THE BROADENING OF SPECTRAL LINES BY AUTOIONIZATION, RADIATIVE T---ETC(U)

V L JACOBS, J DAVIS

UNCLASSIFIED NRL-MR-4365
A comprehensive quantum mechanical theory of atomic spectral line shapes is developed within the framework of the Lippmann - Schwinger scattering theory adapted to the tetradic (Liouville) - space representation of density matrices. An expression for the spontaneous electric-dipole emission rate is derived which has the same general form as the results obtained by Baranger and by Kolb and Griem for overlapping lines. The frequency-dependent relaxation operator which occurs in this expression involves not only the collisional broadening interaction but also the interactions which give rise to autoionization and to both spontaneous and induced...
20. Radiative transitions. Explicit expressions for the widths are obtained in the isolated-line approximation by evaluating the lowest-order nonvanishing contribution to the diagonal matrix elements of the relaxation operator. The total width is obtained as the sum of the rates for all inelastic transitions from the initial and final states comprising the spectral line and the term involving the square of the difference between the elastic scattering amplitudes. Finally, we present a qualitative discussion on how the partial widths vary with the states comprising the spectral line and with the physical properties of the surrounding particles and radiation field.
CONTENTS

I. Introduction ................................................. 1

II. Density Matrix Description .................................. 3

III. Spontaneous Emission of Electric-Dipole Radiation .......... 9

IV. The Isolated Line Approximation ............................. 13

V. Relative Importance of Various Broadening Mechanisms .......... 23

VI. Acknowledgments ............................................ 30

VII. References ................................................ 31
I. INTRODUCTION

The shape of atomic spectral lines is seldom determined exclusively by the spontaneous radiative or autoionization processes. In addition to the Doppler effect, spectral lines can be appreciably broadened as a result of collisions with the surrounding particles, classical electromagnetic fields, and radiation fields. Although natural broadening is neglected in most theories of collision or pressure broadening, it is desirable to have a comprehensive theoretical framework in which all broadening mechanisms can be treated on an equal footing. Such a theory could be applied to calculate the shapes of satellite lines which are associated with resonance lines of multiply-charged ions in the x-ray emission spectra of high-temperature plasmas. Since the satellite lines are prominent features over a wide range of electron densities \(10^{10} - 10^{24} \text{ cm}^{-3}\), it is desirable to take into account their natural width due to spontaneous radiative decay and autoionization. Another application of a more general theory would be the simultaneous treatment of collision and radiation broadening.

Most treatments of collision or pressure broadening are based on the evaluation of the autocorrelation function of the dipole-moment operator. The modern quantum theory of spectral line broadening by plasmas has been developed from this point of view by Baranger\(^1-3\) and independently by Kolb and Griem\(^4\). Collision broadening by neutral gases has been treated by Fano\(^5\) using a variation of the autocorrelation-function approach which is directly applicable to other broadening mechanisms. The key quantity in Fano's treatment is the frequency-dependent relaxation operator introduced by Zwanzig\(^6\). This operator obeys a Lippman-Schwinger type equation which is defined in the Liouville-space
representation, in which density matrices are described by vectors. In the present investigation, we derive the general form of the overlapping line-shape formula by evaluating the Liouville-space expression for the spontaneous electric-dipole emission rate between states of the complete quantum mechanical system, which consists of the atomic system, the surrounding particles, and the radiation field. The key quantity in this treatment is the Liouville-space scattering operator, employed in the investigations of Ben-Reuven and co-workers, which is defined in terms of the Zwanzig relaxation operator.

The determination of the spectral line shape is reduced to the problem of evaluating the matrix elements of the relaxation operator, whose real and imaginary parts correspond to the line shifts and widths, respectively. In this investigation, the explicit expressions for the widths are obtained in the isolated-line approximation by evaluating the lowest-order nonvanishing contribution to the diagonal matrix elements of the relaxation operator. We show that the total width is the sum of the rates for all inelastic transitions from the initial and final atomic states comprising the spectral line and a term involving the square of the difference between the elastic scattering amplitudes. In a treatment of the isolated line shape problem based on the autocorrelation function, Lambropoulos obtained only the inelastic contributions and did not allow for autoionization processes.

The remainder of this paper is organized as follows: In section II, we review the general theory of transition probabilities within the framework of the density matrix description of atomic states. In section III, the spontaneous emission rate for electric-dipole radiation is evaluated and shown to have the same form as the general formula for overlapping lines. In section IV, explicit expressions are obtained for the widths associated with various broadening mechanisms in the isolated-line approximation. The variations of these widths
with the atomic states and with the properties of the surrounding particles and radiation field are discussed in section V.

II. DENSITY MATRIX DESCRIPTION

A very general theory of transition probabilities can be developed within the framework of the density matrix description of states in quantum mechanics. In order to describe radiative transitions of an atomic system which is interacting with the surrounding particles through collisions, it is necessary to first consider the total Hamiltonian

$$H = H^S + H^P + H^R + V^{SP} + V^{SR} + V^{PR},$$

where $H^S$ is the Hamiltonian for the isolated atomic system, $H^P$ is the Hamiltonian for the perturbing particles, and $H^R$ is the Hamiltonian for the free radiation field. The interaction between the atomic system and the surrounding particles is represented by $V^{SP}$, while the interaction between the atomic system and the radiation field is denoted by $V^{SR}$. The interaction $V^{PR}$, which describes radiation processes involving the perturbing particles, is usually neglected in the theory of spectral line shapes. However, a treatment of perturber radiation interfering with line radiation has been given by Burgess.

Eigenstates of $H^S$, $H^P$, and $H^R$ will be denoted by $|a⟩$, $|b⟩$, ..., $|p⟩$, $|q⟩$, ..., and $|n⟩$, $|m⟩$ ..., respectively. The index $p$ represents the quantum states of all the surrounding particles, and the index $n$ is understood to represent a set of photon occupation numbers, one for each mode of the radiation field. The notation can be further compressed by introducing the single index $α$ to represent $a$, $p$, $n$; and the index $β$ for $b$, $q$, $m$, etc. We shall assume that the zero-order Hamiltonian eigenvalue problem

$$H^0 |α⟩ = ω_α |α⟩,$$

where

$$H^0 = H^S + H^P + H^R,$$

...
can be solved. The solution of (2) can be most easily obtained if correlations between the perturbing particles are neglected. In the theory of spectral line broadening by plasmas\textsuperscript{13}, $|a\rangle$ and $|b\rangle$ are often taken to be eigenstates of the atomic system in the presence of the electric-microfield produced by the plasma ions, which is usually assumed to be time-dependent on the time scale of the radiative transition. In order to describe autoionization, part of the electrostatic interaction between the atomic electrons must be removed from $H^S$ and included in equation (6) below.

A general eigenstate of $H^O$ can be represented by the density operator

$$\rho_I = \rho_S \rho_P \rho_R$$  \hspace{1cm} (4)

for the initial state. The separable form implies that initially there are no correlations between the atomic system, the perturbing particles, and the radiation field. The matrices corresponding to $\rho_S$, $\rho_P$, and $\rho_R$ are assumed to be diagonal in the representations $a$, $p$, and $n$, respectively. For example, the atomic density matrix $\rho_S$ can then be expanded in the form

$$\rho_S = \sum_a \rho_a |a\rangle \langle a|.$$  \hspace{1cm} (5)

In order to describe the transitions between the eigenstates of $H^O$ which are induced by the interaction

$$\nu = \nu^{SP} + \nu^{SR} + \nu^{PR},$$  \hspace{1cm} (6)

it is necessary to investigate the time evolution of the density operator $\rho(t)$ for the complete interacting system. After introducing the Liouville-space operator

$$\mathcal{L} = \kappa^{-1} (H I^* - I H^*),$$  \hspace{1cm} (7)

the Schrödinger equation can be written in the form:\textsuperscript{5}
\[
\frac{d}{dt} \rho(t) = -i \mathbf{L} \rho(t). \tag{8}
\]

The matrix elements of equation (8) are given by
\[
\frac{d}{dt} \rho_{ab}(t) = -i \sum_{cd} \mathbf{L}_{ab,cd} \rho_{cd}(t) \tag{9}
\]

where
\[
\mathbf{L}_{ab,cd} = \hbar^{-1} \left( H_{ac} \delta_{bd} - \delta_{ac} H_{b}^* d \right). \tag{10}
\]

The asterisk denotes complex conjugation. The solution to equation (8) can be expressed formally by
\[
\rho(t) = \mathcal{U}(t,t_0) \rho(t_0), \tag{11}
\]

where \(\mathcal{U}(t,t_0)\) is the Liouville time-evolution operator defined by
\[
\mathcal{U}(t,t_0) = \exp \left[ -i \mathbf{L} (t-t_0) \right]. \tag{12}
\]

In order to define the transition probability, it is necessary to introduce the final state projection operator \(P_f\). The spontaneous emission of a single photon with wave number \(k\) and polarization \(\lambda\) can be described by the final state projection operator
\[
P_f = \sum_b \left| b \right> \left< b \right| \times \left| 1_k^+ \lambda \right> \left< 1_k^+ \lambda \right| \times I_B, \tag{13}
\]

where \(I_B\) is the unit operator in all other degrees of freedom which will be treated as bath states, to be defined below. The probability per unit time for the transition \(i \rightarrow f\) can then be expressed by
\[
A(i \rightarrow f) = \lim_{t \to \infty} \frac{d}{dt} \lim_{t_0 \to -\infty} \text{Tr} \left\{ P_f \mathcal{U}(t,t_0) \rho_i(t_0) \right\}. \tag{14}
\]
which is the Liouville-space analogue of the definition given by Goldberger and Watson 14.

Equation (14) provides a general description of a collision process in which the asymptotic states are eigenstates of the Hamiltonian $H^0$ for the non-interacting systems. It has been pointed out 9 that the same description can be applied to relaxation processes which involve an initial stationary state (e.g., a state with only diagonal non-zero density matrix elements).

The asymptotic time limits can be taken to obtain the steady-state transition rate 10

$$A(i + f) = -i \operatorname{Tr} \left\{ P_f \, \mathcal{K}(i0) \, \rho_i \right\}. \quad (15)$$

The Liouville operator $\mathcal{K}(z)$ is the tetradic-space analogue of the $T$ matrix occurring in ordinary scattering theory. The Liouville-space analogue of the Lippman-Schwinger equation can be written in the form

$$\mathcal{K}(z) = V + V \mathcal{L}(z) \, V, \quad (16)$$

where $\mathcal{L}(z)$ is the Green's operator

$$\mathcal{L}(z) = (z - \mathcal{L})^{-1}. \quad (17)$$

The Liouville operator $\mathcal{L}$ has been divided according to

$$\mathcal{L} = \mathcal{L}^0 + V, \quad (18)$$

where $\mathcal{L}^0$ and $V$ are the Liouville operators corresponding to $H^0$ and $V$, respectively. The limit $z \to 0$ of the complex variable $z$ in equation (15) corresponds to reaching the origin from above the real axis. The initial state density operator $\rho_i$ and the final state projection operator $P_f$ in equation (15) must correspond to stationary states of $\mathcal{L}^0$, i.e.

$$\mathcal{L}^0 \, \rho_i = \mathcal{L}^0 \, P_f = 0 \quad (19)$$
In the density matrix formulation of relaxation processes\textsuperscript{6,9,10}, the transition amplitude is expressed in terms of a scattering operator $\chi^r(z)$, which is defined on a subspace of states incorporating only the interaction between the uncorrelated atomic system and the relevant mode $k^+\lambda$ of the radiation field. The scattering operator $\chi^r(z)$ is then explicitly expanded in a power series of the relevant interaction $U^r$. The remaining interactions are introduced as relaxation effects by a self-energy operator. The distinction between the "free" states of the relevant degrees of freedom and the states in which they are correlated to the remaining (bath) degrees of freedom can be formally achieved by using the Liouville-space projection operators $P$ and $Q = 1 - P$ which were introduced by Zwanzig\textsuperscript{6}. These projection operators play a role similar to that of the dyadic projection operators introduced in the theory of nuclear reactions by Feshbach\textsuperscript{15}. Their precise definition will be given below later.

The relevant part of the interaction can be defined by

$$\mathcal{U}^r = P U^SR_{k^+\lambda} P.$$  \hspace{1cm} (20)

The relevant scattering operator $\chi^r(z)$ obeys the Lippmann-Schwinger type equation

$$\chi^r(z) = \mathcal{U}^r + \mathcal{U}^r \mathcal{H}^r(z) \mathcal{U}^r,$$  \hspace{1cm} (21)

where

$$\mathcal{H}^r(z) = \left[ z - P^L P - P R(z) P \right]^{-1}. \hspace{1cm} (22)$$

The self-energy operator $\mathcal{R}(z)$ is given by

$$\mathcal{R}(z) = \mathcal{U} + \mathcal{U} Q (z - Q L Q)^{-1}. \hspace{1cm} (23)$$

Equations (22) and (23) are analogous to the results obtained for the ordinary (dyadic) Greens operator by Mower\textsuperscript{16}. The transition rate can be evaluated in terms of $\chi^r(+\iota 0)$ as follows\textsuperscript{10}. 

-7-
The Zwanzig projection operator can be defined in the double-space notation introduced by Baranger\textsuperscript{1-3} as follows:

\[ \mathcal{P} = | \rho_B \rangle \langle I_B |, \]  

(25)

where B denotes the bath degrees of freedom. The effect of \( \mathcal{P} \) on a Liouville operator is to average it over these degrees of freedom. The double-space vectors in the subspace spanned by \( \mathcal{P} \) will be denoted by \( |a_b, n m \rangle \rangle \), where \( |a_b \rangle \rangle \) represents the operator \( |a \rangle \langle a| \) in the space of eigenstates of \( H^S \) and \( |n m \rangle \rangle \) represents \( |n \rangle \langle m| \) in the space of relevant photon occupation number states. The initial state density operator \( \rho^r_i \) and the final state projection operator \( P^r_f \), which are defined on the relevant degrees of freedom, can be expanded in the form

\[ \rho^r_i = \sum_a \rho_a |a a, 0 0 \rangle \rangle \]  

(26)

and

\[ P^r_f = \sum_b |b b, 1 1 \rangle \rangle. \]  

(27)

The single-photon spontaneous emission rate can now be expressed in terms of the matrix elements of \( \mathcal{T}^r(+ i 0) \) as follows:

\[ A(i \rightarrow f) = -i \langle \langle P_f | \mathcal{T}^r(+ i 0) | \rho_i \rangle \rangle \]

\[ = -i \sum_a \sum_b |b b, 1 1 \rangle \langle \langle a a, 0 0 | \mathcal{T}^r(+ i 0) | a a, 0 0 \rangle \rangle \rho_a. \]  

(28)

The summations over \( a \) and \( b \) are to be taken over all bound and continuum states of the atomic system. In practice, these summations are restricted to include only a few discrete states.
III. SPONTANEOUS EMISSION OF ELECTRIC-DIPOLE RADIATION

In this section, we evaluate the spontaneous emission rate for single-photon electric-dipole radiation. The objective of this evaluation is to demonstrate that the formula for overlapping lines\textsuperscript{1-5}, which is usually obtained by starting with the autocorrelation function of the electric-dipole moment operator, can be derived from the general quantum mechanical expression for the transition rate in the Liouville-space representation of density matrices. Alternative density matrix formulations of the line shape problem have been given by Cooper\textsuperscript{17} and Davis\textsuperscript{18}. In these formulations the time dependence of the density matrix is treated directly.

Since the second term in equation (21) gives the only nonvanishing contribution to the scattering operator $\mathcal{T}^r(z)$, the rate for the spontaneous emission of dipole radiation is given by

$$A_r(\omega) = -i \sum_a \sum_b \rho_a \rho_b$$

$$X \left\langle bb, 1 | \mathcal{U}^r \mathcal{H}^r(\pm \Omega) \mathcal{U}^r | aa, 0 0 \right\rangle$$

$$= -i \sum_a \sum_b \rho_a \sum_c \sum_d \sum_e \sum_f \rho_a \rho_b \rho_c \rho_d \rho_e \rho_f$$

$$X \left\langle bb, 1 | \mathcal{U}^{SR}_{k\lambda} | cd, n_c n_d \right\rangle$$

$$X \left\langle cd, n_c n_d | \mathcal{H}^r(\pm \Omega) | ef, n_e n_f \right\rangle$$

$$X \left\langle ef, n_e n_f | \mathcal{U}^{SR}_{k\lambda} | a a, 0 0 \right\rangle.$$
where all summations over the relevant degrees of freedom are indicated explicitly in the second version.

The matrix elements of the Liouville interaction operator \( V_{SR}^{k \lambda} \) can be evaluated using the definition which follows from equation (7). The matrix elements of the dyadic interaction operator \( V_{SR}^{k \lambda} \) are evaluated using the expansion

\[
V_{SR}^{k \lambda} | \text{a, n} \rangle = \sum_{b} \left\{ V_{ba} (k) \sqrt{n} | \text{b, n - 1} \rangle + V_{ba} (-k) \sqrt{n + 1} | \text{b, n + 1} \rangle \right\},
\]

where

\[
V_{ba} (k) = \frac{e}{m} \sqrt{\frac{2 \pi \hbar}{\omega}} \left\langle b \mid \sum_{j} \hat{\epsilon} \cdot \hat{p}_{j} e^{i \hat{k} \cdot \hat{r}} \mid a \right\rangle.
\]

We then obtain the result

\[
\langle \langle e_f, n_e n_f | U_{SR}^{k \lambda} | a a, 0 0 \rangle \rangle =
\]

\[
\mathcal{N}^{-1} \left\{ V_{ea} (-k) \delta_{n_e, 1} \delta_{f, a} \delta_{n_f, 0} - \delta_{e, a} \delta_{n_e, 0} V_{fa} (-k) \delta_{n_f, 1} \right\}.
\]

We obtain a similar expression for \( \langle \langle b b, 1 1 | U_{SR}^{k \lambda} | c d, n_c n_d \rangle \rangle \), except for the presence of terms corresponding to \( n_c \) or \( n_d = 2 \). These terms can be neglected because they do not contribute to the dominant spontaneous emission process.

The frequency-dependent transition rate \( A_{\lambda}(\omega) \) given by equation (29) can now be written as a sum of four terms. These terms may be combined in pairs by utilizing the Hermitian property

\[
V_{ab} (k) = V_{ba}^{*} (k)
\]
and the Liouville conjugation symmetry relationship\textsuperscript{9}

\[
\mathcal{H}_{ab, cd}^{*}(z) = \mathcal{H}^{*}_{ba, dc}(-z),
\]

(34)

which is a consequence of the microscopic reversibility of the time evolution operator under time reversal.

The frequency-dependent transition rate can now be written in the form

\[
A_{\omega}(\omega) = -\frac{2}{\hbar} \text{Im} \sum_{a} \sum_{b} \sum_{c} \sum_{d} V_{db}(k) \mathcal{H}^{(k)}_{bd, ac}(\omega) V_{ac}(k) \rho_{a}
\]

(35)

\[
= -\frac{2}{\hbar} \text{Im} \text{Tr} \left[ V(k) \mathcal{H}(\omega) \rho V(k) \right],
\]

where the reduced Green's operator \( \mathcal{H}(\omega) \) is defined by

\[
\mathcal{H}_{bd, ac}(\omega) = \left\langle \left. bd, 0 \left| \mathcal{H}^{(+i\omega)} \right| ac, 0 \right\rangle \right|
\]

(36)

\[
= \left\langle \left. bd, 1 \left| \mathcal{H}^{(+i\omega)} \right| ac, 0 \right\rangle \right|

\]

The second term, in which the photon occupation numbers are interchanged in the bra, does not give a sharp resonant contribution to the spectral line shape and will therefore be omitted.

Using the relationship

\[
\mathcal{F} L^{0} \mathcal{P} | ab, nm \rangle = \left[ \omega_{a} - \omega_{b} + (n-m) \omega \right] | ab, nm \rangle
\]

(37)

and neglecting the relaxation operator \( \mathcal{R} \), the reduced tetradic Greens operator generally defined by equation (36) has matrix elements given by

\[
\mathcal{H}_{bd, ac}(\omega) = \lim_{n \to 0} (\omega_{c} - \omega_{a} + \omega + i n)^{-1} \delta_{b \delta_{a}} \delta_{d c}.
\]

(38)

Consequently, the frequency-dependent transition rate in the absence of relaxation effects simplifies to
\[ A_{\gamma}(\omega) = \frac{2\pi}{\hbar^2} \sum_{a} \sum_{b} | V_{ab}(\vec{k}) |^2 \rho_a \delta(\omega - \omega_a + \omega_b), \quad (39) \]

which is the familiar Fermi Golden Rule formula for spontaneous emission from the levels \( a \) to the levels \( b \).

In order to obtain the differential transition rate for the emission of a photon per unit solid angle and angular frequency intervals, equation (35) must be multiplied by the density of final states factor \( \omega^2/(2\pi c)^3 \). This factor would have been introduced automatically if the final state projection operator (27) had been more carefully defined to take into account the continuous distribution of modes of the radiation field. Using the electric-dipole approximation

\[ V_{ab}(\vec{k}) = -i \sqrt{2\pi \hbar \omega} \left\langle b | \vec{D}_\lambda | a \right\rangle, \quad (40) \]

where \( D_\lambda \) denotes the component of the total atomic dipole-moment operator in the direction of polarization, the differential photon emission rate for a given polarization \( \lambda \) is obtained in the form

\[ A_{\gamma}^\lambda(\omega, \Omega) = \frac{\omega^3}{2\pi \hbar c^3} L_\lambda(\omega, \Omega). \quad (41) \]

The line shape function \( L_\lambda(\omega, \Omega) \), which depends on the angles \( \Omega \) of photoemission, is given by

\[ L_\lambda(\omega, \Omega) = -\frac{1}{\pi} \text{Im Tr} \left[ D_\lambda \mathcal{H}(\omega) \rho D_\lambda \right]. \quad (42) \]

Usually, one is interested only in the total spontaneous emission rate per unit frequency interval

\[ A_{\gamma}(\omega) = \sum_{\lambda} \int d\Omega A_{\gamma}^\lambda(\omega, \Omega) = \frac{4}{3} \frac{\omega^3}{\hbar c^3} L(\omega), \quad (43) \]

where

\[ L(\omega) = -\frac{1}{\pi} \text{Im Tr} \left[ \vec{D} \cdot \mathcal{H}(\omega) \rho \cdot \vec{D} \right]. \quad (44) \]
Equation (44) has the same form as the results derived by Baranger, by Kolb and Griem, and by Fano. Note, however, that equation (44) is more general than the formula usually employed for overlapping lines in that no restriction has as yet been imposed on the subspace of the atomic states over which the trace is to be taken. Consequently, equation (44) provides a general framework for the description of the entire emission spectrum due to all bound-bound transitions, including transitions involving autoionizing levels.

IV. The Isolated Line Approximation

If only the diagonal matrix elements $\mathcal{H}_{ab, ab}(\omega)$ of the reduced Green's operator $\mathcal{H}(\omega)$ are included in the line shape formula (44), we obtain

$$L(\omega) = -\frac{1}{\pi} \text{Im} \sum_a \sum_b \frac{|\tilde{D}_{ab}|^2 \rho_a}{\omega - \omega_a + \omega_b - \mathcal{R}_{ab, ab}(\omega)},$$

(45)

where

$$\mathcal{R}_{ab, ab}(\omega) = \left\langle a b, 0 1 \left| \mathcal{P} \mathcal{R}(+i0) \mathcal{P} \right| a b, 0 1 \right\rangle.$$

(46)

From the definition of the tetradic relaxation operator $\mathcal{R}(z)$ given by equation (23), it follows that

$$\mathcal{R}^+(z) = \mathcal{R}(z^*)$$

(47)

where the Hermitian conjugation of $\mathcal{R}$ is defined by

$$\left( \mathcal{R}^+ \right)_{\alpha\beta, \alpha'\beta'} = \mathcal{R}^{*}_{\alpha'\beta', \alpha\beta}.$$

(48)

Consequently, the procedure of Goldberger and Watson can be followed to obtain
\[
\lim_{\eta \to 0} \mathcal{R}(x + i \eta) = \Delta(x) + i \Gamma(x) |z, (49)
\]

where \(\Delta(x)\) and \(\Gamma(x)\) are Hermitian operators. \(\mathcal{R}_{ab,ab}^{(\omega)}\) can therefore be expressed in the form

\[
\mathcal{R}_{ab,ab}^{(\omega)}(\omega) = \Delta_{ab,ab}^{(\omega)} \Gamma_{ab,ab}^{(\omega)} |z, (50)
\]

where \(\Delta_{ab,ab}^{(\omega)}\) gives the shift associated with the line \(a \rightarrow b\) and \(\Gamma_{ab,ab}^{(\omega)}\) corresponds to the full-width at half maximum.

It is clear that equation (45) corresponds to an incoherent superposition of the Lorentzian profiles associated with each atomic line \(a \rightarrow b\). The isolated line approximation is expected to be valid provided that the separations of the lines are large compared with their widths. In this section, we evaluate the lowest-order nonvanishing contributions to the widths arising from autoionization, spontaneous and induced radiative transitions, and collisions.

The width and shift operators \(\Gamma(x)\) and \(\Delta(x)\) can be expressed in terms of the relaxation operator \(\mathcal{R}(x + i \eta)\) by following the procedure used by Goldberger and Watson\(^{14}\). From the definition (23) and the identity

\[
(z - \mathcal{Q} \mathcal{Q}^{-1}) \gamma \mathcal{R} = (z - \mathcal{Q} \mathcal{Q}^{-1}) \mathcal{R}(z), (51)
\]

which is analogous to the dyadic relationship used by Mower\(^{15}\), we obtain the results

\[
\Gamma(x) = 2\pi \lim_{\eta \to 0} \mathcal{R}(x + i \eta) = \frac{\mathcal{K}^{\pm}(x + i \eta) \gamma \delta(x - \mathcal{Q} \mathcal{Q}^{-1}) \mathcal{Q} \mathcal{R}(x + i \eta)}{(x + i \eta)} (52)
\]

and

\[
\Gamma(x) = \mathcal{K}_{e} \gamma - \frac{P}{2\pi} \int_{-\infty}^{\infty} \frac{\Gamma(x') dx'}{x' - x}, (53)
\]

where \(P\) denotes the Cauchy principal value. Similar results have also been derived by Lambropoulos\(^{10}\).
The lowest-order nonvanishing contribution to the width operator \( \Gamma(x) \), which is obtained from the approximation of \( \mathcal{R}(z) \) by \( \mathcal{U} \), is

\[
\Gamma(z) = 2\pi \mathcal{U} \mathcal{Q} \delta(x - Q \mathcal{L}^0 Q) \mathcal{Q} \mathcal{U}.
\] (54)

Using the compressed notation introduced in section II for the eigenstates of \( \mathcal{L}^0 \), the general matrix elements of \( \Gamma(x) \) can be expressed in the form

\[
\left\langle \left\langle \alpha \beta \mid \Gamma(x) \mid \alpha' \beta' \right\rangle\right\rangle = \frac{2\pi}{\hbar^2} \sum_{\alpha'' \beta''} \delta(x - \omega_{\alpha''} + \omega_{\beta''})
\]

\[
x \left\langle \left\langle \alpha \beta \mid \mathcal{V} \mathcal{I}^* - \mathcal{I} \mathcal{V}^* \mid \alpha'' \beta'' \right\rangle\right\rangle \left\langle \left\langle \alpha'' \beta'' \mid \mathcal{V} \mathcal{I}^* - \mathcal{I} \mathcal{V}^* \mid \alpha' \beta' \right\rangle\right\rangle
\]

\[
= \frac{2\pi}{\hbar^2} \sum_{\alpha''} \mathcal{V} \delta_{\alpha \alpha''} \mathcal{V} \mathcal{I} \delta_{\beta' \beta''} \delta(x - \omega_{\alpha''} + \omega_{\beta''})
\]

\[
+ \frac{2\pi}{\hbar^2} \sum_{\beta''} \mathcal{V}^* \delta_{\beta \beta''} \mathcal{V}^* \mathcal{I} \delta_{\alpha \alpha''} \delta(x - \omega_{\alpha} + \omega_{\beta''})
\]

\[
- \frac{2\pi}{\hbar^2} \mathcal{V} \delta_{\alpha \alpha'} \mathcal{V}^* \mathcal{I} \delta_{\beta \beta'} \delta(x - \omega_{\alpha} + \omega_{\beta'});
\] (55)

where we have made the approximation \( Q = 1 \), discussed by Ben-Reuven 19.

To obtain the line width, we must evaluate the expression

\[
\Gamma_{ab, ab}(\omega) = \left\langle \left\langle a b, 0 1 \mid \mathcal{P} \Gamma (+10) \mathcal{P} \mid a b, 0 1 \right\rangle\right\rangle, \quad (56)
\]
It is now necessary to introduce the bath degrees of freedom \( n \) and \( p \), where \( n \) represents the set of all photon occupation numbers and \( p \) is the quantum state of the perturbing particles. The frequency-dependent width can then be expressed in the form

\[
\Gamma_{ab, a'b'}(\omega) = \sum_n \sum_{n'} \sum_p \sum_{p'} \langle a b, 0 1, n n' p p' | \Gamma(0) | a b, 0 1, n' n p p \rangle \rho_n^p \rho_{n'}^p.
\] (57)

It is necessary to make the somewhat artificial distinction between the relevant and bath photon modes in order to obtain the frequency-dependence, which occurs in the delta functions in equation (55). In subsequent equations, reference to the relevant photon mode numbers will be omitted whenever it becomes unnecessary.

After substituting equation (55) into equation (57), the lowest-order non-vanishing contribution to the isolated line width can be obtained in the form

\[
\Gamma_{ab, a'b'}(\omega) = \frac{2\pi}{\hbar^2} \sum_{a''} \sum_{a'''} \sum_p \sum_{p''} \left| \langle a n p | \mathcal{V} | a'' n'' p'' \rangle \right|^2 \times \delta(\omega - \omega_{a''} + \omega_{a'''} - \omega_p + \omega_{n''} - \omega_{n'''} + \omega_{p''}) \rho_n^p \rho_{n''}^{p''} \\
+ \frac{2\pi}{\hbar^2} \sum_{b''} \sum_{b'''} \sum_p \sum_{p''} \left| \langle b n p | \mathcal{V} | b'' n'' p'' \rangle \right|^2 \times \delta(\omega - \omega_{a} + \omega_{b''} - \omega_{p} + \omega_{n''} - \omega_{n'''} + \omega_{p''}) \rho_n^p \rho_{p''} \\
+ \frac{2\pi}{\hbar^2} \sum_p \sum_{p'} \left| \langle a_p \mathcal{V} a_{p'} \rangle - \langle b_p \mathcal{V} b_{p'} \rangle \right|^2 \times \delta(\omega - \omega_a + \omega_b - \omega_p + \omega_p) \rho_{p'} \\
+ \frac{2\pi}{\hbar^2} \sum_p \sum_{p'} \left| \langle a_p \mathcal{V} a_{p'} \rangle - \langle b_p \mathcal{V} b_{p'} \rangle \right|^2 \times \delta(\omega - \omega_a + \omega_b - \omega_p + \omega_p) \rho_{p'}.
\] (58)
The first two terms describe all inelastic transitions from the initial and final states of the radiating atomic system. Note that the states of the radiation field are not involved in the last term, which describes only elastic collisions. This is a consequence of the assumption that photons are emitted only by the atomic system and the neglect of correlations between the atomic system and the perturbers. The radiation emitted by the perturbers can be described by the use of higher-order perturbation theory.

A. Autoionization

Autoionization results from the electrostatic interaction $V^S$ between a discrete state and the continuum states of the atomic system having the same energy. The definition of the zero-order atomic states can be made in an unambiguous manner by using the dyadic projection operators introduced by Feshbach $^{15}$ or by the alternative procedure introduced by Fano $^{20}$. Autoionization can be included in the present theory by transferring $V^S$ from $H^0$ to $V$, as discussed in Section II. The width arising from the possibility of autoionization from both the initial and final states of the radiating atomic system is obtained from the inelastic terms in equation (58) and can be written as

\[
\Gamma^{A}_{ab,ab}(\omega) = \frac{2\pi}{\hbar^2} \sum_{c} |\langle a | V^S | c \rangle|^2 \delta(\omega - \omega_c + \omega_b)
\]

\[
+ \frac{2\pi}{\hbar^2} \sum_{c} |\langle b | V^S | c \rangle|^2 \delta(\omega - \omega_a + \omega_c),
\]

where the sum over $c$ ranges over all continuum states which are allowed by the energy and symmetry selection rules. If we now make the approximation $\omega = \omega_a - \omega_b$, in which the frequency-dependence is neglected, we obtain the result
\[ \Gamma_{a,b}^{A} = \frac{2\pi}{\hbar^2} \sum_{c} \left| \langle a | v^{S} | c \rangle \right|^2 \delta(\omega_{a} - \omega_{c}) + \frac{2\pi}{\hbar^2} \sum_{c} \left| \langle a | v^{S} | c \rangle \right|^2 \delta(\omega_{b} - \omega_{c}), \]

which is the sum of the autoionization rates from the initial and final atomic levels. In its most general form, equation (60) gives the autoionization contribution to the line width for a radiative transition between two autoionizing levels a and b.

It is well-known that the mixing between the discrete and continuum states produces an asymmetric profile. However, to the best of the authors' knowledge, no theories have been developed which incorporate the effects of radiative transitions and collisions into the asymmetric line shape function. In the limit where the line profile parameter \( q \) introduced by Fano becomes large, the line shape function approaches a Lorentzian function, which is generalized in the present investigation in order to treat all broadening mechanisms on an equal footing.

B. Radiative Transitions

The width arising from all radiative transitions from the states a and b which is obtained from the inelastic terms in equation (58) is given by

\[ \Gamma^{(R)}_{a,b} = \frac{2\pi}{\hbar^2} \sum_{a''} \sum_{a'''} \sum_{n''} \sum_{n''} \left| \langle a | v^{SR} | a'' n'' \rangle \right|^2 \]

\[ \times \delta(\omega_{a''} + \omega_{a'''} - \omega_{n''} - \omega_{n''}) \rho_{n} \]

\[ + \frac{2\pi}{\hbar^2} \sum_{b''} \sum_{b'''} \sum_{n''} \sum_{n''} \left| \langle b | v^{SR} | b'' n'' \rangle \right|^2 \]

\[ \times \delta(\omega_{b''} + \omega_{b'''} - \omega_{n''} + \omega_{n''}) \rho_{n} \]
We now introduce the expansion of the interaction $V^{SR}$ in terms of modes $k\lambda$ of the radiation field and replace the summation over $k$ in this expansion by integrations over the frequency $\omega_k$ and the solid angle $\Omega_k$. In the electric-dipole approximation, the total radiative contribution to the width is given by

\[
\Gamma_{ab,ab}^{R}(\omega) = \frac{1}{2\pi \hbar c^3} \sum_{\lambda} \int \int \omega_k \, d\omega_k \, d\Omega_k \left\{ \sum_{a'' \neq a} \left| \langle \mathbf{r} | D_{\lambda} | a'' \rangle \right|^2 \left( \omega_a - \omega_{a''} \right)^2 \left( \delta(\omega - \omega_{a''} + \omega_b - \omega_k) \right) \right. \\
+ \left. \left( \langle n_{\lambda} (\omega_k, \Omega_k) \right) \delta (\omega - \omega_a + \omega_b - \omega_k) \right. \\
+ \left. \langle n_{\lambda} (\omega_k, \Omega_k) \right) \delta (\omega - \omega_a + \omega_b + \omega_k) \right. \\
+ \left. \sum_{b'' \neq b} \left| \langle \mathbf{r} | D_{\lambda} | b'' \rangle \right|^2 \left( \omega_b - \omega_{b''} \right)^2 \left( \delta(\omega - \omega_a + \omega_{b''} + \omega_k) \right) \right. \\
+ \left. \langle n_{\lambda} (\omega_k, \Omega_k) \right) \delta (\omega - \omega_a + \omega_{b''} + \omega_k) \right. \\
+ \left. \langle n_{\lambda} (\omega_k, \omega_k) \right) \delta (\omega - \omega_a + \omega_{b''} - \omega_k) \right. \right\},
\]

where $\langle n_{\lambda} (\omega_k, \Omega_k) \rangle$ denotes the average number of photons per unit frequency, per unit solid angle, which have polarization $\lambda$.

The contribution from all spontaneous radiative transitions can be obtained from the terms which are independent of the average number of photons by performing the integration over $\Omega_k$ and the sum over $\lambda$. If we now make the approximation $\omega = \omega_a - \omega_b$, we obtain the result.
where

\[ \Theta(x) = \begin{cases} 1 & x > 1 \\ 0 & x < 1 \end{cases} \]  

(64)

which is the sum of all spontaneous emission rates out of the states a and b comprising the spectral line.

The contribution from all induced radiative transitions is conventionally expressed in terms of the specific intensity \( I_\lambda(\omega) \) which is related to the average number of photons by

\[ \langle n_\lambda(\omega_\lambda) \rangle = \frac{8\pi^2 c^2}{h \omega^3} I_\lambda(\omega_\lambda). \]  

(65)

If we again make the approximation \( \omega = \omega_a - \omega_b \), we obtain the result

\[ r^R_{ab} = \frac{4\pi^2}{\hbar^2 c} \sum_\lambda \oint d\eta_k \left[ \langle a | D | a'' \rangle \right]^2 \left[ I_\lambda(\omega_a - \omega_a', \eta_k) \Theta(\omega_a - \omega_a') + I_\lambda(\omega_a' - \omega_a, \eta_k) \Theta(\omega_a' - \omega_a) \right] \]

\[ + \sum_{b'' \neq b} \left| \langle b | D | b'' \rangle \right|^2 \left[ I_\lambda(\omega_b - \omega_b', \eta_k) \Theta(\omega_b - \omega_b') + I_\lambda(\omega_b' - \omega_b, \eta_k) \Theta(\omega_b' - \omega_b) \right], \]

(66)

which gives the contribution to the width from all stimulated radiative emission and absorption processes out of the states a and b. Results similar to those
given by equations (63) and (66) have also been obtained by Lambropoulos\textsuperscript{11}. The starting point in his work is the autocorrelation function.

C. Collisions

The contribution to equation (58) from collisions between the perturbers and the radiating atomic system is given by

\[ \Gamma^{c}_{a b, a' b'} (\omega) = \frac{2\pi}{\hbar^2} \sum_{a''} \sum_{p} \sum_{p''} \left| \left\langle a \left| V^{SP} \right| a'' p'' \right\rangle \right|^2 
\times \delta (\omega - \omega_a + \omega_{b'} - \omega_{p'} + \omega_{p'\prime}) \rho_{p'\prime} \rho 
+ \frac{2\pi}{\hbar^2} \sum_{b''} \sum_{p} \sum_{p''} \left| \left\langle b \left| V^{SP} \right| b'' p'' \right\rangle \right|^2 
\times \delta (\omega - \omega_a - \omega_{b'} - \omega_{p'} + \omega_{p'\prime}) \rho_{p'\prime} \rho 
+ \frac{2\pi}{\hbar^2} \sum_{p} \sum_{p'} \left| \left\langle a \left| V^{SP} \right| a_{p'\prime} \right\rangle - \left\langle b \left| V^{SP} \right| b_{p'\prime} \right\rangle \right|^2 
\times \delta (\omega - \omega_a - \omega_{b'} - \omega_{p'} + \omega_{p'\prime}) \rho_{p'\prime} \rho . \]

(67)

We now treat electron collisions in the binary-collision approximation, and set

\[ \omega = \omega_a - \omega_b, \]

which corresponds to the impact approximation in the theory of spectral line broadening\textsuperscript{13}.

It is conventional in electron-atom collision theory to introduce the inelastic scattering cross section.
\[ \sigma (a \rightarrow a'', \mathbf{p}) = \frac{2\pi}{\hbar^2 V_e} \sum_{\mathbf{p}''} \left| \left\langle a \mathbf{p} \Big| V^c | a'' \mathbf{p}'' \right\rangle \right|^2 \delta (\omega_a - \omega_a'' - \omega_{p''} + \omega_p) \] (68)

and the elastic scattering amplitude
\[ f_a (\mathbf{p}, \mathbf{p}'') = -\frac{m_e}{2\hbar^2} \left\langle a \mathbf{p} \bigg| V^c \bigg| a'' \mathbf{p}'' \right\rangle, \] (69)

where \( \mathbf{p} \) denotes the wave vector describing the electron scattering by the atomic system, and \( V^c \) is the appropriate collisional interaction. These quantities can be defined without the use of lowest-order perturbation theory by replacing \( V^c \) by the scattering matrix \( T \).

The isolated-line width produced by electron collisions is then obtained in the form
\[ \Gamma^c_{ab} = N_e \sum_{\mathbf{p}} \rho (\mathbf{p}) \left( \sum_{a''} V_e \sigma (a'' \rightarrow a, \mathbf{p}) + \sum_{a''} V_e \sigma (b \rightarrow a'', \mathbf{p}) \right) \]
\[ + V_e \int d\Omega \sum_{\mathbf{p}''} \left| f_a (\mathbf{p}, \mathbf{p}'') - f_b (\mathbf{p}, \mathbf{p}'') \right|^2, \] (70)

where \( N_e \) is the electron density and \( V_e \) is the electron velocity. The density matrix \( \rho (\mathbf{p}) \) corresponds to the electron velocity distribution. This result is in agreement with the work of Baranger 1-3. Note that the total width due to collisions is the sum of the rates for all inelastic transitions and a term involving the square of the difference between the elastic scattering amplitudes.

We have shown that the approximation of \( R(z) \) by \( V \) implies that the isolated line width can be obtained by adding up the partial widths arising from auto-ionization, spontaneous and induced radiative transitions, and collisions, as expressed by equations (60), (64), (66), and (70).
\[ \Gamma_{ab,ab} = \Gamma_{ab,ab}^A + \Gamma_{ab,ab}^{SR} + \Gamma_{ab,ab}^{IR} + \Gamma_{ab,ab}^C \]  

(71)

Although the final results have been worked out explicitly in the approximation \( \omega = \omega_a - \omega_b \), it should be emphasized that the general theory predicts that each partial width will have a frequency dependence.

5. RELATIVE IMPORTANCE OF VARIOUS BROADENING MECHANISMS

In order to determine the relative importance of various broadening mechanisms for a particular spectral line, the expressions obtained for the isolated line widths in section IV must be evaluated using realistic atomic wavefunctions. In this section, we discuss approximate evaluations of the widths which reveal their dependence on the states \( a \) and \( b \) comprising the line and on the physical properties of the surrounding particles and radiation field. The shifts, which are not considered in this investigation, may be important, particularly in the case of induced transitions.

Even when the atomic lines are assumed to be isolated, the upper and lower levels \( a \) and \( b \) usually consist of degenerate magnetic sublevels. The result of superimposing the contributions from each allowed magnetic component can be shown\(^{13,17}\) to be the same as the expression obtained from equation (45) after both \( |\mathbf{b}_{ab}|^2 \) and \( \kappa_{ab,a,b}(\omega) \) are summed over the magnetic sublevels of \( b \) and averaged over the magnetic sublevels of \( a \). The density matrix \( \rho_a \) must be replaced by the number density of atoms in the state \( a \), regardless of the magnetic sublevel quantum numbers. In the remainder of this section, all atomic states will be specified by giving only the principal and angular momentum quantum numbers of the active electron.
A. Autoionization

Consider the autoionization process corresponding to the ejection of an \( n \ell \)-electron accompanied by the deexcitation \( n \aa \rightarrow n \aa \) of one of the remaining bound electrons. Using quantum-defect theory, the autoionization rate for large \( n \) can be related to the cross section for the excitation \( n \cc \rightarrow n \aa \) induced by electron impact. In a previous paper, we showed that the autoionization rate due to the dipole part of the electrostatic interaction can be approximated by

\[
A_{\aa}(n \aa, n \cc, n \aa) = \frac{E_H}{a} \left( \frac{2n}{\sqrt{3}} \right) \frac{2(2\ell + 1)}{2(2\ell_a + 1) 2(2\ell + 1)}
\]

\[
\left( \frac{E_H}{E_a - E_c} \right) f(n \cc \rightarrow n \aa) \sum_{\ell'} g \left( \varepsilon_{\ell'} + \varepsilon_a \right) \bigg| \varepsilon_a = 0 ,
\]

where \( f(n \cc \rightarrow n \aa) \) is the oscillator strength and \( g \left( \varepsilon_{\ell'} + \varepsilon_a \right) \bigg| \varepsilon_a = 0 \) is the threshold value of the partial-wave Gaunt factor for the free-free transition of an electron in the field of the ion with residual charge \( Z \). In the non-relativistic approximation employed here, equation (70) is practically independent of \( Z \). The \( n^{-3} \) dependence which is predicted is a well-known property. The \( \ell \)-dependence is difficult to derive, but experience in evaluating the free-free Gaunt factor indicates that the autoionization rates decrease rapidly with increasing \( \ell \), typically like \( \ell^{-5} \).

B. Spontaneous Radiative Transitions

The properties of radiative transition rates are so well-known that only a brief discussion will be necessary. Using the asymptotic behavior of the hydrogenic oscillator strengths for large \( n \), the spontaneous radiative decay rate due to all downward transitions can be estimated by

-24-
\[ A_r(n) = 1.6 \times 10^{10} \frac{Z^4}{n^{9/2}} \text{ sec}^{-1}. \] (73)

It should be kept in mind that equation (73) describes only \( \Delta n \neq 0 \) transitions, for which the radiative decay rates increase like \( Z^4 \) with increasing \( Z \). The \( n^{-9/5} \) dependence indicates that the dominant radiative decay mode for a doubly-excited state with the outer-electron in a high-\( n \) level will be the radiative deexcitation of the inner-electron.

The total spontaneous radiative width is estimated by

\[ \Gamma_{SR}^{ab, ab} = 1.6 \times 10^{10} Z^4 \left(n_a^{-9/2} + n_b^{-9/2}\right), \] (74)

with the understanding that \( n_a \) and \( n_b \) refer to the lowest values for which a dipole transition can occur.

In medium- and high-\( Z \) ions, the most prominent emission lines have radiative decay rates which are at least comparable to any autoionization rates. Consequently, equation (74) can be used as an estimate for the total natural width. It should be emphasized that prominent absorption lines can arise from transitions to autoionizing levels whose spontaneous radiative decay rates are relatively small.

C. Induced Radiative Transitions

The result obtained after averaging equation (66) over the magnetic quantum numbers can be written in the single-particle approximation as
\[ \Gamma_{ab, \ell_a \ell_b} = \frac{4 \pi e^2}{3 \hbar c} \sum_{\lambda} \int d\Omega_k \left( \sum \max(\ell_{a_{n}}^{ab}, \ell_{a_{n}}^{ab}) \left| \left\langle n_{a_{n}}^{ab} | r^2 | n_{a_{n}}^{ab} \right\rangle \right|^2 \right. \]

\[ \times \left[ I_{\lambda}(\omega_{a_{n}} - \omega_{a_{n}}^{ab}, \Omega_k) \Theta(\omega_{a_{n}} - \omega_{a_{n}}^{ab}) + I_{\lambda}(\omega_{a_{n}} - \omega_{a_{n}}^{ab}, \Omega_k) \Theta(\omega_{a_{n}} - \omega_{a_{n}}^{ab}) \right] \]

\[ + \sum \max(\ell_{b_{n}}^{b_{n}^{ab}}, \ell_{b_{n}}^{b_{n}^{ab}}) \left| \left\langle n_{b_{n}}^{b_{n}^{ab}} | r^2 | n_{b_{n}}^{b_{n}^{ab}} \right\rangle \right|^2 \]

\[ \times \left[ I_{\lambda}(\omega_{b_{n}} - \omega_{b_{n}}^{ab}, \Omega_k) \Theta(\omega_{b_{n}} - \omega_{b_{n}}^{ab}) + I_{\lambda}(\omega_{b_{n}} - \omega_{b_{n}}^{ab}, \Omega_k) \Theta(\omega_{b_{n}} - \omega_{b_{n}}^{ab}) \right] \]  

\text{(75)}

We note that the electron impact width associated with inelastic transitions can be obtained in the Bethe approximation from equation (75) simply by replacing \( \frac{1}{c} \int \frac{E}{\lambda} \int d\Omega_k I_{\lambda}(\omega, \Omega_k) d\Omega_k \) by \( e^2 N_e \int \frac{f(V_e)}{V_e} \frac{2\pi}{\sqrt{3}} \).

The application of the \( r^2 \) sum rules given by Bethe and Salpeter is prevented by the photon-frequency dependence of the radiation field intensity \( I_{\lambda}(\omega, \Omega) \). If we define a total intensity by

\[ I(\omega) = \sum_{\lambda} \int d\Omega_k I_{\lambda}(\omega, \Omega_k) \]  

and replace \( I(\omega) \) by a suitable frequency-independent average intensity \( I \), the summations over the complete sets of states \( n_{a_{n}}^{ab} \) and \( n_{b_{n}}^{b_{n}^{ab}} \) can be carried out, and the absorption and stimulated emission contributions can be combined to give the result
\[ \langle n_a | r^2 | n_a \rangle + \langle n_b | r^2 | n_b \rangle ) \].

(77)

The expectation values of \( r^2 \) will now be approximated by using the hydrogenic formula

\[ \langle n^l | r^2 | n^l \rangle = \frac{n^2}{2Z^2} (5n^2 + 1 - 3l (l + 1)) a_o^2 \]

(78)

\[ \approx \frac{2n^4}{Z^2} a_o^2 \]

where the last version corresponds to an \( l \)-independent approximation.

Let us consider a thermal equilibrium distribution of radiation characterized by a radiation temperature \( T_R \). We assume that this radiation is incident from one side only, so that the integration over \( \Omega_k \) reduces to multiplication by \( 2\pi \). We choose the frequency-independent intensity

\[ \bar{I} = \frac{(kT_R)^3}{2\pi c^2 n^2} \]

(79)

which is about \( 2/3 \) the maximum value of the Planck distribution function corresponding to \( T_R \). The estimated width due to induced radiative transitions is then given by

\[ \Gamma^{IR}_{ab, ab} = \frac{a_0^3}{3} \left( \frac{E_H}{\hbar} \right) \left( \frac{kT_R}{E_H} \right)^3 \left( \frac{n_a^4 + n_b^4}{Z^2} \right) \]

(80)

\[ = 2.67 \times 10^9 \left( \frac{kT_R}{E_H} \right)^3 \left( \frac{n_a^4 + n_b^4}{Z^2} \right) \text{ sec}^{-1}. \]

In contrast to equation (74) for \( \Gamma^{SR}_{ab, ab} \), \( \Gamma^{IR}_{ab, ab} \) increases with increasing principal quantum number and decreases with increasing residual charge. The \( (kT_R)^3 \) dependence on the radiation temperature is also an important property.
Unless the induced radiative transitions have frequencies within the region of the maximum in $I(\omega)$, equation (80) is probably an overestimate of the actual width.

D. Electron Collisions

It has been pointed out by Griem\textsuperscript{25} that the Bethe approximation for the electron-impact width due to inelastic transitions can be modified to take into account some of the effects of elastic scattering. This is done by beginning the integral over the Maxwellian distribution at zero velocity rather than starting it at the excitation threshold. The result can be written as

$$\Gamma_{\text{el}} = \frac{32}{3\sqrt{3}} \pi^{3/2} a_o N_e \left( \frac{E_H}{a} \right) \sqrt{\frac{E_H}{kT_e}} \left\{ \sum \frac{\max(\xi_a, \xi_b)}{2 \xi_a + 1} \left| \left\langle n_{a}, n_{a}' \right| r^2 \left| n_{a}, n_{a}' \right\rangle \right|^2 g\left(\frac{3kT_e}{2|E_a - E_{a}'|}\right) \right\}$$

$$\left\{ \sum \frac{\max(\xi_{b}, \xi_{b}')}{2 \xi_b + 1} \left| \left\langle n_{b}, n_{b}' \right| r^2 \left| n_{b}, n_{b}' \right\rangle \right|^2 g\left(\frac{3kT_e}{2|E_{b} - E_{b}'|}\right) \right\},$$

where $g$ now denotes the thermal average of the free-free Gaunt factor.

If the Gaunt factors in equation (81) are now replaced by suitable average values, the $r^2$ sum rules can be employed to obtain the result\textsuperscript{25}.
\[ \Gamma_{ab, a b} = \frac{32}{3\sqrt{3}} \pi^{3/2} a_o N_e \left( \frac{E_H}{\hbar} \right) \sqrt{\frac{E_H}{kT}} \]

\[ \chi \left\{ \left| \langle n_a l_a | r^2 | n_a l_a \rangle \right| \right\} \left| \langle n_b l_b | r^2 | n_b l_b \rangle \right| g \left( \frac{3kT_e}{2\Delta E_a} \right) \quad \text{(82)} \]

where \( \Delta E_a \) and \( \Delta E_b \) should be chosen to be the smallest threshold energies. We now use equation (78) and set the average Gaunt factors equal to unity, which is a good approximation for \( \Delta n=0 \) transitions in multiply-charged ions. The electron impact width is then estimated by

\[ \Gamma_{ab, a b} = \frac{64}{3\sqrt{3}} \pi^{3/2} a_o N_e \left( \frac{E_H}{\hbar} \right) \sqrt{\frac{E_H}{kT_e}} \left( \frac{n_a^4 + n_b^4}{z^2} \right) \quad \text{(83)} \]

\[ = 2.1 \times 10^{-7} N_e \sqrt{\frac{E_H}{kT_e}} \left( \frac{n_a^4 + n_b^4}{z^2} \right) \]

As expected, the electron impact width has the same \( n \)- and \( Z \)- dependences as the width due to induced radiative transitions. The \( z \)-dependences would also be the same if they had been retained in the approximation for the expectation values of \( r^2 \). The condition for thermal radiation broadening to be dominant over electron impact broadening is found from equations (80) and (83) to be

\[ \left( \frac{kT_R}{E_H} \right)^3 \gg 7.9 \times 10^{-17} N_e \sqrt{\frac{E_H}{kT_e}} \quad \text{(84)} \]
ACKNOWLEDGMENTS

Helpful discussions with H. R. Griem and A. Ben Reuven are gratefully acknowledged. This work has been supported by the Office of Naval Research.
REFERENCES


DISTRIBUTION LIST

Agostini, Pierre
C.E.A.
CEN SACLAY
B.P. 2 Fig-sur-Yvette
France

Aspect, Alain
Universite Paris Sud
B. P. 43
91416 Orsay Cedex
France

Alvarez, Ignacio
Instituto de Fisica
U.N.A.M.
Apdo. Pstal 20-364
Mexico

Baird, P. E. G.
Clarendon Laboratory
University of Oxford
Parks Road
Oxford, England

Armstrong, Lloyd
Department of Physics
Johns Hopkins University
Baltimore, Md. 21318

Bayfield, J. E.
Dept. of Physics
University of Pittsburgh
100 Allen Hall
Pittsburg, Pa. 15260

Baum, Gunter G.
Univ. Bielefeld
Fakultat fur Physik
48 Bielefeld
West Germany

Andrews, H. R.
Atomic Energy of Canada
Research Company
Chalk River, St. 49
Ontario, Canada KOJ 1 JO

Becker, Uwe
Tech. Univ. Berlin
Rundellstr. 5, D-1000
Berlin 37
West Germany

Baer, Thomas
JILA
University of Colorado
Boulder, Colorado 80302

Berry, Gordon
Argonne National Laboratory
9700 S. Cass Avenue
Argonne, Illinois 60439

Barnett, C. F.
Oak Ridge National Laboratory
P. O. Box X
Bldg. 6003
Oak Ridge, Tennessee 37830

Biemont, Emile
Inst. D'Observatoire
Universite de Liege
B-4200 Cointe-Ourgée
Belgium

Berkner, Klaus H.
Lawrence Berkeley Laboratory
University of California
Berkeley, California 94720
Bhaskar, Nat
Columbia University
Columbia Radiation Lab.
New York, New York 10027

Commins, E.D.
Department of Physics
Univ. of California
Berkeley, California 94720

Bose, Norbert
University of Toronto
60 St. George Street
Toronto, M5S 1A7
Canada

Couillaub, Bernard
Stanford University
Stanford, California 94720

Brandenberger, John
Lawrence University
Physics Department
Appleton, Wisconsin 54911

Breining, Marianne
Univ. of Tennessee
101 West Vance Road
Oak Ridge, Tennessee 37830

Buchinger, Fritz
Mainz University
CERN
CERN Div. EP
Geneve, Switzerland

Burnett, K.
JILA
University of Colorado
Boulder, Colorado 80309

Carlson, Nils
Physics Department
Stanford University
Stanford, California 94305

Cheng, K. T.
Building 203
Argonne National Lab.
Argonne, Illinois 60439

Chang, Edward S.
Department of Physics
Univ. of Massachusetts
Amherst, Massachusetts 01003

Church, David
Department of Physics
Texas A&M University
College Station, Texas 77843

Chu, Shih-I
Dept. of Chemistry
University of Kansas
Lawrence, Kansas 66045

Clark, Charles W.
SRC Daresbury Laboratory
Daresbury, Warrington
England WA4 4AD

Cisneros, Carmen Dra.
Instituto de Fisica,
U.N.A.M.
Apdo. Postal 20-364
Mexico

Contin, Ralph
Physics Department
University of Michigan
Ann Arbor, Michigan 48109

Crampton, Stuart
Williams College
Williamstown, Massachusetts 01267
<table>
<thead>
<tr>
<th>Name</th>
<th>Institution</th>
<th>Address</th>
</tr>
</thead>
<tbody>
<tr>
<td>Bouchiat, C.</td>
<td>Ecole Normale Sup.</td>
<td>24 Rue Lhomond 24, 75231 Paris Cedex 05, France</td>
</tr>
<tr>
<td>Dehmelt, Hans G.</td>
<td>Department of Physics</td>
<td>Univ. of Washington, Seattle, Washington 98195</td>
</tr>
<tr>
<td>Bryant, Howard C.</td>
<td>Physics and Astronomy</td>
<td>University of New Mexico, Albuquerque, New Mexico 87131</td>
</tr>
<tr>
<td>Dietrich, Andre</td>
<td>Tech. Univ. Berlin</td>
<td>D 1 Berlin 37, Rondellstrasse 5, West Germany</td>
</tr>
<tr>
<td>Cagnac, Bernard</td>
<td>Univ. P et M Curie</td>
<td>4 Place Jussieu, Paris 53, France</td>
</tr>
<tr>
<td>Egan, Patrick O.</td>
<td>Physics Department</td>
<td>Yale University, 216 Prospect Street, New Haven, Connecticut 06520</td>
</tr>
<tr>
<td>Chan, J. T.</td>
<td>Physics Department</td>
<td>Univ. of Arkansas, Fayetteville, Arkansas 72701</td>
</tr>
<tr>
<td>Farley, John</td>
<td>Physics Department</td>
<td>Univ. of Arizona, Tucson, Arizona 85721</td>
</tr>
<tr>
<td>Childs, William J.</td>
<td>Physics Department</td>
<td>Argonne Laboratory, Argonne, Illinois 60439</td>
</tr>
<tr>
<td>Franz, Frank</td>
<td>Dept. of Physics</td>
<td>Indiana University, Bloomington, Indiana 47405</td>
</tr>
<tr>
<td>Clark, David A.</td>
<td>Univ. of New Mexico</td>
<td>P.O. Box 608, Albuquerque, New Mexico 87544</td>
</tr>
<tr>
<td>Dagenais, Mario</td>
<td>Harvard University</td>
<td>Cambridge, Massachusetts 02138</td>
</tr>
<tr>
<td>Cooke, William E.</td>
<td>Department of Physics</td>
<td>Univ. of So. California, University Park, Los Angeles, California 90007</td>
</tr>
<tr>
<td>Ducas, Theodore</td>
<td>MIT Rm. 26-237</td>
<td>Cambridge, Massachusetts 02139</td>
</tr>
<tr>
<td>Ezekiel, Shaoul</td>
<td>MIT</td>
<td>Room 26-255, Cambridge, Massachusetts 02139</td>
</tr>
<tr>
<td>Czajkowski, N</td>
<td>University of Windsor</td>
<td>Windsor, Ontario, Canada N9B 3P4</td>
</tr>
</tbody>
</table>
Feld, Michael
MIT
Room 6-009
Cambridge, Massachusetts 02139

Freund, Robert S.
Bell Laboratories
600 Mountain Avenue
Murray Hill, New Jersey 07974

Freeman, Richard
Bell Laboratories
4D-321
Crawfords Corner Road
Holmdel, New Jersey 07733

Fritsch, Wolfgang
Department of Physics
Kansas State Univ.
Manhattan, Kansas 66506

Dalgarno, Alex
Center for Astrophysics
60 Garden Street
Cambridge, Massachusetts 02138

Genenx, Edmond
Universitat Bern
Sid' erstrasse
3012 Berne
Switzerland

Davis, William A.
Fusion Energy Division
Oak Ridge National Lab.
Box Y, Building 9201-2
Oak Ridge, Tennessee 37830

Goldman, Samuel A.
University of Windsor
Windsor, Ontario
Canada N9B 3P4

Deslattes, Richard D.
National Science Foundation
Washington, D. C. 20234

Greytak, T. J.
Department of Physics
MIT
Cambridge, Massachusetts 02139

Drake, Gordon
Department of Physics
University of Windsor
Windsor, Ontario
Canada N9B 3P4

Gupta, R.
University of Arkansas
Department of Physics
Fayetteville, Alaska 72701

Fabre, Francoise
C.E.A.
CEN SACLAY
B.P. 2 Gif-sur-Yvette
France

Hanne, G. Friedrich
Physikal Institut
Corrensstrasse
4400 Muenster
West Germany

Fortson, Norval
University of Washington
Seattle, Washington

Giacobino, Elisabeth
Universite Paris VI
4 Place Jussieu
75230 Paris Cedex 05
France
<table>
<thead>
<tr>
<th>Name</th>
<th>Institution</th>
<th>Address</th>
</tr>
</thead>
<tbody>
<tr>
<td>Hughes, Vernon</td>
<td>Department of Physics</td>
<td>Yale University, 217 Prospect Street, New Haven, Connecticut 06520</td>
</tr>
<tr>
<td>Hagmann, S.</td>
<td>Department of Physics</td>
<td>Kansas State University, Cardwell Hall, Manhattan, Kansas 66506</td>
</tr>
<tr>
<td>Jacobson, Harry C.</td>
<td>Dir. of Special Programs</td>
<td>University of Tennessee, Knoxville, Tennessee 37916</td>
</tr>
<tr>
<td>Hansch, Theodore</td>
<td>Department of Physics</td>
<td>Stanford University, Stanford, California 94305</td>
</tr>
<tr>
<td>Jeys, Thomas H.</td>
<td>Rice University</td>
<td>Space Phys. and Astronomy, 6100 South Main, Houston, Texas 77005</td>
</tr>
<tr>
<td>Harris, S.E.</td>
<td>Edward L. Ginzton Lab.</td>
<td>Stanford University, Stanford, California 94305</td>
</tr>
<tr>
<td>Johnson, W. R.</td>
<td>Physics Department</td>
<td>Univ. of Notre Dame, Notre Dame, Indiana 46556</td>
</tr>
<tr>
<td>Hegstrom, Roger A.</td>
<td>Dept. of Chemistry</td>
<td>Wake Forest University, Winston-Salem, North Carolina 27109</td>
</tr>
<tr>
<td>Kelly, Hugh R.</td>
<td>Physics Department</td>
<td>Univ. of Virginia, Charlottesville, Va. 22901</td>
</tr>
<tr>
<td>Geltman, Sidney</td>
<td>J.I.L.A.</td>
<td>Univ. of Colorado, Boulder, Colorado 80309</td>
</tr>
<tr>
<td>Kim, Young Soon</td>
<td>Physics &amp; Astronomy</td>
<td>Univ. of Pittsburgh, Pittsburgh, Pa. 15260</td>
</tr>
<tr>
<td>Gidley, David W.</td>
<td>Randall Lab.</td>
<td>University of Michigan, 500 E. University, Ann Arbor, Michigan 48109</td>
</tr>
<tr>
<td>Hill, Robert M.</td>
<td>SRI International</td>
<td>333 Ravenswood Avenue, Menlo Park, California 94025</td>
</tr>
<tr>
<td>Gould, Harvey</td>
<td>Bldg. 70, Room 257</td>
<td>Laurence Berkeley Lab., Berkeley, California 94720</td>
</tr>
<tr>
<td>Hall, John</td>
<td>JILA</td>
<td>Natl. Bureau of Standards, Boulder, Colorado 80309</td>
</tr>
<tr>
<td>Huldt, Sven</td>
<td>University of Tennessee</td>
<td>Physics and Astronomy, Knoxville, Tennessee 37916</td>
</tr>
<tr>
<td>Name</td>
<td>Institution</td>
<td>City, State, Zip</td>
</tr>
<tr>
<td>------------------</td>
<td>--------------------------------------</td>
<td>----------------------</td>
</tr>
<tr>
<td>Junker, B. R.</td>
<td>ONR Code 421, 800 North Quincy Street</td>
<td>Arlington, Va. 22217</td>
</tr>
<tr>
<td>Lennard, W.N.</td>
<td>Atom. Ener. of Canada Ltd. C.R.N.L.</td>
<td>Chaulk River, Ontario</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Canada KOJ 1J0</td>
</tr>
<tr>
<td>Humberston, J. W.</td>
<td>Univ. College London, Gower Street</td>
<td>London WCIE 6BT</td>
</tr>
<tr>
<td></td>
<td></td>
<td>England</td>
</tr>
<tr>
<td>Jaecks, Duane H.</td>
<td>Physics Department, University of Nebraska</td>
<td>Lincoln, Nebraska 68588</td>
</tr>
<tr>
<td>Knight, Randall</td>
<td>60 Garden Street, Center for Astrophysics</td>
<td>Cambridge, Massachusetts 02138</td>
</tr>
<tr>
<td>Johnson, Charles E.</td>
<td>Physics Department, North Carolina State University</td>
<td>Raleigh, North Carolina 27607</td>
</tr>
<tr>
<td>Litster, David</td>
<td>MIT, Room 13-2030, Cambridge, Massachusetts 02139</td>
<td></td>
</tr>
<tr>
<td>Kelleher, Daniel</td>
<td>National Bur. of Standards, Room A167, Bldg. 221, Washington, D.C. 20234</td>
<td></td>
</tr>
<tr>
<td>Lu, K.T.</td>
<td>Chem. Division K105, Argonne National Lab., Argonne, Illinois 60439</td>
<td></td>
</tr>
<tr>
<td>Kim, Y.-K.</td>
<td>Argonne National Lab., Argonne, Illinois 60439</td>
<td></td>
</tr>
<tr>
<td>Koch, Peter</td>
<td>Physics Department, Yale University, 217 Prospect Street, New Haven, Connecticut 06520</td>
<td></td>
</tr>
<tr>
<td>Klien, H.</td>
<td>Clarendon Laboratory, Parks Road, Oxford, England</td>
<td></td>
</tr>
<tr>
<td>Lin, C.D.</td>
<td>Department of Physics, Kansas State University, Manhattan, Kansas 66506</td>
<td></td>
</tr>
<tr>
<td>Kleppner, Daniel</td>
<td>M.I.T., Room 26-231, Cambridge, Massachusetts 02139</td>
<td></td>
</tr>
<tr>
<td>Larson, Daniel J.</td>
<td>Department of Physics, Univ. of Virginia, Charlottesville, Va. 22903</td>
<td></td>
</tr>
<tr>
<td>Lubell, Michael S.</td>
<td>Department of Physics, Yale University, New Haven, Connecticut 06520</td>
<td></td>
</tr>
</tbody>
</table>
Marrus, Richard  
Physics Department  
Univ. of California  
Berkeley, California 94720

Lutz, H. O.  
Univ. Bielefeld  
Fakultat fur Physik  
D-4800 Bielefeld 1  
West Germany

Meschede, Dieter  
Univ. of Colorado  
JILA  
Boulder, Colorado 80309

Mann, Joseph B.  
Los Alamos Sci. Lab.  
Los Alamos, New Mexico 87545

Mizushima, Masataka  
Physics Department  
Univ. of Colorado  
Boulder, Colorado 80309

Mariani, David  
Physics Department  
Yale University  
217 Prospect Street  
New Haven, Connecticut 06520

Lundeen, Stephen R.  
Lyman Laboratory  
Harvard University  
Department of Physics  
Cambridge, Massachusetts 02138

Meyer, Fred W.  
Oak Ridge National Lab.  
P.O. Box X  
Bldg. 6003  
Oak Ridge, Tennessee 37830

Metcalf, Harold J.  
Physics Department  
State University of NY  
Stony Brook, New York 11794

Mitsui, H. P.  
Physics Department  
Meerut College  
Meerut, 250001  
India

Meyerhof, W.E.  
Department of Physics  
Stanford University  
Stanford, California 94305

Marrow, Hajime  
Dept. of Physics  
Hiroshima University  
Hiroshima 730  
Japan

Mohr, Peter  
Dept. of Physics  
Yale University  
New Haven, Connecticut 06520

Nayfeth, M. H.  
Univ. of Illinois  
Dept. of Physics  
Urbana, Illinois 61801

Morgenstern, Reinhard  
University Utrecht  
Princetonplein 5  
NL 3508 TA Utrecht  
Netherlands

Odabasli, Halis  
Department of Physics  
Bosphorus University  
Babek, Istanbul  
Turkey
Padial, Nely
JILA
1510 Eisenhower Drive
Boulder, Colorado 80303

Parkinson, W.H.
Harvard College Observ.
60 Garden Street
Cambridge, Massachusetts 02138

Peek, James M.
P. O. Box 5800
Sandia Laboratory
Albuquerque, New Mexico 87185

Peterson, Randolph S.
Box U-46
Dept. of Physics
Univ. of Connecticut
Storrs, Connecticut 06268

Pindzola, Michael S.
Oak Ridge National Lab.
Phys. Div., Bldg. 6003
Oak Ridge, Tennessee 37830

Paul, Derek
Physics Department
University of Toronto
Toronto M5S 1A7 Canada

Pipkin, Francis M.
Lyman Laboratory
Harvard University
Cambridge, Massachusetts 02138

Pratt, Richard H.
Department of Physics
Univ. of Pittsburgh
Pittsburgh, Pennsylvania 15260

Read, F. H.
Physics Department
Univ. of Manchester
Manchester M13 9P1
England

Rich, Arthur
Physics Department
Univ. of Michigan
Ann Arbor, Michigan 48104

Rob, Akiva
Univ. of Pittsburgh
Physics Department
Pittsburgh, Pa. 15260

Rozsnyai, Balazs F.
L-71, P. O. Box 808
Lawrence Livermore Lab.
Livermore, California 94550

Sandner, Wolfgang
SRI International
332 Ravenswood Avenue
Menlo Park, California 94025

Sellin, Ivan A.
Oak Ridge National Lab.
P. O. Box X
Oak Ridge, Tennessee 37830

Rosen, Arne
Department of Physics
Chalmers Univ. of Tech.
S-41296 Goteborg
Sweden

Schawlow, Arthur L.
Department of Physics
Stanford University
Stanford, California 94405
Shakeshaft, Robin
Department of Physics
Texas A & M University
College Station, Texas 77843

Sweetman, Eric
Physics Dept.
Univ. of Michigan
Ann Arbor, Michigan 48109

Rau, A.R.P.
Physics & Astronomy
Louisiana State University
Baton Rouge, Louisiana 70803

Taylor, Barry N.
National Bureau of Standards
Bldg. 220, Room B258
Washington, D.C. 20234

Rosner, S. David
Department of Physics
Univ. of Western Ontario
London, Ontario N6A 3K7 Canada

Shugart, Howard A.
Department of Physics
Univ. of California
Berkeley, California 94720

Rudd, M. Eugene
University of Nebraska
Behlen Lab. of Physics
Lincoln, Nebraska 68588

Sinclair, Rolf
Physics Division
Natl. Sci. Foundation
Washington, D.C. 20550

Shore, Bruce W.
Lawrence Livermore Lab.
P. O. Box 808
Livermore, California 94550

Smith, Ken
Rice University
Space Phys. and Astronomy
6100 South Main
Houston, Texas 77005

Silver, Joshua D.
Univ. of Oxford
Parks Road
Oxford OX1 3PU
England

Spruch, Larry
New York University
Physics Department
4 Washington Place
New York, New York 10003

Smith, Winthrop W.
Department of Physics
Univ. of Connecticut
Storrs, Connecticut 06268

Talman, J. D.
Department of Chemistry
Univ. of West. Ontario
London, Ontario N6A 3K7 Canada

Sternheimer, Rudolph M.
Dept. of Physics
Brookhaven National Lab.
Upton, New York 11973

Siegel, Augustin
Dept. of Physics
Stanford University
Stanford, California 94305
Smith, A.C.H.
Dept. of Phys. and Astro.
Univ. College London
London WC1E 6BT
England

Smith, Stephen J.
JILA/NBS
Campus Box 440
Univ. of Colorado
Boulder, Colorado 80309

Souder, Paul
Department of Physics
Yale University
New Haven, Connecticut 06520

Stebblings, Ronald F.
Dept. of Space and Astron.
Rice University
Houston, Texas 77001

Stolterfoht, N.
Hahn-Mfitner-Institut
D-1 Berlin 39
100 Glienickerstr.
West Germany

Tanaka, Hiroski
Sophia University
Faculty of Sci. and Tech.
Chiyoda-Ku, Kiocho 7
Tokyo, Japan 102

Trager, F.
Physikalisches Institut
Der Univ. Heidelberg
D-6900 Heidelberg 1
West Germany

Venanzi, Carol
Rutgers University
P.O. Box 939
Piscataway, New Jersey 08854

Walls, Fred L.
National Bureau of Standards
325 Broadway
Boulder, Colorado 80303

Watanabe, Shinichi
Dept. of Physics
University of Chicago
1100 East 58th Street
Chicago, Illinois 60637

Weinberger, Ervin
University of Toronto
255 Huron Street
Toronto, Ontario
Canada M5S 1A7

Wing, William
Phys. and Optical Sciences
University of Arizona
Tucson, Arizona 85721

Wuilleumier, Francois J.
23 Rue la Abesse Roche
Villebon Sur Yvette
91120 Palaiseau, France

Vuskovic, Leposava
Jet Propulsion Lab.
4800 Oak Grove Drive
Pasadena, California 91103

Westerveld, Willem
Department of Physics
N. Carolina State Univ.
Raleigh, N. C. 27650

Williams, W. L.
Physics Department
University of Michigan
3044 Randall Lab.
Ann Arbor, Michigan 48109
Winkler, Peter  
Physics Department  
University of Nevada  
Reno, Nevada 89557

Yaris, Robert  
Chemistry Department  
Washington University  
St. Louis, Missouri 63130

Victor, G.A.  
Center for Astrophysics  
60 Garden Street  
Cambridge, Massachusetts 02138

Wadzinski, Henry T.  
Bedford Res. Assoc.  
2 DeAngelo Drive  
Bedford, Massachusetts 01730

Younger, Stephen M.  
National Bureau of Standards  
Washington D.C. 20234

Zimmermann, Dieter  
Inst. fur Strahlungs-Un  
Kernphysik  
Technische Univ. of Berlin  
D-1002 Berlin,  
West Germany

Zorn, Jens C.  
University of Michigan  
Physics Department  
Ann ARbor, Michigan 48104

Defence Technical Information Center  
Cameron Station  
5010 Duke Street  
Alexandria, Va. 22314

Naval Research Laboratory  
Washington, D.C. 20375

Code 4700 - 25 copies  
Code 4707 - 50 copies