Analysis of Conventional and Reflective Butler Matrices with Imperfect Components

J. P. Shelton and J. K. Hsiao

Target Characteristics Branch
Radar Division

March 18, 1980

NAVAL RESEARCH LABORATORY
Washington, D.C.

Approved for public release; distribution unlimited.
**REPORT DOCUMENTATION PAGE**

**REVIEW INSTRUCTIONS BEFORE COMPLETING FORM**

1. **REPORT NUMBER**
   - NRL Report 8392

2. **GOVT ACCESSION NUMBER**
   - AD-A083474

4. **TITLE (and sub-title)**
   - ANALYSIS OF CONVENTIONAL AND REFLECTIVE BUTLER MATRICES WITH IMPERFECT COMPONENTS

7. **AUTHOR(s)**
   - J.P. Shelton and J.K. Hsiao

9. **PERFORMING ORGANIZATION NAME AND ADDRESS**
   - Naval Research Laboratory
   - Washington, DC 20375

10. **PROGRAM ELEMENT, PROJECT, TASK AREA & WORK UNIT NUMBERS**
    - 61153N; R021-05-41; 63-0624-6-0

11. **CONTRACT OR GRANT NUMBER(S)**

12. **REPORT DATE**
    - March 18, 1980

13. **NUMBER OF PAGES**
    - 45

15. **SECURITY CLASS. (of this report)**
    - UNCLASSIFIED

16. **DISTRIBUTION STATEMENT (of this report)**
    - Approved for public release; distribution unlimited.

17. **DISTRIBUTION STATEMENT (of the abstract entered in Block 20, if different from report)**

18. **SUPPLEMENTARY NOTES**

19. **KEY WORDS**
   - Multibeam antenna
   - Butler matrix
   - Array antenna
   - Multiplex beam form
   - Network

20. **ABSTRACT**
    - In a reflective Butler matrix the input ports are coincident with the output ports. Hence, all error components produced by imperfect hybrid couplers accumulate at the single set of input/output ports, rather than being distributed at the separate sets of input and output ports as in the conventional Butler matrix. This report gives a precise scattering analysis of both conventional and reflective Butler matrices made up of imperfect hybrid couplers, and the effect on the performance of such matrices is computed. A computer program for carrying out the analysis is also given.
CONTENTS

INTRODUCTION ........................................... 1
SCATTERING MATRIX OF A 3-dB HYBRID COUPLER .... 1
SCATTERING AND TRANSFFER MATRICES OF A
BUTLER NETWORK ...................................... 3
PATTERNS OF AN ARRAY FED BY
A BUTLER NETWORK ............................... 8
CONCLUSIONS ........................................... 11
REFERENCES ............................................ 11
APPENDIX – Computer Program for Analysis .......... 17
ANALYSIS OF CONVENTIONAL AND REFLECTIVE BUTLER MATRICES
WITH IMPERFECT COMPONENTS

INTRODUCTION

A Butler matrix that forms a cluster of beams evenly distributed in the \( \sin \theta \) space is not usually symmetric with respect to a plane midway between the input and output ports. However, by properly adjusting the phase shifts and interconnections one may modify a conventional Butler matrix to be symmetric. Such a matrix may also be folded on itself on the line of symmetry, so that the input and output ports are identical. Such a network not only reduces the number of components required; it also becomes a reflection-type system in which the feed positions are in the plane of the aperture. The synthesis of this network was described previously [1,2]. In this report, we analyze the performance of both conventional and reflective Butler matrices. In particular, we investigate the effect of reflected waves on the beam-forming performance. In a conventional Butler matrix, since the input and output ports are separate, the reflected waves emerging from the input ports have no effect on the beam-forming performance. Multiply reflected waves may emerge from output ports; however, their amplitudes are generally small, and their effects are relatively insignificant. In a reflective Butler matrix, the reflected waves accumulate at the input/output ports; hence, the aperture distribution at the antenna array is significantly modified, and this may degrade the beam-forming performance. These effects are investigated, and computer simulated results are presented together with a listing of the computer program.

SCATTERING MATRIX OF A 3-dB HYBRID COUPLER

The basic building block of a Butler matrix is a 3-dB hybrid coupler. For the ideal hybrid coupler, energy fed into any one of the input ports will be split into two equal components, one with a phase shift of 90° relative to the other. However, practical hybrid couplers will in general exhibit amplitude and phase errors in their transfer coefficients. These amplitude and phase errors will affect the transfer coefficients of both reflective and conventional Butler matrices in the same way. That is, the errors in the overall network input/output transfer coefficients will be the same for both conventional and reflective networks. Practical hybrid couplers will also have nonzero reflection and transfer coefficients to the isolated port. For the conventional network, to a first order, the error components due to these effects will appear at the network inputs. For the reflective network, with its inputs and outputs sharing a single set of ports, all error components affect the input/output transfer coefficients.

Thus, the two types of hybrid coupler errors are forward and reverse. Our investigation will be concentrated on the reverse-error components, and we shall assume that there

Manuscript submitted January 4, 1980.
is no amplitude or phase error in the forward-transfer coefficients of the 3-dB coupler. The following analysis is based on the assumption that, when an incident wave of unit amplitude is applied to one of the input ports, two waves of amplitude $\alpha$ will emerge from the two output ports, one with a $90^\circ$ phase shift and the other with no phase shift. Similarly, waves of amplitude $\beta$ will be reflected to the two input ports. As shown in Fig. 1(a), when an incident wave of unit amplitude is applied at port 12, reflected waves of $-\beta$ and $-j\beta$ appear at ports 11 and 12 respectively and waves of $-j\alpha$ and $\alpha$ appear at ports 21 and 22. For conservation of energy, one has

$$2\alpha^2 + 2\beta^2 = 1. \quad (1)$$

The isolation factor is defined as the power ratio of the reflected wave to the incident wave. In this case, the isolation is

$$I = \beta^2. \quad (2)$$

Accordingly, in terms of the isolation factor,

$$\alpha = \sqrt{0.5 - I}. \quad (3)$$

If the parameters in Fig. 1(b) are used, the reflected waves are related to the incident waves by the matrix equation

$$\begin{bmatrix} b_{11} \\ b_{12} \\ b_{21} \\ b_{22} \end{bmatrix} = \begin{bmatrix} -j\beta & -\beta \\ -\beta & -j\beta \\ \alpha & -j\alpha \\ -j\alpha & \alpha \end{bmatrix} \begin{bmatrix} a_{11} \\ a_{12} \\ a_{21} \\ a_{22} \end{bmatrix}, \quad (4)$$

where $a_{11}, a_{12}, a_{21},$ and $a_{22}$ are incident waves and $b_{11}, b_{12}, b_{21},$ and $b_{22}$ are scattered waves at ports 11, 12, 21, and 22 respectively.

Let

$$b_1 = \begin{bmatrix} b_{11} \\ b_{12} \end{bmatrix}, \quad b_2 = \begin{bmatrix} b_{21} \\ b_{22} \end{bmatrix}, \quad (5a)$$

$$a_1 = \begin{bmatrix} a_{11} \\ a_{12} \end{bmatrix}, \quad a_2 = \begin{bmatrix} a_{21} \\ a_{22} \end{bmatrix},$$
Matrix Eq. (4) can now be simplified to the form

\[
\begin{bmatrix}
  b_1 \\
  b_2
\end{bmatrix} =
\begin{bmatrix}
  S_{11} & S_{12} \\
  S_{21} & S_{22}
\end{bmatrix}
\begin{bmatrix}
  a_1 \\
  a_2
\end{bmatrix}
\] (6)

SCATTERING AND TRANSFER MATRICES OF A BUTLER NETWORK

A Butler network can be represented by a block diagram as shown in Fig. 2. Blocks in regions 1 and 3 represent the 3-dB couplers described in the previous section, and a phase-shift transfer network is located in region 2. A number of similar networks are
connected in cascade to form a complete conventional Butler network. The scattering matrix for regions 1 and 3 is

\[
\begin{bmatrix}
    b_{11} & -j\beta & -\beta & 0 & 0 & \cdots & \alpha & -j\alpha & 0 & 0 & \cdots & 0 \\
    b_{12} & -\beta & -j\beta & 0 & 0 & \cdots & -j\alpha & \alpha & 0 & 0 & \cdots & 0 \\
    \vdots & \vdots & \vdots & \vdots & \vdots & \ddots & \vdots & \vdots & \vdots & \vdots & \ddots & \vdots \\
    b_{1n} & 0 & 0 & -j\beta & -\beta & 0 & 0 & \cdots & \alpha & -j\alpha & 0 & \cdots \\
    b_{21} & 0 & 0 & -j\alpha & 0 & 0 & \cdots & -j\beta & -\beta & 0 & 0 & \cdots \\
    b_{22} & -j\alpha & -j\beta & 0 & 0 & \cdots & -\beta & -j\beta & 0 & 0 & \cdots \\
    \vdots & \vdots & \vdots & \vdots & \vdots & \ddots & \vdots & \vdots & \vdots & \vdots & \ddots & \vdots \\
    b_{2n} & 0 & 0 & -j\alpha & 0 & 0 & \cdots & -\beta & -j\beta & 0 & 0 & \cdots \\
    \vdots & \vdots & \vdots & \vdots & \vdots & \ddots & \vdots & \vdots & \vdots & \vdots & \ddots & \vdots \\
    \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots \\
\end{bmatrix}
\begin{bmatrix}
    a_{11} \\
    a_{12} \\
    \vdots \\
    a_{1n} \\
    a_{21} \\
    a_{22} \\
    \vdots \\
    a_{2n}
\end{bmatrix}
\]

Define

\[
b_1 = \begin{bmatrix}
    b_{11} \\
    b_{12} \\
    \vdots \\
    b_{1n}
\end{bmatrix},
b_2 = \begin{bmatrix}
    b_{21} \\
    b_{22} \\
    \vdots \\
    b_{2n}
\end{bmatrix}
\]

\[
a_1 = \begin{bmatrix}
    a_{11} \\
    a_{12} \\
    \vdots \\
    a_{1n}
\end{bmatrix},
a_2 = \begin{bmatrix}
    a_{21} \\
    a_{22} \\
    \vdots \\
    a_{2n}
\end{bmatrix}
\]

\[
S_{11} = S_{22} = \begin{bmatrix}
    -j\beta & -\beta & 0 & 0 & \cdots & \cdots \\
    -\beta & -j\beta & 0 & 0 & \cdots & \cdots \\
    0 & 0 & -j\beta & -\beta & 0 & 0 & \cdots \\
    0 & 0 & -\beta & -j\beta & 0 & 0 & \cdots \\
    \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots \\
    0 & 0 & \cdots & \cdots & -j\beta & -\beta \\
    0 & 0 & \cdots & \cdots & -\beta & -j\beta
\end{bmatrix}
\]

4
Fig. 2 — Block diagram of a Butler network
and

\[
S_{12} = S_{21} = \begin{bmatrix}
\alpha & -j\alpha & 0 & 0 & \cdots & \cdots \\
-j\alpha & \alpha & 0 & 0 & \cdots & \cdots \\
0 & 0 & \alpha & -j\alpha & 0 & 0 \\
0 & 0 & -j\alpha & \alpha & 0 & 0 \\
\cdots & \cdots & \cdots & \cdots & \cdots & \cdots \\
0 & 0 & \cdots & \cdots & \alpha & -j\alpha \\
0 & 0 & \cdots & \cdots & -j\alpha & \alpha 
\end{bmatrix}.
\tag{8d}
\]

Equation (7) can now be simplified to

\[
\begin{bmatrix}
\mathbf{b}_1 \\
\mathbf{b}_2
\end{bmatrix} =
\begin{bmatrix}
S_{11} & S_{12} \\
S_{21} & S_{22}
\end{bmatrix}
\begin{bmatrix}
\mathbf{a}_1 \\
\mathbf{a}_2
\end{bmatrix}.
\tag{9}
\]

The scattering matrix in region 2, which is a phase-shift and transfer network, can be represented as

\[
\begin{bmatrix}
\mathbf{d}_1 \\
\mathbf{d}_2
\end{bmatrix} =
\begin{bmatrix}
R_{11} & R_{12} \\
R_{21} & R_{22}
\end{bmatrix}
\begin{bmatrix}
\mathbf{c}_1 \\
\mathbf{c}_2
\end{bmatrix},
\tag{10}
\]

where \(\mathbf{d}_1, \mathbf{d}_2, \mathbf{c}_1,\) and \(\mathbf{c}_2\) are vectors such that

\[
\mathbf{d}_1 = \begin{bmatrix}
d_{11} \\
d_{12} \\
\vdots \\
d_{1n}
\end{bmatrix}, \quad \mathbf{d}_2 = \begin{bmatrix}
d_{21} \\
d_{22} \\
\vdots \\
d_{2n}
\end{bmatrix},
\tag{11a}
\]

\[
\mathbf{c}_1 = \begin{bmatrix}
c_{11} \\
c_{12} \\
\vdots \\
c_{1n}
\end{bmatrix}, \quad \mathbf{c}_2 = \begin{bmatrix}
c_{21} \\
c_{22} \\
\vdots \\
c_{2n}
\end{bmatrix}.
\tag{11b}
\]

Matrices \(R_{11}\) and \(R_{22}\) are zero, and matrices \(R_{12}\) and \(R_{21}\) have identical elements. These matrices describe the phase shifts and interconnections from one row of couplers to the
next. Their elements depend on the configuration of the Butler network. As an example, the $R$ matrix of the 4-port Butler network shown in Fig. 3 is

$$
R_{12} = R_{21} = \begin{bmatrix}
1 & 0 & 0 & 0 \\
0 & 0 & e^{-j\frac{\pi}{4}} & 0 \\
0 & e^{-j\frac{\pi}{4}} & 0 & 0 \\
0 & 0 & 0 & 1
\end{bmatrix}
$$

(12)

Fig. 3 — Four-port Butler network

Since we are interested in the overall scattering matrix of this network, we must first convert the scattering matrix in each region to a transfer matrix, which in turn can be multiplied to form the overall transfer matrix of the whole network. A transfer matrix can be represented as

$$
\begin{bmatrix}
\begin{bmatrix}
b_2 \\
a_2
\end{bmatrix} \\
\begin{bmatrix}
a_1 \\
b_1
\end{bmatrix}
\end{bmatrix} =
\begin{bmatrix}
T_{11} & T_{12} \\
T_{21} & T_{22}
\end{bmatrix}
\begin{bmatrix}
\begin{bmatrix}
a_1 \\
b_1
\end{bmatrix}
\end{bmatrix},
$$

(13)

where $a_1$ and $b_1$ are the incident and reflected waves at the left hand ports and $a_2$ and $b_2$ are similar waves at the right hand ports.

It can be shown that a matrix $T$ is related to an $S$ matrix by the following relations [3,4]:

$$
T_{11} = S_{21} - S_{22} S_{12}^{-1} S_{11},
$$

(14a)

$$
T_{12} = S_{22} S_{12}^{-1},
$$

(14b)
\[ T_{21} = -S_{12}^{-1}S_{11}, \]  
(14c)

and

\[ T_{22} = S_{12}^{-1}. \]  
(14d)

The overall transfer matrix is

\[ T = \prod_{i=1}^{k} T_i \]  
(15)

where \( T_1, T_2, \ldots, T_k \) are transfer matrices in regions 1, 2, ..., \( k \).

The overall transfer matrix can be converted to a scattering matrix by the relations

\[ S_{11} = -T_{22}^{-1}T_{21}, \]  
(16a)

\[ S_{12} = T_{22}^{-1}, \]  
(16b)

\[ S_{21} = T_{11} - T_{12}T_{22}^{-1}T_{21}, \]  
(16c)

and

\[ S_{22} = T_{12}T_{22}^{-1}. \]  
(16d)

Since \( S_{12} = S_{21} \), one may use the simpler relation of Eq. (16b) instead of Eq. (16c).

Elements of matrix \( S_{21} \) (or \( S_{12} \)) represent the transmitted waves at the output ports when a unit incident wave is applied at any one of the input ports. Therefore, matrix \( S_{21} \) is the transfer function of a conventional Butler network. Elements of matrix \( S_{11} \) (or \( S_{22} \)) represent the reflected waves at the input ports when a unit incident wave is applied at any one of the input ports. In a reflective Butler network both the reflected waves and transmitted waves emerge from the same set of ports. Therefore, the scattering matrix of such a network is the sum of matrices \( S_{12} \) and \( S_{11} \), or

\[ S = S_{11} + S_{12}. \]  
(17)

In deriving this relation, we have made the assumption that the symmetry plane of a reflective Butler network exhibits an open-circuit unity reflection coefficient.

**Patterns of an Array Fed by a Butler Network**

Figure 4 shows a schematic diagram of a reflective Butler network, which has half the components of a conventional Butler network. There are \( n \) ports, since ports a11,
$a_1, a_2, \ldots, a_n$ are identical with ports $a_{21}, a_{22}, \ldots, a_{2n}$. Using previously developed notation and setting $[b_2] = [a_2] = 0$, this can be represented as

$$[b_1] = [S_{11} + S_{12}][a_1].$$  \hspace{1cm} (18)

![Diagram of Reflective Butler network]

Fig. 4 - Reflective Butler network

The vector input of $[a_1]$ can be represented, for the case of an incident plane wave received by a linear array, by

$$a_{1k} = A_k \exp [j(k - 1)u]$$  \hspace{1cm} (19),

where $u = 2\pi d \sin \theta / \lambda$,

with $\lambda$ = wavelength, $\theta$ = angle of incidence from the normal to the array, and $d$ = element spacing.
In the subsequent discussion, we shall assume that the array has a uniform illumination function, that is, that $A_k = 1$. The scattering matrix $[S_{11}] + [S_{12}]$ is computed as a function of isolation factor $I$. Radiation patterns of the network-fed array are represented by two types of plot. One shows the main beams formed by several ports of the reflective Butler network, and the other shows the complete array pattern of one port of the network, in the range $0 \leq u \leq 180^\circ$.

Figure 5 shows the array patterns of an eight-port reflective Butler network. Figure 5a shows four of the main beams for variation of the isolation factor of the 3-dB hybrid from 10 dB to 40 dB. Figure 5b shows the array pattern when the main beam is at $u = 22.5^\circ$ for the same range of isolation factor. Figure 6 shows the corresponding patterns for a 16-port reflective network. From these figures, it can be seen that the null filling level is roughly equal to the isolation factor of the 3-dB couplers. That is, for the case of 10-dB isolation, the pattern is filled to a level of about 10 dB below its peak; and for the case of 40 dB isolation, the pattern is filled to a level of about 40 dB below its peak.

Tables 1 and 2 show computed results for eight-port and 16-port reflective Butler networks, respectively. The isolation factors in dB are listed in the first column. The transmitted power is the percentage of incident power, averaged over all inputs and outputs, that would emerge from the outputs for the conventional Butler-network configuration. The remaining power emerges from the input ports. It is seen that the transmitted power decreases as the isolation decreases and as the number of rows of couplers in the network increases. For the reflective-network configuration, the input and output ports are combined, and the components emerging from these ports are also combined. The RMS amplitude and phase errors describe the effects of these spurious components on the combined outputs and are defined by

$$
\Delta b = \left[ \frac{\sum_{k=1}^{N} \sum_{k'=1}^{N} \left| s_{k'k} \right|^2 - \bar{s}}{N^2} \right]^{1/2}
$$

and

$$
\Delta \phi = \left[ \frac{\sum_{k=1}^{N} \sum_{k'=1}^{N} \left( \phi_{k'k} - \phi_{k'k} \right)^2}{N^2} \right]^{1/2},
$$

where $\Delta b$ and $\Delta \phi$ are the RMS amplitude and phase errors, respectively, $s_{k'k}$ is an element of the scattering matrix $S$,

$$
\bar{s} = \frac{1}{N^2} \sum_{k=1}^{N} \sum_{k'=1}^{N} |s_{k'k}| \sqrt{N^2},
$$
\( \phi_{kk} \) is the phase of \( s_{kk} \), and \( \phi'_{kk} \) is the phase of \( s_{kk} \) for the ideal network with no errors. The error components increase with the number of rows of couplers and with decreasing isolation.

A computer program for carrying out these calculations is listed in the appendix. In addition to providing for imperfect reverse parameters of the hybrid couplers, the program provides for imperfect forward parameters and for errors in the interconnecting transmission lines.

CONCLUSIONS

An exact analysis procedure has been developed that is applicable to both conventional and reflective Butler networks with imperfect components. The analytical procedure has been programmed for computation of results for conventional and reflective Butler networks of arbitrary size. Results are presented for eight-port and 16-port reflective networks using hybrid couplers with varying degrees of isolation. The results are given in the form of radiation-pattern factors that would be obtained from a linear antenna array fed by the network and also in terms of the RMS phase and amplitude errors of the network transfer coefficients.

REFERENCES


Fig. 5a — Main-beam pattern of an eight-port reflective Butler network; isolation factor varies from 10 dB to 40 dB
Fig. 5b — Array pattern of an eight-port reflective Butler network; isolation factor varies from 10 dB to 40 dB; main beam at \( \theta = 22.5^\circ \)
Fig. 6a — Main beam pattern of a 16-port reflective Butler network; isolation factor varies from 10 dB to 40 dB
Fig. 6b — Array pattern of a 16-port reflective Butler network; isolation factor varies from 10 to 40 dB; main beam at $\theta = 11.25^\circ$
Table 1 — Computed Statistical Parameters for Eight-Port Reflective Network

<table>
<thead>
<tr>
<th>Isolation (dB)</th>
<th>Transmitted Power (percent of incident)</th>
<th>RMS Amplitude Error (percent)</th>
<th>RMS Phase Error (degrees)</th>
</tr>
</thead>
<tbody>
<tr>
<td>10</td>
<td>54.85</td>
<td>30.25</td>
<td>38.69</td>
</tr>
<tr>
<td>15</td>
<td>80.76</td>
<td>19.42</td>
<td>23.53</td>
</tr>
<tr>
<td>20</td>
<td>93.20</td>
<td>12.30</td>
<td>12.97</td>
</tr>
<tr>
<td>25</td>
<td>97.77</td>
<td>7.30</td>
<td>7.16</td>
</tr>
<tr>
<td>30</td>
<td>99.29</td>
<td>4.21</td>
<td>3.99</td>
</tr>
<tr>
<td>35</td>
<td>99.77</td>
<td>2.39</td>
<td>2.24</td>
</tr>
<tr>
<td>40</td>
<td>99.93</td>
<td>1.35</td>
<td>1.25</td>
</tr>
</tbody>
</table>

Table 2 — Computed Statistical Parameters for 16-Port Reflective Network

<table>
<thead>
<tr>
<th>Isolation (dB)</th>
<th>Transmitted Power (percent of incident)</th>
<th>RMS Amplitude Error (percent)</th>
<th>RMS Phase Error (degrees)</th>
</tr>
</thead>
<tbody>
<tr>
<td>10</td>
<td>44.92</td>
<td>47.29</td>
<td>50.58</td>
</tr>
<tr>
<td>15</td>
<td>76.30</td>
<td>31.96</td>
<td>30.95</td>
</tr>
<tr>
<td>20</td>
<td>91.49</td>
<td>20.41</td>
<td>12.66</td>
</tr>
<tr>
<td>25</td>
<td>97.19</td>
<td>11.67</td>
<td>6.91</td>
</tr>
<tr>
<td>30</td>
<td>99.10</td>
<td>6.61</td>
<td>3.84</td>
</tr>
<tr>
<td>35</td>
<td>99.71</td>
<td>3.73</td>
<td>2.16</td>
</tr>
<tr>
<td>40</td>
<td>99.91</td>
<td>2.10</td>
<td>1.22</td>
</tr>
</tbody>
</table>
Appendix

COMPUTER PROGRAM FOR ANALYSIS

This computer program computes the coupling coefficients from the input ports to the output ports and the power transmitted and reflected; it also plots the array radiation pattern if it is desired. The type of Butler matrix analyzed by this program can be either a conventional or a reflective type as described in this report. For this program three input data cards are required. The first data card enters the following fixed-point (15 format) data:

- **NPT** — Number of ports of the Butler matrix to be computed.
- **NROW** — Number of rows of this network.
- **KLL** — Absolute value of KLL represents the beam index whose pattern is to be plotted. If KLL = 0, there is no plot. If KLL is less than 0, the program plots the array pattern and also plots all main beams formed by the Butler matrix network.
- **LPRINT** — Printout control. If LPRINT = 0, the program prints all detailed output at each computation step.

The second data card, which is also in a fixed-point 15 format, specifies the number of ports in each basic coupling network in each row. This implies that identical coupling networks are used in each row. However, coupling networks of different ports may be used in different rows.

The third input data card, which has a F10.6 floating-point format, specifies the coupling coefficients of the 3-dB coupler used as the basic building block of the Butler matrix network. These coefficients are read in the sequence A1, B1, C1, D1. These numbers are related to the coupling coefficient of the 3-dB coupler by the relations (see Fig. 1a)

\[ \beta_1 = 10^{-0.05 \times A1} \]
\[ \beta_2 = 10^{-0.05 \times B1} \]
\[ \alpha_1 = 10^{-0.05 \times C1} \]
\[ \alpha_2 = 10^{-0.05 \times D1} \]

and
PROGRAM RFBMTX
C THIS PROGRAM FIRST FIGURES OUT BUTLER MATRIX CONNECTION AND PHASE
C ANGLE, COMPUTES THE TRANSFER FUNCTION AND THEN PLOT THE PATTERN
C MATRIX LIMIT TO THE SIZE OF 64
C COMPILED ON JULY 13,1976 BY J. K. HSIAO
C REVISED ON AUGUST 18,1976 BY J. K. HSIAO
C ABSOLUTE VALUE OF KLL REPRESENTS THE BEAM INDEX WHOSE PATTERN IS
C TO BE PLOTTED
C KLL=0 NO PLOT
C KLL CRATER THAN 0 PLOT PATTERN ONLY
C KLL LESS THAN 0 PLOT BOTH PATTERN AND MAIN BEAMS
C LLL=1, FULL MATRIX, LLL=0 REFLECTIVE MATRIX
C LPRINT !=0, PRINT ALL DETAILED OUTPUTS
C IF LPRINT NOT EQUAL 0 NO MATRIX MULTIPLICATION RESULT IS PRINTED
C IF LPRINT LT 0 PRINT ONLY THE TRANSFER FUNCTION

COMMON/C54/AL1,A2,B1,B2
COMMON/C54/AL4/16/,B1(16)
DIMENSION NOP(16),NBK(16)
DIMENSION MCC8,64),PHA(8,64)
DIMENSION S11(32,32),S12(32,32),S21(32,32),S22(32,32)
COMPLEX S11,S12,S21,S22
CALL PLOTS(PLTAY,500,0.)
NMAX=32
KC=0
READ 100,NTP,NROW,KLL,LPRINT,LLL
IF(NTP.EQ.0)GO TO 2
READ 100,NBP(I),I1,NROW)
FORMAT(6I5)
READ 101, A12,81,82
FORMAT(8F10.6)
IF(KC.EQ.0)CALL ORIGTW14.,0.)
NRI=NROW+1
CALL NTWK(NTP,NRI,NBP,MC,PHA)
IF(KLL.GT.0)GO TO 4
CALL HLFMTX(NTP,NRI,NBP,MC,PHA)
CALL TRFMIX(MMMAX,NTP,NRI,NBP,MC,PHA,S11,S12,S21,S22)
LLL=0
CALL PRTOUT(N1F,S21,S11,LLL,NNAX,LLTFP,LPRINT)
LTFP=1
IF(KLL.EQ.0)GO TO 1
CALL PATERN(NTP,S21,S11,KLL,NPAV,MAX)
GO TO 1
CALL ENDPY
END
SUBROUTINI PRIOUT(NTP,TRFF,TRFB,LL,NIX,NTP,LPRINT)
C LLGT=0 FOR BLOCK, AND LL IS THE BLOCK NUMBER
C LL=0 FOR OVERAL TRANSFER FUNCTION
DIMENSION TRFF(NMX,NMX),TRFB(NMX,NMX)
COMMON/C6/AMPT(32,32),ANGL(32,32),ANGT(32),TRFF2(32,32),TR(32,32)
COMMON/APPI,A2,Bl,82,AP2,3?,ANGL,32,ANCT,32),TRFF2(32,32),TR(32,32)
COMMON/32,32,ANGL,32,AMPRMS(32),ANGRMS(32)
C SUMR(1824)
C COMPLEX TRFF,TRFB,TRFF2,TR,SR
KC=0
PI=3.1415926536
RAC=180./PI
K6=6
LLL=0
IF(A1.LE.0. OR.LL.GT.0)LLL=1
PRINT 101,LL
101 FORMAT(//,20X,'THIS IS THE TRANSFER FUNCTION OF BLOCK',15)
GO TO 1
1 PRINT 111
111 FORMAT(//,20X,'OVERAL TRANSFER FUNCTION')
PRINT 106
106 FORMAT(//,20X,'OVERAL TRANSFER FUNCTION')
IF(A1.GT.0.)GO TO 000
PRINT 119
119 FORMAT(//,10X,'ZERO REFLECTION')
C GENERATE TRANSFER FUNCTION FOR AN IDEAL BUTLER MATRIX
PRINT 124,NTP,A1
124 FORMAT(//,20X,'NUMBER OF PORTS',I5,5X,'ISOLATION(DB)',F10.4,1//)
CALL TRFIDOL(NTP)
125 IF(LPRINT.GT.0)GO TO 4
126 IF(LPRINT.GT.0)GO TO 4
PRINT 107,(AMPT(I,J),J=1,NTP),I=1,NTP)
PRINT 117
107 FORMAT(10X,20F4.1)
PRINT 117
117 FORMAT(10X,20F4.1)
PRINT 111
111 FORMAT(10X,20F4.1)
PRINT 110
110 FORMAT(//,20X,'AMPLITUDE OF FORWARD TRANSFER FUNCTION')
PRINT 102
102 FORMAT(//,20X,'AMPLITUDE OF FORWARD TRANSFER FUNCTION')
PRINT 117
117 FORMAT(//)
PRINT 75
75 FORMAT(//,20X,'PHASE ANGLE OF FORWARD TRANSFER FUNCTION')
SHELTON AND HSIAO

0046 PRINT 117
0047 GO TO 75
0048 73 PRINT 104
0049 104 FORMAT(//'20X,'AMPLITUDE OF REFLECTIVE TRANSFER FUNCTION')
0050 PRINT 117
0051 GO TO 75
0052 74 PRINT 105
0053 105 FORMAT(//'20X,'PHASE ANGLE OF REFLECTIVE TRANSFER FUNCTION')
0054 PRINT 117
0055 GO TO 75
0056 75 PRINT 109
0057 109 FORMAT(//'20X,'AMPLITUDE OF THE RESULTANT TRANSFER FUNCTION')
0058 PRINT 117
0059 GO TO 75
0060 77 PRINT 110
0061 110 FORMAT(//'20X,'PHASE ANGLE OF THE RESULTANT TRANSFER FUNCTION')
0062 PRINT 117
0063 75 GO TO J=I,NTP
0064 GO TO J=I,NTP
0065 GO TO J=I,NTP
0066 61 ANGT(J)=CABS(TRFF(I,J))
0067 ANGT2=ANGT(J)**2
0068 SUM=SUM+ANGT2
0069 SUMR(J)=SUMR(J)+ANGT2
0070 GO TO 70
0071 62 IF(PLNT .GT.0)GO TO 70
0072 ANGTIJ=ANGT(TRFF(I,J)*RAC
0073 GO TO 70
0074 63 ANGT(J)=CABS(TRFB(I,J))
0075 ANGT2=ANGT(J)**2
0076 SUM=SUM+ANGT2
0077 SUMR(J)=SUMR(J)+ANGT2
0078 GO TO 70
0079 64 IF(PLNT .GT.0)GO TO 70
0080 ANGT(J)=CANG(TRFB(I,J))*RAC
0081 GO TO 70
0082 65 TR(I,J) =TRFF(I,J)+TRFB(I,J)
0083 ANGT(J)=CABS(TR(I,J))
0084 ANGT2=ANGT(J)**2
0085 SUM=SUM+ANGT2
0086 SUMR(J)=SUMR(J)+ANGT2
0087 IF(LLL .GT.0)GO TO 70
0088 AMP(I,J)=(ANGT(J)-AMPT(I,J))/AMPT(I,J)
0089 GO TO 70
0090 66 ANGT(J)=CANG(TR(I,J))*RAC
0091 IF(LLL .GT.0)GO TO 70
0092 AG =ANGT(J)-ANGL(I,J)
0093 ANGL(J,J)=AG
0094 IF(CABS(AG).LE.100.)GO TO 70
0095 NSIGN=1

20
0096 IF(AG.GT.0.)NSIGN=-1
0097 ANGL(1,J)-NSIGN*(360.-ABS(AG))
0098 70 CONTINUE
0099 IF(LPRINT .GT.0.)GO TO 67
0100 PRINT 107,(ANGT(J),J=1,NTP)
0101 107 FORMAT(10X,8F10.4)
0102 67 CONTINUE
0103 KMOD=MOD(K+7)
0104 IF(KMOD.LE.0)GO TO 60
0105 PRINT 122,SUM
0106 122 FORMAT(/,10X,"TOTAL POWER OUTPUT",F10.4)
0107 PRINT 123,(SUMR(I),I=1,NTP)
0108 123 FORMAT(/,10X,"POWER FROM EACH PORT",/(10X,10F10.4))
0109 60 CONTINUE
0110 IF(LPRINT .GT.0.)GO TO 7
0111 IF(LPRINT .GT.0.)GO TO 8
0112 DO 50 L=1,2
0113 GO TO (51,52)L
0114 51 PRINT 120
0115 120 FORMAT(/,20X,"ERROR FUNCTION",/,20X,"AMPLITUDE")
0116 GO TO 53
0117 52 PRINT 121
0118 121 FORMAT(/,20X,"PHASE ANGLE")
0119 53 DO 50 I=1,NTP
0120 GO TO (54,55)L
0121 54 PRINT 107, (AMP(I),I=1,NTP)
0122 GO TO 50
0123 55 PRINT 107, (ANG(I),I=1,NTP)
0124 50 CONTINUE
0125 IF(LPRINT .LT.0)RETURN
0126 PRINT 111
0127 111 DO 59 L=1,2
0128 DO 57 I=1,NTP
0129 IF(L.GT.1.AND.I.GT.1)GO TO 58
0130 #NGS=0.
0131 AMP=0.
0132 ANGX=0.
0133 AMPX=0.
0134 58 DO 56 J=1,NTP
0135 AMP=AMPS+AMP(J)
0136 ANG=ANGS+ANG(J)
0137 IF(AMP(J).LT.1.)AMPX=AMP(J)
0138 56 IF(ABS(ANG(J)).LT.ABS(ANGX))ANGX=ANG(J)
0139 IF(L.GT.1)GO TO 57
0140 AMPAV(I)=AMPS/NTP
0141 ANGAV(I)=ANGS/NTP
0142 #MX(I)=AMPX
0143 ANXI(I)=ANGX
0144 57 CONTINUE
0145 IF(L.GT.1)GO TO 59
SHELTN AND HSIAO

0146 PRINT 117
0147 PRINT 107, (AMPAY(K), K=1, NTP)
0148 PRINT 117
0149 PRINT 107, (ANPAY(K), K=1, NTP)
0150 PRINT 117
0151 PRINT 107, (ANXP(K), K=1, NTP)
0152 PRINT 117
0153 PRINT 107, (ANX(K), K=1, NTP)
0154 PRINT 117
0155 CONTINUE
0156 AMPS=AMPS/NTP**2
0157 ANGS=ANGS/NTP**2
0158 ANGSST=0.
0159 AMPSSST=0.
0160 DO 80 I=1, NTP
0161 ANGS=0.
0162 AMPSS=0.
0163 DO 81 J=1, NTP
0164 AMPSS=AMPSS+(AMPX(J, I)-AMPAY(I))**2
0165 ANGSST=ANGSS+(ANGLX(J, I)-ANGAY(I))**2
0166 AMPSSST=AMPSSST+(AMPX(J, I)-AMPAY(I))**2
0167 ANGSST=ANGSSST+(ANGCLX(J, I)-ANGAY(I))**2
0168 AMPS=(AMPX(I)-AMPAY(I))* .1
0169 ANGST=ANGSS+(ANGCLX(I)-ANGAY(I))* .1
0170 CONTINUE
0171 PRINT 107, (AMPRMS(K), K=1, NTP)
0172 PRINT 117
0173 PRINT 107, (AMGRMS(K), K=1, NTP)
0174 AMPS=SQR2(AMPSSST/NTP**2)
0175 ANGSST=SQR2(ANGSSST/NTP**2)
0176 PRINT 117
0177 PRINT 107, AMPS, ANGS, AMPX, ANGX, AMPSS, ANGSS
0178 IF (LL. GT. 0) RETURN
0179 IF (LLPRINT .NE. 0) RETURN
0180 IF (LLPRINT .GT. 0) RETURN
0181 111 FORMAT (1HL)
0182 112 Format (1HL)
0183 LL=K6/2
0184 "DO 10 L=1, L3"
0185 GO TO (11, 12, L3)
0186 112 PRINT 112
0187 GO TO 14
0188 17 PRINT 113
0189 113 PRINT 113
0190 GO TO 14
0191 13 PRINT 114
0192 114 PRINT 114
0193 14 DO 30 I=1, NTP
0194 30 DO 30 J=1, NTP
0195 SR=CMPX(0., 0.)
0196 DO 40 K=1,NTP
0197 GO TO (41,42,43)L
0198 41 SR = SR + TRFF(I,K) * CONJG(TRFF2(J,K))
0195 GO TO 40
0200 42 SR = SR + TR(I,K) * CONJG(TRFF2(J,K))
0201 GO TO 40
0202 43 SR = SR + TRF8(I,K) * CONJG(TRFF2(J,K))
0203 40 CONTINUE
0204 ANGL(I,J) = CANG(SR) * RAC
0205 AMPT(I,J) = CABS(SR)
0206 DO 20 K=1,2
0207 GO TO (21,22)K
0208 21 PRINT 115
0209 115 FORMAT(20X, "AMPLITUDE", /)
0210 GO TO 73
0211 22 PRINT 116
0212 116 FORMAT(20X, "PHASE ANGLE", /)
0213 DO 20 T=1,NTP
0214 GO TO (24,25)K
0215 24 PRINT 107, (AMPT(I,J), J=1,NTP)
0216 GO TO 20
0217 25 PRINT 107, (ANPT(I,J), J=1,NTP)
0218 20 CONTINUE
0219 10 CONTINUE
0220 IF ( KC.GT.0) RETURN
0221 IF (A1.LE.0.) RETURN
0222 PRINT 118
0223 118 FORMAT(1H1,10X, "REFLECTION MATRIX IS USED", /)
0224 DO 5 I=1,NTP
0225 5 TRFF2(I,J) = TRFF(I,J)
0226 KC = KC + 1
0228 GO TO 6
0229 END
SUBROUTINE TRFMIX(NM, NN, NR1, NB1, NBK, MC, PHA, S11, S12, S21, S22)

DIMENSION NBP(16), NBK(16)

DIMENSION MC(NR1, NN), PHA(NR1, NN)

DIMENSION S11(NM, NM), S12(NM, NM), S21(NM, NM), S22(NM, NM)

COMMON/C$51q411(32, 32), T12(32, 32), T21(32, 32), T22(32, 32)

COMMON/C$61R11(32, 32), R12(32, 32), R21(32, 32), R22(32, 32)

COMMON/S$61S11(8, 8), S12(8, 8), S21(8, 8), S22(8, 8)

COMMON/C$61MCT(32)

COMMON/C$61R11(32, 32), R12(32, 32), R21(32, 32), R22(32, 32)

C FIRST INDEX ROW
C SECOND INDEX COLUMN

DO 10 I=1, NR1
C TRANSFER MATRIX IN CONNECTION REGION

DO 11 L=1, NN

LL=MCL(I, L)

11 MCT(LL)=L

PRINT 102, (MCL(!, L), L=1, NN)

PRINT 102*(MC(L), L=1, NN)

FORMAT((10X, 8F10.4))

DO 20 J=1, NN

T11(J, K)=CMPLX(0., 0.)

T12(J, K)=CMPLX(0., 0.)

T21(J, K)=CMPLX(0., 0.)

T22(J, K)=CMPLX(0., 0.)

IF(MCT(J).NE.K) GO TO 20

T11(J, K)=AR(PHA(I, J))

T22(J, K)=CONJG(T11(J, K))

CONTINUE

PRINT 100, ((T11(M, N), N=1, NN), M=1, NN)

PRINT 100, ((T22(M, N), N=1, NN), M=1, NN)

IF(1.GT.1) GO TO 21

DO 22 K=1, NN

R11(J, K)=T11(J, K)

R12(J, K)=CMPLX(0., 0.)

R21(J, K)=CMPLX(0., 0.)

R22(J, K)=CMPLX(0., 0.)

22 R22(J, K)=T22(J, K)

GO TO 23

21 CALL MXM11(NM, NN), T11, T12, T21, T22, R11, R12, R21, R22)

23 PRINT 100, ((CR11(M, N), N=1, NN), M=1, NN)

PRINT 100, ((CR12(M, N), N=1, NN), M=1, NN)

PRINT 100, ((CR21(M, N), N=1, NN), M=1, NN)

PRINT 100, ((CR22(M, N), N=1, NN), M=1, NN)

100 FORMAT((10X, 8F10.4))

10 FORMAT(/, 4X, "REFERENCE")

C TRANSFER MATRIX IN BLOCK REGION

24
NFRL REPORT 8392

0046     NP=NP(I)
0047     IF(1.LE.1)GO TO 26
0048     IF(NP.LE.I)GO TO 27
0049     CALL BLK8(NP,S11,S12,S21,S22)
C     RES.ET S MATRIX
0050     DO 24 J=1,NN
0051     DO 24 K=1,NN
0052     24 S11(J,K)=CMPLX(0.,0.)
0053     S12(J,K)=CMPLX(0.,0.)
0054     S21(J,K)=CMPLX(0.,0.)
0055     S22(J,K)=CMPLX(0.,0.)
0056     DO 25 J=1,NN,NP
0057     DO 25 JJ=1,NP
0058     25 J1=JJ-I
0059     DO 25 KK=1,NP
0060     K1=KK-I
0061     S11(J1,J1+J*K1)=S11(JJ, KK)
0062     S12(J1,J1+J*K1)=S12(JJ, KK)
0063     S21(J1,J1+J*K1)=S21(JJ, KK)
0064     S22(J1,J1+J*K1)=S22(JJ, KK)
0065     25 CONTINUE
0066     PRINT 100,(S11(M,N),N=1,NN),M=1,NN)
0067     PRINT 100,(S12(M,N),N=1,NN),M=1,NN)
0068     PRINT 100,(S21(M,N),N=1,NN),M=1,NN)
0069     PRINT 100,(S22(M,N),N=1,NN),M=1,NN)
C     INVERS E S MATRIX
0070     CALL INVS(NM,NN,N,S12)
0071     CALL STYRF(NM,NN,S11,S12,S21,S22,T11,T12,T21,T22)
0072     PRINT 100,(S11(M,N),N=1,NN),M=1,NN)
0073     PRINT 100,(S12(M,N),N=1,NN),M=1,NN)
0074     PRINT 100,(S21(M,N),N=1,NN),M=1,NN)
0075     PRINT 100,(S22(M,N),N=1,NN),M=1,NN)
0076     27 DO 50 J=1,NN
0077     DO 50 K=1,NN
0078     T11(J,K)=S11(J,K)
0079     T12(J,K)=S12(J,K)
0080     T21(J,K)=S21(J,K)
0081     T22(J,K)=S22(J,K)
0082     50 CONTINUE
0083     CALL INVS(NM,NN,N,R22)
0084     DO 40 J=1,NN
0085     DO 40 K=1,NN
0086     S17(J,K)=R22(J,K)
0087     S21(J,K)=R22(J,K)
0088     S22(J,K)=R22(J,K)
0089     S22(J,K)=CMPLX(0.,0.)
0090     DO 40 L=1,NN
0091     S11(J,K)=S11(J,K)-R22(J,K)*R21(L,K)
0092     S22(J,K)=S22(J,K)*R21(L,K)
0093     40 CONTINUE
0094     PRINT 100,(S11(M,N),N=1,NN),M=1,NN)
0095     PRINT 100,(S17(M,N),N=1,NN),M=1,NN)
0096     PRINT 100,(S21(M,N),N=1,NN),M=1,NN)
0097     PRINT 100,(S22(M,N),N=1,NN),M=1,NN)
0098     RETURN
0099     END

25
SUBROUTINE STRF(NM,NN,MR1,NBP,MBK,MC,PHA,S11,S12,S21,S22)

DIMENSION NRP(N16),NBK(16)

DIMENSION MC(NR1,NN),PHA(NR1,NN)

DIMENSION S11(NN,NN),S12(NN,NN),S21(NN,NN),S22(NN,NN)

COMMON/GS/ T11(8,8), T12(8,8), T21(8,8), T22(8,8), R11(8,8), R12(8,8),
R21(8,8), R22(8,8), SPACE(T168)

DIMENSION MCT(32)

DIMENSION S11(2,2), S12(2,2), S21(2,2), S22(2,2)

COMPLEX S11,S12,S21,S22

CALL TWOPT(S11,S12,S21,S22)

C 1ST INDEX,COLUMN
C 2ND INDEX,ROW

DO 10 II=1,NN

LL=MCT(II)

10 CONTINUE

DO 20 J=1,NN

IF(C1.GT.1) GO TO 21

IF(C2.GT.1) GO TO 26

R11(J,K)=T11(J,K)

R12(J,K)=T12(J,K)

R21(J,K)=T21(J,K)

R22(J,K)=T22(J,K)

GO TO 23

CALL MCTXL TM(1,NM,MR1,T11,T12,T21,T22,R11,R12,R21,R22)

C TRANSFER MATRIX IN BLOCK REGION
C RESET S MATRIX

DO 23 J=1,NN

IF(C1.GT.1) GO TO 27

GO TO 23

DO 27 J=1,NN

IF(C1.EQ.16) GO TO 26

IF(C1.EQ.11) GO TO 26

DO 23 J=1,NN

IF(C1.GT.1) GO TO 27

GO TO 23

DO 27 J=1,NN

IF(C1.GT.1) GO TO 27

GO TO 23

DO 27 J=1,NN
SUBROUTINE STTRF(NM,NN,T11,T12,T21,T22)

C THIS SUBROUTINE INVERSES S-MATRIX AND STORES IN T

DIMENSION S11(NM,NM),S12(NM,NM),S21(NM,NM),S22(NM,NM)
DIMENSION T11(NM,NM),T12(NM,NM),T21(NM,NM),T22(NM,NM)

COMPLEX S11,S12,S21,S22

DO 30 J=1,NN
  DO 30 K=1,NN
    T22(J,K)=S12(J,K)*R22(J,K)
    T12(J,K)=COMPLEX(0.,0.)
  DO 30 L=1,NN
    T21(J,K)=S22(J,K)*R12(J,K)
    T11(J,K)=S11(J,K)*R11(J,K)
  DO 30 L=1,NN
    T12(J,K)=T12(J,K)-S12(J,L)*T11(J,L)*R22(J,L)
    T11(J,K)=T11(J,K)-S22(J,L)*T21(J,L)*R12(J,L)
  DO 30 K=1,NN
  DO 30 J=1,NN
RETURN
END

SUBROUTINE INVS2(NM,NN,S12)

CALL STTRF(NM,NN,S11,S12,S21,S22,T11,T12,T21,T22)

DO 50 J=1,NN
  DO 50 L=1,NN
    T11(J,K)=S11(J,K)
    T12(J,K)=S12(J,K)*R22(J,K)
    T21(J,K)=S21(J,K)*R12(J,K)
    T22(J,K)=S22(J,K)*R11(J,K)
  DO 50 K=1,NN
  DO 50 L=1,NN
RETURN
END
SUBROUTINE PTXMLT(NMNN,R11,R12,R21,R22,T11,T12,T21,T22)

C THIS SUBROUTINE MULTIPLE SUBMATRICES R*T THEN STORE THE RESULT
C IN R
C S=R*T
C S11=R11*T11+R12*T21
C S12=R11*T12+R12*T22
C S21=R21*T11+R22*T21
C S22=R21*T12+R22*T22

DIMENSION T11(NM,NM),T12(NM,NM),T21(NM,NM),T22(NM,NM)

DIMENSION R11(NM,NM),R12(NM,NM),R21(NM,NM),R22(NM,NM)

DIMENSION RI1(NM,NM),R12(NM,NM),R21(NM,NM),R22(NM,NM)

COMPLEX TT1,TT2,TT11,TT21,TT12,TT22

PRINT 101

PRINT 100,(RI1(M,N),N=1,NN),P=1,NN)

PRINT 100,(T11(M,N),N=1,NN),P=1,NN)

PRINT 100,(T12(M,N),N=1,NN),P=1,NN)

PRINT 100,(T21(M,N),N=1,NN),P=1,NN)

PRINT 100,(T22(M,N),N=1,NN),P=1,NN)

PRINT 101

100 FORMAT//,(10X,8I0.4))

101 FORMAT//(10X,8F10.4))

DO 10 J=1,NN

DO 10 K=1,NN

TT1(J,K)=CMPLX(0.,0.)

TT2(J,K)=CMPLX(0.,0.)

DO 10 L=1,NN

TT1(J,K)=TT1(J,K)+R11(J,L)+T11(L,K)+R12(J,L)+T21(L,K)

TT2(J,K)=TT2(J,K)*R11(J,L)*T12(L,K)+R12(J,L)*T22(L,K)

CONTINUE

DO 20 J=1,NN

DO 20 K=1,NN

R11(J,K)=TT1(J,K)

R12(J,K)=TT2(J,K)

DO 20 J=1,NN

DO 20 K=1,NN

T11(J,K)=CMPLX(0.,0.)

T12(J,K)=CMPLX(0.,0.)

DO 20 L=1,NN

TT1(J,K)=TT1(J,K)+R21(J,L)+T11(L,K)+R22(J,L)+T21(L,K)

TT2(J,K)=TT2(J,K)+R21(J,L)+T12(L,K)+R22(J,L)+T22(L,K)

CONTINUE

DO 30 J=1,NN

DO 30 K=1,NN

R21(J,K)=TT1(J,K)

R22(J,K)=TT2(J,K)

DO 30 J=1,NN

DO 30 K=1,NN

T11(J,K)=R11(J,K)

T12(J,K)=R12(J,K)

T21(J,K)=R21(J,K)

T22(J,K)=R22(J,K)

CONTINUE

END
NRL REPORT 8392

0001 SUBROUTINE PATTERN (NTP,TRFF,TRFB,KLL,NPAV,NMX)
C ABSOLUTE VALUE OF KLL REPRESENTS THE BEAM INDEX WHOSE PATTERN IS
C TO BE PLOTTED
C KLL=0 NO PLOT
C KLL GREATER THAN 0 PLOT PATTERN ONLY
C KLL LESS THAN 0 PLOT BOTH PATTERN AND MAIN BEAMS
0002 COMMON/CS/PLTAY(500)
0003 COMMON/CS5/PEAK(64,100),PAV(64),PAV(64),KIND(64),
C KIND(64),PEAKDB(100),SPACE(1372)
0004 COMMON/C56/CONTA(4096),INTA(4096)
0005 COMPLEX TRFF,TRFB,S
0006 COMMON/CTF,TRFB,S
0007 COMMON/COINTA(4096),SINTA(4096)
0008 COMMON COMPLEX TRFF,TRFB,S
0009 FORMAT(1X)
0010 PRINT 104
0011 THR=PI/180
0012 KPAIR=TRANS(KLL)
0013 NTP2=NTP/2
C PLOT FRAME
0014 YSL=80.
0015 NT=YSL
0016 MX=100.
0017 NX=MX
0018 HN=5.
0019 ST=2.
0020 XM=10.
0021 YM=5.
0022 YS=2.
0023 YS=Y5+YM
0024 NTA=20*NTP
0025 NT1=NT1+1
0026 TAINC=P1/NTA
0027 PNOR=NP
0028 CALL PHTAN(TAINC,K)
0029 KL=1
0030 IF(KLL.LT.0)KL=2
0031 NTA1=N+1
0032 DO 1 J=1,NTP
0033 DO 1 I=1,NTP
0034 1 TRFF(I,J)=TRFF(I,J)+TRFB(I,J)
0035 DO 20 IL=1,KL
0036 IF(KLL.LT.0)Go TO 25
0037 CALL PLOT(XM*100+J,XM*100-I,3)
0038 NTA=NTA1
0039 ASSIGN=1
0040 CALL PDATE(XM,YM,KSL,YSL,ST,HN,HD,NY)
0041 DO 20 K=1,NTP2
0042 K=0
0043 KFLAG=0

29
SHELTON AND HISAIO

0044  KMI=1
0045  KPCONT=1
0046  KMIND=1
0047  KMA=0
0048  LEDGE=0
0049  GO TO 30
0050  IF(IJ.GT.1)GO TO 23
0051  NSIGN=-1
0052  IF(IJ.GE.NTA1)NSIGN=1
0053  I=IJ-NTA1+NSIGN
0054  GO TO 24
0055  23 I=IJ
0056  24 II=1ABS(I)
0057  II=II-1
0058  PAR=0.
0059  PAI=0.
0060  IF(J1.LT.2)GO TO 21
0061  IF(J1.LT.KIND1(K).OR.I.GT.KIND2(K))GO TO 30
0062  IF(J1.EQ.KIND1(K))GO TO 24
0063  21 DO 40 J=1,NTP
0064  S=TRPC(J,K)
0065  30 IF(J1.LE.1)GO TO 31
0066  JMOD=MOD(J1,IKA)
0067  IF(JMOD.EQ.0)JMOD=IKA
0068  PAR=CENTA(JMOD)+REAL(S)-SINTA(JMOD)*AIMAG(S)+NSIGN*PAR
0069  PAI=CENTA(JMOD)+AIMAG(S)+SINTA(JMOD)*REAL(S)+NSIGN*PAI
0070  40 CONTINUE
0071  00 CONTINUE
0072  PAI=PAI**2+PAI**2
0073  IF(IJ.EQ.2)LEDGE=1
0074  IF(KMA.LT.0)GO TO 31
0075  IF(J1.LE.PEAK(K,KPCONT))GO TO 34
0076  32 KMI=1
0077  IF(IJ.LT.2)LEDGE=1
0078  IF(KMA.LE.0)GO TO 31
0079  IF(PAT1.LE.PEAK(K,KPCONT))GO TO 34
0080  KIND1(K)=KMIND
0081  KFLAG=1
0082  KPCONT =KK+1
0083  34 KK=KK+1
0084  PEAK(K+KK)=PAT1
0085  KMA=0
0086  GO TO 31
0087  IF(KFLAG.GT.0)KIND2(K)=I-1
0088  33 KMA=1
0089  IF(KMI.LE.0)GO TO 31
0090  35 KMIND=I-1
0091  IF(KFLAG.GT.0)KIND2(K)=I-1
NRL REPORT 8392

0092  KMI=0
0093  31  PAT1=PAT
0094  C  PLOT PATTERN FOR A GIVEN BEAM
0095  IF(KM.EQ.1)GO TO 30
0096  22  DB=Z(PAT)
0097  Y=(1.+DB/YSL)*Y+SY
0098  IF(CY.GT.YSM)Y=YSM
0099  IF(CY.LT.-SY)Y=SY
0100  P=-1
0101  X=PMNTA
0102  IF(II.EQ.1)GO TO 3
0103  30  CONTINUE
0104  IF(IL.GE.2)GO TO 20
0105  IF(KMA.GT.0)GO TO 42
0106  3  CALL PLOT(X,Y,Z)
0107  GO TO 30
0108  CALL PLOT(X,Y,3)
0109  CONTINUE
0110  IF(IFL.GE.2)GO TO 20
0111  IF(KPCONT.EQ.1)KNT02(K)=NTA1
0112  IF(KPCONT.EQ.0)KNT02(K)=NTA1
0113  C  DELETE THE MAIN Lobe
0114  GO TO 43
0115  42  IFL.LE.0)GO TO 43
0116  43  DO 44 I=1,KK
0117  44  IF(KPCONT.GT.0)GO TO 53
0118  102  IF(PEAK(K,1).LE.0)PEAK(K,1)=PAT1
0119  103  DO 102 L=1,KK
0120  104  IF(PEAK(K,L).GT.PMAX(K))PMAX(K)=PEAK(K,L)
0121  105  PSUM=PSUM+PEAK(K,L)
0122  CONTINUE
0123  106  MAX(K)=0
0124  107  DO 106 L=1,KK
0125  108  IF(PEAK(K,L).GT.PMAX(K))PMAX(K)=PEAK(K,L)
0126  109  PSUM=PSUM+PEAK(K,L)
0127  CONTINUE
0128  110  IF(PEAK(K,L).GT.PMAX(K))PMAX(K)=PEAK(K,L)
0129  111  PSUM=PSUM+PEAK(K,L)
0130  112  DO 111 L=1,KK
0131  113  IF(PEAK(K,L).GT.PMAX(K))PMAX(K)=PEAK(K,L)
0132  114  PSUM=PSUM+PEAK(K,L)
0133  CONTINUE
0134  115  IF(PEAK(K,L).GT.PMAX(K))PMAX(K)=PEAK(K,L)
0135  116  PSUM=PSUM+PEAK(K,L)
0136  117  CONTINUE
0137  118  IF(PEAK(K,L).GT.PMAX(K))PMAX(K)=PEAK(K,L)
0138  119  PSUM=PSUM+PEAK(K,L)
0139  CONTINUE
0140  120  RETURN
0141  END
SUBROUTINE HLFMTKC(NTP,NR1,NBP,NBK,MC,PHA)
DIMENSION NBP(16),NBK(16)
DIMENSION MC(NR1,NTP),PHA(NR1,NTP)
COMMON/C$3/MCT(64)
DIMENSION ANG(64),ATEMP(64)
NN=NR1/2
LL=(NR1+1)/2-NN
C LL=1 NUMBER OF ROWS IS EVEN
C LL=0 NUMBER OF ROWS IS ODD
CALL PHASUM(NR1,NTP,NBP,MC,PHA,ANG)
DO 10 I=1,NTP
   II=I
   JJ=11 J=1,NR1
   KK=MCC(J,II)
   IF(J.EQ.NN)KKP=KK
   II=KK
10 CONTINUE
C FIND THE JOINT POINT THEN STORE IN MCT ARRAY
DO 12 J=1,NN
   KKS=MCC(J,II)
   MCT(KKP)=KKS
   AVG=PHAC(J,IMC)+PHAC(JJ,II))/2.
22 PHAC(J,IMC)=AVG
21 PHAC(JJ,II)=AVG
12 CONTINUE
C AVERAGE THE PHASE ANGLES FOR SYMMETRICAL MATRIX
DO 20 I=1,NN
   JJ=NR1-J+1
   IMC=MCC(J+1)
   AVG=(PHAC(J,IMC)+PHAC(JJ,J))/2.
20 CONTINUE
C CORRECT PHASE ANGLE OF THE MIDDLE ROW WHEN THE NUMBER OF ROWS IS EVEN
IF(LL.LE.0.)GO TO 1
NN=NN+1
20 CONTINUE
C CORRECT THE PHASE ANGLE BY ADDING THE SAME EXTRA PHASE TO EACH PORT IN A BLOCK
NMP=NBPC(N1)
NMB=NBK(N1)
DO 21 M=1,NMB
   DO 22 J=1,NMP
      KK=IMB+J
      AA=0.
22 AA=AA+PHA(N1,KK)
21 CONTINUE
C CORRECT THE PHASE ANGLES FOR THE CASE WHEN THE NUMBER OF ROWS IS 32
SUBROUTINE PHASUM(NR1, NTP, NBP, MC, PHA, AS)
DIMENSION NBP(16)
DIMENSION MC(NR1, NTP), PHA(NR1, NTP), AS(NTP)
DIMENSION L(2,64), A(64)

C SET THE PHASE SHIFT OF THE BOTTOM ROW

NN = NBP(NROW)
DO 1 J = 1, NN
L(AJ) = J
AS(J) = PHA(1, I)
KK = MN
DO 10 I = 1, NROW
II = NROW - I
IF(II .LE. 0) GO TO 17
DO 12 J = 1, NTP
L(AJ) = L(AJ)+1
CONTINUE
NMD = MOD(JJ*NN)
DO 30 K = 1, NN
JJ = JJ + K
A(JJ) = ALL(JJ) + (1, I) + A(JJ)
CONTINUE
KK = KN
RETURN
END
SUBROUTINE NTWK(NTP,NRI,NBP,NBK,MC,PHA)

C******THIS SUBROUTINE FINDS THE CONNECTION OF A BUTLER MATRIX OR FFT
C GIVEN THE NUMBER OF ROWS AND THE NUMBER OF PORTS IN EACH BLOCK IN
C EACH ROW
C******COMPILED BY J. K. HSIAO
C******FIRST VERSION IS COMPILED ON MAY 3, 1976
C******NTP, NUMBER OF TOTAL INPUT PORTS OR SAMPLES
C******NROW, NUMBER OF ROWS REQUIRED TO PERFORM THE TRANSFORMATION
C******NBK, AN ARRAY STORES THE NUMBER OF PORTS IN EACH BLOCK AT EACH
C ROW. EACH BLOCK IN A ROW HAS THE SAME NUMBER OF PORTS
C******NBK, AN ARRAY STORES THE NUMBER OF BLOCKS IN EACH ROW.
C******MC, A TWO DIMENSIONAL ARRAY STORES THE CONNECTIONS OF THE NETWORK.
C FIRST INDEX OF THE ARRAY REPRESENTS THE NUMBER OF CURRENT ROW. THE
C LOCATION OF THE SECOND INDEX REPRESENTS THE PHYSICAL LOCATION OF
C THE PREVIOUS ROW WHILE THE CONTENTS OF IT IS THE CONNECTION TO THE
C CURRENT ROW
C******MC(NRINTP),PHAC(NRINTP)

DIMENSION PHA(NRINTP,NTP)
DIMENSION NFTS(64),NBK(16),NBP(16)

C COMPUTES THE NUMBER OF PORTS IN EACH BLOCK
NTP2=NTP/2
10 I=INRI
NBK(I)=NTP/NBP(I)

C**** NFTS ARRAY STORES THE LOCATION OF THE SAMPLES IN EACH BEAM(OR
C FREQUENCY SAMPLE). THE STRUCTURE IS CHARACTERIZED BY TWO NUMBERS,
C NTS, NUMBER OF TIME SAMPLES(OR INPUT PORTS) AND NFS, NUMBER OF
C FREQUENCY SAMPLES(OR NUMBER OF BEAMS). FOR EXAMPLE, NFTS((3-1)*
C NFS2) IS THE PHYSICAL LOCATION OF THE FIRST TIME SAMPLE IN THE
C THIRD FREQUENCY GROUP( OR OF THE THIRD BEAM), THIS IS REPRESENTED
C BY LMC
C
C SET THE INITIAL NFTS ARRAY
I=1,NTP
NFTS(I)=I

C**** NTS1 IS THE PREVIOUS VALUES OF THE NUMBER OF TIME SAMPLES(OR INPUT
C PORTS)
C**** NFS1 IS THE PREVIOUS VALUE OF THE NUMBER OF FREQUENCY SAMPLES(OR
C BEAMS)
C**** NFS2 IS THE CURRENT VALUE
C
C SFT THE INITIAL VALUES OF NTS AND NFS
I=1,NTP
NFS1=1
NFS2=I
DO 20 I=1,NRI

C WM THE NUMBER OF BLOCKS OF THE CURRENT ROW

34
C  THE NUMBER OF PORTS IN EACH BLOCK OF THE CURRENT ROW
  MM=NBNK(I)
  NN=NBBP(I)
C  SET NTS2 AND NFS2
  NTS2=NTS1/NN
  NFS2=NTP/NTS2
C**** THE ACTUAL REQUIRED PHASE GRADIENT BETWEEN SUCCESSIVE ELEMENT FOR
C  THE FIRST BEAM IS
  PAG=PI/NFS2
C*** THE AVAILABLE PHASE GRADIENT FOR THE FIRST BEAM IN EACH BLOCK IS
  PSG.PI/NN
  KK=0
  DO 30 J=1,MM
      MODJ=MOD(J,NFS1)
      IF(MODJ.EQ.0)MODJ=NFS1
      JJ=(J-1)/NFS1+1
      PAG=PAG*(MODJ*2-1)
      DO 30 K=1,NN
          KI=K-1
          KK=KK+1
          LMC=(MOD.I-1)*NTS1.*(K-1)*NTS2+JJ
          MC=MCLC(*LMC)KK
          IF(KK.LE.NTP)GO TO 31
          KK=KK-1
          CONTINUE
          NFTS(KK)=(J-1)*NFS1+MC*(K-1)*NN+JJ
          CALL CONTINUE
          RETURN
          END

C  RECORDING THE FREQUENCY SAMPLE OR BEAM POSITION INTO NFTS ARRAY
  NTS1=NTS2
  NFS1=NFS2
  MNS=MM/NTS1
  DO 40 J=1,NFS1
      JMOD=MOD(J,NMS)
      IF(JMOD.EQ.0)JMOD=MNS
      JJ=(J-1)/MNS+1
      DO 40 K=1,NTS1
          KK=KK+1
          NFTS(KK)=(K-1)*NFS1+(JMOD-1)*NN+JJ
          CONTINUE
          RETURN
          END
SUBROUTINE FRAME(XM, YM, XSL, YSL, SY, XN, YN)
COMMON/CS1/PLTAY(500)

MLAB=HN+.035
MLAS=MLAB+.035
MLAB=4.*MLAB/7.
XSCL=XSL/NX
YSCL=YSL/NY
DY=YM/NY
Y=SY
NT=NY+1
CALL PLOT(0., SY, 3)
CALL PLOT(XM, SY, 2)
CALL PLOT(XM, YMSY, 2)
CALL PLOT(0., YMSY, 2)
CALL PLOT(0., SY, 2)
DO 10 I=1,2
10 Y=SY
IF(CI.GT.1)GO TO 12
X1=0.
X2=.-2
X3=.-1
GO TO 13
12 X1=XN
X2=XN+.2
X3=XN+.1
13 DO 10 J=1, NNY
CALL PLOT(X1, Y, 3)
MOOT=MOD(J-1,10)
IF(MO0Y.NE.0)GO TO 11
CALL PLOT(X2, Y2, 2)
IF(CI.GT.1)GO TO 10
A=YSCL*(J-1-NNY)
CALL NUMBER(-6.5*WLAB, Y-HLAB/2., HLAA, 0., 4HFF3.0)
GO TO 10
10 CALL PLOT(X3, Y3, 2)
11 CALL PLOT(X3, Y3, 2)
12 CALL PLOT(X3, Y3, 2)
13 CALL PLOT(X3, Y3, 2)
14 CALL PLOT(X3, Y3, 2)
15 CALL PLOT(X3, Y3, 2)
16 CALL PLOT(X3, Y3, 2)
17 CALL PLOT(X3, Y3, 2)
18 CALL PLOT(X3, Y3, 2)
19 CALL PLOT(X3, Y3, 2)
20 CALL PLOT(X3, Y3, 2)
21 CALL PLOT(X3, Y3, 2)
22 CALL PLOT(X3, Y3, 2)
23 CALL PLOT(X3, Y3, 2)
24 CALL PLOT(X3, Y3, 2)
25 CALL PLOT(X3, Y3, 2)
26 CALL PLOT(X3, Y3, 2)
27 CALL PLOT(X3, Y3, 2)
28 CALL PLOT(X3, Y3, 2)
29 CALL PLOT(X3, Y3, 2)
30 CALL PLOT(X3, Y3, 2)
31 CALL PLOT(X3, Y3, 2)
32 CALL PLOT(X3, Y3, 2)
33 CALL PLOT(X3, Y3, 2)
34 CALL PLOT(X3, Y3, 2)
35 CALL PLOT(X3, Y3, 2)
36 CALL PLOT(X3, Y3, 2)
37 CALL PLOT(X3, Y3, 2)
38 CALL PLOT(X3, Y3, 2)
39 CALL PLOT(X3, Y3, 2)
40 CALL PLOT(X3, Y3, 2)
41 CALL PLOT(X3, Y3, 2)
42 CALL PLOT(X3, Y3, 2)
43 CALL PLOT(X3, Y3, 2)
44 CALL PLOT(X3, Y3, 2)
45 CALL PLOT(X3, Y3, 2)
46 CALL PLOT(X3, Y3, 2)
47 CALL PLOT(X3, Y3, 2)
48 CALL PLOT(X3, Y3, 2)
49 CALL PLOT(X3, Y3, 2)
50 CALL PLOT(X3, Y3, 2)
51 CALL PLOT(X3, Y3, 2)
52 CALL PLOT(X3, Y3, 2)
53 CALL PLOT(X3, Y3, 2)
54 CALL PLOT(X3, Y3, 2)
55 CALL PLOT(X3, Y3, 2)
56 CALL PLOT(X3, Y3, 2)
57 CALL PLOT(X3, Y3, 2)
58 CALL PLOT(X3, Y3, 2)
59 CALL PLOT(X3, Y3, 2)
60 CALL PLOT(X3, Y3, 2)
61 CALL PLOT(X3, Y3, 2)
NRL REPORT 8392

0062 CALL SYMBOL(+5*X-17.5*Wlab,-5.*Wlab*SY*Wlas+22*PARAMETER U IN DEG
0063 CREFS0.9ZZ)
0064 35 CALL SYMBOL(-7.*Wlab*YM/2+SY-15.*Wlab*KLAS,18 ARRAY PATTERN (DB),
0065 *90.18)
0064 32 CALL PLOT(0,0,3)
0065 END

0001 SUBROUTINE SIMCX ISORIGINNRATMCTANSLK)
0002 C IOENT NUMBER - F1002ROO
0003 C TITLE - COMPLEX MATRIX INVERSION, SOLUTION OF LINEAR EQUATIONS
0004 C IDENT NAME - FI-NRL-SIMCX
0005 C LANGUAGE - FORTRAN
0006 C COMPUTER - CDC-3300
0007 C CONTRIBUTOR - JANET P. HASON, CODE 7813, RESEARCH COMPUTATION
0008 C CENTER, MIS DIVISION
0009 C ORGANIZATION - NRL - NAVAL RESEARCH LABORATORY - WASHINGTON, U.C.
0010 C DATE - 16 DECEMBER 1970
0011 C PURPOSE - TO SOLVE THE COMPLEX MATRIX EQUATION AX=B WHERE A IS A
0012 C SQUARE COEFFICIENT MATRIX AND B IS A MATRIX OF CONSTANT
0013 C VECTORS, THE DETERMINANT AND INVERSE OF A ARE ALSO
0014 C OBTAINED.
0015 C002 SUBROUTINE SIMCX(IS,ORIG,NN,MAT,ICT,ANS,LK)
0006 C004 DIMENSION MAT(1),ORIG(1),ANS(1),ICT(1),LK(1)
0015 C006 FORMAT(X,2E12.6)
0016 C008 FORMAT(1H1,6X,THE INVERSE (BY COLUMNS))
0017 C008 FORMAT(5X,1H1,VALUES OF THE UNKNOWNS)
0018 C008 FORMAT(5X,1H1,IDENTITY MATRIX)
0019 C008 B3=(-1.0,0.0)
0020 C008 B4=(0.0,0.0)
0021 C008 ICT=MCT
0022 C008 J=ICT
0023 C008 MT=MCT+1
0024 C008 NCI=ICT+MCT
0025 C008 PUT ORIGINAL MATRIX INTO MAT
0026 IF(IS.EQ.0)GO TO 39
0027 ICI=MCI+1
0028 NCI=ICT
0029 39 DO 2 J=1,ICT
0030 DO 2 I=1,MCI
0031 2 CONTINUE
0032 IF(IS.NE.0)GO TO 30
0033 PUT IDENTITY MATRIX INTO RIGHT HALF OF MAT
0034 DO 32 J=MT,MCI
0035 DO 32 I=1,MCI
0036 32 MAT(I,J)=0.0
0037 DO 33 J=1,MCI
0038 33 MAT(I,J)=B3
0039 C FORM TRANGULARIZED MATRIX

37
SHELTON AND HSIAO

0034 30 JCT=MCT-1
0035   DO 3 J=1,JCT
0036   KK=J+1
0037   GOTO 25
0038 24 DO 4 K=KK,MCT
0039     BB=MA[K,J]/MAT(J,J)
0040     DO 5 L=J,MCT
0041     B10=BB*MAT(J,L)
0042     5 MAT(K,L)=MAT(K,L)-B10
0043     4 CONTINUE

C VALUE OF DETERMINANT

0044 3 B11=BI1*MAT(J,J)
0045 0 B11=B11*MAT(MCT,MCT)
0046 0 LOW=MCT
0047 0 M0=-1

C TO DO ONE OR MORE BACK SOLUTIONS

0048 0 DO 6 MINC=MT,MCT
0049   IF(JM=1)GO TO 5
0050   IX=0

C BACK SOLUTION

0051 0 DO 6 INH=LOW,M0
0052   M=IABS(JMIN)
0053 0 B0=MAT(M,MINC)
0054 0 B2=MA[K,M]
0055 0 B4=(0.0,0.0)
0056   IF(CY).EQ.0.0.AND.CY.EQ.0.0)GO TO 13
0057   29 MAT(M,MINC)=B0/B2
0058   ANS(M)=B0/B2
0059   6 CONTINUE
0060   DO 40 J=MT,MCT
0061   JJ=J-MCT
0062   DO 40 I=1,MCT
0063   ORG(I,JJ)=MAT(I,J)
0064   40 CONTINUE
0065   GO TO 41

C CHECK FOR SINGULARITY AND TO SEE IF FIRST TERM = 0

0066 25 JV=J
0067   IF(CY).EQ.0.0.OR.CY.EQ.0.0)GO TO 12
0068   IF(JV.EQ.JSING)GO TO 14

38
C PRINT TITLE - THE INVERSE
0106   34 PRINT 26
0107   GO TO 43
C PRINT TITLE - VALUES OF UNKNOWNS
0108   41 PRINT 28
0109   43 GO 36 JJ=RT,MCT
0110   PRINT 27
0111   GO 38 II=1,MCT
0112   38 PRINT 10, MAT(II, JJ)
C PRINT VALUE OF DETERMINANT
0113   PRINT 10,B11
0114   IF(IS,NE,0)GO TO 45
C PRINT IDENTITY MATRIX
0115   PRINT 35
0116   GO 36 K=1,MCT
0117   PRINT 27
0118   GO 36 I=1,MCT
0119   SUM=(O.O,O.O)
0120   GO 37 J=1,MCT
0121   37 SUM=ORIG(K,J)*MAT(J,MCT+I)+SUM
0122   36 PRINT 10;SUM
0123   RETURN
0124   END

C PRINT SUBSTITUTIONS BACK INTO ORIGINAL MATRIX
0094   20 CONTINUE
0095   45 GO 20 NNV=1,IS
0096   PRINT 27
0097   44 PRINT23
0098   GO 20 LL=1,MCT
0099   B13=ORIG(LL,MCT)*MAT(LL,MCT+NNV)*B13
0100   GO 19 MM=1,MCT
0101   19 B13=ORIG(LL,MCT)*MAT(LL,MCT+NNV)+B13
0102   B15=ORIG(LL,MCT+NNV)
0103   PRINT21,B15,B13
0104   20 CONTINUE
0105   RETURN

0886   13 PRINT15
0881   PRINT 100, J,(MAT(K,J),K=1,MCT)
0892   100 FOR MAT(IUX,IX,8F10.4)
0883   PRINT 21,(MAT(I+J),I=1,MCT),J=1,MCT
0884   RETURN
0885   14 JV=JV+1
0886   CC2=MAT(JV,J)
0887   IF(CX*(I),EQ.0.0.AND.CX2(2),EQ.0.0)GO TO 11
0888   GO 17 JJ=J,MCT
0889   B6=MAT(J,J)
0890   MAT(J,JJ)=MAT(JV,JJ)
0891   17 MAT(JV,JJ)=B6
0892   B11=-B11
0893   12 CONTINUE
0894   GOTO 24

FORMAT(10XI598Fl0.4)
PRINT((MAT.(I,J),SL~INCT)PJ=1,8INCT)
RETURN

NRL REPORT 8392
SUBROUTINE BLK(NM, NN, S11, S12, S21, S22)

DIMENSION NBP(16), NBK(16)
DIMENSION MC(8, 16), PHA(8, 16)
DIMENSION S11(NM, NM), S12(NM, NM), S21(NM, NM), S22(NM, NM)
COMMON/CSS/Al(8, 8), T11(8, 8), T12(8, 8), T21(8, 8), T22(8, 8), R11(8, 8), R12(8, 8),
C R21(8, 8), R22(8, 8), SPACE(7160)
COMMON/CSS/T11(8, 8), T12(8, 8), T21(8, 8), T22(8, 8), R11(8, 8), R12(8, 8), R21(8, 8),
R22(8, 8), SPACE(7160)
COMMON/CSS/T11(8, 8), T12(8, 8), T21(8, 8), T22(8, 8), R11(8, 8), R12(8, 8), R21(8, 8),
R22(8, 8), SPACE(7160)

COMPLEX S11, S12, S21, S22
COMPLEX T11, T12, T21, T22, R11, R12, R21, R22

IF(NM .LT. 2) GO TO 9

CALL TWOPT(S11, S12, S21, S22, NM)
RETURN

1 = 0
II = II + 1
GO TO 2

II = 0
II = II + 1
GO TO 3

II = 0
II = II + 1
GO TO 3

CALL NTWK(NM, II, NBK, MC, PHA)
CALL STRF(NM, II, NBK, MC, PHA, S11, S12, S21, S22)
RETURN
END

SUBROUTINE INVS1(NM, NN, S12)

COMMON/CSS/Al(32), T(32, 64), SPACE(4032)
DIMENSION S12(NM, NM)
COMPLEX A1, T, S12
CALL SIMCX(0, S12, NM, T, NN, A1, 1)
RETURN
END

SUBROUTINE INVS2(NM, NN, S12)

COMMON/CSS/Al(6), T(8, 16), SPACE(7920)
DIMENSION S12(NM, NM)
COMPLEX A1, T, S12
CALL SIMCX(0, S12, NM, T, NN, A1, 1)
RETURN
END
SUBROUTINE TWOPT (S11, S12, S21, S22, M)
DIMENSION S11(M,M), S12(M,M), S21(M,M), S22(M,M)
COMMON S11, S12, S21, S22

BC0 = B + C + D

IF (BC0 .GT. 0.) GO TO 1

AR = 0.*A*(-A*0.5)

IF (A .LE. 0.) AR = 0.

A1 = SQRT(S-AR*AR)

A2 = SQRT(S-AR*AR)

B1 = AR

B2 = AR

GO TO 2

1 B1 = 10.*A*(-AR*0.5)

IF (A .LE. 0.) B1 = 0.

B2 = 10.*A*(-AR*0.5)

A1 = 10.*(-A*0.5)

A2 = 10.*(-D*0.5)

S11(1,1) = B1*CMPLX(0.,-1.)

S11(2,2) = B1*CMPLX(0.,-1.)

S22(1,1) = B1*CMPLX(0.,-1.)

S22(2,2) = B1*CMPLX(0.,-1.)

S11(1,2) = B2*CMPLX(-1.,0.)

S11(2,1) = B2*CMPLX(-1.,0.)

S22(1,2) = B2*CMPLX(-1.,0.)

S22(2,1) = B2*CMPLX(-1.,0.)

S11(1,1) = A1*CMPLX(1.*0.)

S11(2,2) = A1*CMPLX(1.*0.)

S22(1,1) = A1*CMPLX(1.*0.)

S22(2,2) = A1*CMPLX(1.*0.)

S12(1,2) = A2*CMPLX(0.*)

S12(2,1) = A2*CMPLX(0.*)

S21(1,2) = A2*CMPLX(0.*)

S21(2,1) = A2*CMPLX(0.*)

RETURN
END
SUBROUTINE TRT(NTPTRFFLL@NNX)
COMMON/C66/APMT(32,32),ANGL(32,32),ANGT(32),TRID(32,32),SPACE(4064)
DIMENSION TRFF(32,32),ANGL(32,32),ANGTC32,TRFC32,SPACEC4064
DIMENSION TRFFCNMXNMX)
COMPLEX TRFP.1RI0
IF(tL.GT.0)6O TO Ia
RETURN
Go 10 J=INTP
TRKD(TJ)sTRFF(IJ)
RETURN
END

SUBROUTINE TRFIDL(NTP)
COMMON/C66/APMT(32,32),ANGL(32,32),ANGT(32),TRFF2(32,32), TR(32,32)
COMPLEX TRFF2,TR
PI=3.1415926536
PI2=PI*2.0
RTA=180.0/PI
A=SQRT(1./NTP)
DO 10 I=1,NTP
PP=(I-1)*P
PP=P*(I-.5)*2.
DO 10 J=1,NTP
PP=AI!C0(PPP!2)
RE=A*COS(PPP)
RI=A*SIN(PPP)
TRFF2(I,J)=CMPLX(RE,RI)
TR(1,J)=A
S11:AT A(T1) -SIN (I A)
TA=TA*TAINC
TF(TA.GE.P12)RUTURN
RETURN
END

SUBROUTINE PHASAN (TAINC,I)
COMMON/C66/CONTAC(4096),SINTA(4096)
PI=3.1415926536
PI2=PI*2.
TA=-O.
I=0
CONTAC(I)=COS(TA)
SINTA(I)=SIN(TA)
TA=TA+TAINC
IF(TA.GE.P12)RETURN
GO TO 1
END

42
COMPLEX FUNCTION AR(AUG)
AMP=I
AG=AUG
RE=AMP*COS(AG)
RI=AMP*SIN(AG)
AR=CMPLX(RE,RI)
RETURN
END

FUNCTION CANG(SR)
COMPLEX SR
A1=REAL(SR)
A2=AIMAG(SR)
CANG=ATAN2(A2,A1)
RETURN
END