USE OF QUADTREES FOR EDGE ENHANCEMENT

Sanjay Ranade
Computer Vision Laboratory
Computer Science Center
University of Maryland
College Park, MD 20742

ABSTRACT

A quadtree approximation of a \(2^n \times 2^n\) gray level image tends to enhance edges that are strong in the original image and suppress those that are weak. An edge detector applied to such an approximation produces edges that are in general stronger than those obtainable from the original, though slightly displaced. Methods of combining the edge maps obtained from the original image and its quadtree approximation are examined. It is shown that the choice of a suitable combination rule results in an edge map which is superior to that obtainable by conventional means.

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1. Introduction

A $2^n \times 2^n$ binary image can be represented by a quadtree by successively subdividing it into four quadrants [1]. Quadrants which contain both white and black areas give rise to branch or "gray" nodes in the tree, while those which are either all black or all white are black or white leaf nodes and are not further subdivided. A quadtree representation is useful, for example, for efficient computation of moments [2], or for the approximation of shapes [3].

An approximate quadtree representation can also be used for $2^n \times 2^n$ gray level images. In this case quadrants through which edges pass, or that have a busy texture, will correspond to branch nodes in the tree, while those that are sufficiently homogeneous will be represented by leaf nodes. Clearly, the measure of homogeneity used for classifying a quadrant as a leaf or branch is important not only for the size of the tree but also for the quality of the image when it is reconstructed from the tree. If the measure is too strict, the size of the tree will be prohibitively large, whereas if it is too loose, the quality of the reconstructed "g-image" will be poor. For a discussion of different measures of homogeneity and their effects on the size of the resulting quadtrees and the quality of the corresponding q-images see [4]. The usefulness of the quadtree representation is not, however, limited to image compression. This paper considers its use for detecting and enhancing edge information in an image.
2. **Edge detection**

Edge detection is a fundamental part of image analysis since it can aid in locating regions or objects in the image. Edge detection techniques in current use (e.g., see [5]) are based on examining a fixed neighborhood of a point, and for this reason often produce "noise" or spurious edges, i.e., edges that would not be regarded as such in the context of the whole image. However, since noise edges typically tend to be weak, they can effectively be removed or disregarded by thresholding. Unfortunately, a global threshold cannot in general be calculated, since there is no way to predict the edge response for an arbitrary image. A common technique used to enhance edges is non-maximum suppression [5], while another which is applicable only to straight edges is described in [6].

Edge information can also be obtained from the quadtree approximation of an image, either by traversing the tree and examining the neighbors of leaf nodes, or by applying an edge operator to the q-image. This information is characterized in four ways. Firstly, since the gray level of a leaf in the quadtree is uniform, edges will occur only on the boundaries of leaves. Secondly, all the edges will be continuous. Thirdly, weak edges in the original image will tend to be suppressed while strong ones will be enhanced. This is because the reasonably homogeneous regions, which we can expect to contain the
weak edges, correspond to leaf nodes that are of approximately constant gray level. Fourthly, a great majority of the edges will be either vertical or horizontal, with the result that curved edges in the original image will be represented by step edges, giving the resulting edge picture a false, blocky appearance.

Figure 1 shows three pairs of images and Figure 2 shows their respective histograms. Figure 1(a) shows the image of a tank and its corresponding q-image. Note that the q-image is a good approximation to the original image. This is confirmed by the histograms in Figures 2(a). (As an aside also note that a valley which is barely discernible in the first histogram is significantly enhanced in the second. The bottom of this valley corresponds to a threshold separating the tank from the background, suggesting the usefulness of the quadtree representation for segmentation [7]).

Figure 1(b) shows the results of applying the Prewitt edge operator to the two images in (a). The first image shows that there are numerous weak edges and proportionately fewer strong ones. In the second image a large proportion of the weak edges have been eliminated, and those that remain are much better defined, though "blocky" as mentioned above. Comparison of the two histograms in Figure 2(b) confirms this observation. Figure 1(c) shows the results of applying non-maximum suppression
based on edge direction to the two images in (b). In both images this process results in the thinning of edges. However a large number of spurious edges still remain, as is evident from Figure 3 which shows the images in Figure 1(b) and 1(c) thresholded at 0. Table 1 shows the total numbers of edge points in these images.

The images in Figure 1(c) are rather unsatisfactory as edge maps of the object in Figure 1(a): the first because of the large number of noise edges and the lack of an algorithmic method to obtain a single global threshold for deleting them, and the second because of the false grid-like appearance of the edges. The following section investigates approaches to combining the edge response from the original and q-images so as to obtain an edge map that is superior to both.
3. **Edge enhancement**

From the discussion in the previous section it is apparent that an edge detector applied to the q-image will tend to suppress weak edges and enhance strong ones present in the original image. These edges are likely to be displaced, since edges can only occur at boundaries of leaves, i.e., if in the original an edge appears in an area corresponding to the interior of a leaf, it will appear in the q-image at the nearest leaf boundary. However, if the q-image approximates the original one reasonably well, the displacements are likely to be small. This fact enables us to use edges obtained from the q-image ("q-edges") as a guide to preserve strong edges obtained from the original image.

Suppose that \( E_0 \) represents the edge response from the q-image and \( E_1 \) represents the edge response from the original image. \( E_0 \) and \( E_1 \) can be combined by some function \( f_0(E_0, E_1) \) to give a new edge map \( E_2 \). The function \( f_0 \) must be chosen such that points having high values in both \( E_0 \) and \( E_1 \) are enhanced, while those with low values in both \( E_0 \) and \( E_1 \) are suppressed. However, since edges in the q-image are likely to be displaced, \( f \) must be modified to include a neighborhood, rather than a single point in \( E_1 \). Thus a point in the new image \( E_2 \) is given by

\[
E_2(x,y) = f(E_0(x,y),g(E_1(x,y)))
\]

(1)

\( g \) must reflect the requirements that the likelihood of a q-edge
corresponding to a real edge at \((x,y)\) decreases as its distance from \((x,y)\). Thus \(g\) could take the form

\[
g_1(E_1(x,y)) = \frac{M}{d}
\]

where \(M\) is the maximum \(q\)-edge value in an \(mxm\) neighborhood about \((x,y)\), and \(d\) is the distance of this point from \((x,y)\).

If the direction of the edge at \((x,y)\) is known, then we can define \(g = g_2\), in which \(M\) is based on only those points that are in the appropriate edge direction in the \(mxm\) neighborhood about \((x,y)\). Another form of \(g\), \(g = g_3\), is simply an average of the \(q\)-edge values in the \(mxm\) neighborhood about \((x,y)\).

The function \(f_1(e_0,e_1)\) where \(e_0 = E_0(x,y)\) and \(e_1 = E_1(x,y)\), must reflect the requirement that the likelihood of a strong edge increases when both \(e_0\) and \(e_1\) are high, and decreases when they are both low. Thus a suitable form for \(f\) is

\[
f_1 = 1 - \frac{1}{1+e_0+e_1} = \frac{e_0+e_1}{1+e_0+e_1}
\]

However, from (3) it is seen that even if \(e_1=0\), \(f\) may still have a non-zero value. This results in edge points appearing in the new map \(E_2\), even though none were present in the corresponding positions in \(E_1\). Re-defining \(f\) so that

\[
f_2 = \begin{cases} 0 & \text{if } e_1 = 0 \\ 1 - \frac{1}{1+e_0+e_1} & \text{if } e_1 > 0 \end{cases}
\]

will ensure that no new edges appear in \(E_2\). However, it is still possible that weak edges that are very near strong ones
may also be enhanced. Since the edge directions at all points in $E_1$ are known, a remedy for this broadening effect is to apply non-maximum suppression to the new edge map $E_2$. Another approach to prevent the broadening effect is to modify $f$ so that the value of a point in $E_1$ is allowed to decrease (leading to the suppression of weak edges) but not to increase (preventing broadening). $f$ then takes the form

$$f_3 = \min(E_1(x,y), f_2)$$

An alternative form of $f$ could be

$$f_5 = \min(E_1(x,y), f_4)$$

where

$$f_4 = 1 - \frac{1}{1 + e_{1}e_{1}}.$$

The relation (1) can be generalized so that

$$E_{i+1}(x,y) = f(E_0(x,y), g(E_1(x,y)))$$

i.e. the edge enhancement process can be iterated. This kind of iterative enhancement will only be useful in the case the function $g$ is an average of the $m \times m$ $g$-edge values in the neighborhood about $(x,y)$, since for $g=g_1$ or $g_2$, iterations after the first will have no effect.

In the discussion that follows, $f = f_3$, $g = g_3$, and a $3 \times 3$ neighborhood is used. Figure 4(a) shows an image $E_2$ where $E_0$ and $E_1$ are the images in Figure 1(b) and 1(c) respectively. Figure 5(a) shows Figure 4(a) thresholded at 0, and Figure 6(a) shows the histogram of the non-zero points in Figure 4(a). Comparison of Figure 4(a) with Figure 1(c) shows that a large proportion of the weak edges have been eliminated. Contrasting
the histogram in Figure 6(a) with that in 2(c) confirms this clearly—whereas in Figure 1(c) weak edges by far outnumber the strong ones, in 4(a) the numbers of strong and weak responses are more even. Note from Table 1 that the total number of edge points in Figure 4(a) for the case \( g=g_2 \) is only about one third of the number of edge points when non-maximum suppression is applied to the edge response for the original image in Figure 1(a). This clearly demonstrates the effectiveness of this kind of edge enhancement scheme. The corresponding results for the case \( f=f_2 \) and \( g=g_2 \) are shown in Figures 4(b), 5(b), and 6(b) respectively. The effectiveness of iterative edge enhancement using the relation (7), \( f=f_3 \) and \( g=g_3 \) are shown in Figure 7. Figure 7(a) is an image of a part of the Frederick, Md., airport. Figure 7(b) is the result of non-maximum suppression applied to the Prewitt edge detector response for 7(a), thresholded at 0. Figures 7(c) to 7(g) are the successive results of five iterations of the enhancement process, and Figure 7(h) is a plot of the number of edge points surviving after each iteration. Note that iteration 0 corresponds to Figure 7(c). The number of edge points surviving after each iteration diminishes significantly.
4. Discussion

Table 1 shows additional results for the tank image for the cases where $f = f_2$ and/or $g = g_2$. It is seen that the best result is obtained for $f = f_3$ and $g = g_3$. This can be explained by noting first of all that the averaging implicit in $g_3$ tends to suppress weak values more effectively than $g = g_2$. The fact that $f_3$ is better than $f_2$ is not evident after the first iteration but becomes apparent after further iterations as is seen from Table 2. This is because $f_2$ allows the value of a point in $E_2$ to increase and the broadening thus caused becomes increasingly noticeable with successive iterations.

It should be noted that although iterative enhancement using $f = f_3$ and $g = g_3$ is quite effective, too many iterations cause edges that were originally continuous to break to an unacceptable degree. This effect, though not excessive, can be seen in Figures 7(c) to 7(g).
5. Conclusion

It has been shown that a quadtree approximation of a $2^n$ by $2^n$ gray level image can be useful in eliminating much of the noise obtained when a conventional edge operator is applied to the original image. The usefulness stems from the observation that strong edges in the original tend to be enhanced in the q-image. An edge operator applied to the q-image yields clean, stronger and continuous, though possibly somewhat displaced, grid-like edges. These edge responses can be combined with those obtained from the original image to give a new edge map that is superior to both.

Clear, noise-free edges are critical to several segmentation schemes such as that described in [8]. The application of quadtrees of gray level images to image segmentation will be considered in a later paper.
References


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<th>$f,g$</th>
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<td>$f=f_3$, $g=g_3$</td>
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Table 1. Results of the first iteration of the enhancement process applied to the image in Figure 1(c).

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<tr>
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<td>520/427</td>
</tr>
<tr>
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<td>318/263</td>
<td>313/260</td>
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<tr>
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<td>4</td>
<td>208/192</td>
<td>179/172</td>
</tr>
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</table>

Table 2. Results of successive iterations of the enhancement process applied to the image in Figure 1(c) using $g=g_3$ before and after non-maximum suppression.
Figure 1. (a) Image of a tank and its corresponding q-image; (b) the results of applying the Prewitt edge operator to (a); (c) the results of applying non-maximal suppression to (b).

Figure 2. Histograms of the images in Figure 1. Note that only non-zero values are histogrammed in (b) and (c).
Figure 3. (a) Images in Figure 1(b) thresholded at 0.
(b) Images in Figure 1(c) thresholded at 0.

Figure 4. (a) Resulting image $E_2$ for the case $f=f_2$.
(b) Resulting image $E_2$ for the case $f=f_3$. 
Figure 5. (a) Figure 4(a) thresholded at 0.
(b) Figure 4(b) thresholded at 0.

Figure 6. (a) Histogram of non-zero values for Figure 4(a).
(b) Histogram of non-zero values for Figure 4(b).
Figure 7. (a) Image of a part of Frederick Airport. (b) Result of non-maximum suppression applied to the Prewitt edge detector output for 7(a) thresholded at 0. (c) to (g) Results of successive iterations of the enhancement process thresholded at 0.
Number of surviving edge points.

- - - Runway

- - Tank

Iteration #
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**Author(s):** Sanjay Ranade

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