PARAMETRIC INVESTIGATION OF RADOME ANALYSIS METHODS:

COMPUTER-AIDED RADOME ANALYSIS USING GEOMETRICAL OPTICS AND LORENTZ RECIPROCITY

By

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**Abstract:** A Fortran computer program is described for computing the effects of a tangent ogive radome on the receiving patterns and boresight directions of a monopulse antenna. The analytical method based on ray tracing, main program, and 34 subroutines are thoroughly documented. Four test cases and their results in the form of printouts, pattern plots, and near-field graphs are presented.
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Chapter 1
INTRODUCTION AND SUMMARY

1-1. Introduction

This Volume II of this final technical report of four volumes documents a ray tracing radome analysis computer program written in Fortran IV for use on the Cyber 70/74 computing system at Georgia Institute of Technology and the IBM 3033 computing system at Johns Hopkins University Applied Physics Laboratory. The program was developed at Georgia Institute of Technology over the past four years; however, considerable development work in computer aided radome analysis has taken place here prior to that time [1-7].

This analysis package was used during the research carried out under this grant to analyze the antennas and radomes using the fast receiving formulation as described in Volume I. Its documentation was done in conjunction with the on-going radome technology program at JHU/APL under the cognizance of R. C. Mallalieu (APL Contract 601053). It is intended to serve as part of a technology base for the radome technical community.

The report is organized by chapters according to the approximate order in which the subprograms are called, and each chapter describes one subprogram. Each chapter is essentially self-contained since it is meant to serve as the complete documentation on a single subroutine. References are provided at the end of each chapter. In some cases, figures are duplicated in different chapters for completeness. Each chapter is terminated with the listing of the subroutine.
Chapter 2 describes the main program and instructions for its use. Chapters 3 through 28 describe the thirty four subroutines required for execution, including those for producing Calcomp pattern plots and threedimensional plots. Appendices A through D present computed results for four test cases for use in verifying correct operation on other systems. These results were obtained on the Cyber 70/74 computing system at Georgia Tech. The remaining part of this chapter describes background of the program development and summarizes the features of the computer analysis.

This report comprises Volume II of four volumes. Volume I describes the salient results of this overall investigation to determine the accuracies and ranges of validity of various analysis methods. Volume III documents the additional software required to analyze radomes using a surface integration method. Volume IV presents the experimental results obtained and is meant to serve as a data base for other investigators seeking to verify the accuracy of their computer codes.

1-2. Background

Development of the radome analysis computer program (RACP) was initiated in 1971 in an effort to include the effects of the radome on a ground mapping radar [1]. A three-dimension geometry and vector field formulation were used. A plane wave spectrum (PWS) representation of the radiation from the antenna greatly facilitated the computations since the Fast Fourier Transform (FFT) could be used. The program was used to compute power patterns on the ground for many different cases of antenna/missile orientations. From these data, the effects of the radome on pattern shape, power loss and VSWR were determined.
Monopulse tracking antennas were next introduced into the computer analysis to evaluate radome materials and shapes for seeker systems in the 8-18 GHz band [2]. Tangent ogive shapes of various fineness ratios were analyzed. Monolithic and multilayer wall structures were used. Algorithms were developed to compute boresight errors from the sampled data difference patterns in two orthogonal planes. A modification of this program was also used to conduct a trade-off and development study for the Multipurpose Missile (MMP), later known as ASALM [3].

The next step in the development of RACP came in 1977 with the introduction of a conical scan tracking antenna into the analysis [4]. This antenna necessitated a reformulation of the analysis from the transmitting formulation used earlier to a receiving formulation. The big advantage offered by the latter is that the antenna response can be calculated for only one direction of arrival of the target return (plane wave). In the former, the FFT automatically computes "responses" for many directions of arrival and, hence, is computationally slower. Subsequent versions of the program have used the same receiving formulation with monopulse and other types of antenna models.

The computed results obtained with the receiving and transmitting formulations are not always the same [5]. A computed-aided analysis which utilizes the Huygens-Fresnel principle [6, 7] is generally considered to be more accurate than the two methods already mentioned, but requires considerably more computation time that may not be warranted in all cases. A research program is now underway at Georgia Tech whose objective is to establish the accuracies and ranges of validity of these three methods of radome analysis [5].
1-1. Description of the Analysis

The current version of the ray tracing analysis computer program utilizes a receiving formulation based on the Lorentz reciprocity theorem [1]. A plane wave of selectable linear or circular polarization is assumed incident on the outside of the radome and is represented by a system of parallel rays. There is one ray for each sample data point in the antenna aperture inside the radome. Each ray is traced from the point where it impinges on the outside surface to the corresponding aperture point. The electric and magnetic fields $E_i'$, $H_i'$ associated with each ray are weighted by the flat panel transmission coefficients $T_i'$, $T_i$ as determined by the unit normal $\mathbf{n}$, the direction of propagation $\mathbf{k}$, and the dielectric properties of the radome wall. The weighted incident fields $E_i'$, $H_i'$ at each aperture point are then used in the following integral to obtain the complex voltage response $V_r$ of the antenna as

$$V_r = C \iint \left( E_T X H_i - E_i X H_T \right) \cdot z \, dx \, dy$$

where $E_T$, $H_T$ are the aperture fields when the antenna is transmitting, $C$ is a complex constant, and $z$ is the unit vector normal to the xy (aperture) plane. For digital computer implementation, the integral in Equation (1) reduces to a double summation, and the equal-area elements $dx \, dy$ is compiled and can be absorbed into the constant $C$.

In its present form, the program accommodates only one radome shape, viz., the tangent cone. The length, diameter and fineness ratio are, of course, all variable in the input data. Monolithic and multi-layer wall configurations can be analyzed; however, only uniform wall configurations whose properties do not vary from point to point on the
wall can be handled. Provisions are made to allow for a metal tip on the radome whose effect is aperture blockage.

The geometry subroutines provide for three separate coordinate systems and the point and vector transformations among them. A reference coordinate system is provided to orient the antenna/radome combination with respect to other bodies. The coordinate systems for the antenna and the radome comprise the other two systems. Boresight error and pattern computations are carried out and expressed in the antenna coordinate system.

The primary outputs of the program are boresight error (mrad.), boresight error slope (deg./deg.), gain loss, and when selected, principal plane patterns. Outputs include both printing and plotting (Calcomp). Plotting options allow for selection of aperture fields with and without the radome. A feature is also provided to either obtain or suppress intermediate calculated results for debugging purposes.

Boresight error calculations for monopulse antennas are carried out by setting the first target return at a known direction within a few degrees of true boresight. The responses in the two difference channels and the sum channel are then computed and stored. Another set of responses for a return 180° away from the first is computed next. The two sets of data are then used to construct a linear tracking model in the two orthogonal planes, and the process is repeated until a boresight null is indicated. The true direction of arrival of the plane wave at this point represents the boresight error directly.

The current subroutine used to characterize the antenna permits selection of various polarizations and two aperture distributions. A uniform, circular aperture distribution having vertical, horizontal or
circular (LHR or RHC) polarizations is one combination. The second
distribution is a tapered rectangular distribution having vertical polar-
ization as found in flat plate antennas. This basic subroutine would not
be difficult to modify to accommodate other distributions, such as rec-
tangular aperture with cosine taper.

Computation time is independent of radome size but depends on the
number of samples used in the aperture. For 256 sample points (16 X 16
array), the time to compute the received voltages in the three channels
is 1.5 seconds.

The program is organized as a main program and a number of sup-
porting subroutines, all written in Fortran IV. The complete program,
including plotting software, contains thirty four subroutines. The core
storage required for the complete program, including all library and
system I/O routines, is just over 46,000 (decimal) words. Integer, real
and complex variables and arrays are utilized. Single, double and three-
dimensional data arrays are present. Only single precision variables and
computations are required with the 60-bit word available on the Cyber 70
at Georgia Tech.

References
1. E. B. Joy and G. K. Huddleston, "Radome Effects on Ground
   Mapping Radar", Contract DAAB01-72-C-0598, U. S. Army
2. E. B. Joy, G. K. Huddleston, R. L. Bassett and C. L. Gorton,
   "Analysis and Evaluation of Radome Materials and Configurations
   for Advanced rf Seekers", Contract DAAB01-73-C-0760, December


Chapter 2

PROGRAM PTFRACP

2-1. Purpose: PTFRACP is a Fortran computer program used to analyze the effects of a tangent ogive radome on the performance of a monopulse aperture antenna. It consists of a main program and 14 subroutines. It uses complex arithmetic and requires 57121 octal words of core memory for execution on the CCR Cyber 70 system (4-bit words) at Georgia Institute of Technology. Execution time to compute boresight error on the Cyber 70 is approximately two seconds per look direction when the antenna aperture is represented by 16 x 16 = 256 sample data points. Execution time to compute transmitting and receiving patterns and aperture near fields, and to compute the necessary Calcomp commands for two- and three-dimensional plotting, is approximately 35 seconds for one look direction.

The computer-aided radome analysis uses a receiving formulation based on the Lorentz reciprocity theorem as described earlier [1,2]. The voltage produced at the terminals of a linear antenna by an incident plane wave is given by

\[ V_r(k) = \int (E_T \times H_R - E_R \times H_T) \cdot \hat{n} \, da \tag{1} \]

where \( E_T, H_T \) are the fields produced on the surface \( S \) enclosing the antenna when the antenna is transmitting; \( E_R, H_R \) are the incident fields produced on \( S \) by the incident plane wave or perturbations thereof; \( k \) is a unit vector which points from the antenna toward the direction from which the plane wave arrives; and \( \hat{n} \) is a unit vector normal to the surface \( S \) and pointing
outward. The fields $E_T$, $H_T$ are taken to be those produced in the planar aperture when the antenna is transmitting in the absence of the radome.

The geometrical optics approximation

$$H_T = \frac{n \times E_T}{\eta} \tag{2}$$

is used to generate the magnetic field in the aperture from the aperture illumination specified by $E_T$. Rays are traced from each sample point in the aperture in the direction $k$ to the inner radome wall. The plane wave fields associated with each ray are weighted with the flat panel insertion voltage transmission coefficients as determined by the radome wall configuration, the angle of incidence, and the plane of incidence. The individual contributions are summed up as indicated in Equation (1).

The parameters of the tangent ogive radome are indicated in Figure 1. The outside base diameter $D_{os}$ and fineness ratio $F_{os}$ determine the outside length, according to

$$F_{os} = L_{os} / D_{os} \tag{3}$$

A similar relation holds for the inside dimensions; viz.,

$$F_{is} = L_{is} / D_{is} \tag{4}$$

The radius of curvature of the outside wall $R_{os}$ is given by

$$R_{os} = F_{os} D_{os} / \sin \left(\pi - 2 \tan^{-1}(2F_{os})\right) \tag{5}$$
Figure 2-1. Tangent Ogive Radome Geometry.
and the dimension \( B \) is given by

\[
B = R_{OS} - D_{OS}/2
\]  

(6)

The placements of a bulkhead (bottom disk) and metal tip (top disk) can be specified by \( Z_{BOT} \) and \( Z_{TOP} \), respectively. The thickness, dielectric constant, and loss tangent of the wall may also be specified for up to \( N=5 \) layers. The radome is assumed to be a body of revolution with uniform wall dimensions independent of location. The dashed cylindrical shape of a diameter \( D \) in Figure 2-1 was used earlier to simulate a laser-induced defect and is not pertinent here.

The subroutine which generates the antenna aperture fields represents two types of antennas: circular aperture with uniform illumination and any one of four polarizations (vertical, horizontal, RHC, LHC); flat plate antenna with tapered illumination and vertical polarization. For each antenna, the fields are computed for one of three selected channels: sum, azimuth difference, elevation difference. Inputs include the number of samples \( N_X, N_Y \) and the aperture diameter \( D_{AP}/\lambda \) in wavelengths.

The antenna/radome orientation is specified according to the parameter defined in Figure 2-2. The angle \( \phi \) selects the plane of scan of the radome tip with respect to the antenna coordinate system: \( \phi = 0^\circ \) selects the azimuth plane; \( \phi = 90^\circ \) selects the elevation plane. The angle \( \theta \) on the tip in the selected plane.

The subroutine computes bore sight errors in the azimuth and elevation plane of the antenna. The radome orientation is specified by \( \phi \) and \( \theta \). To find target return plane wave is made to arrive from the direction.
Figure 2-2. Coordinate Systems Used in Radome Analysis.
where $\theta_{os}$ is the initial specified offset angle; e.g., $2^\circ$. The voltage received by each channel is computed and stored. The second return is made to arrive from

$$k_2 = x_A (-\sin \theta_{os}) + y_A (-\sin \theta_{os}) + z_A \sqrt{1 - 2 \sin^2 \theta_{os}}$$ (8)

and the voltages are again computed. The data from these two points are used to construct a linear tracking model in the two planes, and a direction of arrival $k$ is predicted which will yield null indications in both planes. The process is repeated until a desired error tolerance is satisfied or a maximum number of iterations is exceeded. Upon completion, the output $k$ indicates the direction from which the plane arrives which yields an electrical boresight indication. If $\alpha$ and $\beta$ represent the boresight error angles in the azimuth and elevation planes, respectively, then they are related to the direction $k = x_A k_x + y_A k_y + z_A k_z$ by

$$\sin \alpha = \frac{k_x}{\sqrt{1 - k_y^2}}$$ (9)

$$\sin \beta = \frac{k_y}{\sqrt{1 - k_x^2}}$$ (10)

where

$$k_z = \sqrt{1 - k_x^2 - k_y^2}$$ (11)
Options are also provided whereby principal plane patterns as shown in Figure 2-3 and additional outputs around boresight can be computed and printed. These options are useful when preparing software for a new type of antenna and to ensure correct operation whenever curious results are obtained.

2-2. Usage:

```
DATA APIN/0.0/
DATA ZBOTIN 0.00/
DATA RADIUS/0.0/  
DATA THETA, PHI, AGAM3A/0.0,90.0,0.0/
DATA NX, NY, NXE, NYE, NXY/16,16,256,1,512/
DATA NREC, NS, MX, MY/32,16,16,1/
READ (5,6) TITLE          
READ (5,*) GRAF3D, GRAFSA, GRAFTR, GRAFRV, SUPPRS, IPENC  
READ (5,*) NFINE, NPHI, NTHE, DIAOS, RA, RR, ZTOPIN, FREQ, OSANG          
READ (5,*) LMAX, DMRA, IOPT, RAPMAX, VAIRM, IPOL, ICASE, N, IPWR        
READ (5,*) DINI, ER(I), TD(I) (I=1,N)  
READ (5,*) FINR(I) (I=1,NFINE)  
READ (5,*) PHI(I) (I=1, NPHI)  
READ (5,*) Theta(I) (I=1,NTHE)  
```

2-3. Arguments

a. Inputs. Units of arguments on input are distances in inches, angles in degrees, and frequency in gigahertz, unless otherwise noted. Units of arguments passed to subroutines are centimeters, radians, and
Figure 2.3 Coordinate System for Far Field Patterns
gigahertz. An asterisk is used to denote those DATA arguments that do not normally need to be changed by the user.

APIN* - Height of a cylindrical base section of the tangent ogive radome. It is no longer included in the ray tracing algorithms and should not be changed from its zero value.

ZBOTIN - Distance from base of tangent ogive radome to missile bulkhead (Figure 2-1).

RADIUS* - The radius R used in the far field factor $e^{-jkR/R}$ by Subroutine FAR. Do not change.

THETAA* - Angle $\theta_a$ between z-axis and the position vector $r_a$ to the antenna origin. This angle was used in earlier work to locate the antenna origin in the reference system using spherical coordinates $(r_a, \theta_a, \phi_a)$. Do not change. See Chapter 7.

PHIA* - Angle $\phi_a$ between the projection of $z_a$ axis onto the xy-plane and the x-axis. Do not change.

AGAM3A* - Angle between $z_A$-axis and $z$-axis in Figure 2-2. Do not change.

NX, NY - Integer powers of two equal to the number of sample points in the antenna aperture: e.g., 16, 32, 64, etc. Changing NX and NY necessitates compatible changes in Lines 16-18.

NXE, NYE - Integer powers of two which specify the expanded number of sample points desired when computing the transmitting patterns of the antenna by inverse Fourier transforming the aperture fields.
Subroutine JOYFFT provides this capability of increased resolution in one or both dimensions. Changes in NXE, NYE necessitate compatible changes in Lines 16, 20, 22, and 23. Note that NXE=NXE=NX*NY and either NX=NX or NY=NY.

 NXY - Integer power of two used by Subroutine JOYFFT for dimension of complex working array XYFFT. Note that MX*NX*NY and MY*NY*NXY. See below for MX and MY.

 NREC - Integer power of two equal to the number of points at which to compute the receiving pattern in either principal plane. The received voltage is computed at points \( \theta_i \) equally spaced in \( \sin \theta \), where \( \theta \) is the angle measured from the \( z \)-axis as indicated in Figure 2-1, where \( \sin \theta = -KMAX + (I-1) \times 2 \times KMAX / NREC \), and where \( KMAX = \sin \theta_{\text{max}} < 1.0 \).

 NS - Not used. It was originally used by Subroutine RECBS. Do not remove.

 MX, MY - Integer powers of two equal to the magnification factors desired in the \( k_x \) and \( k_y \) (E-plane) directions, respectively, of the transmitting antenna patterns. Note that the restrictions MX*NY*NX and MY*NY*NXY must be observed. The data cited above indicated increased resolutions in the NX direction of MX=16 and no magnification (MY=1) in the NY direction. Consequently, note that NXE-MX*NX=NXe.
TITLE - A Hollerith string of up to 72 characters which describes briefly the analysis being done. A format of 18A4 is specified and should work for machines with word length greater than or equal to 32 bits. The dimension of TITLE (Line 31) should be at least 18.

GRAF3D - A logical variable used to control the plotting of the incident fields on the antenna aperture. This feature has been removed from the program, and GRAF3D should always be FALSE.

GRAFSA - A logical variable which (if TRUE) controls the plotting of the transmitting power patterns of the antenna as follows: E-plane sum, E-plane difference equation (A_EL), H-plane sum, and H-plane difference azimuth (A_AZ). The radome is absent.

GRAFTR - A logical variable which controls the plotting of the amplitude and phase of the antenna aperture fields in the following order:

\[ E_X, E_Y, E_{XEL}, E_{YEL}, E_{XAZ}, E_{YAZ} \]

GRAFkW - A logical variable which controls the plotting of the receiving patterns of the antenna with radome in the same order as specified under GRAFSA above.

SUPPRS - A logical variable which controls the printing of numerous results as illustrated in the test data in Section 2-6 below. When TRUE, the printing of these numerous results are suppressed. This feature
is convenience to aid in debugging new portions of software prior to making production runs.

IPENCD - An integer variable which selects pen and paper for the Calcomp. This variable may be system dependent. For the Cyber 70, IPENCD=00 yields ballpoint pen and 11" wide plain paper; IPENCD=40 yields a heavier ink pen and the same paper.

NPINE - Integer variable equal to the number of fineness ratios to be considered for the tangent ogive radome; e.g., NPINE=1.

NPHI - Integer variable equal to the number of scan planes; e.g., NPHI=2.

NTHE - Integer variable equal to the number of angles in each scan plane at which to compute boresight errors, etc. Note: The program is set up to iterate on fineness ratio, scan plane, and scan angle as outer loop, middle loop, and inner loop, respectively. Therefore, for each of NPINE fineness ratios, the analysis will be done for NTHE scan angles in NPHI different scan planes.

DIA - Real variable equal to the outside base diameter (in.) of the radome. See Figure 2-1.

RA - Real variable equal to the distance (in.) from the gimbal point to the antenna aperture.

RR - Real variable equal to the distance (in.) from the gimbal point to the base of the radome.
ZTOPIN - Real variable equal to the distance (in.) from the base of the radome to the face of a metal tip on the radome.

FREQ - Real variable equal to the frequency of operation in gigahertz.

OSANG - Real variable equal to the offset angle in degrees at which the first target return is to arrive on the antenna; e.g., OSANG=3.0.

LMAX - Integer variable equal to the maximum number of iterations allowed by Subroutine RECBS in computing boresight error; e.g., LMAX=5.

DMRAD - Real variable equal to the tolerance in milliradians allowed on computing boresight error; e.g., DMRAD=0.1.

IOPT - Integer variable which selects the polarization of the incident plane wave as follows:
   1. Linear, elevation component
   2. Linear, azimuth component
   3. Right hand circular
   4. Left hand circular

RAPMAX - Real variable equal to the maximum radius (in.) of the antenna aperture. See Figure 3-1.

VAIRM - Real variable equal to the maximum amplitude of sum channel received voltage without radome. Any real value can be entered for this variable since a subsequent program modification (Lines 326-328) causes VAIRM to be computed automatically.

IFOL - Integer variable which selects the polarization of the antenna when ICASE=1 according to the
same code as used above for IOPT.

ICASE - Integer variable which selects one of two types of antenna apertures for the analysis: ICASE=1 or 2 selects a circular aperture with uniform illumination; ICASE=3 selects a flat plate antenna with programmed illumination. See Subroutine HACNF in Chapter 3.

N - Integer variable equal to the number of layers (up to 5) in the radome wall. For cases where more than 5 layers are required, the dimensional arrays on Line 37 must be changed to NN=N+1.

IPWR - Integer variable which selects the component for which to compute the transmitting power patterns as follows:
1. Elevation Component
2. Azimuth Component
3. Total power

DIN,ER,TU - Subscripted real variables equal to the thickness (in.), dielectric constant ($\varepsilon_r$), and loss tangent (tan $\delta$) of each layer of the radome wall. $I=1$ corresponds to the first layer and is the layer on the exit side of the wall. Layer $N$ is the first layer encountered by the incident plane wave. See Subroutine WALL.

FINR - Subscripted real variable equal to NFINE fineness ratios.

PHI - Subscripted real variable equal to NPHI angles (degrees) which specify the scan planes.
THETA - Subscripted real variable equal to NT HE angles (degrees) which specify the scan angles in the scan plane.

b. Outputs. The parameters of analysis which are computed and outputted by the program depend on whether SUPPRS is true. In what follows, it is assumed that SUPPRS=FALSE so that all possible outputs are obtained. Since many of the original input parameters are printed directly, only those parameters not already explained above will be included below. Additional clarification may be found in Section 2-6.

TABLE - Logical variable which, if TRUE, causes a look-up table to be used in computing transmission coefficients. When SUPPRS=FALSE, an abbreviated table of transmission coefficients of the radome wall is printed by Subroutine WALL with variables as explained immediately below.

ANGLE - Real variable equal to the angle of incidence (degrees) of the plane wave on a plane sheet of infinite extent having the layered configuration specified for the radome wall. The entries in the table are computed at 250 equal increments in \( \sin \theta_i \), but only every fifth result is printed.

TPERI,TPARI - Complex variables equal to the voltage insertion transmission coefficients of the sheet for the two cases of \( E_i \) perpendicular to the plane of incidence (\( T_{\perp} \)) and \( E_i \) parallel to the plane of incidence (\( T_{\parallel} \)). In the printed table, the power transmission coefficients \( |T_{\perp}|^2 \) are...
$|T_{\parallel}|$ are printed; adjacent to each, the phases of $T_{\parallel}$ and $T_{\perp}$ are also printed.

**$K_{\parallel,\perp}$** - Complex variable equal to the reflection coefficients $k_{\parallel}$, $k_{\perp}$ of the plane dielectric sheet. Actually, $|k_{\parallel}|$ and $|k_{\perp}|$ are printed, accompanied by the phases $k_{\parallel}$ and $k_{\perp}$.

**$K_{\rm{MAX}}$** - Real variable equal to the folding wavenumber associated with sampling the aperture fields according to $K_{\rm{MAX}} = \lambda_A/(\lambda_x/\lambda)$, where $\lambda_x$ is the distance between samples. See Subroutines HACNE and EPTA.

**$K_{\rm{MIN}}$** - Real variable equal to $\lambda_A/2\lambda$.

**$K_{\rm{M}}, K_{\rm{Y}}$** - Real variables equal to the folding wavenumbers of the principal plane patterns after magnification for increased resolution. $K_{\rm{M}} K_{\rm{MAX}}=K_{\rm{Y}}/K_{\rm{M}}$ applies to the $H$-plane, $K_{\rm{Y}}=K_{\rm{MAX}} N_{\theta}/(N_{\varphi} N_{\theta})$ and applies to the $E$-plane. Usually, the expanded dimension $N_{\varphi}$ and magnification factor $K_{\theta}$ are selected so that $K_{\theta} < K_{\theta Max}$.

Also, $N_{\varphi}$ and $MY$ are usually selected so that $MY < K_{\rm{MAX}}$.

**$\text{MIN}, \text{MAX}$** - Real variables equal to the minimum and maximum values of the amplitude of the complex arrays containing the aperture fields as processed by Subroutine NORM in preparation for 3D plotting by Subroutine PLOT.
ROS - Real variable equal to the radius of curvature of the outside shape of the tangent ogive radome.

ROS - Real variable equal to the distance B in inches defined in Figure 2-1.

FINOS - Real variable equal to the fineness ratio of the radome as based on the outside dimensions.

FINIS - Real variable equal to the fineness ratio of the radome as based on the inside dimensions.

The following variables are printed when the receiving patterns are computed and printed:

ICUT - Integer variable which defines the E-plane (ICUT=1) or H-plane (ICUT=2) pattern. See Figure 2-3.

ICOMP - Integer variable which defines the field component of the plane wave incident on the receiving antenna: ICOMP=1 for elevation component; ICOMP=2 for azimuth component.

KMAX - Real variable equal to the sine of the maximum angle off broadside for which the received voltage is computed.

NREC - Integer variable (power of 2) equal to the number of points at which the receiving pattern is computed. The pattern is computed at NREC points spaced equally in $k_{xy} = \sin \theta$ according to $A k_{xy} = 2 KMAX/NREC$.

DK - Real variable equal to $2 KMAX/NREC$. 
ANGMAX = real variable equal to $\sin^{-1}(K_{MAX})$.

The receiving pattern is computed at 580 points and summed after subroutine MAGFET to 58 points equally spaced in time over the range $(-K_{MAX}, K_{MAX})$. The parameters are printed: range in degrees, amplitude in decibels, and phase in degrees. Only every fourth point on the 296 points is printed. The receiving patterns are printed in the following order:

E-Plane: [E1', E1]

H-Plane: [H1', H1]

Subroutine KB2M maintains a count NMAX of the number of rays actually traced from points in the aperture to the target wall. When KB2M fails, NMAX is reset to zero.

Subroutine KB2M computes the bore sight error of the antenna at the point traced by the ray. When KB2M fails, the following parameters are printed:

K1, K2 = real subscripted variables containing the direction cosines $(k_x, k_y, k_z)$ of the last and next to last true directions to the target. One of these variables is equal to K, the subscripted variable containing the direction cosines of the last target return.

AZTM, ELTM = real variables equal to the bore sight error in the H-plane and E-plane associated with the last target return $(k_x', k_y', k_z')$. Expressed in milliradians, these errors are computed according to

\[
AZTM = \sin^{-1}\left(\frac{k_x}{\sqrt{1-k_y^2}}\right) \times 1000, \\
ELTM = \sin^{-1}\left(\frac{k_y}{\sqrt{1-k_x^2}}\right) \times 1000.
\]
Let \( k = \frac{x}{A_x} + \frac{y}{A_y} + \frac{z}{A_z} \). Then AZM is the angle between the \( z_A \)-axis and the projection of \( k \) onto the \( x_A, y_A \) (azimuth) plane. ELTM is the angle between the \( z_A \)-axis and the projection of \( k \) onto the \( y_A, z_A \) (elevation) plane.

**MEAS, MECHE** - Real variables equal to the monopulse error slopes in the azimuth and elevation channels, expressed in units of volts per degree, where the maximum signal received by the two channels is considered to be one volt.

**CAL, ULI** - Four real gratings equal to the received tracking functions \( L_{\text{CAL}} \) corresponding to the target returns \( K_{\text{CAL}} \) and \( K_{\text{ULI}} \). Given \( L_{\text{CAL}} \), \( L_{\text{ULI}} \), \( K_{\text{CAL}} \), and \( K_{\text{ULI}} \), the following equations are solved:

\[
I_{\text{CAL}} = K_{\text{CAL}} \text{ and } I_{\text{ULI}} = K_{\text{ULI}}.
\]

**MAX** - Real variable equal to the maximum amplitude of the received signal channel voltages.

**ELTM** - Integer variable equal to the number of elevation channel returns in any automatic search to compute boresight error.

Subroutine MEBS also computes and prints six additional monopulse error gratings to define the adjacent boresight direction \( k_{\text{adj}} \). The directions \( k_{\text{adj}} \) chosen are in the plane \( x, y \) and are spaced one milliradian apart over the range -90° to 90° and centered on the direction \( k \). The variables printed are as follows:

**ANG** - Real variable equal to the angle in milliradians between \( k_{\text{adj}} \) and \( k \).

**VRAGE** - Real variable equal to 1 if \( k_{\text{adj}} \) for the target return is in the direction \( k \) for the azimuth and elevation channels, respectively.
DAZ, DEL - Amplitude and phase (degrees) of the complex voltages received on the \( A_{AZ} \) and \( A_{EL} \) channels, respectively, for target return \( k \).

SLEAZ, SLEEL - Average values of the monopulse error slopes (volts/degree) in the azimuth and elevation channels, respectively, obtained by a linear approximation of the tracking functions based on their values at \( AN = 3 \) mrad. For example,

\[
SLEAZ = \frac{VRAZ(3 \text{ mrad}) - VRAZ(-3 \text{ mrad})}{15} \text{ (degrees)}
\]
b. Supporting Subroutines. Thirty four supporting subroutines are required by RTFRACP. The purpose of each one is briefly described below.

1. **HACNF**—Computes complex vector aperture electric fields of antenna for all three monopulse channels at NX x NY sample points.

2. **ORIENT**—Computes matrices ROTATE and TRANSLATE used for coordinate transformations by Subroutines POINT and VECTOR.

3. **POINT**—Transforms a point \( P(x_A, y_A, z_A) \) in antenna system to the same point \( P(x_R, y_R, z_R) \) in radome coordinate system, and vice versa.

4. **VECTOR**—Transforms a vector from radome to antenna coordinate system, and vice versa.

5. **INPW**—Computes the rectangular electric field components of a plane wave incident from the direction \( k_z \) in antenna coordinate. The power density of the plane wave is unity.

6. **HAW**—Computes the voltage received by each channel of the antenna for a plane wave \( E_{X ', y ', z } \) incident at the range from the direction \( k_A(z, k) \). Subroutine RTFRACP calls the following subroutines:

   - **POINT**, **RAD**, **RADM**.

   These routines are required to calculate angles:

   - **HAW**, **HFF**, **THFA**, **RDF**, **HFFN**, **RAD**, **RADM**.
dielectric wall with unit inner normal \( \hat{n} \). The unit vectors \( \hat{\mathbf{x}}, \hat{\mathbf{n}} \) are used to resolve the incident plane wave into vector components perpendicular and parallel to the plane of incidence, and to determine the angle of incidence. RXMIT calls Subroutines WALL and AMPHS.

(9) WALL--Computes the voltage insertion transmission coefficients of flat panel model of the radome wall as function of the sine of the incidence angle.

(10) AXR--Computes real vector cross product \( \mathbf{C} = \mathbf{A} \times \mathbf{B} \).

(11) CAXB--Computes the complex vector cross product \( \mathbf{C} = \mathbf{A} \times \mathbf{B} \).

(12) RECBS--Computes boresight errors of antenna enclosed by the radome for the specified orientation, fineness ratio, etc. RECBS calls Subroutines INCPW, RECM, and AMPHS.

(13) RECPTN--Computes receiving patterns of all three channels. RECPTN calls Subroutines INCPW and RECM.

(14) OGIVE--Computes point of intersection of ray and ogive by solving a quartic equation. OGIVE calls Subroutines CBRT, SQRT, and XY.

(15) CBRT--Computes cube root.

(16) SQRT--Computes square root with test for negative argument.

(17) OGIVE--Computes the unit inward normal vector to the ogive surface at the point \( \mathbf{P}(x_R, y_R, z_R) \).

(18) XY--Called by Subroutine OGIVE to compute the \( x_R \) and \( y_R \) components at the point of intersection of a ray on the ogive surface.

(19) XE--Computes the point of intersection of a ray and the inner wall representing the bulkhead inside the radome.
(20) BISKH--Computes unit normal vector to bulkhead \((n + k)\).

(21) TDisk--Computes the point of intersection of a ray and the base of the metal tip on the radome.

(22) TDISKH--Computes unit normal vector to metal tip \((n - z_K)\).

(23) PAK--Computes the amplitude of the power pattern from the complex plane wave spectrum \(A_x(k_x, k_y), A_y(k_x, k_y)\) of an antenna.

(24) AMPLH--Converts a complex number from rectangular to polar form. This subroutine utilizes the intrinsic function ATAN2. The amplitude produced is linear (not decibels), and the phase is in degrees on the range \((-180, 180)\).

(25) DBP--Converts a real, two-dimensional array from linear to logarithmic values in decibels on the range 0 to -40 dB.

(26) NORMAL--Normalizes a two-dimensional real array to values between 0 and 1.

(27) CNPLTH--Plots single dimensional far field patterns on axes patterned after standard pattern recorder paper. CNPLTH calls Subroutine PSI in addition to the usual Calcomp subroutines.

(28) PSI--Used by Subroutine CNPLTH to compute the azimuthal angle \(\phi\).

(29) PLT3DH--Yields three-dimensional plots of the data in the two-dimensional real array FIELD. PLT3DH calls Subroutines PLT, NOAM as well as the usual Calcomp subroutines.

(30) PLTT--Used by Subroutine PLT3DH to eliminate moving the pen for hidden lines.
A. DDA.--Computes the Fast Fourier Transform of a one-dimensional complex array having 2**EXP elements. Proper selection is machine dependent.

B. MTDD.--Provides increased resolution of a sample function.

C. FTFT.--Adapts more extended Fourier Transform technique.

D. JHTTF.--Provides increased resolution of a selected portion of an extended Fourier transform. JOINT with

E. BROK.--Uses subroutine JOINT to ensure that a given

F. SUBR.--......

**REFERENCES**

The following should be the program listing in Section I.3.1 and

The next number on the right-hand margin of that listing.

**The Yes.**

**Explanation.**

**Table:**

| Yes.  | All variables contained with the letter F in the
|-------| women's chart.

**Table:**

| Yes.  | All are variables and array descriptions. State
|-------| variables that are to be visible. The amount of
|-------| state is to be a maximum of one byte. The
|-------| amount of state is to be a maximum of
|-------| that only one file is available, only one file
|-------| and that only one file is to be used.

**Table:**

| Yes.  | Subsequent use is a subsequent state to transient
|-------| variables to support a not directly defined by
|-------| DEFS. The labels are generated from the names of
|-------| the variable which receive the value, and
|-------| the label is terminated with the letter F to end
|-------| the process. Thus, DEFS denotes variables end
|-------| and of the DATA DEFS.
Lines 40-42: Declare namelists for printing data. These namelists are no longer used except for occasional debugging purposes.

Lines 43-57: Set data in DATA statements as described above in Section 2-3.

Lines 61-62: Set SMAX and VMAX to unity to prevent division by zero.

Lines 63-64: Read and write TITLE according to I8A4 format.

Lines 65-67: Read input data using free-field format.

Line 68: Compute sine of the offset angle $\theta_{OS}$.

Line 69: Set TABLE=FALSE so that normalizing factor VAIM can be computed (lines 819-829) via a call to Subroutines REC and RMIT. In the latter, TABLE=FALSE causes $T_1$, $T_{||}$ to be set to unity as in the case of no radome.

Lines 71-75: Write input data.

Lines 76-77: Read input data and set VAIM needlessly.

Lines 78-104: Comments explaining input variables.

Line 105: Set NN=N+1: Number of wall layers plus one.

Line 106: Initialize DINC: total thickness of radome wall in inches.

Lines 107-109: Read wall data and compute total thickness.

Line 110: Compute DIAM: inside base diameter of the radome in inches.

Lines 111-112: Compute indices of the center element of near-field arrays corresponding to $x^2+y^2=0$. 
Lines 113-114: Write array dimensional data.

Lines 115-122: Read fineness ratios, scan planes, and scan angles.

Lines 123-126: Compute wavelength in inches and centimeters.

\[ \text{Compute } \lambda = \frac{2 \pi}{\lambda_{cm}}. \]

Lines 127-128: Call RXMIT and compute table of transmission coefficients versus sine of incidence angle. The first call to RXMIT builds the table. Subsequent calls use the table if TABLE=TRUE.

Line 129: Compute DAPWL= diameter of antenna aperture in wavelengths.

Lines 130-139: Convert variables in inches to centimeters for input to subroutines. Some variables are multiply defined to avoid conflicts in labeled common; e.g., ZBOT and Z1. Note that DIACM is the inside diameter of the radome in centimeters.

Lines 140-144: Convert angles from degrees to radians using RAD=\(\pi/180\).

Lines 145-151: Compute near fields of three channel monopulse antenna using Subroutine HACNF.

Lines 152-158: Set KYMAX=KMAX, compute magnified folding wavenumbers KXM, KYM, and print results.

Lines 159-177: Initialize Calcomp plotter, if required. The commented initialization (lines 164-174) applies to the IBM 3033 system at JHU/APL.

Note: Lines 176-178 are used to plot the near fields of the antenna and/or the transmitting principal plane power patterns.
Lines 178-179: Initialize the maximum values FMXEL, FMXDAZ of the E- and H-plane patterns so that when used initially as inputs to Subroutine FAR, the resulting pattern will be normalized with respect to its own maximum and FMXEL and FMXDAZ will be set equal to these respective maxima. On subsequent calls to FAR, the resulting patterns will be normalized with respect to FMXEL and FMXDAZ. Hence, the relative gain of the difference and sum patterns will be correctly displayed in the graphs.

Line 180: Iterate for each of three monopulse antenna channels.

Lines 181-190: Equate complex arrays EXT, EYT to the selected near field and compute the amplitude NF of EXT.

Line 191: Assume transmitting near fields are to be plotted (GRAFTR=T).

Line 193: Call Subroutine PLT3DH to plot the amplitude of EXT. The inputs XSIZE=6., YSIZE=2.5, HEIGHT=2.5 yield a 3D plot that will fit on an 8½" x 11" report page. The inputs NF, NX, NY specify the real array to be plotted and its dimensions. The input NMZ=.TRUE. directs the subroutine to normalize NF so that its values be between 0 and 1. The input LDR=.FALSE. indicates that the array NF contains linear values rather than logarithmic values (decibels).

Lines 194-201: Compute and plot phase of EXT on a scale of -180 degrees to +180 degrees. Note that Line 190 ensures that the real array NF contains these phase values scaled to the required 0 to 1 range.
Line 202-213: Repeat amplitude and phase 3D plots for EYT.

Line 216: Assume GFAFSA=T so that principal plane patterns are plotted.

Line 219: If IP=3, go to Line 243 and plot H-plane patterns; otherwise, plot E-plane patterns.

Line 222: Call Subroutine JOYFFT to calculate the inverse Fourier transform of the $x_A$-component of near field EXT to produce the plane wave spectrum XEEL from which the radiation field can be computed. In the process of computing the transform, provide increased resolution from $x_X \times x_Y$ points to $x_E \times x_{AE}$ points through the point $(x_C, x_Y)$ in the array EXT. In the $k_x$ direction, the plane wave spectrum is magnified by $x_Y$; it is magnified by $x_X$ in the $k_y$ direction. The array FFTXY is a working array.

Line 223: Repeat for EYT to produce the plane wave spectrum YEEL for the $y_A$-component of field.

Line 224: Call Subroutine FAR to calculate the E-plane elevation (IPWR=3) power pattern FSEL of the near field at equal samples in sine over the range $(-KXM, KXM - 2K)$. If FMXEL is 0 (and it is for IP=1), normalize FSEL with respect to its own maximum.

Line 226: Call Subroutine DBPV and convert the power pattern to decibels on a scale of 0 to -40 dB.

Lines 227-233: Scale the values in FSEL to the range of 0 to 1 for plotting.
Line 231: Call Subroutine CNPLTH and plot the power pattern. If \( KXM < 1 \), the pattern is plotted over the angular range corresponding to \( \sin^{-1}(KXM) \); if \( KXM > 1 \), the angular angle is \((-90^\circ, 90^\circ)\). Subroutine CNPLTH actually plots conical cuts corresponding to \( k_x = \text{constant} \) or \( k_y = \text{constant} \) as specified by inputs \( KXC, KYC \). In the call here, \( KXC = KYC = 0 \) so that a principal pattern is produced.

Lines 232-236: Write a figure title for the plot and establish a new origin for the next plot.

Line 237: If \( IP = 2 \), the E-plane patterns are finished.

Lines 238-242: Since JOYFFT changes the input arrays \( EXT, EYT \), it is necessary to recompute them so that increased resolution can be obtained in the plane wave spectra in the H-plane.

Lines 243-258: Repeat computation and plotting for H-plane power patterns.

Line 260: Iterate the radome analysis for NFINE fineness ratios.

Line 261: Set FINE = outside fineness ratio.

Lines 262-266: Calculate and write \( R_{OS}, R, F_{OS}, F_{IS} \) as defined in Figure 2-1 for the radome geometry.

Line 267: Compute RDML = distance from the base of the radome to the theoretical tip on the inside of the radome.

Lines 268-272: If \( ZTOPIN < RDML \), the radome has a metal tip, and a message is written to that effect.

Lines 273-283: Compute parameters needed by Subroutine OGIVE to describe the radome shape. \( R \) and \( B \) are in centimeters.
and apply to the inside dimensions. AP, the height of the cylinder in centimeters, is not used. \( \text{RTSQ} \) = square of the radius of the top disk. \( \text{RBSQ} \) = square of the radius of the bottom disk (bulkhead). The other variables, \( \text{BSQ}, \text{RINV}, \text{RSQI}, \text{RP}, \) and \( \text{RP2} \), are precalculated here to speed later computations in OGIVE.

Line 285: Compute conversion factor DPMR for converting milli-radians to degrees.

Lines 286-288: Initialize the "last" values of boresight error in azimuth (AZL) and elevation (ELL) and the "last" value THL of scan angle. These variables are used later to compute boresight error slope in degrees per degree from the present and last values of boresight error.

Lines 289-292: Write title for analysis results.

Lines 293-295: Write parameters of radome wall.

Lines 296-299: Write heading for table of boresight error and gain data.

Lines 300-303: Write this same data to logical unit 7 for subsequent storage as a disk file, if desired.

Line 309: Iterate the radome analysis for \( NPHI \) scan planes.

Lines 310-312: Compute \( \phi_r \) in radians as required by Subroutine ORIENT.

Line 313: Iterate the analysis for \( NTHE \) scan angles in each scan plane.

Lines 314-316: Compute \( \theta_r \) in radians as required by Subroutine ORIENT.

Line 317: Call Subroutine ORIENT and compute the rotation matrix \( \text{ROTATE} \) and translation matrix \( \text{TRANSL} \) required for coordinate transformations using Subroutines \text{POINT} \ and \text{VECTOR}.
Line 318: On the first iteration, TABLE is false so that the maximum amplitude of the received voltage on the sum channel is computed without the radome.

Line 319-322: Set the direction cosines of the incident plane wave so that it arrives from the $z_A$ direction.

Line 323: Call Subroutine INCPW and compute the rectangular components $PW$ of the incident plane wave having polarization specified by IOPT.

Lines 324-325: Set TSUP=T and TABLE=F so that an air radome wall be used and so that printing by Subroutines RXMIT and RECM will be suppressed.

Lines 326-327: Call Subroutine RECM and compute the complex voltages $VR$ received on the sum, difference elevation, and difference azimuth channels, respectively, corresponding to $VR(I), I=1,3$.

Line 328: Compute $V_{AIRM} = |VR(I)|$.

Line 329: Set TABLE=T so that on subsequent iterations $V_{AIRM}$ will not be recomputed, and so that the table of transmission coefficients will be utilized when RXMIT is called.

Line 330: If SUPPRS=F, compute and print the E-plane and H-plane receiving power patterns of the antenna with the radome in place.

Lines 333-334: Iterate in J for E-plane (ICUT=1) and H-plane (ICUT=2) patterns.

Line 335: Set the desired far field component.
Lines 336-337: Set $\text{KMAX} = \sin^{-1}(\theta) = 0.06$. If $\text{KMAX}$, as computed by HACNP, is less than $\text{KMAX}$, then use the smaller as the maximum angle in the principal plane at which to compute the pattern.

Line 338: Set the temporary logical variable TSUP=T so that printing will be suppressed.

Lines 339-340: Call Subroutine RECPHit and compute the complex received voltages on each of three channels at NREC points over the range (-KMAX, KMAX - DK).

Lines 341-344: Increase the resolution and print results for all three channels. Do not print results that are known to be identically zero.

Lines 345-346: Transfer the received voltage into a one-dimensional array VREC.

Line 347: If NREC>NXE, there is no need to increase the resolution.

Line 348: Call Subroutine MAGFT to increase the resolution of VREC from NREC points to NXE points. The result is contained in complex array NYFPT on output.

Lines 349-353: Compute linear power pattern.

Line 354: Select NX2-larger of NXE and NREC.

Lines 355-356: Write heading for printed results from Subroutine NORMH.

Line 357: Call Subroutine NORMH to normalize the NX2 values in real array MVREC to be between zero and one. The input argument LDBL:FALSE, since the values are not in degrees.
Line 358: Call Subroutine DBPV to convert the power pattern in MVREC to decibels.

Lines 360-360: Write correct heading for E-plane or H-plane.

Line 361: Compute the increment in sinθ at which the power pattern has been computed and resolved.

Lines 362-366: Scale the power pattern to have values between 0 and 1. If SUPPRN=0, compute the angle "ANL; and the phase of the pattern, and print the results for every fourth angle.

Line 372: If GRAPRNT, plot the receiving power patterns.

Lines 373-376: Call Subroutine CNPLTH and plot the receiving patterns in turn. Write an appropriate figure title following each pattern plot. Re-origin the plotter pen for subsequent plots. The result of Lines 330-343 is four principal plane patterns: E-plane sum, E-plane ΔEL, H-plane sum, H-plane ΔAZ.

Lines 384-390: Call Subroutine PECBS and compute the boresight errors AZT, ELT in the azimuth and elevation planes of the antenna as caused by the radome. On output, the real array KA contains the direction cosines of the last target return and, hence, gives the true direction to the target at the time that the tracking functions in the azimuth and elevation planes indicated the electrical boresight direction.

Line 397: If this is the first iteration in scan angle, do not attempt to compute boresight error slope.
Lines 1-15: Compute boresight error slope (degrees/degree) in azimuth and elevation channels.

Lines 16-35: Set the "last" values of boresight errors and scan angle to the current values in preparation for next iteration.

Line 393: Compute loss in maximum gain of the antenna sum channel due to the radome.

Lines 400-404: Write results to logical units 6 and 7.

Lines 405-409: Write maximum amplitude of received sum voltage VABN without radome.

Line 411: Terminate plotting software.

STOP

END

Test Cases

Four test cases are presented in Appendices A, B, C, and D to demonstrate correct operation of the radome analysis computer program STERACF.

Appendices A and B present the test data and results for a circularly (RHCP) polarized antenna and five-layer tangent ogive radome at a frequency of 11.80285GHz ($\lambda$=1.0 inch). The diameter of the aperture is 11.842. The outside diameter of the radome is 16.267 inches. The fineness ratio is 3.00. In Appendix A, the program is exercised without plotting, and printing is minimized. In Appendix B, all plotting and printing options are exercised.

Appendices C and D present the test data and results for a vertically polarized flat plate antenna of diameter 5.1992\(\lambda\). All other parameters of the analysis are the same as in Appendices A and B. Appendix C contains the
The transmitting and receiving patterns at normal incidence are not in agreement contrary to expectations. The discrepancy is due to the fact that the receiving patterns have a first-order variation characteristic of the geometrical optics approximation used for large. On the other hand, the transmitting patterns have a second-order variation characteristic. An assumption of only magnetic current sources in the aperture. The difference is significant only for angles away from I-normal.
A step-by-step solution for each test case on the model was as follows:

- [Step 1]
- [Step 2]
- [Step 3]

The results showed that [details here].
THIS PAGE TRACS FORMULATION RADOM OE ANALYSIS COMPUTER PROGRAM.

exact as geometric optics and lorentz reciprocity

TO COMPUTE THE DEFLECTIONS AND AURSIGHT CHOPS OF A MONOPULSE

INTERACT WITH A TARGET JPEG RADEME TO AN INCIDENT PLANE

ELECTROMAGNETIC WAVE OF SPECIFIED POLARIZATION (TARGET RETURN).

PFIRAC WAS DEVELOPED AT GEORGIA INSTITUTE OF TECHNOLOGY, ATLANTA

IS A RESEARCH CONTRACT NO. F046153 (ROBERT C.

SUCCESSFUL FOR USE IN A SIGNIFICANT RADOMOE TECHNOLOGY PROGRAM

FOR THE DEPARTMENT OF THE NAVY.

THIS VERSION IS FOR EXECUTION ON CYBER 7274. WITH ONLY MINOR

SYNTAX CHANGES, IT HAS BEEN IMPLEMENTED ON THE IBM 3633 AT JHU-API.

01 PROGRAM PFIRAC(INLUT, OUTPUT, TAPE=INPUT, TAPE=OUTPUT, TAPE7)

02 IMPLICIT REAL(K)

03 REAL G1(0,1), VNFG(0,1), VAM(3)

04 COMPLEX SUMX(0,4), SUMY(0,4), Y02X(16,16), Y02Y(16,16), DELX(16,16)

05 COMPLEX DA2X(16,16), DA2Y(16,16), X02Y(16,16), E02Y(16,16)

06 COMPLEX VH(25), VEC(132,31), VREC(32)

07 REAL FF3(25,1), FFSFO(0,1), NOH(13), P1(3)

08 COMPLEX FVAT(3), FWI(3)

09 COMPLEX XE(25), Y2(25), X2Y(25)

10 COMPLEX XEFL(1,25), Y2EF(1,25)

11 EQUIVALENCE XE(XE(1,1), XEFL(1,1))

12 EQUIVALENCE Y2(Y2(1,1), Y2EF(1,1))

13 EQUIVALENCE FF3(1,1), FFSF(1,1), NOH(13), P1(3)

14 EQUIVALENCE GRAFT, GRAP15, GRAFF, TABLE, SUPPR, TSUP

15 INTEGRAL I10F(132)

16 REAL ESTATE(13), TANSL(3), TITLE(1)

17 REAL FNP(20), FI(20), THETA(20)

18 COMMON/TU00172411,PTSO

19 COMMON/TACO/721,71

20 COMMON/LSK/7707, P30

21 COMMON/LSK/7707, RP5

22 COMMON/LSK/11,11, AD(6), TO(6), Tz, VLUL, 1, N, NO. D(6), 7B, TK

23 COMMON/LSK/11,11, RP1, AD, PINV, 9, RP1, RP2
C
NAMESLIST/GECM/PE,PA,AFIN,ZACTIN,NX,NYE,NVE,NXY,MY,MYG,NYC
NAMESLIST/ECDATA/KYMAX,KXM,KHY
NAMESLIST/NEWLMAX,PAPAD,IOPT,PPMAX,VAIRM
C BOUNDARY VALUES NEEDED BY SUB= TRACE (INCHES, CONVERT TO CM BELOW)
C Z1=ZP COORDINATE OF BOTTOM DISK
C Z2=ZP COORDINATE OF TOP DISK (Z1,Z2 IN CM)
C APIN IS HEIGHT OF CYLINDER IN INCHES, CONVERT TO CM BELOW
C DATA APIN/0.0/
C ZACTIN IS ZP COORD OF BOTTOM DISK (BULKHEAD) IN RADOME COORD IN INCH
C DATA ZBOTIN/0.0/
C KXMAX,KYMAX ARE OUTPUTS OF NEAR FIELD SUBR
C INITIALIZE CONSTANTS
C DATA RADIUS/100/
C DATA THETA,PHI,AGAM3A/0.0,90.0,0.0/
C DATA PI/3.1415926535898/
C************
C DATA NX,NY,NXE,NYD,NXY/16,16,256,1,512/
C DATA NEQ,NS,MY,MY/32,16,16,1/
C************
C
C READ IN DESCRIPTION OF RADOME WALL
SMAX=1.0
VMAX=1.0
READ(5,6)TITLE
WRITE(6,6)TITLE
READ(*,*)GRAFT,GRAFSA,GRAFTK,GRAFR,SUPPR,SIPRCC
6 FORMAT(1L6)
READ(5,*)NFISH,NPHI,NTHA,NIAUS,PA,PZ,TOPIK,FREQ,OSANG
SINGS=SIN(OSANG*PI/180.)
TABLE=.FALSE.
C TABLE IS SET FALSE SO THAT NORMALIZING FACTOR CAN BE COMPUTED.
WRITE(6,205)GRAFT,GRASF,KRAFT,GRARFW,TABLE
205 FORMAT(" GRAFT="*,L2," GRAFS ="*,L2," GRAFTK="*,L2," GRAFRW="*,L2, "," TABLE="*,L2)
WRITE(6,270)NFISH,NPHI,NTHA,CSANG
270 FORMAT(" NFISH="*,I5," NPHI="*,I5," NTHA="*,I5," CSANG="*,F5.2/
READ(5,*)LMAX,PAPAD,IOPT,PAPAX,VAIRD,IPOL,ICASE,N,IPWR

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IF (VAIRM.LE.0) VAIRM=1.0
C DIACS=OUTSIDE DIAMETER OF BASE OF TANGENT OGIVE RADOME
C VAIRM=MAXIMUM DEGC VOLTAGE W/O RADOME AT KX=0., KY=0.
C NFNE=NO. OF FINENESS PATIOS
C NPHI=NUMBER OF SCAN PLANES
C NTHF=NUMBER OF ANGLES IN EACH SCAN PLANE
C DIAIN=INSIDE BASE DIAMETER OF RADOME IN INCHES
C ZTOP=PI COUPC (IN) OF TOP DISK (METAL TIP)
C FREQ=FREQUENCY IN GHZ
C GGRA3D=.TRUE., GIVES 3D PLOTS OF INCIDENT FIELDS ON APERTURE (DELETED)
C GGRA=, .TRUE., GIVES SA PLOTS OF RECEIVING PATTERNS (AZ & EL)
C GGRA=, .TRUE., GIVES SA PLOTS OF TRANSMITTING PATTERN WITHOUT RADOME
C SUPRSE=.TRUE., suppresses the printing of numerous results
C RAPMAX=MAX RADIUS OF ANTENNA APERTURE IN INCHES.
C IOPT SELECTIONS POLARIZATION OF INCIDENT PLANE WAVE.
C =1 ELEV (VERTICAL)
C =2 AZIMUTH (HORIZONTAL)
C =3 RHC
C =4 LHC
C IPOL SELECTIONS POLARIZATION OF ANTENNA WHEN ICASE=1:
C = SAME COOE AS FOR IOPT
C ICASE=1 OR 2 FOR CIRC APERTURE, UNIFORM ILLUMINATION
C =3 FOR FLAT PLATE WITH SPECIFIED ILLUM, VERT POL (CASE III)
C N=nUMBER OF LAYERS IN RADOME WALL
C OSANG=ANGLE IN DEG IN 45 PLANE OFF BORESIGHT OF FIRST TARGET RETURN
C USED BY SUBR PROPS IN GETTING INITIAL DATA.
C IP=WR=1 FOR POWER IN ELEV COMP OF FAR FIELD PATTERN
C =2 FOR AZIMUTH COMP=3 FOR TOTAL POWER.
C NN=NN+1
C DINC=1.
C DO 5 I=1,N
C PFAA(E,*),CJN(I),EP(I),TD(I)
C 5 DINC=DINC(I)+DINC
C DIAIN=DIA0S=DINC*2.
C NX=N/2+1
C NY=N/2+1
C WRITE(E,4) NX, NY, NXE, NYE, NXY, MX, MY
C FORMAT(" NX, NY, NXE, NYE, NXY, MX, MY", 7I4)
C READ FINENESS  IGS FOR THIS RUN—BASED ON OUTSIDE DIMENSIONS
DO 13 I=1,NFIN
13 READ(F*,*)FING(I)
C READ ORIENTATION FOR THIS RUN (DEGREES)
DO 14 I=1,NPH
14 READ(F*,*)PHI(I)
DO 15 I=1,NTH
15 READ(F*,*)THETA(I)
C COMPUTE WAVELENGTH
WLINC=29.7928/(FREQ*2.54)
WLCM=WLIN*2.54
BETA=2.*PI/WLCM
C INITIALIZE TABLE OF KMN COEFFICIENTS
CALL RXMIT(FMT,MT,KX,NORM,PHI,PHAL,TABLE,SUPPR,HNS,BETA)
OEPWL=2.*PI/WLINC
C CONVERT TO CENTIMETERS AND RADIANS
Z40T=Z40TIN*2.54
Z1=Z10T
RSOMAX=12.54*FAPMAX)**2
7TOP=7TOPIN*2.54
7R=7TOP
Z2=ZTOP
RA=RAS*2.54
RR=RRAS*2.54
DIACH=DICH*2.54
PAD=PI/180.0
6 FORMAT(14,4)
THETAA=THETA*PAD
PHI1=PHI4*PAD
ACG3A=ACG3*PAD
C COMPUTE FIELDS OF ANTENNA WHEN XMITTING
CALL HACNF(SUMX,NY,NZ,1,IFOL,1,UFHDOM,DXWLM,KMAX,ICASE)
CALL HACNF(SUMX,NY,NZ,1,IFOL,2,UFHDOM,DXWLM,KMAX,ICASE)
CALL HACNF(SUMX,NY,NZ,1,IFOL,1,UFHDOM,DXWLM,KMAX,ICASE)
CALL HACNF(SUMX,NY,NZ,1,IFOL,2,UFHDOM,DXWLM,KMAX,ICASE)
CALL HACNF(SUMX,NY,NZ,3,IFOL,1,UFHDOM,DXWLM,KMAX,ICASE)
CALL HACNF(SUMX,NY,NZ,3,IFOL,2,UFHDOM,DXWLM,KMAX,ICASE)
KMAX=KMAX
KYM=KMAX*X*NY/X/NX
KYM=KMAX*NYE/XY/LY
WRITE(6,3) KMAX,Y NWL,KXH,KYM
3 FORMAT(" KMAX=",F8.5," XY SPACING=",F8.5,
" WAVELENGTH=", KYN=",F8.5")

C INITIALIZE PLOTTER SOFTWARE
IF (GRAF30.0,P,GRAFSA,0).GRAFTR,GR,A,GRAFRV) GO TO 200
GO TO 205
203 CONTINUE

C---------------------CALCOMP INITIALIZATION---------------------

C CALL TITL36("PACO M ANALYSIS COMPUTER PROGRAM",
C ** G.K. HUDDLESTON **
C * " GEORGIA INSTITUTE OF TECHNOLOGY"
C CALL INIT36("DAY")

C CALL PLCT(C,-3,-3)
C CALL PLOTS(I=1,512,3,IFEND)

C

C

IF (GRAFTR,CP,GRAFSA) GO TO 201
GO TO 205

201 IF=EL=0.
F=DA7=0.
DO 35 IP=1,3
DO 35 J=1,3
IF (IP.EQ.1) EYT(I,J)=SUMX(I,J)
IF (IP.EQ.1) EYT(I,J)=SUMY(I,J)
IF (IP.EQ.2) EYT(I,J)=DELX(I,J)
IF (IP.EQ.2) EYT(I,J)=DELY(I,J)
IF (IP.EQ.3) EYT(I,J)=DZX(I,J)
IF (IP.EQ.3) EYT(I,J)=DZY(I,J)
NE(T,J)=GAPS(EYT(I,J))
35 CONTINUE
IF (.NOT. GRAFT) GO TO 215
C PLOT 3D NEAR FIELDS X-COMPONENTS
   CALL PLT3DH(6.,2.5,2.5,NF,NX,NY,.TRUE.,.FALSE.)
C PLOT PHASE ALSO
   DO 42 I=1,NX
       DO 40 J=1,NY
           NF(I,J)=0.
           CALL AMPHS(EXT(I,J),PLF,AIF)
           NF(I,J)=(AIF+180.)/360.
   40 CONTINUE
   CALL PLT3DH(6.,2.5,2.5,NF,NX,NY,.FALSE.,.FALSE.)
C PLOT 3D NEAR FIELDS Y-COMPONENTS
   DO 45 I=1,NX
       DO 40 J=1,NY
           NF(I,J)=CABP(FYT(I,J))
   45 CONTINUE
   CALL PLT3DH(6.,2.5,2.5,NF,NX,NY,.TRUE.,.FALSE.)
C PLOT PHASE ALSO
   DO 50 I=1,NX
       DO 50 J=1,NY
           NF(I,J)=J.
           CALL AMPHS(FYT(I,J),RLF,AIF)
           NF(I,J)=(AIF+180.)/360.
   50 CONTINUE
   CALL PLT3DH(6.,2.5,2.5,NF,NX,NY,.FALSE.,.FALSE.)
   IF (GRAFSA) GO TO 215
   GO TO 30
215 CONTINUE
   IF (IP.EQ.3) GO TO 220
C CALC FL CUT OF SUM
C NOTE THAT JCYFFT CHANGES EXT, EYT.
   CALL JCYFFT(EXT,NX,NY,NY,MY,MX,NXC,NYC,XEEL,NYE,NXE,XYFFT,NXY,3)
   CALL JCYFFT(EYT,NX,NY,NY,MY,MX,NXC,NYC,YEEL,NYE,NXE,XYFFT,NXY,3)
   CALL FAR(FFSFL,XXFL,JEEL,LYE,NX,FREO,KYM,KXM,RADIUS,IPWR,FMXEL)
C SA PLOTS OF ELEVATION RESULTS
   CALL DARP(FFSFL,NYE,NXE,1)
   DO 216 I=1,NYE
       DO 216 J=1,NXE
   216 CONTINUE
218 CONTINUE
219 CONTINUE
220 CONTINUE
221 CONTINUE
222 CONTINUE
223 CONTINUE
224 CONTINUE
225 CONTINUE
226 CONTINUE
227 CONTINUE
228 CONTINUE
FFSEL(I,J)=1.0+FFSEL(I,J)/40.
216 CONTINUE
CALL CNPLTH(FFSEL,NXE,KXM,C...C)
CALL SYMBOL(5,6,5,14000,37,HFIGURE)  TRANSMITTING ELEVATION P0
$ WRITE(0,33)
RPWR=REAL(IPWR)
CALL NUMBER(999,999,14,RPWR,C,..C)
CALL PLOT(8,5,0,..-3)
IF (IP.EQ.2) GO TO 30
C RECOMPUTE SUMX, SUMY FOR JOYFFT:
CALL HACNF(EXT,IX,NY,1,IPOL,1,DPWH,OXWL,KXMAX,ICASE)
WRITE(6,219) IPWR
219 FORMAT("" IPWR OF PATTERN="",I2)
CALL HACNF(EXT,IX,NY,1,IPOL,2,DPWH,OXWL,KXMAX,ICASE)
226 CALL JOYFFT(EXT,IX,NY,MY,NXC,NYG,YE,NXE,NYE,XYFFT,NXY,3)
CALL JOYFFT(EXT,IX,NY,MY,NXC,NYG,YE,NXE,NYE,XYFFT,NXY,3)
CALL FAR(FFS,YE,NXE,NYE,FREQ,KXM,KYM,RADIUS,IPWR,FMXDAZ)
C SA PLOTS OF AZIMUTH RESULTS
CALL DBPW(FFS,NXE,NYE,1)
DO 10 I=1,NYE,1
DO 10 J=1,NXE
FFS(I,J)=1.0+FFS(I,J)/40.0
10 CONTINUE
CALL CNPLTH(FFS,NXF,KXM,0,..0)
226 CALL SYMBOL(5,6,5,14000,37,HFIGURE)  TRANSMITTING AZIMUTH POWER
$ WRITE(0,37)
CALL NUMBER(999,999,14,RPWR,0,..J)
CALL PLOT(8,5,0,..-3)
30 CONTINUE
205 CONTINUE
C
DO 100 NG=1,NF1NE
FINE=FIRNG(NG)
C CALCULATE INSICE FINENES RATIO
PIN=FINE*DIAS/(SIN(P1-2.*ATAN(2.*FINE)))
BIN=PIN-DIAS/2.
FINE=SQRT((PIN-CINCH)**2-BIN**2)/CIAIN
WRITE(6,22) PIN,BIN,FINE(NG),FINE
266
DIM = FINE/10
IF (ZTOP/IN GT DIM) WRITE(*,25) ZTOP/IN
2 FORMAT(//" THIS RADDOME HAS A TOP DISK AT ZTOP=", E12.5)
COMPUTE PARAMETERS NEEDED BY SUBR OEDGE
R = FINE*DIAC/((SIN(PI-2.*ATAN(2.*FINE)))
B = 1.-DIAC/2.
AP = APIN*2.54,
R752 = (SQR2*(R**2-(ZTOP-AP)**2)-B)**2
R75O = (SQR2*(R**2-(ZTOP-AP)**2)-B)**2
B5O = 3**2
RINV = 1./R
RSQ1 = R**2
RF = RSQ1 - R5O
RP2 = RSQ1 + R5O

DEMR = 163./(PI*10.3.)
AZL = 0.
ELL = 0.
THEL = 0.
WRITE(6,2) TITLE, FINRING, DIACS, ZTOP/IN, FREQ, RA, RR, DAPWL, IPOL.
$CASE, IOPT
DO 8 I=1,N
8 WRITE(6,7) I, CIN(I), ER(I), TC(I)
7 FORMAT(2X,13,F13.5,F10.3,F9.4)
WRITE(6,9)
9 FORMAT(//" PHI THETA BSEEL BSEAZ SLP EZ SLP PAZ GAIN"//
1 " (DEG) (DEG) (MRAO) (DEG/DEG) (DEG/DEG) (DB)"//)
WRITE(7,2) TITLE, FINRING, DIACS, ZTOP/IN, FREQ, RA, RR, DAPWL, IPOL.
$CASE, IOPT
DO 1A I=1,N
1A WRITE(7,7) I, CIN(I), ER(I), TC(I)
WRITE(7,9)
9 FORMAT(//" RESULTS OF RALCME ANALYSIS"//
1 IN" " FINESS =RATIO="", FED.2, 2X,
2 " DIAMETER=", F9.5, " IN. LENGTH=", F9.5, " IN."") FREQUENCY="",
53 3F7,3, "GIZ/ 
54 " FA", FA, 5, " IN, FR", FA, 5, " IN, ANTEENA D=" F8.4, 
55 "AVELENGTHS/ " IPCL="I2", ICASE="I2", IOPT="I2// 
56 " LAYER "THICKNESS(IN)/*1 1 N/AW//",/ 
57 TO 100 IPHI=1, NPHI 
58 PHIP=PHI(IPHI) 
59 PHIP=PHIP+1,PC. 
60 NPHI=PHIR*RAD 
61 DO 100 ITHE=1, NTHE 
62 THEAL=THETA(ITHE) 
63 THEBAR=THETAR+0.5*THETAL 
64 THEBAR=THETAR*PAD 
65 CALL ORIENT(RA,THETAA,PHIA,FR,THETAR,PHIR,AGAM3A,ROTATE,TRANSL 
66 IF (TABLE) GO TO 23 
67 C COMPUTE NORMALIZING FACTORS 
68 KA(1)=0. 
69 KA(2)=6. 
70 KA(3)=1. 
71 CALL INCPO(KA,PIW,ICPT) 
72 TSUP=TRUE. 
73 TABLE=.FALSE. 
74 CALL PDECblems,KA,XX,NX,KMAX,FREQ,ROTATE,TRANSL 
75 SUMX,SUMY,SUMX,DECX,DELY,DAYZ,DAZY,VR,TABLE,TSUP,RSQMAX 
76 VAIIM=CA1S(VR(1)) 
77 TABLE=.TRUE. 
78 23 IF (.NOT.TABLE) GO TO 24 
79 GO TO 350 
80 CONTINUE 
81 DC 320 J=1,2 
82 IGUT=J 
83 ICOMF=IOPT 
84 KMAX=.999 
85 IF (KMAX,GT,KMAX) KMAX=KMAX 
86 TSUP=TRUE. 
87 CALL PECPT(SUMX,SUMY,SUMX,DECX,DELY,DAYZ,DAZY,DX,NX,IGUT,ICOMF,KMAX, 
88 FREQ,VR,ECX,KMAX,KMAX,FREQ,ROTATE,TRANSL,TABLE,TSUP,RSQMAX) 
89 DO 325 K=1,3 
90 ICHAN=44 
91 305 
92 306 
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IF ((IGUT, FG, I) .AND. (ICHAN, EQ, 3)) GO TO 325
IF ((IGUT, FG, I) .AND. (ICHAN, EQ, 2)) GO TO 325
324 IF (26 I=1, NREC)
26 VREC(I) = VREC(I, ICHAN)
IF (NREC, I, NXT) GO TO 31
CALL MAGFFT(VREC, NREC, XYFFT, NXX)
DO 305 I = 1, NXX
305 MVREC(I) = CAPS(XYFFT(I)**2)
GO TO 33
31 DO 32 1 = 1, NXX
32 MVREC(I) = CAPS(MVREC(I)**2)
33 NXX = NXX(NXX, NREC)
WRITE(6, 356)
356 FORMAT(/'** MIN AND MAX VALUES OF REC***G PATTERN: */')
CALL NORMV(MVREC, NXX, 1, .FALSE.)
CALL JBPV(MVREC, NXX, 1, 1)
IF (J.EQ.1) WRITE(6, 357)
IF (J.EQ.2) WRITE(6, 358)
DOX = 2*KMAX/NXX
DO 307 I = 1, NXX, 1
IF (SUPPO) GO TO 307
ANG = ASIN(-KMAX*(I-1)*DK)*180./PI
CALL AMPHS(XYFFT(I), AMP, PHS)
IF (NREC, GN, NXX) CALL AMPHS(MVREC(I), AMP, PHS)
IF (MD(I, 4), 0) WRITE(6, 359) ANG, MVREC(I), PHS
MVREC(I) = 1.0+MVREC(I)/4C
357 MVREC(I) = 1.0+MVREC(I)/4C
358 FORMAT(/** REC***G PATTERN, EL CUT, EL COMP (DRA)**/)
359 FORMAT(/** REC***G PATTERN, AZ CUT, EL COMP (DB)**/)
360 FORMAT(F9.1, 5X, FA, 3, 3X, FA, 1)
IF (I .NOT. CAFERV) GO TO 320
CALL CNPLTF(MVREC, NXX, KMAX, 5, 3, 3)
IF (J.EQ.1) CALL SYM90L(.5, 6.3, 140, 43, 15) RECVG POWER PA
$TPFPN-ELEV PLAN E, 0.43$ RECVG POWER PA
IF (J.EQ.2) CALL SYM90L(.5, 6.5, 140, 41, 15) RECVG POWER PA
$TPERK-AZ PLAN E, 0.41$ RECVG POWER PA
CALL FLOT(.5, 6, 0.0) CONTINUE
325 CONTINUE
320 CONTINUE
C COMPUTE SUB-SIGHT GROUP
275 CONTINUE
    CALL RC383(SUMX, SUMY, DELX, DELY, JAZX, JAZY, NX, NY,
    IF (LMAX, NNC, FCM, DMRAD, ROTATE, TRANS, FREQ, KM, KY, KM, KMAX
    i TABLE, SINC, KA, AZT, ELT, RMSQ, VX, VX, VX, SUPPRS)
    IF (ITHEQ, 1) GO TO 300
    SLPAZ = (AZT - AZT) * DPMR / (THETAL - THL)
    SLPEL = (ELT - ELL) * DPMR / (THETAL - THL)
    330 AZT = AZT
    ELT = ELL
    THL = THETAL
    GAINM = 2C. * ALOG1C (SMAX / VAIHM)
    WRITE (6, 11) HPIM, THETAL, ELT, AZT, SLPEL, SLPAZ, GAINM
    WRITE (7, 11) HPFM, THETAL, ELT, AZT, SLPEL, SLPAZ, GAINM
    11 FORMAT (1X, F5.1, F6.1, F8.2, F9.4, F10.4, F7.1)
    C GRAF3D OPTION HAS BEEN REMOVED.
100 CONTINUE
    WRITE (6, 135) VAIRHM
    105 FORMAT (/" PERCEIVED SUB VOLTAGE WITHOUT OVERCOMEm", E12.5//)
    IF (GRAF3D.0, GRAFSA.0, GRAFT.0, GRAFR.0) CALL PLOT(0, 0, 999)
    STOP
    END
Chapter 3
SUBROUTINE HACNF

3-1. Purpose: To compute near-field aperture distributions for two types of three-channel monopulse antennas: (1) circular aperture with uniform amplitude and phase distributions; (2) flat plate antenna with a programmed amplitude distribution and uniform phase. Four polarizations can be selected for the circular aperture. The flat plate antenna is vertically \( y_A \) polarized only.

3-2. Usage: CALL HACNF (E, NX, NY, ICHAN, IPOL, IXY, DAPWL, DXWL, KMAX, ICASE)

3-3. Arguments

\( E \) - Complex array of NX by NY elements which, on output, contains the values of the specified (IXY) rectangular component (\( x_A \) or \( y_A \)) of the electric field distribution over the specified (ICASE) antenna aperture having the specified (IPOL) polarization for the specified (ICHAN) channel of a three-channel monopulse antenna.

\( NX, NY \) - Even integer number of points in a rectangular array at which the aperture distribution is computed in the \( x_A \) and \( y_A \) directions, respectively. The point \( 1 = NX/2 + 1, J = NY/2 + 1 \) corresponds to \( x_A = 0, y_A = 0 \).
FSW - Integer control variable with values 1, 2, or 3 which selects the sum, elevation difference, or azimuth difference channel, respectively.

FVAR - Integer control variable which selects the antenna polarization as follows:
- Vertical (Y) polarization
- Horizontal (X) polarization
- Right-hand circular
- Left-hand circular

FXY - Integer control variable having values 1 or 2 to select the x_A or y_A component of aperture electric field.

DAWL - Diameter, in wavelengths, of the antenna aperture.

DAXL - Spacing in wavelengths, between samples in aperture in x_A and y_A directions (output).

EMAX - Maximum value of normalized wavenumber corresponding to EMAX = 1./(2.*DAWL) (output).

ICASE - Integer control variable having values 1 or 2 to specify a circular aperture antenna with uniform amplitude and phase. If ICASE=3, a flat plate antenna having a programmed amplitude distribution (see Table 3-2) with vertical polarization is selected.

Notes and Method

ICASE integers NX, NY must each be equal to each other and to an integer power of two, e.g., NX = NY = 16. In addition, when ICASE=3 (flat plate antenna), NX and NY must each be 16.
b. The actual shape of the circular aperture, as approximated by a rectangular array of sample points, is shown in Figure 3-1 for the case of $\text{NX} = \text{NY} = 16$. Row 1 and Column 1 of the array contain null elements. The elements inside and on the boundary of the aperture may contain non-zero values as shown in Table 3-1 for the various cases when ICHAN = 1 (sum channel). Note that specification of $D_{\text{AP}}$ in Figure 3-1 determines the sample spacings according to

$$\Delta x_A = \Delta y_A = \frac{D_{\text{AP}} \cos \alpha}{(N - 2)} = \frac{D_{\text{AP}} \cos \alpha}{(N - 2)}$$

where $\alpha = \tan^{-1}(3/7)$.

The aperture distributions for three monopulse channels are formed by phasing the elements in the four quadrants of the aperture appropriately. The sum channel distribution is formed by assigning equal phases to all elements. The azimuth difference channel is formed by multiplying all elements in Quadrants II and III of the sum distribution by minus one and by zeroing all elements along $x_A = 0$. For the elevation difference channel, Quadrants III and IV are negated, and all elements along the line $y_A = 0$ are made zero for symmetry reasons.

The phasing chosen models a tracking antenna and provides outputs in two orthogonal channels from which the direction of arrival of a target return can be mathematically determined. Let $k$ be a unit vector which points from the antenna origin toward the direction from whence the plane wave (target return) emanates; i.e.,
FIGURE 3-1. APPROXIMATION OF CIRCULAR APERTURE BY RECTANGULAR GRID OF SAMPLE POINTS.
<table>
<thead>
<tr>
<th>IPOL</th>
<th>IXY</th>
<th>Value</th>
<th>Polarization Type</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1</td>
<td>$0 + j0$</td>
<td>Vertical</td>
</tr>
<tr>
<td>1</td>
<td>2</td>
<td>$1 + j0$</td>
<td>&quot;</td>
</tr>
<tr>
<td>2</td>
<td>1</td>
<td>$1 + j0$</td>
<td>Horizontal</td>
</tr>
<tr>
<td>2</td>
<td>2</td>
<td>$0 + j0$</td>
<td>&quot;</td>
</tr>
<tr>
<td>3</td>
<td>1</td>
<td>$0 + j1$</td>
<td>RHC</td>
</tr>
<tr>
<td>3</td>
<td>2</td>
<td>$1 + j0$</td>
<td>&quot;</td>
</tr>
<tr>
<td>4</td>
<td>1</td>
<td>$0 - j1$</td>
<td>LHC</td>
</tr>
<tr>
<td>4</td>
<td>2</td>
<td>$1 + j0$</td>
<td>&quot;</td>
</tr>
</tbody>
</table>
Define the tracking functions for this plane wave as

\[
f_1(k, k_0) = \frac{A_1(k, k_0)}{F(k, k_0)}
\]

where \( A_1 \) represents the output of the elevation (\( \theta \)) or azimuth (\( \phi \)) difference channel and \( F \) represents the sum channel output. Then for small \( k_x > 0 \), the phase of \( f_1 \) is \( +\pi/2 \); for small \( k_x < 0 \), the phase of \( f_1 \) is \( -\pi/2 \). Similarly, for small \( k_y > 0 \), \( \arg(f_1) = \pi/2 \); for small \( k_y < 0 \), \( \arg(f_1) = -\pi/2 \). Hence, the change in phase by \( \pi \) in either channel represents the boresight direction of the antenna, and tracking is done using the imaginary parts of the tracking functions rather than their real parts.

c. The shape and sampling grid used to model the flat plate antenna are shown in Figure 3-2. In Subroutine \( \text{ECNP} \), the integers \( NX \) and \( NY \) must both equal 1, and only linear polarization (\( y_A \)) is applicable to the flat plate antenna (ICASE=3). The phasing of the four quadrants is done as described above to model the three monopulse channels so that tracking can be simulated. Note that specification of \( D_A \) determines the sample spacing according to

\[
S_x = \frac{D_A \cos \theta}{N_x} \quad S_y = \frac{D_A}{N_y} \quad (x = 1, 2)
\]
FIGURE 3-2. GEOMETRY OF FLAT PLATE ANTENNA.
where \( a = \tan^{-1}(A,w) \).

The phase of each sample point in Figure 1-2 for the sum channel is made equal, but the amplitudes are tapered in the \( x_A \) and \( y_A \) directions as shown in Table 1-2. The amplitude distribution is separable and symmetrical so that

\[
E_{\alpha_A}(x_A,y_A) = a(x_A)h(y_A) \quad E_{\beta_A}(-x_A,y_A) = E_{\gamma_A}(x_A,-y_A) \quad (5)
\]

It is noted that samples 10, 15, 14, and 16 are actually specified in the program, and samples 9, 14, 15, and 16 are obtained from them by averaging.

6. Program Flow

Line 10: Assign complex values to ERA to use in generating vertical, horizontal, RHC, and LHC polarization according to H0L.

Line 11-31: Compute the angles \( a \) and the upper bound \( R_{\text{max}} \) of the radius of the circular aperture.

Line 6-91: Ensure that H0L has correct values of 1, 2, 3, or 4.

Line 62: IF H0L is not 4, write error message and stop the program.

Line 63: Ensure that INX 1 or 2.

Line 64: IF INX is not even, stop the program.

Line 65: Test value of ICASE: if ICASE 5 generate fields of flat plate antenna (Lines 97-83); otherwise, generate fields of circular aperture (Lines 86-88).

Line 66-84: Assign complex field value to each sample point \((x_A,y_A,\cdots)\) in the aperture according to the values shown in Table 1-2. If \( \sqrt{x_A^2 + y_A^2} > R_{\text{max}} \), make the
Table 3-2. Symmetrical Amplitude Distribution for Flat Plate Antenna

<table>
<thead>
<tr>
<th>Sample No.</th>
<th>$X_A$</th>
<th>Amplitude</th>
<th>$Y_A$</th>
<th>Amplitude</th>
</tr>
</thead>
<tbody>
<tr>
<td>9</td>
<td>0</td>
<td>1.0280</td>
<td>0</td>
<td>1.0280</td>
</tr>
<tr>
<td>10</td>
<td>$\Delta x$</td>
<td>1.0280</td>
<td>$\Delta y$</td>
<td>1.0280</td>
</tr>
<tr>
<td>11</td>
<td>2$\Delta x$</td>
<td>.9120</td>
<td>2$\Delta y$</td>
<td>.9120</td>
</tr>
<tr>
<td>12</td>
<td>3$\Delta x$</td>
<td>.7959</td>
<td>3$\Delta y$</td>
<td>.8000</td>
</tr>
<tr>
<td>13</td>
<td>4$\Delta x$</td>
<td>.6077</td>
<td>4$\Delta y$</td>
<td>.6155</td>
</tr>
<tr>
<td>14</td>
<td>5$\Delta x$</td>
<td>.4194</td>
<td>5$\Delta y$</td>
<td>.4256</td>
</tr>
<tr>
<td>15</td>
<td>6$\Delta x$</td>
<td>.2097</td>
<td>6$\Delta y$</td>
<td>.2125</td>
</tr>
<tr>
<td>16</td>
<td>7$\Delta x$</td>
<td>0.0</td>
<td>7$\Delta y$</td>
<td>0.0</td>
</tr>
</tbody>
</table>
field value zero (Line 40). Multiply the non-zero elements by CFAC(IPOL) to generate the correct polarization (Line 38).

Lines 42-43: Compute sample spacing $\Delta x_A / \lambda$ and go to statement 60.

Lines 44-46: Error message and STOP.

Line 47-48: Flat plate antenna--if NX#16, write error message and STOP (Lines 109-111).

Line 49: Compute sample spacing $\Delta x_A / \lambda$.

Line 50: Ensure NX=NY

Lines 51-54: Zero all elements in the aperture. If IXY=1 ($x_A$-component), go to statement 60.

Lines 55-62: Assign tapered amplitude values to eight "even" elements in Quadrant III.

Lines 63-71: Compute amplitude values for the "odd" elements in Quadrant III.

Lines 72-75: Compute amplitude values for elements 3-9 along $y_A=0$ line and along $x_A=0$ line.

Lines 76-79: Generate symmetrical amplitude values in Quadrant IV.

Lines 80-83: Generate symmetrical amplitude values in Quadrants I and II.

Line 84: Compute $k_{\text{xmax}}$.

Lines 85-89: Test to determine if the sum channel data generated should be phased to produce the aperture distribution for a specified difference channel (ICHAN).

Lines 90-98: Form aperture distribution for difference elevation channel by zeroing all elements along $y_A=0$ and negating all elements for $y_A<0$. RETURN.
Lines 99-107: Form aperture distribution for difference azimuth channel by zeroing all elements along $x_A=0$ and negating all elements for $x_A<0$. RETURN.


END

3-6. Test Case: See discussion in Chapter 2.

3-7. References


3-8. Program Listing: See following pages.
SUBROUTINE HACNF(NX,NY,ICHAN,IPOL,IXY,DAPML,DXML,KMAX,ICASE)
C SUBR HACNF COMPUTES ELECTRIC FIELD COMPONENTS OVER A CIRCULAR APERTURE
C OF RADIUS RMAG=(NX/2-1)/COS(2.2/7) AND RETURNS SAME IN E(NX,NY).
C NX MUST EQUAL NY AND MUST BE EVEN.
C ICHAN=1 FOR SUM CHANNEL, IPOL=1 FOR VERT-Y POL. IXY=1 FOR X-COMP.
C IXY=2 FOR HORIZ-X POL =2 FOR Y-COMP.
C IXY=3 FOR HORIZ-Y POL =2 FOR Y-COMP.
C =4 FOR LHC POL.
C DAPML=DIAMETER OF APERTURE IN WAVELENGTHS (INPUT)
C DXML=SAMPLE SPACING IN APERTURE (OUTPUT)
C KMAX=MAXIMUM WAVELENGTH (OUTPUT)
C ICASE=1 OR 2 FOR UNIFORM, CIRCULAR APERTURE (CASE I AND II)
C =3 FOR FLAT-PLATE ANTENNA, VERTICAL POL (CASE III).
C
C COMPLEX E(IXY),CFAC(4)
C REAL KMAX

DATA CFAC/(1.,0.),(1.,0.),(-1.,0.),(-1.,0.)/
C ANG=ATAN(2./7.)
C IF (ICASE.EQ.3) ANG=ATAN(4./6.)
C RMAX=(NX/2-1)/COS(ANG)*.001
C IF (IPOL.GT.4) IPOL=4
C IF (IPOL.LT.1) IPOL=1
C IF (NX.NE.NY) GO TO 15
C IF ((IXY.LT.1).OR.(IXY.GT.2)) IXY=2
C IF (MOD(NX,2).NE.0) GO TO 15
C IF (ICASE.EQ.3) GO TO 25
C DO 10 I=1,NX
C X=FLOAT((-NX/2)+I-1)
C DO 10 J=1,NY
C Y=FLOAT((-NY/2)+J-1)
C R=SQRT(X**2+Y**2)
C IF (R.GT.RMAX) GO TO 9
C IF ((IPOL.EQ.1).AND.(IXY.GT.1)) GO TO 9
C IF ((IPOL.EQ.2).AND.(IXY.GT.1)) GO TO 9
C IF (RMAG,FY=(1,0),EX=(0,1)) I.E., EX LEADS EY BY 90 DEG.
C IF LHC,EY=(1,0),FY=(0,-1) I.E., EX LAGS EY BY 90 DEG.
C E(I,J)=(1,0)
C IF (IPOL.LT.3).CFAC(IXY.EQ.2)) GO TO 10
C F(I,J)=E(I,J)*CFAC(IPOL)
C
GO TO 10
9 E(I,J)=(. ., .)
10 CONTINUE
DXL=(CAPW/2,)*COS(ANG)/(NX/2-1)
GO TO 60
15 WRITE(6,21)
20 FORMAT('//', NX, NE, NY OR NX NOT EVEN IN SUBR MAGNF')
STOP
C THE FOLLOWING IS FOR CASE III (IGSE=2):
25 IF (NX,NE,16) GO TO 90
30 NXL=(CAPW/2,)*COS(ANG)/(NX/2-2)
NY=NX
DO 26 I=1,NX
DO 26 J=1,NY
26 E(I,J)=(0,0,0)
IF (IXY.EQ.1) GO TO 60
E(6,4)=(.2224,0,0)
E(4,4)=(.4250,0,0)
E(4,6)=(.2228,0,0)
E(6,6)=(.5218,0,0)
E(4,8)=(.8060,0,0)
E(8,8)=(1.4194,0,0)
E(6,6)=(.7959,0,0)
S(8,9)=(1.628,0,0)
DO 30 J=4,6,2
DO 30 I=3,9,1
IF ((MOD(J,2),EQ.0),AND,(MOD(I,2),EQ.0)) GO TO 30
F(I,J)=(E(I-1,J)+E(I+1,J))/2.
30 CONTINUE
DO 35 I=3,9,1
DO 35 J=3,9,1
F(I,J)=(E(I,J-1)+E(I,J+1))/2.
35 CONTINUE
DO 40 I=3,9
40 F(I,J)=F(I,J)
DO 45 J=3,9
45 F(9,J)=F(8,J)
DO 50 J=3,9
50 CONTINUE
00 50 I=1,c
05 E(I+1)=E(I,1)
50 CONTINUE
05 55 I=1,15
05 55 J=1,6
05 E(I,J)=E(I,I-J)
55 CONTINUE
60 H *X=1.*(2.*I*X*L)
IF (ICHA.N,E.G.1) GOTO 75
IF ((XY,E.G.1) .AND. (CASE,E.G.1)) RETURN
IF ((XY,E.G.1) .AND. (IPOL,E.G.1)) RETURN
IF ((XY,E.G.1) .AND. (IPOL,E.G.2)) RETURN
IF (ICHA.N,E.G.2) TO 75
C LOAD ELEVATION DIFFERENCE CHANNEL
J=NY/2+1
55 E(I,J)=E(I,J)
JMAX=NY/2
50 70 J=1,JMAX
50 70 I=1,15
70 E(I,J)=-E(I,J)
RETURN
C LOAD AZIMUTH DIFFERENCE CHANNEL
75 I=NY/2+1
50 80 J=1,15
80 E(I,J)=E(I,J)
I.MAX=NY/2
50 80 J=1,15
50 80 I=1,15
80 E(I,J)=-E(I,J)
RETURN
C GAPWL=5.0 FOR CASE III
90 WRITE(6,95)
95 FORMAT(/"***ERROR EXIT! NY NOT EQUAL TO 15 IN SUBR MACHF***/")
STOP
END
Chapter 4

SUBROUTINE ORIENT

4-1. Purpose: To compute the rotational matrix of direction cosine
ROTATE and the translational matrix TRANSL required to carry out
coordinate and vector transformations between antenna coordinate
system \((x_A', y_A', z_A')\) and radome coordinate system \((x_R', y_R', z_R')\).

4-2. Usage: CALL ORIENT (RA, THETA, PHIA, RR, THETAR, PHIR, ACARIA,
ROTATE, TRANSL)

4-3. Arguments

RA,          - Spherical coordinates (cm, radians) of the
THETA        - origin of the antenna coordinate system with
PHIA         - respect to the reference coordinate system
(x,y,z) as indicated in Figure 4-1. Note that
the origin of the reference system coincides with
the origin point, which is located on the axis of
symmetry \(y_R\) of the radome.
RR,          - Spherical coordinates (cm, radians) of the origin
THETAR,      - of the radome coordinate system with respect to
PHIR         - the reference system.
ACARIA       - Angle (radians) between the \(A\) and \(R\) axes.
ROTATE       - Real array of 6 x 3 elements which contains
               output the matrix of direction cosine \(H_{il}\)
               explained below.
TRANSL       - Real array of three elements which contains
               output the translation matrix \(T_{il}\) as explained
               below.
Figure 4-1. Coordinate Systems Used in Radome Analysis.

Reference System: \((X, Y, Z)\)

Antenna System: \((X_A, Y_A, Z_A)\)

Radome System: \((X_R, Y_R, Z_R)\)
4-4. Comments and Method

a. The coordinate systems of Figure 2-2 are obtained from those shown in Figure 4-1 by setting THETA = 0, PHIA = θ/2, and AGAM3A = 0. When this is done, the z and z_A axes coincide, the x and x_A axes are parallel, and the y and y_A axes are parallel. The angles φ_p and φ_L in Figure 2-2 are related to φ_r and φ_L as

\[
\phi_r = \phi_p + \pi \quad (1)
\]

\[
\phi_L = \pi - \phi_L \quad (2)
\]

The unit vectors \( \vec{x}, \vec{y}, \vec{z} \) were chosen to coincide with the spherical coordinate unit vectors \( \vec{r}, \vec{\theta}, \vec{\phi} \) associated with the point \( Q_0: (r_0, \theta_0, \phi_0) \); hence, \( \vec{x} \) always lies in the plane of scan as indicated in Figure 4-2. This observation is important if the properties of the radome wall are not symmetric with respect to rotation about the \( z_r \)-axis, such as in the case of circumferential variations in wall thickness, nonuniform heating, etc.

b. The details of the coordinate system transformations are described in Reference 1 and summarized below. The transformation of the point \( P \) in antenna coordinates \( (x_A, y_A, z_A) \) to radome coordinates \( (x_R, y_R, z_R) \) is given by

\[
\begin{pmatrix}
    x_R \\
    y_R \\
    z_R
\end{pmatrix} =
\begin{pmatrix}
    R \\
    R
\end{pmatrix}
\begin{pmatrix}
    x_A \\
    y_A \\
    z_A
\end{pmatrix} +
\begin{pmatrix}
    T_x \\
    T_y \\
    T_z
\end{pmatrix}
\]

(3)
The transformation of a vector

\[ \mathbf{F} \times R = x_A \mathbf{F} + y_A \mathbf{F} + z_A \mathbf{F} = x_R \mathbf{F} + y_R \mathbf{F} + z_R \mathbf{F} \]  \hspace{1cm} (4)

is given by

\[
\begin{pmatrix}
F_{xR} \\
F_{yR} \\
F_{zR}
\end{pmatrix}
= R_{11}
\begin{pmatrix}
F_{xA} \\
F_{yA} \\
F_{zA}
\end{pmatrix}
\hspace{1cm} (5)
\]

In the above, \( |R_{12}| \) is the matrix of direction cosines which describes the rotation of the radome coordinate system with respect to the antenna system; \( |T_{\perp}| \) describes the translation of the radome origin \( Q \) with respect to the origin \( Q_0 \) of the antenna system. In fact, setting \( (x_A = 0, y_A = 0, z_A = 0) \) in Equation (3) shows that \( (T_{x}, T_{y}, T_{z}) \) represents the location, in radome coordinates, of the antenna origin.

The matrix \( |R_{12}| \) can be expanded and written explicitly as

\[
\begin{pmatrix}
\cos \phi_1 & \cos \theta_1 & \cos \psi_1 \\
\cos \phi_2 & \cos \theta_2 & \cos \psi_2 \\
\cos \phi_3 & \cos \theta_3 & \cos \psi_3
\end{pmatrix}
\hspace{1cm} (6)
\]
where

\[ a_1 = \text{angle between } x_A \text{ and } x_R = L x_A, x_R \]  
\[ a_2 = L x_A', y_R \]  
\[ a_3 = L x_A', z_R \]  
\[ \beta_1 = L y_A', x_R \]  
\[ \beta_2 = L y_A', y_R \]  
\[ \beta_3 = L y_A', z_R \]  
\[ \gamma_1 = L z_A', x_R \]  
\[ \gamma_2 = L z_A', y_R \]  
\[ \gamma_3 = L z_A', z_R \]  

The inverse transformations are given by

\[
\begin{pmatrix}
    x_A \\
    y_A \\
    z_A
\end{pmatrix} = \begin{bmatrix} R_{13} & \epsilon \end{bmatrix} \begin{pmatrix}
    x_R \\
    y_R \\
    z_R
\end{pmatrix} + \begin{pmatrix}
    T_X \\
    T_Y \\
    T_Z
\end{pmatrix}
\]
where \([R_{ij}]^T\) denotes the transpose of \([R_{ij}]\); i.e., \([R_{ij}]^T = [R_{ji}]\) since rows and columns are interchanged. Also, since \([R_{ij}]\) is a unitary matrix, its inverse is equal to its transpose.

To facilitate the specification of a particular antenna/radome orientation, the reference coordinate system \((x, y, z)\) is used. Transformations from the reference system to the antenna system are described by

\[
\begin{pmatrix}
    x_A \\
    y_A \\
    z_A
\end{pmatrix} = \begin{pmatrix}
    x - r_a \sin \theta \cos \phi_a \\
    y - r_a \sin \theta \sin \phi_a \\
    z - r_a \cos \theta
\end{pmatrix} \quad (10)
\]

while transformations from reference system to radome system are described by

\[
\begin{pmatrix}
    x_R \\
    y_R \\
    z_R
\end{pmatrix} = \begin{pmatrix}
    x - r \sin \theta_0 \cos \phi_r \\
    y - r \sin \theta_0 \sin \phi_r \\
    z - r \cos \theta_0
\end{pmatrix} \quad (11)
\]

where \([R_{ij}]\) and \([R_{ji}]\) represent the rotations of the two systems with respect to the reference system. When these two separate transformations
are combined, there results

\[
\begin{pmatrix}
  R_{ij}
\end{pmatrix} = \begin{pmatrix}
  p_{ij}
  \end{pmatrix} \begin{pmatrix}
  Y_{ij}
\end{pmatrix}^T \tag{12}
\]

\[
\begin{pmatrix}
  T_x \\
  T_y \\
  T_z
\end{pmatrix} = \begin{pmatrix}
  p_{ij}
  \end{pmatrix} \begin{pmatrix}
  X_x - X_i \\
  Y_y - Y_i \\
  Z_z - Z_i
\end{pmatrix} \tag{13}
\]

where \( X_a, Y_a, \) etc., are defined in Equations (11) and (11); i.e.,

\[ X_a = r_a \sin \phi_a \cos \theta_a \]

4-6. Program Flow: See listing below and compare directly to method described above. Note that in \( \text{GAM}(I,J), \) the index \( I \) represents the row number in \( \{ Y_{ij} \} \), and index \( J \) represents the column number.

4-6. Test Case: See discussion in Chapter 7.

4-7. Reference


4-6. Program Listing: See following page.
SIMULATION INPUT, THETA, PHI, RHO, THETA, PHI, GAMMA, AGMA3A,
- ROTATE, TRANSL
DIMENSION REAL GAM (3,3), RHO (3,3)
DIMENSION T (3)
PSI=THETA-AGMA3A
GAM (1,1)=SIN (PHIA)
GAM (1,2)=-COS (PHIA)
GAM (1,3)=".
GAM (2,1) = SIN (PSI) * SIN (THETA) * COS (PHIA) + COS (PSI) * COS (THETA)
- * COS (PHIA)
GAM (2,2) = SIN (PSI) * SIN (THETA) * SIN (PHIA) + COS (PSI) * COS (THETA)
- * SIN (PHIA)
GAM (2,3) = SIN (PSI) * COS (THETA) - COS (PSI) * SIN (THETA)
GAM (3,1) = COS (PSI) * SIN (THETA) * COS (PHIA) - SIN (PSI) * COS (THETA)
- * COS (PHIA)
GAM (3,2) = COS (PSI) * SIN (THETA) * SIN (PHIA) - SIN (PSI) * COS (THETA)
- * SIN (PHIA)
GAM (3,3) = COS (AGMA3A)
RHO (1,1) = COS (THETA) * COS (PHIA)
RHO (1,2) = COS (THETA) * SIN (PHIA)
RHO (1,3) = -SIN (THETA)
RHO (2,1) = SIN (PHIA)
RHO (2,2) = COS (PHIA)
RHO (2,3) = L.
RHO (3,1) = -SIN (THETA) * COS (PHIA)
RHO (3,2) = -SIN (THETA) * SIN (PHIA)
RHO (3,3) = COS (THETA)
YA = PA * SIN (THETA) * COS (PHIA)
YA = PA * SIN (THETA) * SIN (PHIA)
ZA = PA * COS (THETA)
XP = PA * SIN (THETA) * COS (PHIA)
YR = PA * SIN (THETA) * SIN (PHIA)
ZR = PA * COS (THETA)

C COMPUTE THE ROTATE ARRAY BY MULTIPLYING THE RHO ARRAY
AND THE TRANSPOSE OF THE GAM ARRAY.
DO ? T=1,3
DO ? J=1,3
ROTATE(I,J)=0.
50 7 <= I,J
51 ROTATE(I,J)=ROTATE(I,J)+RHO(I,K)*GAM(J,K)
52 CONTINUE

C COMPUTE TRANSL ARRAY
54 T(1)=X(I)-X(J)
55 T(2)=Y(I)-Y(J)
56 T(3)=Z(I)-Z(J)
57 DO 10 I=1,3
58 TRANSL(I)=0.0
59 DO 10 J=1,3
60 TRANSL(I)=TRANSL(I)+RHO(I,J)*T(J)
61 CONTINUE
62 RETURN
63 END
Chapter 5

SUBROUTINE POINT

1. Purpose: To transform a point P in antenna coordinates \((x_A, y_A, z_A)\) to radome coordinates \((x_R, y_R, z_R)\), and vice versa.

2. Usage: CALL POINT (I, PT, ATOR, T, PO)

3. Arguments

- \(P\) - Real (input) array of three elements representing the Cartesian coordinates (cm) of the point to be transformed; e.g., \(P(1) = x_A, P(2) = y_A, P(3) = z_A\).
- \(PT\) - Real (output) array of three elements representing the Cartesian coordinates (cm) in the other coordinate system; e.g., \(PT(1) = x_R\), etc.
- \(ATOR\) - Logical input variable which controls direction of transformation: \(ATOR = .TRUE.\), for antenna to radome (see Equation (3-3)); \(ATOR = .FALSE.\), for radome to antenna coordinate transformation (Equation (3-3)).
- \(T\) - Real (input) array of 3 x 5 elements representing the \text{ROTATE} array computed by Subroutine \text{ORIENT}.
- \(PO\) - Real (input) array of three elements representing the \text{TRANS} array computed by Subroutine \text{ORIENT}.

4. Comments and Method

- Subroutines required: Subroutine \text{ORIENT} must be called prior to the first call to \text{POINT} so that \(T\) and \(PO\) are available.
- For method, see Subroutine \text{ORIENT} in Chapter 4.
- Subroutine \text{POINT} then utilizes Equations (3-3) and (1-8).
5-6. Test Case: See Chapter 2.


5-8. Program Listing: See following page.
CONVERSION FROM FOCUS TO ANTENNA COORDINATES

CONVERSION FROM ANTENNA TO FOCUS COORDINATES
Chapter 6

SUBROUTINE VECTOR

6-1. Purpose: To transform a vector \( F \) in antenna coordinates to radar coordinates, and vice versa.

6-2. Usage: CALL VECTOR (V, VT, ATOR, T)

6-3. Arguments

<table>
<thead>
<tr>
<th>Argument</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>V</td>
<td>Real (input) array of three elements representing the rectangular components of the vector to be transformed; e.g., ( V(1) = F_x^A, V(2) = F_y^A, V(3) = F_z^A ).</td>
</tr>
<tr>
<td>VT</td>
<td>Real (output) array of three elements representing the rectangular components of the vector in the other coordinate system; e.g., ( VT(1) = F_x^R, ) etc.</td>
</tr>
<tr>
<td>ATOR</td>
<td>Logical input variable which controls the direction of the transformation: ( ATOR = .TRUE. ) for antenna-to-radar (Equation 6-1); ( ATOR = .FALSE. ) for radar-to-antenna (Equation 6-2).</td>
</tr>
<tr>
<td>T</td>
<td>Matrix ( H ) defined in Chapter 4 as computed by Subroutine ORIENT.</td>
</tr>
</tbody>
</table>

6-4. Comments and Notes:

1. Subroutine required: Subroutine ORIENT must be called prior to first call to VECTOR so that \( T \) will be available.

2. For method, see Subroutine ORIENT in Chapter 4.

3. Freeman, How: computer listing below directly to equations (6-1) and (6-2).

6-7. References: See Chapter 4.

6-8. Program Listing: See following page.
Chapter 7

SUBROUTINE INCPW

7-1. Purpose: To compute the rectangular vector components of the electric field of a plane- electromagnetic wave propagating in the direction \( -k_A \) and incident on the \( z_A = 0 \) plane of the antenna coordinate system, where \( (x_A = 0, y_A = 0, z_A = 0) \) is used as the plane origin.

7-2. Usage: CALL INCPW (KA, EI, IOPT)

7-3. Arguments

- **KA**: Real input array of three elements containing the direction cosines of the direction \( k_A \) from whence the plane wave emanates; e.g., \( KA(1) = k_{x_A} \), \( KA(2) = k_{y_A} \), \( KA(3) = k_{z_A} \) where \( k_A = x_A k_{x_A} + y_A k_{y_A} + z_A k_{z_A} \).

- **EI**: Complex output array of three elements containing the three rectangular components of the electric field \( E_z \) emanates such that \( |E_z| = 1 \).

- **IOPT**: Integer input variable which specifies:
  - 0: Right hand from positive x-axis (default).
  - 1: Right hand from negative x-axis.

The subroutine is named INCPW.
^\varepsilon = \text{elevation component}

^\alpha = \text{azimuth component}

Figure 1. Coordinate System for Far Field Patterns
will see the tip of the \( \mathbf{E} \) vector trace out a circle in a plane of equal phase, with the direction of rotation being clockwise for right-hand circular and counterclockwise for left-hand circular.

b. Spherical to rectangular coordinate transformations [1] are used to define the rectangular vector components \( E_x, E_y, E_z \) in terms of the transverse spherical components \( E_r, E_{\theta}, E_{\phi} \). Let \( r = k = x k_x + y k_y + z k_z \) represent the direction from whence the plane wave emanates, where \( k_x, k_y, k_z \) are the direction cosines of \( r = k \). Then, with reference to Figure 7-1, there results

\[
E = \begin{bmatrix}
-x k_y \\
-x k_z \\
-x k_z \\
\end{bmatrix} + \begin{bmatrix}
y(0) \\
z \\
z(0) \\
\end{bmatrix} \sqrt{1 - k_y^2}
\]

except at \( k_y = \pm 1 \), where these equations reduce to

\[
\begin{align*}
E_x &= -z \\
E_y &= x \\
E_z &= z
\end{align*}
\]

The expressions for the field components for the two cases of interest are summarized in Table 7-1. The corresponding rectangular field can be obtained from

\[ H = (\mathbf{E} \times \mathbf{k}) \cdot \nu. \]
<table>
<thead>
<tr>
<th>IOPT</th>
<th>$E_x$</th>
<th>$E_y$</th>
<th>$E_z$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>$-k \frac{ky}{\sqrt{1-k_y^2}}$</td>
<td>$\frac{\sqrt{1-k_y^2}}{y}$</td>
<td>$-k \frac{k_y}{\sqrt{1-k_y^2}}$</td>
</tr>
<tr>
<td>2</td>
<td>$k_z/\sqrt{1-k_y^2}$</td>
<td>0</td>
<td>$-k \frac{k_y}{\sqrt{1-k_y^2}}$</td>
</tr>
<tr>
<td>3</td>
<td>$(k_z-k_x \frac{3\pi}{2})/\sqrt{2(1-k_y^2)}$</td>
<td>$e^{\frac{3\pi}{2}\sqrt{1-k_y^2}/\sqrt{2}}$</td>
<td>$(-k \frac{k_x e^{\frac{3\pi}{2}}}{y z})/\sqrt{2(1-k_y^2)}$</td>
</tr>
<tr>
<td>4</td>
<td>$(k_z-k_x \frac{3\pi}{2})/\sqrt{2(1-k_y^2)}$</td>
<td>$e^{-\frac{3\pi}{2}\sqrt{1-k_y^2}/\sqrt{2}}$</td>
<td>$(-k \frac{k_x e^{-\frac{3\pi}{2}}}{y z})/\sqrt{2(1-k_y^2)}$</td>
</tr>
</tbody>
</table>

7-6. Test Case: See Chapter 2.

7-7. References


7-8. Program Listing: See following pages.
SUBROUTINE INCPW(KA, EI, IOPT)
C KA=NEGATIVE OF DIR OF PROP^N OF INCIDENT PLANE WAVE (ANT COORD)
C EI= ELECTRIC FIELD VECTOR OF INCIDENT PLANE WAVE (OUTPUT)
C IOPT=1 MAKES EI ELEVATION COMPONENT ONLY
C =2 MAKES EI AZIMUTH COMPONENT ONLY
C =3 FOR RHCP POLARIZATION (DEFINE WRT DIR OF PROP OF INC WAVE)
C *+ FOR LHCP
C POWER OF INCIDENT WAVE IS UNITY.
COMPLEX EI(3), CIA
REAL KA(3), IE, IA
1 FORMAT("" ERROR IN SUBP INCPW IOPT= ", I3//)
C COMPUTE ELEVATION COMPONENT ONLY:
R=1.-KA(3)**2
IF (R.LT.0.) Q=0.
R=SQRT(R)
RY=1.-KA(2)**2
IF (RY.GT.0.) GO TO 5
GO TO 40
5 RY=SQRT(RY)
GO TO (10, 20, 30, 40) IOPT
C CORRECTIONS TO LOCPS 10, 20, 30, 40 MADE JAN 78 BY GKH.
10 IE=1.
EI(1)=-KA(2)*KA(1)*IE/RY
EI(2)=RY*IE
EI(3)=-KA(2)*KA(3)*IE/RY
RETURN
C COMPUTE AZIMUTH COMPONENT ONLY:
20 IA=1.
EI(1)=+KA(3)*IA/RY
EI(2)=CMPLX(0., 0.)
EI(3)=-IA*KA(1)/RY
RETURN
C COMPUTE PHC:
30 IE=7.07
CIA=CMPLX(0., 1.)*IE
IF (IOPT.EQ.4) CIA=CMPLX(0., -1.)*IE
EI(1)=(-KA(2)*KA(1)*CIA-KA(3)*IE)/RY
EI(2)=CIA*RY
C
EI(3) = (-KA(2) * KA(3) * CIA - IE * KA(1)) / RY
RETURN
40 GO TO (50, 60, 70, 71), ICPT
50 EI(1) = (0, 0, 0)
   EI(2) = (0, 0, 0)
   EI(3) = -KA(2)
RETURN
60 EI(1) = (1, 0, 0)
   EI(2) = (0, 0, 0)
   EI(3) = (0, 0, 0)
70 IE = .707
   CIA = CMPLX(0, 1) * IE
   IF (IOPT.EQ.4) CIA = CMPLX(0, -1) * IE
   EI(1) = IE
   EI(2) = (0, 0, 0)
   EI(3) = -KA(2) * CIA
RETURN
END
Chapter 8

SUBROUTINE RECM

8-1. Purpose: To compute the complex voltages produced at the terminals of the three channels of a radome enclosed monopulse antenna by a plane wave of specified polarization and direction of arrival.

8-2. Usage: CALL RECM (PWI, KA, NX, NY, KXMAX, KYMAX, FGHZ, ROTATE, TRANSL, SUMX, SUMY, DELX, DELY, VR, TABLE, SUPPRS, RSQMAX)

8-3. Arguments

PWI - A complex array of three elements containing $E_x$, $E_y$, $E_z$ of the incident plane wave. See Subroutine INCPW.

KA - A real array of three elements containing the direction cosines $k_x^A$, $k_y^A$, $k_z^A$ of the unit vector $k^A$ which points from the antenna origin in the direction from whence the plane wave emanates.

NX, NY - The even integer number of sample points in $x_A$ and $y_A$ directions used to represent the antenna aperture fields.

KXMAX, KYMAX - Real variables which represent the normalized folding wavenumbers corresponding to the sample distances $\Delta x_A$, $\Delta y_A$ according to $\Delta x_A = \lambda / (2 \times KXMAX)$, $\Delta y_A = \lambda / (2 \times KYMAX)$, where $\lambda$ is the free space wavelength.

FGHZ - Frequency in gigahertz of the monochromatic plane wave.
ROTATE,TRANS - Real matrices of direction cosines and translation distances used to carry out coordinate transformations of points and vectors from antenna to radome coordinate systems, and vice versa. See Subroutine ORIENT.

SUMX,SUMY - Two dimensional (NX X NY) complex arrays of the x and y vector components of the antenna aperture fields for the sum channel of a three-channel monopulse antenna. The element at I=NX/2+1, J=NY/2+1, corresponds to that at x_A=0, y_A=0 in the aperture. The general correspondence is given by

\[
\begin{align*}
x_A &= -x_{\text{max}} + (I-1)\Delta x_A = (I-\text{MIDNX})\Delta x_A \\
y_A &= -y_{\text{max}} + (J-1)\Delta y_A = (J-\text{MIDNY})\Delta y_A
\end{align*}
\]

where \( x_{\text{max}} = \Delta x_A * \text{NX}/2 \) and \( y_{\text{max}} = \Delta y_A * \text{NY}/2 \).

Also see Subroutine HACNF.

DELX,DELY - Antenna aperture fields for the difference elevation channel.

DAZX,DAZY - Antenna aperture fields for the difference azimuth channel.

VR - Complex array of three elements which on output contains the complex terminal voltage of the antenna for the sum, elevation difference, and azimuth difference channels, respectively.
TABLE
- Logical variable required by Subroutine RXMIT:
  if TRUE, a look-up table is used to calculate the
  transmission coefficients of the radome wall; if
  FALSE, these coefficients are calculated exactly
  for each angle of incidence specified.

SUPPRS
- Logical variable used to control the printing of
  results from Subroutine RXMIT: if FALSE, a table
  of power transmission and reflection coefficients
  for equal increments in the sine of the incidence
  angle is printed. The phases of the complex
  voltage transmission and reflection coefficients
  of the radome wall are also printed.

RSQMAX
- Real variable denoting the maximum radius of the
  antenna aperture such that any point \((x_A^2 + y_A^2) > RSQMAX\)
  is omitted from the ray tracing and summation pro-
  cedure used to compute the received voltages \(VR\).

8-4. Comments and Method

a. Subroutines Required: TRACE, VECTOR, POINT, RXMIT, CAXB.

b. Method: The voltage \(V_R\) induced at the terminals of a linear
  antenna by a "received" electromagnetic plane wave \(E_R, H_R\) is given by
  the Lorentz reciprocity theorem as [1]

\[
V_R(k) = C \int_S \left( \mathbf{E}_T \times \mathbf{H}_R - \mathbf{E}_R \times \mathbf{H}_T \right) \cdot \mathbf{k} \, \text{d}a \tag{1}
\]

where \(k\) is the unit vector which points in the direction from whence the
plane wave emanates and where \(E_T, H_T\) are the electromagnetic fields of
the antenna as produced on the closed surface $S$ which surrounds the antenna when it is transmitting. The unit vector $\mathbf{n}$ is the normal to $S$ pointing into the region not containing the antenna, and $C$ is a complex constant.

When the closed surface $S$ is the $z_A^A = 0$ plane, $\mathbf{n} = z_A^A$, $da = dx_A^A dy_A^A z_A^A dx_A^A dy_A^A$ and the integral in (1) can be approximated by

\[
V_R(k) = C \Delta x_A^A \Delta y_A^A \sum \sum_1^\infty \left( E_{tx}^A H_{ry}^A - E_{ty}^A H_{rx}^A - E_{rx}^A H_{ty}^A + E_{ry}^A H_{tx}^A \right)
\]

where $\Delta x_A^A$, $\Delta y_A^A$ are equal sample distances in $x_A^A$ and $y_A^A$ and where the rectangular vector components of the fields on the $z_A^A = 0$ plane are given generically by

\[
E_T^A = x_A^A E_{tx}^A + y_A^A E_{ty}^A + z_A^A E_{tx}^A
\]

It is assumed that the fields $E_T^T$, $H_T^T$ on $S$ with the radome in place are unperturbed by the radome. Also, $E_T$ is specified according to the aperture distribution and polarization desired as is usually done in antenna analysis. The corresponding magnetic field $H_T$, however, presents something of a vexation in that a non-Maxwellian aperture field can result if $H_T$ is specified independently of $E_T$ and Maxwell's equation $H_T = \nabla \times E_T^T - j \omega \mu$. On the other hand, specification of $H_T$ independently of $E_T$ is tantamount to specifying magnetic and electric current sheets in the antenna aperture which produce two independent solutions to Maxwell's equations whose sum yields the total fields. This latter approach is
taken here when the geometrical optics approximation

\[ H_T = \frac{z_A \times E_T}{\eta} \]  

(4)

is utilized, where \( \eta = \sqrt{\mu/\varepsilon} = 377 \text{ ohms} \). Also, the magnetic field \( H_R \) is given by a similar formula

\[ H_R = \frac{-k \times E_R}{\eta} \]  

(5)

where \( -k \) is the direction of propagation of the incident plane wave.

Combining the results of Equations (4) and (5) into (2), and designating the origin \( x_A = y_A = z_A = 0 \) as the phase reference for the complex fields, there results

\[ V_R(k) = C' \sum_{l,m} \left\{ (E_{Tx_l} E_{Rx_m} + E_{Ty_l} E_{Ry_m})(1 + k_{zA}) - (k_{xA} E_{Tx_l} + k_{yA} E_{Ty_l} E_{Rz_m}) \right\} \]

\[ e^{j \frac{2\pi}{\lambda} (k_{xA} x_A + k_{yA} y_A)} \]

(6)

where

\[ k = x_A k_{xA} + y_A k_{yA} + z_A k_{zA} \]  

(7)

\[ k_{xA}^2 + k_{yA}^2 + k_{zA}^2 = 1 \]  

(8)
and where the exponential factor accounts for the phase of the incident wave. It is noted that \( k_xA, k_yA, k_zA \) are direction cosines of \( \hat{k} \); hence, 
\[ k_{zA} = \cos \theta \], where \( \theta \) is the polar angle measured from the \( z_A \)-axis in the usual spherical coordinate system. The \((1+\cos \theta)\) term in Equation (6) is characteristic of the geometrical optics approximation of Equation (4) [2]. The other factors have been absorbed into complex constant \( C' \).

The effects of the radome on the received voltage given by Equation (6) are accounted for by tracing a ray from each aperture element \( A_xA_yA_z \) in the direction \( \hat{k} \) and weighting the field \( E_R \) associated with the ray by the complex insertion transmission coefficients \( T_\perp, T_\parallel \) of the radome wall as shown in Figure 8-1. These coefficients depend on the incidence angle \( \theta_1 \) and the plane of incidence defined by \( \hat{k} \) and the unit inward normal \( \hat{n}_R \) to the radome wall at each point of incidence for each ray as illustrated in Figure 8-2. The ray tracing is carried out in the direction \( \hat{k} \), and the direction of propagation of each ray is assumed to be the same on both sides of the wall, an assumption that mandates use of the insertion transmission coefficients defined for an infinite sheet by

\[
T_\perp = \frac{E_\perp (P)}{E_\perp^i (P)} \quad (9)
\]

\[
T_\parallel = \frac{E_\parallel (P)}{E_\parallel^i (P)} \quad (10)
\]

where the numerator in each case is the field at point \( P \) with the sheet in place and the denominator is the field at the same point with the sheet removed.
Figure 8-1. Illustration of the Fast Receiving Method of Radome Analysis
Figure 8-2. Plane Wave Propagation Through an Infinite Plane Sheet
The ray tracing is carried out in radome coordinates \((x_R, y_R, z_R)\), and transformations of points and vectors from antenna coordinates \((x_A, y_A, z_A)\) to radome coordinates, and vice versa, are required. (These transformations are described in detail in Subroutines ORIENT, POINT, and VECTOR.)

Let \((x_A, y_A, 0)\) be the point in the aperture from which the ray (line) emanates in the direction \(k\). Convert this point and unit vector to the radome coordinate system. Find the intersection \((x_{RI}, y_{RI}, z_{RI})\) of this ray with the inner radome surface as described by \(f(x_R^2 + y_R^2 z)\) = constant since it is a surface of revolution (Subroutines TRACE and OGIVE). Compute the unit inward normal \(\hat{n}_R\) to the surface

\[
\hat{n}_R = -\frac{\nabla f}{\left|\nabla f\right|} = x_R \hat{x}_R + y_R \hat{y}_R + z_R \hat{z}_R
\] (11)

and convert it to antenna coordinates

\[
\hat{n}_R = n_A = x_A \hat{x}_A + y_A \hat{y}_A + z_A \hat{z}_A
\] (12)

Use \(n_A\) and \(k\) to determine the plane of incidence, angle of incidence, and the transmitted plane wave \(E'_R, H'_R\) (see Subroutine RXMIT) for this ray. Substitute into Equation (6) and sum the results to obtain the following expression for the received voltage

\[
V_R(k) = C'' \sum_{j=1}^{L} \left\{ -\left(1 + k z_A \right) (E'_R x_T \hat{x}_R + E'_R y_T \hat{y}_R + E'_R z_T \hat{z}_R) + (k x_A E'_{Tx} + k y_A E'_{Ty} + k z_A E'_{Tz}) \right\}
\]

\[
\cdot e^{-j \frac{2\pi}{\lambda} \left( k x_A x_A \hat{x}_A + k y_A y_A \hat{y}_A \right)}
\] (13)
where a sign change and $\eta^{-1}$ have been absorbed into $C''$.

Equation (13) with $C'' = 1$ is used in Subroutine RECM to compute the received voltage on each of the three monopulse channels. Note that the received field $E'_R, H'_R$ at each point $(x'_A, y'_A, 0)$ is the same for all three channels so that three summations can be carried simultaneously to maximize computational speed. In each summation, only the data corresponding to $E_{Tx}', E_{Ty}'$ for the sum, elevation difference, and azimuth difference channels need to be changed. Note also that Equation (13) can be rewritten as

$$V_R(k) = \sum_{1}^{m} \left[ E_{Tx} \left( \eta H'_R - E'_R \right) - E_{Ty} \left( \eta H'_R + E'_R \right) \right] e^{\frac{j}{\lambda} \left( k x_A x'_A + k y_A y'_A \right)}$$

(14)

where $\eta H'_R, \eta H'_R$ are given by Equation (5).

8-5. Program Flow

Line 12: Initialize the ray counter NRAY.

Lines 13-18: Compute $\lambda$ (cm), $k_0 = 2\pi/\lambda$, $\Delta x'_A$, $\Delta y'_A$, and the midpoint of the NX X NY data arrays corresponding to $x'_A = 0, y'_A = 0$.

Lines 21-24: Set $z'_A = PA(3)$ and precalculate $k x'_A$ and $k y'_A$.

Transform $k$ to radome coordinates $k R = k_{xR} x'_R + k_{yR} y'_R$ and $x'_R, y'_R$ in preparation for the ray tracing.

Lines 26-28: Initialize the summations VR(1), VR(2), VR(3) for the received voltages on the sum, difference elevation, and difference azimuth channels, respectively.

Lines 30-33: Iterate for each aperture point $x'_A = PA(1)$. precalculate $x'_A x'_A$ and $k k y'_A$ outside the subsequent loop for $y'_A$.
Lines 34-40: Iterate for each aperture point $y_A = PA(2)$. Compute
\[ x_A^2 + y_A^2 = RSQ; \] if point is outside $RSQMAX$, omit from computation.

Lines 41-47: Transform $(x_A, y_A, 0)$ to radome coordinates and trace ray to radome inner surface to find unit inward normal $\hat{n}_R$. If metal tip or bulkhead is encountered by ray, omit this ray from computation of received voltages.

Lines 48-52: Transform $\hat{n}_R$ to antenna coordinates and compute the transmitted plane wave $PWT = (E'_R, E'_y, E'_z)$.

Lines 53-57: Compute phase $PC = e^{j \frac{2\pi}{\lambda} (kx_A + ky_A)}$ and apply to $E'_R, E'_y, E'_z$.

Lines 58-71: Form $n'_R = E'_R \times k$ and use Equation (14) to add the contribution of this ray to the received voltage on each of the three channels.

Lines 72-73: Increment ray counter and continue the iteration until all aperture points have been used. Upon completion, NRAY equals the number of rays used in the summation for each received voltage.

Lines 74-75: If SUPPRS is false, write NRAY.

RETURN
END

8-6. Test Case: See Chapter 2.

8-7. References

2. Microwave Antenna Theory and Design, ed. by S. Silver,

Program Listing: See following pages.
SUBROUTINE REC(PWI, KA, NX, NY, XXMAX, KYMAX, FGHZ, ROTATE, TRANSL)
2  SUMX, SUMY, DFLY, DELX, DAZY, NAZY, VP, TABLE, SUPPRS, RSOMAX)
C NOTE: MXI, M XI MAGNETIC FIELDS HAVE NOT BEEN DIVIDED BY CTAG.
REAL XXMAX, KYMAX, LAMBDA, KO, ROTATE (3, 3), TRANSL (3)
REAL VR (3), KA (3), NAR (3), PR (3), PA (3), PR (3)
LOGICAL ATORG, EROT, METAL, TABLE, SUPPRS
COMPLEX SUMX (NX, NY), SUMY (NX, NY), DELX (NX, NY), DELY (NX, NY)
COMPLEX DAZY (NX, NY), NABZ (NY, NY), VR (3)
COMPLEX PWI (3), PWT (3), HPWT (3), PC
DATA ATOR, RTOA, TRUE, FALSE,
DATA TUPD/E, 2831653071796/, NDC/C/
1 NRAY=0
LAMBDA=29.97925/FGHZ
KO=TUPD/LAMBDA
DX=LAMBDA/(2*XXMAX)
DY=LAMBDA/(2*KYMAX)
MIONX=NX/2+1
MIONY=NY/2+1
C PSOMAX IS THE SQUARE OF THE MAXIMUM RADIUS OF THE APERTURE
PA (3)=0.
PHKA1=KO*KA(1)
PHKA2=KO*KA(2)
CALL VECTOR(KA, KR, ATOR, ROTATE)
C VR(1)=(0., C.)
VP(1)=(0, C.)
VP(3)=(0., 0.)
C ITERATE FOR EACH APERTURE POINT
DO 10 L=1, NX, 1
  PA(1)=(L-MIONX)*DX
3 APA=PA(1)*PA(1)
  PHA=PA(1)*PHKA1
  PA(2)=(M-MIONY)*DY
C C IF APERTURE POINT IS OUTSIDE CIRCULAR APERTURE, OMIT FROM CALCULATION
C
C
C
C
C
RSQ = AP + PA(2) * PA(2)
IF (RSQ.GT.RSQMAX) GO TO 10
4 CALL POINT (FX, FY, Z, ROTATE, TRANSL)

TRACE FAY TO FIRST INTERSECTION POINT.
NOTE: ALL APERTURE POINTS MUST BE CONTAINED WITHIN RADOME.

CALL TRACE (PK, K, PHP1, NIAL, METAL)
IF (METAL) GO TO 16
CALL VECTOR (NIP, NIA, RTOA, ROTATE)

C TABLE OF XMN OCEFS IS FORMED ON FIRST CALL TO XMFIN
C
5 CALL XMFIN (FWI, PHT, KA, NIA, PIR, TABLE, SUPPRS, KO)
PHASE = PAKA + PA(2) * PHKA2
PC = CEXP (CMFLX (C1, C2, ANOD (PHASE, TUP1)))
PH1 = PH1(1) * PC
PH2 = PH2(2) * PC
PH3 = PH3(3) * PC
C FORM MAGNETIC FIELD
CALL GA3 (PHT, KA, HPHT)

C COMPUTE CONTRIBUTION TO RECEIVED VOLTAGE ON EACH CHANNEL;
C
VR1 = EYT * HPHT(1) - EXT * HPHT(2) * PWT(1) * HWT

1
C 1 = PWT(2) * X + VR(2)
VR(1) = VR(1) + SUMX (LM1) * (HPHT(2) - PWT(1)) - SUMY (LM1) * (HPHT(1) + PWT(2))
VR(2) = VR(2) + DELX (LM1) * (HPHT(2) - PWT(1)) - DELY (LM1) * (HPHT(1) + PWT(2))
VR(3) = VR(3) + OAYZ (LM1) * (HPHT(2) - PWT(1)) - OAZY (LM1) * (HPHT(1) + PWT(2))

C GEOMETRIC OPTICS APPROXIMATION IS USED ABOVE IN EXPRESSIONS
C FOR RECEIVED VOLTAGE. I.E., HT = ZHAT X ET IN APERTURE.
C DIVISION BY ETA1 IS NOT DONE TO SAVE COMPUTATION TIME.
C
NRA = NRT + 1
10 CONTINUE

2
C IF (NOT. SUPPRS) WRITE C, NRT
C
IF (C ? SUPPRS) WRITE (C, 16) NRA
16 FORMAT (1, NUMBER OF PAYS USED IN COMPUTING APERTURE FIELD =", I10)
Chapter 9

SUBROUTINE TRACE

9-1. Purpose: To direct the ray tracing to find the intersection of a ray emanating from a point inside the radome and the inner radome surface. All dimensions are in centimeters. Radome coordinates are implied.

9-2. Usage: CALL TRACE (PO, K, P, N, METAL)

COMMON/TRACC/Z2, ZI

9-3. Arguments

<table>
<thead>
<tr>
<th>Argument</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>PO</td>
<td>Real input array of three elements containing the point (PO(x_0, y_0, z_0)) from which the ray emanates.</td>
</tr>
<tr>
<td>K</td>
<td>Real input array of three elements containing the direction cosines of the ray; i.e., (K(1) = k_x, K(2) = k_y, K(3) = k_z).</td>
</tr>
<tr>
<td>P</td>
<td>Real output array of three elements containing the point of intersection (P(x, y, z)) of the ray and the inner radome surface.</td>
</tr>
<tr>
<td>N</td>
<td>Real output array containing the direction cosines of the unit inward normal vector to the radome inner surface at (P(x, y, z)); i.e., (N(1) = n_x, N(2) = n_y, N(3) = n_z) where (n = x n_x + y n_y + z n_z).</td>
</tr>
<tr>
<td>METAL</td>
<td>Logical output variable which indicates any opaque surfaces encountered, such as a metal tip or bulkhead: (METAL = .TRUE.) indicates that (P(x, y, z)) lies on such an opaque surface.</td>
</tr>
</tbody>
</table>
22 - Real input variable which designates the $z_R$ coordinate (cm) of the intersection of the ogive section of the radome, and the metal tip (if any); must be set in main program prior to the first call to TRACE.

21 - Real input variable which designates the $z_R$ coordinate (cm) of the intersection of the ogive section and the bulkhead of the air frame; must be set in main program prior to the first call to TRACE.

9-4. Comments and Method

a. Subroutines required: OGIVE, TDISK, BDISK, OGINEN, TDISKN, BDISKN

b. The inner surface of the radome is represented by three distinct surfaces as indicated in Figure 9-1: a planar bottom disk (bulkhead), a tangent ogive, and a planar top disk (base of a metal tip). The ray is traced to the ogive surface first to find a point of intersection $P(x, y, z)$:

1. If $z_1 < z < z_2$, then the ogive section of the radome was struck, the unit normal is computed (OGINEN), METAL is set to .FALSE., and the program returns.

2. If $z > z_2$, it is assumed that the ray encountered the top disk before impinging on the ogive surface (which actually extends beyond the $z_2$ coordinate). The ray is then traced to find its intersection with the plane $z = z_2$. If the top disk is indeed struck, then METAL is set .TRUE., and $n = -z$ is returned.
Figure 9-1. Tangent Ogive Radome Geometry.
(3) If \( z < z_1 \) from (1) above, it is assumed that the bottom disk was struck by the ray before it encountered the ogive surface. The ray is traced again to find its intersection with the plane \( z = z_1 \). If this bottom disk is indeed struck, then METAL = .TRUE. and \( \hat{n} \) = \( \hat{z} \) is returned.

The steps in (2) and (3) above appear to be unnecessary; however, they are included to ensure that the ray tracing procedure works correctly and to alert the user if it does not. For example, if incorrect variable values are passed to the supporting subroutines, there is a good chance that no intersection will be found with any one of the three surfaces, in which case the following error message is outputted:

"THERE IS A HOLE IN THIS RADOME"

The message is continued with the values of \((x_o', y_o', z_o')\) and \((k_x', k_y', k_z')\). Incorrect values of geometry variables passed to the supporting subroutines, and attempts to trace a ray from a point exterior to the inner radome surface, will prompt the error message and alert the user of his mistake.

9-7. References: None
SUBROUTINE TRACER(F0,K,P,N,METAL)

SUBROUTINE TRACER TRACES A RAY FROM ITS POINT OF ORIGIN P0(3)
TO ITS POINT OF INTERSECTION WITH THE RADOME WALL, P(3)
THE RAY IS TRAVELING IN THE DIRECTION K(3)
REAL P0(3),K(3),P(3),N(3)
LOGICAL METAL,HIT

SET SURFACE INTERSECTION Z VALUES
72 IS THE INTERSECTION OF THE TOP DISK AND THE OGINE
71 IS THE INTERSECTION OF THE OGINE AND THE BOTTOM DISK
COMMON/TRACE/72,71

DETERMINE IF RAY INTERSECTS WITH OGINE SECTION
CALL OGINE(F0,K,P,N,HIT)
IF(.NOT.HIT) GO TO 2
IF(P(3).LE.71) GO TO 2
IF(P(3).GE.72) GO TO 1
GO TO 10

DETERMINE IF RAY INTERSECTS WITH TOP DISK
1 CALL "DISK(F0,K,P,N,HIT)
IF(HIT) GO TO 11
GO TO 13

DETERMINE IF RAY INTERSECTS WITH BOTTOM DISK
2 CALL "DISK(F0,K,P,N,HIT)
IF(HIT) GO TO 12
3 WRITE(F,110)
110 FORMAT(2X,"THERE IS A HOLE IN THIS RADOME")
WRITE(F,111)PC,K
111 FORMAT(2X,"RAY STARTED HERE",3G10.4,"RAY TRAVELED IN THIS DIRECTION
",3G10.4)
RETURN
4 CALL "OGINE(K,N")
METAL=.FALSE.
RETURN
Chapter 10

SUBROUTINE RXMIT

10-1. Purpose: To compute the complex rectangular vector components of the electric field \( \mathbf{E}_T \) transmitted through the radome wall, where it is assumed that the incident field \( \mathbf{E}_I, \mathbf{H}_I \) is locally a plane wave and that the radome wall behaves as an infinite plane dielectric sheet. The direction of propagation of the plane wave \( \mathbf{k} \) and the unit inward normal \( \mathbf{n} \) at the point \( P_1(x, y, z) \) are used to determine the angle of incidence and the plane of incidence of the plane wave. All dimensions are in centimeters. Radome coordinates are implied.

10-2. Usage: CALL RXMIT (PWI, PWT, K, NORM, PI, TABLE, SUPPRS, BETA)

COMMON/TRANSC/DIN(6), ER(6), TD(6), T2, WALTOL, N, NN,
D(6), ZB, TK

10-3. Arguments

\( \text{PWI} \) - Complex input array containing the vector components of the incident electric field; i.e.,
\( \text{PWI}(i_x, i_y, i_z) \).

\( \text{PWT} \) - Complex output array containing the vector components of the transmitted electric field; i.e.,
\( \text{PWT}(x_t, y_t, z_t) \).

\( \text{K} \) - Real input array containing the direction cosines of the direction from whence the plane wave emanates; i.e., \( K_x, K_y, K_z \).
RkM  - Real input array containing the rectangular components of the unit inward normal \( x, y, z \). i.e., RkM \((x, y, z)\).

Fl  - Real input array containing the coordinates \((x, y, z)\) of the point on the radome inner surface where the transmitted plane wave is assumed to emerge i.e., Fl\((x, y, z)\).

TABLE  - Logical input variables: if TRUE, a look-up table is used to compute the insertion voltage transmission coefficients \( T_\perp, T_\parallel \) corresponding to the angle of incidence \( n_\perp \); if FALSE, \( T_\perp, T_\parallel \) are each set to unity to simulate the absence of the radome. Originally, if TABLE = .FALSE., the coefficients \( T_\perp, T_\parallel \) were computed at each point Fl\((x, y, z)\) by a call to Subroutine WALL as in the case of the wall configuration being dependent on position (temperature variables, prescription tables, etc.).

SIPERS  - Logical input variables: if FALSE, a table of transmission coefficients versus \( \sin n_\perp \) is printed.
Actually, \( |T_\perp|^2, |T_\parallel|^2, |R_\perp|^2, |R_\parallel|^2 \) and the phases of \( T_\perp, T_\parallel, R_\perp, R_\parallel \) are printed.

BETA  - Real input variable \( \lambda = 2\pi/\lambda \), where \( \lambda \) is the free space wavelength (cm).

SIG.  - Real input arrays which specify the thickness th, indices, relative dielectric constant \( \epsilon_r \), and loss tangent tan\(^\delta\) of the \( N \) layers comprising the multi-layer radome wall. Layer 1 is the first layer on.
N, the exit side of the wall; layer N is the first layer on the incident side. ER(NN), TD(NN) specify $\varepsilon_r$, $\tan\delta$ of the medium in which the N-layer structure is immersed (normally, free space so that $ER(NN) = 1.0$, $TD(NN) = 0.0$). The real array D contains, after the first call to RXMIT, the thickness in centimeters of each layer.

TZ, Real variables used previously to specify longitudinal and circumferential variations in wall thickness and in the tolerance on thickness. These variables are not active in this version of RXMIT.

10-4. Comment and Method
a. Subroutines required: WALL, AMPHS, AXB
b. The transmission of an incident plane wave through a plane dielectric sheet immersed in free space ($\varepsilon_o = 8.854 \times 10^{-12}$ farads/m, $\mu_o = 4\pi \times 10^{-7}$ henries/m) may be described in terms of the insertion voltage transmission coefficients

$$T_\perp = \frac{E_{t\perp}(P')}{E_{i\perp}(P')} \quad (1)$$

$$T_\parallel = \frac{E_{t\parallel}(P')}{E_{i\parallel}(P')} \quad (2)$$

where $E_{t\perp}, E_{t\parallel}$ are the transmitted fields at P' with the sheet in place, and $E_{i\perp}, E_{i\parallel}$ are the incident fields at the same point in the absence of the sheet. The point P' lies on the colinear extension of the incident ray and is located on the exit side of the sheet.
Since the transmission coefficients $T_L, T_R$ are different, it is necessary to resolve the incident electric field $E_i$ into perpendicular and parallel components; i.e., vector components which are perpendicular to and parallel to the plane of incidence (POI) defined by $k$ and $n_R$ as illustrated in Figure 10-1. The unit vector perpendicular to the POI is given by

$$
\hat{k} = \frac{k \times n_R}{|k \times n_R|} = \frac{k \times n_R}{\sin |k, n_R|}
$$

A unit vector parallel to the POI is given by

$$
\hat{k}_l = \hat{k}_L \times \hat{k}_R
$$

The incident electric field may be written as

$$
E_i = \hat{x} E_{i\hat{x}} + \hat{y} E_{i\hat{y}} + \hat{z} E_{i\hat{z}} = \hat{k}_L E_i + \hat{k}_R E_i
$$

where

$$
E_i = \hat{k}_L \cdot E_i = k_x x + k_y y + k_z z
$$

and

$$
E_i = \hat{k}_R \cdot E_i = k_x x + k_y y + k_z z
$$

and where $k_x, k_y, k_z$ are the vector components of $\hat{k}_L, \hat{k}_R$. In terms of the coordinate system $(x, y, z)$,
Figure 10-1: Plane Wave Propagation Through an Infinite Plane Sheet
\[ E_1 = \hat{x}(E_{x1} + E_{x1}) + \hat{y}(E_{y1} + E_{y1}) + \hat{z}(E_{z1} + E_{z1}) \]  

(8)

where, for example

\[ E_{x1} = \hat{x} \cdot k \cdot E_{x1} = k \cdot E_{x1} \]  

(9)

The transmitted plane wave is then given by

\[ E_T = k \cdot T_1 \cdot E_{x1} + k \cdot T_1 \cdot E_{y1} \]  

(10)

\[ E_T = \hat{x}(T_1 \cdot E_{x1} + T_1 \cdot E_{y1} + T_1 \cdot E_{z1}) + \hat{y}(T_1 \cdot E_{x1} + T_1 \cdot E_{y1} + T_1 \cdot E_{z1}) + \hat{z}(T_1 \cdot E_{x1} + T_1 \cdot E_{y1} + T_1 \cdot E_{z1}) \]  

(11)

10-5. Program Flow

<table>
<thead>
<tr>
<th>Lines</th>
<th>Comments</th>
</tr>
</thead>
<tbody>
<tr>
<td>Line 9:</td>
<td>Set NANGLE = number of entries used in the look-up tables for T1, T1.</td>
</tr>
<tr>
<td>Lines 10-12:</td>
<td>NDO causes initialization of variables and the computation of the look-up tables on the first call to RXMIT (lines 1159).</td>
</tr>
<tr>
<td>Lines 15-16:</td>
<td>Convert layer thicknesses from inches to centimeters.</td>
</tr>
<tr>
<td>Lines 17-59:</td>
<td>Compute look-up tables for T1, T1 at NANGLE points spaced equally in sinh1 over the range (0, 1). If SUPPRES = .FALSE., print a table of transmission coefficients (every fifth point only). If ER(1) = 1.05, set AIR = .TRUE. and compute unity transmission coefficients for the &quot;air&quot; radome (for testing).</td>
</tr>
</tbody>
</table>
Lines 60-78: Compute $\sin \theta_1$.

Lines 79-86: Interpolate in table to compute $T_\perp, T_\parallel$ at $\sin \theta_1$.

Lines 87-100: Normalize the vector $\mathbf{k}_\perp$.

Lines 101-112: Compute $E_{xi}$, $E_{yi}$, $E_{zi}$.

Lines 113-124: Compute $E_{xii}$, $E_{yii}$, $E_{zi}$. 

Lines 125-129: Compute $E_{xt}$, $E_{yt}$, $E_{zt}$ and return.

Lines 130-136: If $\sin \theta_1$ is out of range of the table, write error message, set $T_\perp, T_\parallel$ to unity, and continue.


10-7. References: None

SUBROUTINE KXMIT(PHI, PHT, K, NORM, P1, TABLE, SUPPRS, BETA)
COMPLEX PHI(3), PHT(3), E7PEF, EZPAR
COMPLEX EXPAR, EYPAR, EOPEF, EYPER, TPAI, TPERI, DOT, RPERI, RPARI
REAL K(1), NORM(3), KPER(3), P1(3), AMP(4), PHS(4), KPAR(3)
LOGICAL TABLE, SUPPRS, AIR
COMPLEX TPER(250), TPAI(250), RPER, RPARI
COMMON/TRANSC/DINI(E), EP(6), TO(6), T, WALTOL, N, NO(6), ZB, TK
DATA PI/3.1415926E/
DATA NANGLE/250/
DATA NG0/0/
IF (NDO.EQ.1) GO TO 5
NDO=1
AIR=.FALSE.
IF (ER(1).LT.1.05) AIR=.TRUE.
DO 95 I=1, KK
95 D(I)=DIN(I)*2.54
PI02=PI/2.

C FORM WALL TRANSMISSION TABLES
C
MANGLE=NANGLE-1
ANGLE=MANGLE
RAD=180./PI
IF (.NOT.SUPPRS) WRITE(6,115)
115 FORMAT(22X, "ANGLE","\$PERI**2","\$PARI**2","\$X,
$ "\$PERI**2","\$X,"\$PARI**2"/
116 FORMAT(1X,F5.2,4(3X,F5.3,FP,1))
DO 106 I=1,MANGLE
SINE=(I-1)/ANGLE
IF (AIR) GO TO 91
CALL WALL(BETA, SINE,D,EP,TD, N,N, TPER(I), TPAR(I), RPER, RPARI)
GO TO 92
91 TPER(I)=(1.,0.)
TPAR(I)=(1.,0.)
RPER=(0.,0.)
RPARI=(0.,0.)
92 IF (MOD(I,5).NE.0) GO TO 100
ANG=ASIN(SINE)*RAD
100 CONTINUE
CALL AMPHS(TPER(I),AMP(1),PHS(1))
CALL AMPHS(TPAR(I),AMP(2),PHS(2))
CALL AMPHS(RPER,AMP(3),PHS(3))
CALL AMPHS(RPAR,AMP(4),PHS(4))

C CONVERT TO POWER X,N COEFFICIENTS

DO 95 L=1,4
  AMP(L)=AMP(L)**2
  IF (NOT(SUPPRS)) WRITE(6,116) ANG,((AMP(J),PHS(J)),J=1,4)
102 CONTINUE
  XC=0.
  IF (ER(1).LT.1.0) XC=1.0
  TPER(NANGLE)=COMPLEX(XC,C.)
  TPAR(NANGLE)=COMPLEX(XC,C.)
  ANG=90.
  CALL AMPHS(TPER(NANGLE),AMP(1),PHS(1))
  CALL AMPHS(TPAR(NANGLE),AMP(2),PHS(2))
  CALL AMPHS(RPER,AMP(3),PHS(3))
  CALL AMPHS(RPAR,AMP(4),PHS(4))
  IF (NOT(SUPPRS)) WRITE(6,116) ANG,((AMP(J),PHS(J)),J=1,4)
  IF (NOT(SUPPRS)) WRITE(6,105)
5 CONTINUE

C FIND VECTOR NORMAL TO NORM AND K

CALL AXB(K,NORM,KPER)

C FIND MAGNITUDE OF KPER (THIS IS ALSO THE SINE OF THE INCLUDED ANGLE)

SINE=SQRT(KPER(1)**2+KPER(2)**2+KPER(3)**2)
IF(SINE.GT.1.0) SINE = 1.0
IF(TABLE) GC TO 25
TPERI=(1.,0.)
TPARI=(1.,0.)
RPERI=(0.,0.)
RPARI=(0.,0.)
GO TO 3

C
USE TABLE OF TRANSMISSION COEFFICIENTS

25 RI=SINF*HANGLE+1.C
   IL=RI
   IF((IL.GE.NANGLE).OR.(IL.LT.1)) GO TO 50
   IH=IL+1
   X=RI-IL
   TPERI=(1.0-X)*TPER(IL)+X*TPER(1H)
   TPERI=(1.0-X)*TPER(IL)+X*TPER(IH)

TEST FOR NORMAL INCIDENCE

3 IF(SINF.LT.1F-10) GO TO 2

UNITIZE PERPENDICULAR VECTOR

   SEC=1/SINF
   KPER(1)=KPER(1)*SEC
   KPER(2)=KPER(2)*SEC
   KPER(3)=KPER(3)*SEC
   GO TO 1

2 KPER(1)=1.0
   KPER(2)=0.0
   KPER(3)=0.0

1 CONTINUE

FIND DOT PRODUTC OF INCIDENT ELECTRIC FIELD WITH KPER

   DOT=PWl(1)*KPER(1)+PWl(2)*KPER(2)+PWl(3)*KPER(3)

FIND PERPENDICULAR COMPONENTS OF ELECTRIC FIELD

   EXPD=DOT*KPER(1)
   EYPD=DOT*KPER(2)
   EZPD=DOT*KPER(3)

FIND PARALLEL COMPONENTS OF ELECTRIC FIELD

   EXPD=DOT*KPER(1)
FXPAR=PWI(1)-EXPER
FYPAR=PWI(2)-EYPER
EZPAR=PWI(3)-EZPER
CALL AXB(KPER*K,K,KPAR)
KPAR IS A UNIT VECTOR AS REQUIRED

DOT=PWI(1)*KPAR(1)+PWI(2)*KPAR(2)+PWI(3)*KPAR(3)
EXPAR=DOT*KPAR(1)
EXPAR=DOT*KPAR(2)
EZPAR=DOT*KPAR(3)

FIND X AND Y COMPONENTS OF TRANSMITTED FIELD

PWT(1)=EXPAR*TPARI+EXPER*TPERI
PWT(2)=EYPAR*TPARI+EYPER*TPERI
PWT(3)=EZPAR*TPARI+EZPER*TPERI
RETURN

50 WRITE(6,55) SINE
55 FORMAT(/10X,"SINE="F10.7," IS NOT IN THE WALL TABLE "/)
TPERI=(1.,0.)
TPARI=(1.,0.)
GO TO 3
END
Chapter 11

SUPROUTINE WALL

11-1. Purpose: To compute the transmission and reflection coefficients of a N-layer dielectric sheet having thicknesses $d_n$, dielectric constants $\varepsilon_{rn}$, and loss tangents $\tan\delta_n$ for each layer when a plane wave is incident at angle $\theta_i$.

11-2. Usage: CALL WALL (BETA, SINE, D, ER, TD, N, NN, TPER, TPAR, RPER, RPAR)

11-3. Arguments

- **BETA** - Real input variable $= 2\pi/\lambda$, where $\lambda$ is the free space wavelength.
- **SINE** - Real input variable $= \sin \theta_i$.
- **D**, **ER**, **TD** - Real input arrays containing the thickness (cm), dielectric constant $\varepsilon_r$, and loss tangent $\tan\delta$ of each layer.
- **N** - Integer input variable equal to the number of layers.
- **NN** - Integer input $= N+1$.
- **TPER, TPAR** - Complex output variables equal to the insertion voltage transmission coefficients for the components of the incident electric field perpendicular to and parallel to the plane of incidence, respectively.
- **RPER, RPAR** - Complex output variables equal to the reflection coefficients $R_\perp, R_\parallel$. 

11-4. Comment and Method

a. Layer 1 is the first layer on the exit side of the panel; layer N is the first layer on the incident side. \( T_1, T_N \) have the same value for either side of the panel being the incident side; however, \( R_1, R_N \) are different (in phase) for the two cases.

b. The details of the method are presented in Reference 1 and are reproduced in Appendix E.


11-7. References

SUBROUTINE ALLDATA(TDATA,STMAX,TPER,TPAR,TMAX,NN,TMPER,TMPAR,TMAXP)

3 COMPUTES THE TRANSMISSION AND REFLECTION COEFFICIENTS FOR EACH LAYER, PLANE DIELECTRIC PANEL FOR PLANE PARALLEL POLARIZATIONS.

4 MAXIMUM ANGLE AT WHICH ANGLES FOR PERPENDICULAR AND PARALLEL POLARIZATIONS.

5 PARAMETERS OF THE WALL: N = THE NUMBER OF LAYERS

6 M = N+1 REQUIRED TO DIMENSION ARRAYS

7 D = THICKNESS OF EACH LAYER IN CENTIMETERS

8 E = RELATIVE DIELECTRIC CONSTANT OF EACH LAYER

9 TO = THE LOSS TANGENT FOR EACH LAYER

10 THE TOP AND BOTTOM NORMAL VOLTAGE XMN COEFFICIENTS; TPER, TPAR ARE THE

11 INCEPTION VOLTAGE TRANSITION COEFFICIENTS. IT IS IMPORTANT TO

12 NOTE THAT THE XMN COEFFS ARE THE SAME FOR PLANE WAVE INCIDENT FROM

13 EITHER SIDE OF THE STRATIFIED DIELECTRIC PANEL IMMERSED IN FREE SPACE.

14 HOWEVER, THE REFLECTION COEFFS ARE NOT, THAT IS, FOR COMPUTING RPER,

15 RPAR, THE ORDERING OF XM(NN), XM(TNN) IS IMPORTANT WITH LAYER 1 BEING

16 THE FIRST LAYER ON THE EXIT SIDE, LAYER N BEING THE FIRST LAYER ON THE

17 INCIDENT SIDE. LAYER 1 AND LAYER 2 ARE JUST FREE SPACE LAYERS.

18 OF SEMI-INFINITE SPHERE.

19 C 0,0,1,1,0,0 ARE ALWAYS USED IN THE SUBROUTINE HAVING NN DIM. LIMITS

20 COMPLEX F(C), G(F), P1(0), P2(0), G(E), E, E, R1(0), P2(0), A1, A2, X1, X2,

21 B*Y4, Y1, Y2, Y3, Y4, X1, X2, Z1, Z2, Z3, Z4, P1, P2, P3, P4, Q1, Q2, Q3, Q4

22 COMPLEX TPER, TPAR, TPER, TPAR, U, V, T1, T2

23 DIMENSION F(NN), G(NN), P1(0), P2(0)

24 SRF(NN)=1.

25 TCF(NN)=1.

26 N F0=1, NN

27 DO 1=1, NN

28 C E(I)=COMPLX(EP(I),-FP(I)*TF(I))

29 AB=RFK*7.777777777777771

30 C CALCULATE TOTAL THICKNESS OF WALL IN CM

31 OTOTAL=0.

32 DO 200 I=1, N

33 C OTOTAL=OTOTAL+Q(I)

34 C Q IS THE SINE OF THE ANGLE; SQUARED

35 C I IS THE COSINE OF THE ANGLE

36 S=SINE**SIN

37 C
A = .
G0 TI 7=
77 A = 4S*SQR(T (G^2 - 40))
74 4 = A*SQR(T (G^2 + 4))
G1 = COMPLEX(A, B)
G2 = G1 * G2
G3 = G2 * G1
B = 1, = T = 1, = T = 1
G0 = G0 + 1
71 IF (T = N) 17F, 77, 177
178 SUM = SUM + T (1) / SQR (40)
177 CONTINUE
SP = SQR (AC * AC + E**2)
IF (G^2 - 40) 70, 70, 70.
72 A = .
G0 TC 41
91 A = 4A*SQR(T (G^2 - 40))
91 A = 4A*SQR(T (G^2 + 4))
G11 = COMPLEX (A, B)
R1 (1) = (G (11) - G (11)) / (G (11) + G (11))
SUM = SUM + A
A = 1, = C = D + 1
A = 1, = C = D + 2
GO = = T = 1, = T = 1
A = A1 X (11, C - E1 (11))
65 A2 = A2 X (11, C - E2 (11))
D = 1.0 + 0.5 * D

C

TV1, TV2 ARE NORMAL VOLTAGE X#4 COEFFICIENTS.

TV1 = (1.5 + Y) * TV2

U = TV1 + TV2

U = TV1 * TV2

C

TOP3, TOP4 HERE ARE VOLTAGE X#4 COEFFICIENTS AT EXIT POINT OF RAY.

TOP3 = TOP3

TOP4 = (TOP4 + TOP4) * TOP4

C

MODIFY TRANSMISSION COEFFICIENT FOR INHIBITION

TOP4 = OP4 - (TOP4 + TOP4) * TOP4

U = TOP4

TOP4 = TOP4

D = D

1 CONTINUE

RETURN

END
Chapter 12
SUBROUTINE AXB

12-1. Purpose: To compute the real vector cross product \( \mathbf{C} = \mathbf{A} \times \mathbf{B} \), where \( \mathbf{A}, \mathbf{B}, \mathbf{C} \) are expressed in rectangular components.

12-2. Usage: CALL AXB(A, B, C)

12-3. Arguments

- \( \mathbf{A}, \mathbf{B} \) - Real input arrays containing the rectangular components of \( \mathbf{A} = A_x \hat{x} + A_y \hat{y} + A_z \hat{z} \) and \( \mathbf{B} \); i.e.,
  \[ A(A_x, A_y, A_z), B(B_x, B_y, B_z). \]

- \( \mathbf{C} \) - Real output array containing the rectangular components of the vector \( \mathbf{C} = \mathbf{A} \times \mathbf{B} \); i.e., \( C(C_x, C_y, C_z) \).

12-4. Comment and Method

a. Both input vectors must be real.

b. The computation of \( \mathbf{C} = \mathbf{A} \times \mathbf{B} \) is elementary.


12-6. Test Case: None

12-7. References: None

Chapter 13

SUBROUTINE CAXB

13-1. Purpose: To compute the complex vector cross product \( \mathbf{C} = \mathbf{A} \times \mathbf{B} \), where \( \mathbf{A} \) is a complex vector and \( \mathbf{B} \) is a real vector expressed in rectangular coordinates.

13-2. Usage: CALL CAXB (A, B, C)

13-3. Arguments
- \( \mathbf{A} \) - Complex input array containing the rectangular components of the vector \( \mathbf{A} = x_A \mathbf{A}_x + y_A \mathbf{A}_y + z_A \mathbf{A}_z \); i.e., \( \mathbf{A} (A_x, A_y, A_z) \).
- \( \mathbf{B} \) - Real input array \( \mathbf{B} (B_x', B_y', B_z') \) representing the vector \( \mathbf{B} \).
- \( \mathbf{C} \) - Complex output array \( \mathbf{C} (C_x', C_y', C_z') \) representing the vector \( \mathbf{C} = \mathbf{A} \times \mathbf{B} \).

13-4. Comment and Method: None


13-6. Test Case: None

13-7. References: None

Chapter 14

SUBROUTINE RECS

14-1. Purpose: To compute the angle of arrival $k$ of a plane wave on a monopulse antenna which yields an electrical boresight indication which, due to the radome, may be different from the mechanical boresight along the $z$ axis of the antenna. The antenna aperture lies in the $xy$ plane. All dimensions are in centimeters. Antenna coordinates are implied.

14-2. Usage: CALL RECS (SUMX, SUMY, DELX, DELY, DAZX, DAZY, NX, NY, LMAX, NS, IOPT, VR, DMRAD, ROTATE, TRANSL, FGHZ, KMAX, KMIN, TABLE, SINGO, K, AZTM, ELTM, RSQMAX, VMAX, SMAX, SUPPRS)

14-3. Arguments

SUMX, SUMY - Complex input arrays of NX by NY elements each
DELX, DELY - containing the aperture distributions of the DAZX, DAZY monopulse antenna. See Subroutine HIACNP.
LMAX - Integer input variable which controls the maximum number of iterations that will be done to find the electrical boresight within the tolerance specified by DMRAD.
NS - Inactive integer variable.
IOPT - Integer input variable which selects the polarization of the incident plane wave. See Subroutine IN_PW.
VR - Complex array of three elements representing the received voltage on the sum, elevation difference,
and azimuth difference channels of the antenna, respectively (output).

**DMRAD**
- Real input variable equal to the tolerance to which the electrical boresight in milliradians is to be computed; i.e., 0.1 milliradian.

**ROTATE**
- Variables required by Subroutine RECM.

**TRANSL**
- See Chapter 8.

**FGHZ, KMAX, KYMAX**

**TABLE**

**SINOS**
- Real variable equal to the sine of the angle $\theta_{os}$ (measured from the z-axis) in the $\phi = 45^\circ$ plane ($\phi$ measured from +x toward +y) at which the first target return arrives; e.g., $\theta_{os} = 3$ degrees.

**K**
- Real output array containing the direction of arrival of the final target return; i.e., $K(k_x, k_y, k_z)$.

**AZTM, ELTM**
- Real output variables equal to angles (mrad) in the azimuth and elevation planes of the antenna which specify the direction of arrival of the final target return. If $k$ is the unit vector pointing from the origin in the direction of the final return, then the orthographic projection of this vector onto the xz-plane makes an angle AZTM with the z-axis; its projection onto the yz-plane makes an angle ELTM with the z-axis.
RSQMAX - Real variable needed by Subroutine RECM. See Chapter 8.

VMAX - Unused.

SMAX - Real output variable equal to the magnitude of the received sum voltage for the final return; used to compute loss in antenna gain.

SUPPRS - Logical input variable which controls the computation and printing of additional antenna outputs around the boresight direction. If TRUE, the complex voltage outputs of the difference channels will be computed at one milliradian increments over the range ±3.0 mrad, centered on the direction of the final target return.

14-4. Comments and Method

a. Subroutines required: AMPHS, RECM, INCPW.

b. Subroutine RECBS uses a linear tracking model to determine the direction of arrival \( k = x_k + y_k + z_k \) of a plane wave which will produce null indications in the elevation and azimuth difference channels of the monopulse antenna inside the radome. Subroutine RECM is used to compute the received voltage on each channel for the specified polarization (IOPT) and direction of arrival.

The first target return is made to arrive from the direction

\[
k_1 = x \sin \theta_{os} + y \sin \theta_{os} + z \sqrt{1 - 2 \sin^2 \theta_{os}} \tag{1}
\]

to produce outputs.
\[ U_{AZ} = \text{Im} \left( \frac{V_{AZ}}{V_x} \right) \] (2a)

\[ U_{EL} = \text{Im} \left( \frac{V_{EL}}{V_x} \right) \] (2b)

where \( V_x, V_{AZ}, V_{EL} \) are the complex voltage outputs of the three-channel outputs of the three-channel antenna. The second return is made to arrive from

\[ k_A = x(-\sin \theta) + y(-\sin \theta) + z(\sin^2 \theta) \] (3)


Construct a linear model for each channel independently using the slope/intercept equation for a line; i.e.,

\[ U_{AZ} = M_{AZ} k_A + b_{AZ} \] (4a)

\[ U_{EL} = M_{EL} k_A + b_{EL} \] (4b)

where

\[ M_{AZ} = \frac{(U_{AZ1} - U_{AZ2})}{(k_{x1} - k_{x2})} \] (5a)

\[ M_{EL} = \frac{(U_{EL1} - U_{EL2})}{(k_{y1} - k_{y2})} \] (5b)

\[ b_{AZ} = U_{AZ1} - M_{AZ} k_{x1} \] (6a)

\[ b_{EL} = U_{EL1} - M_{EL} k_{y1} \] (6b)
use this model to estimate the values of \( k_x, k_y \) that will make \( U_{AZ} = U_{EL} = 0 \); i.e.,

\[
\begin{align*}
k_x &= -b_{AZ}/M_{AZ} \\
k_y &= -b_{EL}/M_{EL}
\end{align*}
\]

(7a) \hspace{1cm} (7b)

where the value of \( k_z \) follows from

\[
k_x^2 + k_y^2 + k_z^2 = 1
\]

(8)

The third target return is made to arrive from this direction and the values of \( U_{AZ}, U_{EL} \) are computed via Subroutine RECM. Now, according to the last linear model, a value of \( U_{AZ} \) within the range

\[
\left| U_{AZ} \right| < \left| M_{AZ} \sin^3 \theta_{tol} + b_{AZ} \right|
\]

(9)

would indicate that the null has been found within the tolerance \( \theta_{tol} (=DMRAD) \) specified. If this tolerance is not satisfied for both channels, then the process is repeated until it is or until LMAX is exceeded. In continuing the iterations, \( k_z \) becomes \( k \), and the estimated point becomes \( k_z \).

On the last return, the direction of arrival is specified by \( k \). The angles in the azimuth and elevation planes are given in milliradians by

\[
AZTM = \sin^{-1} \left( \frac{k_x}{1 - k_y^2} \right) \cdot 1000
\]

(10)
\[
M_{\infty} = \frac{M_{\infty}}{L_{\infty}}
\]

where \( L_{\infty} \) is the maximum amplitude returned by the trim calculator. In addition, for a particular purpose, additional output around the important aspects of the solution was calculated. The direct values are expressed by the method

\[
V = \pm \sqrt{k_i + \sqrt{k_i^2 - 4k_j}}
\]

where \( i \) and \( j \) are real and imaginary. At each direction, the real and imaginary \( \pm \sqrt{k_i + \sqrt{k_i^2 - 4k_j}} \) are printed as well as the complex monopole transition function values in the brackets of equations (11). It is noteworthy that the shape of the transition function will change from \(-90^\circ\) to \(+90^\circ\) as the solution in Equation (11) goes from negative to positive values. This behavior is a consequence of the phasing chosen for the aperture distributions for the different annulars in Subroutine DMRAD.

---

**Input Flow**

**Input**

**Output**

**Comments**

Line 11-1a: Initialize variables. Convert DMRAD to radians and compute sine.
Lines 16-30: Compute first two target returns to construct linear tracking model.

Lines 31-36: Compute slopes $A_{AZ} = SLPAZ$, $E_{EL} = SLP_E$ from first two returns.

Line 38: Iterate on linear model up to $LMAX$ times.

Lines 39-50: If the increment in $\Delta k$ is larger than $\sin(DMRAd/1000)$, then use it to compute slopes; if not, use the last computed values of slopes to avoid division by too small a number.

Lines 45-50: Compute intercepts $B_{AZ}$, $B_{EL}$.

Lines 51-52: Compute accuracy criteria based on current slopes and intercepts.

Lines 53-54: Compute direction $k$ that the model indicated will produce nulls $U_{AZ}(0)$, $U_{EL}(0)$ in both planes.

Lines 55-56: Compute $U_{EL}$, $U_{AZ}$ for this direction $k$.

Lines 57-58: Update the linear tracking model by storing the last two points in each channel as $U(1)$, $U(2)$; e.g.,

$$U_{AZ}(1), U_{AZ}(2) \text{ and } U_{AZ}(2) - U_{AZ}(1) = K_1(1) \text{ and } K_2(1) = K_1(1), \text{ etc.}$$

Line 59: At least three iterations are always used.

Lines 60-63: If $A_{AZ}$, $E_{EL}$ are within error bounds, exit the loop; if not, continue to iterate.

Lines 64-66: If $LMAX$ exceeded, inform the user.

Lines 67-68: Compute amplitude $SMAX$ on sum channel for final target return.

Lines 69-70: Compute slopes for final return.

Lines 71-74: Compute hearseight error $AZTH$, $ELTH$.
Lines 103-104: Compute $k_n$ for $k_1$ and $k_2$. 

Lines 105-106: Convert slopes to volts/degree. 

Lines 108-110: If SUFFER = .FALSE., print results. 

Lines 112-117: Compute and print additional outputs around the boresight direction $k$. 

Lines 140-144: Compute and print the slopes of a linear tracking model based on the outputs at $+1$ mill and $-1$ mill (hence, the division by .006 = +1 mill). 

Line 146: RETURN 

Line 148: END 


14-7. References: None.
B \times (1) = V \times (2)
\text{GO TO 77}
\text{JAN} = 2
\text{GO TO 77}
66 \text{UE} (2) = U_A
\text{K} \times (1) = \text{K} \times (2)
\text{JAN} = 1
67 \text{IF} (\text{UE} \times \text{UE} \geq 2) \text{GO TO 79}
\text{UE} (1) = U_F
\text{K} \times (2) = \text{K} \times (2)
\text{JAN} = 2
\text{GO TO 77}
68 \text{UE} (2) = U_F
\text{K} \times (2) = \text{K} \times (2)
\text{JAN} = 1
72 \text{C} = \text{K} \times (1) - \text{K} \times (2)
\text{D} = \text{K} \times (2) - \text{K} \times (2)
\text{IF} (\text{TP} \times \text{LT} \leq 3) \text{GO TO 80}
\text{IF} ((\text{ABS} (\text{UE}) \times \text{LT} \times \text{ACC} \times \text{Z}) \times \text{AND} ((\text{ABS} (\text{UE}) \times \text{LT} \times \text{ACC})) \text{GO TO 85}
86 \text{CONTINUE}
\text{WRITE (6,25)}
25 \text{FORMAT} (// "\text{LMAX EXCEEDED BEFORE ACCURACY CRITERION MET"/})
85 \text{SMAX} = \text{CABS} (\text{VF} (1))
\text{IF} (\text{ABS} (\text{C}) \times \text{DY} \times \text{SROAD}) \text{SLPAZ} = \text{UA} (1) - \text{UA} (2) / \text{C}
\text{IF} (\text{ABS} (\text{D}) \times \text{DY} \times \text{SROAD}) \text{SLPEL} = \text{UE} (1) - \text{UE} (2) / \text{D}
\text{ZTM} = \text{ASIN} (\text{K} \times (1)) / \text{SOPT} (1, - \text{K} \times (2) \times \text{2}) \times \text{1000}
\text{ELTM} = \text{ASIN} (\text{K} \times (2)) / \text{SOPT} (1, - \text{K} \times (2) \times \text{2}) \times \text{1000}
\text{K} \times (1) = \text{SORT} (1, - \text{K} \times (2) \times \text{2})
\text{K} \times (2) = \text{SORT} (1, - \text{K} \times (2) \times \text{2})
\text{C CONVERT SLOPES TO VOLTS/DEG, WHERE THE SIGNAL RECEIVED BY SUM}
\text{C CHANNEL IS CONSIDERED TO BE ONE VOLTS/}
\text{SLPAZ} = \text{SLPAZ} / 57.3
\text{SLPEL} = \text{SLPEL} / 57.3
\text{IF} (\text{SUPPERS} \leq \text{METU} \times \text{4}) \text{WRITE (6,90)}
\text{\text{K} \times 2, AZTM, ELTM, SLPAZ, SLPEL, UA} (1), \text{UE} (1), \text{SMAX, LCTR}
4) \text{FORMAT} (// "\text{FINAL ANSWERS FOR MICRONS SYSTEM: } \\
\text{\text{" } K = " } 3, \text{E}12, \text{,} 5 \\
\text{\text{" } AZTM = " } 3, \text{E}12, \text{,} 5, \text{,} \text{M}40, " \text{ELTM = " } 3, \text{E}12, \text{,} 5, \text{,} \text{M}40, " \text{SOLUTIONS: } \\
\text{\text{" } SLPAZ = " } 3, \text{E}12, \text{,} 5, \text{,} \text{VOLTS/DEG} " \text{SLPEL = " } 3, \text{E}12, \text{,} 5, \text{,} \text{VOLTS/DEG} " \\
\text{\text{" } SLPAZ = " } 3, \text{E}12, \text{,} 5, \text{,} \text{VOLTS/DEG} " \text{SLPEL = " } 3, \text{E}12, \text{,} 5, \text{,} \text{VOLTS/DEG} " \\
\text{\text{" } } 3, \text{E}12, \text{,} 5, \text{,} \text{VOLTS/DEG} " \text{SLPAZ = " } 3, \text{E}12, \text{,} 5, \text{,} \text{VOLTS/DEG} " \\
\text{\text{" } SLPAZ = " } 3, \text{E}12, \text{,} 5, \text{,} \text{VOLTS/DEG} " \text{SLPAZ = " } 3, \text{E}12, \text{,} 5, \text{,} \text{VOLTS/DEG} " \
\text{\text{" } } 3, \text{E}12, \text{,} 5, \text{,} \text{VOLTS/DEG} " \text{SLPAZ = " } 3, \text{E}12, \text{,} 5, \text{,} \text{VOLTS/DEG} " \\
\text{\text{" } } 3, \text{E}12, \text{,} 5, \text{,} \text{VOLTS/DEG} " \text{SLPAZ = " } 3, \text{E}12, \text{,} 5, \text{,} \text{VOLTS/DEG} " \\
\text{\text{" } } 3, \text{E}12, \text{,} 5, \text{,} \text{VOLTS/DEG} " \text{SLPAZ = " } 3, \text{E}12, \text{,} 5, \text{,} \text{VOLTS/DEG} " 
110 \text{\text{" } } 3, \text{E}12, \text{,} 5, \text{,} \text{VOLTS/DEG} " \text{SLPAZ = " } 3, \text{E}12, \text{,} 5, \text{,} \text{VOLTS/DEG} " \\
111 \text{\text{" } } 3, \text{E}12, \text{,} 5, \text{,} \text{VOLTS/DEG} " \text{SLPAZ = " } 3, \text{E}12, \text{,} 5, \text{,} \text{VOLTS/DEG} " \\
112 \text{\text{" } } 3, \text{E}12, \text{,} 5, \text{,} \text{VOLTS/DEG} " \text{SLPAZ = " } 3, \text{E}12, \text{,} 5, \text{,} \text{VOLTS/DEG} " \\
113 \text{\text{" } } 3, \text{E}12, \text{,} 5, \text{,} \text{VOLTS/DEG} " \text{SLPAZ = " } 3, \text{E}12, \text{,} 5, \text{,} \text{VOLTS/DEG} " \\
114
**FORMAT**"**ADDITIONAL** MODULCE OUTPUTS AROUND BORESIGHT**: **/\**

**DC**: \( T = \pm 125 \)

\( \text{K1}(1) = \text{SIN}(-3, + \text{IP} / 1000) \times \text{K1}(1) \)

\( \text{K1}(2) = \text{SIN}(-3, + \text{IP} / 1000) \times \text{K1}(2) \)

\( \text{AMP} = T \times 5 \times 1 \)

\( \text{K1}(1) = \text{SQR}(1, - \text{K1}(1)^* \times \text{K1}(2)^*) \)

**CALL** \( \text{THEX}(\text{K1}, \text{VX}, \text{NY}, \text{XMAX}, \text{XMAX}, \text{FGH7}, \text{HCTATE}, \text{TRANSL}, \)

\( \text{CIMY}, \text{SUPY}, \text{SEL}, \text{CILY}, \text{CILX}, \text{CILY}, \text{VY}, \text{ROLL}, \text{SUPRPS}, \text{PSQMAX}) \)

\( \text{UAT}(1) = \text{AIMAG} \times \text{VF}(1) / \text{VF}(1) \)

\( \text{UAT}(1) = \text{AIMAG} \times \text{VF}(2) / \text{VF}(1) \)

\( \text{IF} \text{(IP, EQ, 1) \ SLF1} = \text{UAT}(1) \)

\( \text{IF} \text{(IP, EQ, 1) \ SLF2} = \text{UAT}(1) \)

**WRITE** \( (0, 9) \ \text{ANG}, \text{UAT}(1), \text{UEL}(1) \)

**FORMAT**"**ANG**"**,9,5,**"**MAKE FROM** BORESIGHT** VRAZ**"**,9,12,5,**"**VOLTS**"*/

\( \text{VF}(1) = \text{VF}(1) / \text{VF}(1) \)

\( \text{VF}(1) = \text{VF}(2) / \text{VF}(1) \)

**CALL** \( \text{AMP} = (\text{VF}(3) / \text{VF}(1), 0) \)

**CALL** \( \text{AMP} = (\text{VF}(2) / \text{VF}(1), 0) \)

**WRITE** \( (0, 9) \ \text{CILG}, \text{E} \)

**FORMAT**","**EILP**"**(AMP, \text{PHS}) = ",9,12,5,**"**DEL(AMP, \text{PHS}) = ",9,2E12,5,**/"

**CONTINUE**

\( \text{SLP1} = (\text{UAT}(1) - \text{SLP1}) / (0,125*57.3) \)

\( \text{SLP2} = (\text{UAT}(1) - \text{SLP2}) / (0,126*57.3) \)

**WRITE** \( (0, 9) \ \text{SLP1}, \text{SLP2} \)

**FORMAT**"**AVERAGE SLPAZ**"**,9,12,5,**"**VOLTS/DEG**"*/

**AVERAGE SLPEL****,9,12,5,**"**VOLTS/DEG"*/**SUM**=1.6 **VOLTS***/

**RETURN**

**END**
Chapter 15

SUBROUTINE RECPTN

15-1. Purpose: To compute the receiving patterns of a monopulse antenna at NREC points in a specified principal plane. A plane wave of specified polarization (ICOMP) is made to be incident on the antenna at equal increments in sinθ over the range (-KMAX, KMAX -DK) in either the elevation plane (ICUT = 1) or azimuth plane. The received voltage in each channel is computed in the presence of the radome and stored for return to the calling program.

15-2. Usage: CALL RECPTN (SUMX, SUMY, DELX, DELY, DAZX, DAZY, NX, NY, ICUT, ICOMP, KMAX, VPREC, KXMAX, KYMAX, FGHZ, ROTATE, TRANSL, TABLE, SUBIRS, RSQMAX)

15-3. Arguments

SUMX, SUMY - Complex input arrays of NX by NY elements containing the aperture field distributions of the monopulse antenna. See Subroutine HACNF.

DELX, DELY - Integer input variable which specifies the principal plane in which the pattern is computed: elevation (ICUT = 1) or azimuth (ICUT = 2).

ICOMP - Integer input variable which specifies the linear polarization of the incident plane wave: elevation component e only (ICOMP = 1) or azimuth component a only (ICOMP = 2). See Figure 15-1 for further clarification.
$\hat{\epsilon} = \text{elevation component}$

$\hat{\alpha} = \text{azimuth component}$

Figure 15-3 Coordinate System for Far Field Patterns
KMAX - Real input variable equal to $\sin \theta_{\text{max}}$, where the pattern is computed over the angular range $(-\theta_{\text{max}}, \theta_{\text{max}})$, but in equal increments in sine so that Fourier interpolation can be applied directly in the wavenumber domain using the Fast Fourier Transform.

NREC - Integer input variable equal to the number of points at which the pattern is computed.

VREC - Complex output array of NREC by 3 elements containing the computed receiving patterns for the sum, elevation difference, and azimuth difference patterns of the monopulse antenna.

KXMAX, KYMAX, - Input variables required by Subroutine RECM. See FGHZ, ROTATE Chapter 8.

TRANSL, TABLE,
SUPPRS, RSQMAX

15-4. Comments and Method

Subroutines INCPW and RECM are used to compute the incident plane wave and the received voltage in each channel for each direction of arrival in the specified plane. For the elevation plane, the direction of arrival is given by

$$k = x(0) + y \sin \theta + z \sqrt{1 - \sin^2 \theta}$$

(1)

where $\theta$ is the angle measured from the z-axis. For the azimuth plane

$$k = x \sin \theta + y(0) + z \sqrt{1 - \sin^2 \theta}$$

(2)
The increments in angle are given by

\[ \Delta \theta = 2 \frac{k_{\max}}{N_{RF}} \]  \hspace{1cm} (1) \]

Values of \( k_{\max} \) correspond to the invisible region and must be excluded from consideration.

1-6. Program Flow: Compare program listing with flow chart in the discussion above.


1-7. References: None

1-8. Program Listings: See following page.
SUBROUTINE RECFT(NUMX, NUMY, DEX, DEY, DAZX, DAZY, NX, NY, ICUT, ICOMP,  
        KMARCH, INC, JN, VXMAX, FGHZ, ROTATE, TRANSL,  
        TABLE, SUPPPS, SIZMAX)  
  1      COMMON TRANSMITTING FEED FIELD ON ZAX=PLANE IS EXT, EYT,  
  2      PAY FACTOR IS USED TO ACCOUNT FOR PARABOE (SUBP, REC).  
  3      WRIT-UP-MAY BE POINTS AT WHICH PATTERN IS COMPUTED  
  4      ICUT= I=1, ICUT= 5 FOR AZ OUT  
  5      ICOMP= FOR AZ COMPONENT, =? FOR 42 COMPONENT  
  6      VVX=1, SPECIF-ES ANGULAR LIMITS (ANALOGUOUS TO XMAX)  
  7      COMPLEX NUMX(NX,NY), NUMY(NX,NY), DEX(NX,NY), DEY(NX,NY)  
  8      COMPLEX DAZX(NX,NY), DAZY(NX,NY)  
  9      COMPLEX V<>(3,NX,NY), FGHZ(NX,NY)  
 10      FGHZ<>, VXMAX, YMAX, XMAX, (3), ROTATE(3,3), TRANSL(3)  
 11      LOCAL T=EL, VT, OLT,  
 12      OLT=1/1.1455265  
 13      VT=120.14:  
 14      IF (ICUT.EQ.1) GT (2, 1) ICUT?  
 15      J=1  
 16      IF VXMAX(I,J)>1.1, VXMAX=1.1, I<NC  
 17      CM=0, K=0,NP=0  
 18      IF (ICUT.EQ.1) U=0  
 19      ANCMAX=ASIN(KMAX)*1PR/PI  
 20      K<IT=(1,1,1) ICUT,ICOMP,KMAX,NP,EK,ANCMAX  
 21      COF=1, NP,  
 22      X<IT(1)=KMAX*(J-1)*P  
 23      X<IT(2)=0*(1,1,1)*NP  
 24      CALL ICMP(EK, INC, ICOMP)  
 25      CALL ICHI<>(CM, N, NX, NY, VXMAX, XMAX, YMAX, FGHZ, ROTATE, TRANSL,  
 26      L<IT(VX:NX, NUMY, DEX:DEY, DAZY:DAZX, V<>, TABLE, SUPPPS, SIZMAX)  
 27      V<>(1,1)=V<>(1,1)  
 28      V<>(1,2)=V<>(1,2)  
 29      VRECI<>(3,3)=VRECI<>(3,3)  
 30      CONTINUE  
 31      HT<IT=(1,1) ICUT,ICOMP,KMAX,NP,EK,ANCMAX  
 32      1: IF MAX"=" 4 RECEIVING PATTERN, DETERMINE FOR 1" NO  
 33      ICUT="127  
 34      ICUT="107" VXMAX="7.37" NRECI<="157" OK=""
Chapter 16

SUBROUTINE OGIVE

16-1. Purpose: To solve for the intersection PH(x, y, z) of a line (ray) and a tangent ogive surface. The ray starts at point PO(x₀, y₀, z₀) and travels in the direction K(kₓ, kᵧ, k₂) = k = kₓkₓ + kᵧkᵧ + k₂k₂. Dimensions are in centimeters. Radome coordinates are implied.

16-2. Usage: CALL OGIVE (PO, K, PH, HIT)

COMMON/OGIVC/RP, BSQ, AP, RINV, B, RSQI, RP2

16-3. Arguments

PO - Real input array containing the point of origin of the ray PO(x₀, y₀, z₀).
K - Real input array containing the direction cosines of the ray K(kₓ, kᵧ, k₂).
PH - Real output array containing the point of intersection PH(x, y, z), if HIT = .TRUE.
HIT - Logical output variable which indicates if an intersection solution was found (TRUE).

The following variables are common with the main program and are precalculated to speed up the ray tracing computations. Refer to Figure 16-1 of the radome geometry for the definitions of k and B.

RP - Real input variable = k² - B².
BSQ - Real input variable = B².
AP - Real input variable = 0. See APIN in Section 2-4.
RINV - Real input variable = 1/R.
B - Real input variable. See Figure 16-1.
Figure 16-1. Tangent Ogre Radome Geometry.
RP1 - Real input variable = $k^2$.

RP2 - Real input variable = $k^2 + b^2$.

16-4. Comment and Method

a. The common variables must be computed in the main program prior to the first call to EIVCM.

b. Subroutines required: CHK1, APK, XY. Real function CHK1(X) computes cube root; APK computes square root with test for negative argument.

c. The intersection of a ray and ogive surface requires the solution of a quartic equation in the parameter $z$ as follows [1]

$$z^4 + c_2 z^2 + c_1 z + C_1 = 0$$  \(1\)

where

$$c_4 = \frac{2(1+U)(2A+V)}{(1+U)^2}$$  \(2\)

$$c_3 = \frac{2(1+U)(-k+A^2+w) + (2A+V)^2 - 4B^2U}{(1+U)^2}$$  \(3\)

$$c_2 = \frac{c_4(2A+V)(-k+A^2+w) - 4B^2V}{(1+U)^2}$$  \(4\)

$$c_1 = \frac{(-k+A^2+w)^2 - 4B^2w}{(1+U)^2}$$  \(5\)

and where
\[ u = \frac{K_j}{I_{Pj}} \]  

(6)

\[ v = \frac{2(K_jP_j + K_iP_i)}{K_i} \]  

(7)

\[ w = P_j + P_i \]  

(8)

\[ \rho = \rho^1 - \rho^2 \]  

(9)

The variables \( k \) and \( \rho \) are defined on Figure 16-1 and by the shape equation:

\[ r = (x^2 + y^2 - c^2 - (\Delta y)^2)^{1/2} \]  

(10)

where \( \Delta y \) is the axis of revolution for the curve shape. The variable \( \Delta y \) provides for an offset along \( z \) of the coordinates for the curve.

Variables are obtained by fitting when the following equations for a ray that passes through the point \( P(x_0, y_0, z_0) \) in the direction \( k = xk_x + yk_y + zk_z \) are substituted into Equation (10):

\[ \frac{x - x_0}{k_x} = \frac{y - y_0}{k_y} = \frac{z - z_0}{k_z} = \text{constant} \]  

(11)

All the parts of the motion equation may be found from the equations:

\[ h = \frac{v_0}{(v_0^2 - 4k_1)^{1/2} - v_1} \]  

(12)
This cubic equation has at least one real root \( p_0 \), given by

\[
\left( -\frac{1}{2}, \sqrt[3]{\frac{27}{4} \cdot \frac{27}{2}} \right)^{1/3} \cdot \left( -\frac{1}{2}, \sqrt[3]{\frac{27}{4} \cdot \frac{27}{2}} \right)^{1/3}
\]

where

\[
\zeta = \frac{1}{2} \left( \frac{3}{4} \pm \sqrt{\frac{3}{4} - \frac{27}{4}} \right)
\]

\[
T = \frac{1}{27} (-2\zeta_3 + \zeta_3^3 (\zeta_0 \zeta_1 - \frac{4}{7} \zeta_0^3) + \zeta_1 \zeta_0^2 (\zeta_0 \zeta_1 - \frac{4}{7} \zeta_0^3) + \zeta_0^2 \zeta_1^2 (\zeta_0 \zeta_1 - \frac{4}{7} \zeta_0^3))
\]

once \( p_0 \) is found, the roots of the quartic equation follow from

\[
\frac{p_{1,2}}{4} = \frac{C_3}{4} + \frac{K_1}{2} + \frac{E}{2}
\]

\[
\frac{p_{3,4}}{4} = \frac{C_3}{4} - \frac{K_1}{2} + \frac{E}{2}
\]

where

\[
K_1 = \sqrt[3]{\frac{1}{4} + C_3 + \zeta_0}
\]

\[
\left( \frac{3}{4} - K_1 - 2C_3 + \frac{4C_3^2 - 8C_3 - \Delta}{4K_1} \right)
\]

\[
\left( \frac{3}{4} - K_1 - 2C_3 + \frac{4C_3^2 - 8C_3 - \Delta}{4K_1} \right)
\]

The correct root \( p_0 \) is chosen if the one with the smallest absolute value and which has the same sign as \( k_{3,0} \). The interception point \( P = (x, y, z) \)
The components of the unit inward normal vector at \( x, y, z \) are given by:

\[
\begin{align*}
N_x &= \frac{x}{\sqrt{x^2 + y^2}} \\
N_y &= \frac{y}{\sqrt{x^2 + y^2}} \\
N_z &= \frac{z}{\sqrt{x^2 + y^2}}
\end{align*}
\]

In the special case of \( x, y, z \), the \( z \) coordinate does not change.

\[
N_x = \frac{x}{E}
\]

The components of the unit normal in the \( x, y \) plane become:

\[
\begin{align*}
x &= x - x_0 \cdot N_x \\
y &= y - x_0 \cdot N_y
\end{align*}
\]
\[ y = y_o + k_y t \]  

(29)

where the parameter \( t \) is the distance along the line from \((x_o, y_o, z_o)\) to \((x, y, z)\). Substituting Equations (27) - (29) into (10) yields the following quadratic equation in \( t \):

\[ t^2 + \frac{1}{L} \left( k x + k y \right) t + (x_o^2 + y_o^2) - \left( \sqrt{R^2 - (z_o - a)^2} - B \right)^2 = R_s^2 \]

(30)

The quadratic formula yields the following solutions to the above equation:

\[ t = \frac{-k(x + y) \pm \sqrt{(k(x + y))^2 - 4(x_o^2 + y_o^2) - (x_o^2 + y_o^2 + R_s^2)}}{2} \]

The unit vectors \( u \) and \( v \) are calculated using Equations (24) through (26).


10-o. Program listings: See following pages.
THE LOCATION OF THE GENERATION CENTER OF THE OIVE
CONCENTRIC.
THE DATA NEEDED FOR THE DATA STATEMENT IS
X = 2*(X(1)) $\ldots$ X = -2*(X(3)) TO 30
Y = 2*(X(1)), 57.1, -157 TO 790
Z = -2*(X(3)), -2*(X(1)) + 2*(R(1) $\ldots$ R(3))
S = 2*(X(2)) + 2*(R(1)) + R(2)
T = 2*(X(2)) + 2*(R(2)) + R(3)
Chapter 17

SUBROUTINE OGIVEN

17-1. Purpose: To compute the unit inward normal vector \( \mathbf{n} = x \mathbf{n}_x + y \mathbf{n}_y + z \mathbf{n}_z \) to the tangent ogive surface at the point \( \mathbf{P}(x, y, z) \).
Dimensions are in centimeters and radome coordinates are implied.

17-2. Usage: CALL OGIVEN (PI, N)

COMMON/OGIVC/RP, BSQ, AP, BINV, P, RSQ1, RP2

(See Chapter 16 for common variables.)

17-3. Arguments

PI - Real input array containing the point \( \mathbf{P}(x, y, z) \)
on the tangent ogive surface at which the unit
normal is desired, as computed by Subroutine OGIVEN.

N - Real output array containing the direction cosines
\( (n_x, n_y, n_z) \) of the unit inward normal vector.

17-4. Comments and Method

The tangent ogive surface is described by

\[
f(r, \phi) = r - \sqrt{r^2 - (z - a)^2} + R = 0 \quad \text{(1)}
\]

where \( r = \sqrt{x^2 + y^2} \) and where \( a \) and \( R \) are defined in Figure 17-1. The unit
inward normal to this surface is given by

\[
\mathbf{n} = -\frac{\partial f}{\partial r} \mathbf{r} - \frac{1}{\sqrt{f'}} \left[ \frac{\partial f}{\partial r} \frac{dr}{dx} + \frac{\partial f}{\partial \phi} \frac{d\phi}{dx} \right] + \frac{\partial f}{\partial \phi} \frac{d\phi}{dy} + \frac{\partial f}{\partial z} \frac{dz}{dy} \frac{dz}{dr} \quad \text{(2)}
\]
where \( \mathbb{K} \) is the celerity operator. Equation (3) can be rewritten as

\[
\mathbb{K} = \frac{1}{2} \left( \frac{\partial}{\partial r} \left( \frac{\partial f}{\partial r} + \frac{\partial f}{\partial \phi} \right) + \frac{\partial^2 f}{\partial \phi^2} \right)
\]

where the differentiation with respect to \( r \) has been done and \( df/dr \) and \( df/d\phi \) are next used. The remaining terms are given by

\[
\frac{df}{dr} = \frac{(\xi - 1)}{\sqrt{1 + \frac{(\xi - 1)^2}{\psi^2 - (\xi - 1)^2}}}
\]

\[
\psi = \frac{(\xi - 1)}{\sqrt{1 + \frac{(\xi - 1)^2}{\psi^2 - (\xi - 1)^2}}}
\]

where \( \psi = \xi + \epsilon \). The direction cosines can be written explicitly as

\[
\cos \beta = \frac{(\xi - 1) \psi}{\sqrt{(\xi - 1)^2 + \psi^2}}
\]

\[
\sin \beta = \frac{\psi}{\sqrt{(\xi - 1)^2 + \psi^2}}
\]

\[
\cos \alpha = \frac{1}{\sqrt{(\xi - 1)^2 + \psi^2}}
\]

\[
\sin \alpha = \frac{(\xi - 1)}{\sqrt{(\xi - 1)^2 + \psi^2}}
\]

where the abbreviations \( \xi = (\xi - 1)^2 \) from Equation (4) has been used.

From Equations (6) to (8) it is seen that the second term is of the form \( (\xi - 1)^2 \) directly to the

\[
\xi = \frac{(\xi - 1)}{\psi}
\]

For further details, see Chapter 5.
17-7. Reference


Chapter 18
SUBROUTINE XV

18-1. Purpose: To compute the x and y coordinates of the intersection point \( P(x, y, z) \) of a line (ray) having direction cosines \( K(k_x, k_y, k_z) \) with a surface of revolution when \( z \) is known. The line passes through the known point \( P(x_0, y_0, z_0) \). All dimensions are in centimeters.

18-2. Usage: CALL XV \((P, E, S, P1)\)

18-3. Arguments:

\( P \) - Real input array containing the known point through which the ray passes; i.e., \( P(x_0, y_0, z_0) \).

\( E \) - Real input array of the direction cosines of the ray; i.e., \( E(k_x, k_y, k_z) \).

\( S \) - Real input variable equal to the known \( z \) coordinate of the intersection as found, for example, from subroutine CEGIE.

\( P1 \) - Real output array containing the desired point of intersection \( P(x, y, z) \).

18-4. Comments and Method:

The parametric equations for a line in space passing through the point \( P(x_0, y_0, z_0) \) and having direction cosines \( (k_x, k_y, k_z) \) are given by

\[
\begin{align*}
x &= x_0 + k_x t \\
y &= y_0 + k_y t \\
z &= z_0 + k_z t
\end{align*}
\]

where \( t \) is a parameter.
where \( s \) is the distance along the line from \( x \), \( y \), \( z \) to \( x_1 \), \( y_1 \), \( z_1 \), and the coordinates \( x \), \( y \), \( z \) follow the Equation 3.5. However, it provides a

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For a detailed analysis on the mechanics of bending under conditions of

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...
III CALCULATE THE Y AND Y COMPONENTS OF AN INTERSECTION POINT PI
FOR "A = O-A" WHEN THE POINT OF EQUATION P, THE DIRECTION OF
INTERSECTION P X P, THE Y COORDINATE OF THE INTERSECTION POINT

\[ P_1 = (x_1, y_1), P_2 = (x_2, y_2) \]

\[ y_1 = \frac{(P_1(3) - P_2(3)) \times y_2}{y_2} \]

\[ y_2 = (P_2(1) + y_2) \times x_1 \]

\[ y_2 = -y_2 \times x_1 \]

\[ P_2 = (x_2, y_2) \]
Chapter 19
SUBROUTINES BDISK, BDISKN, TDISK, TDISKN

19-1. Purpose: To compute the intersection PI(x, y, z) of a line (ray) emanating from the point P(x, y, z) having direction cosines K(kx, ky, kz) with a planar disk at z = zbot or at z = ztop.

Subroutine BDISK is used to compute the unit inward normal $n = z$.
Subroutine TDISKN is used to compute the normal $n = -z$.

19-2. Usage:
CALL BDISK (P, K, PI, HIT)    CALL TDISK (P, K, PI, HIT)
COMMON/BDISKC/ZBOT, RBSQ    COMMON/TDISKC/ZTOP, RTSQ
CALL BDISKN (N)             CALL TDISKN (N)

19-3. Arguments

P - Real input array containing the point P(x, y, z) from which the ray emanates.

K - Real input array of direction cosines K(kx, ky, kz).

PI - Real input array containing the desired point of intersection PI(x, y, z).

HIT - Logical output variable which is TRUE if an intersection is found.

ZBOT - Real input variable equal to the z coordinate of the planar disk.

RBSQ - Real input variable equal to the square of the radius of the planar disk.

N - Real output array containing the direction cosines of the unit inward normal vector; viz., N(kx, ky, kz).
I. Computer and Method

Form the parametric equations for the ray

\[ x = x_0 + k_x t \]  \hspace{1cm} (1a) \\
\[ y = y_0 + k_y t \]  \hspace{1cm} (1b) \\
\[ z = z_0 + k_z t \]  \hspace{1cm} (1c)

and the equation of the plane \( z = z_{1o} \); the parameter \( t \) is given by

\[ t = \frac{(z_{1o} - z_0)}{k_z} \]  \hspace{1cm} (2)

provided \( k_z \neq 0 \). The \( x \) and \( y \) coordinates follow from the above equations; however, if \( (x^2 + y^2) > r_1^2 \) (where \( r_1 \) is the radius of the disk), no intersection is found. Similar statements apply for the top disk.

Hence, Branch B2: Compare the listings below directly to the equations above.

B3a. Test Cases: See Chapter 2.

B3b. Reference: Same

B3c. From Listings: See following pages.
THE EQUATION USED FOR THE HORIZONTAL DISK IS \( z = z_0 \) for \( (x^2 + y^2) < R^2 \)

COMMON : \( (x, y, z, \theta, \phi) \)

\( x = \cos(\theta) \cdot \cos(\phi) \)
\( y = \sin(\theta) \cdot \cos(\phi) \)
\( z = \sin(\phi) \)

LOGICAL HIT
\( Z = \text{if} (z < z_0) \text{ then } 0 \text{ else } 1 \)
\( \text{if} (\text{NOT } Z) \text{ then } 0 \text{ else } 1 \)

\( x = x \cdot k(1) \)
\( y = y \cdot k(2) \)
\( z = z \cdot k(3) \)

\( \text{if } (Z = 0) \text{ then } \text{go to 1} \)

\( \text{RETURN} \)

\( \text{IF } (Z = 1) \text{ then } \text{go to 1} \)

\( \text{RETURN} \)
Chapter 20

SUBROUTINE FAR

20-1. Purpose: To compute the far field pattern in wavenumber coordinates \((k_x', k_y')\) of an antenna whose radiating characteristics are specified by the complex plane wave spectra \(A_x(k_x', k_y'), A_y(k_x', k_y')\). The antenna is located in a plane perpendicular to the z (polar) axis.

20-2. Usage: CALL FAR(FIELD, XFIELD, YFIELD, NX, NY, FGHZ, KMAX, KMAX, RADIUS, HWK, HWK2)

20-3. Arguments:

FIELD - Real output array of NX by NY elements containing the far field power pattern at discrete wavenumbers \(k_x' = \sin \theta \cos \phi, k_y' = \sin \theta \sin \phi\), where \(\theta\) and \(\phi\) are the usual polar and azimuthal angles.

XFIELD, YFIELD - Complex input arrays of NX by NY elements containing the plane wave spectra \(A_x, A_y\) at discrete wavenumbers \(k_x', k_y'\).

NX, NY - Integer input variables equal to the array sizes.

FGHZ - Real input variable equal to the frequency in gigahertz.

KMAX, KMAX - Real input variables equal to the maximum wavenumber associated with the elements of the arrays FIELD, XFIELD, and YFIELD. The element \(i,j\) in these arrays corresponds to the wavenumber coordinate \((-KMAX, -KMAX)\). For any \((i,j)\), the wavenumber coordinates are given by
$KX = (1 - \frac{NX}{2} - 1) \times KXINC$

$KY = (J - \frac{NY}{2} - 1) \times KYINC$

where

$KXINC = 2 \times KXMAX/NX$

$KYINC = 2 \times KYMAX/NY$

**RADIUS** - Real input variable equal to the radius $r$ in centimeters of the sphere on which the far field pattern is computed. This variable effects only the term $e^{-ixr}/r$, and $r$ is set to unity in the calling program for normal use.

**IPWR** - Integer input variable which selects the vector components to be used in computing the power pattern:

1. Elevation component only
2. Azimuth component only
3. Total power
4. Right hand circular polarization
   Left hand circular polarization

**FMAX** - Real input and output variable. On input, if $FMAX = 0$, the program will normalize the array $FIELD$ from zero to one and output the normalizing factor as $FMAX$. If $FMAX > 0$ on input, it will be used as the normalizing factor; on output it will be unchanged.
20-4. Comments and Method

Let $E_x(x, y, 0), E_y(x, y, 0)$ be the tangential electric fields of a rectangular antenna aperture located in the $z$-plane and centered at the origin of the coordinate system. The plane wave spectra of the aperture fields are defined by

\[
A_x(k_x, k_y) = \frac{1}{(2\pi)^2} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} E_x(x', y', 0) e^{-j(k_x x' + k_y y')} \, dx' \, dy'
\]

\[
A_y(k_x, k_y) = \frac{1}{(2\pi)^2} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} E_y(x', y', 0) e^{-j(k_x x' + k_y y')} \, dx' \, dy'
\]

\[
A_z(k_x, k_y) = \frac{-k_A x - k_A y}{k^2}
\]

where

\[
k^2 = k_x^2 + k_y^2 + k_z^2 = k^2 - \left(\frac{2\pi}{\lambda}\right)^2
\]

The electric field \( E_z(x, y, z) \) at any point \((x, y, z) \geq 0\) is given by

\[
E_z(x, y, z) = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} A_x(k_x, k_y) e^{-j\left(k_x x + k_y y\right)} \, dk_x \, dk_y
\]

with

\[
k^2 = k_x^2 + k_y^2 + k_z^2
\]

\[
k = k_x + k_y + k_z
\]
And for the special case of large $r$, the rectangular field components $A_r$ approach their asymptotic values [1]

$$E_{rr}(r, k_{x_0}, k_{y_0}) \sim \frac{-ikr}{r} \cos \lambda_1(k_{x_0}, k_{y_0})$$

where the stationary phase points are given by

$$k_{x_0} = k \sin \alpha \cos \beta$$

$$k_{y_0} = k \sin \alpha \sin \beta$$

$$k_{z_0} = k \cos \alpha$$

In the above equations, $\alpha$ is the polar angle measured from the $z$ axis, and $\beta$ is the azimuthal angle measured from $+x$ toward $+y$ in the conventional spherical coordinate system.

Consider the antenna measurement coordinate system in Figure 3-1, and the wave numbers $k_{x_0}, k_{y_0}, k_{z_0}$ be normalized by $k = k^*$, so that for $k^* = \sqrt{1} = 1$, they represent direction cosines of the direction specified by $(\alpha, \beta)$, or equivalently by $(\rho, \phi)$. In terms of these normalized wavevectors, the unit vectors $e_{\rho}, e_{\phi}$ may be written as

$$e_{\rho} = -\frac{k k_{x_0} - \sqrt{1-k^2} e_1 - k k_{y_0}}{\sqrt{1-k^2}},$$

$$e_{\phi} = -\frac{k k_{z_0}}{\sqrt{1-k^2}}.$$
\( \hat{\epsilon} = \text{elevation component} \)
\( \hat{\alpha} = \text{azimuth component} \)

Figure: Coordinate System for Far Field Patterns
\begin{align*}
E_{1r} &= \cdot \mathbf{E}_{1f} = \cdot \mathbf{E}_{1f} \cdot (x \cdot \mathbf{A}_x + y \cdot \mathbf{A}_y + z \cdot (-k_y \mathbf{A}_x - k_y \mathbf{A}_y) ) \quad (14) \\
E_{2r} &= \cdot \mathbf{E}_{2f} \\
\text{(in subroutine F29, the factor } \lambda \text{ is not used, and the plane wave source } \\
\mathbf{A}_x, \mathbf{A}_y \text{ are provided as computed previously using the Fast Fourier Transform.)}
\end{align*}

It is convenient to recall that the receiving and transmitting
fields are orthogonal and identical, and that the receiving pattern

\[ \mathbf{E}_{2r} \text{ can be written in terms of the far field } \mathbf{E}_{2f} \text{ by } (12) \]

\[ \mathbf{E}_{2r}(k) = \cdot \mathbf{E}_{2f}(k) \quad (16) \]

where \( \cdot \) is the far-field current source (probe) located on the far-field
plane \( z = 0 \). The \( \cdot \) is simply constant which is set to unity for
\( k_1 = k_2 = 0 \). The relative separant of the receiving \( E_{2r} \) pattern is

\[ \mathbf{E}_{2r}(k) = \cdot \mathbf{E}_{2f}(k) \quad (16) \]

The receiving pattern is

\[ \mathbf{A}_x(k) = \cdot \mathbf{A}_y(k) \quad (16) \]

where \( \cdot \) is scalar
and is the receiving pattern when the probe is polarization matched at every point to the test antenna.

In the case of circularly polarized fields, the probe \( \vec{n}_b \) can be expressed as

\[
\text{RHC: } \vec{n}_b = \frac{e^{i} + i e^{-i} \frac{\pi}{2}}{\sqrt{2}}
\]

\[n_b = \frac{e^{-i} + i e^{i} \frac{\pi}{2} + i}{\sqrt{2}} \quad (21)
\]

The receiving patterns in the two cases are then given by

\[
|V_K|^2 = |\vec{n}_b \cdot E_{ff}|^2 \quad (22)
\]

where the appropriate \( \vec{n}_b \) is used.

Subroutine FAR implements the above equations and computes the power patterns for an aperture in an infinite ground plane; i.e., the use of only \( A_X \) and \( A_Y \) is tantamount to the assumption that \( E_{tan} \) outside the finite aperture area is zero. For the extended case of a finite aperture in free space, the tangential magnetic field \( H_{tan} \) also contributes to the radiated field, and the far fields are given by Equations (3-40) - (3-49) of reference 4. In fact, it is only by including the effects of \( H_{tan} \) that the transmitting and receiving formulations for the finite aperture can be shown to be equivalent [1].

The current version of subroutine FAR listed below could be easily modified to include the additional terms. If the geometrical optics approximation for the aperture fields is made, viz.,
then the far-field expressions become

\[
\begin{aligned}
U_{xy}(k_x, k_y) &= 
8\left[ (k_x^2 + 1 - \frac{k_y^2}{k_x^2}) A_x - k_x k_y A_y \right] \\
&\quad + 2\left[ k_y + (k_x^2 + 1 - k_y^2) A_x \right] \\
&\quad + 2\left[ k_x (1 + k_y^2) A_x - k_y (1 + k_x^2) A_y \right]
\end{aligned}
\]

These modifications would involve changes only to Lines 75-77 of

Program Flow: Compare listing below directly to Equations (1) -

(3) above.

Test Case: See Chapter 7.

References:

1. E. N. Clement, The Probe Waveguide: A Numerical Solution of


2. G. K. Batchelor, W. J. Macdonald, and R. M. Norton,

"Parametric Interpretation of Radar Absorption Schemes", IEEE


"The Fourth Annual Symposium on Electromagnetics, Boston, MA

June, June 1970.

3. F. March, "Cordillera Trends for Near-Field Measurements

of Radar", Inst. Electr. and Electronics, Geomagnetic Institute of Teco

A. R., Aragon, 1971, August 17.

4. G. K. Batchelor, "Interference of Transfer Lines and Coaxial

Cable in Radar Antennas", at the 4th.
SUBROUTINE FAR(FIELD, XFIELD, YFIELD, NX, NY, FWHZ, KXMAX, KYMAX, 1
    RADIUS, IFW, IFM)
C
    FIELD IS A TWO DIMENSIONAL REAL ARRAY (NX,NY). ON OUTPUT 2
    IT CONTAINS THE FAR FIELD POWER PATTERN OF A COMPLEX 3
    VECTOR PLANE WAVE SPECTRUM.
C
    XFIELD AND YFIELD ARE TWO DIMENSIONAL COMPLEX ARRAYS WHICH 4
    CONTAIN RESPECTIVELY THE X AND Y COMPONENTS OF A 5
    COMPLEX PLANE WAVE SPECTRUM.
C
    KXMAX AND KYMAX ARE RESPECTIVELY THE MAXIMUM ABSOLUTE 6
    VALUES OF KX AND KY WAVE NUMBERS FOR WHICH THE FAR FIELD 7
    IS CALCULATED. KXMAX AND KYMAX ARE NORMALIZED SUCH THAT 8
    KX=1.0 AND KY=1.0 CORRESPOND TO THE VISIBLE 9
    REGION OF WAVE NUMBER SPACE.
C
    FMAX IS AN INPUT-OUTPUT VARIABLE. IF FMAX IS LESS THAN OR 10
    EQUAL TO ZERO ON INPUT, THE FIELD ARRAY IS NORMALIZED 11
    FROM ZERO TO ONE AND FMAX IS THE NORMALIZING FACTOR. 12
    IF FMAX IS GREATER THAN ZERO ON INPUT IT REMAINS 13
    UNCHANGED AND IS USED AS THE NORMALIZING FACTOR.
C
    IFPW DETERMINES WHICH POWER COMPONENT WILL BE USED IN THE 14
    FAR FIELD CALCULATIONS. IFPW=1 FOR ELEVATION COMPONENTS; 15
    IFPW=2 FOR AZIMUTH COMPONENTS, AND IFPW=3 FOR TOTAL POWER 16
    IFPW=4 FOR RIGHT-HANDED CIRCULAR POLARIZATION COMPONENTS 17
    IFPW=5 FOR LEFT-HANDED CIRCULAR POLARIZATION COMPONENTS 18
    RADIUS SPECIFIES THE RADIUS OF THE FAR FIELD SPHERE IN 19
    CENTIMETERS.
C
    REAL K, KK, KY, KZ, XX, XY, YY, ZZ, XXIC, KYINC, KXMAX, KYMAX 20
    COMPLEX XFIELD(NX,NY), YFIELD(NX,NY), XX, YY, ZZ 21
    COMPLEX HTHTA, HPHI, HY, HZ 22
    IM=1+NX/2 23
    IN=1+NY/2 24
    SP=30/SP(2,1) 25
    HTHTA=COMPLEX(1.0,1.0)/SQR2 26
    HPHI=COMPLEX(0.0,1.0)/SQR2 27
    IF (IPW=2.G,5) HPHI=HPhi 28
    IF (IPW=5.G,5) HPHI=HPhi 29
    GO TO 161
CALCULATE THE POWER PATTERN ON A SPHERE.

\begin{verbatim}
      DO 10 I=1, NV+1
      DO 10 J=1, NV+1
      KY= (J-1)*Y*INCR
      KZ= (I-1)*Z*INCR
      IF (KZ.LT.-1.) GO TO 5
      KZ= 0.5*(KZ)
      ZS=(1.-KZ**2)**.5
      FZ=-C*X*KY*FIELD(I,J) + KY*FIELD(I,J)
      FX=C*X*KY+FIELD(I,J)
      FY=C*Z*FIELD(I,J)
      FIELD(I,J)=FX
      FIELD(I,J)=FZ
      IF (FZ.GT.0.) FIELD(I,J)=ABS(-EX*KY*Z/X + EY*Z = EZ*KY*Z/X)**2
      IF (KZ.GT.0.) FIELD(I,J)=ABS(-EX*KZ/Z = EZ*KX/X)**2
      CONTINUE
      END
\end{verbatim}
IF (FMAX .GT. .0) GO TO 9
70 9 I=1, NY
80 9 J=1, NX
90 = FIELD (I, J) = FIELD (I, J) / FMAX
CONTINUE
CONTINUE
TO 11 I=1, NY
100 11 J=1, NX
110 FIELD (I, J) = FIELD (I, J) * FMAX
CONTINUE
RETURN
END
Chapter 21
SUBROUTINE AMPHS

21-1. Purpose: To convert a complex number \( c = x + jy \) from rectangular to polar form \( c = |c|e^{j\phi} \).

21-2. Usage: CALL AMPHS (C, AMP, PHS)

21-3. Arguments
- \( C \) - Complex input variable containing the rectangular components of the complex number to be converted; i.e., \( C = \text{CMPLX}(X,Y) \).
- \( AMP \) - Real output variable equal to \( \sqrt{x^2 + y^2} \).
- \( PHS \) - Real output variable equal to the phase angle \( \phi \) in degrees.

21-4. Comment
The intrinsic Fortran function ATAN2 is used to compute PHS.


21-6. Test Case: None

21-7. References: None

Chapter 22
SUBROUTINE DBPV

22-1. Purpose: To convert a real array of linear values, normalized to lie between zero and unity, to decibels.

22-2. Usage: CALL DBPV (FIELD, NX, NY, IPV)

22-3. Arguments

FIELD - Real input/output array of NX by NY elements: on input, it contains the values to be converted; on output, it contains the corresponding decibel values on the range (-40, 0). All input values less than \( 10^{-2} \) are set to -40 dB on output.

NX, NY - Integer input variables which specify the size of the array FIELD.

IPV - Integer input variable which specifies whether the input values in FIELD represent power (IPV=1) or voltage (IPV=2). If IPV=1, \( F(I, J) = 10 \log_{10} F(I, J) \) is returned; if IPV=2, \( F(I, J) = 20 \log_{10} F(I, J) \) is returned.

22-4. Comments

It is intended that the input array FIELD be normalized prior to the call to Subroutine DBPV.


22-6. Test Case: None

22-7. References: None

22-8. Program Listing: See following page.
SUBROUTINE C8PV(FIELD,NX,NY,IPV)
C MODIFIED BY GKH 4/74 TO PERMIT POWER (IPV=1) OR VOLTAGE (IPV=2) DB.
C
C SUBROUTINE OR CONVERTS AN INPUT ARRAY (FIELD(NX,NY)) OF
C VOLTAGE OR POWER VALUES TO DECIBELS AND RETURNS DB VALUES IN THE
C SAME ARRAY.
C ALL VALUES OF POWER LESS THAN 40 DB DOWN ARE SET EQUAL TO -40DB
C
DIMENSION FIELD(NX,NY)
DO 1 J = 1,NX
DO 10 I = 1,NY
IF (IFV.EQ.2) FIELD(I,J)=FIELD(I,J)**2
IF (FIELD(I,J),.LE.,1E-4) FIELD(I,J)=1E-4
FIELD(I,J)=10.6*ALOG10(FIELD(I,J))
1 CONTINUE
RETURN
END
Chapter 23

SUBROUTINE NORMH

23-1. Purpose: To normalize a two-dimensional real array of field values so that all values in the array lie between zero and unity.

23-2. Usage: CALL NORMH (FIELD, IMAX, JMAX, LDB)

23-3. Arguments

FIELD - Real array of IMAX by JMAX elements. On input, it contains the field values expressed as non-negative real linear amplitude or as amplitude in decibels. On output, the linear amplitudes are replaced by their scaled values FIELD(I,J)/FMAX, where FMAX is the maximum amplitude value in the array; the logarithmic amplitude values are replaced by (FIELD(I,J)+40.)/40., where -40 decibels is assumed to be the lower bound on the original data.

IMAX, JMAX - The number of elements in FIELD.

LDB - A logical variable set TRUE if the values in FIELD are in decibels.

23-4. Comments and Method

A function \( f(x,y) \) of two variables having minimum value \( f_{\text{min}} \) and maximum value \( f_{\text{max}} \) may be normalized to \( 0 \leq f_n(x,y) \leq 1 \) according to

\[
  f_n(x,y) = \frac{f(x,y) - f_{\text{min}}}{f_{\text{max}} - f_{\text{min}}} \tag{1}
\]
provided that the denominator is not zero. In this procedure, the $f_n=0$ corresponds to $f=f_{\min}$, and $f_n=1$ corresponds to $f=f_{\max}$.

When $f(x,y)$ represents a linear (vice logarithmic) variable, it is desirable to force $f_n$ to be zero if the minimum value of $f$ is actually greater than zero. In this special case, $f_n$ becomes

$$f_n(x,y) = \frac{f(x,y)}{f_{\max}} \quad (2)$$

Equation (2) is also used to treat the special case of $f_{\max} - f_{\min} < 0$; however, if $|f_{\max}| < 1$, $f_{\max}$ is set equal to $\pm 1$, where the sign used is that of $f_{\max}$. This refinement has the effect of producing a constant function whose value lies between zero and unity; without it, $f_n$ would be simply set to unity or division by zero may result.

When $f(x,y)$ represents a logarithmic variable, such as the amplitude in decibels of an electromagnetic field, all of the foregoing discussion applies; however, a minimum value $f_{\min}$ must be imposed. If $f_{\min} < -40$, $f_{\min}$ is set equal to $-80$ (decibels); otherwise, a $-40$ decibel level is assumed. A value of $f_{\max}$ equal to zero decibel is also assumed.

23-5. Program Flow

Lines 9-16: Find minimum $MN$ and maximum $MX$ values of data in $\text{FIELD}$; form their difference $DR=MX-MN$.

Line 17: If array values are in decibels, go to 50.

Line 18: If all values in the array are the same, go to 25 and scale the data to lie between zero and unity (Lines 28-37).
Lines 19-27: If all linear amplitude values in FIELD are not identical, scale the data according to $\text{FIELD}(I,J) = (\text{FIELD}(I,J) - \text{Min. Value})/(\text{Maximum Value} - \text{Minimum Value})$.

Line 38: If values in FIELD are in decibels, and the minimum value is less than -41dB, then assume a -80dB lower bound, go to 60 (Lines 47-52), and scale the data according to $(\text{FIELD}(I,J) + 80.)/80$.

Lines 39-46: Scale the data according to a -40dB lower bound; i.e., $(\text{FIELD}(I,J) + 40.)/40$.

Lines 53-54: Write $MN$ and $MX$.


23-7. References: None.

SUBROUTINE NORM(FIELD, IMAX, JMAX, LDA)
C MODIFIED BY GKH 4/78 TO CAUSE PROPER NORMALIZATION OF BOTH
C LINEAR AND UJ ARRAYS.
C
NORMALIZE FIELD SO THAT ALL VALUES ARE BETWEEN ZERO AND ONE.
C
REAL MN, MX, FIELD(IMAX, JMAX)
LOGICAL LDB
MX=FIELD(1,1)
MN=MX
DO 22 I=1, IMAX
DO 22 J=1, JMAX
MN=AMIN1(MN, FIELD(I, J))
MX=AMAX1(MX, FIELD(I, J))
20 CONTINUE
DR=MX-MN
IF (LDB) GO TO 30
IF (DR, LT, 1E-18) GO TO 25
TMN=MN
IF (MN, GT, 0.) TMN=0.
TCD=DR
IF (MN, GT, 0.) TCR=MX
DO 21 I=1, IMAX
DO 21 J=1, JMAX
FIELD(I, J)= (FIELD(I, J)-TMN)/TCD
21 CONTINUE
GO TO 35
C CASE WHERE ALL VALUES ARE THE SAME:
25 TMX=MX
IF (ABS(MX), LT, 1.0) TMX=SIGN(1., MX)
DO 30 I=1, IMAX
DO 30 J=1, JMAX
FIELD(I, J)=FIELD(I, J)/TMX
C FIELD IS FILLED WITH SAME VALUES SCALLED BETWEEN ZERO AND UNITY.
IF (FIELD(I, J), LT, 0.) FIELD(I, J)=0.
35 CONTINUE
50 CONTINUE
GO TO 50
50 IF (MN, LT, -1.0) GO TO 60
C ASSUMF 4 TO -4.: SCALE

DO 55 I=1,1MAX
   DO 55 J=1,1MAX
      FIELD(I,J)=(FIELD(I,J)+40.)/40.
      IF (FIELD(I,J).LT.0.) FIELD(I,J)=0.
      IF (FIELD(I,J).GT.1.) FIELD(I,J)=1.
   55 CONTINUE
   GO TO 35

50 DO 65 I=1,1MAX
   DO 65 J=1,1MAX
      FIELD(I,J)=(FIELD(I,J)+40.)/40.
      IF (FIELD(I,J).LT.0.) FIELD(I,J)=0.
      IF (FIELD(I,J).GT.1.) FIELD(I,J)=1.
   65 CONTINUE
35 WRITE(6,40) MX,MY
40 FORMAT(//" SUBROUTINE NORM: MIN= ",E10.3," MAX= ",E10.3//)
RETURN
END
Chapter 24

SUBROUTINE CNPLTH AND FUNCTION PSI

24-1. Purpose: To plot (Calcomp) single dimensional far field patterns at constant wavenumber $k_{\text{fix}}$.

24-2. Usage: CALL CNPLTH (FIELD, N, KMAX, KCNTR, KFIX)

$$\psi = \text{ATAN2} \left( \frac{k}{\sqrt{1.} - \frac{K^2 - K_{\text{fix}}^2}{\text{field}}\right)$$

24-3. Arguments

FIELD - Real input array of N elements containing the field values in decibels but normalized so that -40 dB corresponds to 0 and 0 dB corresponds to unity on the normalized scale.

N - Integer input variable which specifies the number of elements in FIELD.

KMAX - Real input variable equal to the half width of the wavenumber range corresponding to the array elements 1 through N of the array FIELD; i.e., the increment in wavenumber corresponding to the distance between the Ith and (I+1)st element is $2 \times KMAX/N$.

KCNTR - Real input variable equal to the wavenumber coordinate of the $(N/2 + 1)$st element of the array FIELD. FIELD(1) has wavenumber coordinate KCNTR - KMAX.

KFIX - Real input variable equal to the fixed value of the other wavenumber coordinate. For example, if $k_x$ varies, then $k_y = KFIX$. 

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24-4. Comment and Method

Let \( F(k_x, k_y) \) represent the far-field power pattern of an antenna where \( k_x \) and \( k_y \) are normalized wavenumbers as defined in Chapter 20. A pattern cut at constant wavenumber is a conical cut about the real axis; e.g., a \( k_x = \) constant cut is a conical cut about the \( x \) axis of the coordinate system.

For principal plane cuts, \( k_x = 0 \) yields an E-plane pattern as defined in Figure 2-3; \( k_y = 0 \) yields an H-plane pattern. For principal plane cuts, KCNTR = 0 and KFIX = 0.

The plotting commands are set up to produce a 4" X 8" rectangular pattern plot on a standard pattern scale. The plot is positioned on the paper to give margins of 2" on the left, 1" on the right, and 2.25" from the bottom and, hence, is suitable for direct use as a figure in a technical report.

24-5. Program Flow: See listing below.

24-6. Test Case: See Chapter 2 and pattern plots in Appendices B and D.

24-7. References: None

24-8. Program Listing: See following pages.
SUBROUTINE CNFLTH(FIELD,N,KMAX,KNTR,KFIX)  
MODIFIED BY GKH 4/28/78 TO GIVE 4 X 8 DATA PLOTS WITH MARGINS  
of 2" FROM LEFT, 2.25 FROM BTM, AND 1" ON RIGHT.  
THIS SUBROUTINE PLOTS SINGLE DIMENSIONAL FAR FIELD PATTERNs  
THE PLOTS ARE CONSTANT WAVEVECTOR PLOTS WHICH CORRESPOND  
to conical far field patterns  
FIELD(N) IS A ONE DIMENSIONAL FAR FIELD POWER PATTERN  
NORMALIZED FROM ZERO (CORRESPONDING TO -40 DB) TO ONE  
(CORRESPONDING TO 0 DB)  
KMAX IS THE HALF WIDTH OF THE WAVEVECTOR REGION OF FIELD  
KNTR IS THE CENTER WAVEVECTOR COORDINATE OF THE INPUT FIELD  
FIELD(1) HAS A WAVEVECTOR COORDINATE KNTR-KMAX  
FIELD(N/2+1) HAS WAVEVECTOR COORDINATE KNTR  
KFIX IS THE FIXED VALUE OF THE OTHER WAVEVECTOR COORDINATE  
REAL FIELD(N),K,KMAX,KNTR,KFIX  
PSIMIN=PSI(KCNT-KMAX,KFIX)  
PSIMAX=PSI(KCNT+KMAX-2*KMAX/N,KFIX)  
PSIMID=PSI(KCNT,KFIX)  
DELPSI=2*AMAX1(PSIMID-PSIMIN,PSIMAX-PSIMID)  
ISCALE=360  
IF(DELPSI.LE.60) ISCALE=60  
IF(DELPSI.LE.10) ISCALE=10  
C INITIALIZE FACTOR TO UNITY AND DRAW LEFT MARGIN FOR GUIDE LATER.  
call factor(1)  
call plot(0,0,0,-3)  
call plot(0,8.5,2)  
call plot(0,0,3)  
c set logical origin of sa plot:  
call plot(2,2.25,-3)  
c plot at .4 scale factor of full size sa plot (10 x 20):  
call factor(.4)  
c draw rectangular perimeter box  
call plot(0,.75,3)  
call plot(0,.1,.75,3)  
call plot(20,.1,.1,10.625,.2)  
call plot(20,0,10.625,2)
DO 6 I=1,2,1
    Y=.75
    CALL PLOT(X,Y,3)
    CALL PLOT(X+1.625,2)
    CALL SYMBOL(X+NY*0.14+0.17,3.85,0.14,27)
    I=90...271
DO 5 J=1,4,1
    CALL PLOT(X,Y,3)
    CALL PLOT(X+0X*.37,Y,2)
    CALL NUMBE(X+0X*.1666=.04,Y=0.07-.014,DA,0.0,0,1)
CONTINUE
5   Y=Y+9.975/2.0C
    Y=14.6667
    DX=-DX
6   77
    78
    79
    80
    81
    82
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    85
    86
    87
    88
    89
    90
    91
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    114

C PLOT CCNF ANGLE AND CENTER OF ROTATION ANGLE
CON=ACOS(KFX)*110031.141592653585
CALL SYMBOL(.6,0.154,0.14,13)CONE ANGLE = .0.0,13
CALL NUMBER(C999.999..0,0,0,1)CONE ANGLE = 0.0,13
CALL SYMBOL(C999.999..0,0,0,1)CENTER RCTATION ANGLE = .0.0
CALL NUMBER(C999.999..0,0,1)

C C PLOT PATTERN
IPEN=*7
DO 10 I=-1,N+1
    K=KCONTR+1)K=0.2,-1)*MAX*2/N
    A=SIXK,KFXI-PSTIMD
    X=10.0+25.0A/ISOGLF
    Y=FITI(LI)
    IF(Y,LT,0.0) Y=0.0
    IF(Y,LT,1.0) Y=1.0
    IF(Y,L0,1.0) Y=0.0
    IF(Y,LT,20.0) Y=20.0
FUNCTION PSI(K, KFIX)
REAL K, KFIX, KZ
KZ=1.0 - K**2 - KFIX**2
IF (KZ.LE.0.) KZ=0.
KZ=SQRT(KZ)
PSI=ATAN2(K, KZ)*180./3.141592653
RETURN
END
Chapter 25

SUBROUTINES PLT3DH AND PLTT

25-1. Purpose: To plot (Calcomp) the two-dimensional array FIELD (I, J).

25-2. Usage: CALL PLT3DH (XSIZE, YSIZE, HEIGHT, FIELD, IMAX, JMAX, NMZ, LDB)

25-3. Arguments

XSIZE, YSIZE, HEIGHT

FIELD, IMAX, JMAX

- Real input array of IMAX by JMAX elements containing the values to be plotted. These values must be normalized to the range (0, 1) before plotting.

NMZ

- Logical input variable. If NMZ = .TRUE., the array FIELD will be normalized with respect to its own maximum value; if NMZ = .FALSE., no normalization will be done.

LDB

- Logical input variable required by Subroutine NORMH (Chapter 23).

25-4. Comments

In Figure 25-1, the axes and labels shown are not produced by the subroutine; these axes are presented to demonstrate the perspective of the plot and to identify its dimensions. Report size plots will be produced suitable for one 8 1/2" X 11" page when FACTOR = 1.0 and
Figure 25-1. Dimensions of Three-Dimensional Plot.
XSIZE = 6.0"
YSIZE = 2.5"
HEIGHT = 2.5"

Margins in this case will be 1.5" on the left, 1" on the right, and 4.25" from the bottom of the plot paper. Margin lines are provided on the plot paper to outline the 8 1/2" x 11" page. Also, the plot itself can be carefully cut from the plot paper and cemented onto a set of axes as has evidently been done in Appendices B and D.


25-6. Test Case: See Chapter 2 and Appendices B and D.

25-7. References: None.

SUBROUTINE PLT3CH(XSIZE,YSIZE,HEIGHT,FIELD,IMAX,JMAX,NMZ,LDB)
C LOB IS REQUIRED BY SUBR NORMH TO SPECIFY IF ARRAY FIELD
C IS IN CB (TRUE) OR NOT (LOB=.FALSE.).
C MODIFIED BY GKH 4/28/79 TO GIVE REPORT SIZE PLOTS WHEN
C XSIZE=6.0, YSIZE=2.5, HEIGHT=2.5, I.E., LEFT MARGIN OF
C 1.5", RIGHT OF 1" AND 4.25" FROM BTM MARGIN.
C
YSIZE IS THE MAXIMUM LENGTH OF THE PLOT IN INCHES.
YSIZE IS THE MAXIMUM WIDTH OF A ZERO PLOT IN INCHES.
THE SUM OF 1/2 INCH + YSIZE + HEIGHT MUST BE LESS THAN
OR EQUAL TO THE PAPER WIDTH.
FIELD(IMAX,JMAX) IS THE TWO-DIMENSIONAL REAL ARRAY TO
BE PLOTTED. IF FIELD IS NOT NORMALIZED ON INPUT, NMZ
MUST BE .TRUE. .
NMZ(NORMALIZE) IS A LOGICAL INPUT VARIABLE. IF ITS VALUE
IS .TRUE. THE VALUES IN FIELD WILL BE REPLACED WITH
THEIR NORMALIZED(ZERO TO ONE) COMPONENTS.
C
REAL FIELD(IMAX,JMAX),HID(128)
LOGICAL NMZ,LOB
REAL LSTX,LSTY,LSTH,LSTHM
IF(NMZ) CALL NORMH(FIELD,IMAX,JMAX,LDB)
XPAGE=0.0
YPAGE=0.0
LSTHM=0.0
NIJ=IMAX+JMAX
RI=IMAX+1.0
RJ=JMAX+1.0
C INITIALIZE FACTOR TO UNITY, DRAW LEFT MARGIN, AND SET LOGICAL ORIGIN.
CALL FACTOR(1.0)
CALL PLOT(G,.0,0,-3)
CALL PLOT(G,.1,.2)
CALL PLOT(G,.2,.3)
CALL PLOT(1.5,3.75,-3)
DO 1 I=1,NIJ
1 WID(I)=.-0.5
DO 7 J=1, JMAX
AJ=J-1.0
DO 7 I=1,IMAX
AI=I-1.
LASTX=XPAGE
YPAGE=(AJ+AI)*YSIZE/(RI+RJ)
LASTY=YPAGE
YPAGE=(AJ*RI+PJ-AI*RJ/RI+PJ)*YSIZE/(RJ+RI)+HEIGHT*FIELD(I,J)
LASTH=LASTH+
LASTH=HID(I+J)
IF(YPAGE-HID(I+J)) 5,5,2
2 IF(I.IE.1) GO TO 3
CALL PLTT(XPAGE,YPAGE,3)
IPEN=2
GO TO 4
3 CALL PLTT(XPAGE,YPAGE,IPEN)
IPEN=2
GO TO 5
4 IF(I.EQ.1) IPEN=3
IF(IPEN.EQ.3) GO TO 6
X1=LASTX*HID(I+J)-LASTH*XPAGE-LASTX*YPAGE+XPAGE*LASTY
X1=HID(I+J)-LASTH*YPAGE+LASTY
X1=X1/X10
Y1=(X1*(HID(I+J)-LASTH)*XPAGE-LASTX*HID(I+J))/(XPAGE-LASTX)
CALL PLTT(X1,Y1,2)
IPEN=3
6 CALL PLTT(XPAGE,YPAGE,IPEN)
7 CONTINUE
DO 4 I=1,NIJ
8 H10(I)=-J,5
DO 16 II=1,IMAX+1
I=IMAX-II+1
AI=I-1
DO 16 J=1,JMAX
AJ=J-1
LASTX=XPAGE
XPAGE=(AJ+AI)*YSIZE/(RI+RJ)
LASTY=YPAGE
YPAGE=(AJ*RI+PJ-AI*RJ/RI+PJ)*YSIZE/(RJ+RI)+HEIGHT*FIELD(I,J)
LASTH=LASTHM
LASTHM=HID (I+J)
IF (YPAGEH-HID (I+J)) 13, 14, 9
9 IF (J .NE. 1) GO TO 10
   CALL PLTI (YPAGE, YPAGE, 3)
   IPEN=2
   GO TO 12
10 IF (IPEN .LE. 2) GO TO 11
   X1=LASTX*YPAGE-LASTY*YPAGE-LASTX*HID (I+J)*XPAGE*HID (I+J-1)
   X10=YPAGE-LASTY-HID (I+J)*HID (I+J-1)
   Y1=X1N/X10
   Y1=(X1*(YPAGE-LASTY)*LASTY*YPAGE-LASTX*YPAGE)/(YPAGE-LASTX)
   CALL PLTI (X1, Y1, 3)
   IPEN=2
11 CALL PLTI (YPAGE, YPAGE, IPEN)
12 HID (I+J)=YPAGE
   GO TO 16
13 IPEN=3
   GO TO 15
14 IF (J .EQ. 1) IPEN=3
15 CALL PLTI (YPAGE, YPAGE, IPEN)
15 CONTINUE
C CONCLUDE PLOT AT RT BTM CORNER OF REPORT PAGE:
   CALL PLOT (XSIZE +1, -3.75, -3)
SUBROUTINE PLTT(X,Y,IPEN)

SUBROUTINE PLTT ELIMINATES MOVING PEN FOR HIDDEN LINES.

XLAST=XN
YLAST=YN
ILAST=IN
XN=Y
YN=Y
IN=IPEN
IF(IPEN.EQ.2.AND.ILAST.EQ.2) CALL PLOT(X,Y,IPEN)
IF(IPEN.EQ.2.AND.ILAST.EQ.3) CALL PLOT(XLAST,YLAST,ILAST)
IF(IPEN.EQ.2.AND.ILAST.EQ.3) CALL PLOT(X,Y,IPEN)
IF(IPEN.NE.2.AND.IPEN.NE.3) CALL FLCT(X,Y,IPEN)
RETURN
END
Chapter 26

SUBROUTINE FFTA

26-1. Purpose: To compute the Discrete Fourier Transform (DFT) or its inverse of a sequence of complex numbers consisting of \(2^N\) elements, where \(N\) is an integer. The Cooley-Tukey algorithm is used to perform computations in place to speed up the computations and to return the transformed values in the input array.

26-2. Usage: CALL FFTA (FIELD, NEXP, IBMISN)

26-3. Arguments

FIELD - Complex array of \(2^{**}\) NEXP elements: on input it contains the sample data to be transformed; on output it contains the transformed data. See below for ordering of data.

NEXP - Integer exponent; e.g., for 64 elements in FIELD, NEXP=6.

IBMISN - Integer parameter which controls operation:

IBMISN = 3 performs the inverse DFT
IBMISN \neq 3 performs the DFT as defined in 4 below.

26-4. Comments and Method

a. Subroutine FFTA is machine-dependent in that the bit reversed number, IFLIP, must be generated using Fortran instructions which are peculiar to a particular machine. Also, the word length must be taken into account. Lines 38-42 of the attached program listing are used to effect the desired operation for the CDC Cyber 70 (60-bit word, numbered
0 through 59 from right to left with Bit 0 being the least significant):

IFLIP=0

DO 4 II=1, IEXP, 1

J=60-II

IFLIP=2*IFLIP+AND(SHIFT(I,I+J),1B)

4 CONTINUE

The SHIFT(I,I+J) operation shifts the bits of the integer I to the left by I+J bit positions. The AND operation strips off the right most bit of the shifted result. E.G., when II=1, the right most bit of I (Bit 0) is extracted from I by the AND(SHIFT) operation. The current value of IFLIP is then shifted one bit to the left by the 2*IFLIP operation. The two results are then added together. A total of NEXP bits are extracted, starting with Bit 0, followed by Bits 1, 2,...(NEXP-1).

The net result of these operations is to take the NEXP-bit binary representation of the array element number I, reverse the order of the bits, and right justify the result. Array elements in FIELD numbered I and IFLIP are then interchanged if I>IFLIP. The first and last elements of FIELD always remain in place. The array elements are rearranged in this manner so that they will be ordered after transforming [1].

b. To explain the ordering of the data in the complex array FIELD, it is convenient to consider the specific example of using FFTA to compute the Fourier transform G(f) of a time function g(t) as defined by

\[
G(f) = \int_{-T_{\text{max}}}^{T_{\text{max}}} g(t) e^{-j2\pi ft} \, dt
\]  

(1)
and as approximated by

\[ G(f) = \sum g(t_i) e^{-j2\pi f t_i \Delta t} \]

where \( t_i \) are the equally spaced points along the t axis when \( g \) is sampled
over the interval \(-T_{\text{max}} \leq t \leq T_{\text{max}}\).

There are \( N = 2^{\text{NEXP}} \) samples in the input array FIELD(I) corresponding
to \( I = 1, N \). The first sample (\( I = 1 \)) corresponds to \( g(-T_{\text{max}}) \). The last
sample (\( I = N \)) corresponds to \( g(T_{\text{max}} - \Delta t) \). The \( I = (N/2 + 1) \)th sample corresponds
to \( g(0) \); i.e., the value of \( g \) at \( t = 0 \). The DFT assumes periodicity of the
sampled data so that the value at \( t = T_{\text{max}} \) is identical to that at \( t = -T_{\text{max}} \).
The sample spacing is

\[ \Delta t = 2 T_{\text{max}} / N \]

and corresponds to a folding frequency \( f_{\text{max}} \) of

\[ f_{\text{max}} = 1/2\Delta t \]

On output, the array FIELD contains the frequency components \( G(f) \)
at \( N \) equally spaced frequencies \( \Delta f \) over the band \(-f_{\text{max}} \leq f \leq f_{\text{max}}\), where
\( I = 1 \) corresponds to \( f = -f_{\text{max}} \), \( I = (N/2 + 1) \) to \( f = 0 \), and \( I = N \) to \( f = f_{\text{max}} - \Delta f \), where

\[ \Delta f = 2 f_{\text{max}} / N \]
and where

\[ T_{\text{max}} = \frac{1}{2\Delta f} \] (6)

Also, by the inversion integral [2],

\[ g(t) = \int_{-T_{\text{max}}}^{T_{\text{max}}} G(f) e^{+j2\pi ft} \, df = \Delta f \sum_p G(f_p) e^{j2\pi f_p t} \] (7)

This version of Subroutine FFTA is written so that division by N is done when the Fourier transform (kernel = \( e^{-j2\pi ft} \)) is computed. When the expression in Equation (3) for \( \Delta t \) is used in (2), there results

\[ G(f_p) = 2 T_{\text{max}} \frac{1}{N} \sum_i g(t_i) e^{-j2\pi f_p t_i} \] (8)

Transposing \( 2 T_{\text{max}} \) and using Equation (6) yields

\[ \Delta f \, G(f_p) = \frac{1}{N} \sum_i g(t_i) e^{-j2\pi f_p t_i} \] (9)

where the righthand side is the definition of the Discrete Fourier Transform as computed by FFTA. Inversely,

\[ g(t_i) = \sum_p \Delta f \, G(f_p) e^{+j2\pi f_p t_i} \] (10)
which is the Inverse DFT as computed by FFTA.

Conversely, if the original data in the input array FIELD are samples of a frequency spectrum \( G(f) \), a similar analysis shows that FFTA computes \( \Delta t \) \( g(t_i) \) as the inverse transform (IBMISN=3); i.e., the time function is modified in amplitude by \( \Delta t \). Of course, when the forward transform (IBMISN=3) is performed on this result, the original sampled data \( G(f_i) \) are obtained in FIELD on output.

From the above considerations, the following conclusions can be drawn concerning the use of FFTA to compute the Fourier transform \( G(f) \) of a windowed time function \( g(t) \):

\[
G(f_p) = 2 T_{\text{max}} \cdot \text{FFTA}(g(t_i)) \tag{11}
\]

\[
g(t_i) = \frac{1}{2 T_{\text{max}}} \cdot \text{IFFTA}(G(f_p)) = \text{IFFTA}(\Delta f G(f_p)) \tag{12}
\]

As an example, let \( g(t) \) be the rectangular pulse function which has constant amplitude \( V \) for \( |t| \leq T_0 \) and which is windowed in the larger time interval \( |t| \leq T_{\text{max}} \). The Fourier transform \( G(f) \) is given by [3]

\[
G(f) = 2 T_0 V \frac{\sin 2\pi ft_0}{2\pi ft_0} \tag{13}
\]

Let \( g(t) \) be sampled at \( N = N_{\text{EXP}} \) points over the interval \( |t| \leq T_{\text{max}} \), and let these sampled points be placed in the array FIELD. Then the spectrum \( G(f) \) will be closely approximated at discrete frequencies \( f_p \) by
where

\[ f_p = -f_{\text{max}} + (I-1) \Delta f \]

and where FIELD is the output of FFTA according to CALL FFTA (FIELD, N, 0).

Proper consideration should be given to the sampling of the time function so that the DFT produces a good estimate of the actual integral transform. For example, if \( t = T_{\text{max}} \) and all samples are constant, then the DFT will produce a single nonzero frequency component at \( f = 0 \) (corresponding to the \((N/2+1)\)th element of FIELD); i.e., a delta function. Such a result follows from the facts that the Fourier transform of a constant \( g(t) = V \) is \( G(f) = V \delta(f) \) and that the DFT assumes a periodicity of the sequence of samples provided to it.

Consider the other extreme. Let the pulse \( g(t) \) be represented by only one sample at \( t = 0 \) in the window \( |t| \leq T_{\text{max}} \). The Fourier transform of \( g(t) = V \delta(t) \) is \( G(f) = V \), a constant.

It is clear from the above considerations that the time function must be properly windowed and properly sampled to produce a good estimate of its transform via the DFT. Simply stated, the time function should be sampled at a rate \( \Delta t \) which is twice the highest frequency contained in the function as interpreted by the DFT.
26-5. Program Flow

Lines 22-24: Compute $N = 2^{N_{\text{EXP}}}$ and set the sign ISN of the exponent in the Fourier kernel.

Lines 26-29: Compute $I_{\text{EXP}} = N_{\text{EXP}}$ from $N$. This is a redundant computation made when the original FFT subroutine was modified to conform to the call to a library version on another computer system.

Lines 30-35: Rearrange the order of the input data so that samples for $t \geq 0$ are placed in the lower half of the array, and those for $t < 0$ are placed in the upper half. For a frequency function, the data are rearranged so that the first $N/2$ points give the components for non-negative frequencies ($I=1$ corresponds to $f=0$), and the last $N/2$ points contain the data for the negative frequencies.

Lines 36-49: Rearrange the data in \texttt{FIELD} so that it will be ordered after transforming as described for Lines 30-35 above.

Lines 50-73: Perform the summation using the Cooley-Tukey algorithm [1].

Lines 74-79: If forward transform is being done, divide all values in \texttt{FIELD} by $N$.

Lines 80-85: Rearrange the output data in \texttt{FIELD} so that it conforms to that used on input; i.e., $f_i = f_{\text{max}} + (I-1)\Delta f$ or $t_i = T_{\text{max}} + (I-1)\Delta t$ as appropriate.

26-6. Test Case

A rectangular pulse function with amplitude $V_o = 100$ was chosen for $g(t)$ with $t_o = 0.1$ second $T_{\text{max}} = 1.60$ seconds, and $N = 2048 = 2^{11}$. The resulting
sample increment $\Delta t$ and folding frequency $f_{max}$ were 0.116 second and 320.0 Hertz, respectively. The comparison of the central nine points of the computed and true frequency spectra were as follows (CDC Cyber 70):

<table>
<thead>
<tr>
<th>$f$ (Hz)</th>
<th>True $G(f)$</th>
<th>Computed $G(f)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>I</td>
<td>Amp.</td>
<td>Phase(°)</td>
</tr>
<tr>
<td>1021</td>
<td>-1.250</td>
<td>18.006</td>
</tr>
<tr>
<td>1022</td>
<td>-0.938</td>
<td>18.863</td>
</tr>
<tr>
<td>1023</td>
<td>-0.625</td>
<td>19.490</td>
</tr>
<tr>
<td>1024</td>
<td>-0.313</td>
<td>19.872</td>
</tr>
<tr>
<td>1025</td>
<td>0.000</td>
<td>20.000</td>
</tr>
<tr>
<td>1026</td>
<td>0.313</td>
<td>19.872</td>
</tr>
<tr>
<td>1027</td>
<td>0.625</td>
<td>19.490</td>
</tr>
<tr>
<td>1028</td>
<td>0.938</td>
<td>18.863</td>
</tr>
</tbody>
</table>

26-7. References


26-8. Program Listing: See following pages. The second listing, Subroutine FFT, is for use on the IBM 3033 at JHU/APL. It employs the subroutine FFTA available on that system library. Use of this subroutine requires the calls in Subroutines JOYFFT and MAGFFT to be changed from CALL FFTA to CALL FFT.
SUBROUTINE FFTA(FIELD,NEXP,IMSLN)

MODIFIED TO SIMULATE FFTA ON IBM 3033 AT APL/JHU
DIVISION BY 4 IS DONE WHEN ISN=-1 (GKH 10 20 76)
CONVERSION DONE 1 JUNE 1976

* THIS SUBROUTINE CALCULATES THE FAST FOURIER TRANSFORM OR
* THE INVERSE FAST FOURIER TRANSFORM OF AN INPUT COMPLEX
* ARRAY FIELD AND RETURNS THE RESULT IN THE SAME ARRAY
* N MUST BE AN INTEGER POWER OF TWO
* ISN IS AN INTEGER WHICH MAY BE EITHER ONE OR MINUS ONE
* IF ISN IS -1 THE FAST FOURIER TRANSFORM IS CALCULATED
* IF ISN IS +1 THE INVERSE FOURIER TRANSFORM IS CALCULATED

REAL FIELD(E12)
INTEGER T,F
N=2**NEXP
ISN=-1
IF (ISN.NE.1) ISN=+1
PI=3.141592653589796
T=0
1 IF(ISN)1,2,1
M=2**T
IF(N-M)1J+2,1
2 N2=N/2
DO 3 I=1,N2+1
K=I+N2
T=FIELD(I)
FIELD(I)=FIELD(K)
3 FIELD(K)=T
N1=N-2
DO 6 I=1,N1+1
IFLIP=4
6 CONTINUE
1 CONTINUE

END
DO 4 II=1,IF1P+1
J=0-II
IFLIP=2*IFLIP+AND(SHFT(I,1+J),1B)
4 CONTINUE
IF(I1.LE.IFLIP) GO TO 5
I1=I1+1
J2=IFLIP+1
T=FIELD(J2)
FIELD(I2)=FIELD(I1)
FIELD(I1)=T
5 CONTINUE
DO 6 I=1,I2*P+1
NFL=2*I
NEL2=NEL/2
NSET=N/NEL
SI=SIN(P12/NEL)
CI=COS(P12/NEL)
DO 96 J=1,NSET,1
INC=(J-1)*NEL
SO=0.C
CC=1.C
CC=1.C
96 CONTINUE
J1=II+INC
J2=J1+NEL2
T=FIELD(J1)
F=FIELD(J2)*CPPLX(CC,ISN*SG)
FIELD(J1)=T+F
FIELD(J2)=T-F
SN=SO*CI+CC*SI
GS=CC*CI-3C*SI
CC=CS
SO=SN
96 CONTINUE
96 CONTINUE
IF(ISN.GT.0) GO TO 8
8 DIVISION BY N IS DONE FOR FORWARD TRANSFORM (GKH 16-20-76)
DO 7 I=1,N,1
SUBROUTINE FFT(FIELD,NEXP,IBMISN)

C MODIFIED TO UTILIZE FFTA ON IBM 3033 AT APL/JHU BY SKM FEB 80.

C******************************************************************************
C
C THIS SUBROUTINE CALCULATES THE FAST FOURIER TRANSFORM OR
C THE INVERSE FAST FOURIER TRANSFORM OF AN INPUT COMPLEX
C ARRAY FIELD AND RETURNS THE RESULT IN THE SAME ARRAY.
C IBMISN (INTEGER) CONTROLS THE DIRECTION OF THE TRANSFORM:
C =1 FOR FORWARD (NEGATIVE EXPONENTIAL) TRANSFORM
C =3 FOR INVERSE TRANSFORM
C SEE JHU/APL SCIENTIFIC SUBR LIBRARY ROUTINE NO. 6.04.051 FOR
C OTHER VALUES OF IBMISN IN SUBR FFTA USED HEREIN.
C ORDERING OF DATA:
C ON INPUT AND OUTPUT, I=1 CORRESPONDS TO MOST NEGATIVE
C ABSCISSA, I=N/2+1 TO ORIGIN, I=N TO MOST POSITIVE ABSCISSA.

C******************************************************************************

COMPLEX FIELD(I)
COMPLEX T,F
N=2**NEXP
DATA *12/6.2831853071796/
2 N2=N/2
C ORDER THE DATA FOR FFTA I.E., NONNEGATIVE ABSCISSAE CORRESPOND
C TO I=1 TO N/2, NEGATIVE ABSCISSAE TO I=N/2+1 TO N.
C0 3 I=1,N2,1
C1 I=1+I+N2
T=FIELD(I)
FIELD(I)=FIELD(K)
3 FIELD(K)=T
4FFTA=NEXP+1
CALL FFTA(FIELD,FFTA,IBMISN)
C REORDER THE DATA FOR OUTPUT.
C0 9 I=1,N2,1
C1 I=1+I+N2
T=FIELD(I)
FIELD(I)=FIELD(K)
FIELD(K)=T
3 CONTINUE
Chapter 27

SUBROUTINE MAGFFT

27-1. Purpose: To increase the resolution of a complex array of data points using Fourier interpolation and the Fast Fourier Transform. The number of points in each array must be an integer power of two.


27-3. Arguments

A, NA - Complex input array of NA = \(2^M < NB\) data points.
B, NB - Complex output array of NB = \(2^N\) data points.

27-4. Comment and Method

a. Subroutines required: FFTA, PWRTWO

b. By Shannon's sampling theorem, a band-limited function is represented by its samples, and it can be reconstructed at any point from them. The computation of the value of the function at a point other than a sample point is called Fourier interpolation. Such interpolation can be used to increase the resolution of a function.

The Fast Fourier Transform (FFT) can be used to facilitate Fourier interpolation. Briefly, the original function \(A(x)\), known at NA points on the range \((-K_M, +K_M)\), is transformed to yield \(E(x) = F(A(k))\) at NA sample points. These NA values of \(E(x)\) are then placed in the center of an array containing \(NB = 2^N > NA\) \(2^M\) points to form the function \(E'(x)\). This function is then inverse transformed to produce \(A(k)\) at NB points over the same range \((-K_M, +K_M)\). (Actually, the range is \((-K_M, +K_M - \Delta k)\) since the FFT considers the sampled function to be periodic outside the known range so that the \(NB + 1\)st point would be the same as the first point in the array.)


SUBROUTINE MAGFFT(A,NA,B,NB)
COMPLEX A(NA),B(NB)
IF (NB.LE.NA) GO TO 1
GO TO 2
1 WRITE(6,3)
3 FORMAT("NB IS LESS THAN OR EQUAL TO NA IN MAGFFT ")
RETURN
2 CONTINUE
NAC=NA/2+1
NJC=NB/2+1
N=NBC-NAC
DO 5 I=1,NB
B(I)=(0.,0.).
5 CONTINUE
CALL PWRWC(INA,INA)
CALL FFT(A,INA,3)
DO 10 I=1,NA
J=N+1
B(J)=A(I)
10 CONTINUE
CALL PWRWC(INP,INP)
CALL FFT(A,INP,1)
RETURN
END
Chapter 28

SUBROUTINE JOYFFT

28-1. Purpose: To compute the two-dimensional Fast Fourier Transform of a complex array of NXI by NYI points and to provide magnification of a specified portion of the transformed data.

28-2. Usage: CALL JOYFFT (INPUT, NXI, NYI, MX, MY, NXC, NYC, OUTPUT, NXO, NYO, XYFFT, NXY, ISN)

28-3. Arguments

INPUT, - Complex input array of NXI by NYI points.
NXI, NYI

MX, MY - Integer input variables, equal to an integer power of two, which specify the magnification in the I and J directions, respectively.

NXC, NYC - Integer input variables which specify the center coordinate I = NXC, J = NYC of the sector to be magnified.

OUTPUT, - Complex output array of NXO by NYO points containing the transformed points of the magnified sector.
NXO, NYO

XYFFT, - Complex working array of NXY points.
NXY

ISN - Integer input variable which specifies the direction of the FFT: ISN = 3 for inverse FFT; ISN = 1 for FFT. See Chapter 26.

28-4. Comment

a. Subroutines required: FFTA, PWRTWO.

PROCEED TO CHAPTER 29 FOR DETAIL
b. All integer input variables must be integer powers of 2 and must satisfy the following restrictions:

(1) \( \text{NXO} \times \text{NYO} \leq \text{NXI} \times \text{NYI} \)

(2) \( \text{NXO} \leq \text{NXI} \text{ or } \text{NYO} \leq \text{NYI} \)

(3) \( \text{MX} \times \text{NXI} \leq \text{NXY} \text{ and } \text{MY} \times \text{NYI} \leq \text{NXY} \)


28-7. References: None

SUBROUTINE JOYFFT INPUT, NXI, NYI, MX, MY, NXC, NYC, OUTPUT, NXO, NYO, 
XYFFT, NXY, ISN) 
COMPLEX INPUT(NXI, NYI), OUTPUT(NXO, NYO), XYFFT(NXY) 

***************************************************************************** 

** SUBROUTINE JOYFFT CALCULATES THE TWO DIMENSIONAL COMPLEX FAST 
** FOURIER TRANSFORM FOR ISN=1 OR THE INVERSE FAST FOURIER 
** TRANSFORM FOR ISN=-1, OUTPUT(NXC, NYC), OR A TWO DIMENSIONAL 
** COMPLEX ARRAY, INPUT(NXI, NYI) 
** JOYFFT ALSO PERMITS CALCULATION OF A MAGNIFIED SECTOR OF THE 
** FFT AS FOLLOWS. MX IS THE MAGNIFICATION FACTOR FOR THE X 
** DIMENSION AND MY IS THE MAGNIFICATION FACTOR FOR THE Y 
** DIMENSION. THE CENER COORDINATE OF THE MAGNIFIED SECTOR, 
** (NXC, NYC) IS SPECIFIED WITH REFERENCE TO THE UNMAGNIFIED FFT 
** THE XYFFT(NXY) ARRAY IS A COMPLEX TEMPORARY STORAGE ARRAY USED 
** TO PERFORM THE MAGNIFIED X AND Y SINGLE DIMENSION FFTS 
** FOLLOWING ARE RESTRICTIONS ON THE INPUT AND OUTPUT PARAMETERS 
** "<" MEANS LESS THAN OR EQUAL TO 
** NXO*NYO<NXI*NYI 
** NXO<NXI OR NYO<NYI 
** MX*NXI<NXY AND MY*NYI<NXY 
** NYI=2**(ANY NON-NEGATIVE INTEGER) 
** NXI=2**(ANY NON-NEGATIVE INTEGER) 
** NXC=2**(ANY NON-NEGATIVE INTEGER) 
** NYC=2**(ANY NON-NEGATIVE INTEGER) 
** MX=2**(ANY NON-NEGATIVE INTEGER) 
** MY=2**(ANY NON-NEGATIVE INTEGER) 
** MAGNIFICATION IS NOT PERMITTED FOR INPUT DIMENSIONS OF ONE 
** THE OUTPUT SECTOR MUST BE CONTAINED IN THE MAGNIFIED FFT 
** NXC/2<MX*PIN*NXC-1, NXI+1-NXC) 
** ANC 
** NYC/2<MYPIN NYC-1, NYI+1-NYC) 
**
C * THE INPUT AND OUTPUT ARRAY MAY BE EQUIVALENCED USING THE
C * FOLLOWING EQUIVALENCENCE STATEMENT IN THE MAIN PROGRAM
C * EQUIVALENCES (INPUT(1,1),OUTPUT(1,1))
C
C NXO*NYO<NXI*NYI
C IF(NXI*NYI.GE.NXO*NYO) GO TO 2
C WRITE(6,1)
C 1 FORMAT(5X,"THE SIZE OF THE OUTPUT ARRAY EXCEEDS THE SIZE OF THE IN
C PUT ARRAY")
C NXI NYI NXO NYO MX MY EACH MUST EQUAL TWO RAISED TO SOME NON
C NEGATIVE INTEGER POWER
C 2 CALL PWRTMC(NXI,INXI)
C CALL PWRTMC(NYI,INYI)
C CALL PWRTMC(NXO,INXO)
C CALL PWRTMC(NYO,INYO)
C CALL PWRTMC(MXI,INMX)
C CALL PWRTMC(MYI,INMY)
C
C IF(NYC.LE.NXI OR .NYC.LE.NYI) GO TO 220
C WRITE(6,221)
C 210 FORMAT(5X,"THE SIZE OF THE FIRST FFT EXCEEDS THE SIZE OF THE INPU
C T ARRAY")
C RETURN
C
C M*NXI<NXY AND M*NYI<NYY
C 220 NY=NXI*MX
C IF(NXY.GE.NX) GO TO 4
C WRITE(6,3) NYX
C 3 FORMAT(I15," THIS DIMENSION IS INSUFFICIENT TO CARRY OUT THE MAGN
C FICATION IN X")
C RETURN
C 4 NY=NYI*NY
C IF(NXY.GE.NY) GO TO 6
C WRITE(6,5) NYX
C 5 FORMAT(I15," THIS DIMENSION IS INSUFFICIENT TO CARRY OUT THE MAGN
C FICATION IN Y")
C
C \#RETURN
C 6 IF(NYO/2.LE.MX*MIN(NYC-1,NXI+1-NXC)) GO TO 8
C WRITE(6,7)
C 7 FORMAT(5X,"THE OUTPUT SECTOR DESIRED IS NOT CONTAINED WITHIN THE C
C CALCULATED FFT")
C RETURN
C 8 IF(NYO/2.LE.MY*MIN(NYC-1,NYI+1-NYC)) GO TO 9
C WRITE(6,7)
C RETURN
C C DETERMINE THE ORDER IN WHICH X AND Y FFTS WILL BE CALCULATED
C 9 IF(NYO.GT.NXO) GO TO 21
C C C PERFORM SINGLE DIMENSION FFTS FOR Y THEN X
C C C LOAD Y VALUES IN XYFFT ARRAY
C NYS=NYI-NYC
C IF(NYI.EQ.1) GO TO 15
C NZ=(NY-NYI)/2
C NXYS=(NYC-1)*NY-NYO/2
C DO 14 I=1,NXI,1
C IF(NZ.EQ.0) GC TO 11
C DO 10 J=1,NZ,1
C XYFFT(J)=(0,0,0,0)
C XYFFT(J+NZ+NYI)=(C,G,0,0)
C 10 CONTINUE
C 11 DO 12 J=1,NYI,1
C XYFFT(J+NZ)=INPUT(I,J)
C 12 CONTINUE
C C C PERFORM SINGLE DIMENSION FFT FOR Y
C CALL PMRTWC(NY,INY)
C CALL FFT4(XYFFT,INY,ISN)
C C C EXTRACT OUTPUT SECTOR OF INTEREST AND STORE IN INPUT ARRAY
C 70 13 J=1,NYO,1
program name

C INPUT (I,J+NYS)=XYFFT(J+NYS)
13 CONTINUE
14 CONTINUE

C PERFORM NYO MAGNIFIED FFTS IN X AND STORE SELECTED SECTORS OF FFTS

C IN OUTPUT ARRAY
15 NZ=(NX-NXI)/2
NXYS=(NXC-1)*NY-NX/2
DO 20 J=1,NY0,1

C LOAD X VALUES IN XYFFT ARRAY
IF(NZ.EQ.0) GO TO 17
DO 16 I=1,NZ,1
XYFFT(I)=(0.,0.,0.,0.)
XYFFT(I+NZ+NXY)=(C.,C.,0.,0.)
16 CONTINUE
17 DO 18 I=1,NXY,1
XYFFT(I+NZW)=INPUT(I,J+NYS)
18 CONTINUE

C PERFORM SINGLE DIMENSION FFTS IN X
CALL PWRTWC(NX,INX)
CALL FFTA(XYFFT,INX,ISN)

C EXTRACT OUTPUT SECTOR OF INTEREST AND STORE IN OUTPUT ARRAY
DO 19 I=1,NX0,1
OUTPUT(I,J)=XYFFT(I+NXY)
19 CONTINUE
20 CONTINUE
RETURN

C PERFORM SINGLE DIMENSION FFTS FOR X THEN Y
21 NYS=NXI-NXC
IF(NXI.EQ.1) GO TO 27
NZ=(NX-NXI)/2
NXYS=(NXC-1)*NY-NX0/2

C LOAD X VALUES IN XYFFT ARRAY
DO 26 J=1,NYI,1
IF(NZ.EQ.0) GO TO 23
DO 22 I=1,NX,1
XYFFT(I)=(0,0,0,0)
XYFFT(I+NZ+NXY)=C(.C,0,0,0)
22 CONTINUE
23 DO 24 I=1,NXI,1
XYFFT(I+NX) = INPLT(I,J)
24 CONTINUE
C
C PERFORM SINGLE DIMENSION FFT IN X
CALL PWPTC(NX,NX)
CALL FFTC(XYFFT,NX,ISN)
C
C EXTRACT OUTPUT SECTOR OF INTEREST AND STORE IN INPUT ARRAY
DO 25 I=1,NX0,1
INPUT(I+NXS,J)=XYFFT(I+NXYS)
25 CONTINUE
26 CONTINUE
C
C PERFORM NC MAGNIFIED FFTS IN Y AND STORE SELECTED SECTOR OF FFT
C IN OUTPUT ARRAY
27 NZ=(NY-NYI)/2
NXYS=(NYC-1)*NY-NY0/2
DO 32 I=1,NX0,1
C
C LOAD Y VALUES IN XYFFT ARRAY
IF(NZ.EQ.0) GO TO 29
DO 29 J=1,NZ,1
XYFFT(J)=(0,0,0,0)
XYFFT(J+NZ+NYI)=(0,0,0,0)
29 CONTINUE
29 DO 30 J=1,NYI,1
XYFFT(J+NZ)=INPLT(I+NXS,J)
30 CONTINUE
C
C PERFORM SINGLE DIMENSION FFT IN Y
CALL PWPTC(NY,NY)
CALL FFTA(XYFFT,INX,INS)

C

C EXTRACT OUTPUT SECTOR OF INTEREST AND STORE IN OUTPUT ARRAY

DO 31 J=1,NY0,1
OUTPUT(I,J)=XYFFT(J+NXY)  
31 CONTINUE
32 CONTINUE
RETURN
END
SUBROUTINE FWTWO(N,I)
I=0
1 M=2**I
IF (N-M) 3,5,2
2 I=I+1
GO TO 1
3 WRITE(*,*) N
4 FORMAT(I15," THIS INTEGER DOES NOT EQUAL TWO RAISED TO A NON NEGA-
5 STIVE INTEGER")
5 RETURN
END
APPENDIX A

Test Case 1 for RTFRACP
TEST DATA TO TEST IMPACT WITHOUT PLOTS (CASE I, F0, RHC, N=5)

DRAF3 = F  DRAFS = F  DRAF2 = F  DRAFREV = F  TABLE = F

NFI = 1  NPHI = 2  NTHETA = 5  OSANG = 2.00

NX, NY, NXE, NYE, MX, MY = 16  16  256  1  512  16  1
XYMAX = KYMAX = .5584  XY SPACING = .76812  WAVELENGTHS
KX = .65194  KY = .24049
TANGENT OGIVE PARAMETERS: ROS(IN)=150.46975  ROS(IN)=142.33625
FINOS=3.000  FINIS=3.07245

RESULTS OF RADCME ANALYSIS

TEST DATA TO TEST IMPACT WITHOUT PLOTS (CASE I, F0, RHC, N=5)
FINENESS RATIO = 3.00  DIAMETER=18.26700 IN.  LENGTH=48.80200 IN.
FREQUENCY = 11.163 GHZ
FA = 7.77700  WR = 7.776979 IN.  ANTENNA D = 11.1840 WAVELENGTHS
IPOL = 3  ICASE = 1  ILOPT = 1

INDF  THICKNESS(IN.)  FR  TANG

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APPENDIX B

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**TABLE OF XYM COEFF. IS FORMED**

**SUBROUTINE: NMX** MINE = 0, MAX = 1.005E+01

**SUBROUTINE: NMX** MINE = 0, MAX = 1.005E+01
POWFR OF PATTERN: 1

SUROUTINE NOR-M: MIN= 0. MAX= 1.00E+01

SUROUTINE NOR-M: MIN= 0. MAX= 1.00E+01

SUROUTINE NOR-M: MIN= 0. MAX= 1.00E+01

SUROUTINE NOR-M: MIN= 0. MAX= 1.00E+01

TANGENT GDTV PARAMETER: R0S(IN)=150.46975 R0S(IN)=142.33625
FINOS=3.000 FINIS= 3.07245

RESULTS OF RANDOM ANALYSIS
TEST DATA TO TEST IMPACF WITH PLOTS (CASE 1, FOG, RHC, N=5)
FINISHES FATTY: 3.6 D I A M E T E R=16.26700 IN. LENGTH=48.80200 IN.
FREQUENCY= 11.103 GHz
NA= 7.74767. P= 39.76876 IN. ANTENNA 0= 11.1840 WAVELENGTHS
IPOL= 3 ICASE= 1 IOPT= 1

LAYER THICKNESS (IN.) ER TAND

| 1 | 1.1525 | 6.200 | 0.090 |
| 2 | 1.1525 | 4.000 | 0.090 |
| 3 | 1.1525 | 4.000 | 0.090 |
| 4 | 1.1525 | 4.000 | 0.090 |
| 5 | 1.1525 | 4.000 | 0.090 |
RECEIVING PATTERN COMPUTED FOR:

ICUT = 1
ICOMP = 1
KMAX = .451
KREP = 32
OK = 4.4694E-01
ANGMAX = 40.61

(ICUT = 1 FOR EL CUT, = 2 FOR AZ CUT
ICOMP = 1 FOR EL COMPONENT, = 2 FOR AZIMUTH)

MIN AND MAX VALUES OF RECEIVING PATTERN:

SUB-OUTING NORM MIN = .455F-03 MAX = .146E+04
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**MIN AND MAX VALUES OF SIMG PATTERN**

**SUBROUTINE ND**

MIN = 0.161E-23  MAX = 0.730E-03
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| -39.1 | -35.537 | -11.4 |
| -38.9 | -36.319 | -1.6 |
| -35.1 | -46.000 | 28.2 |
| -33.7 | -40.830 | 95.9 |
| -32.6 | -38.291 | 163.4 |
| -30.9 | -41.531 | 50.5 |
| -23.6 | -21.935 | -16.9 |
| -28.2 | -23.946 | -31.2 |
| -26.4 | -22.047 | -37.3 |
| -25.6 | -22.637 | -42.1 |
| -24.3 | -21.852 | -44.7 |
| -27.1 | -22.151 | -44.9 |
| -21.4 | -23.636 | -41.1 |
| -27.5 | -23.582 | -37.4 |
| -19.3 | -21.046 | -40.4 |
| -14.1 | -17.659 | -46.9 |
| -16.9 | -14.785 | -52.2 |
| -15.0 | -13.034 | -59.1 |
| -14.4 | -12.291 | -59.0 |
| -13.2 | -13.911 | -50.9 |
| -12.0 | -15.572 | -39.9 |
| -10.8 | -19.217 | -23.4 |
| -9.7 | -19.864 | -26.9 |
| -4.0 | -14.123 | -50.7 |
| -7.3 | -6.073 | -61.9 |
| -6.1 | -7.649 | -66.7 |
| -5.0 | -1.041 | -66.2 |
| -3.4 | -0.010 | -61.9 |
| -2.6 | -0.010 | -57.8 |
| -1.3 | -7.545 | -49.6 |
| -1.3 | -1.343 | -20.5 |
| 0.9 | -9.741 | 76.4 |
| 2.7 | -3.453 | 102.3 |
| 3.2 | -7.744 | 111.1 |
| 4.4 | -1.193 | 116.8 |</p>
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-COFISHING PATTERN COMPUTED FOR:

ICUT = 2
ICOMP = 1
KMAX = 1551
M+ = 32
UK = 4.54
NGMAX = 41

(INPUT: 1 FOR EL OUT, 2 FOR AZ OUT
ICOMP = 1 FOR EL COMPONENT, = 2 FOR AZimuth)
RECEIVING PATTERN COMPUTED FOR:

IGUT= 2
ICOMP= 1
KMAX= 100
WFE= 32

MAX= 1.4e+04
MIN= 42.9

ICUT= 1 FOR SL CUT, = 2 FOR AZ CUT
(ICOMP= 1 FOR SL COMPONENT, = 2 FOR AZ COMPONENT)

MIN AND MAX VALUES OF RECEIVING PATTERN:

SUBROUTINE: V04Rd MIN= 0.571E-23 MAX= 1.4E+04

RECEIVING PATTERN, AZ CUT, SL COMPONENT (dB):

-32.5  -40.1  31.5
-39.9  -36.7  49.3
-36.5  -33.9  59.4
-35.1  -34.7  62.9
-37.7  -39.1  64.0
-32.3  -40.0  -127.1
-35.4  -34.7  -122.6
-29.6  -31.7  -121.6
-28.2  -31.4  -122.9
-26.3  -36.7  -131.5
-25.6  -40.0  -120.7
-23.3  -37.4  77.6
-21.1  -35.3  75.4
### MIN. AND MAX. VALUES OF RECG PATTERN

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<th>Max.</th>
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<td>(152.4)</td>
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<td>(30.3)</td>
<td>(25.7)</td>
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### SUBROUTINE Details

- Min: \(0.125e+02\)
- Max: \(0.999e+03\)

### RECG PATTERN, AZ CUTF, EL COMP (DB)

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### Notes and Observations

- The table presents the minimum and maximum values of the RECG pattern, with values ranging from \(-149.9\) to \(153.3\) for different parameters.
- The subroutine details show a minimum value of \(0.125e+02\) and a maximum value of \(0.999e+03\).

---

**Note:** The values in the table are likely numerical data related to a specific pattern or measurement, possibly from a scientific or engineering context. The exact interpretation would depend on the specific field of study.
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**NUMBER OF RAYS USED IN COMPUTING APERTURE FIELD = 177**

**FINAL ANSWERS FOR MONOPULSE SYSTEM**

**K11** = -4.9308E-02, 4.4512E-02, 99996E+00

**K21** = -1.4815E-02, 4.6031E-02, 99996E+00

**AZM** = -1.48156E+01 MRAD

**ELM** = 4.45135E+01 MRAD

**MES47** = 0.2770E+01 VOLTS/DEG

**MESEL** = 0.2840E+00 VOLTS/DEG

**VAZ1** = -2.1445E-02, 1.6566E-04

**VEL1** = 4.936E-02, 2.4790E-02

**Smax** = 0.3772625959E+03

**LCTR** = 3
### Additional Monocular Outputs Around Bore Sight

**Number of Rays Used in Computing Aperture Field:** 177

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<thead>
<tr>
<th>Angle From Bore Sight</th>
<th>WAZ</th>
<th>VREL</th>
<th>Volts</th>
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<tbody>
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<td>.5714E+00</td>
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<table>
<thead>
<tr>
<th>Angle From Bore Sight</th>
<th>WAZ</th>
<th>VREL</th>
<th>Volts</th>
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<table>
<thead>
<tr>
<th>Angle From Bore Sight</th>
<th>WAZ</th>
<th>VREL</th>
<th>Volts</th>
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</thead>
<tbody>
<tr>
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<thead>
<tr>
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<th>WAZ</th>
<th>VREL</th>
<th>Volts</th>
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<thead>
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<th>VREL</th>
<th>Volts</th>
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<th>VREL</th>
<th>Volts</th>
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<th>VREL</th>
<th>Volts</th>
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<tbody>
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</table>
AVERAGE SLP AZ = 26.435E+00 VOLTS/DEG
AVERAGE SLP FL = 27.353E+00 VOLTS/DEG
SUM = 1.9 VOLT

0.0 14.0 4.45 -4.82 0.0000 0.0000 -1.2

RECEIVED SUM VOLTAGE WITHOUT RADIO = 354.60E+03
Figure B-1. $|E_x|$ or $|E_y|$ of the RHC (ICASE=1) Antenna.
Figure B-2. Phase of $E_{x\Sigma}$ for RHC (ICASE=1) Antenna.
Figure B-3. Phase of $E_{yL}$ for RHC Antenna.
Figure B-4. Transmitting E-Plane Σ Pattern of RHC Antenna Without Radome.
Figure B-5. Transmitting H-Plane Pattern of RHC Antenna Without Radome.
Figure B-6. $|E_x|_{\Delta EL}$ or $|E_y|_{\Delta EL}$ of RHC Antenna.
Figure B-7. Phase of $E_{\text{XGEL}}$ of RHG Antenna.
Figure B-8. Phase of $E_{y\lambda E L}$ of RHC Antenna.
Figure B-10. $|E_{X_{AZ}}|$ or $|E_{Y_{AZ}}|$ of RHC Antenna.
Figure 11. Phase of $E_{\text{XAZ}}$ of RIC Antenna.
Figure B-12. Phase of $E_{ymAZ}$ of RHC Antenna.
Figure B-15. Receiving E-Plane $A_{el}$ Pattern of RHIC Antenna With Radome at $(0^\circ, 14^\circ)$. 
Figure B-16. Receiving H-Plane E Pattern of RHC Antenna With Radome at (0°,14°).
APPENDIX C

Test Case 3 for RTFRACP
TEST DATA TO TEST HAMRACP WITHOUT PLOTS (CASE 3,F0,LINEAR,N=5)

f, f, f, f, f, f, 0,
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### RESULTS OF PADOME ANALYSIS

Test data for test IBMACP without plots (case 3, f0, linear, n=5)

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<th>Frequency (GHz)</th>
<th>Diameter (in.)</th>
<th>Length (in.)</th>
<th>Antenna (wavelengths)</th>
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</table>

Layer thickness (in.) | TAND

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<th>TAND</th>
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APPENDIX D

Test Case 4 for RTFRACP
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<th>B</th>
<th>C</th>
<th>D</th>
<th>E</th>
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**SUBROUTINE NORM: MIN = 0, MAX = 0.**

**SUBROUTINE NORM: MIN = 0, MAX = 1.03E-01**
POWER OF PATTERN = 1

SUBROUTINE NORM: MIN = 0., MAX = 1.

SUBROUTINE NORM: MIN = 0., MAX = .103E+01

SUBROUTINE NORM: MIN = 0., MAX = 0.

SUBROUTINE NORM: MIN = 0., MAX = .103E+01

TANGENT OGIVE PARAMETERS: ROS(IN) = 150.46975 BOS(IN) = 142.33625
FINOS = 3.000 FINIS = 3.07245

RESULTS OF RADOME ANALYSIS
TEST DATA TO TEST IBMRACP WITH PLOTS (CASE 3, F3, LINEAP, N=5)
FINENESS RATIO = 3.00 DIAMETER = 16.26706 IN. LENGTH = 48.80200 IN.
FREQUENCY = 11.43 GHz
PA = 7.74703 IN. RR = 39.76878 IN. ANTENNA D = 5.1992 WAVELIGHTS
IPDL = 1 ICASE = 3 IOPT = 1

LAYER THICKNESS (IN.) ER TAN
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2 .01525 2.400 .0050
3 .01525 6.000 .0090
4 .01525 2.400 .0050
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KMAX = 0.396
NREG = 32
OK = 0.62250E+31
ANGMAX = 84.87

(ICUT = 1 FOR FL CUT, = 2 FOR AZ CUT
ICOMP = 1 FOR EL COMPONENT, = 2 FOR AZIMUTH)

MIN AND MAX VALUES OF RECEIVING PATTERN:

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**RECEIVING PATTERN COMPUTED FOR:**

- **ICUT** = 2
- **ICOMP** = 1
- **KMAX** = 0.996
- **NRED** = 32
- **OK** = 62253E-01
- **ANGMAX** = 34.87

*(ICUT=1 FOR EL CUT, =2 FOR AZ CUT
* (ICOMP=1 FOR EL COMPONENT, =2 FOR AZIMUTH)*
RECEIVING PATTERN COMPUTED FOR:
ICUT = 2
ICOMP = 1
KMAX = 0.996
NREC = 32
DK = 0.6225E-01
ANGMAX = 84.87
(ICUT=1 FOR EL CUT, =2 FOR AZ CUT
(ICOMP=1 FOR EL COMPONENT, =2 FOR AZIMUTH)

MIN AND MAX VALUES OF RECEIVING PATTERN:

SUBROUTINE NORM:
MIN = -333E-03 MAX = 0.229E+03

RECEIVING PATTERN, AZ CUT, EL COMP (DB):

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MIN AND MAX VALUES OF REC"G PATTERN:

SUBROUTINE NORM MIN = .534E-02 MAX = .987E+02

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NUMBER OF RAYS USED IN COMPUTING APERTURE FIELD = 169
NUMBER OF RAYS USED IN COMPUTING APERTURE FIELD = 169
NUMBER OF RAYS USED IN COMPUTING APERTURE FIELD = 169
NUMBER OF RAYS USED IN COMPUTING APERTURE FIELD = 169
NUMBER OF RAYS USED IN COMPUTING APERTURE FIELD = 169

FINAL ANSWERS FOR MONOPULSE SYSTEM:

K11  -0.5396E-02  -0.35755E-04  0.99999E+00
K21  -0.53667E-02  0.14448E-07  0.99999E+00

AZTH=  -0.53667E+01 MRAD
ELTM=   0.14449E-04 MRAD

YESAZ=  0.33465E-01 VOLTS/DEG
MESEL=  0.10196E+00 VOLTS/DEG

UNZI=  -0.11735E-03  0.45944E-07
UFLI=  -0.20894E-03  0.84448E-07

SMAI=  0.1209628595054E+03  LCTR=  3
ADDITIONAL MONOPULSE OUTPUTS AROUND BORESIGHT:

NUMBER OF RAYS USED IN COMPUTING APERTURE FIELD = 169
ANG = -3.0 MRAD FROM BORESIGHT VRAZ = -1.6032E-11 VOLTS VREL = -1.7502E-0

DAZ(AMP,PHS) = .16703E-01 -.75959E+02 DEL(AMP,PHS) = .17507E-01 -.91343E+02

NUMBER OF RAYS USED IN COMPUTING APERTURE FIELD = 169
ANG = -2.0 MRAD FROM BORESIGHT VRAZ = -1.0716E-11 VOLTS VREL = -1.1675E-0

DAZ(AMP,PHS) = .11671E-01 -.65530E+02 DEL(AMP,PHS) = .11674E-01 -.91340E+02

NUMBER OF RAYS USED IN COMPUTING APERTURE FIELD = 169
ANG = -1.0 MRAD FROM BORESIGHT VRAZ = -5.3557E-12 VOLTS VREL = -5.8410E-0

DAZ(AMP,PHS) = .71136E-02 -.44846E+02 DEL(AMP,PHS) = .54426E-02 -.91340E+02

NUMBER OF RAYS USED IN COMPUTING APERTURE FIELD = 169
ANG = 0.0 MRAD FROM BORESIGHT VRAZ = .45944E-07 VOLTS VREL = .84448E-0

DAZ(AMP,PHS) = .47179E-02 .55796E-03 DEL(AMP,PHS) = .94471E-07 .88662E+02

NUMBER OF RAYS USED IN COMPUTING APERTURE FIELD = 169
ANG = 1.0 MRAD FROM BORESIGHT VRAZ = .53599E-02 VOLTS VREL = .58486E-0

DAZ(AMP,PHS) = .71646E-02 .48265E+02 DEL(AMP,PHS) = .58532E-02 .88664E+02

NUMBER OF RAYS USED IN COMPUTING APERTURE FIELD = 169
ANG = 2.0 MRAD FROM BORESIGHT VRAZ = .12746E-31 VOLTS VREL = .11705E-0

DAZ(AMP,PHS) = .11746E-01 .65927E+02 DEL(AMP,PHS) = .11703E-01 .88665E+02

NUMBER OF RAYS USED IN COMPUTING APERTURE FIELD = 169
ANG = 3.0 MRAD FROM BORESIGHT VRAZ = .16033E-31 VOLTS VREL = .17570E-0

DAZ(AMP,PHS) = .16832E-01 .73301E+02 DEL(AMP,PHS) = .17575E-01 .88667E+02

535

532
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Table: Values

- 14.0
- 0.001
- 5.37
- 1.0000
- 0.0000
- -1.6

Received SUM Voltage without Radome = 0.14515E+03
Figure D-1. $|E_x|$ of Flat Plate Antenna (ICASE=3) for Sum, Elevation Difference, and Azimuth Difference Channels.
Figure D-2. $|E_y|^E$ of Flat Plate Antenna.
Figure D-3. Phase of $E_{\gamma\ell}$ of Flat Plate Antenna.
Figure 3-4. Transmitting E-Plane Sum Pattern of Flat Plate Antenna.
Figure D-5. Transmitting H-Plane Sum Pattern of Flat Plate Antenna.
Figure D-6. $|E_{Y\Delta E}|$ of Flat Plate Antenna.
Figure D-7. Phase of $E_{Y\Delta EL}$ of Flat Plate Antenna.
Figure D-8. Transmitting E-Plane $\Delta_{EL}$ Pattern of Flat Plate Antenna.
Figure D-9. $|E_{Y\Delta AZ}|$ of Flat Plate Antenna.
Figure D-10. Phase of $E_{Y\Delta AZ}$ of Flat Plate Antenna.
Figure D-11. Transmitting H-Plane $E_{\text{H}}$ Pattern of Flat Plate Antenna.
Figure D-12. Receiving E-Plane Sum Pattern of Flat Plate Antenna With Radome.
Figure D-13. Receiving E-Plane $\Delta_{EL}$ Pattern of Flat Plate Antenna With Radome.
Figure D-14. Receiving H-Plane Sum Pattern of Flat Plate Antenna With Radome.
Figure D.15. Receiving H-Plane Δθ Pattern of Flat Plate Antenna With Radome.
Appendix E

Plane Wave Transmission Through Multilayered Radome Wall
(Excerpted from Reference 1 cited in Chapter 11.)

The derivation below and the computer program implementation
listed in Appendix F are based on work done by Richmond at Ohio State
University. Although Richmond's matrix formulation for the analysis
of plane multilayers has been previously documented [3], an outline of
the theory is repeated here to provide a convenient reference in
defining the quantities described in the computer program of Appendix F.

Consider a plane electromagnetic wave incident on the surface of
a stack of plane, homogeneous, dielectric slabs of finite thickness
and infinite width surrounded by free space as shown in Figure 7(a).
The wave illustrated has perpendicular polarization (electric field
intensity vector perpendicular to the plane of incidence) and the
symbols $E_i$ and $E_r$ represent the electric field intensities of the
incident and reflected waves at the "incident point " $P$, and $E_t$
represents the electric field intensity of the transmitted wave at the
"normal exit point " $Q$. The reflection coefficient $R$ and the "normal
transmission coefficient " $T_n$ of the multilayer are defined by

$$ R = \frac{E_t(P)}{E_i(P)} \quad \text{(perpendicular polarization)} \quad (171) $$

and

$$ T_n = \frac{E_t(Q)}{E_i(P)} \quad \text{(perpendicular polarization)} \quad (172) $$

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Figure 7. Plane Electromagnetic Wave Incident on Plane Multilayer.
The "insertion transmission coefficient" $T$ is defined as follows

$$T = \frac{E_t(Q)}{E_i(Q)} \quad \text{(perpendicular polarization)} \quad (172)$$

$$= T_n e^{jkd\cos \theta}$$

where $d$ is the total multilayer thickness, $\theta$ is the angle of incidence measured from the normal, and $k$ is the free-space phase constant $\omega \sqrt{\mu_0 \varepsilon_0} = 2\pi / \lambda_0$.

The resultant field in each layer consists of an outgoing wave and a reflected wave. In Figure 1 the complex constants $A_n$ and $C_n$ represent the electric field intensity $E_x$ of the outgoing wave in layer $n$, evaluated at its two boundaries, and $B_n$ and $D_n$ represent the reflected field intensity at the two boundaries.

The field intensity in layer $n$ can be written as

$$E_x = (ae^{-\gamma_n z} + be^{\gamma_n z}) e^{-jky \sin \theta} \quad (173)$$

The propagation constant $\gamma_n$ is expressed in terms of the attenuation constant $\alpha_n$ and the phase constant $\beta_n$ as

$$\gamma_n = \alpha_n + j\beta_n \quad (174)$$

It is assumed that the permeability of each layer is real and the complex permittivity is expressed as
Using the wave equations and Equations (173), (174), and (175), it can be found that

\[ a = \left( \frac{k}{\sqrt{2}} \right) \sqrt{(\mu'_{r} \varepsilon'_{r} - \sin^2 \theta)^2 + (\mu'_{r} \varepsilon'_{r} \tan \delta)^2 - (\mu_{r} \varepsilon'_{r} - \sin^2 \theta)} \quad (176) \]

\[ b = \left( \frac{k}{\sqrt{2}} \right) \sqrt{(\mu'_{r} \varepsilon'_{r} - \sin^2 \theta)^2 + (\mu'_{r} \varepsilon'_{r} \tan \delta)^2 + (\mu_{r} \varepsilon'_{r} - \sin^2 \theta)} \quad (177) \]

where \( \mu_{r} \) and \( \varepsilon'_{r} \) are the relative permeability and permittivity:

\[ \mu_{r} = \mu/\mu_{0} \quad (178) \]

and

\[ \varepsilon'_{r} = \varepsilon'/\varepsilon_{0} \quad (179) \]

Evaluating \( F_{x} \) in Equation (173) at the left and right boundaries of layer \( n \), it can be shown that

\[ A_{n} = C_{n} e^{-\gamma_{n} d_{n}} \quad (180) \]

and

\[ B_{n} = D_{n} e^{\gamma_{n} d_{n}} \quad (181) \]

where \( d_{n} \) is the thickness of layer \( n \). Equations (180) and (181) can be expressed by the following matrix equation:
Let $t_{n+1,n}$ and $r_{n+1,n}$ denote the interface transmission and reflection coefficients for a wave in layer $n+1$ incident on the boundary of layer $n$. Further, let $t_{n,n+1}$ and $r_{n,n+1}$ represent the interface coefficients for a wave in layer $n$ on the boundary of layer $n+1$. In terms of these coefficients, the electric field intensities, evaluated at both sides of the boundary between layers $n$ and $n+1$, are related linearly as follows:

$$C_n = t_{n+1,n}A_{n+1} + r_{n+1,n}D_n$$

(183)

and

$$B_{n+1} = t_{n,n+1}D_n + r_{n+1,n}A_{n+1}$$

(184)

The relations follow from the superposition theorem and the definitions of the interface coefficients.

It can be shown that

$$r_{n,n+1} = -r_{n+1,n}$$

(185)

$$t_{n+1,n} = 1 + r_{n+1,n}$$

(186)

$$t_{n,n+1} = 1 + r_{n,n+1} - r_{n+1,n}$$

(187)
and

\[ t_{n+1, n, n+1} - t_{n, n+1, n+1} = 1 \]  

(188)

by using Equations (185) through (188), Equations (183) and (184) can be arranged as

\[ C_n = (A_{n+1} + r_{n, n+1, n+1})/t_{n, n+1} \]  

(189)

and

\[ D_n = (b_{n+1} - r_{n+1, n+1})/t_{n, n+1} \]  

(190)

These can be expressed in matrix form as

\[
\begin{bmatrix}
C_n \\
D_n
\end{bmatrix}
= \frac{1}{t_{n, n+1}} \begin{bmatrix}
1 & -r_{n+1, n} \\
-r_{n+1, n} & 1
\end{bmatrix}
\begin{bmatrix}
A_{n+1} \\
b_{n+1}
\end{bmatrix}
\]  

(191)

The matrix equations (192) and (191) can be combined to obtain the following:

\[
\begin{bmatrix}
\gamma_{n-1} \\
\gamma_{n}
\end{bmatrix}
= \frac{1}{t_{n-1, n}} \begin{bmatrix}
\gamma_{n-1, n} & \gamma_{n-1, n} \\
\gamma_{n-1, n} & \gamma_{n-1, n}
\end{bmatrix}
\begin{bmatrix}
\gamma_{n-1} \\
\gamma_{n}
\end{bmatrix}
\]  

(192)

Let the two-by-two matrix in Equation (192) be denoted by \( M_n \).
Repeated application of Equation (193) yields the following matrix relationship between the electric field intensities at the incidence and exit surfaces:

\[
M_n = \begin{pmatrix}
\gamma_{dN} & \gamma_{dN}
\end{pmatrix}
\begin{pmatrix}
e^{-\gamma_{dN}} & -r_{n,n-1}e^{-\gamma_{dN}}
-2 & \gamma_{dN}
\end{pmatrix}
\]

(193)

where the dots denote matrix multiplication, N represents the total number of layers, S denotes the matrix

\[
(1/2) M_1 M_2 M_3 \ldots M_N S
\]

and

\[
t = t_0, t_1, t_2, t_3 \ldots t_N, N+1
\]

(196)

In the situation used to define the transmission and reflection coefficients of the structure, a wave of unit amplitude is assumed to

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be incident on one outer surface, so that

\[ A_{n+1} = 1 \]  

(197)

\[ B_{n+1} = R \]  

(198)

\[ C_0 = T_n \]  

(199)

and

\[ D_0 = 0 \]  

(200)

Thus Equation (194) becomes

\[
\begin{bmatrix}
Z_n \\
Y_n \\
M_n \\
L_n
\end{bmatrix} = (1/t) \begin{bmatrix} M_1 & M_2 & M_3 & \ldots & M_n \end{bmatrix} \begin{bmatrix} 1 \\ 0 \end{bmatrix}
\]  

(201)

The solution for "parallel polarization" (electric field intensity parallel to the plane of incidence) is obtained by applying the theorem of duality to the above solution. Thus, the reflection and transmission coefficients are defined by

\[ R_n(t) \] (parallel polarization)  

(202)

and

\[ T_n(p) \] (parallel polarization)  

(203)
The matrix equations given above apply also for parallel polarization, in which case the complex constants $A_n$, $B_n$, $C_n$ and $D_n$ represent the amplitudes of the magnetic field intensities $H_x$ of the traveling waves in layer $n$. Equations (185) through (188) also apply for parallel polarization, in which case the interface reflection and transmission coefficients are defined by the ratio of the magnetic field intensities $H_x$. The interface reflection coefficients are given by

$$r_{n+1,n} = \frac{\mu_n y_{n+1} - \mu_{n+1} y_n}{\mu_n y_{n+1} + \mu_{n+1} y_n} \quad \text{(perpendicular polarization)} \quad (204)$$

and

$$r_{n+1,n} = \frac{\varepsilon_n y_{n+1} - \varepsilon_{n+1} y_n}{\varepsilon_n y_{n+1} + \varepsilon_{n+1} y_n} \quad \text{(parallel polarization)} \quad (205)$$

where $\gamma$ is given by Equations (174), (176), and (177) if the permeability $\mu$ of each layer is real.

After the indicated matrix multiplications of Equation (44) are performed, and the division by $t$, the equation has the form

$$\begin{bmatrix} T_n \\ 0 \end{bmatrix} = \begin{bmatrix} a & b \\ c & e \end{bmatrix} \begin{bmatrix} 1 \\ R \end{bmatrix} \quad (206)$$

Thus,

$$T_n = a + bR = a - \frac{bc}{e} \quad (207)$$

and

$$R = -\frac{c}{e}. \quad (208)$$

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