PRODUCTION OF LORAN-C RELIABILITY DIAGRAMS AT THE DEFENSE MAPPING AGENCY HYDROGRAPHIC/TOPOGRAPHIC CENTER ETC F/G 17/7
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**Title:** Production of LORAN-C Reliability Diagrams at the Defense Mapping Agency.

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Signal limits are computed using Bremmer's field prediction formula (Ref. 1) and an algorithm that predicts the range for a signal of predetermined signal-to-noise ratio.
Ratio propagating along an electrically inhomogeneous transmission path (Ref. 2). Fix uncertainty predictions are based on a formula relating fix uncertainty to (a) crossing angle between lines of position, (b) system standard deviation, and (c) the divergence of hyperbolic lines of position.

Actual range and fix uncertainty may differ from values shown on reliability diagrams, depending on such factors as weather, the occurrence of geomagnetic disturbances, and the user's direction of travel.

Reliability diagrams currently produced show signal limits and fix uncertainties for LORAN-C chains at a scale of 1:5,000,000; a new generation of reliability diagrams could show data at a reduced scale (1:10,000,000) for each LORAN-C triad (one master and two slave transmitters), making more chain and transmitter selection information available to the user.

References Cited in Abstract


PRODUCTION OF LORAN-C RELIABILITY DIAGRAMS
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ABSTRACT
LORAN-C reliability diagrams produced at the Defense Mapping Agency Hydrographic/Topographic Center depict two types of data: (a) the maximum usable groundwave signal limit, which aids the LORAN-C user in determining which transmitters provide coverage in his area of operation, and (b) the predicted uncertainty of a LORAN-C hyperbolic fix.

Signal limits are computed using Bremmer's field prediction formula (Ref. 1) and an algorithm that predicts the range for a signal of predetermined signal-to-noise ratio propagating along an electrically inhomogeneous transmission path (Ref. 2). Fix uncertainty-predictions are based on a formula relating fix uncertainty to (a) the crossing angle between lines of position, (b) system standard deviation, and (c) the divergence of hyperbolic lines of position.

Actual range and fix uncertainty may differ from values shown on reliability diagrams, depending on such factors as weather, the occurrence of geomagnetic disturbances, and the user's direction of travel.

Reliability diagrams currently produced show signal limits and fix uncertainties for LORAN-C chains at a scale of 1:5,000,000; a new generation of reliability diagrams could show data at a reduced scale (1:10,000,000) for each LORAN-C triad (one master and two slave transmitters), making more chain and transmitter selection information available to the user.

INTRODUCTION

The LORAN-C Navigation System

LORAN-C is a pulsed, low frequency (100 kHz) electronic navigation system operated by the U.S. Coast Guard. LORAN-C provides fix data to vessels operating in the northern, northwestern, and central Pacific, the Mediterranean, the northern Atlantic, and the U.S. Coastal Confluence Zone.

LORAN-C is operable in either of two modes: (a) range-range mode, in which a fix is defined by the intersection of two circular lines of position (LOP's) defined by the arrival times of LORAN-C pulses from two synchronized transmitters, and (b) hyperbolic mode, in which each LOP is hyperbolic and is defined by the difference in arrival times of pulses from two synchronized transmitters.

Synchronization of LORAN-C transmissions is maintained by system area monitors which record, at fixed positions, time differences (TD's) in pulse arrivals from each master-slave transmitter pair operating in the area. If TD's measured at a monitor drift excessively from the norm, then the monitor directs the slave transmitter to make a change in pulse timing to compensate for the drift.

The ideas expressed in this paper represent the opinions of the author and do not necessarily reflect official policies of the DMA.

A LORAN-C chain consists of one master transmitter (which initiates the transmission of pulses from other transmitters in the chain) and several secondary (or slave) transmitters; each LORAN-C chain is assigned a unique group repetition interval (GRI). The GRI is the length, commonly expressed in terms of microseconds, of the coded sequence of pulses that comprise the LORAN-C signal format.

General Description of the LORAN-C Reliability Diagram

Shown in Fig. 1 is the LORAN-C reliability diagram for the Gulf of Mexico (GRI 7980). (A list of the currently available LORAN-C reliability and coverage diagrams is in Appendix A.)

Two types of data, depicted on a Lambert conformal conic projection of land-sea interfaces at a scale of 1:5,000,000, are shown in the diagram: (a) the maximum usable groundwave signal limits for signal-to-noise ratios (SNR's) of 1:3 and 1:10 and (b) predicted fix uncertainty.

The signal limit contours define the areas in which the LORAN-C groundwave signal from each transmitter in the chain is of sufficient field strength to be detected by either commercially available receivers (generally capable of extracting signal at SNR's of at least 1:3) or military receivers (generally capable of extracting signal at SNR's of at least 1:10).

Predicted fix uncertainty contours define the areas in which the accuracy of the LORAN-C system, operating in hyperbolic mode, is limited by random errors such as those caused by transmitter instability or variable real-time propagation conditions. Contours representing fix uncertainty of 100, 500, 750, and 1500 ft. are shown in reliability diagrams.

The information shown in reliability diagrams aids the LORAN-C user both in (a) planning chain and transmitter selection for a voyage and (b) making enroute changes in chain and transmitter selection that become necessary when a transmitter's signal becomes unusable due to conditions such as transmitter failure or changes in weather.

Figure 1. LORAN-C reliability diagram for the Gulf of Mexico (GRI 7980). (Signal limits for the master transmitter and for slaves X, Y, and Z are not shown.)

PREDICTING THE MAXIMUM USABLE GROUNDWAVE SIGNAL LIMIT

Attenuation of the LORAN-C Signal

The LORAN-C signal loses energy as it is transmitted along the earth's surface; this loss results from signal front spreading, energy scattering by irregular terrain, energy absorption by the earth and its atmosphere, etc. Factors such as transmission path characteristics, transmitted power, and distance traveled by the signal affect the amount of energy lost in that signal. Transmission path characteristics include (a) physical properties, such as curvature of the earth's surface, and (b) electrical properties (ground conductivity and permittivity, atmospheric refractivity) which vary as functions of weather (Ref. 3), vegetation, and terrain ruggedness.
LORAN-C receivers are designed to sample the LORAN-C pulse 25 microseconds (usec) following its leading edge; this standard sampling point (SSP) occurs, ideally, at 0.506 of the pulse's peak amplitude (Ref. 4). See Fig. 2. For purposes of predicting field strength, the signal level of the LORAN-C pulse is taken to be the root-mean-square (rms) amplitude of a continuous wave whose amplitude is that of the pulse at its SSP (Fig. 2).

At some distance along its transmission path, the groundwave component of the LORAN-C pulse loses so much energy that it becomes indistinguishable from the ambient atmospheric noise. This occurs generally at a SNR of 1:3 for commercial receivers or 1:10 for military receivers. The SNR is the ratio of the LORAN-C signal's field strength at 0.128 of its peak power to the rms field strength of the ambient atmospheric noise, as determined from CCIR report 322 (Refs. 5, 6).

Once the signal limit for each type of terrain segment is predicted, the signal limit of the inhomogeneous transmission path can be predicted using the method derived below (Ref. 2).

Figure 4. Mixed-path approximation of an inhomogeneous transmission path.

Applying Eq. 2 converts a mixed-terrain transmission path to an all sea water transmission path. The sea water signal limit $S_{sea}$ is known, so the mixed-path signal limit can be calculated by converting all sea-equivalent segments up to $S_{sea}$ back to terrain segments, then summing their lengths:

$$
S_{2} = \sum_{i=1}^{A} \left( \frac{G(i) \cdot S_{sea}}{S(i)} \right)
$$

(Eq. 3)

where the $A$'th sea-equivalent segment extends to $S_{sea}$. If the $A$'th segment "straddles" $S_{sea}$, in other words,

$$
\sum_{i=1}^{A} G(i) > S_{sea}
$$

(Eq. 4)
mixed-path signal transmission path. (Note that the fourth terrain type S(\text{sea}) is the residual:

\[ \text{RES} = S(\text{sea}) - \sum_{i=1}^{A-1} G(i) \]  

(Eq. 6).

The mixed-path signal limit \( S(\text{mixed}) \) is predicted by converting the sea-equivalent segments \( G(i) \) through \( G(A-1) \) and RES back to their original lengths \( L(i,j) \) and \( r \), and summing:

\[ S(\text{mixed}) = \sum_{i=1}^{A-1} L(i,j) + r \]  

(Eq. 7)

where \( r = \text{RES}/F(j)_A \) and \( F(j)_A = F(j) \) for \( i=A \).

Thus,

\[ S(\text{mixed}) = \sum_{i=1}^{A-1} L(i,j) + \frac{\sum_{i=1}^{A-1} F(j)_A - F(j)}{F(j)_A} S(\text{sea}) \]  

(Eq. 9)

\[ S(\text{mixed}) = S(\text{sea}) + \sum_{i=1}^{A-1} [F(j)_A - F(j)] L(i,j) \]  

(Eq. 10)

As an example, refer to Tables 2 and 3. In Table 2 are predicted signal limits and sea-equivalency factors for three terrain types; in Table 3 are sea-equivalent lengths \( G(i) \) calculated for each of five segments in a mixed-terrain transmission path. (Note that the fourth terrain segment straddles \( S(\text{sea}) \), hence \( A=4 \).) Using Eq. 10, the mixed-path signal limit is calculated to be 353 nautical miles (n.m.i.). This technique for predicting mixed-path signal limits is represented graphically in Fig. 5.

**Table 2**

<table>
<thead>
<tr>
<th>Terrain Type (i)</th>
<th>Sea-Equivalency Limit S(i)</th>
<th>Signal Factor F(j) (n.m.i.)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1.1</td>
<td>810</td>
</tr>
<tr>
<td>2</td>
<td>8.5</td>
<td>200</td>
</tr>
<tr>
<td>3</td>
<td>1.0</td>
<td>900</td>
</tr>
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</table>

Note: \( S(\text{sea}) = S(3) \).

**Table 3**

<table>
<thead>
<tr>
<th>Segment Length</th>
<th>Terrain Type (j)</th>
<th>Sea-Equivalent Length G(i) (n.m.i.)</th>
<th>Segment Length</th>
<th>Terrain Type (j)</th>
<th>Sea-Equivalent Length G(i) (n.m.i.)</th>
</tr>
</thead>
<tbody>
<tr>
<td>50</td>
<td>1</td>
<td>225</td>
<td>50</td>
<td>2</td>
<td>225</td>
</tr>
<tr>
<td>100</td>
<td>3</td>
<td>100</td>
<td>100</td>
<td>3</td>
<td>100</td>
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<tr>
<td>500</td>
<td>2</td>
<td>2250</td>
<td>500</td>
<td>5</td>
<td>600</td>
</tr>
</tbody>
</table>

Note: \( \text{RES} = 900-(225+100+110) = 465 \) n.m.i.
Figure 7. Fix uncertainty due to LOP instability over distances "a" and "b" is exaggerated by the small crossing angle A.

\[ d^2 = p^2 + q^2 + 2pq \cos \theta \]  
(Eq. 12)

(p and q are the lengths of the parallelograms sides).

Thus, fix uncertainty contours show areas in which the accuracy of the LORAN-C system is limited by the random instability of hyperbolic position lines. Fix uncertainty due to this instability is amplified (a) where crossing angles between LOP's are small and (b) where LOP's diverge.

Fix uncertainty must not be confused with "absolute accuracy" - the accuracy with which a LORAN-C fix can be associated with a distinct geodetic position. The absolute accuracy of a LORAN-C fix is a function of both (a) fix uncertainty and (b) the ability to correctly predict the time required for a LORAN-C signal to propagate from its transmitter to any given geodetic position.

Fix uncertainty is similar to repeatable accuracy (the accuracy with which a vessel can return to a previous position using LORAN-C). In an area of 500 ft. repeatable accuracy, a vessel, using the same LORAN-C TD coordinates, can return to the same position repeatedly within 500 ft., most of the time.

In an area of 500 ft. fix uncertainty, a vessel will be able to position itself to within 500 ft. of a buoy using the TD coordinates of that buoy, provided these TD coordinates have been established by monitoring TD's at the buoy for a sufficient length of time.

Derivation of an Approximate Method for Predicting Fix Uncertainty

Fix uncertainty data depicted on LORAN-C reliability diagrams are computed using a formula derived by Trow and Jessell (Ref. 15). A similar formula is derived by Sitterly (Ref. 16). This formula defines the "95 percent radial error" - the radius of the circle containing about 95% of all fixes associated with a given TD pair.

Shown in Fig. 8 are two LOP's intersecting at an angle A and displaced from their average positions by random errors e(1) and e(2). The major diagonal d of the parallelogram thus formed is the position fix error associated with random errors e(1) and e(2). d can be expressed, according to the Law of Cosines, as

\[ d^2 = p^2 + q^2 + 2pq \cos \theta \]  
(Eq. 12)

(p and q are the lengths of the parallelograms sides).
By the definition of sine,
\[
P = \frac{e(\theta)}{\sin A}, \quad q = \frac{e(\theta)}{\sin A} \quad \text{(Eq. 13)}.
\]

Combining Eqs. 12 and 13 gives
\[
d^2 = \left(\frac{e(\theta)}{\sin A}\right)^2 + \left(\frac{e(\theta)}{\sin A}\right)^2 + \frac{2 e(\theta)^2}{\sin^2 A} \quad \text{(Eq. 14)}.
\]

Eq. 14 relates position fix error \(d\) to (a) random variations that occur in LOP location and (b) LOP crossing angle. The magnitudes of roughly 95% of these position fix errors are smaller than twice their root mean square (twice their standard deviation) 2\(d_{\text{rms}}\). This approximation is made assuming that the fix error \(d\) is normally distributed, which it is not; the probability that \(d\) is smaller than twice its root mean square varies with the crossing angle. Thus, fix uncertainty is approximated as
\[
95\% \text{ radial error} \approx 2d_{\text{rms}} = 2\sqrt{\frac{\sum e(\theta)^2}{\sum \sin^2 A}} \quad \text{(Eq. 15)}.
\]

Substituting from Eq. 14, Eq. 15 becomes
\[
2d_{\text{rms}} = 2\sqrt{\frac{\sum e(\theta)^2}{\sum \sin^2 A} + \frac{\sum e(\theta)^2}{\sin A}} \quad \text{(Eq. 16)}.
\]

Recalling that the standard deviation of a normal distribution of errors is
\[
sd = \sqrt{\frac{\sum e(\theta)^2}{\sum \sin^2 A}} \quad \text{(Eq. 17)}.
\]

and letting the "correlation coefficient" be defined as
\[
C = \frac{\sum e(\theta)^2}{\sum \sin^2 A} \quad \text{(Eq. 18)}.
\]

Eq. 16 becomes
\[
2d_{\text{rms}} = \frac{2}{\sin A} \left(\frac{\sum e(\theta)^2}{\sum \sin^2 A} + \frac{\sum e(\theta)^2}{\sin A}\right)^{1/2} \quad \text{(Eq. 19)}.
\]

The correlation coefficient \(C\) varies between -1 and 1, indicating the degree to which random errors \(e(1)\) and \(e(2)\) are related to one another by some common causal mechanism.

The standard deviations \(sd(1)\) and \(sd(2)\) in LOP location should be expressed in a way that reflects the effects of LOP divergence. Shown in Fig. 10 are two ray path distances \(r(1)\) and \(r(2)\) between each of two transmitters and a LORAN-C receiver (located at \(R\)). Also shown is the line segment tangent to the hyperbolic LOP at \(R\). This segment bisects the angle 2\(B\) between ray paths \(r(1)\) and \(r(2)\). Because of instability, the LOP at \(R\) may be displaced a distance \(\text{DISP}\) to \(R'\); \(\text{DISP}\) can be approximately expressed in terms of a variation \(dr\) in ray path lengths \(r(1)\) and \(r(2)\):
\[
\text{DISP} = \frac{dr}{\sin B} \quad \text{(Eq. 20)}.
\]

The hyperbolic position line at \(R\) is defined by
\[
r(1) - r(2) = \text{constant} \quad \text{(Eq. 21)}.
\]

So the displaced LOP at \(R'\) is defined by
\[
(r(1)+dr)-(r(2)-dr) = r(1)-r(2) + 2dr = \text{constant} \quad \text{(Eq. 22)}.
\]

The distance 2\(dr\) can be expressed as a time difference error \(dt\):
\[
dt = 2dr/c \quad \text{(Eq. 23)}
\]

where \(c\) is the speed of light through the atmosphere.

Combining Eqs. 20 and 23 gives
\[
\text{DISP} = \frac{\sqrt{2}}{\sin B} \quad \text{(Eq. 24)}.
\]

Letting \(\text{DISP}\) be a random error in LOP location (such as \(e(1)\) or \(e(2)\) in Fig. 9) the standard deviation \(sd\) becomes
\[
sd[\frac{1}{\sqrt{2}}] \frac{\sum \text{DISP}^2}{\sin B} = \frac{\sqrt{2} e(1)^2}{\sin B} + \frac{\sqrt{2} e(2)^2}{\sin B} \quad \text{(Eq. 25)}
\]

where \(sd\) is the standard deviation of random time difference errors (also called the "system standard deviation").

Thus, assuming no correlation in random errors (Eq. 16). Eq. 19 (which approximates fix uncertainty) reduces to
\[
2d_{\text{rms}} = \frac{sd(1)}{\sin A} \left(\frac{1}{\sin^2 B(1)} + \frac{1}{\sin^2 B(2)}\right)^{1/2} \quad \text{(Eq. 26)}.
\]

The displacement \(\text{DISP}\) of an LOP can be expressed approximately in terms of the deviations \(dr\) in ray path lengths \(r(1)\) and \(r(2)\). (Taken from Ref. 15.)

**Fix Uncertainty Contours**

For a LORAN-C triad, the crossing angle \(A\) is the sum (or difference) of angles \(B(1)\) and \(B(2)\), as shown in Fig. 11. Thus, for a given location relative to the LORAN-C triad, the fix uncertainty 2\(d_{\text{rms}}\) can be computed using Eq. 26.

Fix uncertainty contours shown on LORAN-C reliability diagrams are derived by first solving Eq. 26 for each node in a spherical grid of latitudes and longitudes in the vicinity of the LORAN-C triad. Next, an interpolation scheme is used to determine the geographic coordinates for contours of 1500, 750, and 500 ft. Fix uncertainty. These coordinates are plotted, using a Lambert conformal conic projection, for each triad in a LORAN-C chain; the resulting fix uncertainty plots are composed for the entire LORAN-C chain.

It is assumed in these computations that the standard deviation of TD errors set is 0.1 usec.
A More Exact Method of Predicting Fix Uncertainty

The error due to random variations in LOP location does not occur as a normal distribution. The actual distribution of errors is described by the relation

$$95\% \text{ radial error} = K_{\text{rms}} = k \sqrt{1 \over A} \sum d^2 \quad (eq.27)$$

where $K$ varies between about 1.73 and 1.96 as a function of the crossing angle and the ratio $sd(1)/sd(2)$. Approximating the 95% radial error as 2.00 drms thus produces a pessimistic prediction of fix uncertainty.

In an approach described by Sitterly (Ref. 16) and investigated in detail by Hiraiwa (Refs. 17, 18) the coefficient $K$ is computed; this two-part process is described below.

(Step 1)

Shown in Fig. 12 are two LOP's intersecting at an angle $A$. The probability that a position fix, associated with the intersection of LOP(1) and LOP(2), will fall within the very thin parallelogram PQ is given by:

$$\text{probability} = \frac{1}{2\pi sd(1)sd(2)} \int_{-\infty}^{v(2)} \exp\left(-u^2 \over 2sd^2(1)\right) \int_{-\infty}^{v(1)} \exp\left(-v^2 \over 2sd^2(2)\right) dv du \quad (Eq. 28).$$

$sd(1)$ and $sd(2)$ are the standard deviations of LOP(1) and LOP(2) from their average locations. $du$ is the thickness of PQ in the direction perpendicular to LOP(1). Lengths $v(1)$ and $v(2)$ are perpendicular to LOP(2) and define the points P and Q. As shown in Fig. 12, $v(1)$ and $v(2)$ are defined, in terms of perpendicular distance $u$ from LOP(1), as

$$v(1) = u \cos A + \sqrt{Z^2 - u^2} \sin A \quad (Eq. 29)$$

$$v(2) = u \cos A - \sqrt{Z^2 - u^2} \sin A \quad (Eq. 30).$$

The probability that a position fix lies within a circle of radius $Z$ can thus be computed using Eqs. 28-30, numerically integrating over the interval $u(1)\rightarrow 0$, $u(2)\rightarrow Z$, and multiplying the results by 2. This is done for various values of $Z$, $sd(1)/sd(2)$, and crossing angle $A$ to determine which combinations of these values produce a probability of 95%.

(Step 2)

Using the same steps by which Eq. 19 is derived, Eq. 27 becomes (for Co)

$$Z = 95\% \text{ radial error} = K_{\text{rms}} \sqrt{sd(1)^2 + sd(2)^2} \quad (Eq. 31).$$

Eq. 31 is solved for $K$ using the various values of $Z$, $sd(1)/sd(2)$, and $A$ determined in Step 1. Shown in Fig. 13 is a graph of $K$ as a function of $sd(1)/sd(2)$ and $A$. 

A comparison of fix uncertainty values computed using $K_{\text{rms}}$ with those computed using $2d_{\text{rms}}$ is shown in Fig. 14.

Reliability diagrams produced in the future by DMA may depict fix uncertainty data computed using $K_{\text{rms}}$.

PRESENTATION OF DATA

Reliability Diagrams

Fix uncertainty and signal limit data are plotted for each LORAN-C chain on a 1:5,000,000 Lambert conformal conic projection of land-sea interfaces. No signal limits are shown outside the 1500 ft. fix uncertainty contour.
Difficulties and impossibilities in predicting signal limits and fix uncertainties

Actual signal limits and fix uncertainties often differ from predicted values, due to a variety of causes, including:

(a) our inability to predict either the occurrence, or the effects on signal propagation, of non-periodic phenomena such as thunderstorms and geomagnetic disturbances;
(b) our inability to accurately predict the effects of periodic phenomena, such as seasonal climate changes, on signal propagation;
(c) inaccuracies in predictions of noise level, groundwave field strength, and fix uncertainty due to assumptions made in the prediction models (for example, the system standard deviation will differ from the assumed 0.1 use, depending on the propagation conditions that exist at a given moment);
(d) limitations in availability and accuracy of data, such as ground conductivities, used in the prediction models.

In addition, variables such as the user's direction of travel affect signal limits and fix uncertainties observed by the user (a LORAN-C receiver may "lock on" to a signal close to the transmitter and track that signal far beyond its predicted limit).

In an effort to make reliability diagrams as accurate and useful as possible, I make the following recommendations:

(a) use the method described in Fig. 6 to predict groundwave signal limits;
(b) use Hiraiwa's method (Eq. 3c) to predict fix uncertainty;
(c) present fix uncertainty and signal limit data for single triads, rather than for entire chains;
(d) where data are available, show absolute accuracy information on reliability diagrams.

Appendix A. Currently Available LORAN-C Coverage and Reliability Diagrams

<table>
<thead>
<tr>
<th>DMA Stock Number</th>
<th>Area (GRI)</th>
</tr>
</thead>
<tbody>
<tr>
<td>WOBZ5130</td>
<td>LORAN-C Coverage Diagram</td>
</tr>
<tr>
<td>WOBZ5131</td>
<td>LORAN-A Coverage Diagram</td>
</tr>
</tbody>
</table>

Reliability Diagrams:
- Mediterranean Sea (7990)
- Norwegian Sea (7970)
- North Pacific (9990)
- Central Pacific (9990)
- Northwest Pacific (9970)
- North Atlantic (7930)
- Gulf of Alaska (7960)
- Canadian West Coast (5990)
- West Coast, U.S.A. (9940)
- Southeast U.S.A. (7980)
- Northeast U.S.A. (99960)
- Great Lakes (8970)
These diagrams are available through agents of the DMA Office of Distribution Services; addresses of those agents are listed in the Department of Commerce, Office of the U.S. Government Catalogue of Maps, Charts, and Related Products (Publication 1-5-1, DMA stock number CAT5111).

APPENDIX B. BREMMER'S FAR FIELD PREDICTION FORMULA

Bremmer's formula (Refs. 1, 9, 10, 19, 20, 21, 22) is used to predict the vertical field of a groundwave propagating along a smooth, spherical, electrically homogeneous earth. The strength of this field (in volts/meter) is calculated as

\[ |E_v| = |E(0)|^2 + |E(2)|^2 \]  

(Eq. 33)

where \( E(0) \) and \( E(2) \) are the real and imaginary components of the groundfield.

\[ E(0) = |-z_0 \frac{1}{2_0 \mu_0} \frac{1}{\sigma_0} \frac{1}{\omega} \sqrt{\pi} \int e^{i(k_0 r - \omega t)} e^{i k_0 z_0} \frac{d^2 I_0}{d r^2} \frac{d z_0}{r} \]  

(Eq. 34)

\[ E(2) = \frac{1}{2} \omega \mu_0 \frac{1}{k_0} e^{i k_0 z_0} \int \frac{d^2 I_0}{d r^2} \frac{d z_0}{r} \]  

(Eq. 35)

\[ E_v = \sqrt{E(0)^2 + E(2)^2} \]  

where \( E(0) \) and \( E(2) \) are the real and imaginary components of the groundwave.

\[ \sigma = \frac{\omega \mu_0}{k_0} \]  

\( k_0 = \sqrt{k_0^2 - k_0^2} \) is the wave number of the atmosphere.

\[ k_0 = \sqrt{k_0^2 - \omega^2 \mu_0 \epsilon_0} \]  

where \( \epsilon_0 \) is the permittivity of free space, \( \mu_0 \) is the permeability of free space, and \( \epsilon_0 \) is the permittivity of the earth. The relative permittivity of the earth is about 15 for land and 80 for sea.

\[ \sigma = \frac{\omega \mu_0}{k_0} \]  

\( \epsilon_0 \) is the conductivity of the earth (in \( \mu \) per meter). \( \epsilon_0 \) and \( \sigma \) vary as functions of weather, soil conditions, etc. Conductivity is a measure of the electrical "absorbancy" of a medium.

\[ \mu_0 = \text{permeability of free space} = 1.26 \times 10^{-6} \]  

Henry/meter.

REFERENCES


