PARAMETRIC INVESTIGATION OF RADOME ANALYSIS METHODS:

COMPUTER-AIDED RADOME ANALYSIS USING THE HUYGENS–FRESNEL PRINCIPLE AND LORENTZ RECIPROCITY

By
G. K. Huddleston, H. L. Bassett, & J. M. Newton

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Atlanta, Georgia 30332

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PARAMETRIC INVESTIGATION OF RADOME ANALYSIS
METHODS: COMPUTER-AIDED RADOME ANALYSIS USING THE
HUYGENS-FRESNEL PRINCIPLE AND LORENTZ RECIPROCITY

A Fortran computer program is described for computing the effects of a tangent ogive radome on the receiving patterns and Loresight directions of a monopulse antenna. A receiving formulation with the inside surface of the radome being the surface of integration is used. Aperture integration is used to compute the near fields of the antenna. The main program and seven subroutines are well documented.
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Chapter 1
INTRODUCTION AND SUMMARY

1-1. Introduction

This Volume III of this final technical report of four volumes documents a surface integration radome analysis computer program written in Fortran IV for use on the Cyber 70/74 computing system at Georgia Institute of Technology and the IBM 3033 computing system at Johns Hopkins University Applied Physics Laboratory. The program was developed at Georgia Institute of Technology over the past three years under grant AFOSR-77-3469 and documented herein under the cognizance of R. C. Mallaleiu (APL Contract 60153).

The analysis package described was used during the research to analyze the antennas and radomes as described in Volumes I and IV. Its documentation was done in conjunction with an on-going radome technology program at JHU/APL. It is intended to serve as part of a technology base for the radome technical community.

This report is organized by chapters, where each chapter describes the main program or one subprogram not already described in Reference 1 (Ray Tracing Formulation). The main program (Chapter 2) described herein differs only slightly from that in Reference 1. Only six new subroutines are required for the surface integration formulation as described in Chapters 3-8. References cited in each chapter are listed therein. Each chapter is terminated with the program listing.

This software is currently being used in a parametric investigation of radome analysis methods, and additional information concerning its speed and accuracy is presented in Volume I [2].
1-2. Description of the Analysis

The basis of analysis is illustrated in Figure 1-1. The inner surface of the radome $S_1$ is chosen as one surface of integration in the Lorentz reciprocity integral (upper left in figure). A surface $S_2$ enclosing the antenna and extending into its interior, as illustrated, comprises the second surface. Together, $S_1 + S_2$ enclose the source-free Volume $V$ indicated; hence, the Lorentz surface integral is identically zero and the integral over $S_1$ equals the negative of the integral over $S_2$.

Consider the surface $S_2$ more closely. The surface integral over $S_2$ is zero except over that part of $S_2$ which is placed across the waveguiding structure that connects the Source "a" to the radiating (flared) part of the antenna. Call this surface $S'_2$. If there can be defined a single dominant mode in the waveguide when Source "a" is activated, then voltage and current $V_a$, $I_a$ can be defined at this terminal plane [4]. When Source "b" is activated, voltage and current $V_b$, $I_b$ at $S'_2$ can also be defined; in fact, $V_b$ is the "received voltage". The received current $I_b$ is related to $V_b$ by a linear impedance relationship

$$V_b = I_b Z_a$$

where $Z_a$ is the impedance seen at $S'_2$ looking toward Source "a" (sinusoidal steady state assumed; time variations of the form $e^{j\omega t}$ understood and suppressed). Also, $I_a$ and $V_a$ are related by

$$V_a = I_a Z_1$$
\[ \oint_{s_1+s_2} (E_a \times H_b - E_b \times H_a) \cdot \hat{n} \, ds = 0 \]

\[ \begin{align*}
\int_{s_1} (E_a \times H_b - E_b \times H_a) \cdot \hat{n} \, ds &= -\int_{s_2} (E_a \times H_b - E_b \times H_a) \cdot \hat{n} \, ds \\
&= V_s I_b + V_b I_a
\end{align*} \]

\[ E(x,y,z) = \frac{1}{4\pi} \oint_S \left[ -j\omega \psi \hat{n} \times H \right] \cdot \hat{n} \times \nabla \psi + (\hat{n} \cdot \nabla) \psi \, dS \]

\[ H(x,y,z) = \frac{1}{4\pi} \oint_S \left[ j\omega \hat{n} \times E \right] \cdot \nabla \psi + (\hat{n} \cdot \nabla) \psi \, dS \]

**FIGURE 1-1. THEORETICAL BASIS OF RADOME ANALYSIS.**
where $Z_1$ is the impedance seen at $S'_2$ looking to the right in Figure 1-1. Combining these results yields the desired expression for the received voltage $V_b$; viz.,

$$V_b = V_{REC} = \frac{Z_1Z_a}{V_a(Z_1 + Z_a)} \int_{S_1} (E_a \times H_b - E_b \times H_a) \cdot \hat{n} \, da \quad (3)$$

Note that the unit normal is directed positively outward from volume $V$ as dictated by Gauss' theorem.

When Source "b" in Figure 1-1 is removed a great distance from the antenna/radome structure, the fields of "b" approach those of an electromagnetic plane wave (target return). The practical analysis approach then takes the form shown in Figure 1-2. The inner radome surface is divided into a number of contiguous elemental areas $AA$, each of which is represented by a sample point $P'$ at its center. The fields $E_T$, $H_T$ at $P'$ are assumed to be those present there in the absence of the radome and are found by aperture integration, the theoretical basis of which is the Huygens-Fresnel principle [5] as stated by the lower integrals in Figure 1-1. The fields $E_R$, $H_R$ at $P'$ are found by applying the normal voltage transmission coefficients [6] to the plane wave incident on the outside at point $P$. The received voltage is found by summing all the contributions as indicated in Equation (3).

The method of analysis indicated by Equation (3) is exact; however, certain approximations are necessarily introduced in its implementation. The fields $E_T$, $H_T$ should correctly include reflections from the inner radome surface. The use of the flat panel transmission coefficients to transform the incident plane wave at $P$ to $P'$ is an approximate method based on the theory of geometrical optics (zero wavelength) and whose
FIGURE 1-2. ILLUSTRATION OF RADOME ANALYSIS METHOD USING INSIDE RADOME SURFACE AS SURFACE OF INTEGRATION IN RECIPROCITY INTEGRAL.
accuracy depends upon the radius of curvature of the radome wall. The accuracy of the method also depends on the size of the samples used to represent the radiating aperture as well as the radome surface. The computational speed of the analysis most certainly depends on the number of these samples.

1-3. References


5. Ibid, Ch. 3.

Chapter 2

PROGRAM SIIRACP

2-1. Purpose: SIIRACP is a Fortran computer program used to analyze the effects of a tangent ogive radome on the performance of a monopulse aperture antenna. It consists of a main program and 28 subroutines, 22 of which are identical to those used in Program RTFRACP [1]. It uses complex arithmetic and requires 66600 octal words of core memory for execution on the CDC Cyber 70 system (60-bit words) at Georgia Institute of Technology. Execution time to compute bore-sight error on the Cyber 70 is approximately 255 seconds per look direction when the small antenna aperture is represented by $7 \times 7 = 49$ sample data points and the radome is represented by 826 sample points; i.e., approximately 1.26 millisecond per aperture sample point per radome sample point.

The computer-aided radome analysis uses a receiving formulation based on the Lorentz reciprocity theorem as described earlier [1,2]. The voltage produced at the terminals of a linear antenna by an incident plane wave is given by

$$V_R(k) = \iiint_S \left( \frac{E_T \times H_R - E_R \times H_T}{n} \right) \cdot \hat{n} \, da \quad (1)$$

where $E_T, H_T$ are the fields produced on the surface $S$ enclosing the antenna when the antenna is transmitting; $E_R, H_R$ are the incident fields produced on $S$ by the incident plane wave or perturbations thereof; $k$ is a unit vector which points from the antenna toward the direction from which the plane wave.
wave arrives; and \( \hat{n} \) is a unit vector normal to the surface \( S \) and pointing into the source-free region. The fields \( E_T, H_T \) are taken to be those produced by the planar aperture on surface \( S \) when the antenna is transmitting in the absence of the radome. The geometrical optics approximation

\[
\frac{n \times E_T}{H_T + \frac{n \times E_T}{n}}
\]

(2)

is used to generate the magnetic field in the aperture from the aperture illumination specified by \( E_T \).

The surface \( S \) is taken to be the inner surface of the radome. At each sample point \( P' \) on this surface, the plane wave fields \( E_R, H_R \) incident from the outside are weighted with the flat panel normal voltage transmission coefficients as determined by the radome wall configuration, the angle of incidence, and the plane of incidence. The fields \( E_T, H_T \) at \( P' \) are found by aperture integration. The individual contributions are summed up as indicated in Equation (1) and was illustrated in Figure 1-2.

The parameters of the tangent ogive radome are indicated in Figure 2-1. The outside base diameter \( D_{os} \) and fineness ratio \( F_{os} \) determine the outside length according to

\[
F_{os} = \frac{L_{os}}{D_{os}} \tag{3}
\]

A similar relation holds for the inside dimensions; viz.,

\[
F_{is} = \frac{L_{is}}{D_{is}} \tag{4}
\]

*By choosing \( n \) this way, the minus sign in Figure (1-1) is removed.
Figure 2-1. Tangent Ogive Radome Geometry.
The radius of curvature of the outside wall \( R_{os} \) is given by

\[
R_{os} = F_{os} D_{os} / \sin \left( \pi - 2 \tan^{-1} \left( 2F_{os} \right) \right)
\]  

(5)

and the dimension \( B \) is given by

\[
B = R_{os} - D_{os} / 2
\]  

(6)

The placements of a bulkhead (bottom disk) and metal tip (top disk) can be specified by \( Z_{BOT} \) and \( Z_{TOP} \), respectively. The thickness, dielectric constant, and loss tangent of the wall may also be specified for up to \( N=5 \) layers. The radome is assumed to be a body of revolution with uniform wall dimensions independent of location. The dashed cylindrical shape of a diameter \( D \) in Figure 2-1 was used earlier to simulate a laser-induced defect and is not pertinent here.

The subroutine which generates the antenna aperture fields represents three types of antennas: circular or square aperture with tapered (\( \cos x \)) illumination and any one of four polarizations (vertical, horizontal, RHC, LHC); flat plate antenna with tapered illumination and vertical polarization. For either antenna, the fields are computed for one of three selected channels: sum, azimuth difference, elevation difference. Inputs include the number of samples \( N_x, N_y \) and the aperture diameter \( D_{AP} / \lambda \) in wavelengths.

The antenna/radome orientation is specified according to the parameters defined in Figure 2-2. The angle \( \phi_p \) selects the plane of scan of the radome tip with respect to the antenna coordinate system: \( \phi_p = 0^\circ \) selects the azimuth plane; \( \phi_p = 90^\circ \) selects the elevation plane. The angle \( \theta_L \) scans the tip in the selected plane.
Figure 2-2. Coordinate Systems Used in Radome Analysis.
The program computes boresight errors in the azimuth and elevation planes of the antenna. The radome orientation is specified by $\phi_p$ and $\phi_L$. The first target return (plane wave) is made to arrive from the direction

$$k_1 = x_A \sin \phi_{os} + y_A \sin \phi_{os} + z_A \sin \phi_{os} - 2 \sin \phi_{os}$$

where $\phi_{os}$ is the initial specified offset angle; e.g., 2°. The voltage received by each channel is computed and stored. The second return is made to arrive from

$$k_2 = x_A (\sin \phi_{os}) + y_A (\sin \phi_{os}) + z_A \sin \phi_{os} - 2 \sin \phi_{os}$$

and the voltages are again computed. The data from these two points are used to construct a linear tracking model in the two planes, and a direction of arrival $k$ is predicted which will yield null indications in both planes. The process is repeated until a desired error tolerance is satisfied or a maximum number of iterations is exceeded. Upon completion, the output $k$ indicates the direction from which the plane arrives which yields an electrical boresight indication. It is noted that represent the boresight error angles in the azimuth and elevation planes, respectively, then they are related to the direction $k = x_A k_x + y_A k_y + z_A k_z$ by

$$\sin \alpha = -\frac{k_x}{\sqrt{1 - k_x^2}}$$

$$\sin \gamma = -\frac{k_y}{\sqrt{1 - k_x^2}}$$

where

$$k_z = k - k_x, k_y, k_z$$
Options are also provided whereby principal plane patterns as shown in Figure 2-3 and additional outputs around boresight can be computed and printed. These options are useful when preparing software for a new type of antenna and to ensure correct operation whenever curious results are obtained.

2-2. Usage:

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<tr>
<td>DATA APIN/0./ 47</td>
</tr>
<tr>
<td>DATA ZBOTIN/0.00/</td>
</tr>
<tr>
<td>DATA RADIUS/1.0/ 52</td>
</tr>
<tr>
<td>DATA THETAA, PHIA, AGAM3A/0.0, 90.0, 0.0/</td>
</tr>
<tr>
<td>DATA NX, NY, NYE, NXY/4, 4, 1, 512/ 56</td>
</tr>
<tr>
<td>DATA MY/1/, NREC/61</td>
</tr>
<tr>
<td>READ (5,6) TITLE 63</td>
</tr>
<tr>
<td>READ (5,*) GRAF3D, GRAFSA, GRAFTR, GRAFRV, SUPPRS, IPENCD, SQUARE</td>
</tr>
<tr>
<td>READ (5,*) NFINE, NPHI, NTHE, DIAOS, RAIN, RRIN, ZTOPIN, FREQ, OSANG</td>
</tr>
<tr>
<td>READ (5,*) LMAX, DMRAD, IOPT, RAPMAX, VAIRM, IPOL, ICASE, N, IPWR, KMAX, NXE</td>
</tr>
<tr>
<td>READ (5,*) DSTMIN, DSPHIN, NTHMIN, NPHIMIN</td>
</tr>
<tr>
<td>READ (5,*) DIN(I), ER(I), TD(I) (I=1,N)</td>
</tr>
<tr>
<td>READ (5,*) FINR(I) (I=1,NFINE) 124</td>
</tr>
<tr>
<td>READ (5,*) PHI(I) (I=1, NPHI) 127</td>
</tr>
<tr>
<td>READ (5,*) THETA(I) (I=1, NTHE) 129</td>
</tr>
</tbody>
</table>
Figure 2.3 Coordinate System for Far Field Patterns

\( \hat{\epsilon} = \) elevation component
\( \hat{\alpha} = \) azimuth component
2-3. Arguments

a. Inputs. Units of arguments on input are distances in inches, angles in degrees, and frequency in gigahertz, unless otherwise noted. Units of arguments passed to subroutines are centimeters, radians, and gigahertz. An asterisk is used to denote those DATA arguments that do not normally need to be changed by the user.

**APIN** - Height of a cylindrical base section of the tangent ogive radome. It is no longer included in the ray tracing algorithms and should not be changed from its zero value.

**ZBOTIN** - Distance from base of tangent ogive radome to missile bulkhead (Figure 2-1).

**RADIUS** - The radius \( R \) used in the far field factor \( e^{-jkr}/r \) by Subroutine FAR. Do not change.

**THETAA** - Angle \( \theta_a \) between \( z \)-axis and the position vector \( r_a \) to the antenna origin. This angle was used in earlier work to locate the antenna origin in the reference system using spherical coordinates \( (r_a, \theta_a, \phi_a) \). Do not change.

**PHIA** - Angle \( \phi_a \) between the projection of \( z_A \)-axis onto the \( xy \)-plane and the \( x \)-axis. Do not change.

**AGAM3A** - Angle between \( z_A \)-axis and \( z \)-axis in Figure 2-2. Do not change.

**NX, NY** - Integer powers of two equal to the number of sample points in the antenna aperture; e.g., 16, 32, 64, etc. Changing NX and NY necessitates compatible changes in Lines 16-18.
NYE (NXE)
- Integer powers of two which specify the expanded number of sample points desired when computing the transmitting patterns of the antenna by inverse Fourier transforming the aperture fields. Subroutine JOYFFT provides this capability of increased resolution in one or both dimensions. Changes in NXE, NYE necessitate compatible changes in Lines 16, 20, 22 and 23. Note that NXE*NYE<NX*NY and either NXE<NX or NYE<NY.

NXY
- Integer power of two used by Subroutine JOYFFT for dimension of complex working array XYFFT. Note that MX*NX<NXY and MY*NY<NSY. See below for MX and MY.

NREC
- Integer variable equal to the number of points at which to compute the receiving pattern in either principal plane. The received voltage is computed at points $\theta_i$ equally spaced in $\sin \theta$, where $\theta$ is the angle measured from the $z$-axis as indicated in Figure 2-3, where $\sin \theta_i = -K_{\text{MAX}} + (I-1)*2*K_{\text{MAX}}/NREC$, and where $K_{\text{MAX}} = \sin \theta_{\text{max}} < 1.0$.

NS
- Not used. It was originally used by Subroutine RECBS. Do not remove.

MX, MY
- Integer powers of two equal to the magnification factors desired in the $k_x$ (H-plane) and $k_y$ (E-plane) directions, respectively, of the transmitting antenna patterns. Note that the restrictions MX*NY<NXY and MY*NY<NXY must be observed. The data cited
above indicates increased resolutions in the NX direction of MX=16 and no magnification (MY=1) in the NY direction. Consequently, note that NXE=MX*NX=256.

**TITLE** - A Hollerith string of up to 72 characters which describes briefly the analysis being done. A format of 18A4 is specified and should work for machines with word length greater than or equal to 32 bits. The dimension of TITLE (Line 31) should be at least 18.

**GRAF3D** - A logical variable used to control the plotting of the incident fields on the antenna aperture. This feature has been removed from the program, and GRAF3D should always be FALSE.

**GRAFSA** - A logical variable which (if TRUE) controls the plotting of the transmitting power patterns of the antenna as follows: E-plane sum, E-plane difference equation (A_EL), H-plane sum, and H-plane difference azimuth (A_AZ). The radome is absent.

**GRAFTR** - A logical variable which controls the plotting of the amplitude and phase of the antenna aperture fields in the following order:

- $E_{XE}$, $E_{YE}$, $E_{XAE}$, $E_{YAE}$, $E_{XAZ}$, $E_{YAZ}$.

**GRAFRV** - A logical variable which controls the plotting of the receiving patterns of the antenna with radome in the same order as specified under GRAFSA above.
SUPPRS - A logical variable which controls the printing of numerous results. When TRUE, the printing of these numerous results are suppressed. This feature is convenient to aid in debugging new portions of software prior to making production runs.

IPENCD - An integer variable which selects pen and paper for the Calcomp. This variable may be system dependent. For the Cyber 70, IPENCD=00 yields ballpoint pen and 11" wide plain paper; IPENCD=40 yields a heavier ink pen and the same paper.

SQUARE - Logical input variable which selects a square aperture (TRUE) in Subroutine TRECNF.

NFINE - Integer variable equal to the number of fineness ratios to be considered for the tangent ogive radome; e.g., NFINE=1.

NPHI - Integer variable equal to the number of scan planes; e.g., NPHI=2.

NTHE - Integer variable equal to the number of angles in each scan plane at which to compute boresight errors, etc. Note: The program is set up to iterate on fineness ratio, scan plane, and scan angle as outer loop, middle loop, and inner loop, respectively. Therefore, for each of NFINE fineness ratios, the analysis will be done for NTHE scan angles in NPHI different scan planes.

DIAOS - Real variable equal to the outside base diameter (in.) of the radome. See Figure 2-1.
RAIN - Real variable equal to the distance (in.) from
the gimbal point to the antenna aperture.

RRIN - Real variable equal to the distance (in.) from the
gimbal point to the base of the radome.

ZTOPIN - Real variable equal to the distance (in.) from
the base of the radome to the face of a metal tip
on the radome.

FREQ - Real variable equal to the frequency of operation
in gigahertz.

OSANG - Real variable equal to the offset angle in degrees
at which the first target return is to arrive on
the antenna; e.g., OSANG=3.0.

LMAX - Integer variable equal to the maximum number of
iterations allowed by Subroutine RECBS in com-
puting boresight error; e.g., LMAX=5.

DMRAD - Real variable equal to the tolerance in milliradians
allowed on computing boresight error; e.g., DMRAD=0.1.

IOPT - Integer variable which selects the polarization
of the incident plane wave as follows:
1. Linear, elevation component
2. Linear, azimuth component
3. Right hand circular
4. Left hand circular

RAPMAX - Real variable equal to the maximum radius (in.)
of the antenna aperture.

VAIRM - Real variable equal to the maximum amplitude of
sun channel received voltage without radome. Any
real value can be entered for this variable since a subsequent program modification (Lines 145-362) causes VAFRM to be computed automatically.

- **IPOL** - Integer variable which selects the polarization of the antenna when ICASE=1 according to the same code as used above for IOPT.

- **ICASE** - Integer variable which selects the type of antenna aperture for the analysis: ICASE=1 or 2 selects a circular or square aperture with tapered illumination; ICASE=3 selects a flat plate antenna with programmed illumination. See Subroutine TRECNF in Chapter 4.

- **N** - Integer variable equal to the number of layers (up to 5) in the radome wall. For cases where more than 5 layers are required, the dimensional arrays on Line 37 must be changed to NN=N+1.

- **IPWR** - Integer variable which selects the component for which to compute the transmitting power patterns as follows:
  1. Elevation Components
  2. Azimuth Component
  3. Total power

- **KMAX** - Real variable equal to the sine of the maximum angle at which receiving patterns are to be computed.

- **NXE** - Integer variable used by JOYFFT as explained above.

- **DSTHIN, DSPHIN** - Real variables equal to the distance between adjacent sample points on the radome surface in the
longitudinal (θ) and circumferential (τ) directions, respectively. See Chapter 3.

NTHMIN, NPHIMIN - Integer variables equal to the minimum acceptable number of radome sample points in the two directions.

DIN, ER, TD - Subscripted real variables equal to the thickness (in.), dielectric constant (ε_r), and loss tangent (tan δ) of each layer of the radome wall. I=1 corresponds to the first layer and is the layer on exit side of the wall. Layer N is the first layer encountered by the incident plane wave. See Subroutine WALL.

FINR - Subscripted real variable equal to NFINE fineness ratios.

PHI - Subscripted real variable equal to NPHI angles (degrees) which specify the scan planes.

THETA - Subscripted real variable equal to NTHE angles (degrees) which specify the scan angles in the scan plane.

b. Outputs. The parameters of analysis which are computed and outputted by the program depend on whether SUPPRS is true. In what follows, it is assumed that SUPPRS=FALSE so that all possible outputs are obtained. Since many of the original input parameters are printed directly, only those parameters not already explained above will be included below. Additional clarification may be found in Section 2-6.

TABLE - Logical variable which, if TRUE, causes a look-up table to be used in computing transmission coefficients. When SUPPRS=FALSE, an abbreviated table
of transmission coefficients of the radome wall is printed by Subroutine WALL with variables as explained immediately below.

**ANGLE** - Real variable equal to the angle of incidence (degrees) of the plane wave on a plane sheet of infinite extent having the layered configuration specified for the radome wall. The entries in the table are computed at 250 equal increments in \(\sin \theta\), but only every fifth result is printed.

**TPERI,TPARI** - Complex variables equal to the normal voltage transmission coefficients of the sheet for the two cases of \(E_n\) perpendicular to the plane of incidence \((T_\perp)\) and \(E_n\) parallel to the plane of incidence \((T_\parallel)\). In the printed table, the **power** transmission coefficients \(|T_\perp|^2\) and \(|T_\parallel|^2\) are printed; adjacent to each, the phases of \(T_\perp\) and \(T_\parallel\) are also printed.

**RPERI,RPARI** - Complex variables equal to the reflection coefficients \(R_\perp, R_\parallel\) of the plane dielectric sheet. Actually, \(|R_\perp|^2\) and \(|R_\parallel|^2\) are printed, accompanied by the phases of \(R_\perp\) and \(R_\parallel\).

**KXMAX** - Real variable equal to the folding wavenumber associated with sampling the aperture fields according to \(K_{\text{XMAX}} = 1.\sqrt{\pi \Delta x/\lambda}\), where \(x\) is the distance between samples.

**DXWL** - Real variable equal to \(\Delta x/\lambda\).

**KXM,KYM** - Real variables equal to the folding wavenumbers of the principal plane patterns after magnification
for increased resolution. \( KXM = \frac{KXMAX \cdot NXE}{(MX \cdot NX)} \)
and applies to the H-plane.

\( KYM = \frac{KYMAX \cdot NYE}{(MY \cdot NY)} \) and applies to the E-plane.

Usually, the expanded dimension \( NXE \) and magnification factor \( MX \) are selected so that \( KXM = KXMAX \).

Also, \( NYE \) and \( MY \) are usually selected so that \( KYM < KYMAX \).

**MIN, MAX** - Real variables equal to the minimum and maximum values of the amplitude of the complex arrays containing the aperture fields are processed by Subroutine NORMH in preparation for 3D plotting by Subroutine PLT3DH.

**ROS** - Real variable equal to the radius of curvature of the outside shape of the tangent ogive radome.

**BOS** - Real variable equal to the distance \( B \) in inches defined in Figure 2-1.

**FINOS** - Real variable equal to the fineness ratio of the radome as based on the outside dimensions.

**FINIS** - Real variable equal to the fineness ratio of the radome as based on the inside dimensions.

The following variables are printed when the receiving patterns are computed and printed:

**ICUT** - Integer variable which defines the E-plane (ICUT=1) or H-plane (ICUT=2) pattern. See Figure 2-3.

**ICOMP** - Integer variable which defines the field component of the plane wave incident on the receiving antenna: ICOMP=1 for elevation component; ICOMP=2 for azimuth component.
KMAX - Real variable equal to the sine of the maximum angle off broadside for which the received voltage is computed.

NREC - Integer variable (power of 2) equal to the number of points at which the receiving pattern is computed. The pattern is computed at NREC points spaced equally in k<sub>xy</sub> = sinθ according to Δk<sub>xy</sub> = 2 KMAX/NREC.

DK - Real variable equal to 2*KMAX/NREC.

ANGMAX - Real variable equal to sin<sup>-1</sup>(KMAX).

The receiving pattern is computed at NREC points and magnified using Subroutine MAGFFT to 256 points equally spaced in sin θ over the range (-KMAX, KMAX-DK). Three parameters are printed: angle in degrees, amplitude in decibels, and phase in degrees. Only every fourth point in the 256 points is printed. The receiving patterns are printed in the following order:

E-Plane: Σ<sub>EL</sub>, Λ<sub>EL</sub>

H-Plane: Σ<sub>AZ</sub>, Λ<sub>AZ</sub>

Subroutine RECBS computes the boresight error of the antenna as produced by the radome. When SUPPRS=FALSE, the following parameters are printed:

K1, K2 - Real subscripted variables containing the direction cosines (k<sub>x1</sub>, k<sub>y1</sub>, k<sub>z1</sub>) of the last and next to last true directions to the target. One of these variables is equal to K, the subscripted variable containing the direction cosines of the last target return.
AZTM, ELTM - Real variables equal to the boresight error in the H-plane and E-plane associated with the last target return \((k_x, k_y, k_z)\). Expressed in milliradians, these errors are computed according to

\[
AZTM = \sin^{-1}(k_x/\sqrt{1-k_y^2}) \times 1000.
\]

\[
ELTM = \sin^{-1}(k_y/\sqrt{1-k_x^2}) \times 1000.
\]

Let \( \hat{k} = \hat{z}_A^x + \hat{y}_A^y + \hat{z}_A^z \). Then AZTM is the angle between the \(z_A\)-axis and the projection of \( \hat{k} \) onto the \(x_A^A\) (azimuth) plane. ELTM is the angle between the \(z_A\)-axis and the projection of \( \hat{k} \) onto the \(y_A^A\) (elevation) plane.

MESAZ, MESEL - Real variables equal to the monopulse error slopes in the azimuth and elevation channels expressed in units of volts per degree, where the maximum signal received by the sum channel is considered to be one volt.

UAZ, UEL - Real subscripted variables equal to the received tracking functions \( I_{\text{mag}}^{(\Lambda/E)} \) corresponding to the target returns \( K1 \) and \( K2 \) above; e.g., \( UAZ(1) = I_{\text{mag}}^{(\Lambda_A^E/\Lambda_A^E)} \) for \( K1 \).

SMAX - Real variable equal to the maximum amplitude of the received sum channel voltage.
LCTR - Integer variable equal to the number of iterations (target returns) used by Subroutine RECBS to compute boresight error.

Subroutine RECBS also computes and prints six additional monopulse outputs around the apparent boresight direction \( \hat{k}_0 \). The directions \( k \) chosen lie in the plane \( k_x = k_y \) and are spaced one milliradian apart over the range \( \pm 3 \) mrad and centered on the direction \( \hat{k}_0 \). The variables printed are as follows:

**ANG** - Real variable equal to the angle in milliradians between \( \hat{k}_0 \) and \( \hat{k} \).

**VRAZ, VREL** - Real variables equal to \( |A/| \) for the target return from direction \( k \) for the azimuth and elevation channels, respectively.

**DAZ, DEL** - Amplitude and phase (degrees) of the complex voltages received on the \( \Delta_{AZ} \) and \( \Delta_{EL} \) channels, respectively, for target return \( k \).

**SLPAZ, SLPEL** - Average values of the monopulse error slopes (volts/degree) in the azimuth and elevation channels, respectively, obtained by a linear approximation of the tracking functions based on their values at \( \text{ANG} = \pm 3 \) mrad. For example,

\[
\text{SLPAZ} = \frac{\text{VRAZ}(3 \text{ mrad}) - \text{VRAZ}(-3 \text{ mrad})}{(.006*57.3)}
\]

The main program always prints the boresight error in azimuth (BSEAZ) and elevation (BSEEL), and the values printed are identical to AZTM and...
ELTM defined above. Main also computes the gain of the antenna in decibels with the radome in place according to

\[ GAIN = 20 \cdot \log_{10}(\frac{S_{MAX}}{V_{AIRM}}) \]

For other than an "air radome", GAIN is negative and indicates a loss in antenna maximum gain due to radome reflections and ohmic (\(\tan \delta\)) losses. The amplitude of received sum voltage, \(V_{AIRM}\), is always printed as the last item prior to termination of the program.

2-4. Comments and Method

a. Method. The method of analysis has been presented in Section 2.1. Additional details of analysis are presented in the descriptions of each subroutine, especially Subroutine RECM.

b. Supporting Subroutines. Twenty eight supporting subroutines are required by SIIRACP, 22 of which are identical to those used by RTFRACP. The purpose of each one is briefly described below. Those subroutines peculiar to SIIRACP and explained in Chapters 3-8 are denoted by asterisks below.

*(1) TCNF--Computes complex vector aperture electric fields of antenna for all three monopulse channels at \(NX \times NY\) sample points.

(2) ORIENT--Computes matrices \(\text{ROTATE}\) and \(\text{TRANSLate}\) used for coordinate transformations by Subroutines POINT and VECTOR.

(3) POINT--Transforms a point \(P(x_A, y_A, z_A)\) in antenna system to the same point \(P(x_R', y_R', z_R')\) in radome coordinate system, and vice versa.

(4) VECTOR--Transforms a vector from radome to antenna coordinate system, and vice versa.
(5) INCPW--Computes the rectangular electric field components of a plane wave incident from the direction \( \mathbf{k} \) in antenna coordinates. The power density of the plane wave is unity.

* (6) RECM--Computes the voltage received by each channel of the antenna for a plane wave \( \mathbf{E} \text{inc}(x', y', z') \) incident on the radome from the direction \( \mathbf{k} \text{rad}(x, y, z) \).

(7) OGIVEN--Computes the unit inward normal to the tangent ogive radome surface at a specified point.

(8) RXMIT--Computes the transmitted electric fields of the plane wave traveling in direction \(-\mathbf{k}\) and incident on a flat dielectric wall with unit inner normal \( \mathbf{n} \). The unit vectors \( \mathbf{k}, \mathbf{n} \) are used to resolve the incident plane wave into vector components perpendicular and parallel to the plane of incidence, and to determine the angle of incidence.

* (9) WALL--Computes the normal voltage transmission coefficients of flat panel model of the radome wall as function of the sine of the incidence angle.

(10) AXB--Computes real vector cross product \( \mathbf{C} = \mathbf{A} \times \mathbf{B} \).

(11) CAXB--Computes the complex vector cross product \( \mathbf{C} = \mathbf{A} \times \mathbf{B} \), where \( \mathbf{A} \) is complex and \( \mathbf{B} \) is real.

(12) RECBS--Computes boresight errors of antenna enclosed by the radome for the specified orientation, fineness ratio, etc.

(13) RECPTN--Computes receiving patterns of all three channels.

* (14) APINT--Computes the fields of specified planar aperture fields using equivalent currents.

* (15) DIPOLES--Computes the fields of electric and magnetic dipoles located on a planar surface as required by Subroutine APINT.
CAXCB--Computes the complex vector product \( C = A \times B \), where \( A \) and \( B \) are complex.

FAR--Computes the amplitude of the power pattern from the complex plane wave spectra \( A_x(k_x,k_y), A_y(k_x,k_y) \) of an antenna.

AMPHS--Converts a complex number from rectangular to polar form. This subroutine utilizes the intrinsic function ATAN2. The amplitude produced is linear (not decibels), and the phase is in degrees on the range \((-180, 180)\).

DBPV--Converts a real, two-dimensional array from linear to logarithmic values in decibels on the range \(-40, -1 \).

NORMH--Normalizes a two-dimensional real array to values between 0 and 1.

CNPLTH--Plots single dimensional far field patterns on axes patterned after standard pattern recorder paper. CNPLTH calls Subroutine PSI in addition to the usual Calcomp subroutines.

PSI--Used by Subroutine CNPLTH to compute the azimuthal angle \( \phi \).

PLT3DM--Yields three-dimensional plots of the data in the two-dimensional real array FIELD. PLT3DM calls Subroutines PLTT, NORMH as well as the usual Calcomp subroutines.

PLTT--Used by Subroutine PLT3DM to eliminate moving the pen for hidden lines.

FFT4--Computes the Fast Fourier Transform of a one-dimensional complex array having \( 2^{**10} \) elements. Proper operation is machine dependent.
(26) MAGFFT--Provides increased resolution of a sampled function using FFT and Discrete Fourier Transform techniques.

(27) JOYFFT--Provides increased resolution of selected portions of a two-dimensional Fourier transform. JOYFFT calls Subroutines PFTA and PWRTWO.

(28) PWRTWO--Used by Subroutine JOYFFT to ensure that a given integer is a power of 2.

2-5. Program Flow

For the following, refer to the program listing in Section 2-6 and the line numbers shown on the right-hand margin of that listing.

<table>
<thead>
<tr>
<th>Line Nos.</th>
<th>Explanation</th>
</tr>
</thead>
<tbody>
<tr>
<td>1-4</td>
<td>All variables beginning with the letter K in the main program are real.</td>
</tr>
<tr>
<td>5-10</td>
<td>Declare variables and array dimensions. Note equivalence statements in Lines 23-30. The dimension of IRCP in Line 23 is may be computer system dependent. Note in Line 31 that only twenty fineness ratios, scan planes, and scan angles can be accommodated.</td>
</tr>
<tr>
<td>49-50</td>
<td>Label common is used as a convenient means to transmit variables to subroutines not directly called by MAIN. The labels are generated from the names of the subroutines which receive the variables, and each label is terminated with the letter C to denote common, e.g., HEC1 denotes variables common to MAIN and Subroutine HEC1.</td>
</tr>
</tbody>
</table>
Declare namelists for printing data. These namelists are no longer used except for occasional debugging purposes.

Set data in DATA statements as described above in Section 2-3.

Set SMAX and VMAX to unity to prevent division by zero.

Read and write TITLE according to 18A4 format.

Read input data using free-field format.

Compute sine of the offset angle \( \theta_0 \).

Set TABLE=FALSE so that normalizing factor VAIRM can be computed via a call to Subroutines RECM and RXMIT. In the latter, TABLE=FALSE causes \( T_1, T_2 \) to be set unity as in the case of no radome.

Write input data.

Read input data and set VAIRM needlessly.

Comments explaining input variables.

Set NN=N+1= Number of wall layers plus one.

Initialize DINCH= total thickness of radome wall in inches.

Read wall data and compute total thickness.

Compute DIAIN= inside base diameter of the radome in inches.

compute radius of the center element of searchfield arrays corresponding to \((x_A, y_A)\).

Write array dimensional data.

Read fineness ratios, scan planes, and scan angles.
130-133 Compute wavelength in inches and centimeters.
Compute \( \beta = \frac{2\pi}{\lambda'} \) cm.

134 Compute DAPW1= diameter of antenna aperture in wavelengths.

135-148 Convert variables in inches to centimeters for input to subroutines. Some variables are multiply defined to avoid conflicts in labeled common; e.g., ZBOT and ZT. Note that DIACM is the inside diameter of the radome in centimeters.

149-153 Convert angles from degrees to radians using \( \text{RAD} = \frac{\pi}{180} \).

154-160 Compute near fields of three channel monopulse antenna using Subroutine TRECN.

161-168 Set KYMAX=KXMAX, compute magnified folding wavenumbers KXM, KYM, and print results.

169-187 Initialize Calcomp plotter, if required. The commented initialization (Lines 175-185) applies to the IBM 3033 system at JHU/APL.

Note: Lines 186-264 are used to plot the near fields of the antenna and/or the transmitting principal plane power patterns.

190-195 Initialize the maximum values FMXEL, FMXDAZ of the E- and H-plane patterns so that when used initially as input to Subroutine FAP, the resulting pattern will be normalized with respect to its own maximum and FMXEL and FMXDAZ will be set equal to these respective maxima. In subsequent calls to FAP, the resulting patterns will be normalized with respect to FMXEL and FMXDAZ. Hence,
the relative gain of the difference and sum patterns will be correctly displayed in the graphs.

191 Iterate for each of three monopulse antenna channels.

192-201 Equate complex arrays EXT, EYT to the selected near field and compute the amplitude $NF$ of EXT.

202 Assume transmitting near fields are to be plotted (GRAPTR=T).

204 Call Subroutine PLT3DH to plot the amplitude of EXT. The inputs XSIZE=6., YSIZE=2.5, HEIGHT=2.5 yield a 3D plot that will fit on a 8½" x 11" report page. The inputs $NF$, $NX$, $NY$ specify the real array to be plotted and its dimensions. The input NMZ=.TRUE. directs the subroutine to normalize $NF$ so that its values be between 0 and 1. The input LDB=.FALSE indicates that the array $NF$ contains linear values rather than logarithmic values (decibels).

205-212 Compute and plot phase of EXT on a scale of -180 degrees to +180 degrees. Note that Line 210 ensures that the real array $NF$ contains these phase values scaled to the required 0 to 1 range.

213-226 Repeat amplitude and phase 3D plots for EYT.

227 Assume GRAFSA=T so that principal plane patterns are plotted.

230 If IP=3, go to Line 254 and plot H-plane patterns; otherwise, plot E-plane patterns.

233 Call Subroutine JOYFFT to calculate the inverse Fourier transform of the $x_A$-component of near field EXT to
produce the plane wave spectrum $X_{EE}$ from which the radiation field can be computed. In the process of computing the transform, provide increased resolution from $NX \times NY$ points to $NYE \times NXE$ points through point $(NXC, NYC)$ in the array $EXT$. In the $k_x$ direction, the plane wave spectrum is magnified by $MY$; it is magnified by $MX$ in the $k_y$ direction. The array $FFTXY$ is a working array.

234 Repeat for $EYT$ to produce the plane wave spectrum $Y_{EE}$ for the $y_A$-component of field.

235 Call Subroutine FAR to calculate the $E$-plane elevation ($IPWR=3$) power pattern $FFSEL$ of the near field at equal samples in $\sin \theta$ over the range $(-KXM, KXM - \Delta K)$. If $FMXEL < 0$ (and it is for $IP=1$), normalize $FFSEL$ with respect to its own maximum.

237 Call Subroutine DBPV and convert the power pattern to decibels on a scale of 0 to -40 dB.

238-241 Scale the values in $FFSEL$ to the range of 0 to 1 for plotting.

242 Call Subroutine CNPLTH and plot the power pattern. If $KXM < 1$, the pattern is plotted over the angular range corresponding to $\sin^{-1}(KXM)$; if $KXM > 1$, the angular range is $(-90^\circ, 90^\circ)$. Subroutine CNPLTH actually plots conical cuts corresponding to $k_x = \text{constant}$ or $k_y = \text{constant}$ as specified by inputs $KXC, KYC$. In the call here, $KXY=KYC=0$ so that a principal pattern is produced.
Write a figure title for the plot and establish a new origin for the next plot.

If IP=2, the E-plane patterns are finished.

Since JOYFFT changes the input arrays EXT,EYT it is necessary to recompute them so that increased resolution can be obtained in the plane wave spectra in the H-plane.

Repeat computation and plotting for H-plane power patterns.

Iterate the radome analysis for NPINE fineness ratios.

Set FINE = outside fineness ratio.

Calculate and write $R_{OS}$, $B$, $F_{OS}$, $F_{IS}$ as defined in Figure 2-1 for the radome geometry.

Compute $RDML$ = distance from the base of the radome to the theoretical tip on the inside of the radome.

If $ZTOPIN < RDML$, the radome has a metal tip, and a message is written to that effect.

Compute parameters needed by Subroutine OGIVE to describe the radome shape. $R$ and $B$ are in centimeters and apply to the inside dimensions. $AP$, the height of the cylinder in centimeters, is not used. $RTSQ$ = square of the radius of the top disk. $RBSQ$ = square of the radius of the bottom disk (bulkhead). The other variables, $BSQ$, $RINV$, $RSQL$, $RP$, and $RP2$, are precalculated here to speed later computations in OGIVE.
310 Compute conversion factor DFMR for converting milli-radians to degrees.

311-314 Initialize the "last" values of boresight error in azimuth (AZL) and elevation (ELA) and the "last" value THL of scan angle. These variables are used later to compute boresight error slope in degrees per degree from the present and last values of boresight error.

315-316 Write title for analysis results.

317-319 Write parameters of radome wall.

320-322 Write heading for table of boresight error and gain data.

323-334 Write this same data to logical unit 7 for subsequent storage as a disk file, if desired.

335 Iterate the radome analysis for NPHI scan planes.

336-338 Compute $\phi_r$ in radians as required by Subroutine ORIENT.

339 Iterate the analysis for NTHE scan angles in each scan plane.

340-342 Compute $\theta_r$ in radians as required by Subroutine ORIENT.

343 Call Subroutine ORIENT and compute the rotation matrix ROTATE and translation matrix TRANSL required for coordinate transformations using Subroutines POINT and VECTOR.

344 On the first iteration, TABLE is false so that the maximum amplitude of the received voltage on the sum channel is computed without the radome.
345-347 Set the direction cosines of the incident plane wave so that it arrives from the $z_A$ direction.

348 Call Subroutine INCPW and compute the rectangular components PWI of the incident plane wave having polarization specified by IOPT.

349-354 Set TSUP=T and TABLE=F so that an air radome wall be used and so that printing by Subroutine RXMIT and RECM will be suppressed.

355-356 Call Subroutine RECM and compute the complex voltages VR received on the sum, difference elevation, and difference azimuth channels, respectively, corresponding to VR(I), I=1,3.

360 Compute \( V_{AIRM} = |VR(1)| \).

362 Set TABLE=T so that on subsequent iterations VAIRM will not be recomputed, and so that the table of transmission coefficients will be utilized when RXMIT is called.

363 If SUPPRS=F, compute and print the E-plane and H-plane receiving power patterns of the antenna with the radome in place.

366 Iterate in J for E-plane (ICUT=1) and H-plane (ICUT=2) patterns.

368 Set the desired far field component.

369 Set the temporary logical variable TSUP=T so that printing will be suppressed.
370-371 Call Subroutine RECPTN and compute the complex received voltages on each of three channels at NREC points over the range (-KMAX, KMAX - DK).

372-375 Increase the resolution and print results for all three channels. Do not print results that are known to be identically zero.

376-377 Transfer the received voltage into a one-dimensional array VREC.

378 If NREC>NXE, there is no need to increase the resolution.

379 Call Subroutine MAGFFT to increase the resolution of VREC from NREC points to NXE points. The result is contained in complex array XYFFT on output.

380-384 Compute linear power pattern.

385 Select NXX= larger of NXE and NREC.

386 Write heading for printed results from Subroutine NORMH.

388 Call Subroutine NORMH to normalize the NXX values in real array MVREC to be between zero and one. The input argument LDB=.FALSE. since the values are not in decibels.

389 Call Subroutine DBPV to convert the power pattern in MVREC to decibels.

390-391 Write correct heading for E-plane or H-plane.

392 Compute the increment in sin0 at which power pattern has been computed and resolved.

393-404 Scale the power pattern to have values between 0 and 1. If SUPPHS=F, compute the angle 0=ANG and the phase
of the pattern, and print the results for every fourth angle.

405 If GRAFRV=T, plot the receiving power patterns.

406-416 Call Subroutine CNPLTH and plot the receiving patterns in turn. Write an appropriate figure title following each pattern plot. Re-origin the plotter pen for subsequent plots. The result of Lines 330-383 is four principal plane patterns: E-plane sum, E-plane $\Delta_{EL}$, H-plane sum, H-plane $\Delta_{AZ}$.

417-419 Call Subroutine RECBS and compute the boresight errors $AZT$, $ELT$ in the azimuth and elevation planes of the antenna as caused by the radome. On output, the real array $KA$ contains the direction cosines of the last target return and, hence, gives the true direction to the target at the time that the tracking functions in the azimuth and elevation planes indicated the electrical boresight direction.

420 If this is the first iteration in scan angle, do not attempt to compute boresight error slope.

421-422 Compute boresight error slope (degrees/degree) in azimuth and elevation channels.

423-425 Set the "last" values of boresight errors and scan angle to the current values in preparation for next iteration.

426-428 Compute loss in maximum gain of the antenna sum channel due to the radome.
429-430 Write results to logical units 6 and 7.

434-435 Write maximum amplitude of received sum voltage VAIRM without radome.

436 Terminate plotting software.

STOP

END

2-6. Test Case

A test case has been delivered to JHU/APL under separate cover.

Typical input data are shown in Table 2-1.

2-7. References


2-8. Program Listing: See following pages.
THIS RADOME ANALYSIS COMPUTER PROGRAM, SIIRACP, WAS PREPARED FOR JOHN HOPKINS APL BY G.K. HUDDELESTON, JANUARY 1980, UNDER THE COGNIZANCE OF ROBERT C. MALLALIEU.

SUBR TRECNF COMPUTES NEAR FIELDS SUITABLE FOR THIS SURFACE INTEGRATION (INNER RADOME SURFACE) ANALYSIS APPROACH. COMPUTED RESULTS ARE ALSO WRITTEN TO TAPE7 FOR LATER USE.

IMPLEMENTATION AT APL/JHU 2/11/80 FOR IBM 3033

*** LIBRARIES LSIIRAC AND MISCFFT ARE REQUIRED FOR EXECUTION ***

PROGRAM SIIRACP(INPUT,OUTPUT,TAPE5=INPUT,TAPE6=OUTPUT,TAPE7)

IMPLICIT REAL(K)
REAL NF(4,4),MVREC(256),KA(3)
COMPLEX SUMX(4,4),SUMY(4,4),DELX(4,4),DELY(4,4)
COMPLEX DAZX(4,4),DAZY(4,4),EXT(4,4),EYT(4,4)
COMPLEX VR(16),VREC36,3),VREC(32)
REAL FFS(256,1),FFSEL(1,256)
COMPLEX XE(256,1),YE(256,1),XYFFT(512),PWI(3)
COMPLEX XEEL(1,256),YEEL(1,256)
EQUIVALENCE(XE(1,1) ,XEEL(1,1))
EQUIVALENCE(YE(1,1) ,YEEL(1,1))
EQUIVALENCE(FFS(1,1) ,MVREC(1) ,FFSEL(1,1))

LOGICAL GRAF3D,GRAFSA,GRAFTR,GRAFTRV,TABLE,SUPPRS,TSUP,SQUARE
INTEGER IBUF(512)

REAL ROTATE(3,3),TRANS(3),TITLE(18)
REAL FINR(20),PHI(20),THETA(20)
COMMON/RECIC/DSTH,DSPHI,NTHMIN,NPHIMIN,AREA,NPOINTS,ROS,RIS,$ZBOTCM,ZTOPCM,BCM,RR
COMMON/TDISKC/ZTOP,RTSQ
COMMON/TRANSC/DIN(6) ,ER(6),TD(6),TZ,WALTOL,N,NN,D(6),ZB,TK
COMMON/OGIVC/RP,BSQ,AP,RINV,B,RSQ1,RP2
NAMELIST/GEOM/RR,RA,APIN,ZBOTIN,NX, NY,NYE,NXY, MX,MY,NXC,NYC
NAMELIST/KDATA/KXMAX,KYMAX,KXM,KYM
NAMELIST/NEW/LMAX,DMRAD,IOPT,RAPMAX,VAIRM

BOUNDARY VALUES NEEDED BY SUBR TRACE (INCHES, CONVERT TO CM BELOW)

Z1=ZR COORDINATE OF BOTTOM DISK
Z2=ZR COORDINATE OF TOP DISK (Z1, Z2 IN CM)
APIN IS HEIGHT OF CYLINDER IN INCHES, CONVERT TO CM BELOW
DATA APIN/0.0/
ZBOTIN IS ZR COORD OF BOTTOM DISK (BULKHEAD) IN RADOME COORD IN INCHES
DATA ZBOTIN/0.00/
KXMAX,KYMAX ARE OUTPUTS OF NEAR FIELD SUBR

INITIALIZE CONSTANTS
DATA RADIUS/1E0/
DATA THETAA,PHIA,AGAM3A/0.0,90.0,0.0/
DATA PI/3.1415926535898/

DATA NX,NY,NYE,NXY/4,4,1,512/
DATA MY/I, NREC/6/

READ IN DESCRIPTION OF RADOME WALL
SMAX=1.0
VMAX=1.0
READ(5,6)TITLE
WRITE(6,6) TITLE
READ(5,*) GRAF3D,GRAFSA,GRAFTR,GRAFRV,SUPPRS,IPENCD,SQUARE
260 FORMAT(4L6)
READ(5,*) NFINE,NPHI,NTHE,OSANG
SINOS=SIN(OSANG*PI/180.)
TABLE=.FALSE.

TABLE IS SET FALSE SO THAT NORMALIZING FACTOR CAN BE COMPUTED.
WRITE(6,265) GRAF3D,GRAFSA,GRAFTR,GRAFRV,TABLE
265 FORMAT(" GRAF3D","L2," GRAFSA","L2," GRAFTR","L2," GRAFRV","L2, 
" TABLE="L2)
WRITE(6,270) NFINE,NPHI,NTHE,OSANG
270 FORMAT(" NFINE="I5," NPHI="I3," NTHE="I3," OSANG="F5.2/)
READ(5,*) LMAX,DMRAD,IOPT,RAPMAX,VAIRM,IPOL,ICASE,N,IPWR,KMAX,NXE

43
MX=NXE/NX
IF (MX.LT.1) MX=1
READ(5,*) DSTHIN, DSPHIN, NTHMIN, NPHMIN
IF (VAIRM.LE.0.) VAIRM=1.0
C DIAOS=OUTSIDE DIAMETER OF BASE OF TANGENT OSGIVE RADOME
C VAIR=MAXIMUM REC'D VOLTAGE W/O RADOME AT KX=0., KY=0.
C NFINE=NO. OF FINENESS RATIOS
C NPHI=NUMBER OF SCAN PLANE
C NTHE=NUMBER OF ANGLES IN EACH SCAN PLANE
C DIAIN=INSIDE BASE DIAMETER OF RADOME IN INCHES
C ZTOPIN=ZR COORD (IN) OF TOP DISK (METAL TIP)
C FREQ=FREQUENCY IN GHZ
C GRAF3D=.TRUE. GIVES 3D PLOTS OF INCIDENT FIELDS ON APERTURE (DELETED)
C GRAFRV=.TRUE. GIVES SA PLOTS OF RECEIVING PATTERNS (AZ & EL)
C GRAFSA=.TRUE. GIVES SA PLOTS OF TRANSMITTING PATTERN WITHOUT RADOME
C SUPPRS=.TRUE. SUPPRESSES THE PRINTING OF NUMEROUS RESULTS
C RAPMAX=MAX RADIUS OF ANTENNA APERTURE IN INCHES.
C IOPT SELECTS POLARIZATION OF INCIDENT PLANE WAVE:
C =1 ELEV (VERTICAL)
C =2 AZIMUUTH (HORIZONTAL)
C =3 RHC
C =4 LHC
C IPOL SELECTS POLARIZATION OF ANTENNA WHEN ICASE=1:
C = SAME CODE AS FOR IOPT
C ICASE=1 OR 2 FOR CIRC APERTURE, UNIFORM ILLUMINATION
C =3 FOR FLAT PLATE WITH SPECIFIED ILLUM, VERT POL (CASE III)
C N=NUMBER OF LAYERS IN RADOME WALL
C OSANG=ANGLE IN DEG IN 45 PLANE OFF BORESIGHT OF FIRST TARGET RETURN
C USED BY SUBR RECBS IN GETTING INITIAL DATA.
C IPWR=1 FOR POWER IN ELEV COMP OF FAR FIELD PATTERN
C =2 FOR AZIMUTH COMP,=3 FOR TOTAL POWER.
C DSTHIN=SAMPLE DISTANCE (IN.) ON RADOME SURFACE IN THETA DIRECTION
C DSPHIN= -DITTO- PHI DIRECTION
C NTHMIN=MINIMUM NUMBER OF SAMPLES IN THETA DIRECTION
C NPHMIN= -DITTO- PHI DIRECTION
NN=N+1
DINCH=0.
DO 5 I=1,N
READ(5,*), DIN(I), ER(I), TD(I)
5 DINCH = DIN(I) + DINCH
DIAIN = DIAIN - DINCH*2.
NXC = NX/2 + 1
NYC = NY/2 + 1
WRITE(6,4) NX, NY, NXE, NYE, NXY, MX, MY
4 FORMAT("NX, NY, NXE, NYE, NXY, MX, MY:")
C READ FINENESS RATIOS FOR THIS RUN--BASED ON OUTSIDE DIMENSIONS
DO 13 I = 1, NFINE
13 READ(5,*), FINR(I)
C READ ORIENTATIONS FOR THIS RUN (DEGREES)
DO 14 I = 1, NPHI
14 READ(5,*), PHI(I)
DO 15 I = 1, NTHE
15 READ(5,*), THETA(I)
C COMPUTE WAVELENGTH:
WLIN = 29.97925/(FREQ*2.54)
WLCM = WLIN*2.54
BETA = 2.*PI/WLCM
DAPWL = 2.*YAPMAX/WL
C CONVERT TO CENTIMETER AND RADIANS
ZBOT = ZBOTIN*2.54
Z1 = ZBOT
RSQMAX = (2.54*RAPMAX)**2
DIACM = DIAIN*2.54
ZTOP = ZTOPIN*2.54
ZB = ZTOP
Z2 = ZTOP
ZTOPCM = ZTOP
ZBOTCM = ZBOT
RA = RAIN*2.54
RR = RRIN*2.54
DSTH = DSTHIN*2.54
DSPHI = DSPHIN*2.54
RAD = PI/180.0
6 FORMAT(18A4)
THETAA = THETAA*RAD
PHIA = PHIA*RAD
DO 30 IP=1,3
DO 35 I=1,NX
DO 35 J=1,NY
IF (IP.EQ.1) EXT(I,J)=SUMX(I,J)
IF (IP.EQ.1) EYT(I,J)=SUMY(I,J)
IF (IP.EQ.2) EXT(I,J)=DELX(I,J)
IF (IP.EQ.2) EYT(I,J)=DELY(I,J)
IF (IP.EQ.3) EXT(I,J)=DAZX(I,J)
IF (IP.EQ.3) EYT(I,J)=DAZY(I,J)
NF(I,J)=CABS(EXT(I,J))
35 CONTINUE
IF (.NOT.GRAFTR) GO TO 215
C PLOT 3D NEAR FIELDS X-COMPONENTS
CALL PLT3DH(6.,2.5,2.5,NF,NX,NY,.TRUE.,.FALSE.)
C PLOT PHASE ALSO
DO 40 I=1,NX
DO 40 J=1,NY
NF(I,J)=0.
CALL AMPHS(EXT(I,J),RLF,AIF)
NF(I,J)=(AIF+180.)/360.
40 CONTINUE
CALL PLT3DH(6.,2.5,2.5,NF,NX,NY,.FALSE.,.FALSE.)
C PLOT 3D NEAR FIELDS Y-COMPONENTS
DO 45 I=1,NX
DO 45 J=1,NY
NF(I,J)=CABS(EYT(I,J))
45 CONTINUE
CALL PLT3DH(6.,2.5,2.5,NF,NX,NY,.TRUE.,.FALSE.)
C PLOT PHASE ALSO
DO 50 I=1,NX
DO 50 J=1,NY
NF(I,J)=0.
CALL AMPHS(EYT(I,J),RLF,AIF)
NF(I,J)=(AIF+180.)/360.
50 CONTINUE
CALL PLT3DH(6.,2.5,2.5,NF,NX,NY,.FALSE.,.FALSE.)
IF (GRAFSA) GO TO 215
GO TO 30
CONTINUE
IF (IP.EQ.3) GO TO 220
C CALCULATE SUM OF 
C NOTE THAT JOYFFT CHANGES EXT,EYT.
CALL JOYFFT(EXT,NX,NY,MY,MX,NXC,NYC,XEEL,NYE,NXE,XYFFT,NXY,3)
CALL JOYFFT(EYT,NX,NY,MY,MX,NXC,NYC,YEEL,NYE,NXE,XYFFT,NXY,3)
CALL FAR(FFSEL,XEEL,YEEL,NXE,NYE,FREQ,KYM,KXM,RADIUS,IPWR,FMXEL)
C PLOTS OF ELEVATION RESULTS
CALL DBPV(FFSEL,NYE,NXE,1)
DO 216 I=1,NYE
DO 216 J=1,NXE
FFS(I,J)=1.0+FFS(I,J)/40.0
CONTINUE
DO 216
CALL CNPLTH(FFS,NXE,KXM,0.,0.)
CALL SYMBOL(.5,6.5,.140000,.39HFIGURE TRANSMITTING ELEVATION PO
RPWR=FLOAT(IPWR)
CALL NUMBER(999.,999.,14,RPWR,0.,0)
CALL PLOT(8.5,0.,-3.
IF (IP.EQ.2) GO TO 30
C RECOMPUTE SUMX,SUMY FOR JOYFFT:
CALL TRECNF(EXT,NX,NY,1,IPOL,1,DAPWL,DXWL,KXMAX,ICASE,SQUARE)
WRITE(6,219) IPWR
219 FORMAT(" IPOWER OF PATTERN="I2)
CALL TRECNF(EYT,NX,NY,1,IPOL,2,DAPWL,DXWL,KXMAX,ICASE,SQUARE)
CALL JOYFFT(EXT,NX,NY,MY,MX,NXC,NYC,XEEL,NYE,NXE,XYFFT,NXY,3)
CALL JOYFFT(EYT,NX,NY,MY,MX,NXC,NYC,YEEL,NYE,NXE,XYFFT,NXY,3)
CALL FAR(FFS,XE,YE,NXE,NYE,FREQ,KXM,KYM,RADIUS,IPWR,FMXDAZ)
C PLOTS OF AZIMUTH RESULTS
CALL DBPV(FFS,NXE,NYE,1)
DO 10 I=1,NXE,1
DO 10 J=1,NYE
FFS(I,J)=1.0+FFS(I,J)/40.0
CONTINUE
CALL CNPLTH(FFS,NXE,KXM,0.,0.)
C PLOTTING AZIMUTH POWER
CALL SYMBOL(.5,6.5,.140000,.39HFIGURE TRANSMITTING AZIMUTH PO
CALL NUMBER(999.,999.,14,RPWR,0.,0)
CALL PLOT(8.5,0.,-3)
30 CONTINUE
205 CONTINUE
C
DO 100 NG=1,NFINE
FINE=FINR(NG)
C CALCULATE INSIDE FINENESS RATIO
RIN=FINE*DIAOS/(SIN(PI-2.*ATAN(2.*FINE)))
ROS=RIN*2.54
BIN=RIN-DIAOS/2.
FINE=SQRT((RIN-DING)**2-BIN**2)/DIAIN
RDML=FINE*DIAIN+APIN
IF (ZTOPIN.LT.RDML) WRITE(6,25) ZTOPIN
20 FORMAT(" TANGENT OGIVE PARAMETERS: "," ROS(IN)="
$ ,F9.5," BOS(IN)=" ,F9.5,/26X," FINOS=" ,F5.3,
$ " FINIS=" ,F8.5," RIN"=",E12.5) 282
25 FORMAT(/" THIS RADOME HAS A TOP DISK AT ZTOPIN= ",E12.5/) 283
C COMPUTE PARAMETERS NEEDED BY SUBR OGIVE
R=FINE*DIACM/(SIN(PI-2.*ATAN(2.*FINE)))
RIS=R
TLIS=DIAIN*FINE
IF (ZTOPIN.GT.TLIS) ZTOPIN=TLIS
ZTOP=ZTOPIN*2.54
ZB=ZTOP
Z2=ZTOP
ZTOPCM=ZTOP
IF (ZTOPCM.GT.RIS) ZTOPCM=RIS
B=R-DIACM/2.
BCM=B
AP=APIN*2.54
RTSQ=R**2-(ZTOP-AF)**2
IF (RTSQ.LT.0.) RTSQ=0.
RTSQ=(SQRT(RTSQ)-B)**2
RBSQ=R**2-(ZBOT-AP)**2
IF (RBSQ.LT.0.) RBSQ=0.
RBSQ=(SQRT(RBSQ)-B)**2
BSQ=B**2
RINV=1./R
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RSQ1=R**2
RP=RSQ1-BSQ
RP2=RSQ1+BSQ
WRITE(6,20) RIN,BIN,FINR(NG),FIRE,RINV

DPMR=180./((PI*1000.)
AZL=0.
ELL=0.
THL=0.
TLOS=DIAOS*FINR(NG)
WRITE(6,2) TITLE,FINR(NG),DIAOS,TLOS,FREQ,RAIN,RRIN,DAPWL,IPOL,

$ICASE,IOPT
DO 8 I=1,N
8 WRITE(6,7) I,DIN(I),ER(I),TD(I)
7 FORMAT(2X,I3,F13.5,F10.3,F9.4)

WRITE(6,9)
9 FORMAT(//" PHI THETA BSEEL BSEAZ SLPEL SLPAZ GAIN"
$" (DEG) (DEG) (MRAD) (MRAD) (DEG/DEG) (DB)"/
WRITE(7,2) TITLE,FINR(NG),DIAOS,ZTOPIN,FREQ,RAIN,RRIN,DAPWL,IPOL,

$ICASE,IOPT
DO 18 I=1,N
18 WRITE(7,7) I,DIN(I),ER(I),TD(I)
WRITE(7,9)
9 FORMAT(1H1,5X," RESULTS OF RADOME ANALYSIS USING INSIDE SURFACE INTEGRATION"/18A4/
F5.4," DIAMETER=".,F8.5,. IN. LENGTH=".,F8.5,. IN."/" FREQUENCY=".,
F8.5."
GHZ "/
4" RA=".,F8.5,. IN. RR=".,F8.5,. IN. ANTENNA D=".,F8.4,
5" WAVELENGTHS"/" IPOL=".,I2., ICASE=".,I2., IOPT=".,I2/
6" LAYER THICKNESS(IN.) ER TAND")/

DO 100 IPHI=1,NPHI
PHIP=PHI(IPHI)
PHIR=PHIR*PHIR

DO 100 ITHE=1,N
THETAL=THETA(ITHE)
THETAR=180.-THETAL
THETAR=THETAR*RAD
CALL ORIENT(RA,THETA,PHIA,RR,THETA,RH,PHIR,AGAM3A,ROTATE,TRANSL)

IF (TABLE) GO TO 23

C COMPUTE NORMALIZING FACTOR:

   KA(1)=0.
   KA(2)=0.
   KA(3)=1.
   CALL INCW(KA,PWI,IOPT)
   TSUP=SUPPRS
   TABLE=.FALSE.
   ZTEMP=ZTOPCM
   ZTOPCM=DIACM*FINE
   IF (ZTOPCM.GT.RIS) ZTOPCM=RIS
   CALL REC(PWI,KA,NX,NY,KXMAX,KYMAX,FREQ,ROTATE,TRANSL,
   $SUMX,SUMY,DELX,DELY,DAZX,DAZY,VR,TABLE,TSUP,RSQMAX)

C SET ZTOPCM BACK TO THE INPUTTED VALUE.

   IF (ZTOPCM.LT.ZTEMP) ZTEMP=ZTOPCM
   ZTOPCM=ZTEMP
   VAIRM=CABS(VR(1))
   WRITE(6,105) VAIRM
   TABLE=.TRUE.

23 IF (.NOT.SUPPRS) GO TO 24

GO TO 350

24 CONTINUE

DO 320 J=1,2
   ICUT=J
   ICOMP=IOPT
   TSUP=.TRUE.
   CALL REC(SUMX,SUMY,DELX,DELY,DAZX,DAZY,NX,NY,ICUT,ICOMP,KMAX,
   $NREC,VREC3,KXMAX,KYMAX,FREQ,ROTATE,TRANSL,TABLE,TSUP,RSQMAX)

DO 325 MM=1,3
   ICHAN=MM
   IF ((ICUT.EQ.1).AND.(ICHAN.EQ.3)) GO TO 325
   IF ((ICUT.EQ.2).AND.(ICHAN.EQ.2)) GO TO 325
   DO 26 I=1,NREC

26 VREC(I)=VREC3(I,ICHAN)

IF (NREC.GE.NXE) GO TO 31
   CALL MAGFFT(VREC,NREC,XYFFT,NXE)

DO 305 I=1,NXE
MVREC(I) = CABS(XYFFT(I))**2
GO TO 33
31 DO 32 I=1,NREC
32 MVREC(I) = CABS(VREC(I))**2
33 NXX = MAX0(NXX, NREC)
WRITE(6,306)
306 FORMAT(/"MIN AND MAX VALUES OF REC""G PATTERN:")
CALL NORMH(MVREC, NXX, 1, .FALSE.)
CALL DBPV(MVREC, NXX, 1, 1)
IF (J.EQ.1) WRITE(6,308)
IF (J.EQ.2) WRITE(6,309)
DK = 2.*KMAX/NXX
IMOD = 4
IF (NREC.GE.NXE) IMOD = 1
DO 307 I=1,NXX,1
IF (SUPPR) GO TO 307
ANG = ASIN(-KMAX+(I-1)*DK)*180./PI
CALL AMPHS(XYFFT(I), AMP, PH-S)
IF (NREC.GE.NXE) CALL AMPHS(VREC(I), AMP, PH-S)
IF (MOD(I,IMOD).EQ.0) WRITE(6,310) ANG, MVREC(I), PH-S
307 MVREC(I) = 1.04 - MVREC(I)/40.4
308 FORMAT(/"REC""G PATTERN, EL CUT, EL COMP (DB):")
309 FORMAT(/"REC""G PATTERN, AZ CUT, EL COMP (DB):")
310 FORMAT(F9.1,5X,F8.3,3X,F6.1)
IF (.NOT.GRAFVR) GO TO 325
CALL CNPLTH(MVREC, NXX, KMAX, 0., 0.)
IF (J.EQ.1) CALL SYMBOL(.5, 6.5, .140, 43HFIGURE RECVG POWER PA
$TERN-ELEV PLANE,0.,.43)
IF (J.EQ.2) CALL SYMBOL(.5, 6.5, .140, 41HFIGURE RECVG POWER PA
$TERN-AZ PLANE,0.,.41)
CALL PLOT(8.5,0.3)
325 CONTINUE
320 CONTINUE
350 CONTINUE
C COMPUTE BORESIGHT ERROR
275 CONTINUE
CALL RECBS(SUMX, SUMY, DELX, DELY, DAZX, DASY, NX, NY, 276 $LMAX, NS, IOPT, VR, DMRAD, ROTATE, TRANSL, FREQ, KMAX, KMAX, 277 414
d6 52
$ TABLE,SINOS,KA,AZT,ELT,RSQMAX,VMAX,SMAX,SUPPRS

IF (ITHE.EQ.1) GO TO 300
SLPAZ=(AZT-AZL)*DPMR/(THETAL-THL)
SLPEL=(ELT-ELL)*DPMR/(THETAL-THL)
300 AZL=AZT
ELL=ELT
THL=THETAL
GAINM=SMAX/VAIRM
IF (GAINM.LT.1E-2) GAINM=1E-2
GAINM=20.*ALOG10(GAINM)
WRITE(6,11) PHIP,THETAL,ELT,AZT,SLPEL,SLPAZ,GAINM
WRITE(7,11) PHIP,THETAL,ELT,AZT,SLPEL,SLPAZ,GAINM
11 FORMAT(1X,F5.1,F6.1,F8.2,F8.2,F9.4,F10.4,F7.1)
C GRAF3D OPTION HAS BEEN REMOVED.
100 CONTINUE
WRITE(6,105) VAIRM
105 FORMAT(//" RECEIVED SUM VOLTAGE WITHOUT RADOME","E12.5//)
IF (GRAF3D.OR.GRAFSA.OR.GRAFTR.OR.GRAFRV) CALL PLOT(0.,0.,999)
STOP
END
BLOCK DATA

COMMON/TRANSC/DIN(6),ER(6),TD(6),TZ,WALTOL,N,NN,D(6),ZB,TK

DATA WALTOL,T,Z/0.,0.,0.,/  

END
Chapter 3

SUBROUTINE RECM

3-1. Purpose: To compute the complex voltages produced at the terminals of the three channels of a radome enclosed monopulse antenna by a plane wave of specified polarization and direction of arrival.

3-2. Usage: CALL RECM (EINC, KA, NX, NY, KXMAX, FREQ, ROTATE, TRANSL, SUMX, SUMY, DELX, DELY, DAZX, DAZY, VREC, TABLE, SUPPRS, RSQMAX)

COMMON/RECIC/DSTH, DSPHI, NTHMIN, NPHIMIN, AREA, NPOINTS, ROS, RIS, ZBOTCM, ZTOPCM, BCM, RR

3-3. Arguments

EINC - A complex array of three elements containing $E_x$, $E_y$, $E_z$ of the incident plane wave. See Subroutine INCPW.

KA - A real array of three elements containing the direction cosines $k_x$, $k_y$, $k_z$ of the unit vector $k_A$ which points from the antenna origin in the direction from whence the plane wave emanates.

NX, NY - The even integer number of sample points in $x_A$ and $y_A$ directions used to represent the antenna aperture fields.

KXMAX, KYMAX - Real variables which represent the normalized folding wavenumbers corresponding to the sample distances $\Delta x_A$, $\Delta y_A$ according to $\Delta x_A = \lambda / (2 \times KXMAX)$, $\Delta y_A = \lambda / (2 \times KYMAX)$, where $\lambda$ is the free space wavelength.
FREQ - Frequency in gigahertz of the monochromatic plane wave.

ROTATE,TRANS - Real matrices of direction cosines and translation distances used to carry out coordinate transformations of points and vectors from antenna to radome coordinate systems, and vice versa. See Subroutine ORIENT.

SUMX,SUMY - Two dimensional (NX X NY) complex arrays of the x and y vector components of the antenna aperture fields for the sum channel of a three-channel monopulse antenna. The element at I=NX/2+1, J=NY/2+1, corresponds to that at $x_A=0$, $y_A=0$ in the aperture. The general correspondence is given by

$$x_A = x_{\text{max}} + (I-1)*x_A = (I-\text{MIDX})*x_A$$

$$y_A = y_{\text{max}} + (J-1)*y_A = (J-\text{MIDY})*y_A$$

where $x_{\text{max}} = \frac{x_A*NX}{2}$ and $y_{\text{max}} = \frac{y_A*NY}{2}$.

Also see Subroutine TREDNE.

DELX,DELY - Antenna aperture fields for the difference elevation channel.

DAZX,DAZY - Antenna aperture fields for the difference azimuth channel.

VREC - Complex array of three elements which on output contains the complex terminal voltage of the antenna.
for the sum, elevation difference, and azimuth difference channels, respectively.

**TABLE**  - Logical variable required by Subroutine RXMIT: if TRUE, a look-up table is used to calculate the transmission coefficients of the radome wall; if FALSE, these coefficients are calculated exactly for each angle of incidence specified.

**SUPPRS** - Logical variable used to control the printing of results from Subroutine RXMIT: if FALSE, a table of power transmission and reflection coefficients for equal increments in the sine of the incidence angle is printed. The phases of the complex voltage transmission and reflection coefficients of the radome wall are also printed.

**RSQMAX** - Real variable denoting the maximum radius of the antenna aperture such that any point \((x_A^2 + y_A^2) > RSQMAX\) is omitted from the summation procedure used to compute the received voltages \(V_{REC}\).

**DSTH,DSPHI** - Real input variables which specify the sample distance in the \(\theta\) and \(\phi\) directions on the radome surface; e.g., \(\lambda/3\).

**NTHMIN,** **NPHIMIN** - Integer input variables which specify the minimum acceptable number of samples \(N_\theta, N_\phi\) in the two directions; e.g., \(N_{MIN}=4\).

**AREA** - Real output variable equal to the surface area of the radome included in the surface integration.
NPOINTS - Integer output variable equal to the number of sample points on the radome surface.

ROS, RIS - Real input variables equal to the generating radii of the inside and outside surfaces of the tangent ogive radome shape (Figure 3-1).

ZBOTCM, ZTOPCM - Real input variables which specify the ZR coordinates of the bulkhead and opaque tip (if any), respectively (Figure 3-1).

BCM, RR - Real input variables defined in Figure 3-1.

3-4. Comments and Method

a. Subroutines Required: APINT, VECTOR, POINT, RXMIT, CAXB, OGINEN, CAXCB.

b. Method: The voltage \( V_R \) induced at the terminals of a linear antenna by a “received” electromagnetic plane wave \( E_R, H_R \) is given by the Lorentz reciprocity theorem as [1]

\[
V_R(k_A) = C \oint_S (E_T \times H_R - E_R \times H_T) \cdot n \, da
\]  

where \( k_A \) is the unit vector which points in the direction from whence the plane wave emanates and where \( E_T, H_T \) are the electromagnetic fields of the antenna as produced on the closed surface \( S \) which surrounds the antenna when it is transmitting. The unit vector \( n \) is the normal to \( S \) pointing into the region not containing any sources, and \( C \) is a complex constant.

When the inside surface of the radome is chosen as (closed) surface of integration, the source-free volume is that inside the radome, excluding the space occupied by the antenna; hence, \( n \) is equal to the unit inward
normal \(\mathbf{n}_{\text{is}}\) to the inside radome surface. The surface can be divided into elemental areas \(\Delta A_{\text{lm}}\), and the received voltage can be approximated by

\[
V_R(k_A) = C \sum_{m=1}^{M} \left( \frac{E_T \times H_R - E_R \times H_T}{n_{\text{is}}} \cdot \mathbf{n}_{\text{is}} \Delta A_{\text{lm}} \right)
\]  

(2)

where the fields are evaluated at the same points \(P_{\text{lm}}\) on the radome surface. The elemental areas \(\Delta A_{\text{lm}}\) differ, in general, from point to point, and must be included under the summation.

It is assumed that the fields \(E_T, H_T\) on \(S\) with the radome in place are the same as those that would exist in the absence of the radome. They are computed at points \(P'\) from their specified aperture values \(E_{\text{ap}}, H_{\text{ap}}\) via the Huygens-Fresnel principle as explained in Chapters 5 and 6. The received fields \(E_R', H_R\) at \(P'\) are computed by applying the flat panel normal transmission coefficients \(T_{n\|}, T_{n\perp}\) to the incident plane wave \(E_1, H_1 = E_1 \times k_A / n\) at the point \(P\) on the outside surface of the radome that is co-linear with the inside point \(P'\) with respect to the unit normal \(\mathbf{n}_{\text{is}}\).

(See Figure 1-2).

The tangent ogive radome surface is divided into elemental (trapezoidal) areas by sections made in the longitudinal (\(\theta\)) and circumferential (\(\phi\)) directions. In both cases, desired sampling intervals \(\Delta S_\theta, \Delta S_\phi\) (e.g., \(\lambda/3\)) are specified as input data. For the \(\theta\) direction of Figure 3-1, the number of samples \(N_\theta\) is given by

\[
N_\theta = \text{MAX} \left\{ \frac{R(\theta_{\text{TOP}} - \theta_{\text{BOT}})}{\Delta S_\theta}, N_{\text{MIN}} \right\}
\]

(3)

where \(R\) is the generating radius of the ogive surface.
FIGURE 3-1. RADOME GEOMETRY FOR DEFINING ELEMENTAL SURFACE AREA IN // DIRECTION.
\[ R = \frac{L}{\sin(\pi - 2 \tan^{-1}(2L/D))} \]  

and where the other variables are defined in Figure 3-1. (A minimum acceptable number of samples \( N_{\text{MIN}} \) is also specified). The angular limits are given by

\[ \theta_{\text{BOT}} = \sin^{-1} \left( \frac{Z_{\text{BOT}}}{R} \right) \]  

\[ \theta_{\text{TOP}} = \sin^{-1} \left( \frac{Z_{\text{TOP}}}{R} \right) \]

Since \( N_\theta \) is an integer, the sample interval \( \Delta S_\theta \) must be recomputed as

\[ \Delta S_\theta = R \left( \theta_{\text{TOP}} - \theta_{\text{BOT}} \right) / N_\theta = R \Delta \theta \]

For iteration in \( i \), a sample point at the center of an elemental area on the radome surface is specified by

\[ \theta = \theta_{\text{BOT}} + \frac{\Delta \theta}{2} + (i-1) \cdot \Delta \theta \]

and the corresponding \( Z_R \) coordinate is given by

\[ Z_R = R \sin \theta \]

The elemental areas are formed in the circumferential (\( \phi \)) direction as indicated in Figure 3-2. Using \( \Delta S_\phi \) as input data, the number of samples \( N_\phi \) in the \( \phi \) direction is given by
FIGURE 3-2 DEFINITION OF ELEMENTAL SURFACE AREA IN ϕ DIRECTION.
where \( \rho \) is defined in Figure 3-1 and is given by

\[
\rho = \sqrt{R^2 - z^2} - b
\]  

(11)

Since \( N_\phi \) is an integer, the sampling distance in \( \phi \) must be recomputed as

\[
\Delta S'_{\phi} = 2\pi \rho / N_\phi
\]  

(12)

The sample point at the center of an elemental area is specified in \( \phi \) by

\[
\phi = \phi_0 + (J-1)\Delta\phi
\]  

(13)

where \( \Delta\phi = 2\pi / N_\phi \) and where \( \phi_0 \) is a specified initial point in \( \phi \). The area \( \Delta A \) of a surface element specified by \( (\theta, \phi) \) is given by

\[
\Delta A = R(\Delta Z - B\Delta\theta)
\]  

(14)

where

\[
\Delta Z = z_2 - z_1 = R[\sin(\theta + \Delta\theta/2) - \sin(\theta - \Delta\theta/2)]
\]  

(15)
It is deemed advantageous to set $i_o$ above in Equation (13) to some midpoint of the illuminated surface of the radome. This is done by transforming the unit vector $k_A$ to radome coordinates; i.e.,

$$\begin{align*}
k_A &= x_A k_x + y_A k_y + z_A k_z = x_R k_xR + y_R k_yR + z_R k_zR
\end{align*}$$

Equation (16)

The angle $i_o$ follows as

$$\phi = \cos^{-1} \left( \frac{k_{xR}}{\sqrt{k_{xR}^2 + k_{yR}^2}} \right)$$

Equation (17)

The computations in $\phi$ proceed first in the counterclockwise (CCW) direction and then in the clockwise (CW) direction as indicated in Figure J-2(a). For the CCW direction, the coordinates of the sample point are given by $(J=1, N^2/2+1)$

$$\begin{align*}
x_R &= x \cos (\phi_o + (J-1)\Delta \phi) \\
y_R &= x \sin (\phi_o + (J-1)\Delta \phi)
\end{align*}$$

Equations (18) and (19)

For the CW direction, there results

$$\begin{align*}
x_R &= x \cos (\phi_o - J\Delta \phi) \\
y_R &= x \sin (\phi_o - J\Delta \phi)
\end{align*}$$

Equations (20) and (21)

where $J$ is incremented from unity to $N^2/2$. (The $z_R$ coordinate is given by Equation (14)).
For each elemental area specified, two tests may be performed to determine if the contribution of the fields on that surface element should be included in the received voltage. The first test consists of ensuring that the sample point \((x_R, y_R, z_R)\) lies forward of the aperture plane of the antenna; i.e., that \(z_A > 0\). The second test (which may be disabled as deemed appropriate) determines if the surface element is directly illuminated by the incident plane wave. The test is performed by computing the angle of incidence \(\theta\) according to

\[
\cos \theta = \frac{n_{is} \cdot \mathbf{k}_A}{k_A}
\]  

If \(\cos \theta < 0\), the point is illuminated and should certainly be included in the summation indicated in Equation (2); if \(\cos \theta > 0\), the point lies in the shadow region, and, under certain circumstances, may be omitted from the computation to save time. The effect of this omission is not completely understood in all cases.

3-5. Program Flow (Refer to Program Listing below)

<table>
<thead>
<tr>
<th>Line Number(s)</th>
<th>Comments</th>
</tr>
</thead>
<tbody>
<tr>
<td>1-39</td>
<td>Declare variables, initialize constants.</td>
</tr>
<tr>
<td>40-52</td>
<td>Initialize Subroutines DIOLES and APINT; compute and write fields (E_r, H_t) at point ((0, 0, 2D^2/3)) for reference.</td>
</tr>
<tr>
<td>53-55</td>
<td>Initialize Subroutine RXMIT.</td>
</tr>
<tr>
<td>56-67</td>
<td>Compute (\theta_{TOP}, \theta_{BOT}, N_p, AS_p, \ldots).</td>
</tr>
<tr>
<td>68-76</td>
<td>Compute (\phi_0) and initialize summation of (V_{REC}).</td>
</tr>
<tr>
<td>77-85</td>
<td>Iterate in (\theta) on radome surface.</td>
</tr>
</tbody>
</table>
86-90 Compute \( z_{\text{ROS}}', z_{\text{RIS}}', z_{\text{ROS}}', z_{\text{RIS}} \) and ensure that surface element does not lie forward of metal tip or aft of bulkhead.

91-97 Compute \( n_1 \) and \( \Delta \).

98-104 Compute \( N_{\text{CCW}}', N_{\text{CW}}' \), and \( \Delta \).

105-106 Compute \( n_2' \) and \( \Delta \).

107-119 Iterate in \( \text{CCW first, CW second}. \)

120-125 Compute inside coordinates \( x_{\text{RIS}}', y_{\text{RIS}}' \).

126-135 Compute unit inward normal \( n_{\text{IS}}' \) and apply illumination test (disabled).

136-141 Convert coordinates of sample point on surface to antenna coordinates to ensure \( z_A' \geq 0 \).

142-152 Compute phase of incident plane wave at outside point \( (x_{\text{ROS}}', y_{\text{ROS}}', z_{\text{ROS}}') \) with respect to the antenna origin. Adjust phase of the specified incident plane wave and store temporarily as \( h_{\text{RP}}' \).

153-156 Compute antenna coordinates of inside point in wavelengths.

157-163 Compute transmitted plane wave \( E'_R, H'_R \) at inside point.

164-171 Use aperture integration to compute the transmitted fields \( E_{T1}, H_{T1} \) of the antenna at the inside point for each channel of the monopulse antenna.

172-176 Disabled statements pertaining to surface integration using the outside radome surface.

177-179 Form the vector cross products \( \mathbf{E}_R' \times \mathbf{H}_R' \), \( \mathbf{E}_R' \times \mathbf{H}_T' \).
Add contribution to received voltage $V_{Ri}$.

Increment AREA.

Increment NCUS = number of points omitted.

Increment NPOINTS.

If SUPPRS=.FALSE., compute and write total surface area, received voltages, number of points used, and number of points omitted.

3-6. Test Case: None

3-7. References


3-8. Program Listing (See following pages)
SUBROUTINE RECMEINC, KA, NX, NY, KXMAX, KYMAX, FREQ, ROTATE, TRANSL,
$ SUMX, SUMY, DELX, DELY, DAZX, DAZY, VREC, TABLE, SUPPRS, RSQMAX)
C SUBR RECMe COMPUTES THE RECEIVED VOLTAGE OF AN ANTENNA INSIDE A TANGENT
C OGIVE RADOME AS PRODUCED BY A PLANE WAVE INCIDENT FROM THE DIRECTION
C SPECIFIED BY KA. THE INSIDE SURFACE OF THE RADOME IS USED AS THE SURFACE
C OF INTEGRATION IN THE RECIPROCITY INTEGRAL, AND THE NORMAL TRANSMISSION
C COEFFICIENT IS USED TO TRANSFER THE INCIDENT PLANE WAVE FROM THE POINT
C ON THE OUTSIDE SURFACE TO THE POINT P' ON THE INSIDE SURFACE, WHERE
C P AND P' ARE COLINEAR WITH THE NORMAL TO EITHER SURFACE.
C THE CALL TO THIS SUBR IS IDENTICAL TO THE CALL TO SUBR RECMe
C USED IN THE RAY TRACING FORMULATION HOWEVER, ADDITIONAL VARIABLES
C ARE NEEDED BY THIS SUBR AND ARE PASSED FROM MAIN PROGRAM VIA LABEL
C COMMON/RECIC/ AS SHOWN BELOW.
COMPLEX ET(3), HT(3), ERP(3), HRP(3)
COMPLEX SI(3), S2(3), U, C, EINC(3), VREC(3)
COMPLEX SUMX(NX, NY), SUMY(NX, NY), DELX(NX, NY), DELY(NX, NY),
$ DAZX(NX, NY), DAZY(NX, NY)
REAL KXMAX, KYMAX, ROTATE(3, 3), TRANSL(3), LAMBDA, NISA(3)
REAL PIR(3), NIS(3), KR(3), KA(3), PT(3), PISR(3), PO(3), PTWL(3)
LOGICAL TABLE, ATOR, RTOA, SUPPRS, INIT
COMMON/RECIC/ DTH, DSPHI, NTMIN, NPHIMIN, AREA, NPOINTS, ROS, RIS,
C INITIALIZE CONSTANTS IN SUBR DIPOLES:
LAMBDA=29.97925/FREQ
BETA=2.*PI/LAMBDA
INIT=.TRUE.
PTWL(1)=0.
PTWL(2)=0.
PTWL(3)=2.*RSQMAX/LAMBDA
CALL APINT(PTWL,SUMX,SUMY,NX,NY,NXMID,NYHID,DXWL,DYWLET,HT,INIT)
WRITE(6,3) PTWL(3),ET,HT
3 FORMAT(" SUBR DIPOLES INITIALIZED BY SUBR REC")
$30X," HT= ",6E12.5/
$ RTD=180./PI
CALL RXMIT(HRP,ERP,KA,NISA,PT,TABLE,SJPPRS.BETA)
CONTINUE

C DETERMINE ANGLE PHI0 OF CENTER OF ILLUMINATED AREA ON RADOME:
CALL VECTOR(KA,KR,ROTATE)
RAD=KR(1)**2+KR(2)**2
IF (RAD.GT.ZERO) GO TO 16
PHI0=0.
GO TO 31
16 PHI0=ACOS(KR(1)/SQRT(RAD))
31 DO 32 I=1,3
32 VREC(I)=(0.,0.)
C SELECT CIRCLE ON SURFACE OF RADOME AT CONSTANT THETA  
TH=THBOT-DTH/2.  
IF ((NDO.EQ.0).AND(.NOT.SUPPRS)) WRITE(6,33)  
33 FORMAT(3X,"THDEG",4X,"PHIDEG",12X,"PT",23X,"NIR")  
DO 20 I=1,NTH  
TH=TH+DTH  
THD=TH*RTD  
SINTH=SIN (TH)  
PIR(3)=ROS*SINTH  
RHOOS=SQRT(ROS**2-PIR(3)**2)-B  
PISR(3)=RIS*SINTH  
IF ((PISR(3).GT.ZTOPCM).OR.(PISR(3).LT.ZBOTCM)) GO TO 20  
RHOIS=SQRT(RIS**2-PISR(3)**2)-B  
NPHIP=TJPI*RHOIS/DSPHI  
IF(NPHIP.GE.NPHIMIN) GO TO 40  
NPHI=NPHIMIN  
GO TO 50  
40 NPHI=NPHIP  
C DIVIDE THE INNER SURFACE INTO NPHI EQUAL PARTS  
50 DSPHIP=TUPI*RHOIS/NPHI  
NPHI2=NPHI/2  
NPHICW=NPHI2  
NPHICCW=NPHI2  
IF(2.*NPHICCW.LT.NPHI) GO TO 55  
GO TO 60  
55 NPHICCW=NPHICCW+1  
60 DPHI=TUPI/NPHI  
DZ=RIS*(SIN(TH+DTH/2.)-SIN(TH-DTH/2.))  
DA=RIS*(DZ-B*DTH)*DPHI  
110 DO 61 J=1,2  
JMAX=NPHICCW  
IF(J1.EQ.2) JMAX=NPHICW  
C SELECT A POINT ON INNER SURFACE OF RADOME AT CONSTANT PHI  
C AND ITERATE IN J, FIRST CCW, THEN CLOCKWISE.  
PHI=PHIO-DPHI  
IF (J1.EQ.2) PHI=PHIO  
120 DO 62 J=1,JMAX
IF (J1.EQ.2) GO TO 41
PHI=PHI+DPHI
GO TO 42
41 PHI=PHI-DPHI
42 CONTINUE
PHID=PHI*RTD
CPhi=COS(PHI)
SPHI=SIN(PHI)
PISR(1)=RHOIS*CPhi
PISR(2)=RHOIS*SPHI
C THE POINT OF INTEREST ON INSIDE SURFACE HAS RADOME COORD PISR(XR,YR,ZR).
C CALL OGIVEN TO FIND INNER UNIT NORMAL NIS TO RADOME SURFACE
CALL OGIVEN(PISR,NIS)
IF ((NDO.EQ.0).AND.(.NOT.SUPPRS)) WRITE(6,56) THD,PHID,PISR,NIS
56 FORMAT(2(2X,F7.2),6E10.3)
C TEST NOW IF THIS POINT IS ILLUMINATED BY PLANE WAVE
C CUS=NIS(1)*KR(1)+NIS(2)*KR(2)+NIS(3)*KR(3)
C IF CUS IS GREATER THAN ZERO, AREA IS NOT ILLUMINATED
C IF(CUS.GT.0.) GO TO 59
C IF(CUS.LT.0.) GO TO 65
C GO TO 59
C CONVERT INSIDE POINT PISR(XR,YR,ZR) TO ANTENNA COORD PT(XA,YA,ZA): 65 CALL POINT(PISR,PT,RTOA,ROTATE,TRANSL)
C TEST TO INSURE THAT POINT XR,YR,ZR IS ILLUMINATED
C IF ZA>0., POINT IS ILLUMINATED
C IF (PT(3).LT.0.) GO TO 59
C COMPUTE PHASE OF INCIDENT PLANE WAVE AT OUTSIDE POINT:
PISR(1)=RHOOS*CPhi
PISR(2)=RHOOS*SPHI
CALL POINT(PISR,PO,RTOA,ROTATE,TRANSL)
PISR=AMOD(BETA*(KAO(1)*PO(1)+KAO(2)*PO(2)+KAO(3)*PO(3)),TUPI)
U=CMPLX(0.,PHS)
C=CEXP(U)
C ADJUST PHASE OF INCIDENT ELECTRIC FIELD AT OUTSIDE POINT AND STORE AS HRP:
HRP(1)=EINC(1)*C
HRP(2)=EINC(2)*C
HRP(3)=EINC(3)*C
C COMPUTE ANTENNA FIELDS AT INSIDE POINT PISR:

\[ \text{PTWL}(1) = \text{PT}(1)/\text{LAMBDA} \]
\[ \text{PTWL}(2) = \text{PT}(2)/\text{LAMBDA} \]
\[ \text{PTWL}(3) = \text{PT}(3)/\text{LAMBDA} \]

C TRANSMIT INCIDENT PLANE WAVE THRU WALL USING NORMAL XMN COEFS:

CALL VECTOR(NIS,NISA,RTOA,ROAT)

IF ((NDO.EQ.0).AND.((NOT.SUPPRS)) WRITE(6,57) THD,PHID,PT,NISA

57 FORMAT(2(2X,F7.2),6E10.3/)

CALL RXMIT(HRP,ERP,KA,NISA,PT,TABLE,SUPPRS,BETA)

C COMPUTE CORRESPONDING MAGNETIC FIELD ETA:

CALL CAXB(ERP,KA,HRP)

DO 58 ICH=1,3

IF (ICH.EQ.1)

CALL APINT(PTWL,SUMX,SUMY,NX,NY,NXMID,NYMID,DXWL,DYWL,ET,HT,INIT)

IF (ICH.EQ.2)

CALL APINT(PTWL,DELX,DELY,NX,NY,NXMID,NYMID,DXWL,DYWL,ET,HT,INIT)

IF (ICH.EQ.3)

CALL APINT(PTWL,DAZX,DAZY,NX,NY,NXMID,NYMID,DXWL,DYWL,ET,HT,INIT)

C SUBR APINT COMPUTES HT ETA.

C THE NEXT TWO STATEMENTS ARE FOR OUTSIDE SURFACE CASE.

C CALL POYNTIN(E,H,S)

C CALL RXMIT(ETR,HTR,STR,NIS,PISP,TABLE,BETA,E_TRP,HTRP)

C***********i***.

C FORM CONTRIBUTION TO RECEIVED VOLTAGE

CALL CAXCB(ET,HRP,S1)

CALL CAXCB(ERP,HT,S2)

VREC(ICH)=VREC(ICH)-((S1(1)-S2(1))*NIS(1)+(S1(2)-S2(2))*NIS(2)+

58 CONTINUE

AREA=AREA+DA

GO TO 62

59 NCUS=NCUS+1

62 CONTINUE

NPOINTS=NPOINTS+JMAX

61 CONTINUE

20 CONTINUE

NDO=1
IF (SUPPRS) RETURN
PERCENT=100.*(1.-FLOAT(NCUS)/FLOAT(NPOINTS))
WRITE(6,25) AREA,VREC,NPOINTS,PERCENT,NCUS
RETURN
END
Chapter 4

SUBROUTINE TRECNF

4-1. Purpose: To compute near-field aperture distributions for four types of three-channel monopulse antennas: (1) circular aperture with tapered amplitude and uniform phase distributions; (2) flat plate antenna with a programmed amplitude distribution and uniform phase; (3) square aperture with cos x amplitude and uniform phase; (4) single element. Four polarizations can be selected for the circular and square apertures. The flat plate antenna is vertically \( y_A \) polarized only.

4-2. Usage: CALL TRECNF (E, NX, NY, ICHAN, IPOL, IXY, DAPWL, DXWL, KXMAX, ICASE, SQUARE)

4-3. Arguments

- **E** - Complex array of NX by NY elements which, on output, contains the values of the specified (IXY) rectangular component \( x_A \) or \( y_A \) of the electric field distribution over the specified (ICASE) antenna aperture having the specified (IPOL) polarization for the specified (ICHAN) channel of a three-channel monopulse antenna.

- **NX, NY** - Even integer number of points in a rectangular array at which the aperture distribution is computed in the \( x_A \) and \( y_A \) directions, respectively. The point \( I=NX/2 + 1, J=NY/2 + 1 \) corresponds to \( x_A=0, y_A=0 \). For the single element case, \( NX=NY=2 \).
ICHAN - Integer control variable with values 1, 2, or 3 which selects the sun, elevation difference, or azimuth difference channel, respectively.

IPOL - Integer control variable which selects the antenna polarization as follows:
1 - Vertical (\( \gamma_A \)) polarization
2 - Horizontal (\( x_A \))
3 - Right-hand circular
4 - Left-hand circular

IXY - Integer control variable having values 1 or 2 to select the \( x_A \) or \( y_A \) component of aperture electric field.

DAPWL - Diameter, in wavelengths, of the antenna aperture.

DXWL - Spacing, in wavelengths, between samples in aperture in \( x_A \) and \( y_A \) directions (output).

KXMAX - Maximum value of normalized wavenumber corresponding to \( KXMAX = 1/(\lambda \cdot DXWL) \) (output).

ICASE - Integer control variable having values 1 or 2 to specify a circular aperture antenna with uniform amplitude and phase. If ICASE=3, a flat plate antenna having a programmed amplitude distribution (see Table 3-4) with vertical polarization is selected.

SQUARE - Logical input variable; if TRUE, square aperture is used.
4-4. Comments and Method

a. The integers NX, NY must each be equal to each other and even; e.g., NX=NY=16. In addition, when ICASE=3 (flat plate antenna), NX and NY must equal 16. If NX=NY=2, the fields of a single element at xA=xA=0 are specified. If NX=NY=32, only the central 15 x 15 elements are non-zero.

b. The actual shape of the circular aperture, as approximated by a rectangular array of sample points, is shown in Figure 4-1 for the case of NX=NY=16. Row 1 and Column 1 of the array contain null elements. The elements inside and on the boundary of the aperture may contain non-zero values as shown in Table 4-1 for the various cases when ICHAN=1 (sum channel).

Note that specification of DA in Figure 4-1 determines the sample spacings according to

\[ \Delta x_A = \Delta y_A = \frac{D_A \cos \alpha}{(N-2)} \]

where \( \alpha = \tan^{-1}(2/7) \).

The aperture distributions for three monopulse channels are formed by phasing the elements in the four quadrants of the aperture appropriately. The sum channel distribution is formed by assigning equal phases to all elements. The azimuth difference channel is formed by multiplying all elements in Quadrants II and III of the sum distribution by minus one and by zeroing all elements along xA=0. For the elevation difference channel, Quadrants III and IV are negated, and all elements along the line yA=0 are made zero for symmetry reasons.

The phasing chosen models a tracking antenna and provides outputs in two orthogonal channels from which the direction of arrival of a target return can be mathematically determined. Let \( \mathbf{k} \) be a unit vector which
FIGURE 1–1. APPROXIMATION OF CIRCULAR APERTURE BY RECTANGULAR GRID OF SAMPLE POINTS.
points from the antenna origin toward the direction from whence the plane wave (target return) emanates; i.e.,

\[ \mathbf{k} = \hat{x}_A k_x + \hat{y}_A k_y + \hat{z}_A k_z \]  

(2)

Define the tracking functions for this plane wave as

\[ f_i(k_x, k_y) = \frac{\Delta_i(k_x, k_y)}{\Xi(k_x, k_y)} \]  

(3)

where \( \Delta_i \) represents the output of the elevation (\( e \)) or azimuth (\( a \)) difference channel and \( \Xi \) represents the sum channel output. Then for small \( k_x > 0 \), the phase of \( f_a \) is +\( \pi/2 \); for small \( k_x < 0 \), the phase of \( f_a \) is -\( \pi/2 \). Similarly, for small \( k_y > 0 \), arg \( (f_e) = \pi/2 \); for small \( k_y < 0 \), arg \( (f_e) = -\pi/2 \).

Hence, the change in phase by \( \pi \) in either channel represents the boresight direction of the antenna, and tracking is done using the imaginary parts of the tracking functions rather than their real parts.

c. The shape and sampling grid used to model the flat plate antenna are shown in Figure 4-2. In Subroutine TRECNF, the integers \( N_X \) and \( N_Y \) must both equal 16, and only linear polarization (\( \hat{y}_A \)) is applicable to the flat plate antenna (ICASE=3). The phasing of the four quadrants is done as described above to model the three monopulse channels so that tracking can be simulated. Note that specification of \( D \) determines the sample spacing according to

\[ \Delta x_A = \Delta y_A = \frac{D \cos \alpha}{N_X \left( \frac{x}{2} - 2 \right)} \]  

(4)

where \( \alpha = \tan^{-1}(4/6) \).
FIGURE 4-2. GEOMETRY OF FLAT PLATE ANTENNA.
Table 4-1. Values of Non-Zero Elements in Circular Aperture (ICHAN=1, ICASE=1 or 2)

<table>
<thead>
<tr>
<th>IPOL</th>
<th>IXY</th>
<th>Value</th>
<th>Polarization Type</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1</td>
<td>(0 + j0)</td>
<td>Vertical</td>
</tr>
<tr>
<td>1</td>
<td>2</td>
<td>(1 + j0)</td>
<td>&quot;</td>
</tr>
<tr>
<td>2</td>
<td>1</td>
<td>(1 + j0)</td>
<td>Horizontal</td>
</tr>
<tr>
<td>2</td>
<td>2</td>
<td>(0 + j0)</td>
<td>&quot;</td>
</tr>
<tr>
<td>3</td>
<td>1</td>
<td>(0 + j1)</td>
<td>RHC</td>
</tr>
<tr>
<td>3</td>
<td>2</td>
<td>(1 + j0)</td>
<td>&quot;</td>
</tr>
<tr>
<td>4</td>
<td>1</td>
<td>(0 - j1)</td>
<td>LHC</td>
</tr>
<tr>
<td>4</td>
<td>2</td>
<td>(1 + j0)</td>
<td>&quot;</td>
</tr>
</tbody>
</table>
The phase of each sample point in Figure 4-2 for the sum channel is made equal, but the amplitudes are tapered in the $x_A$ and $y_A$ directions as shown in Table 4-2. The amplitude distribution is separable and symmetrical so that

$$E_{yA}(x_A, y_A) = g(x_A)h(y_A) = E_{yA}(-x_A, y_A) = E_{yA}(x_A, -y_A)$$

(5)

It is noted that samples 10, 12, 14, and 16 are actually specified in the program, and samples 9, 11, 13, and 15 are obtained from them by averaging.

d. The square aperture is formed by setting to zero Row 1 and Column 1 of the array of Figure 4-1 for symmetry reasons. The values of field at the other points in the aperture are computed to yield a $\cos x$ amplitude taper in the $x_A$ direction and a uniform amplitude in $y_A$; i.e.,

$$E(x, y) = \cos \frac{\pi x}{2 x_{\text{max}}}$$

(5)

where $x_{\text{max}}$ corresponds to the sample at $I=NX$.

3-5. Program Flow

<table>
<thead>
<tr>
<th>Line Nos.</th>
<th>Comments</th>
</tr>
</thead>
<tbody>
<tr>
<td>19</td>
<td>Assign complex values to CFAC to use in generating vertical, horizontal, RHC, and LHC polarization according to IPOL.</td>
</tr>
<tr>
<td>20-22</td>
<td>Compute the angle $\alpha$ and the upper bound $R_{\text{max}}$ of the radius of the circular aperture.</td>
</tr>
<tr>
<td>23-24</td>
<td>Ensure that IPOL has correct values of 1, 2, 3, or 4.</td>
</tr>
</tbody>
</table>
Table 4-2. Symmetrical Amplitude Distribution for Flat Plate Antenna

<table>
<thead>
<tr>
<th>Sample No.</th>
<th>$x_A$</th>
<th>Amplitude</th>
<th>$y_A$</th>
<th>Amplitude</th>
</tr>
</thead>
<tbody>
<tr>
<td>9</td>
<td>0</td>
<td>1.0280</td>
<td>0</td>
<td>1.0280</td>
</tr>
<tr>
<td>10</td>
<td>$\Delta x$</td>
<td>1.0280</td>
<td>$\Delta y$</td>
<td>1.0280</td>
</tr>
<tr>
<td>11</td>
<td>$2\Delta x$</td>
<td>.9120</td>
<td>$2\Delta y$</td>
<td>.9170</td>
</tr>
<tr>
<td>12</td>
<td>$3\Delta x$</td>
<td>.7959</td>
<td>$3\Delta y$</td>
<td>.8060</td>
</tr>
<tr>
<td>13</td>
<td>$4\Delta x$</td>
<td>.6077</td>
<td>$4\Delta y$</td>
<td>.6155</td>
</tr>
<tr>
<td>14</td>
<td>$5\Delta x$</td>
<td>.4194</td>
<td>$5\Delta y$</td>
<td>.4250</td>
</tr>
<tr>
<td>15</td>
<td>$6\Delta x$</td>
<td>.2097</td>
<td>$6\Delta y$</td>
<td>.2125</td>
</tr>
<tr>
<td>16</td>
<td>$7\Delta x$</td>
<td>0.0</td>
<td>$7\Delta y$</td>
<td>0.0</td>
</tr>
</tbody>
</table>
If NX≠NY and SQUARE=False, write error message and stop the program.

Compute indices of midpoint 17.

Ensure that IXY=1 or 2.

If NX and NY are not even, stop the program.

Test value of ICASE: if ICASE=3 generate fields of flat plate antenna (Lines 64-105); otherwise, generate fields of circular or square aperture (Lines 34-60).

Assign complex field value to each sample point \( (x_A, y_A, 0) \) in the circular aperture according to the values shown in Table 4-1. If \( \sqrt{x_A^2 + y_A^2} > R_{\text{max}} \), make the field value zero. Multiply the non-zero elements by CFAC(IPOL) to generate the correct polarization. For the square aperture, zero Column 1 and Row 1, and insert cos x taper (Line 37).

Compute sample spacing \( \Delta x_A / \lambda \) and go to statement 60.

Error message and STOP.

Flat plate antenna-- if NX≠16, write error message and STOP (Lines 131-133).

Compute sample spacing \( \Delta x_A / \lambda \).

Ensure NX=NY

Zero all elements in the aperture. If IXY=1 (\( x_A \)-component), go to statement 60.
Assign tapered amplitude values to eight "even" elements in Quadrant III.

Compute amplitude values for the "odd" elements in Quadrant III.

Compute amplitude values for elements 3-9 along $y_A=0$ line and along $x_A=0$ line.

Generate symmetrical amplitude values in Quadrant IV.

Generate symmetrical amplitude values in Quadrants I and II.

Compute $k_{\text{max}}$ and $d/\lambda$.

Test to determine if the sum channel data generated should be phased to produce the aperture distribution for a specified difference channel (ICHAN).

Form aperture distribution for difference elevation channel by zeroing all elements along $y_A=0$ and negating all elements for $y_A<0$. RETURN.

Form aperture distribution for difference azimuth channel by zeroing all elements along $x_A=0$ and negating all elements $x_A<0$. RETURN.

Error messag for ICASE=3 and NX#16. RETURN.

END
SUBROUTINE TRECNF(E,NX,NY,ICHAN,IPOL,IXY,DAPWL,DXWL,KXMAX,ICASE, &SQUARE)  
C *** MODIFIED JAN 80 FOR SQUARE APERTURE AND FOR SINGLE ELEMENT*** 
C SUBR TRECNF COMPUTES ELECTRIC FIELD COMPONENTS OVER A CIRCULAR APERTURE 
C OF RADIUS RMAX=(NX/2-1)/COS(ATAN(2/7)) AND RETURNS SAME IN E(NX,NY). 
C NX MUST EQUAL NY AND MUST BE EVEN. 
C ICHAN=1 FOR SUM CHANNEL  IPOL=1 FOR VERT-Y POL.  IXY=1 FOR X-COMP. 
C =2 FOR ELEV DIFF  =2 FOR HORIZ-X POL  =2 FOR Y-COMP. 
C =3 FOR AZ DIFF  =3 FOR RHC POL 
C =4 FOR LHC POL 
C DAPWL=DIAMETER OF APERTURE IN WAVELENGTHS (INPUT) 
C DXWL=sample spacing in aperture (OUTPUT) 
C KXMAX=MAXIMUM WAVE NUMBER (OUTPUT) 
C ICASE=1 OR 2 FOR UNIFORM, CIRCULAR APERTURE (ADA M.'S CASE I AND II) 
C =3 FOR FLAT-PLATE ANTENNA, VERTICAL POL (CASE III). 
C =3 FOR FLAT-PLATE ANTENNA, VERTICAL POL (CASE III). 
COMPLEX E(NX,NY),CFAC(4) 
REAL KXMAX 
LOGICAL SQUARE 
DATA CFAC/(I.,0.),(I.,0.),(0.,+I.),(0.,-I.)/ 
ANG=ATAN(2./7.) 
IF (ICASE.EQ.3) ANG=ATAN(4./6.) 
RMAX=(NX/2-1)/COS(ANG)+.001 
IF (IPOL.GT.4) IPOL=4 
IF (IPOL.LT.1) IPOL=1 
IF ((.NOT.SQUARE).AND.(NX.NE.NY)) GO TO 15 
NXMM7=NX/2+1-7 
NXMP7=NX/2+1+7 
NYMM7=NY/2+1-7 
NYMP7=NY/2+1+7 
C FOR NX, NY=32, ONLY THE CENTRAL 15 X 15 ELEMENTS ARE NONZERO. 
IF ((IXY.LT.1).OR.(IXY.GT.2)) IXY=2 
IF (MOD(NX,2).NE.0) GO TO 15 
IF (ICASE.EQ.3) GO TO 25 
TUXMX=FLOAT(NX) 
DO 10 I=1,NX 
X=FLOAT(-(NX/2)+I-1) 
COSX=COS(3.14159265*X/TUXMX) 
DO 10 J=1,NY
IF ((I.EQ.1).OR.(J.EQ.1)) GO TO 9
IF (NX.EQ.16) GO TO 1
IF ((I.LT.NXMM7).OR.(I.GT.NXMP7).OR.(J.LT.NYMM7).OR.(J.GT.NYMP7)) $GO TO 9
IF (SQUARE) GO TO 8
Y=FLOAT(-(NY/2)+J-1)
R=SQRT(X**2+Y**2)
IF (R.GT.RHAX) GO TO 9
IF ((IPOL.EQ.1).AND.(IXY.EQ.1)) GO TO 9
IF ((IPOL.EQ.2).AND.(IXY.EQ.2)) GO TO 9
C IF RHC, EY=(1,0), EX=(0,1) I.E., EX LEADS EY BY 90 DEG.
C IF LHC, EY=(1,0), EX=(0,-1) I.E., EX LAGS EY BY 90 DEG.
E(I,J)=CMPLX(COSX,O.)
IF ((IPOL.LT.3).OR.(IXY.EQ.2)) GO TO 10
E(I,J)=E(I,J)*CFAC(IPOL)
GO TO 10
E(I,J)=(0.,0.)
CONTINUE
IF (NX.EQ.2) GO TO 56
DXWL=(DAPWL/2.)*COS(ANG)/(NX/2-1)
IF (SQUARE) DXWL=(DAPWL/SQRT(2.))/(NX/2-2)
GO TO 60
WRITE(6,20)
20 FORMAT(//"NX.NE.NY OR NX NOT EVEN IN SUBR TRECNS"//)
STOP
C THE FOLLOWING IS FOR ADA M.'S CASE III (ICASE=2):
25 IF (NX.NE.16) GO TO 90
DXWL=(DAPWL/2.)*COS(ANG)/(NX/2-2)
IF (SQUARE) DXWL=(DAPWL/SQRT(2.))/(NX/2-2)
NY=NX
DO 26 I=1,NX
DO 26 J=1,NY
E(I,J)=(0.,0.)
IF (IXY.EQ.1) GO TO 60
E(6,4)=(.2824,0.)
E(8,4)=(.4250,0.)
E(4,6)=(.2888,0.)
E(6,6)=(.5218,0.)
\[
E(8,6) = (0.8060, 0.)
\]
\[
E(4,8) = (0.4194, 0.)
\]
\[
E(6,8) = (0.7959, 0.)
\]
\[
E(8,8) = (1.028, 0.)
\]
\[
\text{DO 30 } J=4,8,2
\]
\[
\text{DO 30 } I=3,8,1
\]
\[
\text{IF } ((\text{MOD}(J,2).EQ.0).\text{AND.}(\text{MOD}(I,2).EQ.0)) \text{ GOTO 30}
\]
\[
E(I,J) = \frac{E(I-1,J)+E(I+1,J)}{2}.
\]
\[
\text{DO 35 } I=3,8,1
\]
\[
\text{DO 35 } J=3,8,2
\]
\[
E(I,J) = \frac{E(I,J-1)+E(I,J+1)}{2}.
\]
\[
\text{DO 40 } I=3,9
\]
\[
\text{DO 45 } J=3,9
\]
\[
E(I,J) = E(I,8)
\]
\[
\text{DO 50 } I=1,6
\]
\[
\text{DO 50 } J=1,6
\]
\[
E(I,9+J) = E(I,9-J)
\]
\[
\text{DO 55 } I=3,15
\]
\[
\text{DO 55 } J=1,6
\]
\[
E(I,9+J) = E(I,9-J)
\]
\[
\text{GO TO 60}
\]
\[
\text{DXWL} = \frac{\text{DAPWL}}{\sqrt{2.}}
\]
\[
KXMAX = \frac{1}{2 \times DXWL}
\]
\[
\text{RETURN}
\]
\[
\text{DO 60 } KXMAX = 1./\left(2. \times DXWL\right)
\]
\[
\text{IF (ICHAN.EQ.1) RETURN}
\]
\[
\text{IF ((IXY.EQ.1).AND.(ICASE.EQ.3)) RETURN}
\]
\[
\text{IF ((IXY.EQ.1).AND.(IPOL.EQ.1)) RETURN}
\]
\[
\text{IF ((IXY.EQ.2).AND.(IPOL.EQ.2)) RETURN}
\]
\[
\text{IF (ICHAN.EQ.3) GO TO 75}
\]
\[
\text{C LOAD ELEVATION DIFFERENCE CHANNEL:}
\]
\[
J = NY/2+1
\]
\[
\text{DO 65 } I=1,NX
\]
65 E(I,J)=(0.,0.)  
JMAX=NY/2  
DO 70 J=1,JMAX  
DO 70 I=1,NX  
70 E(I,J)=-E(I,J)  
RETURN  
C LOAD AZIMUTH DIFFERENCE CHANNEL:  
75 I=NX/2+1  
DO 80 J=1,NY  
80 E(I,J)=(0.,0.)  
IMAX=NX/2  
DO 85 I=1,IMAX  
DO 85 J=1,NY  
65 E(I,J)=-E(I,J)  
RETURN  
C DAPWL=5.047 FOR ADA M.'S CASE III  
90 WRITE(6,95)  
95 FORMAT(/**ERROR EXIT! NX NOT EQUAL TO 16 IN SUBR TRECNF***/)  
STOP  
END
Chapter 5

SUBROUTINE APINT

5-1. Purpose: To compute the electromagnetic fields $\mathbf{E}, \mathbf{H}$ of a rectangular aperture in the $z=0$ plane at a point $P(x,y,z>0)$, where the amplitude and phase of the aperture electric fields $E_{x\mathrm{ap}}, E_{y\mathrm{ap}}$ are specified at $N_x$ by $N_y$ discrete points spaced $d_x/\lambda$ and $d_y/\lambda$ apart. The aperture magnetic fields $H_{x\mathrm{ap}}, H_{y\mathrm{ap}}$ are derived from $E_{x\mathrm{ap}}$ via the geometrical optics approximation.

5-2. Usage: CALL APINT (PFWL, EX, EY, NX, NY, MIDX, MIDY, DXWL, DYWL, E, H, INIT)

5-3. Arguments

PFWL - Real input array of three elements which specifies the Cartesian coordinates in wavelengths of the point $P(x/\lambda, y/\lambda, z/\lambda)$ at where the fields are to be computed; i.e., PFWL(1) = $x/\lambda$, etc.

EX,EY - Complex input arrays of NX by NY elements each which specify the aperture electric field.

NX,NY - Integer input variables equal to the number of sample points in the aperture in the $x$ and $y$ directions, respectively. NX and NY must be even.

MIDX, MIDY - Integer input variables equal to the indices in the arrays EX, EY corresponding to $x=y=0$; i.e., MIDX = NX/2+1, MIDY = NY/2+1.

DXWL, DYWL - Real input variables equal to the sample spacings in wavelengths in the $x$ and $y$ directions, respectively.
EIX - Complex output arrays of three elements each equal to the rectangular vector components of the electric and magnetic fields at $i$; i.e., $E_i = E_x$, etc.

EHT - Logical input variable which controls initialization of Subroutine MIPOLES.

1. Comments and Method

The fields $E_i(x,y,z)$ due to the sampled aperture fields are computed by superposing the individual fields of equivalent electric and magnetic dipoles located at each sample point as explained in Section 1.4.

2. Program Flow

1. Declare variables, initialize constants.

2-14 Compute initial source point, minus $x/y$. Set $x$-coordinate of source points to zero.

3-1e Initialize summations of the fields $E_i$.

3-27 Compute first source point $E(x,y,z)$.

4-3 Compute electric $j_{ik}^E$ and magnetic $j_{ik}^M$ currents according to $j_i^E = j_{ik}^E x y$ and $j_i^M = j_{ik}^M x z$.

5-15 Call Subroutine MIPOLES to compute the fields of the electric and magnetic dipoles $j_i^E$, $j_i^M$ located at the specified source point.

6-14 Add contributions from $1$ to $5$ to the electric component to the fields at $E_{i+1}$, etc.

7. Repeat for all source points.

8. The End of the Program.

9. References and Bibliography.

10. From field data to field solutions.
SUBROUTINE APINT(PFWL, EX, EY, NX, NY, MIDX, MIDY, DXWL, DYWL, E, H, INIT)

C SUBR APINT COMPUTES FRESNEL FIELDS OF RECTANGULAR APERTURE WITH
C APERTURE FIELDS GIVEN BY EX, EY (H FIELDS ARE DERIVED USING G.O. APPROX.)
C FIELDS E, H ARE COMPUTED AT THE POINT PFWL.
C COMPLEX E(3), H(3), EX(NX, NY), EY(NX, NY), JE(2), JM(2), ES(3), HS(3)
C JE, JM ARE ELECTRIC AND MAGNETIC SURFACE CURRENT DENSITIES FOUND FROM
C EAPXZHAT AND ZHAT X HAP.
C LOGICAL INIT
REAL PSWL(3), PFWL(3)
DATA ETA/376.991185/
C NX, NY MUST BE EVEN SO THAT OMITTING ROW 1 AND COL 1 YIELDS SYM APERTURE
C INIT=.TRUE. TO INITIALIZE CONSTANTS IN SUBR DIPOLES
PSWL(1)=(1-MIDX)*DXWL
PSWL(3)=0.
DO 1 L=1,3
E(L)=(0., 0.)
H(L)=(0., 0.)
1 CONTINUE
DO 10 J=2, NY
PSWL(2)=PSWL(1)+DXWL
PSWL(2)=(1-MIDY)*DYWL
DO 10 J=2, NY
10 CONTINUE
JE(1)=-EX(I, J)/ETA
JM(2)=-EX(I, J)
JE(2)=-EY(I, J)/ETA
JM(2)=-EY(I, J)
CALL DIPOLES(JE, JM, PSWL, PFWL, DXWL, DYWL, ES, HS, INIT)
DO 5 L=1, 3
E(L)=E(L)+ES(L)
H(L)=H(L)+HS(L)
5 CONTINUE
10 CONTINUE
RETURN
END
Chapter 6
SUBROUTINE DIPOLES

6-1. Purpose: To compute the electromagnetic fields \( \mathbf{E} = \hat{x} E_x + \hat{y} E_y + \hat{z} E_z \) and \( \mathbf{H} = \hat{x} H_x + \hat{y} H_y + \hat{z} H_z \) at point \( P_f(x'/\lambda, y'/\lambda, z'/\lambda) \) as produced by electric \( \mathbf{j}^e = z \times \mathbf{H} \) and magnetic \( \mathbf{j}^m = \mathbf{E} \times \hat{z} \) surface currents flowing on the planar rectangular surface of dimensions \( \Delta x/\lambda, \Delta y/\lambda \) located at source point \( P_s(x'/\lambda, y'/\lambda, z'/\lambda) \) and oriented in the \( z=z' \) plane. All dimensions are in wavelengths.

6-2. Usage: CALL DIPOLES (JE, JM, PSWL, PFWL, DXWL, DYWL, E, H, INIT)

6-3. Arguments

JE, JM - Complex input arrays of two elements each containing the \( x \) and \( y \) components of the electric and magnetic surface current densities at the center of the planar element as found from \( \mathbf{E} \times \hat{z} \) and \( \hat{z} \times \mathbf{H} \), respectively, where \( \hat{z} \) is the unit normal to the element and \( \mathbf{E}, \mathbf{H} \) are the fields at the center of the element.

PSWL, PFWL - Real input arrays of three elements each which contain the coordinates \( P_s(x'/\lambda, y'/\lambda, z'/\lambda) \), \( P_f(x/\lambda, y/\lambda, z/\lambda) \) of the center of the source element and the point at which the field is to be computed, respectively.

DXWL, DYWL - Real input variables equal to the dimensions \( \Delta x/\lambda, \Delta y/\lambda \) of the rectangular source element.
E, H - Complex output arrays of three elements each containing the fields E, H at the point Pf.
Note that H is computed rather than H above (to save time).

INIT - Logical input variable which controls initialization of various constants for repetitive calls to the subroutine: if TRUE, the constants are computed; if FALSE, the constants are not computed, and their last computed values are used.

6-4. Comments and Method

a. Comment. The source and field points cannot be any closer together than \( r = 0.01 \lambda \). This restriction is necessary to prevent division by zero due to the \( r^{-1} \) variation of the dipole fields as explained below. Actually, field points should be removed to the order of \( r = \sqrt{(\Delta x/\lambda)^2 + (\Delta y/\lambda)^2} \) for validity of the discretized approximation to the physical model.

b. Method. The subroutine computation is motivated by the problem of computing the fields of a rectangular antenna aperture located in the \( z = z' \) plane as illustrated in Figure 1. Let the electric and magnetic fields \( E_{ap} \), \( H_{ap} \) be specified at discrete points \( (x_m, y_n, 0) \). Then, at each point, the equivalent surface current densities \( \mathbf{J}^C \) and \( \mathbf{J}^m \) are given by [1]

\[
\begin{align*}
\mathbf{J}^C &= z \times H_{ap} + x (-H_{ap}) + y H_{ap} = x J^C_x + y J^C_y \\
\mathbf{J}^m &= E_{ap} \times z = x E_{ap} + y (-E_{ap}) = x J^m_x + y J^m_y
\end{align*}
\]
Figure 6-1. Geometry of Rectangular Aperture Antenna Approximated by Elementary Dipoles.
The surface current densities so defined can be discretized for each element $\Delta x \Delta y$ as follows. Consider the current density $J_y^e$. The total current entering the lower boundary and leaving the upper boundary of the element is $J_y^e \Delta x$ and can be regarded as an elementary dipole concentrated at the center of the element. The dipole moment is

$$p_0 = q \xi = \frac{J_y^e \Delta x}{j \omega} \Delta y$$

(3)

where $q$ is the charge and $\xi$ is the separation [2], and where the following relation for the sinusoidal steady state has been used:

$$q = \int I \, dt = \frac{I}{j \omega}$$

(4)

Similar relations hold for the other component of $J_y^e$ and, by duality, for $J_m^e$ as will be summarized below.

The next step in the development is to obtain expressions for the dipole fields of $J_y^e$ and $J_m^e$. To facilitate this step, first consider the fields radiated by electric and magnetic dipoles oriented along the $z_o$ axis as shown in Table 1 [2]. Note that these expressions require $r_o$ in wavelengths, and that $\Delta x$ and $\Delta y$ refer to the element size in the original aperture.

Matters are simplified if the spherical coordinate components of Table 1 are transformed to their corresponding rectangular components according to [3]
Table 6-1. Elementary Dipole Fields of Z-Directed Currents.

<table>
<thead>
<tr>
<th>Electric</th>
<th>Duality</th>
<th>Magnetic</th>
</tr>
</thead>
<tbody>
<tr>
<td>$E^e_{r0} = J^e_{zo} \epsilon_0 \left[ \frac{1}{r_o^2} + \frac{2\pi}{r_o} \right] \cos \theta _0 \ e^{-j2\pi r_o}$</td>
<td>$E^m_{r0} = -J^m_{zo} \epsilon_0 \left[ \frac{1}{r_o^2} + \frac{2\pi}{r_o} \right] \sin \theta _0 \ e^{-j2\pi r_o}$</td>
<td>$H^m_{r0} = J^m_{zo} \mu_0 \left[ \frac{1}{r_o^2} + \frac{2\pi}{r_o} \right] \sin \theta _0 \ e^{-j2\pi r_o}$</td>
</tr>
<tr>
<td>$E^e_{\theta0} = J^e_{zo} \epsilon_0 \frac{2\pi}{r_o^2} \sin \theta _0 \ e^{-j2\pi r_o}$</td>
<td>$E^m_{\theta0} = -J^m_{zo} \epsilon_0 \frac{2\pi}{r_o^2} \sin \theta _0 \ e^{-j2\pi r_o}$</td>
<td>$H^m_{\theta0} = -J^m_{zo} \mu_0 \frac{2\pi}{r_o^2} \sin \theta _0 \ e^{-j2\pi r_o}$</td>
</tr>
<tr>
<td>$H^e_{\phi0} = J^e_{zo} \left( \frac{1}{j2\pi} \right) \eta_0 \ e^{-j2\pi r_o}$</td>
<td>$H^m_{\phi0} = J^e_{zo} \left( \frac{1}{j2\pi} \right) \eta_0 \ e^{-j2\pi r_o}$</td>
<td>$H^m_{\phi0} = J^e_{zo} \left( \frac{1}{j2\pi} \right) \eta_0 \ e^{-j2\pi r_o}$</td>
</tr>
</tbody>
</table>

$r_o$ in wavelengths

$r_o$ in wavelengths
\[ A_x = (A_{x_0} \cos \theta_o) \sin \phi_o + (A_{y_0} \sin \theta_o) \cos \phi_o \] (5a)

\[ A_y = (A_{x_0} \cos \theta_o) \sin \phi_o + (A_{y_0} \sin \theta_o) \cos \phi_o \] (5b)

\[ A_z = (A_{x_0} \cos \theta_o) \cos \phi_o - (A_{y_0} \sin \theta_o) \sin \phi_o \] (5c)

\[ C_x = -(C_{x_0} \sin \theta_o) \sin \phi_o \] (5d)

\[ C_y = (C_{x_0} \sin \theta_o) \cos \phi_o \] (5e)

\[ C_{z_0} = 0 \] (5f)

In the above, the trigonometric function in parentheses comes from the field expressions in Table I; hence, the "minus" superscript indicates the field expression from Table I without the orientation factor \( \cos \phi_o \) or \( \sin \theta_o \), weighting \( e_o \) or \( h_o \), and without the current \( J_{z_0} \) or \( J_{z_0}^e \).

Define direction cosines \( k_{x_0} \), \( k_{y_0} \), \( k_{z_0} \) related to \( \theta_o \), \( \phi_o \) according to

\[ k_{x_0} = \sin \theta_o \cos \phi_o \] (6a)

\[ k_{y_0} = \sin \theta_o \sin \phi_o \] (6b)

\[ k_{z_0} = \cos \theta_o \] (6c)

Then Equations (5) can be rewritten succinctly as

\[ \text{Equations (5)} \]
\[ A_{x_0} = (A_{i_{0}}^{-} + A_{\theta_0}^{-}) k_{x_0} k_{z_0} \] (7a)

\[ A_{y_0} = (A_{i_{0}}^{-} + A_{\theta_0}^{-}) k_{y_0} k_{z_0} \] (7b)

\[ A_{z_0} = (A_{i_{0}}^{-} + A_{\theta_0}^{-}) k_{z_0}^2 - \lambda_\theta \] (7c)

\[ C_{x_0} = - \lambda_\phi k_{x_0} \] (7d)

\[ C_{y_0} = \lambda_\phi k_{y_0} \] (7e)

\[ C_{z_0} = 0 \] (7f)

Similar expressions for cases of x-directed and y-directed dipoles may be derived from those given above merely redefining the axes in Table 1. When this is done, the generalized expressions shown in Table 2 result for all three cases.

When both electric and magnetic currents are present (x-directed and y-directed components) the expressions for \( E \) and \( H \) are obtained by adding the contributions due to each current as given in Table 2. Note that \( A_{i_x}^{-}, A_{i_y}^{-}, \) and \( A_{i_z}^{-} \) are identical for both types and directions of currents so that the expressions for the field components may be written, for example, as follows:

\[
E_x = e_o \left\{ j^e_x [(A_{i_x}^{-} + A_{i_{0}}^{-}) k_x^2 - A_{i_0}^{-}] + j^e_y k_x k_y (A_{i_y}^{-} + A_{\theta_0}^{-}) \right\} \\
- h_0 j^m_y k_z C_{i_0}^{-} \] (8a)
Table 6-2. Rectangular Field Components of Elementary Dipoles

<table>
<thead>
<tr>
<th>Field Component</th>
<th>x-directed</th>
<th>Dipole Orientation</th>
<th>y-directed</th>
<th>z-directed</th>
</tr>
</thead>
<tbody>
<tr>
<td>$A_x$</td>
<td>$-\frac{k^2}{\rho} - A_\theta x$</td>
<td>$A_\phi k_y$</td>
<td>$A_\phi k_z$</td>
<td>$A_\phi k_y k_z$</td>
</tr>
<tr>
<td>$A_y$</td>
<td>$A_\theta k_x y$</td>
<td>$-A_\phi k_y$</td>
<td>$A_\theta k_y z$</td>
<td>$A_\theta k_y k_z$</td>
</tr>
<tr>
<td>$A_z$</td>
<td>$A_\theta k_x z$</td>
<td>$A_\phi k_y k_z$</td>
<td>$A_\theta k_z$</td>
<td>$A_\phi k_y k_z$</td>
</tr>
<tr>
<td>$C_x$</td>
<td>0</td>
<td>$C_\phi k_z$</td>
<td>$-C_\phi k_y$</td>
<td>$C_\phi k_y$</td>
</tr>
<tr>
<td>$C_y$</td>
<td>$-C_\phi k_z$</td>
<td>0</td>
<td>$C_\phi k_x$</td>
<td></td>
</tr>
<tr>
<td>$C_z$</td>
<td>$C_\phi k_y$</td>
<td>$-C_\phi k_x$</td>
<td>0</td>
<td></td>
</tr>
</tbody>
</table>

where:

$$A_{\theta} = (A_x + A_y)$$

$$A_x = \left(\frac{1}{3} + \frac{j2\pi}{r_o^2}\right) e^{-j2\pi r_o/r_0}$$

$$A_y = \left(\frac{1}{3} + \frac{j2\pi}{r_o^2} - \frac{(2\pi)^2}{r_o^2}\right) e^{-j2\pi r_o/r_0}$$

$$A_{\phi} = \left(\frac{1}{3} + \frac{j2\pi}{r_o^2}\right) e^{-j2\pi r_o/r_0}$$

$$C_\phi = \left(\frac{1}{2} + \frac{j2\pi}{r_o^2}\right) e^{-j2\pi r_o/r_0}$$
\[ nH_x = n h_0 \left\{ \sum_j J_j^m (A_{J r} - A_{J o}) k_x^2 - A_{J o}^{-1} \right\} + \sum_j J_j^m k_x k_y (A_{J r} - A_{J o}) \]

\[ + e_o j \sum_j \frac{J_j^u}{n H_x} \frac{k_z C}{\phi} \]

(8b)

Similar expressions may be obtained for the other rectangular components of \( E \) and \( H \) as given in Table 6-3 and by Lines 56-57 and 62-65 of the program listing.

6-6. Program Flow

<table>
<thead>
<tr>
<th>Lines</th>
<th>Comment</th>
</tr>
</thead>
<tbody>
<tr>
<td>15</td>
<td>If INIT=.TRUE., compute constants in Lines 18-29.</td>
</tr>
<tr>
<td>18-29</td>
<td>Compute ((2\pi)^2, j, e_o, h_o, j2\pi, j\pi, h = -h, e_o = e_o / h.)</td>
</tr>
<tr>
<td></td>
<td>Lines 26-27 have been added to cause ( nH_x ) to be computed instead of ( H ) to save time in Subroutine RECM (See Chapter 3).</td>
</tr>
<tr>
<td>30-33</td>
<td>Compute ( r ) in wavelengths; i.e., the distance from the source point to the field point.</td>
</tr>
<tr>
<td>34</td>
<td>If ( r &lt; 01 ), write error message and stop (Lines 67-69).</td>
</tr>
<tr>
<td>35-37</td>
<td>Compute direction cosines ( k_x, k_y, k_z ).</td>
</tr>
<tr>
<td>38-40</td>
<td>Compute exponential phase factor ( e^{-j2\pi r} ).</td>
</tr>
<tr>
<td>41-45</td>
<td>Compute ( A_{J r}, A_{J o}, C_{\phi o}, (A_{J r} + A_{J o}) ), and ( C = (A_{J r} + A_{J o}) k_x^2 - A_{J o}^{-1} ).</td>
</tr>
<tr>
<td>46-49</td>
<td>These commented lines contain only ( 1/r ) terms and can be used to replace lines 41-45.</td>
</tr>
<tr>
<td>50-51</td>
<td>Precalculate ( A_{J r} + A_{J o} ) ( k_x ), ( k_y ), and ( C_{\phi o} k_z ) to facilitate computation of ( E_x ) and ( H_x ).</td>
</tr>
</tbody>
</table>
Table 6-3. Fields of Elementary x-Directed and y-Directed Dipoles

\[ \mathbf{E} = \mathbf{E}^e + \mathbf{E}^m \]

\[ \mathbf{H} = \mathbf{H}^e + \mathbf{H}^m \]

\[ E_x = \varepsilon_o \left[ J^e_x (\hat{k}_x - \hat{A}_0) + J^e_y (k_x \hat{A}_y - \hat{A}_y) \right] - \mu_o j \varepsilon_o J^m_y k_z \]

\[ E_y = \varepsilon_o \left[ J^e_x (k_y \hat{A}_y - \hat{A}_y) + J^e_y (\hat{k}_y - \hat{A}_y) \right] + \mu_o j \varepsilon_o J^m_x k_z \]

\[ E_z = \varepsilon_o \left[ J^e_x (k_z \hat{A}_z - \hat{A}_z) + J^e_y (k_z \hat{A}_y - \hat{A}_y) \right] + \mu_o j \varepsilon_o J^m_y k_x \]

\[ H_x = \varepsilon_o \left[ \frac{J^e_y}{\mu_o} \hat{k}_y + \mu_o \left[ J^m_x (\hat{k}_x - \hat{A}_x) + J^m_y (k_x \hat{A}_y - \hat{A}_y) \right] \right] \]

\[ H_y = -\varepsilon_o \left[ \frac{J^e_x}{\mu_o} \hat{k}_x + \mu_o \left[ J^m_y (k_y \hat{A}_y - \hat{A}_y) + J^m_x (k_x \hat{A}_x - \hat{A}_x) \right] \right] \]

\[ H_z = \varepsilon_o \left[ \frac{J^e_x}{\mu_o} \hat{k}_x + \mu_o \left[ J^m_y (k_x \hat{A}_x - \hat{A}_x) + J^m_z (k_z \hat{A}_z - \hat{A}_z) \right] \right] \]

\[ \frac{(\lambda \mu)^2}{2} \]

\[ \frac{(\varepsilon \mu)^2}{2} \]

Where:

\[ \varepsilon_o = \frac{\varepsilon}{j(2\pi)^2} \]

\[ \mu_o = \frac{\lambda}{j(2\pi)^2} \]
52-53 Compute \( E_x \) and \( H_x \) due to the x-directed and \( y \)-directed electric and magnetic currents: 
\[ J_x^e = JE(1), \]
\[ J_y^e = JE(2), J_x^m = JM(1), J_y^m = JM(2). \]

54 Precalculate \( (\lambda_y^{-2} + \lambda_y^{-2}) k \). 

55 See lines 46-49 above.

56-57 Compute \( E_y \) and \( H_y \).

58-61 Precalculate common variables for \( E_z, H_z \).

62-65 Compute \( E_z \) and \( H_z \).

RETURN

67-69 Error message and halt.

END

6-6. Test Case

Selected test cases shown in Figure 2-15 of Reference 1 were executed. The square, 4" x 4", uniform aperture \( (\lambda = 1.18") \) was sampled at \( M=15, N=15 \) points in the x and y directions, respectively. Cases were done for \( E_{ap} = \gamma(l), H_{ap} = 0 \), and for \( E_{ap} = \gamma(l), H_{ap} = -x(1/\pi) \). In the latter case, the amplitudes obtained were twice as large (as expected). Although exact comparison to the graphical results in Figure 2-15 was not possible, agreement was obtained so far as could be determined. Some benchmarks as computed by Subroutine DIPLOELS are shown in Tables 4 through 6.

6-7. References


6-8. Program Listing. See following pages.
SUBROUTINE DIPOLES(JE,JM,PSWL,PFWL,DXWL,DYWL,E,H,INIT)
C **** MODIFIED 1-23-80 TO INCLUDE ONLY 1/R TERMS **** NULLIFIED 1-24-80**
C SUBR DIPOLES COMPUTES THE RECTANGULAR COMPONENTS OF THE FIELDS E,H OF
C ELECTRIC AND MAGNETIC DIPOLES LOCATED AT PSWL(X',Y',Z')
C AND ORIENTED IN THE X' AND Y' DIRECTIONS. THE FIELDS ARE COMPUTED AT
C THE POINT PFWL(X,Y,Z). ALL DIMENSIONS ARE IN WAVELENGTHS. MKS SYSTEM
C IS USED. FREE SPACE (ETA=377 OHMS) IS ASSUMED.
COMPLEX JE(2),JM(2),E(3),H(3),JAY,HO,EO,CPHS,JPI,JAY2PI
COMPLEX ARO,ATO,CPO,ARTO,CT,ARTOK,CPOK,EOH,HOE
REAL PSWL(3),PFWL(3),KX,KY,KZ
LOGICAL INIT
C DXWL,DYWL=X' AND Y' DIMENSIONS OF THE RECTANGULAR ELEMENT OVER WHICH
C CURRENT DENSITIES JE AND JM FLOW TO MAKE THE DIPOLES.
DATA TUPI/6.283185301/,ETA/376.9911185/
IF (INIT) GO TO 1
GO TO 2
C COMPUTE EO,HO (SEE DERIVATION DATED 7-23-79):
1 TUPI2=TUPI**2
JAY=(0.,1.)
EO=DXWL*DYWL*ETA/(JAY*TUPI2)
HO=DXWL*DYWL/(JAY*TUPI2*ETA)
JAY2PI=JAY*TUPI
JPI=JAY*TUPI/2.
HOE=-HO*ETA
EOH=EO/ETA
HO=HO*ETA
EOH=EOH*ETA
C THE ABOVE TWO LINES CAUSE ETA*H TO BE COMPUTED FOR USE IN RECI.
INIT=.FALSE.
2 X=PFWL(1)-PSWL(1)
Y=PFWL(2)-PSWL(2)
Z=PFWL(3)-PSWL(3)
R=SQRT(X*X+Y*Y+Z*Z)
IF (R.LT..01) GO TO 90
KX=X/R
KY=Y/R
KZ=Z/R
PHS=AMOD(TUPI*R,TUPI)
106
CPHS=CMPLX(0., -PHS)
CPHS=CEXP(CPHS)
AR0=CPHS*(1./R**3+JAY2PI/R**2)
ATO=.5*(AR0-CPHS*TUP12/R)
CPO=JPI*AR0*R
ART0=AR0+ATO
CT=AR0*KX**2-ATO*(1.-KX**2)
AR0=-.5*CPHS*TUP12/R
CPO=JPI*CPHS*JAY2PI/R
C ART0=ATO
CT=-ATO*(1.-KX*KX)
ART0K=ART0*KX*KY
CPOK=CPO*KZ
E(1)=E0*(JE(1)*CT+JE(2)*ART0K)+JM(2)*HOE*CPOK
H(1)=HO*(JM(1)*CT+JM(2)*ART0K)+EOH*JE(2)*CPOK
CT=AR0*KY**2-ATO*(1.-KY**2)
E(2)=E0*(JE(1)*ART0K+JE(2)*CT)-JM(1)*HOE*CPOK
H(2)=HO*(JM(1)*ART0K+JM(2)*CT)-JE(1)*EOH*CPOK
ART0K=ART0*KX*KZ
ART0K=ART0*KX*KZ
CPOK=CPO*KZ
CPO=CP0*KX
E(3)=E0*(JE(1)*ART0+JE(2)*ART0K)+HOE*(JM(1)*CPOK
$-JM(2)*CPO)
H(3)=HO*(JM(1)*ART0+JM(2)*ART0K)+EOH*(JE(1)*CPOK
$$JE(2)*CPO)
RETURN
90 WRITE(6,91)
91 FORMAT("**** R.LT..01 WAVELENGTH IN SUBR DIPOLES--STOP****")
STOP
END
### Table 6-4. Fields Computed by Subroutine DIPOLES Along z-Axis for 4" x 4" Uniform Aperture ($\lambda=1.18$)

<table>
<thead>
<tr>
<th>n</th>
<th>X</th>
<th>Y</th>
<th>Z</th>
<th>XYZ</th>
<th>HY</th>
<th>HZ</th>
<th>EY</th>
<th>HSEEG</th>
<th>AMPGo</th>
<th>PHSEEG</th>
<th>AMPGo</th>
<th>PHSEEG</th>
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<td>177.4</td>
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<td>135.2</td>
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<td>3.94</td>
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<td>49.6</td>
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<td>10.1</td>
<td></td>
<td></td>
<td></td>
<td></td>
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<td>4</td>
<td>4.89</td>
<td>4.14</td>
<td>-40.0</td>
<td>189.2</td>
<td>-0.1</td>
<td>38.7</td>
<td>-40.0</td>
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<td>157.1</td>
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Table 6-5. Fields Computed by Subroutine DIPOLES Along x-Axis at z=8 inches.

<table>
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<th>Field</th>
<th>E.x</th>
<th>E.y</th>
<th>E.z</th>
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</thead>
<tbody>
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<th>Z</th>
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<th>B</th>
<th>C</th>
<th>D</th>
<th>E</th>
<th>F</th>
<th>G</th>
<th>H</th>
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<td>15</td>
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<td>450</td>
<td>450</td>
<td>450</td>
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</tr>
</tbody>
</table>

Note: The table provides values for x, y, z, A, B, C, D, E, F, G, and H for different fields computed by the subroutine DIPOLES along the x-axis at z=8 inches.
Table 6-6. Fields Computed by Subroutine DIPOLES Along x-Axis at z=24 inches.

<table>
<thead>
<tr>
<th>N</th>
<th>X</th>
<th>Y</th>
<th>Z</th>
<th>W</th>
<th>FX</th>
<th>FY</th>
<th>EFX</th>
<th>EFY</th>
<th>EFZ</th>
</tr>
</thead>
<tbody>
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<td>1</td>
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<td>-181.0</td>
<td>-10.0</td>
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<td>0.3</td>
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<td>-141.0</td>
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<td>-49.8</td>
<td>-42.6</td>
<td>-66.7</td>
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</tr>
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<td>0.5</td>
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</tr>
<tr>
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<td>1.0</td>
<td>-40.0</td>
<td>-65.0</td>
<td>-12.0</td>
<td>-54.5</td>
<td>-44.0</td>
<td>-76.0</td>
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</tr>
<tr>
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<td>-40.0</td>
<td>-45.0</td>
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<td>-45.0</td>
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Table 6-7. Fields Computed by Subroutine DIPOLES Along x-Axis at z=48 inches.

1 TEST PROGRAM FOR USING ELEMENTARY SOURCES FOR COMPUTING FRESNEL FIELDS
(SEE WALTER (1965), pp. 55-57)
MAX = 4.06  MIN = 4.72  M = 15  N = 15  LAMBDA = 1.19
FIELD IS -10: FB AT Z = 74.6 INCHES
APERTURE FIELDS: EAP = (1.0, 0.0, 0.0)  (1.0, 0.0, 0.0)  HAP = (0.0, 0.0, 0.0)
-90, HAP = .97472 + .231908 F = -2  IAX IS 1  PRWHE = F, CL  I = 48.00

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<th>Y</th>
<th>Z</th>
<th>ANGRA</th>
<th>PHASEG</th>
<th>AMPRA</th>
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Chapter 7

SUBROUTINE WALL

7-1. Purpose: To compute the normal transmission \( T_n, T_{n'} \) and reflection coefficients of a \( N \)-layer dielectric sheet having thicknesses \( d_n \), dielectric constants \( \epsilon_{r_n} \), and loss tangents \( \tan \delta_n \) for each layer when a plane wave is incident at angle \( \theta_i \).

7-2. Usage: CALL WALL (BETA, SINE, D, ER, TD, N, NN, TN1, TN2, RPER, RPAR)

7-3. Arguments

- **BETA** - Real input variable = \( 2\pi/\lambda \), where \( \lambda \) is the free space wavelength.
- **SINE** - Real input variable = \( \sin \theta_i \).
- **D**, **ER**, **TD** - Real input arrays containing the thickness (cm), dielectric constant \( \epsilon_r \), and loss tangent \( \tan \delta \) of each layer.
- **N** - Integer input variable equal to the number of layers.
- **NN** - Integer input = \( N+1 \).
- **TN1, TN2** - Complex output variables equal to the normal voltage transmission coefficients for the components of the incident electric field perpendicular to and parallel to the plane of incidence, respectively.
- **RPER, RPAR** - Complex output variables equal to the reflection coefficients \( R_1, R_\parallel \).
7-4. Comment and Method

a. Layer 1 is the first layer on the exit side of the panel; layer N is the first layer on the incident side. $T_1$, $T_N$ have the same value for either side of the panel being the incident side; however, $R_1$, $R_N$ are different (in phase) for the two cases.

b. The details of the method are presented in Appendix E of Reference 1.


7-6. Test Case: None.

7-7. References


7-8. Program Listing: See following pages.
SUBROUTINE WALL(BETA,SINE,D,ER,TD,N,NN,TN1,TN2,RPER,RPAR)  
C SUBROUTINE WALL COMPUTES THE TRANSMISSION AND REFLECTION  
C COEFFICIENTS FOR AN N LAYER, PLANE DIELECTRIC PANEL FOR PLANE  
C WAVE INCIDENT AT SINE(ANGLE) FOR PERPENDICULAR AND  
C PARALLEL POLARIZATIONS.  
C PARAMETERS OF THE WALL: N= THE NUMBER OF LAYERS  
C NN= N+1 REQUIRED TO DIMENSION ARRAYS  
C D= THICKNESS OF EACH LAYER IN CENTIMETERS  
C ER= RELATIVE DIELECTRIC CONSTANT OF EACH LAY  
C TD= THE LOSS TANGENT FOR EACH LAYER  
C TN1,TN2 ARE THE NORMAL VOLTAGE XMN COEFFICIENTS; TPER,TPAR ARE THE  
C INSERTION VOLTAGE TRANSMISSION COEFFICIENTS. IT IS IMPORTANT TO  
C NOTE THAT THE XMN COEFS ARE THE SAME FOR PLANE WAVE INCIDENT FROM  
C EITHER SIDE OF THE STRATIFIED DIELECTRIC PANEL IMMERSED IN FREE SPACE;  
C HOWEVER, THE REFLECTION COEFS ARE NOT. THAT IS, FOR COMPUTING RPER,  
C RPAR, THE ORDERING OF ER(NN),TD(NN) IS IMPORTANT WITH LAYER 1 BEING  
C THE FIRST LAYER ON THE EXIT SIDE, LAYER N BEING THE FIRST LAYER ON THE  
C INCIDENT SIDE. LAYER NN AND LAYER 0 ARE JUST FREE SPACE LAYERS  
C OF SEMI-INFINITE DEPTH.  
C E,G,R1,R2, ARE ARRAYS USED IN THE SUBROUTINE HAVING NN DIM"L LIMITS  
C COMPLEX E(6),G(6),R1(6),R2(6),EE,RR1,RR2,AA1,AA2,X1,X2,  
$X3,X4,Y1,Y2,Y3,Y4,U1,U2,U3,U4,V1,V2,V3,V4,P1,P2,P3,P4,Q1,Q2,Q3,Q4  
C COMPLEX TPER,TPAR,RPER,RPAR,U,V,TN1,TN2  
C DIMENSION ER(NN),TD(NN),D(N)  
C ER(NN)=1.0  
C TD(NN)=0.0  
C DO 50 I=1,NN  
50 E(I)=CMPLX(ER(I),-ER(I)*TD(I))  
C CALCULATE TOTAL THICKNESS OF WALL IN CM  
C DTOTAL=0.0  
C DO 200 I=1,N  
200 DTOTAL=DTOTAL+D(I)  
C S IS THE SINE OF THE ANGLE Squared  
C C IS THE COSINE OF THE ANGLE  
S=SINE*SINE
C=SQRJT(1.0-S)
AD=ER(1.-S)
ET=ER(1)*TD(1)
SR=SQRJT(AD*AD+ET*ET)
IF(SR-AD) 76,76,77
76 A=0.
GO TO 78
77  A=AB*SQRJT(SR-AD)
78  B=AB*SQRJT(SR+AD)
G(1)=CMPLX(A,B)
GG=CMPLX(0.0,BETA*C)
EE=1.0
SUM=0.
SUM=SUM+D(1)/SQRJT(AD)
RR1=(G(1)-GG)/(G(1)+GG)
RR2=(EE*G(1)-E(1)*GG)/(EE*G(1)+E(1)*GG)
DO 84 I=1,N
II=I+1
AD=ER(II)-S
ET=ER(II)*TD(II)
IF (I-N) 176,177,177
176 SUM=SUM+D(II)/SQRJT(AD)
177 CONTINUE
SR=SQRJT(AD*AD+ET*ET)
IF(SR-AD) 79,79,80
79  A=0.
GO TO 81
80  A=AB*SQRJT(SR-AD)
81  B=AB*SQRJT(SR+AD)
G(II)=CMPLX(A,B)
R1(II)=(G(II)-G(II))/(G(II)+G(II))
84  R2(II)=(E(II)*G(II)-E(II)*G(II))/(E(II)*G(II)+E(II)*G(II))
SUM=SUM+D
AA1=1.0-RR1
AA2=1.0-RR2
DO 85 I=1,N
AA1=AA1*(1.0-R1(II))
85  AA2=AA2*(1.0-R2(II))
AA1=1.0/ AA1
AA2=1.0/ AA2
U=-G(1)*D(1)
V=G(1)*D(1)
X1=CEXP(U)
X4=CEXP(V)
X2=-RR1*X4
X3=-RR1*X1
Y1=X1
Y4=X4
Y2=-RR2*Y4
Y3=-RR2*Y1
DO 105 I=2,NN
IF(I-NN) 95,90,1
90 U1=1.0
U2=-R1(N)
U3=-R1(N)
U4=1.0
V1=1.0
V2=-R2(N)
V3=-R2(N)
V4=1.0
GO TO 100
95 II=I-1
U=-G(I)*D(I)
V=G(I)*D(I)
U1=CEXP(U)
U4=CEXP(V)
U2=-R1(II)*U4
U3=-R1(II)*U1
V1=U1
V4=U4
V2=-R2(II)*V4
V3=-R2(II)*V1
100 P1=X1*U1+X2*U3
P2=X1*U2+X2*U4
P3=X3*U1+X4*U3
P4=X3*U2+X4*U4
Q1 = Y1*V1 + Y2*V3
Q2 = Y1*V2 + Y2*V4
Q3 = Y3*V1 + Y4*V3
Q4 = Y3*V2 + Y4*V4
X1 = P1
X2 = P2
X3 = P3
X4 = P4
Y1 = Q1
Y2 = Q2
Y3 = Q3
Y4 = Q4
RPER = -X3/X4
C TN1, TN2 ARE NORMAL VOLTAGE XMN COEFFICIENTS.
RPAR = -Y3/Y4
TN1 = (X1 + X2*RPER)*AA1
U = CMPLX(0.0, -SUM*BETA)
U = CEXP(U)
C TPER, TPAR HERE ARE VOLTAGE XMN COEFFICIENTS AT EXIT POINT OF RAY.
TPER = TN1*U
TN2 = (Y1 + Y2*RPAR)*AA2
TPAR = TN2*U
C MODIFY TRANSMISSION COEFFICIENTS FOR INSERTION
U = CMPLY(0.0, BETA*DTOtal*C)
U = CEXP(U)
TPER = TN1*U
TPAR = TN2*U
1 CONTINUE
300 RETURN
END
Chapter 8

SUBROUTINE CAXCB

8-1. Purpose: To compute the complex vector cross product \( \mathbf{C} = \mathbf{A} \times \mathbf{B} \),
where \( \mathbf{A} \) and \( \mathbf{B} \) are complex vectors expressed in rectangular coordinates.

8-2. Usage: CALL CAXCB (A, B, C)

8-3. Arguments

\( \mathbf{A} \) - Complex input array containing the rectangular components of the vector \( \mathbf{A} = x_\mathbf{A} + y_\mathbf{A} + z_\mathbf{A} \); i.e., \( \mathbf{A} (A_x, A_y, A_z) \).

\( \mathbf{B} \) - Complex input array \( \mathbf{B} (B_x, B_y, B_z) \) representing the vector \( \mathbf{B} \).

\( \mathbf{C} \) - Complex output array \( \mathbf{C} (C_x, C_y, C_z) \) representing the vector \( \mathbf{C} = \mathbf{A} \times \mathbf{B} \).

8-4. Comment and Method: None

8-5. Program Flow: See listing below.

8-6. Test Case: None.

8-7. References: None.

8-8. Program Listing: See following page.
SUBROUTINE CAXCB(A,B,C)
COMPLEX A(3),B(3),C(3)
C SUBR CAXCB COMPUTES THE VECTOR CROSS PRODUCT C=AXB OF
C TWO COMPLEX VECTORS A AND B
C(1)=A(2)*B(3)-A(3)*B(2)
C(2)=A(3)*B(1)-A(1)*B(3)
C(3)=A(1)*B(2)-A(2)*B(1)
RETURN
END