 ELECTRICAL ENGINEERING EAST
UNIVERSITY PARK, PA 16802
(NO CLASSIFIED TO THIS ADDRESS)
OICY ATTN IONOSPHERIC RESEARCH LAB

PHOTOMETRICS, INC.
442 MARKET ROAD
LEXINGTON, MA 02173
OICY ATTN IRVING L. KOFSKY

TECHNOLOGY INTERNATIONAL CORP
75 WIGGINS AVENUE
BEDFORD, MA 01730
OICY ATTN A. P. BOQUIST

TRW DEFENSE & SPACE SYS GROUP
ONE SPACE PARK
REDONDO BEACH, CA 90278
OICY ATTN R. K. PLEBCH
OICY ATTN S. ALTSMOUSER
OICY ATTN J. DEE

Visidyne
5 Corporate Place
South Bedford St.
Burlington, Mass 01803

SRI INTERNATIONAL
333 KAVENDOHO AVENUE
PESLO PARK, CA 94025
OICY ATTN DONALD NEILSON
OICY ATTN ALAN BURNS
OICY ATTN G. SMITH
OICY ATTN E. L. COBB
OICY ATTN DAVID A. JOHNSON
OICY ATTN WALTER G. CHESTER
OICY ATTN CHARLES L. RIND
OICY ATTN WALTER JAYE
OICY ATTN M. BARON
OICY ATTN RAY L. LEAGABRAND
OICY ATTN G. CARPENTER
OICY ATTN G. PRICE
OICY ATTN J. PETERSON
OICY ATTN R. HANG, JR.
OICY ATTN V. GOMEZ
OICY ATTN D. McDANIEL
Please distribute one copy to each of the following people:

Naval Research Laboratory
Washington, D.C. 20375
Dr. J. M. Kane - Code 4101
Dr. J. A. Land - Code 4141
Dr. J. Estabrook - Code 4187
Dr. J. G. Goodwin - Code 4180

Science Applications, Inc.
1250 Prospect Plaza
La Jolla, California 92037
Dr. D. J. Kimball
Dr. J. L. Linton
Dr. R. Sachs

Director of Space and Environmental Laboratory, NASA
Boulder, Colorado 80302
Dr. A. Glenn Jean
Dr. C. A. Adams
Dr. D. J. Anderson
Dr. K. Davies
Dr. R. F. Donnelly

A. F. Geophysics Laboratory
L. O. Hudson Field
Bedford, Mass. 01730
Dr. T. Akins
Dr. J. Swider
Mrs. R. Raglin
Dr. J. M. Foroke
Dr. J. J. Keneshea
Dr. J. Aarons
Dr. R. Harcisi

Office of Naval Research
Arlington, Virginia 22217
Dr. H. Mulloney

Commander
Naval Electronics Laboratory Center
San Diego, California 92152
Mr. R. Rose - Code 5321

U.S. Army Aberdeen Research and Development Center
Ballistic Research Laboratory
Aberdeen, MD
Dr. J. Heimerl

Commander
Naval Air Systems Command
Department of the Navy
Washington, D.C. 20360
Dr. T. Seebo

Harvard University
Harvard Square
Cambridge, Mass. 02138
Dr. M. B. McDonald
Dr. G. L. Lindzen

Pennsylvania State University
University Park, Pennsylvania 16802
Dr. J. S. Hitch
Dr. R. E. Rohraugh
Dr. L. A. Carpenter
Dr. M. Lee
Dr. A. Gilmore
Dr. P. Bennett
Dr. F. A. Cleaves

University of California, Los Angeles
405 Hilgard Avenue
Los Angeles, California 90024
Dr. H. V. Coroniti
Dr. C. Kennedy

University of California, Berkeley
Berkeley, California 94720
Dr. M. Hudson

Utah State University
4th N. and 8th Streets
Logan, Utah 84322
Dr. P. M. Banks
Dr. R. Harris
Dr. E. Baker

Cornell University
Ithaca, New York 14850
Dr. W. E. Swartz
Dr. R. Cohen
Dr. D. Farley
Dr. W. Kelley

NASA
Goldman Space Flight Center
Greenbelt, Maryland 20771
Dr. S. Chandra
Dr. K. Ramo
Dr. J. I. Benzoni

Princeton University
Princeton, New Jersey 08544
Dr. F. Perkins
Dr. E. Friece

Institute for Defense Analysis
400 Army/Navy Driv
Arlington, Virginia 22202
Dr. E. Bauer

University of Maryland
College Park, MD 20742
Dr. K. Papadopoulos
Dr. E. O. Ott

University of Pittsburgh
Pittsburgh, Pa. 15261
Dr. N. Zabusky
Dr. M. Biondi

Defense Documentation Center
Camden Station
Alexandria, VA 22314
(12 copies if open publication otherwise 2 copies)

12CY Att 1C

University of California
Los Alamos Scientific Laboratory
J-10, MS-466
Los Alamos, New Mexico 87545
M. Pongratz
D. Simons

L. Duncan
S. Peter Cary
Massachusetts Institute of Technology
Plasma Fusion Center
Library, W36-282
Cambridge, MA 21139