WHEN CAN COST-REDUCING R&D BE JUSTIFIED?
A SIMPLE EXPLANATORY MODEL

H. Bruno W. Augenstein
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The Rand Corporation
Santa Monica, California 90406
An important class of expensive and lengthy R&D projects seeks to reduce the subsequent costs of operating systems. Some energy R&D projects are of this kind. Despite the confident assertions of technologists that the R&D undertaken will produce a large technological advance, and hence a given large amount of system operating cost reduction at a later time, it can often be economically much wiser, even when the large technological advance is achievable, to aim deliberately for a far lesser technological advance—if this lesser objective is attainable earlier and at lower R&D cost.

This result is intuitively plausible if one accepts discounting, or the fact that there is a time preference for money. That is, a dollar available this year has more value to us than a dollar available next year—an observation which is trivial, but nevertheless is sometimes ignored or dismissed by, or seems unfamiliar to, advocates of particular R&D projects. To make this notion quantitative, and the possible tradeoffs more easily discernible, generally requires that many parameters and parameter interactions be accounted for in systematic exploration of discounting questions, even in the simplest reasonable models of cost-reducing R&D. Such a very simple model is developed in this paper. Enough detail is provided so that readers can trace through for themselves examples which may particularly interest them.

I. INTRODUCTION

Research and Development (R&D) may produce technology which reduces operating costs of a system. A case in point is the application of energy technology R&D to cut future costs of energy generation.

When is such R&D economically justifiable? On economic grounds, the operating savings should exceed the R&D costs. Justifications on other grounds may sometimes be proposed. Such justifications raise difficult, complex, and controversial questions which are commented on in Section V.

Two widely used methods for comparing the savings and R&D costs exist (there are others).* One method is the discounted cash flow (DCF) method. This assumes investment and return cash flows for a project are known, and seeks a discount rate which appropriately relates these

*Often, private firms favor DCF, regulated firms RR methods.
cash flows. That discount rate—the internal rate of return—can be interpreted as the opportunity cost of money used for the project.

Another method is the revenue requirement (RR) method. This uses some set discount rate, and seeks to compute the revenue requirement equal to the present worth of all costs, including a minimum acceptable return for investors. The relations and differences between these two methods are amply discussed in the literature. Inflation is generally handled very simply in the two methods.

In this paper we use essentially the DCF method; the cash flows are supposed known. Discounted cash flow analysis used by industry can set the discount rate \( r_b \cdot f_b + r_s \cdot f_s \), where \( f_b \) is the bond fraction of capital, \( f_s \) the stock fraction, \( r_b \) the bond interest, \( r_s \) the required return on equity. But, in general, discussions of what the "proper" discount rate ought to be tend to be inconclusive. We consequently emphasize conditions, constraints, requirements and choices brought about by discounting, and treat sensitivity questions over a range of discount values, with other economically important parameters included. This emphasis is appropriate, because many of the factors in DCF analysis are of course imperfectly known in principle, and can vary with time (e.g., we clearly cannot accurately predict future inflation rates, R&D costs or operating savings). Usually we therefore inter alia make the assumption of constant discount (and inflation) rates. A simple, transparent, but suggestive model of DCF analysis applied to cost-reducing R&D is then especially useful, because of the many parameters and parameter interactions present. Such a model is the limited aim of this paper.

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II. THE COST-SAVINGS MODEL

One familiar simplification we use throughout is to consider the present value $A_p$ of an amount $A$ discounted at a rate of $d$ percent per year for $t$ years as $A_p = Ae^{-dt}$. This adds to the transparency of our analysis without excessively forcing the implications of the model results. The simplification is arithmetically exact if we assume discounting infinitely often, and is generally adequately good even without that assumption, for the types of problems we deal with.

The well-known reasons for this are as follows. First, discounting very many times or infinitely often is simply analogous to the case of compounding interest many times per year. If we compound interest, at a rate $r$, but $q$ times per year, the amount $P_n$ of a principal $P$ in $n$ years is: $P_n = P\left(1 + \frac{r}{q}\right)^{nq}$, which by the definition of $e$, the base of natural logarithms, $\lim_{q\to\infty} e^{-nq} = e^{-nd}$. Correspondingly, when we discount at a rate $d$, but $q$ times per year, standard discount factors like $(1 + d)^{-n}$ become $\left(1 + \frac{d}{q}\right)^{-nq} = e^{-nd}$ as $q \to \infty$. We will make the implicit assumption in this paper that all discounting is to be done very many times per year or continuously, so that the exponential is used. Second, exponential discounting is quite often a pretty good and useful approximation even without the assumption of discounting infinitely often. Consider a representative problem of calculating the present value, $\Sigma_1$, of a total of a unit amount added each year for $n$ years when discounted at a rate of $d$ percent. The standard calculation, with discounting once per year, gives $\Sigma_1 = \frac{1}{d}\left(1 - \frac{1}{(1 + d)^n}\right)$; exponential discounting gives $\Sigma_2 = \frac{1}{d}\left(1 - e^{-nd}\right)$;
standard discounting, but say 50 times per year, gives in effect
\[ F_3 = \frac{1}{d} \left( 1 - \frac{1}{(1 + \frac{d}{50})^{50n}} \right), \]
the unit amounts being in this case assumed added in amounts of \( \frac{1}{50} \) fifty times during the year. If we put \( d = 10 \) percent, the following typical table results:

<table>
<thead>
<tr>
<th>Years</th>
<th>( n = )</th>
<th>2</th>
<th>5</th>
<th>10</th>
<th>20</th>
<th>30</th>
</tr>
</thead>
<tbody>
<tr>
<td>( F_1 )</td>
<td></td>
<td>1.736</td>
<td>3.791</td>
<td>6.145</td>
<td>8.514</td>
<td>9.427</td>
</tr>
<tr>
<td>( F_2 )</td>
<td></td>
<td>1.813</td>
<td>3.935</td>
<td>6.321</td>
<td>8.647</td>
<td>9.502</td>
</tr>
<tr>
<td>( F_3 )</td>
<td></td>
<td>1.811</td>
<td>3.932</td>
<td>6.318</td>
<td>8.644</td>
<td>9.501</td>
</tr>
</tbody>
</table>

The difference between \( F_1 \) and \( F_2 \) is between 4.2 percent and 0.8 percent, while clearly \( F_3 \) is always very close to \( F_2 \).

The simple DCF model used starts with the assumption that in real, constant, undiscounted dollars the money flows as the diagram shows.
In the absence of any R&D the yearly operating cost of the system would continue to be $C_0$. But at $t = 0$ we initiate an R&D program, lasting $T$ years, with a triangular funding flow reaching a maximum of $C_R$ at $t = T/2$, and dropping to 0 at $t = T$. The cumulative R&D funding is an S-shaped curve. The technology produced by the R&D allows us to cut the operating cost to $\lambda C_0$, where $\lambda < 1$. That technology cannot be introduced instantaneously, however, taking $T_M - T$ years to introduce fully. The effective introduction rate is supposed linear between $t = T$ and $t = T_M$.

It will be appreciated that this is a simple model indeed for attempting to capture what is ordinarily a very complicated situation. The picture in more detail underlying the model for the case of energy systems is this. We assume that the yearly operating cost $C_0$ results from the operation of many individual operating units. The product of the R&D effort allows us to reduce the energy generation cost of each unit via the factor $\lambda$ (where $\lambda < 1$ for the specific examples pursued here), or by an amount saved proportional to $(1 - \lambda)$. Reductions in $\lambda$ are to be considered brought about by, for example, reductions in the capital cost of a new unit, by lowering of required total fuel cycle costs, by reduction in indirect costs, and the like. In any case, a new unit once built embodying the new R&D technology will produce energy at $\lambda x$ (cost of current unit). Once the R&D is accomplished (time $T$), we begin to replace current units with the new ones, so that some savings begin to be produced after time $T$, since $\lambda < 1$. That replacement rate could be constrained by many circumstances—perhaps we don't want to scrap the older current units.
prematurely; perhaps the industrial capacity to build new units con-
strains us to produce only a few new units per year; perhaps there is
some well-defined start-up time; and so on. For whatever reasons,
the transition to a new configuration again producing energy by many
new units, but now at a unit cost \( \lambda x \) (cost of current units), requires
some time equal to \( T_M - T \). The simple model used captures this effect
via the assumed linear variation between a total cost of \( C_o \) at time \( T \)
and the lesser cost of \( \lambda C_o \) at time \( T_M \). More complicated forms—e.g.,
logistic curves—for introduction description can be used, but add
complexity without any essentially new phenomena. Finally, the level
of \( C_o \) itself may not be the complete cost of energy generation, unless
the new technology captures the entire market. Otherwise, \( C_o \) is to be
taken as that part of the costs of the energy system which the market
penetration of the new technology could reasonably be expected to
affect.

In undiscounted dollars we can compare the cost of the R&D and
the savings in operating costs, shown in the diagram as shaded areas.
Discounting both of these at a rate \( d \), via a factor \( e^{-dt} \), then gives
us a discounted R&D cost, \( R_D \), to compare to a discounted savings, \( S_D \).
We want: \( S_D = N R_D \), where \( N = 1 \) would give "breakeven" on discounted
costs and savings. That is, at \( N = 1 \) the total project is "solvent"—
disbursements (R&D costs) and net receipts (savings) balance. In the
real world, however, to justify a prudent R&D investment, we would
generally want a predicted value of \( N \) greater than 2 or 3, or even
more, to account for the usual circumstance that we tend to mispredict
both \( R_D \) (on the low side) and \( S_D \) (on the high side), as innumerable
historical examples show.

Different parties can incur costs and realize savings.
Now put: \( \alpha = dT; \beta = \frac{T}{a}. \) Then it is easy to show that:

\[
R_D = 2 C_R T \left( \frac{1 - e^{-\alpha/2}}{\alpha} \right)^2 \equiv 2 C_R T \cdot E(\alpha)
\]  

\[
S_D = (1 - \lambda) C_0 T \left( \frac{e^{-\alpha} - e^{-\beta \alpha}}{\alpha^2 (\beta - 1)} \frac{e^{-\alpha} - e^{-\beta \alpha}}{\alpha} \right)
\]

If the savings integral is carried out to some large time \( t = T \). We can generally let \( t \to \infty \), so that:

\[
S_D = (1 - \lambda) C_0 T \left( \frac{e^{-\alpha} - e^{-\beta \alpha}}{\alpha^2 (\beta - 1)} \right) \equiv (1 - \lambda) C_0 T \cdot G(\alpha, \beta).
\]  

We can combine these to get the relationship:

\[
\frac{C_0}{C_R} \left( \frac{e^{-\alpha} - e^{-\beta \alpha}}{\beta - 1} \right) \cdot \left( \frac{1}{1 - e^{-\alpha/2}} \right)^2 \equiv \frac{2N}{1 - \lambda} \equiv \frac{C_0}{C_R} \cdot F(\alpha, \beta).
\]

The functions \( E, F, G \) in (1), (2), and (3) are roughly plotted in Figs. 1 and 2. Note that (3) allows us to treat the case, among others, where the predicted value of \( \lambda \) is not achieved. With everything else the same, the result is simply a change in \( N \).

More or less obvious extensions of these relations are clearly possible. Suppose, for example, that the original undiscounted costs themselves have an exponential growth like \( C_0 e^{g t} \). The \( g \) term might, e.g., arise because of changes in real, constant dollar costs; because of demand changes (for energy systems); and the like. Then, with obvious assumptions on how \( \lambda \) behaves, we can in effect calculate

*The internal rate of return is usually defined when \( N = 1 \).
Figure 1

\[ \frac{(1 - e^{-\alpha/2})^3}{\alpha^3} = E(\alpha) \]

\[ \frac{(s - a - \beta \alpha)}{\alpha^2 (\beta - 1)} = G(\alpha, \beta) \]
Figure 2

\[ F(\alpha, \beta) = \left( \frac{e^{-\alpha} - \alpha \beta}{\beta - 1} \right) \cdot \left( \frac{1}{1 - 2e^{-\alpha/s_0} - \alpha} \right) \]
by using a "virtual" discount rate \( d^* = d - g \). All our subsequent examples, however, deal with the case \( g = 0 \). Similarly, the case where \( (1 - \lambda) \) has an exponential factor \( e^{ht} \) can be handled. It is also easy to handle the case where the end of R&D and the initiation of the technology introduction period are not coincident in time. Finally, with a little more care, we can handle the case where, for whatever reason, \( \lambda \) is greater than unity at time \( T \), but the \( C_0 \) term is rising so that at time \( T' > T \) the new plants begin to produce positive savings. In such a case (which we do not consider here), the initial part of the savings integral is negative (before \( T' \)), and a useful strategy would be to delay the introduction of the new technology appropriately.

III. DISCUSSION OF THE MODEL

Equations (1), (2), and (3) provide the simple, transparent basis for discussing when cost-reducing R&D is economically justifiable. We observe first that \( R_D, S_D, \) and \( S_{D_\infty} \) have the evidently correct limiting behavior:

\[
R_D \rightarrow \frac{C_R T}{2} \text{ as } \alpha \rightarrow 0 \text{ (e.g., if } d \rightarrow 0); \\
S_{D_1} \rightarrow (1 - \lambda) C_O (t - T) \text{ as } \beta \rightarrow 1, \alpha \rightarrow 0; \\
S_{D_\infty} \rightarrow (1 - \lambda) C_O T \cdot \frac{e^{-\alpha}}{\alpha} \text{ as } \beta \rightarrow 1, \alpha \neq 0.
\]

There are now many parameters (the handling of inflation is implicit in the model) explicitly related by the model---\( C_O; C_R; T; T_M; T; \lambda; N; d; \alpha = dT; \beta = \frac{T}{T}; \) and \( R_D, S_{D_\infty} \). The multiplicity of these
parameters is what makes it useful to have simple analytic formulations so that extensive parameter spaces can be easily explored and implications drawn therefrom. Otherwise, the combinatorial number of combinations of parameters to consider often serves to inhibit exploration of the parameter spaces, and leads to inadequate consideration of the implications of discounting. In general, these implications turn out not to be excessively sensitive if other reasonable functional forms for the cost and savings streams are used (see Appendix).

A number of relationships can be derived analytically from the equations (where maxima and minima occur; tradeoffs among parameters which equivalently perturb important conclusions, etc.). The relationships, even with this simple model, are generally cumbersome, however, and it is most often simpler to work with Figs. 1 and 2 directly.

Together, equations (1), (2), and (3), and Figs. 1 and 2, can provide insights into the justification of cost-reducing R&D. Three of the related classes of problems approachable include:

A. Simple explorations of the parameter space, to understand better the effects of discounting and means for alleviating such effects.

B. Exploration of the requirements placed on a subset of parameters (e.g., \( \lambda \) and \( N \)), when other parameters are assigned explicit values.

C. Exploration of some of the simplest aspects of R&D strategy, assuming we were actually to know some functional relationships between, for example, \( \lambda \), \( C_R \), and \( T \).

Examples of A, B and C are given in the following section.
IV. SOME SAMPLE RESULTS

A. (a) Suppose, to take a reasonably realistic case, we take an energy R&D program lasting $T = 30$ years, and during that time to spend one percent of the relevant energy generation cost ($C_0 = 100$); hence $C = 50$. Suppose the technology is complex enough so that $T_M - T = 30$ years are spent in introducing that technology; hence $\beta = 2$. Suppose that the achieved $\lambda = 0.75$, and that $N$ is to be 2; hence $\frac{2N}{T - \lambda} = 16$. What values of $d$ will permit this?

Equation (3) gives: $16 = 50F(\alpha, 2)$, so $F(\alpha, 2) = 0.32$, which from Fig. 2 is achieved when $\alpha = 1.9$. Hence $d \left(= \frac{\alpha}{T}\right)$ must be $\leq 0.063$, which is very low indeed.

(b) Suppose all else is the same, except that $\lambda = 0$—that is, the R&D is assumed so effective that all energy is ultimately generated cost-free. How high can $d$ be? Here $\frac{2N}{T - \lambda} = 4$, so that $F(\alpha, 2) = 0.08$, which is achieved when $\alpha = 3$. Hence, $d$ cannot be higher than 0.10, even in this extreme case. Because $d$'s of 0.10 to 0.15 are often postulated in the literature, such an R&D program could scarcely be justifiable on economic grounds.

(c) Suppose we propose to make $T = 15$ years, while keeping the total undiscounted R&D cost (30) the same—i.e., $C_R = 4$. If it still takes 30 years to introduce the technology, now $\beta = 3$. We set $\lambda = 0.75$, and calculate $N$ for $d = 0.063$, 0.10 (i.e., $\alpha = 0.95$ and 1.5, respectively). Then $F(0.95, 3) = 1.15$, $F(1.5, 3) = 0.36$, so that $\frac{2N}{0.25} = 8N = 28.75$ and 9, respectively, or $N = 3.6, 1.13$, respectively. At $d = 0.063$ the project is now
much more certain of economic justification, while even at \( d = 0.10 \) the project is still at least marginally attractive.

(d) Suppose we increase the total cost of the project to 40, with \( T = 15 \) (hence, \( C_R \) is now 5.33 and \( C_{o/C_R} = 18.8 \)) so as to permit \( T_M - T \) to be 20 years (hence \( \beta \) is now 2.33). Again set \( \lambda = 0.75 \), and calculate \( N \) for \( d = 0.063, 0.10 \). Then \( F(0.95, 2.33) = 1.5 \), \( F(1.5, 2.33) = 0.52 \), so that \( N = 3.53, 1.22 \). This somewhat further improves the project acceptability at \( d = 0.10 \).

Cases (c) and (d) show the potential benefits of time reductions in the R&D and technology introduction phases, even if it means spending for R&D at much higher rates.

(e) Suppose we can reduce the R&D cost (reduce \( C_R \)); what increases in \( d \) can we then tolerate? Use A (a) as the base case, but now put \( C_R = 1.5, 1.0, 0.5 \). For \( C_R = 1.5 \), everything else the same, we have again \( 2N/T - \lambda = 16 = 100/1.5 \) \( F(\alpha, 2) \). Therefore \( F(\alpha, 2) \approx 0.24 \), which gives \( \alpha \approx 2.2, d \approx 0.073 \). The whole table of values is then:

<table>
<thead>
<tr>
<th>( C_R )</th>
<th>2.0</th>
<th>1.5</th>
<th>1.0</th>
<th>0.5</th>
</tr>
</thead>
<tbody>
<tr>
<td>( d )</td>
<td>0.063</td>
<td>0.073</td>
<td>0.083</td>
<td>0.100</td>
</tr>
</tbody>
</table>

B. (a) Suppose we use Case A (a) as a base case, and now consider the possible consequences of deliberately taking a much smaller technology step earlier and cheaper. Specifically, assume we could in a period of \( T = 10 \) years achieve the much more conservative \( \lambda = 0.94 \) by spending at the same maximum rate (hence \( C_R = 2; C_{o/C_R} = 50 \)), so that the total undiscounted spending is
10 instead of 30. Suppose that this lower level of technology could also be introduced faster, so that $T_M - T = 20$ years (hence $\beta = 3$). Again at $d = 0.063$ and $d = 0.10$ we compute $N$. Then $\alpha = dT = 0.63$ and 1.0, respectively, $1 - \lambda = 0.06$, and $F(0.63,3) = 2.7$, $F(1,3) = 1.05$, giving $N$ values of: 4.05, 1.58, respectively. The absolute savings, $S_D$, for this case and the case of A (a), for $d = 0.063$, are computed using Eq. (2) and Fig. 1. The results are shown below:

This case: $S_D = 28.7$

Case A (a): $S_D = 26.4$

Thus, in this case the project is not only much more certain (since $N = 4.05$ instead of 2), but also the absolute discounted savings are greater than in Case A (a). In addition, we could now tolerate $d = 0.10$ (since $N > 1$).

According to this example, there can be considerable economic wisdom in the notion of taking, by choice, smaller technology steps earlier (even if you spend at the same rate), which runs counter to the predilections of most technologists.* In expensive, long duration projects, an early deliverable can be very important. When discounting is used, there can be higher payoff from a smaller stream of undiscounted savings which starts early than from a much larger stream of undiscounted dollars which starts later. $\lambda$-constraints for this to occur are defined by (2).

(b) An interesting type of problem is this: we have an R&D project which is expensive, and its $C_R$ is high. Therefore there

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*Further, our predictions of R&D outcomes can be more confident in this case.
can be pressures from higher management to "stretch out" the project—i.e., increase \( T \), so that the maximum and average R&D expenditure rates decrease (to make the project "less visible," for example). What are the ramifications of this? Here we need typically to specify the presumed behavior of \( C_R \), \( N \) or \( \lambda \), or combinations of these. Consider, therefore, two kinds of problems.

1. Assume \( N, \lambda \) fixed. What is the required bound on \( C_R \)?
2. Assume \( S_D, C_R \) fixed. What is the required bound on \( \lambda \)?

For both problems we use A (a) as the base case. From this base case we go to \( T = 40 \) years, keeping \( T_M - T = 30 \). Hence now:
\[
\alpha = dT = 2.52, \quad \beta = 1.75.
\]

**Problem 1.**

\[
\frac{2N}{1 - \lambda} = 16 \text{ again. } F(\alpha, \beta) = F(2.52, 1.75) = 0.17. \text{ Therefore:}
\]

\[
\frac{2N}{1 - \lambda} = \frac{C_D}{C_R} \left( 0.17 \right) = 16, \text{ so } C_R = 1.06. \text{ Consequently, in this case we would have to reduce the maximum and average annual R&D spending rates almost by half, \( 1.06 \), hence the total undiscounted R&D spending by \( \sim 30 \) percent, and yet achieve the same \( \lambda \) in 40 years instead of 30 years. Note, however, that the absolute value of } R_D \text{ decreases by } \sim 44 \text{ percent in this case.}
\]

**Problem 2.**

\( S_D \) from A (a) is:
\[
0.25 \times 100 \times 30 \times G(\alpha, \beta) = 750 \times G(1.9, 2.0) = 750 \times 0.035 = 26.3. \text{ In Problem 2, } G(\alpha, \beta) = G(2.52, 1.75) = 0.0135; \text{ hence: } (1 - \lambda) \times 100 \times 40 \times 0.0135 = 26.3 =
\]
$54(1 - \lambda)$, so that $\lambda = 0.51$. Consequently, in this case the improvement in the achieved value of $\lambda$ (i.e., the value of $1 - \lambda$) would have to nearly double, to simply achieve the same $S_{D_\infty}$ as at $T = 30$ years.

There seems, unfortunately, to be little persuasive evidence to support the notion that a "stretched out" program would often operate in either of these ways—that is, achieve either the drastic reduction in total spending to achieve the same $\lambda$, or the dramatic increases in technology reflected by the higher $1 - \lambda$ values. This is because, on the contrary, not untypical institutional responses to a "stretched out" program appear to resist cutting the infrastructure support costs, if at all possible; and, to the extent that cost reductions are achieved, to cut back at least proportionately on the central technology improvement program. Similarly, the prospect of nearly doubling the achieved $(1 - \lambda)$ would often be complicated by the fact that many large, long duration R&D projects seek in the first place to achieve nearly the "technological asymptote" possible anyway, and would find it difficult to increase the $(1 - \lambda)$ values very substantially even with the expenditure of 33 percent more in undiscounted dollars and time.

C. Suppose we had available some general relations between $C_R$, $T$, $T_M$, and $\lambda$. Such relations would, likely, be technology-specific; would depend on the maturity of the technology; would be significantly influenced by management style, policy environment,
and the 'like; and so on. Despite a great deal of observation and considerable empirical research, we unfortunately still have no adequately useful understanding of such general relations (if indeed they exist). If, for example, we had some relations expressing achievable \( \lambda \)'s as functions of \( C_R \), \( T \), and \( T_M \), equations (1), (2), and (3) could be used to explore, by analysis, at least some aspects of R&D strategies.

To illustrate this, we assume the following artificial but not implausible example, and explore some of its consequences. This example is akin to but goes considerably farther than Example B.

We assume that the technologists tell us that for a program spending 30 over 30 years they could produce \( (1 - \lambda) = 0.30 \). They tell us they could also engage on any one of a set of independent, equally possible smaller programs, which would give an S-shaped \( \lambda, T \) curve reflected by the table below:

<table>
<thead>
<tr>
<th>R&amp;D program taking ( T ) years</th>
<th>6</th>
<th>12</th>
<th>18</th>
<th>24</th>
<th>30</th>
</tr>
</thead>
<tbody>
<tr>
<td>( (1 - \lambda) ) achieved in ( T ) year</td>
<td>0.02</td>
<td>0.08</td>
<td>0.22</td>
<td>0.28</td>
<td>0.30</td>
</tr>
</tbody>
</table>

Normally, we would suspect that one should have a higher confidence of actually achieving the smaller \( (1 - \lambda) \) values. Here, however, we assume that each of the assertions of the technologists is correct—they could achieve the stated \( (1 - \lambda) \) values at the stated times.
For the corresponding R&D program costs, undiscounted, we take two alternatives which would likely bound the situation. $C_0$ is always assumed 100. Alternative 1 would assume the costs incurred to be linear with $T$. Alternative 2 would assume costs proportional to the area under the $(1 - \lambda), T$ curve. The cost versus $T$ relations would then be:

<table>
<thead>
<tr>
<th>$T$</th>
<th>6</th>
<th>12</th>
<th>18</th>
<th>24</th>
<th>30</th>
</tr>
</thead>
<tbody>
<tr>
<td>Alt 1</td>
<td>6.0</td>
<td>12.0</td>
<td>18.0</td>
<td>24.0</td>
<td>30.0</td>
</tr>
<tr>
<td>Alt 2</td>
<td>0.4</td>
<td>2.4</td>
<td>8.4</td>
<td>18.4</td>
<td>30.0</td>
</tr>
</tbody>
</table>

We still need to choose $d$ and $\beta$. We assume $d = 0.08, \beta = 1.67$; ($\beta$ might itself be some nonconstant function). Finally, we use equations (1) and (2) and Fig. 1 directly to compute $R_D$ and $S_D$. For this we need to compute the equivalent $C_R$ for Alternative 1 and Alternative 2 (since we assume that any R&D program once selected a priori out of the above set of possibilities has the appropriate triangular undiscounted funding flow). We have the following $C_R$ table:

<table>
<thead>
<tr>
<th>$T$</th>
<th>6</th>
<th>12</th>
<th>18</th>
<th>24</th>
<th>30</th>
</tr>
</thead>
<tbody>
<tr>
<td>Alt 1</td>
<td>2.0</td>
<td>2.0</td>
<td>2.0</td>
<td>2.0</td>
<td>2.0</td>
</tr>
<tr>
<td>Alt 2</td>
<td>0.13</td>
<td>0.40</td>
<td>0.93</td>
<td>1.53</td>
<td>2.00</td>
</tr>
</tbody>
</table>

Finally, the $\alpha, T$ table is just:

<table>
<thead>
<tr>
<th>$T$</th>
<th>6</th>
<th>12</th>
<th>18</th>
<th>24</th>
<th>30</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\alpha$</td>
<td>0.48</td>
<td>0.96</td>
<td>1.44</td>
<td>1.92</td>
<td>2.40</td>
</tr>
</tbody>
</table>
The objective of the following computations is to calculate, for Alternative 1 and Alternative 2, the values of $R_D$ and $S_{D_{m\infty}}$ as a function of the particular $\lambda$-objective selected (and correspondingly therefore of $T$ and $C_R$). We will assume that a prudent program planner will want a value of $S_{D_{m\infty}}$ above some minimum, and a value of $R_D$ below some maximum. These constraints will then illuminate for that prudent planner which particular one of the set of independent R&D programs the technologists propose as equally possible should be selected and initiated.

The whole table of $R_D$, $S_{D_{m\infty}}$ values, filled in, then is approximately:

<table>
<thead>
<tr>
<th>$T$</th>
<th>6</th>
<th>12</th>
<th>18</th>
<th>24</th>
<th>30</th>
</tr>
</thead>
<tbody>
<tr>
<td>Alternative 1 - $R_D$ =</td>
<td>4.7</td>
<td>7.6</td>
<td>9.2</td>
<td>9.9</td>
<td>10.2</td>
</tr>
<tr>
<td>$S_{D_{m\infty}}$ =</td>
<td>13.3</td>
<td>28.0</td>
<td>41.2</td>
<td>28.8</td>
<td>16.7</td>
</tr>
<tr>
<td>Alternative 2 - $R_D$ =</td>
<td>0.3</td>
<td>1.5</td>
<td>4.3</td>
<td>7.6</td>
<td>10.2</td>
</tr>
<tr>
<td>$S_{D_{m\infty}}$ =</td>
<td>13.3</td>
<td>28.0</td>
<td>41.2</td>
<td>28.2</td>
<td>16.7</td>
</tr>
</tbody>
</table>

Assume the prudent program manager now requires that a discounted savings of at least $S_{D_{m\infty}} = 35$ must be realized. In Alternative 1 the cost $R_D \approx 8$, the target $\lambda = 0.88$; in Alternative 2 the cost $R_D \approx 2.5$, and the target $\lambda$ again $\approx 0.88$. In both cases, the $\lambda$ goal selected and the program undertaken would be far short of the extremes suggested as possible. If the program manager required an $S_{D_{m\infty}}$ of only 28, then $T \approx 12$ and the target $\lambda$ would need to be only $\approx 0.92$; again, far short of the value $\lambda = 0.70$ which the technologists assert for $T = 30$. 
Using similar procedures we can carry out calculations which illuminate the notion of multi-stage R&D (take an R&D step and introduce it into an operating system; after some time, take a new R&D step and introduce it) and the like. These examples are not pursued here.

V. SOME CONCLUSIONS

Equations (1), (2), and (3), and Figs. 1 and 2, provide a convenient, albeit approximate, way of exploring the large parameter spaces involved when one considers the problem of justifying R&D whose goal is to find means of reducing operating costs of systems. Because conceptual approximations (even though they are generally good ones) are involved, the results reached and the conclusions derived should be viewed as suggestive, rather than necessarily definitive. But since there can be no absolutely definitive solution to the problem anyway--because, inter alia, one would for this have to have perfect foresight to predict future cost and savings streams and the outcomes of R&D exactly--even approximate treatments which must perforce use a priori estimations of costs, savings, and technological achievements of R&D can be quite useful in exploring the parameter spaces.

While the model is addressed particularly to energy contexts, it likely has wider application. Some of the model's notions may be useful in more generalized situations--e.g., defense-related R&D--wherein comparable critical choices must also be made between an early realization of modest technology, or a significantly later realization of considerably more advanced technology. However, the translation of this kind of simple model into a defense context is not a trivial or obvious step, in other than cost-reduction related issues.
A conclusion reached in this paper is that it is often economically wise, if one uses discounting calculations, to seek deliberately for far less in the way of technological goals than the technologists tell you is possible (even if the technologists are wholly correct), if you can thereby save either time, money, or both. This is often an uncomfortable position for technologists to face. It is the nature of the R&D institutional framework, on many occasions, to resist conclusions which suggest that the maximum accessible technology be not always sought for. A response made by technologists can take the forms of an argument that their project should not be subjected to discounting, or an argument for a specifically selected discount rate low enough to make the answers more palatable, or an argument that there exist countervailing circumstances for their project which outweigh results of discounting calculations.

If one could make any case at all for not subjecting a project to discounting, that project would have to have an absolutely unique outcome whose nonattainment would be catastrophic. Projects in the energy field, as one instance, are not of this character, because there appear always to be several possible approaches to given objectives, each favored by some particular R&D community. There are, for example, a number of possible ways for the United States to arrive at nationally controlled, domestically secure "inexhaustible" or "virtually inexhaustible" (foreseeably lasting for 10 centuries, say) energy supplies, which is certainly a far-reaching objective. Discounting calculations are then one clear means of helping to establish preferences among available alternatives for those routes which warrant priority. When there are alternatives, a decision to pursue one route may tend to
foreclose other routes or the opportunity to take other beneficial actions. This represents an opportunity cost, shutting out a body of possibly comparably beneficial projects. An appropriate discount rate to use would presumably have to consider the totality of these comparably beneficial projects, and not be singularly low in this context.

There remains one quite important set of additional issues which need comment, which are part of a larger problem area, and which unfortunately appear to have no final, definitive resolution. The case is often made that a certain technology has "non-economic" benefits. Such benefits might be of many sorts: enhanced safety characteristics; lower nuclear proliferation risk; more benign environmental impacts; increased public acceptance; better conservation or extension of resources; a more secure resource base; and the like. Such "non-economic" benefits are then, presumably, another set of justifications for investing in the technology under consideration. There is no doubt a credible basis for assertions of "non-economic" benefits, and the subject is one of great and justified interest. But we nevertheless need to be very careful of these arguments. Classifying a benefit as "non-economic" reflects either a genuine difficulty or inability to quantify such benefits appropriately, or a lack of desire to do so. Nevertheless, even if only implicitly, we usually do in effect endeavor to quantify such benefits, by such behavior as, for example, not being willing to pay an arbitrarily high price to secure a more benign environmental impact. Use of a single comparison metric is in many ways the least troublesome procedure. The "non-economic" benefits can, the writer
believes, in many or most cases, even if more or less clumsily, be quantified and couched in understandable economic terms, as one useful comparison metric, and so in effect be included in the \( \lambda c_0 \) estimates.

But suppose we choose not to try to quantify such benefits. Then what we give up is some way of roughly ordering an array of alternative possible technologies in terms of the relative degree to which these technologies might satisfy some specific set of goals implied by a particular kind of "non-economic" benefit. We also, in effect, give up the notion of comparison metrics for weighing which of two "non-economic" benefits is "more important," and by "how much," to what party. Without quantifying the benefits at least crudely, we cannot even have a generally acceptable algorithm for simply ordering, in terms of relative dominance, a set of alternative technologies vis-a-vis a specified "non-economic" benefit. The best we appear then to be able to do is to say that in terms of some specific "non-economic" benefit a number of such alternatives are crudely equivalent. If this is the case, the economic arguments again furnish us with a helpful necessary tool for helping to choose among the alternatives, even though now we may not be wholly persuaded that it is a sufficient tool which encompasses all that is important.

These kinds of arguments would, again, not be relevant only if it turned out that there existed some one unique technology satisfying in some singular, unique way a specific, vitally important "non-economic" objective, as is often claimed by technologists when they assert the merits of their particular R&D project, instead of there being available a set of alternative technologies. In general, there is reason
to believe that a set of alternatives will always or almost always exist. In this case, economic arguments, even in the presence of "non-economic" benefits, do not substantially lose their force and importance. In short, accepting the notion of "non-economic" benefits cannot be a license to simply forego economic comparisons. Final decisions on technology choices may well not be solely based on economic arguments, however. This can be the situation particularly if intuitive judgments on the relative importance of members of some set of several "non-economic" arguments contribute powerfully to final decisions. This is just the situation wherein the problems of the "preference relation paradoxes" can begin to come into play, however. This is another reason why even very simple comparison metrics have utility, and economics provides one such metric.

These questions of "non-economic" benefits are part of the large general problem, which goes far beyond the subject of this paper, of computing and comparing appropriately the private rate of return of a project (i.e., revenues or benefits that accrue to, and costs that are borne by, some small, well-defined group, such as a set of firms and investors) with the true social rate of return (i.e., revenues or benefits available to all of society and costs borne by the entire economy). When there are differences between these two rates of return for a project, attributable to "externalities," the rational decision-maker is presumed to act differently according to where he sits. If, for example, one could show a large social rate of return and a small private rate of return, the private investor might well argue that he should act only if there is a subsidy (perhaps provided by the government)
which reflects the benefits available to society as a whole. The governmental decisionmaker might well agree to this argument, and provide a subsidy. The inherent difficulty is of course that in order to compare private and social rates of return, we want in the first place to be able to estimate each, bringing us back to the need for at least rough quantification which characterizes both kinds of return in a more or less standard, common way. Whether we have adequate means for meeting that need is an open question.

To summarize, there are a number of large scale R&D projects proposed or under way in the energy field—which, as now planned, will have high costs \( C_0/C_R \) can be in the range of 25-100, for example), take several decades to consummate, several more decades to implement on an adequately large commercial scale, and have relatively modest goals for the effective \( 1 - \lambda \) values attained. Such projects could quite often be difficult to justify, with reasonable discount rate values, on the basis of the simple model of this paper. On the other hand, the same projects, restructured to have significantly earlier deliverables with more modest R&D goals at some fraction of the cost of the more ambitious version of the project, could have a much firmer economic justification. To accomplish this, we need more often to differentiate sharply between maximal technological embellishment and most useful end results in R&D programs, and to accommodate the needs to reduce R&D to practice earlier, accepting the pain of perturbing an otherwise orderly and congenially long time horizon R&D program. The net economic benefits can be substantial. Incentives for and accommodation to such required actions can of course be different for governmental versus privately sponsored R&D.
In this connection, it is interesting to consider an even simpler model—which essentially removes much of the structure (and hence some of the realism) from the cost and savings streams, but still gives useful insights into the sort of behavior which occurs. Such a model, using the previous symbols, is represented by the diagram:

This very simplest model then gives the following analogues of Eqs. (1), (2), and (3), again putting $\alpha = dT$, $\beta = \frac{T_M}{T}$:

1. $R_D = \frac{C_RT}{\alpha} \left(1 - e^{-\alpha}\right)$

2. $S_{D_N} = (1 - \lambda)C_O T \left(\frac{e^{-\beta \alpha}}{\alpha}\right)$

3. $\frac{N}{1 - \lambda} = \frac{C_O}{C_R} \left(\frac{e^{-\beta \alpha}}{1 - e^{-\alpha}}\right)$

The gross behavior of this very simplest model is like that of the earlier one, but in addition the simpler forms of 1., 2., and 3. makes it convenient and easy to derive analytically a number of results.
which bear on sensitivity questions. Thus, we can find pertinent "influence coefficients" when various parameters are varied. For example:

\[
\frac{\partial S_D}{\partial \lambda} = \left( \frac{C_0}{\alpha} \right) (e^{-\beta \alpha});
\]

\[
\frac{\partial S_D}{\partial \beta} = \left( \frac{C_0}{\alpha} \right) (1 - \lambda) \left( \frac{e^{-\beta \alpha}}{\alpha - 1} \right) = \left( \frac{1 - \lambda}{\alpha - 1} \right) \cdot \frac{\partial S_D}{\partial \lambda};
\]

\[
\frac{\partial S_D}{\partial d} = \left[ \frac{1 - \lambda}{d} \cdot (1 + \beta \alpha) \right] \cdot \frac{\partial S_D}{\partial \lambda}.
\]

If then, to take a specific case, we put: \(\lambda = 0.7\); \(\alpha = 2\); \(\beta = 2\);

\(d = 0.10\), we get, for example: \(\frac{\partial S_D}{\partial \beta} = 0.60 \frac{\partial S_D}{\partial \lambda}; \frac{\partial S_D}{\partial (d)} = 15 \frac{\partial S_D}{\partial \lambda}\),

showing the comparative effects on \(S_D\) of changes in \(\beta, \lambda\) and \(d\). This is a useful, very simple model to comprehend.

We can also proceed in the opposite direction, and try to refine the earlier model somewhat. This is again easy to do. For example, we can approximate the undiscounted cost stream by a function like \(R(t) = C_R t^n e^{-bt}\), the undiscounted savings stream by a function like \(S(t) = C_S \left\{ (1 - \lambda) - (1 - \lambda)e^{-c(t - T)} \right\}\). Both these can be easily integrated when multiplied by the discount factor \(e^{-dt}\), and values of \(n, b, c\) chosen to reflect some realism in the shapes of the cost streams and savings streams.

These, and even more complicated but still tractable, refinements add nothing basically new to the behavior of the earlier model, however. The earlier model represents a convenient compromise between some structure in the cost and savings streams, and excessive complication.
which does not add conceptually to discussion of the sorts of problems
we are interested in. In any case, we need to avoid a spurious,
apparently increased, precision in the mere arithmetic of such problems.
No matter how seemingly precise the arithmetic is, we are still approxi-
mating any real situation by the assumptions that we can predict future
cost and savings streams, outcomes of R&D, and so on. There is no
point, therefore, in basically purposeless model elaboration for our
kinds of questions.

The very simplest model of this Appendix also makes clear the
notion of "virtual" discounting noted on p. 10. Suppose the original
undiscounted costs grow like \( C_0 e^{bt} \), and the new undiscounted costs grow
like \( \lambda C_0 e^{ht} \), allowing for exponential behavior in \( \lambda \) as well. Taking
the difference of these two, multiplying by the discount factor \( e^{-dt} \),
and integrating from \( T_M \) to \( \infty \) gives the discounted savings as

\[
S_{D_\infty} = \frac{C_0}{d_1} e^{-d_1 T_M} - \frac{\lambda C_0}{d_2} e^{-d_2 T_M}, \text{ with } d_1 = d - b, d_2 = d - h. \text{ With }
\]

\[\alpha_1 = d_1 T, \alpha_2 = d_2 T, \beta = \frac{T_M}{T} \text{ we have:} \]

\[
S_{D_\infty} = \frac{C_0}{\alpha_1} e^{-\alpha_1 T_M} \left[ 1 - \frac{\alpha_1}{\alpha_2} \lambda e^{\beta (\alpha_1 - \alpha_2)} \right], \text{ showing that if } b = h, \text{ or}
\]

\[\alpha_1 = \alpha_2, \text{ the net result is just to introduce the "virtual" discount}
\]
rate \( d_1 \), which is \( < d \) if \( b > 0 \), and so increases \( S_{D_\infty} \). If, on the
other hand, \( h < b \), so that \( \alpha_1 < \alpha_2 \), the term in brackets also changes
to still further increase \( S_{D_\infty} \). Many types of functional behavior
approximate exponentials (over limited intervals), so that such
formulations are convenient.